• TAREA #6 FABIAN LEONARDO CAMARGO BERNATE 2021100S048

• Transcripción video #1

State feedback control System design first example

• Desarrollo por realimentación de estados 7° edición Norman. S. Nise.

example. 12.1.

Diseñar un Sistema de control con la Siguiente Planta:

• $G(s) = \dfrac{20(s+5)}{s(s+1)(s+4)}$ • overshut = $OS\% = 9.5\%$.
• $ts = 0.74$ segundos.
tiempo de establecimiento

• abrir Sistema:

$$U(s) \longrightarrow \boxed{\dfrac{1}{s^3+5s^2+4s}} \xrightarrow{X_1(s)} \boxed{0s^2+20s+100} \longrightarrow Y(s).$$

$$\frac{X_1(s)}{U(s)} = \frac{1}{s^3+5s^2+4s}$$

$$(s^3+5s^2+4s)X_1(s) = U(s).$$

$$\downarrow \mathcal{L}^{-1}$$

$$\dddot{x}_1 + 5\underbrace{\ddot{x}_1 + 4\dot{x}_1}_{} = U(s)$$

$q_1 = x_1 \quad q_2 = \dot{x}_1 \quad q_3 = \ddot{x}_1$

$\dot{q}_1 = \dot{x}_1 = q_2.$

$\dot{q}_2 = \ddot{x}_1 = q_3$

$\dot{q}_3 = \dddot{x}$

$$\dot{X}_3 + 5X_3 + 4X_2 = U$$

$$\dot{X}_3 = U - 5X_3 - 4X_2$$

$$y(s) = (b_2s^2 + b_1s + b_0)X_1(s).$$

$$= (0s^2 + 20s + 100)X_1(s)$$

$$\mathcal{L}^{-1}$$

$$y(s) = 20\underbrace{\dot{X}_1}_{X_2} + 100\underbrace{X_1}_{X_1}.$$

$$y = 20X_2 + 100X_1.$$

Expresión espacio de estados

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{bmatrix} = \begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ 0 & -4 & -5\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix} + \begin{bmatrix}0\\ 0\\ 1\end{bmatrix}u$$

$$y = \begin{bmatrix}100 & 20 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}$$

• $OS\% = 9.5\%$

$$\%OS = e^{-(\zeta\pi/\sqrt{1-\zeta^2})} \times 100.$$

$$0.095 = e^{-(\zeta\pi/\sqrt{1-\zeta^2})}$$

$$(\ln(0.095))^2 = \frac{(-\zeta\pi)^2}{(\sqrt{1-\zeta^2})^2}$$

$$\ln^2(0.095) = \frac{\zeta^2\pi^2}{1-\zeta^2.}$$

$$\ln^2(0.095) - \ln(0.095)\zeta^2 = \zeta^2\pi^2.$$

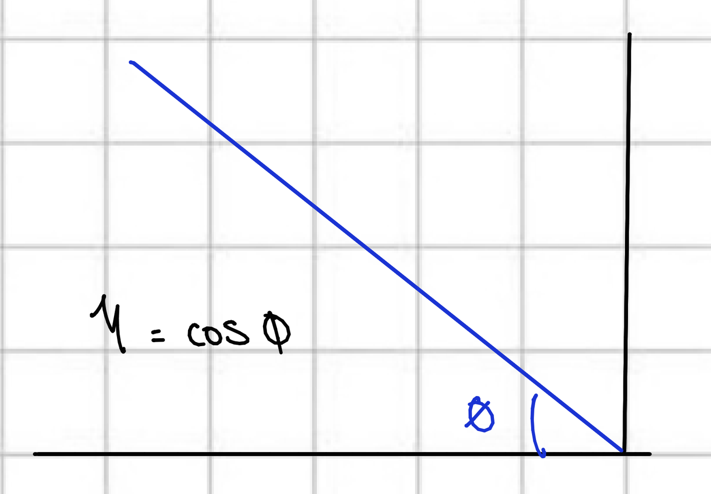
$$\ln^2(0.095) = \zeta^2\pi^2 + \ln(0.095).\zeta^2.$$

$$\ln^2(0.095) = \zeta^2(\pi^2 + \ln(0.095))$$

$$\zeta = \frac{\ln(0.095)}{\sqrt{\pi^2 + \ln(0.095)}.} = \underline{0.5996.}$$

$$S = \sigma + j\,wd.$$

$$= \zeta w_n.$$

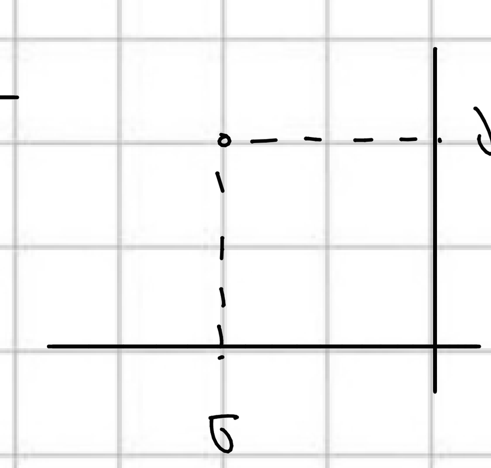


$\arccos(0.5996) \Rightarrow 53.16°$ $\quad \Phi = 53.16°$

$t_s = 0.74$ segundos $\quad t_s = \dfrac{4}{\sigma}$

$\sigma = 4/0.74 = 5.405$

$S = \sigma + j\,wd.$



$\sigma = \zeta \omega_n$

$5.405 = 0.5976\, \omega_n$

$\omega_n = 9.02\ \text{rad/seg.}$

$\sigma = \zeta \omega_n = -5.41.$

$\omega_d = \omega_n \sqrt{1-\zeta^2}$

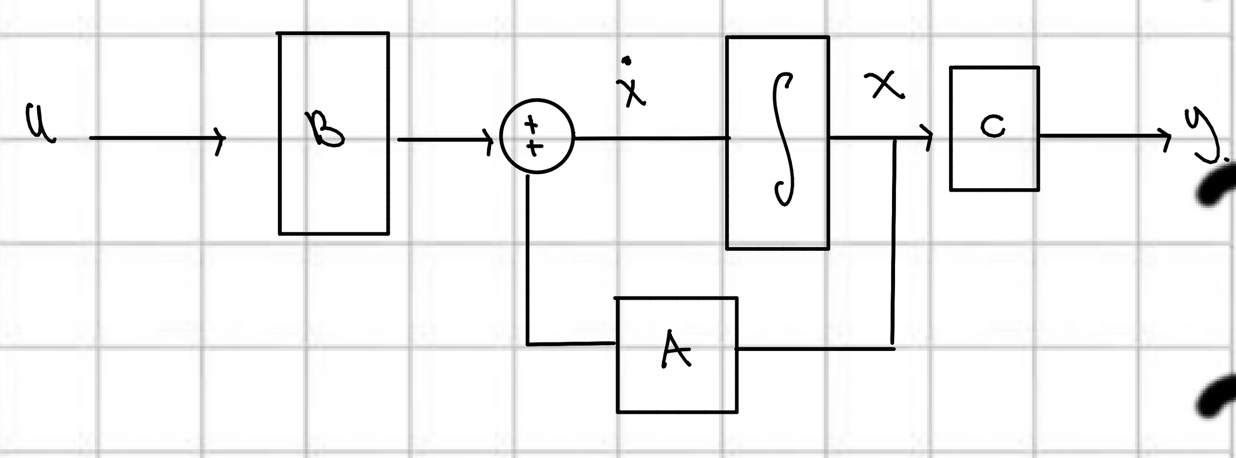
$\Updownarrow$

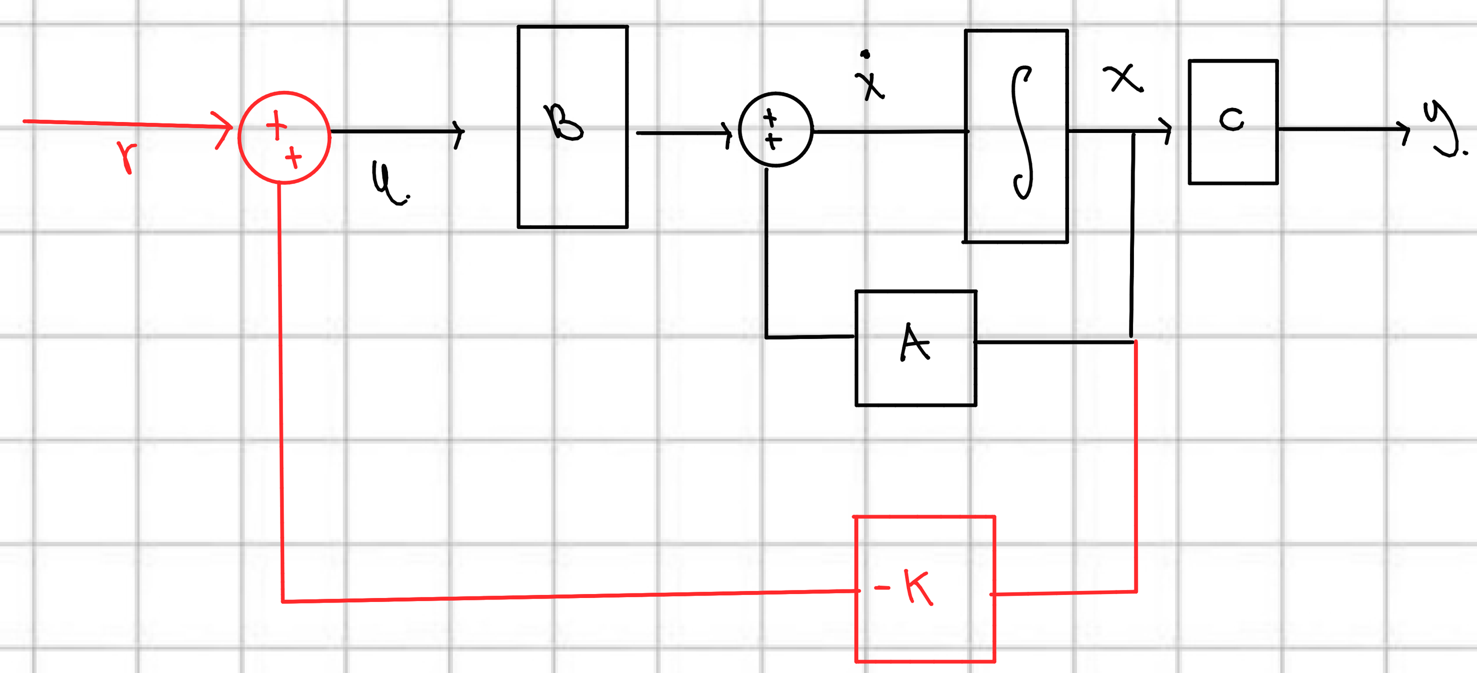
$\tan\phi = \dfrac{\omega_d}{5.41} \quad 5.41 \tan\phi = \omega_d.$

$\omega_d = 7.21.$

- $\dot{x} = Ax + Bx$

  $y = Cx$



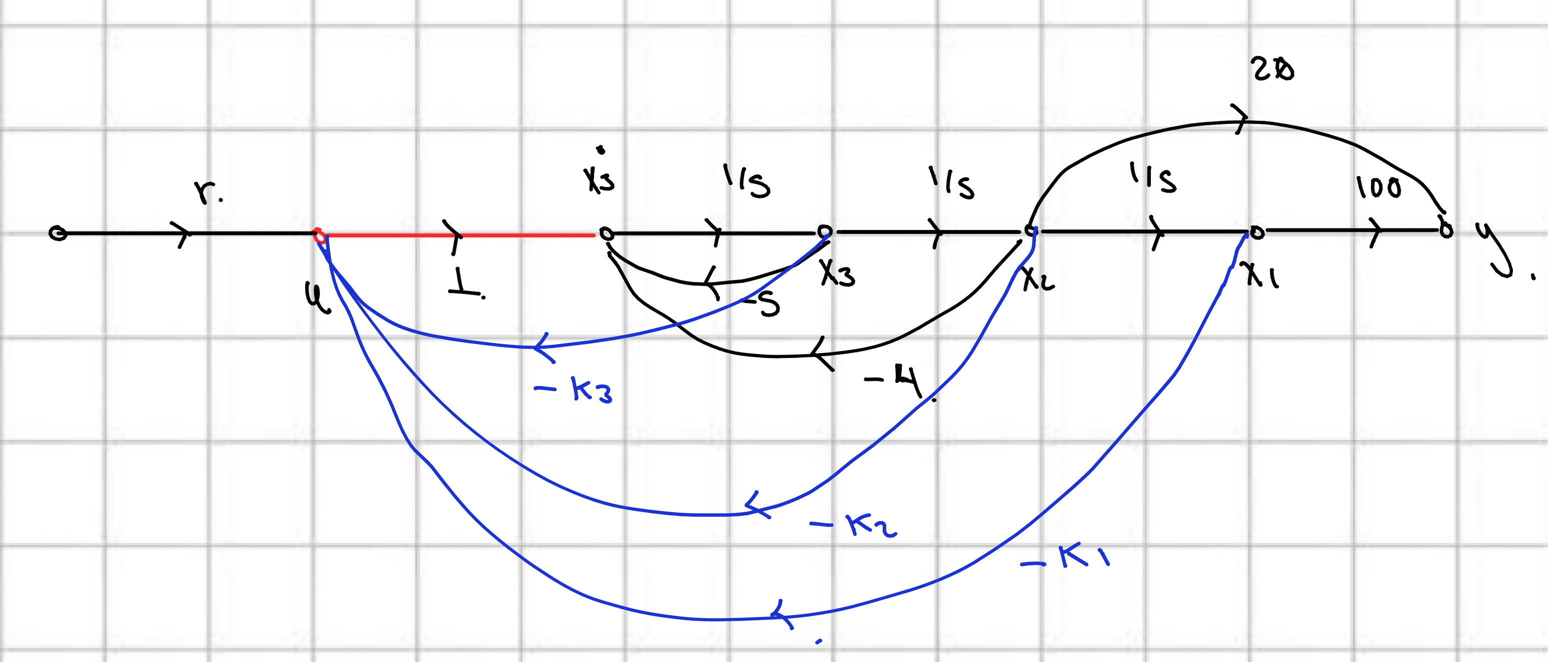


$$\dot{x} = Ax + Bu.$$
$$= Ax + B(-Kx + r)$$
$$= Ax - BKx + Br.$$
$$\boxed{\dot{x} = (A - BK)x + Br.}$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -4 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 100 & 20 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

- Diagrama de flujo de Señal.



$$\dot{x}_3 = -4x_2 - 5x_3 + u.$$

$$\dot{x}_3 = -4x_2 - 5x_3 + [-K_3x_3 - K_2x_2 - K_1x_1] + r.$$
$$= -4x_2 - 5x_3 - K_3x_3 - K_2x_2 - K_1x_1 + r$$
$$= -K_1x_1 - (4+K_2)x_2 - (5+K_3)x_3 + r.$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -K_1 & -(4+K_2) & -(5+K_3) \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} r.$$

$$\det(sI - (A - BK)) = s^3 + (5+\underline{K_3})s^2 + (4+\underline{K_2})s + \underline{K_1} = 0$$