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Quiz 5: Integrales

$$\begin{aligned}
 1) \int \operatorname{Sen}(t) \ln(\cos(t)) dt & \quad u = \cos(t), \quad du = -\operatorname{Sen}(t) dt \\
 & = \cos(t) - \cos(t) \ln(\cos(t)) + C \\
 & = -\int \ln(u) du = -u \ln(u) + \int du \\
 & \quad \left| \begin{array}{l} w = \ln(u) \quad dv = du \\ dw = \frac{1}{u} du \quad v = u \end{array} \right. \\
 & = -u \ln(u) + u
 \end{aligned}$$

$$\begin{aligned}
 2) \int \cos^4(x) dx & = \int (\cos^2(x))^2 dx = \int \left(\frac{1 + \cos(2x)}{2} \right)^2 dx \\
 & = \int \frac{(1 + \cos(2x))^2}{4} dx = \frac{1}{4} \int 1 + 2\cos(2x) + \cos^2(2x) dx \\
 & = \frac{1}{4} \left[\int dx + 2 \int \cos(2x) dx + \int \frac{1 + \cos(4x)}{2} dx \right] \\
 & = \frac{1}{4} \left[\int dx + 2 \int \cos(2x) dx + \frac{1}{2} \int dx + \frac{1}{2} \int \cos(4x) dx \right] \\
 & = \frac{x}{4} + \frac{x}{8} + \frac{1}{2} \int \cos(2x) dx + \frac{1}{8} \int \cos(4x) dx
 \end{aligned}$$

$$\begin{aligned}
 u &= 2x \\
 du &= 2 dx
 \end{aligned}$$

$$\begin{aligned}
 u &= 4x \\
 du &= 4 dx
 \end{aligned}$$

$$\frac{1}{2} \int \cos(u) du = \frac{1}{2} \operatorname{Sen}(u)$$

$$\frac{1}{4} \int \cos(u) du = \frac{1}{4} \operatorname{Sen}(u)$$



$$= \frac{3}{8}x + \frac{1}{2} \left(\frac{1}{2} \text{Sen}(2x) \right) + \frac{1}{8} \left(\frac{1}{4} \text{Sen}(4x) \right) + C$$

$$= \frac{12x + 8\text{Sen}(2x) + \text{Sen}(4x)}{32} + C$$

$$3-) \int \frac{e^{2x}}{\sqrt{1-e^x}} dx \quad u = e^x \quad du = e^x dx$$

$$= \int \frac{(e^x)^2}{\sqrt{1-e^x}} dx \Rightarrow \int \frac{u}{\sqrt{1-u}} du \quad v = 1-u \quad dv = -du$$

$$\Rightarrow \int \frac{v-1}{\sqrt{v}} dv = \int \sqrt{v} dv - \int \frac{1}{\sqrt{v}} dv = \int v^{1/2} dv - \int v^{-1/2} dv$$

$$= \frac{v^{3/2}}{3/2} - \frac{v^{1/2}}{1/2} + C = \frac{2}{3} v^{3/2} - 2 v^{1/2} + C$$

$$\Rightarrow \frac{2}{3} (1-u)^{3/2} - 2 (1-u)^{1/2} \Rightarrow \frac{2}{3} (1-e^x)^{3/2} - 2 (1-e^x)^{1/2} + C$$

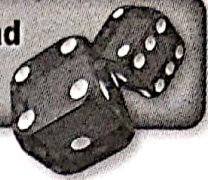
$$= \frac{2(1-e^x)^{3/2} - 6(1-e^x)^{1/2}}{3} + C = \frac{2}{3} \sqrt{1-e^x} (e^x + 2) + C$$

$$4-) 2\pi \int_0^1 (y+1) \sqrt{1-y} dy \quad u = 1-y \quad du = -dy$$

$$-u+1 = y$$

$$\Rightarrow -2\pi \int (u+2) \sqrt{u} du = -2\pi \left[\int u^{3/2} du + \int 2u^{1/2} du \right]$$

$$= -2\pi \left[\frac{u^{5/2}}{5/2} + \frac{2u^{3/2}}{3/2} \right] = -2\pi \left[\frac{2u^{5/2}}{5} + \frac{4u^{3/2}}{3} \right]$$



$$= \frac{4\pi}{5} u^{5/2} - \frac{8\pi}{3} u^{3/2}$$

$$\Rightarrow \left[\frac{4\pi}{5} (1-y)^{5/2} - \frac{8\pi}{3} (1-y)^{3/2} \right]_0^1$$

$$= \left(\frac{4\pi}{5} \overset{0}{(1-1)^{5/2}} - \frac{8\pi}{3} \overset{0}{(1-1)^{3/2}} \right) - \left(\frac{4\pi}{5} (1-0)^{5/2} - \frac{8\pi}{3} (1-0)^{3/2} \right)$$

$$= -\left(\frac{4\pi}{5} - \frac{8\pi}{3} \right) = -\frac{12\pi}{15} + \frac{40\pi}{15} = \frac{28\pi}{15}$$