Día	Mes	Año	Hora	Institución			Código	Materia	
Alumno									
Curso	Bimestre	Semestre	Salón					CALIFICACIÓN	
Profesor			Hoja No _____ de _____						

Fabian Camilo Peña Lozano

TALLER 3

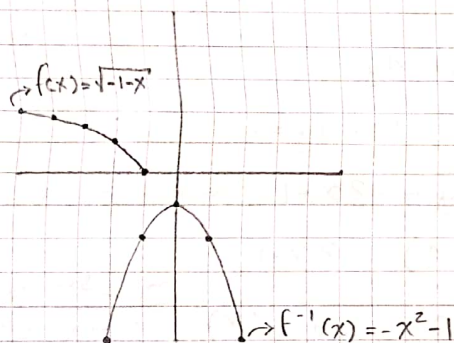① a.) $f(x) = x^3$, inyectiva? Si, dado que $f(2) = 8 \neq -8 = f(-2)$

$f(x) = x^2$, inyectiva? No, dado que $f(2) = 4 = f(-2)$

b.)

c.) $f(x) = x^2 + 2$

$f^{-1}(x) = \sqrt{x-2}$

d.) $f(x) = \sqrt{-1-x}$ $\qquad$ $f^{-1}(x) = -x^2 - 1$

e.) $Ln(x) = 5 \Rightarrow e^{ln(x)} = e^5 \Rightarrow x = e^5$

② a.) $f(x) = 1 + \sqrt{2+3x}$

$$y = 1 + \sqrt{2+3x} \implies x = 1 + \sqrt{2+3y}$$

$$x - 1 = \sqrt{2+3y}$$

$$(x-1)^2 = 2 + 3y$$

$$(x-1)^2 - 2 = 3y$$

$$y = \frac{(x-1)^2 - 2}{3} = f^{-1}(x)$$

b.) $f(x) = e^{2x-1}$

$$y = e^{2x-1} \implies x = e^{2y-1}$$

$$\ln(x) = \ln(e^{2y-1})$$

$$\ln(x) = 2y - 1$$

$$\ln(x) + 1 = 2y$$

$$y = \frac{\ln(x) + 1}{2} = f^{-1}(x)$$

c.) $y = \ln(x+3) \implies x = \ln(y+3)$

$$e^x = e^{\ln(y+3)}$$

$$e^x = y + 3$$

$$y = e^x - 3 = f^{-1}(x)$$

d.) $f(x) = \frac{4x-1}{2x+3}$

$$y = \frac{4x-1}{2x+3} \implies x = \frac{4y-1}{2y+3}$$

$$x(2y+3) = 4y - 1$$

$$2xy + 3x + 1 - 4y = 0$$

$$2xy - 4y = -3x - 1$$

$$y(2x-4) = -3x - 1$$

$$y = \frac{-3x-1}{2x-4} = f^{-1}(x)$$

e.) $y = \frac{e^x}{1+2e^x} \implies x = \frac{e^y}{1+2e^y}$

$$x(1+2e^y) = e^y$$

$$x + 2xe^y = e^y$$

$$x = e^y - 2xe^y$$

$$x = e^y(1 - 2x)$$

$$e^y = \frac{x}{1+2x}$$

$$\ln(e^y) = \ln\left(\frac{x}{1+2x}\right)$$

$$y = \ln\left(\frac{x}{1+2x}\right) = f^{-1}(x)$$

f-) $y = \dfrac{1-e^{-x}}{1+e^{-x}} \Rightarrow x = \dfrac{1-e^{-y}}{1+e^{-y}}$

$$x(1+e^{-y}) = 1-e^{-y}$$

$$x + xe^{-y} = 1-e^{-y}$$

$$xe^{-y} + e^{-y} = 1-x$$

$$e^{-y}(x+1) = 1-x$$

$$e^{-y} = \dfrac{1-x}{x+1}$$

$$\ln(e^{-y}) = \ln\left(\dfrac{1-x}{1+x}\right)$$

$$y = -\ln\left(\dfrac{1-x}{1+x}\right)$$

③

a-) $\displaystyle\lim_{x\to 1} \dfrac{x-1}{x^2-1} = \lim_{x\to 1} \dfrac{(x-1)}{(x+1)(x-1)} = \lim_{x\to 1} \dfrac{1}{x+1} = \dfrac{1}{2}$

b-) $\displaystyle\lim_{t\to 0} \dfrac{\sqrt{t^2+9}-3}{t^2} =$

Por derecha $\Rightarrow t=0.001 \Rightarrow \dfrac{\sqrt{(0.001)^2+9}-3}{(0.001)^2} = \dfrac{1.6\times 10^{-7}}{0.000001} = 0.166...$

Por izquierda $\Rightarrow t=-0.001 \Rightarrow \dfrac{\sqrt{(-0.001)^2+9}-3}{(-0.001)^2} = \dfrac{1.6\times 10^{-7}}{0.000001} = 0.166...$

c-) $\displaystyle\lim_{x\to 3} \dfrac{x^2-3x}{x^2-9} = \lim_{x\to 3} \dfrac{x(x-3)}{(x+3)(x-3)} = \lim_{x\to 3} \dfrac{x}{x+3} = \dfrac{3}{6} = \dfrac{1}{2}$

Para $x = 2.999 \Rightarrow \dfrac{2.999}{2.999+3} = \dfrac{2.999}{5.999} = 0.499$

Para $x = 2.9999 \Rightarrow \dfrac{2.9999}{2.9999+3} = \dfrac{2.9999}{5.9999} = 0.4999$

Para $x = 3.01 \Rightarrow \dfrac{3.01}{3.01+3} = \dfrac{3.01}{6.01} = 0.5008$

Para $x = 3.001 \Rightarrow \dfrac{3.001}{3.001+3} = \dfrac{3.001}{6.001} = 0.50008$

d.) $\lim\limits_{x \to -3} \dfrac{x^2-3x}{x^2-9} = \lim\limits_{x \to -3} \dfrac{x(x-3)}{(x+3)(x-3)} = \lim\limits_{x \to -3} \dfrac{x}{x+3}$

Para $x=-2.999 \Rightarrow \dfrac{-2.999}{-2.999+3} = \dfrac{-2.999}{0.001} = -2999$

Para $x=-2.9999 \Rightarrow \dfrac{-2.9999}{-2.9999+3} = \dfrac{-2.9999}{0.0001} = -29.999 \to -\infty$

Para $x=-3.01 \Rightarrow \dfrac{-3.01}{-3.01+3} = \dfrac{-3.01}{-0.01} = 301$

Para $x=-3.001 \Rightarrow \dfrac{-3.001}{-3.001+3} = \dfrac{-3.001}{-0.001} = 3.001 \to \infty$

e.) $\lim\limits_{x \to 5+} \dfrac{x+1}{x-5}$

Para $x=5.01 \Rightarrow \dfrac{5.01+1}{5.01-5} = \dfrac{6.01}{0.01} = 601$

Para $x=5.001 \Rightarrow \dfrac{5.001+1}{5.001-5} = \dfrac{6.001}{0.001} = 6001$

Para $x=5.0001 \Rightarrow \dfrac{5.0001+1}{5.0001-5} = \dfrac{6.0001}{0.0001} = 60001 \to \infty$

f.) $\lim\limits_{x \to 5-} \dfrac{x+1}{x-5}$

Para $x=4.99 \Rightarrow \dfrac{4.99+1}{4.99-5} = \dfrac{5.99}{-0.01} = -599$

Para $x=4.999 \Rightarrow \dfrac{4.999+1}{4.999-5} = \dfrac{5.999}{-0.001} = -5999$

Para $x=4.9999 \Rightarrow \dfrac{4.9999+1}{4.9999-5} = \dfrac{5.9999}{-0.0001} = -59999 \to -\infty$

g.) $\lim\limits_{x \to 1} \dfrac{2-x}{(x-1)^2}$

Para $x=0.99 \Rightarrow \dfrac{2-0.99}{(0.99-1)^2} = \dfrac{1.01}{(-0.01)^2} = \dfrac{1.01}{0.0001} = 10.100$

Para $x=0.999 \Rightarrow \dfrac{2-0.999}{(0.999-1)^2} = \dfrac{1.001}{(-0.001)^2} = \dfrac{1.001}{0.000001} = 1'001.000 \to \infty$

Para $x=1.01 \Rightarrow \dfrac{2-1.01}{(1.01-1)^2} = \dfrac{0.99}{(0.01)^2} = \dfrac{0.99}{0.0001} = 9.900$

Para $x=1.001 \Rightarrow \dfrac{2-1.001}{(1.001-1)^2} = \dfrac{0.999}{(0.001)^2} = \dfrac{0.999}{0.000001} = 999.999 \to \infty$

Observaciones

h) $\lim\limits_{x \to 5} 2x^2 - 3x + 4 = 2(5)^2 - 3(5) + 4 = 50 - 15 + 4 = 39$

i) $\lim\limits_{x \to -2} \dfrac{x^3 + 2x^2 - 1}{5 - 3x} = \dfrac{(-2)^3 + 2(-2)^2 - 1}{5 - 3(-2)} = \dfrac{-8 + 8 - 1}{5 + 6} = \dfrac{-1}{11}$

j) $\lim\limits_{x \to 1} \dfrac{x^2 - 1}{x - 1}$

Para $x = 0.99 \Rightarrow \dfrac{(0.99)^2 - 1}{0.99 - 1} = \dfrac{0.9801 - 1}{-0.01} = \dfrac{-0.0199}{-0.01} = 1.99$

Para $x = 0.999 \Rightarrow \dfrac{(0.999)^2 - 1}{0.999 - 1} = \dfrac{0.998001 - 1}{-0.001} = \dfrac{-0.001999}{-0.001} = 1.999 \to 2$

Para $x = 1.01 \Rightarrow \dfrac{(1.01)^2 - 1}{1.01 - 1} = \dfrac{1.0201 - 1}{0.01} = \dfrac{0.0201}{0.01} = 2.01$

Para $x = 1.001 \Rightarrow \dfrac{(1.001)^2 - 1}{1.001 - 1} = \dfrac{1.002001 - 1}{0.001} = \dfrac{0.002001}{0.001} = 2.001 \to 2$

k) $\lim\limits_{h \to 0} \dfrac{(3 + h)^2 - 9}{h}$

Para $h = -0.01 \Rightarrow \dfrac{(3 - 0.01)^2 - 9}{-0.01} = \dfrac{(2.99)^2 - 9}{-0.01} = \dfrac{8.9401 - 9}{-0.01} = \dfrac{-0.0599}{-0.01} = 5.99$

Para $h = -0.001 \Rightarrow \dfrac{(3 - 0.001)^2 - 9}{-0.001} = \dfrac{(2.999)^2 - 9}{-0.001} = \dfrac{8.994001 - 9}{-0.001} = \dfrac{0.001999}{-0.001} = 5.999 \to 6$

Para $h = 0.01 \Rightarrow \dfrac{(3 + 0.01)^2 - 9}{0.01} = \dfrac{(3.01)^2 - 9}{0.01} = \dfrac{9.0601 - 9}{0.01} = \dfrac{0.0601}{0.01} = 6.01$

Para $h = 0.001 \Rightarrow \dfrac{(3 + 0.001)^2 - 9}{0.001} = \dfrac{(3.001)^2 - 9}{0.001} = \dfrac{9.006001 - 9}{0.001} = \dfrac{0.006001}{0.001} = 6.001 \to 6$

l) $\lim\limits_{x \to -1} (x^2 + x)(3x^2 + 6) = ((-1)^2 - 1)(3(-1)^2 + 6) = (1 - 1)(3 + 6) = 0$

m) $\lim\limits_{t \to -2} \dfrac{t^4 - 2}{2t^2 - 3t + 2} = \dfrac{(-2)^4 - 2}{2(-2)^2 - 3(-2) + 2} = \dfrac{16 - 2}{8 + 6 + 2} = \dfrac{14}{16} = \dfrac{7}{8}$

n)

④

a) $\lim\limits_{x \to \infty} \dfrac{3x^2 - x - 2}{5x^2 + 1x + 1} = \lim\limits_{x \to \infty} \dfrac{3 - \dfrac{x}{x^2}^{\to \infty} - \dfrac{2}{x^2}^{\to \infty}}{5 + \dfrac{4x}{x^2}^{\to \infty} + \dfrac{1}{x^2}^{\to \infty}} = \dfrac{3}{5}$

b) $\lim\limits_{x \to \infty} \dfrac{1}{2x+3} = \lim\limits_{x \to \infty} \dfrac{\dfrac{1}{x}^{\to \infty}}{2 + \dfrac{3}{x}^{\to \infty}} = \dfrac{0}{2} = 0$

c) $\lim\limits_{x \to \infty} \dfrac{3x+5}{x-4} = \lim\limits_{x \to \infty} \dfrac{3 + \dfrac{5}{x}^{\to \infty}}{1 - \dfrac{4}{x}^{\to \infty}} = 3$

d) $\lim\limits_{x \to \infty} \dfrac{1 - x - x^2}{2x^2 - 7} = \lim\limits_{x \to \infty} \dfrac{\dfrac{1}{x^2}^{\to \infty} - \dfrac{1}{x}^{\to \infty} - 1}{2 - \dfrac{7}{x^2}^{\to \infty}} = \dfrac{-1}{2}$

e) $\lim\limits_{y \to \infty} \dfrac{2 - 3y^2}{5y^2 + 4y} = \lim\limits_{y \to \infty} \dfrac{\dfrac{2}{y^2}^{\to \infty} - 3}{5 + \dfrac{4}{y}^{\to \infty}} = \dfrac{-3}{5}$

f) $\lim\limits_{x \to \infty} \dfrac{\sqrt{1 + 4x^6}}{2 - x^3} = \dfrac{\sqrt{\lim\limits_{x \to \infty} 1 + 4x^6}}{\lim\limits_{x \to \infty} 2 - x^3}$

$= \dfrac{\sqrt{\lim\limits_{x \to \infty} \dfrac{1}{x^6} + 4}}{\lim\limits_{x \to \infty} \dfrac{2}{x^3} - 1} = -\sqrt{4}$

g) $\displaystyle \lim_{x\to\infty} \frac{\sqrt{x+3x^2}}{4x-1} = \frac{\sqrt{\displaystyle\lim_{x\to\infty} x+3x^2}}{\displaystyle\lim_{x\to\infty} 4x-1}$

$$= \frac{\sqrt{\displaystyle\lim_{x\to\infty} \frac{1}{x}^{\,0}+3}}{\displaystyle\lim_{x\to\infty} 4-\frac{1}{x}^{\,0}} = \frac{\sqrt{3}}{4}$$

h) $\displaystyle \lim_{x\to\infty} \frac{x+3x^2}{4x-1} = \lim_{x\to\infty} \frac{\frac{1}{x}^{\,0}+3}{\frac{4}{x}^{\,0}-\frac{1}{x^2}^{\,0}} = \infty$

i) $\displaystyle \lim_{x\to\infty} \frac{x+x^2}{3-x} = \lim_{x\to\infty} \frac{\frac{1}{x}^{\,0}+1}{\frac{3}{x^2}^{\,0}-\frac{1}{x}^{\,0}} = -\infty$