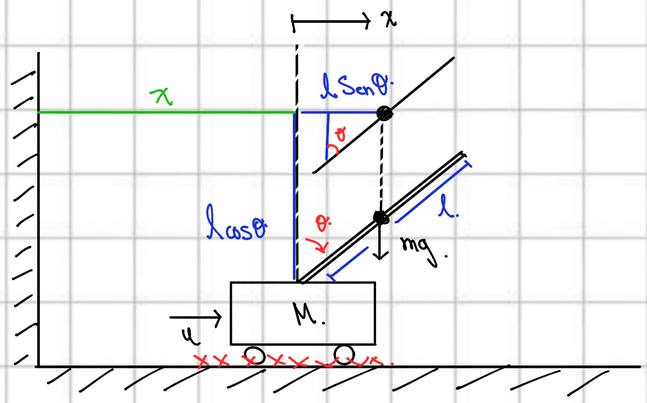
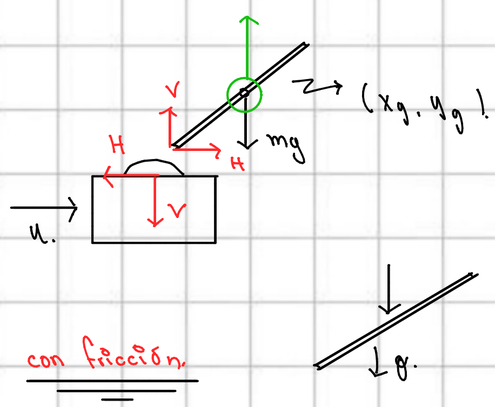
TAREA #4 Fabián Leonardo Camargo Beltrán.

- Pendulo invertido



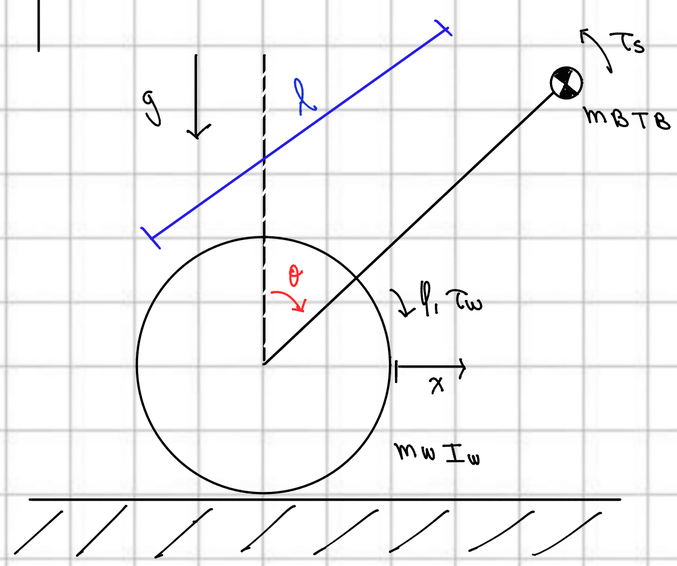


con fricción

$x_g = x + l\,Sen\theta$

$y_g = l\cos\theta$

- Diagrama de cuerpo libre



- Variables: M: masa del carro m = masa del pendulo
  - x = posición del carro
  - $x_p = x + l\,Sen\theta$ posición del pendulo

- Analisis Eje x:

$F = u - b\dot{x}$

$F = ma \longrightarrow u - b\dot{x} = ma \qquad u - b\dot{x} = M\ddot{x} + m\ddot{x}_p$

$u - b\dot{x} = M\ddot{x} + m\dfrac{d^2}{dt^2}(x + l\,Sen\theta)$

Proceso de la derivada

- $\dfrac{m\,d^2(x + l\,Sen\theta)}{dt^2}$

$\dfrac{m\,d^2(x + l\,Sen\theta)}{dt^2} \rightarrow m\ddot{x} + \dfrac{m\,d(l\cos(\theta)\dot{\theta})}{dt}$

$= m\ddot{x} + \dfrac{m\,d^2(l\,Sen\theta)}{dt^2}$

$= m\ddot{x} + \dfrac{ml\,d(\cos\theta\cdot\dot{\theta})}{dt} = m\ddot{x} + ml[-Sen\theta\,\dot{\theta}\dot{\theta} + \cos\theta\,\ddot{\theta}]$

$= m\ddot{x} - ml\,Sen\theta\,\dot{\theta}^2 + ml\cos(\theta)\ddot{\theta}$

Reemplazando este resultado se tiene que:

- $u - b\dot{x} = M\ddot{x} + m\ddot{x} - ml\,Sen\theta\,\dot{\theta}^2 + ml\cos(\theta)\ddot{\theta}$
- $u - b\dot{x} = (M+m)\ddot{x} - ml\,Sen\theta\cdot\dot{\theta}^2 + ml\cos(\theta)\ddot{\theta}$

- Analisis en el eje y.

$F = mg\,Sen\theta \qquad x_p = x + l\,Sen\theta \qquad y_p = l\cos\theta$

$F = ma \longrightarrow mg\,Sen\theta = m\cos\theta\dfrac{d^2}{dt^2}x_p - m\,Sen\theta\dfrac{d^2}{dt^2}y_p$

$mg\,Sen\theta = m\cos\theta\dfrac{d^2}{dt^2}(x + l\,Sen\theta) - m\,Sen\theta\dfrac{d^2}{dt^2}(l\cos\theta)$

$mg\,Sen\theta = m\cos\theta + m\ddot{x} - ml\,Sen\theta\,\dot{\theta}^2 + ml\cos(\theta)\ddot{\theta} - ml\,Sen\theta(-Sen\theta\cdot\ddot{\theta} - \cos\theta\cdot\dot{\theta}^2)$

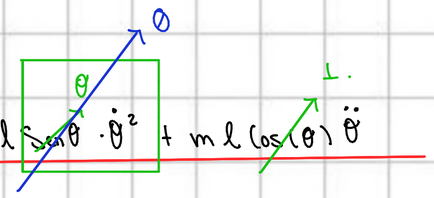
$mg\,Sen\theta = m\ddot{x}\cos\theta + ml\cos^2\theta\cdot\ddot{\theta} - ml\,Sen\theta\cdot\cos\theta\cdot\dot{\theta}^2 + ml\,Sen^2\theta\ddot{\theta} + ml\,Sen\theta\cos\theta\cdot\dot{\theta}^2$

Dividir sobre m y considerar la expresión $\cos^2\theta + Sen^2\theta = 1$

$g\,Sen\theta = \ddot{x}\cos\theta + l\ddot{\theta}$

- Linealización para angulos pequeños $\begin{cases} Sen\theta \approx \theta \\ \cos\theta \approx 1 \\ \theta\dot{\theta}^2 = 0 \end{cases}$

- Aplicar aproximaciones.



$u - b\dot{x} = (M+m)\ddot{x} - ml\,Sen\theta\cdot\dot{\theta}^2 + ml\cos(\theta)\ddot{\theta}$

- $u - b\dot{x} = (M+m)\ddot{x} + ml\ddot{\theta}$

$g\,Sen\theta = \ddot{x}\cos\theta + l\ddot{\theta}$

- $g\theta = \ddot{x} + l\ddot{\theta}$

teniendo las ecuaciones 1 y 2.

1. $u - b\dot{x} = (M+m)\ddot{x} + ml\ddot{\theta}$
2. $g\theta = \ddot{x} + l\ddot{\theta}$

teniendo las ecuaciones 1 y 2.

1. $u - b\dot{x} = (M+m)\ddot{x} + ml\ddot{\theta}$
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- Espacio de estados.

- $u - b\dot{x} = M\ddot{x} + m\ddot{x} + ml\ddot{\theta}$

$u - b\dot{x} = M\ddot{x} + m(\ddot{x} + l\ddot{\theta})$ Reemplazando

$\longrightarrow u - b\dot{x} = M\ddot{x} + mg\theta \longrightarrow \ddot{x} = \frac{u}{M} - \frac{b\dot{x}}{M} - \frac{mg\theta}{M}$

- Variables de estado

$x_1 = \theta \qquad x_2 = \dot{\theta} \qquad x_3 = x \qquad x_4 = \dot{x}$

- $\dot{x_4} = \frac{u}{M} - \frac{b}{M}x_4 - \frac{mg}{M}x_1$

$g\theta = \ddot{x} + l\ddot{\theta} \longrightarrow g\theta = \frac{u}{M} - \frac{b}{M}\dot{x} - \frac{mg}{M}\theta + l\ddot{\theta}$

$l\ddot{\theta} = g\theta - \frac{1}{M}u + \frac{b}{M}\dot{x} + \frac{mg}{M}\theta$

$\ddot{\theta} = \frac{g}{l}\left(1 + \frac{m}{M}\right)\theta - \frac{u}{Ml} + \frac{b}{Ml}\dot{x}$

$\dot{\theta} = g\frac{M+m}{Ml}\theta - \frac{u}{Ml} + \frac{b}{Ml}x_4$

$\dot{x_2} = g\frac{M+m}{Ml}x_1 - \frac{u}{Ml} + \frac{b}{Ml}x_4$

- Formato Matricial.

$$\begin{bmatrix} \dot{x_1} \\ \dot{x_2} \\ \dot{x_3} \\ \dot{x_4} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{g(M+m)}{Ml} & 0 & 0 & \frac{b}{Ml} \\ 0 & 0 & 0 & 1 \\ -\frac{mg}{M} & 0 & 0 & \frac{-b}{M} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{-1}{Ml} \\ 0 \\ \frac{1}{M} \end{bmatrix} u$$

- Salida.

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$