• TAREA #5. FABIAN LEONARDO CAMARGO BERNATE

• Encontrar la respuesta al escalón de los sistemas con estas ecuaciones de planta.

$$G(s) = \frac{9}{s^2 + 2s + 9} = \frac{W_n^2}{s^2 + 2\gamma W_n s + W_n^2}$$

• $W_n^2 = 9 \longrightarrow W_n = \sqrt{9} = 3.$

• $2\gamma W_n s = 2s \longrightarrow \gamma = 1/3.$ $\quad 0 < \gamma < 1$ Subamortiguado

Respuesta al Impulso $1/s$.

$$G(s) = \frac{1}{s}\left[\frac{9}{s^2 + 2s + 9}\right] = \frac{9}{s^3 + 2s^2 + 9s}$$

$$G(s) = \frac{9}{(s + 1 - 2{,}83j)(s + 1 + 2{,}83j)(s)}$$

• Fracciones Parciales

$$\frac{9}{(s + 1 - 2{,}83j)(s + 1 + 2{,}83j)(s)} = \frac{A}{s + 1 - 2{,}83j} + \frac{B}{s + 1 + 2{,}83j} + \frac{C}{s}$$

$B = A^*$

$$A = (s + 1 - 2{,}83j)\frac{9}{(s + 1 - 2{,}83j)(s + 1 + 2{,}83j)(s)}\Bigg|_{s=-1+2{,}83j}$$

$$A = \frac{9}{(-1 + 2{,}83j + 1 + 2{,}83j)(-1 + 2{,}83j)}$$

$$A = \frac{9}{(5{,}66j)(-17 2{,}83j)}$$

$$A = \frac{9}{-5{,}66j + 16{,}0178} \cdot \frac{-5{,}66j - 16{,}0178}{-5{,}66j - 16{,}0178}$$

$$A = -0{,}5 + 0{,}18j.$$

$$B = A^* = -0{,}5 - 0{,}18j.$$

• $C = s\,X(s)\Big|_{s=0} = s\left[\frac{9}{(s + 1 - 2{,}83j)(s + 1 + 2{,}83j)(s)}\right]$

$$C = 1.$$

• Hallar la expresión en el tiempo de:

$$\frac{-0{,}5 + 0{,}18j}{s + 1 - 2{,}83j} + \frac{-0{,}5 - 0{,}18j}{s + 1 + 2{,}85j} + \frac{1}{s + 0}$$

(1) (2) (3)

• Sacar la transformada Inversa. Propiedades.

• $\frac{1}{s + \alpha - \beta j} = e^{-\alpha t}\cos(\beta t) + j\,e^{-\alpha t}\,\text{Sen}(\beta t).$

• $1/s = 1.$

(1)
$$y(t) = (-0{,}5 + 0{,}18j)\,e^{-t}\cos(2{,}83t) + j\,e^{-t}\,\text{Sen}(2{,}83t)$$

(2)
$$+(-0{,}5 - 0{,}18j)\,e^{-t}\cos(-2{,}83t) + j\,e^{-t}\,\text{Sen}(-2{,}83t) \quad (3)\ +\ 1$$

• Simplificar

$$y(t) = -0{,}5\,(e^{-t}\cos(2{,}83t) + j\,e^{-t}\,\text{Sen}(2{,}83t))$$

$$+ 0{,}18j\,(e^{-t}\cos(2{,}83t) + j\,e^{-t}\,\text{Sen}(2{,}83t))$$

$$-0{,}5\,(e^{-t}\cos(-2{,}83t) + j\,e^{-t}\,\text{Sen}(-2{,}83t))$$

$$-0{,}18j\,(e^{-t}\cos(-2{,}83t) + j\,e^{-t}\,\text{Sen}(-2{,}83t))$$

• Propiedad ... $\cos(x) = \cos(-x)$

$\text{Sen}(-x) = -\text{Sen}(x)$

$$y(t) = -e^{-t}\cos(2{,}83t) - j\,e^{-t}\,\text{Sen}(-2{,}83t)$$

$$-0{,}36\,e^{-t}\,\text{Sen}(2{,}83t) + 1.$$

Se toma la Respuesta Real

$$y(t) = -e^{-t}\cos(2{,}83t) - 0{,}36\,e^{-t}\,\text{Sen}(2{,}83t) + 1$$

Punto 2.

$$G(s) = \frac{9}{s^2 + 9.} = \frac{W_n^2}{s^2 + 2\eta W_n s + W_n^2.}$$

$W_n^2 = 9 \qquad W_n = \sqrt{9} = 3.$

$2\eta W_n s = 0 \qquad \eta = 0$ Vibra siempre

- Respuesta al escalón.

$$G(s) = \frac{1}{s}\left(\frac{9}{s^2 + 9}\right) = \frac{9}{s^3 + 9s.}$$

$$G(s) = \frac{9}{(s-3j)(s+3j)(s)}$$

- Fracciones Parciales:

$$\frac{9}{(s-3j)(s+3j)(s)} = \frac{A}{s-3j} + \frac{B}{s+3j.} + \frac{C}{s.}$$

$$A = (s-3j)X(s)\Big|_{s=3j.} = \cancel{s-3j.}\ \frac{9}{\cancel{(s-3j)}(s+3j)(s)}$$

$$A = \frac{9}{(3j+3j)3j} = \frac{9}{(6j)(3j).} = \frac{9}{-18} = -0,5$$

$$B = (s+3j)X(s)\Big|_{s=-3j.} = \cancel{s+3j.}\ \frac{9}{(s-3j)\cancel{(s+3j)}(s)}$$

$$\frac{9}{(-3j-3j)-3j} = \frac{9}{(-6j)(-3j).} = \frac{9}{-18} = -0,5$$

$$C = sX(s)\Big|_{s=0} \qquad \cancel{s}.\ \frac{9}{(\cancel{s-0}3j)(\cancel{s+0}3j)(\cancel{s})}$$

$$C = 9/9 = 1.$$

$$G(s) = \frac{-0,5}{s-3j} \quad \frac{-0,5}{s+3j} + 1/s.$$

- Para hallar la ecuación en el tiempo:

$$G(s). = -0,5\left[\frac{1}{s-3j} + \frac{1}{s+3j}\right] + \frac{1}{s.}$$

- $\frac{1}{s+\alpha-\beta j} = e^{-\alpha t}\cos(\beta t) + j\,e^{-\alpha t}\,\mathrm{Sen}(\beta t).$

- $1/s = 1.$

$$y(t) = -0,5\left((\cos(3t) + j\,\mathrm{Sen}(3t)) + (\cos(-3t) + j\,\mathrm{Sen}(-3t).\right) + 1$$

$$y(t) = 1 - \cos(3t.$$