Fabian Camilo Peña

Taller 5.

Observaciones

# SUCESIONES

1.

a-) $\left\{\frac{n}{n+1}\right\}_{n=1}^{\infty} = \frac{1}{1+1}, \frac{1}{2+1}, \frac{1}{3+1}, \frac{1}{4+1}, \dots = \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \dots$

b-) $\left\{\frac{(-1)^n(n+1)}{3^n}\right\}_{n=0}^{\infty} = \frac{(-1)^0(0+1)}{3^0}, \frac{(-1)^1(1+1)}{3^1}, \frac{(-1)^2(2+1)}{3^2}, \frac{(-1)^3(3+1)}{3^3}, \dots$

$= \frac{(1)(1)}{(1)}, \frac{(-1)(2)}{3}, \frac{(1)(3)}{9}, \frac{(-1)(4)}{27}, \dots = 1, \frac{-2}{3}, \frac{1}{3}, \frac{-4}{27}, \dots$

c-) $\left\{\sqrt{n-3}\right\}_{n=3}^{\infty} = \sqrt{3-3}, \sqrt{4-3}, \sqrt{5-3}, \sqrt{6-3}, \dots = 0, 1, \sqrt{2}, \sqrt{3}, \dots$

d-) $\left\{\cos\left(\frac{n\pi}{6}\right)\right\}_{n=0}^{\infty} = \cos\left(\frac{0\cdot\pi}{6}\right), \cos\left(\frac{1\pi}{6}\right), \cos\left(\frac{2\cdot\pi}{6}\right), \cos\left(\frac{3\cdot\pi}{6}\right), \dots$

$= \cancel{\cos(0)} \to 1, \cos\left(\frac{\pi}{6}\right), \cos\left(\frac{\pi}{3}\right), \cos\left(\frac{\pi}{2}\right), \dots$

2. $\frac{3}{5}, \frac{-4}{25}, \frac{5}{125}, \frac{-6}{625}, \frac{7}{3125}, \dots = \left\{\frac{(-1)^{n+1}n+2}{5^n}\right\}_{n=1}^{\infty}$

$-1 \quad +1 \quad -1 \quad +1$

$5\times5 \quad 25\times5 \quad 125\times5$

3.

a-) $\lim_{n\to\infty} \frac{n}{n+1} = \lim_{n\to\infty} \frac{\cancel{\frac{n}{n}}^{1}}{\cancel{\frac{n}{n}}^{1} + \cancel{\frac{1}{n}}^{0}} = 1$

b-) $\lim_{n\to\infty} \frac{\ln(n)}{n} = 0$, dado que $n$ crece más rápido que $\ln(n)$ y

$\lim_{n\to\infty} \frac{1}{n} = 0$

4.

a-) $a_n = \frac{2^n}{2n+1}, n \geq 0 \Rightarrow \frac{2^0}{2(0)+1}, \frac{2^1}{2(1)+1}, \frac{2^2}{2(2)+1}, \frac{2^3}{2(3)+1}, \frac{2^4}{2(4)+1}$

$= 1, \frac{2}{3}, \frac{4}{5}, \frac{8}{7}, \frac{16}{9}$

b-) $a_n = \frac{n^2-1}{n^2+1}, n \geq 0, \Rightarrow \frac{(0)^2-1}{(0)^2+1}, \frac{(1)^2-1}{(1)^2+1}, \frac{(2)^2-1}{(2)^2+1}, \frac{(3)^2-1}{(3)^2+1}, \frac{(4)^2-1}{(4)^2+1}$

$= -1, 0, \frac{3}{5}, \frac{8}{10}, \frac{15}{17}$

c-) $a_n = \frac{2n}{n^2+1}, n \geq 0 \Rightarrow \frac{2(0)}{(0)^2+1}, \frac{2(1)}{(1)^2+1}, \frac{2(2)}{(2)^2+1}, \frac{2(3)}{(3)^2+1}, \frac{2(4)}{(4)^2+1}$

$= 0, 1, \frac{4}{5}, \frac{3}{5}, \frac{8}{17}$

d-) $a_n = \frac{3^n}{1+2^n}, n \geq 0 \Rightarrow \frac{3^0}{1+2^0}, \frac{3^1}{1+2^1}, \frac{3^2}{1+2^2}, \frac{3^3}{1+2^3}, \frac{3^4}{1+2^4}$

$= \frac{1}{2}, 1, \frac{9}{5}, 3, \frac{81}{17}$

## SERIES

a-) $\sum_{n=1}^{\infty} 2^{2n} 3^{1-n} = \sum 4^n 3^{-(n-1)} = \sum \frac{4^n 4 4^{-1}}{3^{n-1}} = 4 \sum \frac{4^{n-1}}{3^{n-1}} = 4 \sum \left(\frac{4}{3}\right)^{n+1}$ $|r|>1$ $\Rightarrow$ Diverge

b-) $\sum_{n=1}^{\infty} \frac{n^2}{5n^2+4} \Rightarrow \lim_{n\to\infty} \frac{n^2}{5n^2+4} = \lim_{n\to\infty} \frac{\cancel{n^2}/\cancel{n^2}}{\cancel{5n^2}/\cancel{n^2} + \cancel{4}/\cancel{n^2}} = \frac{1}{5} \neq 0 \Rightarrow$ La serie diverge

c-) $\sum_{n=1}^{\infty} \frac{1}{n(n+1)} \Rightarrow \lim_{n\to\infty} \frac{1}{n^2+n} = \lim_{n\to\infty} \frac{1/n^2}{n^2/n^2 + n/n^2} = 0 \Rightarrow$ La serie converge

## INTEGRALES

a-) $\int x^2 dx = \frac{x^3}{3} + C$

b-) $\int_{-1}^{5} (1+3x)dx = \int_{-1}^{5} dx + \int_{-1}^{5} 3x dx = x\Big]_{-1}^{5} + \frac{3}{2}x^2\Big]_{-1}^{5} = (5+1) + \frac{3}{2}(5^2 - (-1)^2)$

$= 6 + 3/2(25-1) = 6+36 = 42$

c-) $\int_{-2}^{0} (x^2+x)dx = \int_{-2}^{0} x^2 dx + \int_{-2}^{0} x dx = \frac{x^3}{3}\Big]_{-2}^{0} + \frac{x^2}{2}\Big]_{-2}^{0} = \left[\frac{(0)^3}{3} - \frac{(-2)^3}{3}\right] + \left[\frac{(0)^2}{2} - \frac{(-2)^2}{2}\right]$

$= 8/3 - 2 = 2/3$

d-) $\int_0^1 (x^3 - 3x^2)dx = \int_0^1 x^3 dx - \int_0^1 3x^2 dx = \frac{x^4}{4}\Big]_0^1 - x^3\Big]_0^1 = \left[\frac{(1)^4}{4} - \cancel{\frac{(0)^4}{4}}\right] - \left[(1)^3 - \cancel{(0)^3}\right]$

$= 1/4 - 1 = -3/4$

e-) $\int_1^4 (x^2+2x-5)dx = \int_1^4 x^2 dx + \int_1^4 2x dx - 5\int_1^4 dx = \frac{x^3}{3} + x^2 - 5x\Big]_1^4$

$= \left[\frac{(4)^3}{3} - \frac{(1)^3}{3}\right] + \left[(4)^2 - (1)^2\right] - \left[5(4) - 5(1)\right] = \left(\frac{64-1}{3}\right) \cancel{+15} \cancel{-15} = \frac{63}{3} = 21$

f-) $\int_0^2 (2x - x^3)dx = \int_0^2 2x dx - \int_0^2 x^3 dx = x^2\Big]_0^2 - \frac{x^4}{4}\Big]_0^2$

$= \left[(2)^2 - \cancel{(0)^2}\right] - \left[\frac{(2)^4}{4} - \cancel{\frac{(0)^4}{4}}\right] = 4 - \frac{16}{4} = \frac{16-16}{4} = 0$

g-) $\int_0^1 x^3 dx = \frac{x^4}{4}\Big]_0^1 = \left[\frac{(1)^4}{4} - \cancel{\frac{(0)^4}{4}}\right] = \frac{1}{4}$

h-) $\int_0^1 \frac{1}{1+x^2} dx = \tan^{-1}(x)\Big]_0^1 = \tan^{-1}(1) - \tan^{-1}(0) = \pi/4$

i-) $\int_0^2 xe^{-x} dx \quad u = x, du = dx \quad dv = e^{-x}dx, v = -e^{-x} \quad uv - \int v du$

$= \left[-xe^{-x} + \int e^{-x} dx\right]_0^2 = \left[-xe^{-x} - e^{-x}\right]_0^2 = -e^{-x}(x+1)\Big]_0^2$

$= -e^{-2}(2+1) + e^0(1) = -3e^{-2} + 1$

j-) $\int_0^3 \frac{1}{x+4}\,dx \quad u = x+4 \quad du = dx \quad \int \frac{1}{u}\,du = \ln(u)$

$= \int_0^3 \frac{1}{x+4}\,dx = \ln(x+4)\Big]_0^3 = \ln(7) - \ln(4) = \ln\left(\frac{7}{4}\right)$

k-) $\int_0^2 (x^3 - 3x + 3)\,dx = \int_0^2 x^3 dx - \int_0^2 3x\,dx + 3\int_0^2 dx = \frac{x^4}{4}\Big]_0^2 - \frac{3x^2}{2}\Big]_0^2 + 3x\Big]_0^2$

$= \left(\frac{2^4}{4} - \cancel{\frac{0^4}{4}}\right) - \left(\frac{3(2)^2}{2} - \cancel{\frac{3(0)^2}{2}}\right) + \left(3(2) - 3(0)\right) = 4 - 6 + 6 = 4$

l-) $\int_\pi^{2\pi} (x - 2\operatorname{Sin}(x))dx = \int_\pi^{2\pi} x\,dx - 2\int \operatorname{Sin}(x)\,dx = \left[\frac{x^2}{2} + 2\operatorname{Cos}(x)\right]_\pi^{2\pi}$

$= \left(\frac{(2\pi)^2}{2} - \frac{(\pi)^2}{2}\right) + 2\left(\operatorname{Cos}(2\pi) - \operatorname{Cos}(\pi)\right) = \frac{4\pi^2 - \pi^2}{2} + 2(1+1) = 4 + \frac{3\pi^2}{2}$

m-) $\int_{\pi/6}^{\pi/2} \operatorname{Csc}(x)\operatorname{Cot}(x)\,dx = -\operatorname{Csc}(x)\Big]_{\pi/6}^{\pi/2} = -\operatorname{Csc}(\pi/2) + \operatorname{Csc}(\pi/6) = 1$

n-) $\int_{\pi/4}^{\pi/3} \operatorname{Csc}^2(x)\,dx = -\operatorname{Cot}(x)\Big]_{\pi/4}^{\pi/3} = -\operatorname{Cot}(\pi/3) + \operatorname{Cot}(\pi/4) = \frac{-1}{\sqrt{3}} + 1$

ñ-) $\int_0^1 (1+x^3)\,dx = \int_0^1 dx + \int_0^1 x^3 dx = x + \frac{x^4}{4}\Big]_0^1 = (1-0) + \left(\frac{1^4}{4} - \cancel{\frac{0^4}{4}}\right) = 1 + \frac{1}{4} = \frac{5}{4}$

o-) $\int (10x^4 - 2\sec^2(x))\,dx = 10\int x^4 dx - 2\int \operatorname{Sec}^2(x)\,dx$

$= \frac{10x^5}{5} - 2\operatorname{Tan}(x) + C = 2x^5 - 2\operatorname{Tan}(x) + C$

p-) $\int \frac{\operatorname{Cos}\theta}{\operatorname{Sen}^2\theta}\,d\theta = \int \operatorname{Cot}(\theta)\operatorname{Csc}(\theta)\,d\theta = -\operatorname{Csc}(\theta) + C$

q-) $\int e^x \operatorname{Sen}(x)\,dx \qquad u = \operatorname{Sen}(x),\ du = \operatorname{Cos}(x)\,dx \quad dv = e^x dx,\ v = e^x$

$= e^x \operatorname{Sen}(x) - \int e^x \operatorname{Cos}(x)\,dx$

$u = \operatorname{Cos}(x),\ du = -\operatorname{Sen}(x)dx\ dv = e^x,\ v = e^x$

$= e^x \operatorname{Sen}(x) - \left[e^x \operatorname{Cos}(x) + \int e^x \operatorname{Sen}(x)\,dx\right]$

$= e^x \operatorname{Sen}(x) - e^x \operatorname{Cos}(x) - \int e^x \operatorname{Sen}(x)\,dx = \frac{1}{2}\left(e^x \operatorname{Sen}(x) - e^x \operatorname{Cos}(x)\right)$

r-) $\int_0^2 \left(2x^3 - 6x - \frac{3}{x^2+1}\right)dx = 2\int_0^2 x^3 dx - 6\int_0^2 x\,dx - 3\int_0^2 \frac{1}{x^2+1}\,dx$

$= \frac{2x^4}{4} - \frac{6x^2}{2} - 3\tan^{-1}(x) = \frac{x^4}{2} - 3x^2 - 3\tan^{-1}(x) = \left(\frac{2^4}{2} - \cancel{\frac{0^4}{2}}\right) - \left(3(2)^2 - \cancel{3(0)^2}\right)$

$-\left(3\tan^{-1}(2) - \cancel{3\tan^{-1}(0)}\right) = 8 - 12 - 3\tan^{-1}(2) = -4 - 3\tan^{-1}(2)$

s-) $\int e^{5x} dx \Rightarrow u = 5x,\ du = 5dx \Rightarrow \frac{1}{5}\int e^u du = \frac{e^u}{5} + C \Rightarrow \frac{e^{5x}}{5} + C$

t-) $\int_0^1 \frac{\ln(x)}{x} dx \quad u = \ln(x) \quad du = \frac{1}{x} dx$

$$\int u\, du = \frac{u^2}{2}$$

$$= \frac{(\ln(x))^2}{2}\Big]_0^1 = \xcancel{\frac{\ln(1)^2}{2}} \xrightarrow{} 0 - \xcancel{\frac{\ln(0)^2}{2}} \xrightarrow{} \text{indefinido} =$$

u-) $\int \cos^3\theta \,\mathrm{Sen}\,\theta\, d\theta \quad u = \cos\theta \quad du = -\mathrm{Sen}\,\theta\, d\theta$

$$-\int u^3 du = -\frac{u^4}{4}$$

$$= -\frac{\cos^4\theta}{4} + C$$

v-) $\int \frac{\mathrm{Sen}(2x)}{1+\cos^2(x)} dx \quad u = \cos^2(x)+1 \quad du = -2\,\mathrm{Sen}(x)\cos(x)dx$

$= -2\,\mathrm{Sen}(2x)\, dx$

$$-\int \frac{1}{u} du = -\ln(u) + C$$

$$= -\ln(\cos^2(x)+1) + C$$

w-) $\int \frac{\mathrm{Sec}^2(x)}{\tan^2(x)} dx = \int \frac{1}{\cancel{\cos^2(x)}} \cdot \frac{\cancel{\cos^2(x)}}{\mathrm{Sen}^2(x)} dx = \int \csc^2(x) dx$

$$= -\cot(x) + C$$

x-) $\int \frac{x}{x^2+4} dx \quad u = x^2+4,\ du = 2x\, dx \Rightarrow \frac{1}{2}\int \frac{1}{u} du = \frac{1}{2}\ln(u) + C$

$$= 1/2(\ln(x^2+4)) + C$$