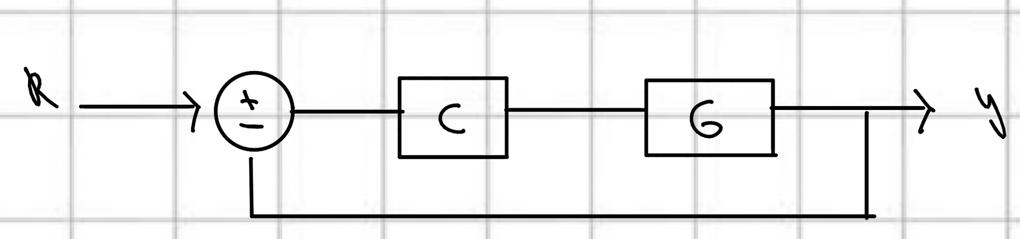
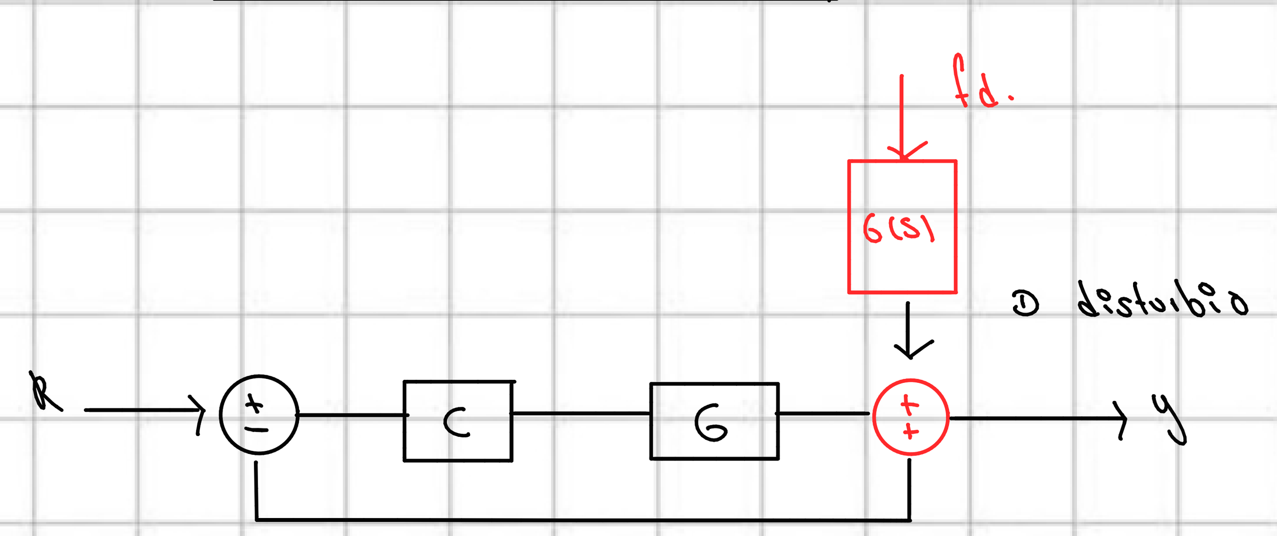
18 - abril - 2024.

- Libro feedback control of dynamics Systems Franklin 8° edición

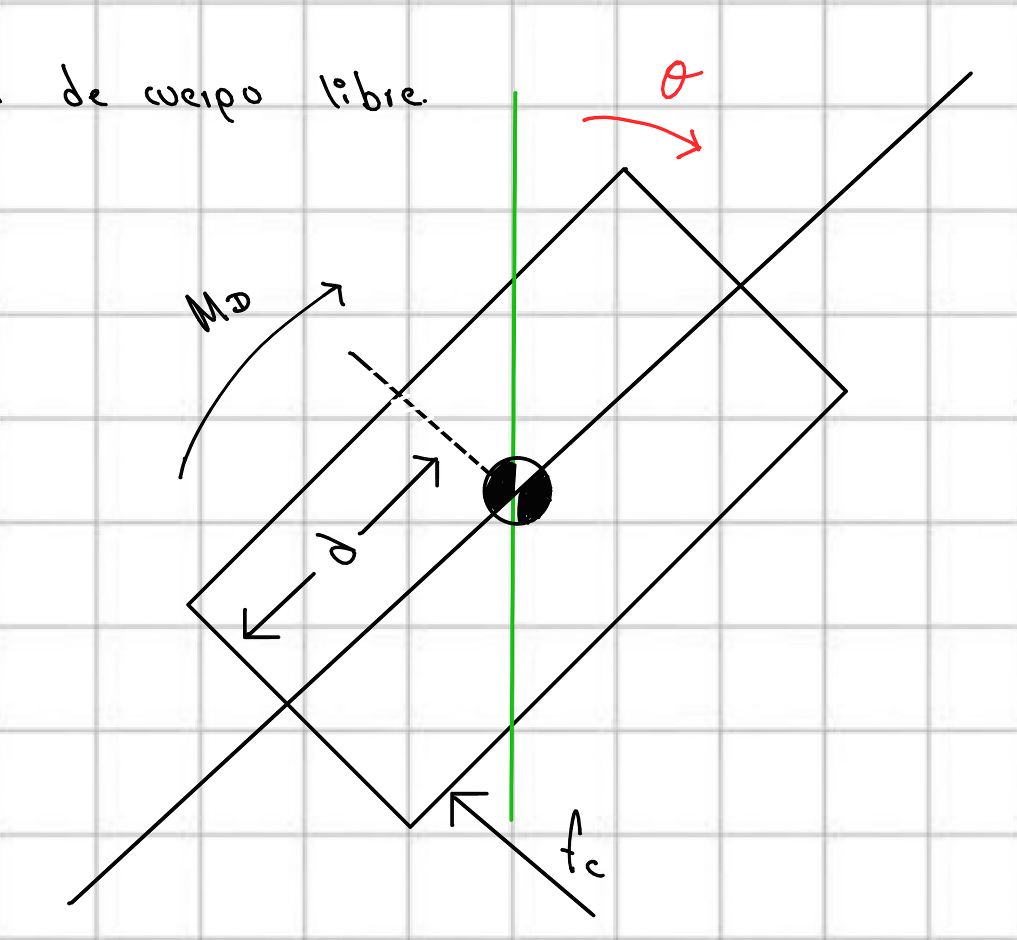
- $M = I\alpha$ momento de inercia

- 

- 

- TAREA # 3 Fabian Leonardo Camargo Benate.

- Diagrama de cuerpo libre

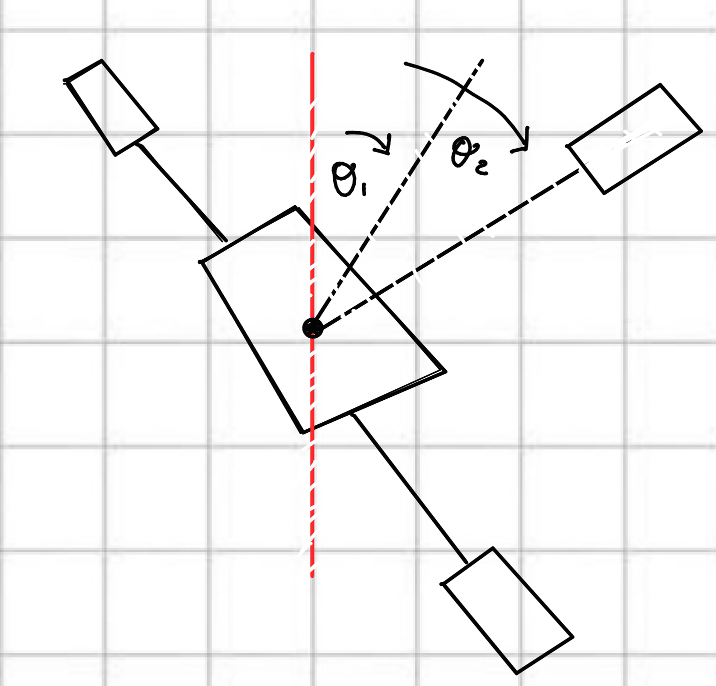


$$\underbrace{F_c d + M_D}_{u} = I\ddot{\theta}$$

$$\frac{\theta(s)}{U(s)} = \frac{1}{I s^2}$$

$$U(s) = I s^2 \theta(s)$$

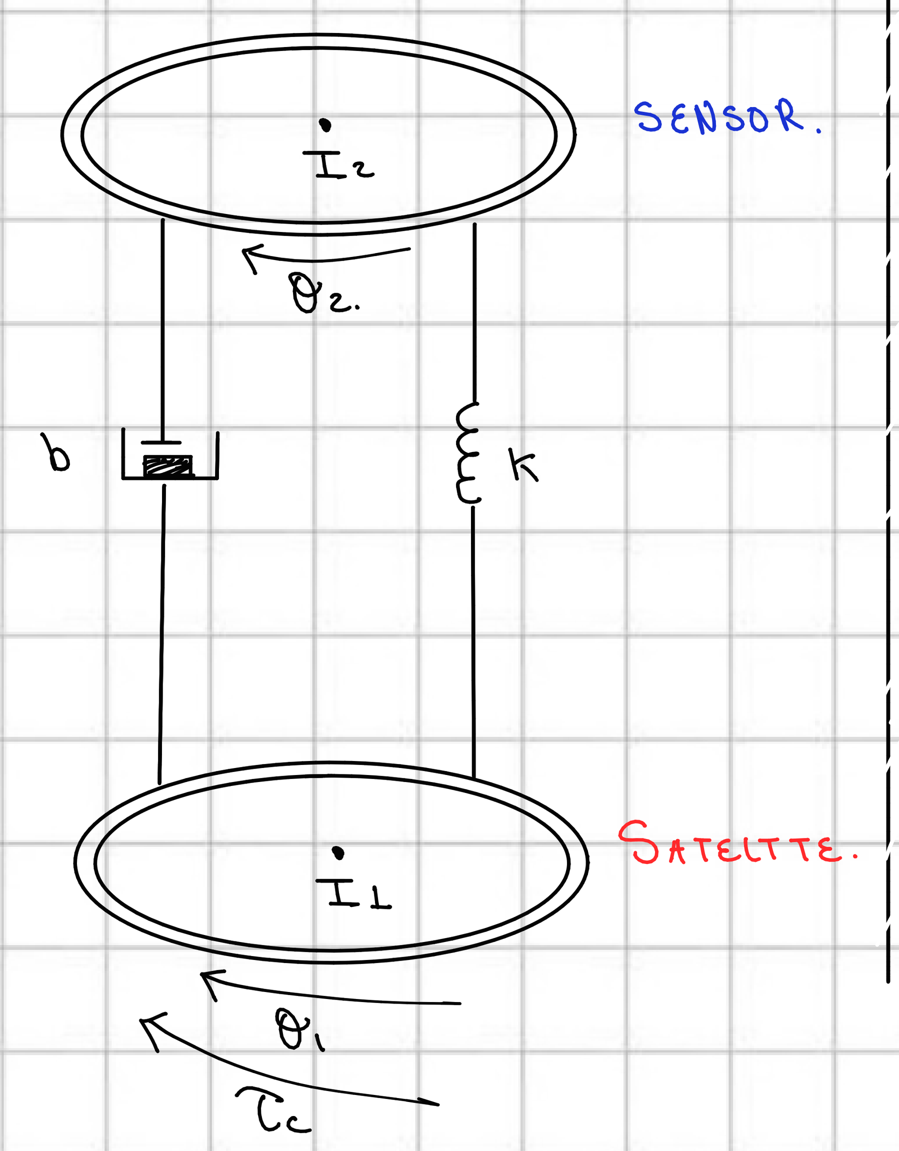
- Ejemplo: Satelite.

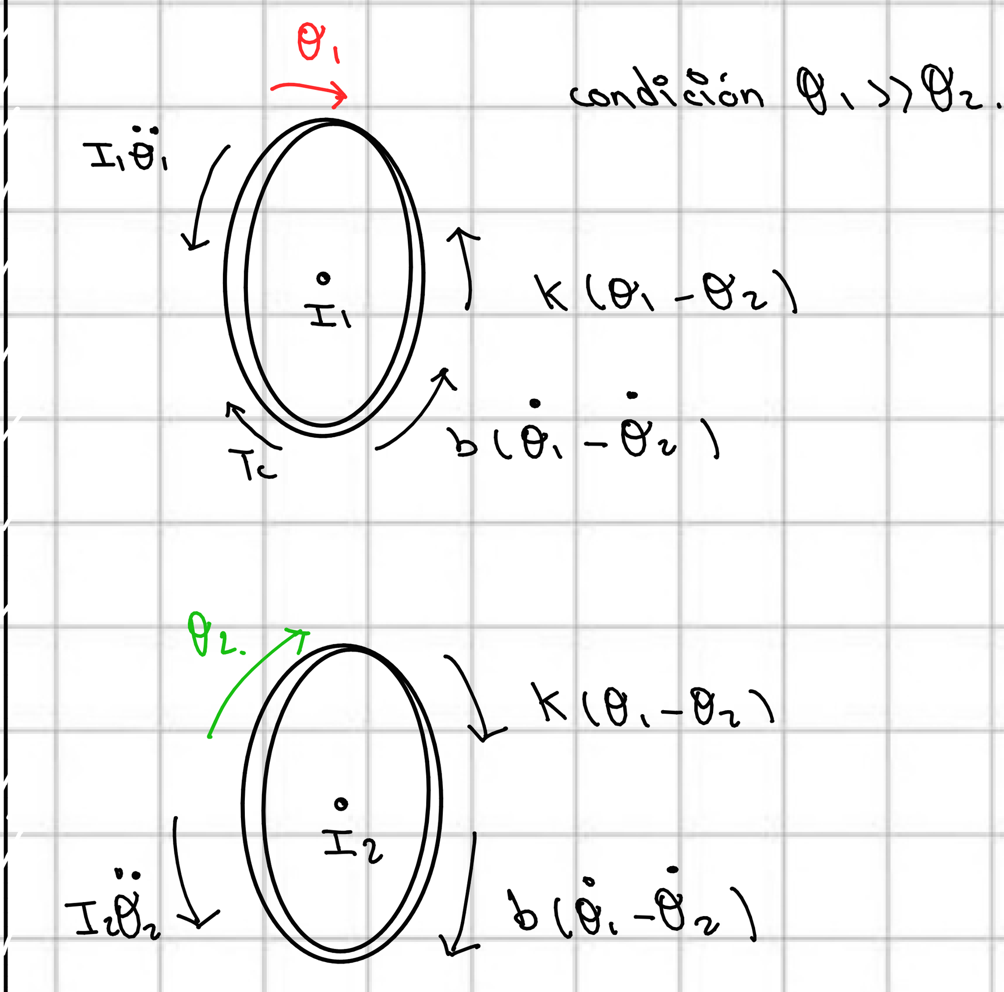


θ_2 : con respecto al Sensor

θ_1 : con respecto a la masa inercial.

Modelado del Sistema





- ecuaciones.

$$T_c - K(\theta_1 - \theta_2) - b(\dot{\theta}_1 - \dot{\theta}_2) = I_1\ddot{\theta}_1$$

$$b(\dot{\theta}_1 - \dot{\theta}_2) + K(\theta_1 - \theta_2) = I_2\ddot{\theta}_2$$

- Ecuaciones. Variables de estado

1. $T = I_1\ddot{\theta}_1 + K\theta_1 + b\dot{\theta}_1 - K\theta_2 - b\dot{\theta}_2$ → $q_1 = \theta_1$ $q_2 = \dot{\theta}_1$
2. $I_2\ddot{\theta}_2 + b\dot{\theta}_2 + K\theta_2 - b\dot{\theta}_1 - K\theta_1 = 0$ → $q_3 = \theta_2$ $q_4 = \dot{\theta}_2$

- Derivar variables de estado.

$\dot{q}_1 = \dot{\theta}_1 = q_2.$ $\dot{q}_3 = \dot{\theta}_2 = q_4.$ $y_1 = \theta_1 = q_1$

$\dot{q}_2 = \ddot{\theta}_1$ $\dot{q}_4 = \ddot{\theta}_2$ $y_2 = \theta_2 = q_3.$

- Despejando $\ddot{\theta}_1$ y $\ddot{\theta}_2$.

1. $\ddot{\theta}_1 = \frac{1}{I_1}\left(T - K\theta_1 - b\dot{\theta}_1 + K\theta_2 + b\dot{\theta}_2\right)$
2. $\ddot{\theta}_2 = \frac{1}{I_2}\left(b\dot{\theta}_1 + K\theta_1 - K\theta_2 - b\dot{\theta}_2\right)$

- Reemplazando

1. $\dot{q}_2 = \frac{1}{I_1}\left(T - Kq_1 - bq_2 + Kq_3 + bq_4\right)$
2. $\dot{q}_4 = \frac{1}{I_2}\left(bq_2 + Kq_1 - Kq_3 - bq_4\right)$

• Expresiones en el espacio de estados.

$$\dot{q}_1 = \dot{\theta}_1 = q_2.$$
$$\dot{q}_2 = \frac{1}{I_1}\left(\tau - K q_1 - b q_2 + K q_3 + b q_4\right)$$
$$\dot{q}_3 = \dot{\theta}_2 = q_4.$$
$$\dot{q}_4 = \frac{1}{I_2}\left(b q_2 + K q_1 - K q_3 - b q_4\right)$$

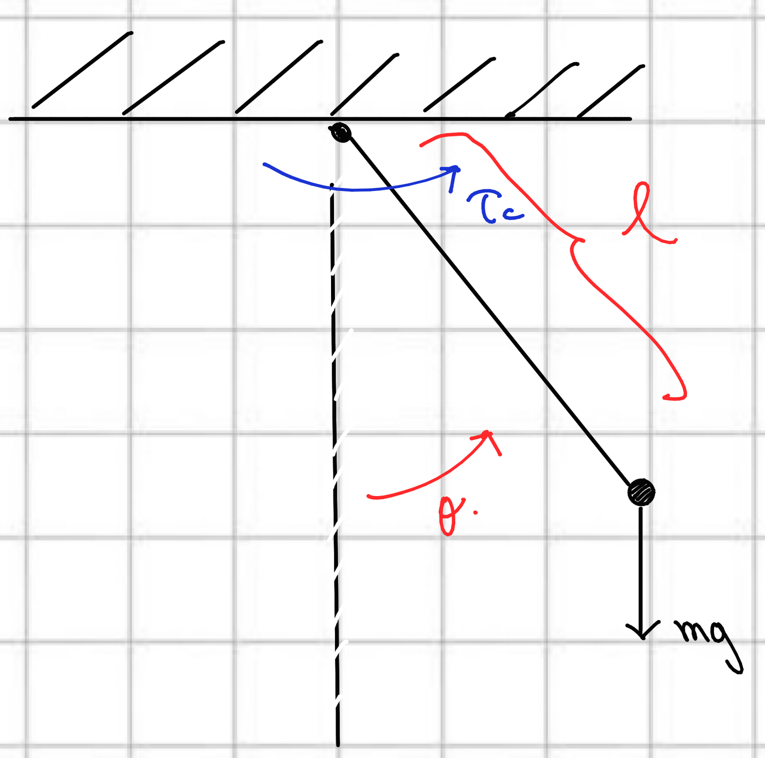
• Formato matricial

$$\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \\ \dot{q}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{-K}{I_1} & \frac{-b}{I_1} & \frac{K}{I_1} & \frac{b}{I_1} \\ 0 & 0 & 0 & 1 \\ \frac{K}{I_2} & \frac{b}{I_2} & \frac{-K}{I_2} & \frac{-b}{I_2} \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{I_1} \\ 0 \\ 0 \end{bmatrix} \tau$$

• Salida

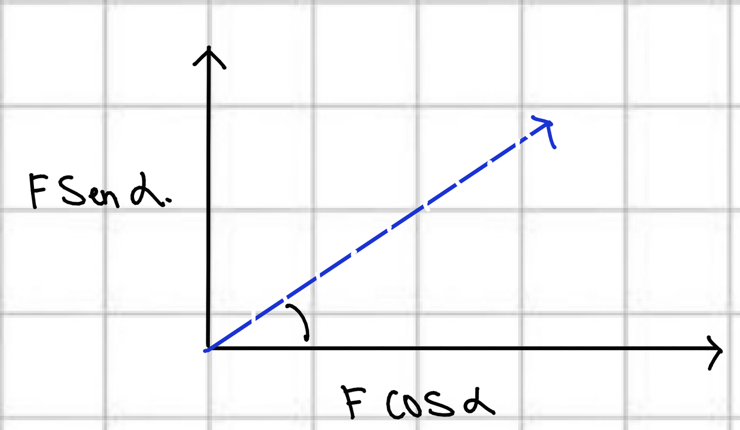
$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix}$$

• Punto 2. Pendulo.



momento de inercia.

• $I = m l^2.$



• $\tau_c - mg\, l \operatorname{Sen}\theta = I\ddot{\theta}$

$$\ddot{\theta} = \frac{\tau_c}{I} - \frac{mgl}{I}\operatorname{Sen}\theta.$$

$$\ddot{\theta} = \frac{\tau_c}{ml^2} - \frac{g}{l}\operatorname{sen}\theta.$$

• Repaso $\tau = \vec{d} \times \vec{F}$

$|\tau| = |d|\,|F| \operatorname{Sen}\theta.$

$|\tau| = I\, mg \operatorname{Sen}\theta.$

• Linealización con angulos pequeños $\operatorname{Sen}\theta \approx \theta$ y $\operatorname{Cos}\theta \approx 1$

1. $\ddot{\theta} = \frac{\tau_c}{ml^2} - \frac{g}{l}\theta$

• Variables de estado.

• $q_1 = \theta$ • $q_2 = \dot{\theta}$

$\longrightarrow$ $\dot{q}_1 = \dot{\theta} = q_2$

$\dot{q}_2 = \ddot{\theta}$ despejar.

• Reemplazando

$$\dot{q}_2 = \frac{\tau_c}{ml^2} - \frac{g}{l} q_1$$

• Formato matricial Espacio de estados.

$$\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ \frac{-g}{l} & 0 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1/ml^2 \end{bmatrix} \tau_c$$

• Salida.

$$y = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \end{bmatrix}$$