Día	Mes	Año	Hora	Institución		
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Curso	Bimestre	Semestre	Salón	Hoja No ___ de ___	CALIFICACIÓN	
Profesor						

TALLER 4

1.

a.) $f(x)=\frac{1}{x^2}$ $\quad f'(x)=\frac{-2}{x^3}$

b.) $f(x)=\sqrt[3]{x^2}=x^{2/3}$ $\quad f'(x)=2/3\, x^{2/3-3/3}=\frac{2}{3}x^{-1/3}=\frac{2}{3\sqrt[3]{x}}$

c.) $f(x)=xe^x$ $\quad f'(x)=xe^x+e^x=e^x(x+1)$

d.) $f(t)=\sqrt{t}(a+bt)$ $\quad f'(t)=1/2\, t^{1/2-1}(a+bt)+\sqrt{t}(b)$

$$=\frac{a+bt}{2\sqrt{t}}+b\sqrt{t}$$

e.) $f(x)=\frac{x^2+x-2}{x^3+6}$ $\quad f'(x)=\frac{(2x+1)(x^3+6)-(x^2+x-2)(3x^2)}{(x^3+6)^2}$

$$=\frac{2x^4+12x+x^3+6-3x^4-3x^3+6x^2}{x^6+12x^3+36}$$

$$=\frac{-x^4-2x^3+6x^2+12x+6}{x^6+12x^3+36}$$

f.) $f(x)=(3x^2-5x)e^x$ $\quad f'(x)=(6x-5)e^x+(3x^2-5x)e^x$

$$=e^x(6x-5+3x^2-5x)$$

$$=e^x(3x^2-x-5)$$

g.) $f(x)=\frac{e^x}{x^2}$ $\quad f'(x)=\frac{e^x x^2-2xe^x}{x^4}=\frac{e^x(x^2-2x)}{x^4}$

h.) $g(x)=\frac{3x-1}{2x+1}$ $\quad g'(x)=\frac{3(2x+1)-2(3x-1)}{(2x+1)^2}=\frac{\cancel{6x}+3-\cancel{6x}+1}{4x^2+4x+1}$

$$=\frac{4}{4x^2+4x+1}$$

i.) $g(x)=(x+2\sqrt{x})e^x$ $\quad g'(x)=(1+2(1/2)x^{-1/2})e^x+(x+2\sqrt{x})e^x$

$$=e^x\left[1+\frac{1}{\sqrt{x}}+x+2\sqrt{x}\right]$$

j.) $f(x)=\frac{e^x}{1+x}$ $\quad f'(x)=\frac{e^x(1+x)-(1)e^x}{(1+x)^2}=\frac{xe^x}{x^2+2x+1}$

k.) $f(t)=\frac{2t}{4-t^2}$ $\quad f'(t)=\frac{2(4-t^2)-(-2t)(2t)}{(4-t^2)^2}=\frac{8-2t^2+4t^2}{t^4-8t^2+16}=\frac{2t^2+8}{t^4-8t^2+16}$

l.) $f(x) = \dfrac{x^2+1}{x^2-1}$ $\quad f'(x) = \dfrac{2x(x^2-1) - 2x(x^2+1)}{(x^2-1)^2} = \dfrac{\cancel{2x^3} - 2x - \cancel{2x^3} - 2x}{x^4 - 2x^2 + 1}$

$$= \frac{-4x}{x^4-2x^2+1}$$

m.) $f(x) = \dfrac{\sqrt{x}}{2+x}$ $\quad f'(x) = \dfrac{(1/2)x^{-1/2}(2+x) - (1)\sqrt{x}}{(2+x)^2}$

$$= \frac{2+x/2\sqrt{x} - \sqrt{x}}{x^2+4x+4} = \frac{\frac{2+x}{2\sqrt{x}} - \sqrt{x}}{x^2+4x+4}$$

n.) $f(x) = \ln(3x^2)\sqrt{x^2+x}$

$$f'(x) = \left[\left(\frac{1}{3x^2} 6x\right)\sqrt{x^2+x}\right] + \left[\ln(3x^2)\frac{1}{2}\frac{1}{\sqrt{x^2+x}}(2x+1)\right]$$

$$= \frac{2}{x}\sqrt{x^2+x} + \frac{\ln(3x^2)(2x+1)}{2\sqrt{x^2+x}} = \frac{4(x^2+x)}{2x\sqrt{x^2+x}} + \frac{\ln(3x^2)x(2x+1)}{2x\sqrt{x^2+x}}$$

$$= \frac{4(x+1) + \ln(3x^2)(2x+1)}{2\sqrt{x^2+x}}$$

2

a.) $f(x) = x^2\sin(x)$ $\quad f'(x) = 2x\sin(x) + x^2\cos(x)$

$$= x(2\sin(x) + x\cos(x))$$

b.) $f(x) = e^x\cos(x)$ $\quad f'(x) = e^x\cos(x) - e^x\sin(x)$

$$= e^x(\cos(x) - \sin(x))$$

c.) $f(x) = x\cos(x) + 2\tan(x)$

$$f'(x) = (1)\cos(x) - x\sin(x) + 2\sec^2(x)$$

$$= \cos(x) - x\sin(x) + 2\sec^2(x)$$

d.) $f(x) = 2\sec(x) - \csc(x)$ $\quad f'(x) = 2\sec(x)\tan(x) + \csc(x)\cot(x)$

e.) $g(t) = t^3\cos(t)$ $\quad g'(t) = 3t^2\cos(t) - t^3\sin(t)$

f.) $f(u) = e^u(\cos(u) + \csc(u))$

$$f'(u) = e^u(\cos(u) + \csc(u)) + e^u(-\sin(u) - \csc(u)\cot(u))$$

$$= e^u(\cos(u) + \csc(u) - \sin(u) - \csc(u)\cot(u))$$

3

a.) $y = \sin(4x)$ $\quad y' = 4\cos(4x)$

b.) $y = \sqrt{4+3x}$ $\quad y' = \dfrac{3}{2\sqrt{4+3x}}$

c.) $y = ((1+x^2))e^{3x}$ $\quad y' = 2xe^{3x} + 3(1+x^2)e^{3x}$

d-) $y = \tan(\sin(x))$ $\qquad y' = \sec^2(\sin(x))\cos(x)$

e-) $y = e^{\sqrt{x}}$ $\qquad y' = \dfrac{e^{\sqrt{x}}}{2\sqrt{x}}$

f-) $y = \ln(2x)\sqrt{2-e^x}$

$$y' = \frac{\cancel{2}}{\cancel{2}x}\sqrt{2-e^x} - \frac{\ln(2x)e^x}{2\sqrt{2-e^x}} = \frac{\sqrt{2-e^x}}{x} - \frac{e^x\ln(2x)}{2\sqrt{2-e^x}}$$

4

a-) $f(x) = (5x^6+2x^3)^4$ $\qquad f'(x) = 4(5x^6+2x^3)^3(30x^5+6x^2)$

b-) $f(x) = ((1+x+x^2)^9)e^{x^2}$

$$f'(x) = 9(1+x+x^2)^8(1+2x)e^{x^2} + (1+x+x^2)^9\,2x\,e^{x^2}$$
$$= e^{x^2}\left[9(1+x+x^2)^8(1+2x) + 2x(1+x+x^2)^9\right]$$

c-) $g(x) = (x^2+1)^3(x^2+2)^6$

$$g'(x) = 3(x^2+1)^2\,2x\,(x^2+2)^6 + (x^2+1)^3\,6(x^2+2)^5\,2x$$
$$= 6x(x^2+2)^6(x^2+1)^2 + 12x(x^2+2)^5(x^2+1)^3$$
$$= 6x\left[(x^2+2)^6(x^2+1)^2 + 2(x^2+2)^5(x^2+1)^3\right]$$

d-) $h(t) = (t+1)^{2/3}(2t^2-1)^3$

$$h'(t) = \frac{2}{3}(t+1)^{-1/3}(2t^2-1)^3 + (t+1)^{2/3}\,3(2t^2-1)^2\,4t$$
$$= \frac{2(2t^2-1)^3}{3\sqrt[3]{t+1}} + 12t(2t^2-1)^2\sqrt[3]{(t+1)^2}$$

e-) $f(t) = (3t-1)^4(2t-1)^{-3} = \dfrac{(3t-1)^4}{(2t-1)^3}$

$$f'(t) = \frac{4(3t-1)^3\,3(2t-1)^3 - (3t-1)^4\,3(2t-1)^2(2)}{(2t-1)^6}$$
$$= \frac{12(3t-1)^3(2t-1)^3 - 6(3t-1)^4(2t-1)^2}{(2t-1)^6}$$

f-) $f(x) = \dfrac{x}{x+1}$ $\qquad f'(x) = \dfrac{(1)(x+1)-x}{(x+1)^2} = \dfrac{1}{(x+1)^2}$

5

a-) $f(x) = x^3-3x^2+1, \quad -1/2 \le x \le 4$

$f'(x) = 3x^2-6x = 0$ $\qquad f''(x) = 6x-6 = 0$

$x(3x-6) = 0$ $\quad \begin{cases} x=0 \\ x=2 \end{cases}$ $\qquad \boxed{x=1} \to$ Punto de inflexión

$f'(-1/2) = 3(-1/2)^2 - 6(-1/2) = 9/4 \Rightarrow (+)$

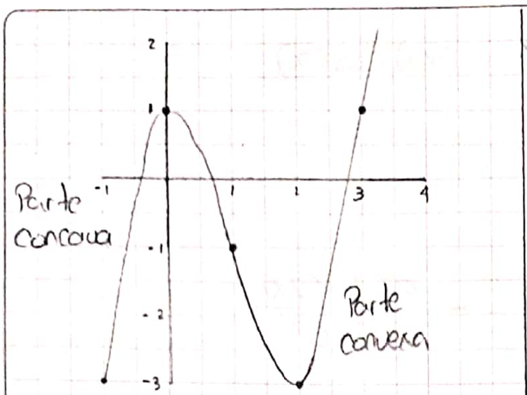
$f'(1/2) = 3(1/2)^2 - 6(1/2) = -9/4 \Rightarrow (-)$

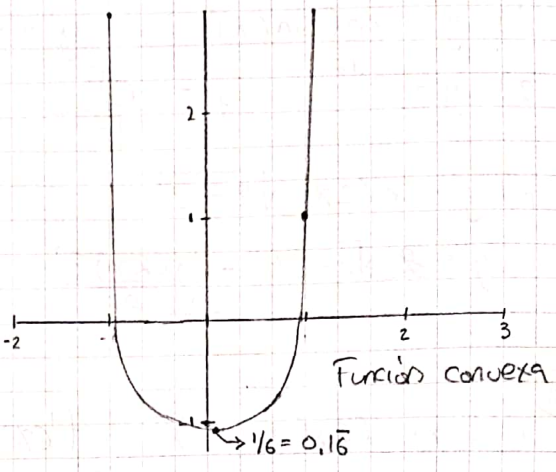
$f'(3/2) = 3(3/2)^2 - 6(3/2) = -9/4 \Rightarrow (-)$

$f'(3) = 3(3)^2 - 6(3) = 9 \Rightarrow (+)$

	$+$	$-$	$-$	$+$	
$-1/2$	0	1	2	3	4

$0 \downarrow$ máximo local $\qquad 2 \downarrow$ mínimo local





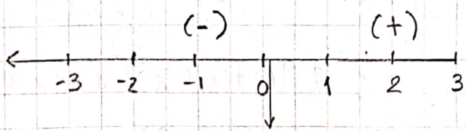
b-) $f(x) = 3x^2 - x - 1 \quad , \quad x \leq 3$

$f'(x) = 6x - 1 = 0$

$\boxed{x = 1/6}$

$f'(2) = 6(2) - 1 = 11 \Rightarrow (+)$
$f'(-1) = 6(-1) - 1 = -7 \Rightarrow (-)$

$f''(x) = 6 \Rightarrow$ No hay punto de inflexion, función convexa



$1/6 = 0,1\bar{6} \Rightarrow$ Mínimo local

6

a-) $\lim\limits_{x \to 1} \dfrac{\ln(x)}{x-1} = \lim\limits_{x \to 1} \dfrac{1}{x} = 1$

b-) $\lim\limits_{x \to \infty} \dfrac{e^x}{x^2} = \lim\limits_{x \to \infty} \dfrac{e^x}{2x} = \lim\limits_{x \to \infty} \dfrac{e^x}{2} = \dfrac{e^\infty}{2} \to \infty$

c-) $\lim\limits_{x \to \infty} \dfrac{\ln(x)}{\sqrt{x}} = \lim\limits_{x \to \infty} \dfrac{1/x}{1/2\sqrt{x}} = \lim\limits_{x \to \infty} \dfrac{2\sqrt{x}}{x} = \lim\limits_{x \to \infty} \dfrac{1}{\sqrt{x}} = \dfrac{1}{\sqrt{\infty}} \to 0$

d-) $\lim\limits_{x \to 0} \dfrac{\operatorname{Tan}(x) - x}{x^3} = \lim\limits_{x \to 0} \dfrac{\operatorname{Sec}^2(x)}{3x^2} = \lim\limits_{x \to 0} \dfrac{2\operatorname{Sec}^2(x)\operatorname{Tan}(x)}{6x} =$

e-) =
f-)

$= \lim\limits_{x \to 0} \dfrac{4\operatorname{Sec}^2(x)\operatorname{Tan}(x) + 2\operatorname{Sec}^4(x)}{6} = \lim\limits_{x \to 0} \dfrac{2\operatorname{Sec}^2(x)[2\operatorname{Tan}(x) + \operatorname{Sec}^2(x)]}{6} = \dfrac{1}{3}$

g-) $\lim\limits_{x \to 4} \dfrac{x^2 - 2x - 8}{x - 4} = \lim\limits_{x \to 4} \dfrac{2x - 2}{1} = 2(4) - 2 = 6$

h-) $\lim\limits_{x \to -2} \dfrac{x^3 + 8}{x + 2} = \lim\limits_{x \to -2} 3x^2 = 3(-2)^2 = 12$

Observaciones