Diplomado en **Probabilidad** e **Inferencia Básica**

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Quiz 5: Integrales

1-) $\int Sen(t) \ln(Cos(t))\, dt \qquad u = Cos(t)\ ,\ du = -Sen(t)\, dt$

$= -\int \ln(u)\, du = -u\ln(u) + \int du = -u\ln(u) + u$

$w = \ln(u) \qquad dv = du$

$dw = \frac{1}{u} du \qquad v = u$

$= Cos(t) - Cos(t)\ln(Cos(t)) + C$

2-) $\int Cos^4(x)\, dx = \int (Cos^2(x))^2\, dx = \int \left(\frac{1+Cos(2x)}{2}\right)^2 dx$

$= \int \frac{(1+Cos(2x))^2}{4}\, dx = \frac{1}{4}\int 1 + 2Cos(2x) + Cos^2(2x)\, dx$

$= \frac{1}{4}\left[\int dx + 2\int Cos(2x)\, dx + \int \left(\frac{1+Cos(4x)}{2}\right) dx\right]$

$= \frac{1}{4}\left[\int dx + 2\int Cos(2x)\, dx + \frac{1}{2}\int dx + \frac{1}{2}\int Cos(4x)\, dx\right]$

$= \frac{x}{4} + \frac{x}{8} + \frac{1}{2}\int Cos(2x)\, dx + \frac{1}{8}\int Cos(4x)\, dx$

$u = 2x \qquad\qquad u = 4x$

$du = 2\, dx \qquad\qquad du = 4\, dx$

$\frac{1}{2}\int Cos(u)\, du = \frac{1}{2} Sen(u) \qquad \frac{1}{4}\int Cos(u)\, du = \frac{1}{4} Sen(u)$

$$= \frac{3}{8}x + \frac{1}{2}\left(\frac{1}{2}\operatorname{Sen}(2x)\right) + \frac{1}{8}\left(\frac{1}{4}\operatorname{Sen}(4x)\right) + C$$

$$= \frac{12x + 8\operatorname{Sen}(2x) + \operatorname{Sen}(4x)}{32} + C$$

3-) $\int \frac{e^{2x}}{\sqrt{1-e^x}}dx \qquad u = e^x \qquad du = e^x dx$

$$= \int \frac{(e^x)^2}{\sqrt{1-e^x}}dx \Rightarrow \int \frac{u}{\sqrt{1-u}}du \qquad v = 1-u \qquad dv = -dv$$

$$\Rightarrow \int \frac{v-1}{\sqrt{v}}dv = \int \sqrt{v}\,dv - \int \frac{1}{\sqrt{v}}dv = \int v^{1/2}dv - \int v^{-1/2}dx$$

$$= \frac{v^{3/2}}{3/2} - \frac{v^{1/2}}{1/2} + C = \frac{2}{3}v^{3/2} - 2v^{1/2} + C$$

$$\Rightarrow \frac{2}{3}(1-u)^{3/2} - 2(1-u)^{1/2} \Rightarrow \frac{2}{3}(1-e^x)^{3/2} - 2(1-e^x)^{1/2} + C$$

$$= \frac{2(1-e^x)^{3/2} - 6(1-e^x)^{1/2}}{3} + C = \frac{2}{3}\sqrt{1-e^x}\,(e^x+2) + C$$

4-) $2\pi\int_0^1 (y+1)\sqrt{1-y}\,dy \qquad u = 1-y \qquad du = -dy$

$-u+1 = y$

$$\Rightarrow -2\pi\int (u+2)\sqrt{u}\,du = -2\pi\left[-\int u^{3/2}du + \int 2u^{1/2}du\right]$$

$$= -2\pi\left[\frac{-u^{5/2}}{5/2} + \frac{2u^{3/2}}{3/2}\right] = -2\pi\left[\frac{-2u^{5/2}}{5} + \frac{4u^{3/2}}{3}\right]$$

$$= \frac{4\pi}{5} u^{5/2} - \frac{8\pi}{3} u^{3/2}$$

$$\Rightarrow \frac{4\pi}{5}(1-y)^{5/2} - \frac{8\pi}{3}(1-y)^{3/2} \Big]_0^1$$

$$= \left(\frac{4\pi}{5}\cancel{(1-1)}^{5/2} - \frac{8\pi}{3}\cancel{(1-1)}^{3/2}\right) - \left[\frac{4\pi}{5}(1-0)^{5/2} - \frac{8\pi}{3}(1-0)^{3/2}\right]$$

$$= -\left(\frac{4\pi}{5} - \frac{8\pi}{3}\right) = -\frac{12\pi}{15} + \frac{40\pi}{15} = \frac{28\pi}{15}.$$