MAS Data Science

Master Thesis

Bayes Network for Commodity Prices

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Abstract

Over the past twelve months, a broader public has become aware of the volatility of commodity and energy markets. Recent months have shown once again the importance of early and correct market positioning as a large energy company or trading unit. The war in Ukraine, political tensions between the United States and China, a slow re-opening of the Chinese economy after long Covid-restrictions and rising interest rates, as well as recession fears repeatedly caused nervous moments in the energy markets. The markets are moving rapidly. It is no longer possible for a single person to process all the information and assess the fair value of a product for all trading products at all times. The amount of data to be processed and the dynamics of the market forces are simply too great.

This is exactly where this project comes in and tries to develop a network model that always calculates the current fair value with the associated uncertainty of the estimate in real-time for all trading products. This fair value model should allow traders to place their offers perfectly priced on the market. The algorithm provides a good estimate for the price expectation in markets with low trading volumes. In the future, the system will also enable the financial evaluation of trading positions and the associated risks (value at risk) in real-time.

In a first step, fair value models from the academic literature are examined and compared with fair value methodologies of the two major financial market data providers Bloomberg and Refinitiv. An analysis of the strengths and weaknesses of these models reveals that essential elements are not represented. On the one hand, causal cross-commodity effects are neglected; on the other hand, only point estimates for the fair value of traded products are given.

From this, the thesis aims to develop a proof-of-concept for a network model that addresses these vulnerabilities. For the modelling of the fair values of the traded products, normal distributions are assumed because of their many pleasant mathematical properties. The Bayes theorem in statistics deals with the updating of an initial probability density function (prior) by new information and the resulting new distribution function (posterior). In combination with normal distributions, closed-form mathematical solutions exist for these Bayes updates, which are used here to update the modelled fair value of a product after new trade information. Between two trades, the model receives no new information, which increases the uncertainty of the fair value estimate. This effect is modelled by a diffusion process. After the fair value distribution function of a traded product (Pt) has been adjusted by Bayes and diffusion according to a trade entered into, this effect must propagate through the network of further modelled products. For this propagation process, a Q-learning reinforcement learning algorithm is used, which receives as input the relative price change of the traded product and the calculated Pearson correlation coefficients of the settlement of the products. Using an Epsilon-Greedy algorithm and a defined reward function, the code finds the best possible actions to model the cross-commodity effects.

For the final evaluation a network consisting of around 200 trading products, covering a time horizon of about three years is chosen. The product set contains power, gas, coal and CO2 certificates as a basket of commodities. Six front months, 5 front quarters and 3 front years are differentiated among the products. For power the three delivery profiles Base, Peak and Offpeak are taken into account. The results on a test data set of more than 15'000 trades show that the developed model achieves a significantly higher total log-likelihood value per trade than the defined reference case, where the last trade is always the expectation for the fair value of the future. A comparison of the quality of the fair value point forecast and the predictive power compared to the Bloomberg and Refinitiv models also shows advantages for the developed model. Given these findings, the proof of concept phase can be classified as successful. However, the work still shows clear potential for improvement in every aspect of the model, especially a refined model for modelling the new information available at the moment of a new trade and more powerful algorithms for the network propagation update should still be tested.

Contents

Acronyms

CET Central European Time

CV confidence value

DDPG Deep Deterministic Policy Gradient

GUI graphical user interface

ICE Intercontinental Exchange

LL log-likelihood

OTC over the counter

PA price of ask

PB price of bid

PI input price

PO output price

PPO Proximal Policy Optimization

Pt traded product

SAC Soft Actor-Critic

TRPC Thomson Reuters / Refinitiv Power Composite

1 Introduction

It is no secret that electricity markets are extremely volatile and characterized by non-storability, seasonal patterns, possible negatives and difficult spot price dynamics. All these impose considerable challenges for quantitative research involving rather sophisticated mathematics. International trading companies are active in different commodity sectors. Typically, the trading activities include gas, electricity, coal, oil and CO2 certificates. The main focus is on maturities up to approximately three years into the future, which is also called the liquid horizon.

The energy crisis in 2022, characterised by very high commodity prices and resulting extreme margin calls as well as extreme volatility, further reinforced the need for optimal bid placement and adjustment on the electronic platforms.

In commodity trading, new information on the individual trading products is constantly available during the course of the day. Trades are concluded, new orders are placed on the market or new quotes are published by exchanges. As an individual trader, who is also involved in operational processes (such as the dispatch of exchange nominations), it is no longer possible to determine the current fair value of all products over various maturities without the support of algorithms.

In addition, the different trading products are not independent of each other. There are time dependencies within a product group. For example, a higher price of the calendar product in power also influences the quarterly products. On the other hand, cross-commodity influences must also be taken into account. Since, for example, gas-fired power plants also serve to cover the electricity demand, a higher gas price is also transferred to the electricity market to a certain extent. This product dependencies as well as the cross-commodity interactions add another layer of complexity to the topic.

Some data providers nowadays offer real-time fair value models to their customers. The quality of two such models (from Bloomberg and Refinitiv) will be analysed in this project. In the course of this project, it will also be examined which modelling proposals for fair values in real-time trading can be found in academic publications.

After a thorough analysis of the literature and the strengths and weaknesses of the external data providers' models, the main focus is on developing our own fair value model.Within this project, a Bayesian network will be developed to support a trading unit in the real-time fair value pricing of the different commodity trading products. Modelling the fair value of a product not just as a point estimate but with an underlying probability density function enables the detection of conspicuous trades. It also allows the trader to quantify uncertainty in illiquid markets and to adjust fair value through network effects even in markets where there is currently no trading.

Following this brief introduction, Chapter 2 discusses approaches from the literature and from external data providers to model fair values of commodities. With the weaknesses discovered, the requirements for the own model to be developed and the overall project objectives are defined in Chapter 3. In Chapter 4, the most important theoretical concepts are presented, which are relevant for the model selection in Chapter 5. Subsequently, the results are presented and a benchmarking against the models of the external data providers is carried out. Finally, a discussion of the results and a short outlook complete the thesis.

2 Literature Review

2.1 Fair Value Models in Bayesian Context in Literature

In general, it can be stated that little scientific literature could be found on the topic of real-time fair value modelling in the commodity sector. Here is a short list of found examples:

Fair Value Models in Finance

- The only references that can be found on the subject of fair value modelling are superficial descriptions of the methodology used, by large commodity exchange platforms such as Intercontinental Exchange (ICE). An example link is provided here `https://www.theice.com/market-data/pricing-and-evaluations/fairvalue`. ICE uses this methodology to create end-of-day fair value curves of the trading products offered on their platform. In contrast to this, this project aims to provide real-time fair value estimates.

Bayesian network approaches are discussed in the literature but rather in long-term forecasting of commodity prices as the following listing indicates:

Bayes Networks in Finance

- Schwarz [12] uses a dynamic Bayesian network to forecast crude oil prices. But instead of market trades or order book data, his inputs are fundamental features such as the recent economic development.
- Another academic paper shows an outstanding performance of a hybrid Bayesian network approach in forecasting crude oil prices compared to other approaches such as Markov chain Monte Carlo, random forest, support vector machine, neural networks or GARCH models. But with a prediction horizon of several days ahead, also this paper does not cover exactly the field of research of this project. [4]

2.2 Bloomberg Fair Value Curves

The following section describes in detail the methodology Bloomberg uses to produce its fair value curves. The text draws heavily on the user manual prepared by Bloomberg on the subject.

Overview

Bloomberg's commodity fair values provide a complete and consistent forward curve for a range of commodities reflecting the most recent market prices available combined with the observed curve structure in end-of-day settlement prices. All the tenors of all the fair value curves are updated throughout the trading day, ensuring that cross-tenor and cross-commodity spreads between curves are maintained consistently. Fair values are derived from a range of market prices. [1]

Types of Fair Values

The fair value pricing engines follow a systematic cycle as illustrated in Figure 1:

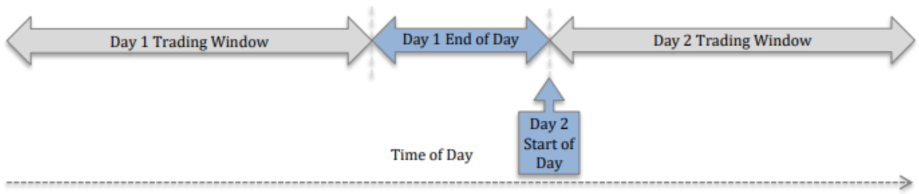

Figure 1: Daily Bloomberg Commodity Fair Value Pricing Cycle [1]

Bloomberg differentiates three main types of fair value curves:

- End-of-day: The fair value field is populated with the end-of-day price derived from the fair value pricing logic. The fair value end-of-day price is aligned with market convention for the calculation of daily settlement reference prices.
- Start-of-day: The initial price for a fair value curve at the start of each trading day is set as the most recent iteration of the prior trading day end-of-day price before the the start of the current trading day, adjusted to reflect market price updates received after the settlement time. For the start-of-day fair values the most recent market input is used for each curve, with these prices included using the same logic as applied for intraday updates to the fair values.
- Intraday: For the duration of each day's trading window the fair value prices are regularly updated to reflect the most recent market prices and quotes from third-party exchange and over the counter (OTC) sources. For tenors not actively priced in the market the default term structure profile from the start day curve is used to imply the price consistent with the latest market prices received on the more active tenors.

Since the project here focuses on real-time fair value calculations, the intraday part is of great relevance. However, since Bloomberg also gives great weight to the price structure of the previous day in the intraday updates, the end-of-day section will also be explained in more detail in the next sections.

Fair Value Methodology

The daily fair value pricing routine is comprised of two key elements;

1. Intraday updates to the curves reflecting the most recent market data available whilst the market is trading
2. Creation of end-of-day curves after the market closes

The end-of-day set of curves from one day is used to seed the start-of-day pricing for the intraday updates the following day, providing a complete default term structure profile. [1]

A) End-of-day Fair Values

The end-of-day process fulfills four distinct objectives:

1. The final set of fair values for the day, for use in historic analysis
2. Where appropriate, settlement prices for contracts derived directly from exchange-traded contracts
3. The initial curve to seed the fair value intraday pricing for the following trading day
4. The arbitrage-free default term structure referenced throughout the following trading day by the intraday pricing process

All of the end-of-day fair values, which are used to create a default curve shapes for use in the following trading day, are derived from exchange settlement and end-of-day OTC prices for Base and Peak power contracts. Offpeak end-of-day prices are implied from the associated Peak and Base contracts. [1]

There are three main elements in the creation of an end-of-day curve:

1. Tenors priced directly from market data:
   Some tenors are priced directly based on market prices from either exchange or OTC sources. To ensure consistency across these different sources only OTC prices falling within the window used to derive the exchange settlement prices are used in the creation of end-of-day fair values. For most continental power markets this period is 15:50 to 16:00 Central European Time (CET), however the Belgium and the Netherlands, which are less liquid markets, a wider period of 15:45 to 16:00 CET is used.
   Where both exchange and OTC prices are available, these are used in equal weight to create the end-of-day fair value price. Where more than one exchange provides prices for a power market these are combined using the following logic: If one or more exchange has open interest greater than zero then weigh by their relative open interest and for any exchange that has no open interest (zero or blank) assign it a weight of 1.
   To augment the tenors priced explicitly by the market, some tenors can be unambiguously implied from the combination of other contracts using a simple bootstrapping logic as shown in Figure 1. For example, if the Q4 price plus the October and November prices are known, the December price can be derived as:

$$p_{December} = (92p_{Q4} – 31p_{October} – 30p_{November})/31 \tag{1}$$

   A similar approach can be applied to yearly and quarterly products.

2. Extrapolation of the curve:
   Often the exchange quotes prices further out than there is currently open interest. In such cases the fair value curve using the seasons with open interest will be extrapolated in preference to relying on the published exchange settlement prices. The extrapolation logic takes the nominal year-on-year change for each season, derived from the furthest dated pairs or equivalent seasons with open interest, as the assumed incremental change between successive seasons.
   An example: If a settlement curve had duration of 6 years and open interest out to year 4, the year 5 and 6 season prices would be derived as follows:

   - Winter delta (Wd) = winter 4th year – winter 3rd year
   - Summer delta (Sd) = summer 4th year – summer 3rd year
   - Winter 5th year = winter 4th year + Wd
   - Winter 6th year = winter 5th year + Wd
   - Summer 5th year = summer 4th year + Sd
   - Summer 6th year = summer 5th year + Sd

3. Shaping strips:
   The processes to derive fair value directly from market data and the extrapolation of the curve will typically result in some parts of the curve only having season values or just seasons and quarters, whereas monthly prices for the whole curves and quarter prices for all periods beyond the quarter in delivery are needed. The missing quarter prices need to be implied from the associated seasons and likewise the missing month prices must be derived from the equivalent quarters. The curve shaping methodology utilises empirical data on the relative pricing of different forward curve tenors historically, in this case over the past 36 months of forwards trading data. [1]

B) Intraday Fair Value Updates

The intraday fair values are published during the trading hours of each market. The continental power market fair values are updated using trades and quotes for the power market directly. Baseload fair values are anchored on recent trades and market data quotes.
The intraday fair value calculation takes the market input price (PI); trades and bid/ask quotes within the last λ minutes, where $\lambda = 10$, each of which has a confidence value (CV). Fair values should not sit outside the price of bid (PB) and price of ask (PA) if possible. The fair value engine seeks to keep the initial prices as close as possible to input prices, weighted by confidence value, for all contracts which have a valid input price, whilst also maintaining the shape from a previous curve night's close. The higher the confidence in the PI, the less important it is to maintain shape. [1]

Calculate the Relative Weight of the Trade versus the Bid-Ask-Spread Midpoint

Bloomberg uses the following formula in Figure 2 to determine the weight of trades in comparison to the bid-ask-spread.

W is the relative value of the trade compared to the mid (when both are valid).

- Very recent trade = very high W.
- Very tight (and recent) bid/offer and a relatively old trade = low W

Equation 4. $\mathbf{W = [(1 - Y)*(t/T)]/[(1-Y)*(t/T) + Y*(x/X)]}$

Equation 5. $\mathbf{Y = [(PA - PB)/K]^{0.4}}$ where;

W = Weight of trade price

t = Age of trade (in seconds)

T = Limiting age of trade (600 seconds)

x = Age of youngest bid/ask

X = Limiting age of oldest of the most recent bid/ask

PA = Price of ask

PB = Price of bid

K = Limiting width of bid/ask spread

Figure 2: Bloomberg Relative Weight of Trade versus Midpoint [1]

Defining Input Price

Bloomberg applies the following general rules for calculating the PI:

- Valid inputs are trades and bid/ask pairs within the last 600 seconds.
- The further ago the trade, the less valuable it is in setting the current fair value
- The wider the bid/ask spread, the less valuable the mid price is in setting the current fair value
- Mid prices derived from bid-ask pairs are only valid if the spread is not excessively wide;
- Decay the trade level towards the current mid as time progresses. The tighter the bid/ask (relative to the hard limit), the more the engine should value mid-over trade, so the steeper the decay. In the case where there's an invalid bid/offer (spread is too wide), but a trade in the last 10mins, the trade is used.
- If no valid bid/ask, use trade, except if the trade is outside a more recent bid/offer, use closest of bid or offer.
- If no valid bid/ask, and no trade within the time cap, PI = previous fair value

Figure 3 summarizes the calculation of the model input price PI as weighted sum of trade and mid-price.

Equation 6. $\mathbf{PI = PT*W + PM*(1 - W)}$ where;

PT = Trade price

W = Relative value of trade

PM = Mid Price [(PA + PB)/2]

Figure 3: Bloomberg Input Price Calculation [1]

<u>Calculate Input Price Confidence</u>

CV is the confidence in the PI. As the trade gets older, or the bid-ask spread gets wider/older, the CV decreases from 1 down to 0

Equation 7. $\mathbf{CV = 1 - [\min(t/T, 1)]*[1 - (1 - (PA - PB)/K)^2*(1\text{-}(x/X))^3]}$ where;

W = Weight of trade value

X = Limiting age of recent bid/ask (600 seconds)

t = Age of last trade (in seconds)

PA = Price of recent ask

T = Limiting age of last trade (600 seconds)

PB = Price of recent bid

x = Age of most recent bid/ask (oldest of the two legs)

K = Limiting width of bid/ask spread

Need to handle edge cases;

- No valid trade, bid/offer only: the formula will weigh the trade component of the CV as zero
- Trade only, no valid bid/offer: as above

Figure 4: Bloomberg Confidence in Input Price [1]

<u>Calculate the Fair Value of Output Prices</u>

Once the set of input prices and confidence weightings in those input prices are estimated, the output fair values can be derived. One of the key objectives when updating the fair value curve is to retain the integrity of the term structure of the curve whilst also reflecting recent market updates to some tenors. This is achieved by seeking to minimise the change in the first derivative of the forward curve profile by minimising the change in the shape of the forward curve.

Each tenor for a product gives three errors:

- The square of the difference between output price (PO) and PI, so that the fair value reflects recent market data.
- The change in the shape (viewed as the average of the squared change in the spread to the next and previous contract of the same type (month/quarter/season/year), weighted by the number of days in the tenor.
- The distance the fair value sits outside the recent market bid/ask (decayed by the age of the bid/ask).

Bloomberg weights these errors appropriately and tries to minimise the total error across all contracts. The errors of the individual contracts are weighted by the number of days of the corresponding trading product.

Equation 8. $\mathbf{\min \Sigma(CV_i*E_p + (1 - CV_i)*E_s + E_a)*D_i}$ [for i = 1 to n] where;

D_i = Days in the month contract

E_s = Change shape versus the prior day end of day curve

E_p = Squared difference between PO and PI

E_a = Distance the fair value falls outside the bid/ask range

For quarters, seasons and year strips the output prices are derived as the day weighted average of the relevant month PO prices.

Equation 9. $\mathbf{E_p = (PO_i - PI_i)^2}$

Equation 10. $\mathbf{Es = ([(PO_{i-1} - PO_i) - (PI_{i-1} - PI_i)]^2 + [(PO_{i+1} - PO_i) - (PI_{i+1} - PI_i)]^2)/2}$

Equation 11. $\mathbf{Ea = PI_i*(\max[(PO_i - PA_i)*(T_A/X),0] + \max[(PB_i - PO_i)*(T_B/X),0])}$ where;

T_A = Age of ask (in seconds)

T_B = Age of bid (in seconds)

Figure 5: Bloomberg Fair Value Output Price Calculation [1]

Peak and Offpeak Prices

As Peak contracts are relatively illiquid compared to the Baseload contracts the intraday Peak fair values are derived by adjusting the level of the previous day's end-of-day Peak prices to account for updates in the current day's Baseload fair value relative to the equivalent Baseload end-of-day prices. Offpeak intraday prices are implied from the associated intraday Peak and Base fair values [1]

Equation 12. $PP_I = PP_E + (BP_I - BP_E)^*\tau$ where;

PP_I = Intraday peak price | BP_I = Intraday base price

PP_E = End of day peak price | BP_E = Intraday peak price

τ = Peak price adjust factor derived from a linear regression of base vs peak prices of the form; PP = (BP) + ϵ using the end of day fair values taken over a rolling 50 day window.

Figure 6: Bloomberg Peak Price Calculation [1]

Critical Evaluation

After a detailed description of the approaches used by Bloomberg, a brief critical assessment will be presented:

- The described fair value calculation procedure uses an interesting combination of bid-ask-spread and trade information for determining the input price and the confidence in this particular price.
- Bloomberg punishes fair values outside of current bid-ask spread boundaries, which makes total sense. Fair values outside the bid-ask spread would suggest completely inefficient markets.
- Bloomberg gives the previous end-of-day curve structure a lot of weight. Throughout the day new information is being processed by the market participants, which can make yesterday's curve term structure completely redundant.
- No cross-commodity effect modelling is described in the Bloomberg fair value calculation handbook, which is clearly problematic for rather illiquid trading products.
- The data provider uses up to 3 years of history for determining term structures of products. The chosen period can be assumed to be too long given the given the large number of fundamental, structural changes in recent months.
- Bloomberg reduces the weight of a bid / ask information with time, which can be seen controversial. The mathematical proof that a valid bid / ask information becomes less relevant over time is missing.
- The Bloomberg fair value curves seem to be updated according to a fixed time-schedule rather than at specific trade times.
- Bloomberg uses a ten-minute window for determining the relevant input prices. This time window width can be of interest when calculating the rewards of the model developed in this project.
- Currently Bloomberg fair value curves are not available for all power markets. Data sets for countries such as Switzerland and Italy cannot be downloaded for now.

2.3 Refinitiv Fair Value Curves

In the following sub-chapter, Refinitiv's fair value methodology is being presented.
Overview

The Refinitiv fair value curves are independently sourced and crafted to arrive at a theoretical price level for each individual traded product and shape an extended forward curve for every maturity type. These are published in real-time throughout the day as the markets change, so there is access to indicative forward-curve data for decision-making.[8]

According to the data provider their fair value curves serve the following purposes:

- Price continuity - what is the theoretical price of a "not traded" listed maturity?
- Price discovery - what is the theoretical price of any listed maturity at any point in time?
- Curve extension - what is the theoretical price of unlisted longer maturities?
- Trading opportunities - spot price arbitrage opportunities quicker

In contrast to Bloomberg, the power fair value curves of Refinitiv include all countries in Europe covering France, Germany, Belgium, Czech Republic, Spain, UK, Hungary, Italy, Netherlands, Poland and Nordic Region. The gas section includes all major European hubs (NBP, ZEE, TTF, THE, PEG, PSV). The fair value assessments are provided for all maturity types comprising of hourly, daily, weekly, monthly, quarterly and yearly maturities for the next 4 years forward for Power and Gas.[8]

Fair Value Methodology

Refinitiv conducted research to determine the most fitting model to apply for shaping the forward curve entailed studying and comparing three different approaches namely:

- Time-series regression analysis
- Stochastic diffusion-based approach
- Market trader practice

The conscious choice was to use a right blend of statistics that is time-series regression analysis and market trader practice. Whilst the market trader practice reflects the market and is simple, complementary and offers flexibility the market expects; the time-series regression analysis helps identify statistically the parameters and assumptions to be used within the model. Engaging with some customers early on during the research helped Refinitiv verify using market trader practice as the favored approach. By using this approach Refinitiv provides an indicative fair value price while it also lets the customers make their substitutions and adjustments. [8]

The key concept of the quantitative approach chosen is to shape the curves around real-time market data. Thus the primary source for computing the fair values are the real-time prices from Thomson Reuters / Refinitiv Power Composite (TRPC). Although TRPC is the prime source, in instances where the gap in the available OTC prices is too big, the shape of the exchange futures curve is used to add on a few more data points to the input.
Historical prices (settlement) from OTC (forwards) and exchange (spot fixings) are used to identify the price patterns, and seasonality effects, for computing ratios and for shaping the curves. The period and size of the historical prices to consider for the analysis is also an important parameter by itself. Observation of the historical prices also helps determine how the price of a quarter is a statistically distributed (ratio) between the months of that quarter. [8]

Product Ratios

The foremost step in the quantitative approach is pulling together the historical prices from both OTC and exchanges. The size and period of the historical prices are critical for determining the best fit calibration. The historical prices are observed for the different maturity periods – years, quarters, months, weeks and days. From the historical price analysis the shaping factor ratio for each maturity type is methodically determined. However, the shaping factor ratio is not determined exclusively from the historical prices. Recent history like yesterday's close or the close of the benchmark curve representing the current market is also taken into consideration for deriving the ratios.

The price patterns of a maturity type across different intervals are also analysed (year/quarters and quarter/months). The ratios are computed by taking into consideration the number of days in a month and reaggregating them and adjusting the weight, similar to the bootstrapping mentioned in the Bloomberg fair value section. Refinitiv has a proprietary dynamic method for the distribution of weights between the historical ratio and the current ratio. Indicators such as volatility and relevance of the market prices allow adjusting the weight distributed to the ratios. From the shaping factor ratios it can be determined how the prices of one maturity period are statistically distributed among the other maturities within that period. An example is presented in the following equation:

$$P_{Q1} = r_1 P_{Jan} + r_2 P_{Feb} + r_3 P_{Mar} \tag{2}$$

r_1, r_2, r_3 represent the quarter/month ratios for the first quarter of a year and P indicates the prices of the different contracts. [8]

Non-Arbitrage Condition

One most important consideration that needs to be taken into account while determining the maturity split is the non-arbitrage or the arbitrage-free condition. The non-arbitrage condition is defined as follows:

- A period cannot be split into sub-periods and generate arbitrage.
- If P is the price for a period and P_i the price of each sub-periods of period P, then P must be equal to the sum of $(P_i * s_i)$ for all $i = 1$ to n. s_i represent the respective shares of the sub-periods at the total time window of the contract. Subsequently, the sum of s_i must be 1.

The objective of Refinitiv is to respect the arbitrage-free condition at all times. However, during periods when the market is not liquid enough, adherence to the arbitrage-free condition may not be possible. During such periods, a technique called the cap-and-floor is used to adjust the price of a period which is not as recent as the prices of the other periods. The price adjustment is limited by the cap and floor technique to ensure the price is within a range of what it was prior to the adjustment. For example: If Jan, Feb and Mar are the periods constituting a quarter maturity and the Mar price is not as recent as the Jan and Feb price; then the Mar price is adjusted to a limit using the cap-and-floor technique.
Therefore there is a constant co-movement of the prices capped-and-floored to a limit during intraday when there is no liquidity. In general, settlement prices are arbitrage free but intraday prices cannot be guaranteed to be arbitrage-free for a certain time period before they become arbitrage free.[8]

Fair Value Relationships

Relevant real-time prices are sourced from TRPC. The fair value curves are as real-time as the TRPC curves are. The maximum delay observed is 5 minutes. In instances where the gap in the available OTC prices is too big, the shape of the exchange futures curve, not the actual prices, is used to add on a few more data points to the input. This is done more often than not for shaping the far end of the curve rather than the front of the curve.
In addition to the TRPC (and exchange) prices, there are also other inputs for price discovery. Spread data (e.g. location spread) is used as an input where ever it is favorable for liquidity. For instance, as France and other Continental European countries like Germany are closely correlated in electricity prices, the inter-country spreads are used if it brings in more fresh prices than the actual TRPC prices. Likewise, the correlation between the Baseload price and Peakload price for a country allows using both Base and Peak prices as an input. Live order from broker platforms that is the best bid and ask during the day also forms a part of the input for price discovery. When there is a pertinent fresh pair of bid and ask and while the latest trade price is not current, then the bid/ask pair is a better reflection of the market. A filtering mechanism is applied to filter out the bid/ask that are not in range. Nevertheless, the current latest trade price always takes precedence over the orders, so that the curve fits to the actual trade rather than the orders.[8]

Bootstrapping

Once all the available real-time prices are aligned, the gaps in the prices that need to be filled are highlighted. An aggregation method like simple bootstrapping is applied and also by using the shaping factor ratios established by the models, the fair value prices are derived. Bootstrap left and bootstrap right refer to techniques used to derive the price for a maturity when the prices for other immediate maturities are available. Hence at any point in time the liquidity of the available maturities is used to derive the immediate surrounding maturities by constant dispatch of the information across the maturities. At all times during bootstrapping, the non-arbitrage condition is respected.[8]

Critical Evaluation

- The data provider offers a wide range of different maturities within their fair value section. Even days and hourly products can be retrieved. For this project the focus lies on the longer-term maturities.

- Refinitiv presents a dynamic weighting of historic and current data for choosing the optimal year/quarter or quarter/month price ratio. The volatility and the relevance of the new trade information determine the exact choice of the applied ratio and are interesting thoughts for the development of the model.

- According to the data provider, the current latest trade price always has priority over the order book information. This translates into a statement that trades generally contain more information than the current order book.

- The company applies a very strict non-arbitrage condition. As Refinitv itself states, this condition is not fulfilled in all market situations in real-time and thus must be critically questioned from a modelling perspective.

- In contrast to Bloomberg, Refinitiv introduces a cross-country relationship of the fair values (e.g. power in France and Germany) but still misses the cross-commodity aspects when updating fair value curves.

- Refinitiv offers no fair value curve for traded CO_2 certificates, which can be detrimental in volatile CO_2 certificate price environments.

- As Refinitiv states, a delay of up to 5 minutes is possible in their fair value calculation methodology which can be disadvantageous in hectic market situations.

3 Main Objectives of the Thesis

3.1 Modelling Goals

The presented fair value models use interesting concepts and approaches, but they also combine some severe weaknesses, which will be addressed in the model development stage of this project:

- The fair value model should address the uncertainty in estimating fair prices. In contrast to the presented external models, a fair value probability density distribution should be modeled. Point predictions for fair values are not sufficient for the requirements in a trading environment.
- The modelling of cross-commodity effects is crucial for increasing the predictive power of the fair value model.
- The arbitrage-free condition should not be enforced.
- The model should be able to handle all typically traded products in commodity trading houses (power, gas, coal, oil, CO2 certificates) and offer the flexibility to add new commodities or additional maturities at any time.

To pick up these points a fair value model is chosen that models the different trading products as a Bayesian network with underlying probability distributions. The construct should process new information (trades, offers) in real-time and update the corresponding probability distributions using Bayes' theorem. As time progresses without new market information, diffusion processes should also be modelled. These diffusion processes lead to the fact that without new information, the probability distributions widen based on the "random walk" principle. When a new trade is being processed, the information affects the network. Starting point of this network propagation mechanism is the product, that has traded last. A major focus of the project will be on modelling the different types of connections between products of the network. Connections of the type "maturity" for edges within a product category (e.g.quarters and years in Germany) and "correlation" for edges connecting different commodities (e.g. calendar product in gas and electricity in Germany) or markets (Q2 power in Germany and France) can be distinguished. The overall network concept can also be seen in Figure 7.

The main goal of the project is to create a working prototype for a real-time deployment in energy trading and thus achieve a successful proof-of-concept of the applied method. The model should be compared against simple reference cases. The discussed fair value models from Bloomberg and Refinitv serve as benchmarks for the developed model.

The main functionalities of the developed algorithms should be found in one single library, that can be integrated and used from different applications. In addition, a simple graphical user interface (GUI) should be developed to track model performance also visually.

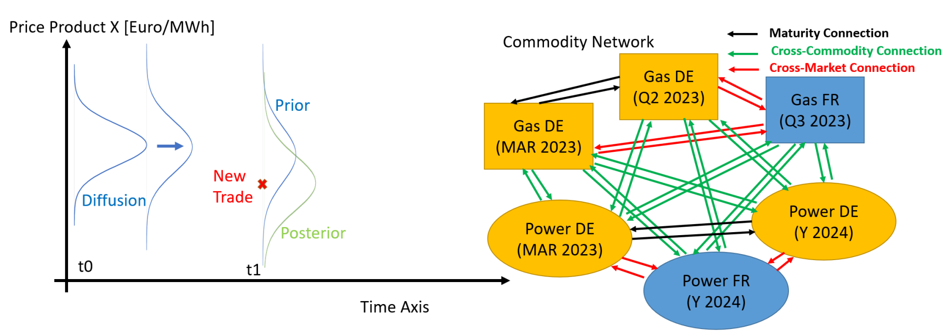

Figure 7: Commodity Price Fair Value Model Concept Sketch

3.2 Network Commodities

For developing and testing the fair value mode the following 199 trading products were used. The distribution among different commodities can be seen in Table 1. To avoid unnecessary computational effort, products that have no liquidity and thus cannot be traded anyway were excluded from the network.

Commodity	Markets	Profiles	Maturities	Number of Products
Power	DE FR CH IT	Baseload Peak Offpeak	front months (6) front quarters (5) front years (3)	168
Gas	TTF	Baseload	front months (6) front quarters (5) front years (3)	14
Coal	API2	Baseload	front months (6) front quarters (5) front years (3)	14
CO2 Certificate	EUA	Baseload	front years (3)	3

Table 1: Simulated and Evaluated Trading Products

The selected products cover the liquid trading horizon of three years and overlap on the time axis as it can be seen in Figure 8.

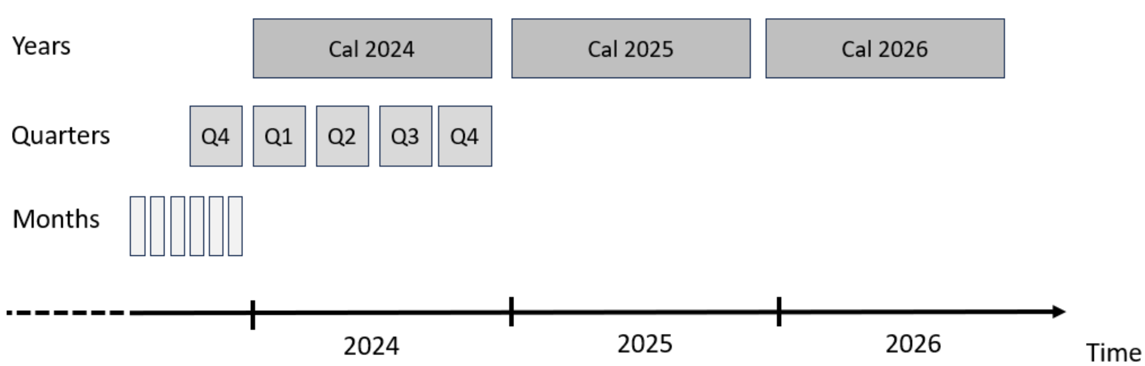

Figure 8: Time Axis of Activated Commodities for Model Development

Around 15'000 trades correspond to an average trading day of the selected trading product group.

4 Theoretical Background

4.1 Correlation Coefficients

An essential part of the modelling of cross-commodity price interactions are correlation coefficients. Correlation coefficients can be determined based on different data sets.

End-of-Day Settlement Data

This data is more easily available and more straightforward to work with. End-of-day settlements are best suited for modelling long-term trends and relationships and do not address intraday volatility. The correlation can be computed by using the Pearson correlation coefficient, which measures the linear relationship between two datasets or the Spearman's rank correlation coefficient.
The Pearson correlation coefficient is typically used when both variables being studied are normally distributed. This coefficient is affected by extreme values. [10] The formula of the Pearson coefficient is presented in the following equation

$$r_{Pearson} = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^{n}(x_i - \bar{x})^2 \sum_{i=1}^{n}(y_i - \bar{y})^2}} \tag{3}$$

where

- x_i and y_i are the individual sample points indexed with i
- $\bar{x}$ and $\bar{y}$ are the means of the x and y data sets
- n is the number of point in each data set

In comparison, Spearman's rank coefficient, which is robust against extreme values, is given by the following formula

$$r_{Spearman} = 1 - \frac{6\sum d_i^2}{n(n^2-1)} \tag{4}$$

where d_i is the rank difference of each observation
There exists a wide range of further interesting correlation coefficients (e.g. distance correlations), but they will not be discussed further in the context of this project.

Asynchronous Trade Data

Asynchronous trade data is more granular and reflects real-time changes in the market, so it may provide a more accurate picture of the correlation if short-term dynamics are of interest. The data set is especially useful, if the commodities in question have markets that are open at different times, as this can create a kind of "lag" in the correlation that won't be picked up if you're just looking at end-of-day prices. To work with this kind of data, more sophisticated statistical methods (e.g., time-series analysis, vector autoregression) are required to deal with the asynchronicity.

4.2 Probability Density Functions

Since the project wants to model probability distributions around fair values, probability density functions play a central role.

4.2.1 Normal Distribution and Gaussian Function

Because of its many very advantageous mathematical properties related to Bayes' theorem or the log-likelihood function, the normal distribution function is used in this project framework to model commodity prices. The distribution can be described with two parameters: μ the mean of the distribution and σ which can be interpreted as the standard deviation.

A normally distributed variable can be expressed as part of the Gaussian function family that is defined in the following form:

$$f(x, a, b, c) = a * exp(-\frac{(x-b)^2}{2c^2}) \tag{5}$$

When this general form is used for representing the probability density function of a normally distributed random variable with expected value $\mu = b$ and variance $\sigma^2 = c^2$. In this case, the Gaussian is of the form:

$$f(x, \mu, \sigma) = \frac{1}{\sigma\sqrt{2\Pi}} * exp(-\frac{1}{2}\frac{(x-\mu)^2}{\sigma^2}) \tag{6}$$

4.2.2 Log-Likelihood

From the current probability density function of a trade product, it is possible to evaluate the likelihood - i.e. the value of the probability density function - for each trade. A good fair value model should always adjust so that the incoming trades total the highest possible likelihood values.

In mathematics, however, log-likelihood is often used instead of likelihood. The main reasons for this are numerical stability, the simplification of exponential terms, the possibility of working with sums instead of products and the avoidance of an overflow with very high likelihoods.

4.2.3 Potential Probability Distribution Alternatives

Besides the normal distribution there exist several other distributions that can be interesting in financial topics. Some of them are listed below:

- <u>Log-normal distribution</u>
  Commodity or stock market prices are typically rather log-normal than normal distributed. One main reason for this phenomenon is the general absence of negative prices. The suggestion to use log-normal distributions can also be found in the literature. [13] However, since the developed model does not aim to make any long-term predictions and permanently receives new information, the choice of the normal distribution is nevertheless justifiable, given clear mathematical and numerical advantages.

- <u>Gamma distribution</u>
  One advantage of the gamma distribution is its flexibility in terms of possible shapes. The function is only defined for positive values which makes the function suitable for modelling prices, rates or durations. Besides interesting aggregation possibilities. Similar to the normal distribution, the gamma distribution is the conjugate prior for several likelihood distributions. [9]

- <u>t-distribution</u>
  Another distribution that has a similar structure as the normal distribution but heavier tails.

- <u>Bi-modal function</u>
  So far, only one-modal distributions have been discussed. In a trading context, it can be doubted that the fair value of a commodity can be represented by a one-modal distribution. Trades are typically executed at the best bid or ask level, with a relatively large spread in between. In this

context, a bi-modal distribution might even be a better approximation. Literature covers for example option pricing with a local bi-modal distribution of the underlying. [6]

4.3 Bayes Theorem

Bayes' theorem is a fundamental principle in the field of statistics and probability. It describes the relationship of conditional probabilities of statistical quantities. In simple terms, Bayes' theorem provides a way to update prediction probabilities given new data.

The theorem states that the conditional probability of an event A given B is equal to the conditional probability of B given A times the probability of A, divided by the probability of B. In other words, it allows to update our initial beliefs (the prior probabilities) based on new evidence to obtain refined beliefs (the posterior probabilities). The following mathematical expression can be stated

$$P(A|B) = \frac{P(B|A) \cdot P(A)}{P(B)} \tag{7}$$

where

- $P(A|B)$ is the posterior probability of event A given event B has occurred
- $P(B|A)$ is the likelihood, the probability of event B given A has occurred
- P(A) is the prior probability of event A, the probability of event A before any specific information about event B is known
- P(B) is the evidence, the total probability of event B

[2]

The Bayes theorem is particularly important in Bayesian inference, a branch of statistics that deals with updating probabilities as more data or information becomes available and therefore a core component of this project. The developed model uses new information (e.g. trades) to update the prior distributions of the fair value of a commodity. In Subsection 5.2.1 elegant and computationally efficient ways will be presented, how to update our normal distributions based on this theorem.

4.4 Diffusion in Financial Markets

Diffusion in its initial form is a naturally occurring physical movement process due to the inherent motion of the particles involved. Over time, it leads to the complete mixing of two or more substances and thus increases the entropy of the system. [3] A similar concept is often used in financial market models: Without new information the uncertainty arises due to the potential stochastic behaviour of the market prices. This phenomenon is often referred to as the "random walk" principle or Brownian motion in a more physical context. In the Brownian motion context one often starts at a specific point (e.g. price) and models the potential future variance after a specified time interval t. Einstein stated the following relationship [3]

$$\sigma^2_{Brownian} = 2Dt \tag{8}$$

where

- D is the diffusion constant
- t is the elapsed time

The main takeaway from the equation above is the linear relationship between variance and time. Applying the Brownian motion findings to our modelling context where commodity prices are being modeled as normal distributions. The following relationship is introduced to account for the rising uncertainty in financial markets without new information (e.g. trades):

$$\sigma_{Product}(t + \Delta t) = \sigma_{Product}(t) + D\sqrt{\Delta t} \tag{9}$$

This is a modelling assumption and cannot be directly mathematically derived. However, speaking in order of magnitudes the relationship between the variance of the modeled trading products and the time elapsed is linear. In this project the diffusion rate D is assumed to be constant throughout the day. The diffusion can be re-calibrated overnight when the commodity price network is not active due to closed exchange platforms.

Approaches to Estimate the Diffusion Constant

Various approaches can be used to estimate the diffusion constant for the individual commodity prices modelled. The following list is intended to provide an overview:

- **Use historical market data**
  - **Standard deviation of "returns" over a specified period of time**(unconditional variance)
    - Close-to-close prices
    - Close-to-open prices (interesting, since no trades overnight, pure diffusion)
  - **(R)GARCH model (generalized retrogressive conditional heteroscedasticity)** (conditional variance) [15]
    - Maximum likelihood estimation for model parameters
      - Nelder Mead Simplex algorithm
      - Broyden-Fletcher-Goldfarb-Shanno (BFGS) algorithm
    - Bayesian estimation of model parameters / Bayesian interference
    - Gradient-based parameter optimization
  - **Applying sequence of Bayes updates and diffusion phases and determine optimal diffusion constant** (closed-form solution or in numerical iterative way)
- **Using option's implied volatility** [7]
- **Stochastic differential equations**

Although implied option volatilities would be exciting because they involve the market's expectation of future movements, this approach is not applicable here. The required option market data is not available within this project setup. In addition, stochastic differential equations are computationally too expensive to be applied in such a big data real-time context. This leaves only the possibility of estimating the diffusion constant via historical market data. Within the framework of this project, the focus is on using the Bayes-diffusion process chain.

In this context, the first step will be to attempt to determine a mathematically closed solution for the optimal diffusion constant D. If this is not possible, the optimal diffusion constant will be determined in a numerical iterative way. This approach guarantees that the resulting diffusion constant D fits excellently with the other modelling blocks.

RGARCH model as described by Hamilton [5] would be another promising alternative for estimating the diffusion constant from historical market data, but will be neglected within this Master thesis.

4.5 Reinforcement Learning

A promising approach for updating the commodity network when new information about a single product is available and the correlation coefficients between products are known, is to use a reinforcement learning approach where the system learns for itself the relationships between the commodities, such that the overall model performance is maximized. Reinforcement learning system variants exist for discrete and continuous action spaces. In this project, a discrete action space method will be used, but some potential future improvement alternatives from the continuous action space are listed in Subsubsection 4.5.2.

4.5.1 Q-Learning Algorithm

An early breakthrough in reinforcement learning was the development of an off-policy control algorithm known as Q-learning. In this case, the learned action-value function,Q, directly approximates q_*, the optimal-value function, independent of the policy being followed. Q-Learning is a model-free reinforcement learning algorithm that tries to find the optimal action-selection policy using a value-based method. The value of an action in a particular state is learned by executing a particular action, receiving a positive or negative reward, and updating a Q-value table using the Bellman equation. Over time and through exploration and exploitation, it converges to the optimal policy that guides the agent to the action yielding the maximum cumulative reward for each state. [14]

Q-learning (off-policy TD control) for estimating $\pi \approx \pi_*$

Algorithm parameters: step size $\alpha \in (0, 1]$, small $\varepsilon > 0$
Initialize $Q(s, a)$, for all $s \in \mathcal{S}^+, a \in \mathcal{A}(s)$, arbitrarily except that $Q(\textit{terminal}, \cdot) = 0$

Loop for each episode:
    Initialize S
    Loop for each step of episode:
        Choose A from S using policy derived from Q (e.g., ε-greedy)
        Take action A, observe R, S'
        $Q(S, A) \leftarrow Q(S, A) + \alpha\big[R + \gamma \max_a Q(S', a) - Q(S, A)\big]$
        $S \leftarrow S'$
    until S is terminal

Figure 9: Q-learning algorithm principle [14]

In Figure 9 the relevant terms can be defined as follows:

- α is the learning rate
- $Q(S, A)$ is the action-value function for a state S and an action A
- γ can be interpreted as the discount factor, which reduces returns in the future
- R is the reward of the chosen action

ϵ-Greedy-Algorithm

Reinforcement learning algorithms are always in an exploration-exploitation dilemma. The algorithm should exploit the maximum rewards by performing the current best action but on the other hand, the code should explore potentially even more profitable options. One way to deal with this conflict of interest, is applying the ϵ-greedy method. The ϵ-greedy methods choose randomly a small fraction of the time. [14] A Q-learning algorithm can conveniently be combined with an ϵ-greedy approach to define the next action.

4.5.2 Alternatives with Continuous Action Space

This paragraph provides potentially interesting continuous action space reinforcement learning system algorithms:

- Deep Deterministic Policy Gradient (DDPG) is an off-policy algorithm that uses deep neural networks.
- Proximal Policy Optimization (PPO) is an off-policy method that has become popular due to its effectiveness and efficiency.
- Soft Actor-Critic (SAC) is another off-policy algorithm that seeks to optimize a trade-off between maximizing reward and entropy, which encourages exploration.

Thanks to their greater action sets and memories, these types of models could be tested for further model improvement in the future.

5 Methodology

5.1 Fair Value Models

This model section describes the used distribution functions for the different parts of the model.

5.1.1 Commodity Fair Value Model

The actual fair values of the individual trading products are modelled by using a normal distribution assumption. Therefore the two parameters μ and σ are sufficient to describe the fair value of trading product at any time.

5.1.2 New Information Fair Value Model

Here, the assumptions for the distribution that is used for the Bayes update is described. Since new information is made available to the model, the subscripts "newinfo" are used. To have a consistent modelling framework normal distributions are used as well for this part.

For the Bayes model input price $\mu_{newinfo}$ several options have been developed:

- using directly the trade price p_t
- using the mid-price between best bid and ask from the order book p_{mid}
- using a combination of trade price p_t and a weighted bid-ask price based on the probabilities of the next trade being on the bid or ask

Since the actual trade price should not be completely ignored, the second proposal is not a valid option. Due to limited time resources during this proof-of-concept phase, only the first option has been used so far in the test runs. But a more detailed discussion of a possible implementation of the third suggestion using logistic regression is discussed in Subsection 5.5.
Similar to the modelling options for $\mu_{newinfo}$, concepts for modelling the uncertainty, or more precisely the standard deviation of the new information $\sigma_{newinfo}$, have been prepared:

- use historical settlement data to derive a fair estimate of a product's volatility σ_0
- a more sophisticated approach is based on the current bid-ask spread which is normalized by a typical bid-ask spread and then multiplied by the previously described historical volatility σ_0

For this proof-of-concept prototype, the simple first option was used. More details about the second approach can as well be found in Subsection 5.5.

5.2 Bayes Update Model

In Subsection 5.1 a model for the normal distribution characterizing the new trade and order book information at the moment of a new trade has been presented. The approximately known volatility and mean of the new information simplify the mathematical complexity for the following Bayes update significantly. This update mechanism is described in the following paragraphs.

5.2.1 Chosen Bayes Approach

If the volatility is assumed to be known, it is possible to calculate mean and standard deviation of the posterior normal distribution as follows in closed-form [11]:

- σ_n is the posterior standard deviation
- $\sigma_{initial}$ is the standard deviation of the prior
- $\sigma_{newinfo}$ is the standard deviation of the observation points (in this context: the new trade / order book information).
- n is the number of Bayes updates that are being performed
- $\mu_{newinfo}$ corresponds to the mean of the observed new data points, also referred to as $\bar{x}$

The posterior σ_n can be calculated as follows:

$$\sigma_n^2 = \frac{\sigma_{newinfo}^2 \sigma_{initial}^2}{n\sigma_{initial}^2 + \sigma_{newinfo}^2} = \frac{1}{\frac{n}{\sigma_{newinfo}^2} + \frac{1}{\sigma_{initial}^2}} \tag{10}$$

The corresponding posterior μ_n is given by the following formula:

$$\mu_n = \sigma_n^2 (\frac{\mu_{initial}}{\sigma_{initial}^2} + \frac{n\overline{x}}{\sigma_{newinfo}^2}) \tag{11}$$

If in other cases, volatility cannot be assumed to be known, literature suggests using a normal-gamma distribution function for a Bayes update. There exists a closed-form solution as well. [11]

5.2.2 Implicit Volume Dependency of the Observations Variance

Another interesting modelling aspect is the question of trade volumes and their effect on the Bayes update. Performing one "big size" trade of volume v [MW] should have the same effect as performing v individual trades with volume 1 [MW]. This condition is automatically fulfilled when applying the closed-form solution Bayes updates, but the question is: What is the resulting standard deviation ratio depending on the traded volume v? What implicit observation variance volume dependency is assumed within the model?

The relationship can be derived mathematically by equating the Bayes update formulae for the variances in both cases.

$$\sigma_v^2 = \frac{1}{\frac{v}{\sigma_{SmallTrades}^2} + \frac{1}{\sigma_{initial}^2}} = \frac{1}{\frac{1}{\sigma_{BigTrade}^2} + \frac{1}{\sigma_{initial}^2}} \tag{12}$$

$$\frac{v}{\sigma_{SmallTrades}^2} + \frac{1}{\sigma_{initial}^2} = \frac{1}{\sigma_{BigTrade}^2} + \frac{1}{\sigma_{initial}^2} \tag{13}$$

$$\sigma_{BigTrade} = +/- \frac{\sigma_{SmallTrades}}{\sqrt{v}} \tag{14}$$

Unsurprisingly, the implied standard deviation for a high-volume trade (relative to a minimum volume trade of 1 MW) is smaller. A high-volume trade contains more information and should produce a stronger effect on the Bayes update. The relationship between the two standard deviations is described by a square-root of volume factor.

5.3 Diffusion Constant Model

The main scope of this section is to determine an optimal diffusion constant for each traded product using an iterative Bayesian diffusion update process chain. In the first subsection, the effort to find a mathematical closed-form solution for the optimal diffusion constant is presented. In the following section, the results of the iterative solution procedure are presented.

5.3.1 Mathematical Closed-Form Solution for Bayes-Diffusion-Chain

The most important abbreviations for the presented mathematical solutions are summarized below:

- D optimal diffusion constant that maximizes the likelihood for the Bayes theorem update and diffusion process chain
- p_i trade execution price, ith element in chronological list
- μ_i mean of normal distribution before ith trade information Bayes update is performed
- σ_i standard deviation of normal distribution before the ith trade information Bayes update is performed
- n is the total number of trades available to determine the optimal diffusion constant
- Δt_i is the time interval between two subsequent trades

The general solution is to maximise the loglikelihoods over all trades in the training data set. The total loglikelihood of the Bayesian diffusion process chain over the n-elements of the data set has to be expressed as a function of diffusion constant D. Then, in a further step, the function found can be derived according to D and the expression can be equated with 0 to determine a maximum. Since the problem is mathematically complex, an attempt is first made to determine an optimal diffusion constant between two individual trades. Afterwards, an attempt is made to determine the optimal diffusion constant for the entire process chain.

Optimal Diffusion Constant for one Diffusion Step between two Trades

The normal distribution function of a particular commodity has in our case the following form (an infinitesimal moment before the Bayesian update of trade i):

$$f(p_i, \mu_i, \sigma_i) = \frac{exp(-\frac{1}{2}(\frac{p_i-\mu_i}{\sigma_i})^2)}{\sigma_i\sqrt{2\pi}} \tag{15}$$

Since the logarithm function itself is a continuous strictly increasing function over the range of the likelihood, the values which maximize the likelihood will also maximize its logarithm (the log-likelihood itself is not necessarily strictly increasing).

$$\mathcal{L}(p_i, \mu_i, \sigma_i) = \prod_{t=1}^{n} f(p_i, \mu_i, \sigma_i) \tag{16}$$

$$\ln(\mathcal{L}(p_i, \mu_i, \sigma_i)) = -\frac{n}{2}\ln(2\pi\sigma_i^2) - \frac{1}{2}\sum_{t=1}^{n}\frac{(p_t-\mu_i)^2}{\sigma_i^2} \tag{17}$$

Typically just one new trade information is available at a specific time i, but a trade of size $n[MW]$ is modelled as n trades with identical price and standard deviation and size $1[MW]$. Therefore the the sum symbol can be eliminated.

$$\ln(\mathcal{L}(p_i, \mu_i, \sigma_i)) = -\frac{n}{2}\ln(2\pi\sigma_i^2) - \frac{n}{2\sigma_i^2}(p_i-\mu_i)^2 \tag{18}$$

In a next step, the log-likelihood at the moment of a new trade is evaluated (just before the moment of the next Bayesian update). So it can assumed $\mu_i = \mu_{i-1}$ (not affected by diffusion process) and $\sigma_i = \sigma_{i-1} + D\sqrt{\Delta t_i}$ (diffusion acting on standard deviation)

$$\ln(\mathcal{L}(p_i,\mu_{i-1},\sigma_{i-1},D)) = -\frac{n}{2}\ln(2\pi(\sigma_{i-1}+D\sqrt{\Delta t_i})^2) - \frac{n}{2(\sigma_{i-1}+D\sqrt{\Delta t_i})^2}(p_i-\mu_{i-1})^2 \tag{19}$$

$$\ln(\mathcal{L}(p_i,\mu_{i-1},\sigma_{i-1},D)) = term1 - term2 \tag{20}$$

$$term1 = -\frac{n}{2}\ln(2\pi(\sigma_{i-1}+D\sqrt{\Delta t_i})^2) = -\frac{n}{2}\ln(2\pi(\sigma_{i-1}^2+2\sigma_{i-1}D\sqrt{\Delta t_i}+D^2\Delta t_i)) \tag{21}$$

$$term1 = -\frac{n}{2}\ln(2\pi\sigma_{i-1}^2+4\pi\sigma_{i-1}D\sqrt{\Delta t_i}+2\pi D^2\Delta t_i)) \tag{22}$$

$$term2 = \frac{n}{2}\frac{1}{(\sigma_{i-1}+D\sqrt{\Delta t_i})^2}(p_i-\mu_{i-1})^2 \tag{23}$$

Now, the derivative with respect to the diffusion constant D has to be taken:

$$\frac{dterm1}{dD} = -\frac{n}{4}\frac{1}{(\pi\sigma_{i-1}^2+2\pi\sigma_{i-1}D\sqrt{\Delta t_i}+\pi D^2\Delta t_i)}(4\pi\sigma_{i-1}\sqrt{\Delta t_i}+4\pi D\Delta t_i) \tag{24}$$

$$\frac{dterm1}{dD} = -n\frac{(\sigma_{i-1}\sqrt{\Delta t_i}+D\Delta t_i)}{(\sigma_{i-1}^2+2\sigma_{i-1}D\sqrt{\Delta t_i}+D^2\Delta t_i)} = -n\frac{(\sigma_{i-1}\sqrt{\Delta t_i}+D\Delta t_i)}{(\sigma_{i-1}+D\sqrt{\Delta t_i})^2} \tag{25}$$

$$\frac{dterm2}{dD} = \frac{-2n}{2}\frac{(p_i-\mu_{i-1})^2}{(\sigma_{i-1}+D\sqrt{\Delta t_i})^3}(\sqrt{\Delta t_i}) = -n\frac{(p_i-\mu_{i-1})^2}{(\sigma_{i-1}+D\sqrt{\Delta t_i})^3}(\sqrt{\Delta t_i}) \tag{26}$$

$$\frac{d\ln(\mathcal{L}(D))}{dD} = \frac{dterm1}{dD} - \frac{dterm2}{dD} \tag{27}$$

$$\frac{d\ln(\mathcal{L}(D))}{dD} = -\frac{n(\sigma_{i-1}\sqrt{\Delta t_i}+D\Delta t_i)}{(\sigma_{i-1}+D\sqrt{\Delta t_i})^2} + \frac{n(p_i-\mu_{i-1})^2}{(\sigma_{i-1}+D\sqrt{\Delta t_i})^3}(\sqrt{\Delta t_i}) \tag{28}$$

$$\frac{d\ln(\mathcal{L}(D))}{dD} = n(\frac{-(\sigma_{i-1}\sqrt{\Delta t_i}+D\Delta t_i)(\sigma_{i-1}+D\sqrt{\Delta t_i})+(p_i-\mu_{i-1})^2\sqrt{\Delta t_i}}{(\sigma_{i-1}+D\sqrt{\Delta t_i})^3}) \tag{29}$$

$$\frac{d\ln(\mathcal{L}(D))}{dD} \overset{!}{=} 0 \tag{30}$$

Expression is zero when the nominator is zero. Since n is strictly bigger than zero, it is not affecting the optimal diffusion constant for this particular update step.

$$0 = -(\sigma_{i-1}\sqrt{\Delta t_i}+D\Delta t_i)(\sigma_{i-1}+D\sqrt{\Delta t_i})+(p_i-\mu_{i-1})^2\sqrt{\Delta t_i} \tag{31}$$

$$0 = -\sqrt{\Delta t_i}(\sigma_{i-1}+D\sqrt{\Delta t_i})^2+(p_i-\mu_{i-1})^2\sqrt{\Delta t_i} \tag{32}$$

$$0 = -(\sigma_{i-1}+D\sqrt{\Delta t_i})^2+(p_i-\mu_{i-1})^2 \tag{33}$$

$$(\sigma_{i-1}+D\sqrt{\Delta t_i})^2 = (p_i-\mu_{i-1})^2 \tag{34}$$

$$(\sigma_{i-1}+D\sqrt{\Delta t_i}) = \sqrt{(p_i-\mu_{i-1})^2} \tag{35}$$

$$D\sqrt{\Delta t_i} = \sqrt{(p_i-\mu_{i-1})^2} - \sigma_{i-1} \tag{36}$$

$$D = \frac{(p_i-\mu_{i-1})-\sigma_{i-1}}{\sqrt{\Delta t_i}} \tag{37}$$

D needs to be greater or equal than zero. This is the optimal diffusion constant D for the time period between t_i and t_{i-1} before a next Bayes update is being performed. Since square root operations were performed, solutions with switched signs also exist.

Optimal Diffusion Constant for all Time Steps / Trades

Unfortunately, it has not been possible within the framework of this project to specify a closed solution for this case. The Bayes update formulas result in complex nested terms with dependencies within a sum symbol, which could not be resolved. The optimal diffusion constant resulting from such a Bayesian diffusion process chain must therefore be determined numerically.

5.3.2 Iterative Solution

In a next step, the diffusion constants for the trading products are determined numerically via an iterative procedure. The following steps are carried out:

- Initial μ and σ of the normal distribution function of the trading product are being determined by using settlement information of the last 30 days.
- The potential numerical range of the diffusion constant is being set (1e-17 up to 100).
- The initial increment step for D is equal 1e-18. The value is increased by a factor of 10 as soon as the simulated diffusion constant increases by a power of ten. This trick reduces computational time by simultaneously applying equal relative incremental steps throughout the possible diffusion constant range.
- Afterwards, there is a complete run through the Bayes-diffusion update chain for all possible diffusion constants within the allowed range. If the resulting total log-likelihood is bigger than the previous maximum, the best diffusion constant to date is stored.

Bayes updates of the normal distributions lead to smaller standard deviations of the modeled commodities. Therefore, higher optimal diffusion constants for the most liquid products can be expected. In addition, highly volatile products, where risk premium is quickly priced in, should also have higher optimal diffusion constants.

The resulting range of optimal diffusion constants is immense: The minimum order of magnitude is 1e-17 up to a maximum value of 7. Based on the reasons mentioned above, it is no surprise at all that four winter gas contracts are at the top of the ranking followed bis French power winter contracts. Winter gas contracts currently combine supply risk fears and a general high trading volume on the commodity gas. French power (especially Peak contracts) are less liquid, but not less sensitive to bad news regarding the availability of the French nuclear reactor fleet.

5.4 Network Propagation Model

This sub-chapter deals with the chosen modelling for the propagation of information after a trade into the entire network. The basis for this model is formed by calculated correlation coefficients, which are discussed first.

5.4.1 Correlation Settings

As discussed in subsection 4.1 the Pearson correlation coefficient is influenced by extreme values. Due to the fact that the most extreme price jumps typically happen on a very short-term time scale (e.g. intraday or balancing markets) and our commodity network only operates with maturities from the front month onwards, the use of the Pearson correlation coefficient is permissible.The model uses a look-back window of 30 days.

In the following paragraph a future improvement for the model is presented: In the standard Pearson correlation, each pair of observations contributes equally to the final correlation coefficient. For the usage of correlations to update the commodity network, a higher weight of the most recent observation points is desired. The exponentially weighted correlation involves applying a set of weights to the observations that decrease exponentially for older data. This can be done by defining a decay factor in the range (0,1), and then each weight for a data point becomes the decay factor raised to the power of the age of the data point.
The steps to compute the exponentially weighted correlation are as follows:

1. Calculation of the differences from the mean for each of the two variables
2. Instead of a direct summation of the products, multiplication by the corresponding weight is performed in the numerator.
3. Similar adjustment of the denominator is necessary as well.

Here the chosen time-weighting function is presented

$$w(i) = 1 - \frac{\lambda^{(n-i)}}{\sum_{j=1}^{n} \lambda^{(n-j)}} \tag{38}$$

where

- λ is a decay factor which satisfies the condition $0 < \lambda < 1$
- n corresponds to the number of days back from now

Figure 10 compares the effects on the daily weights for the correlation when comparing Pearson standard approach with the exponentially weighted variant.

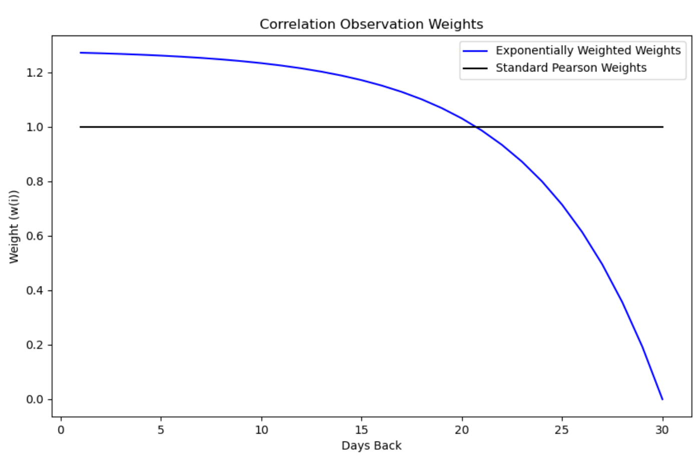

Figure 10: Different Options for Weighting Historic Days in Correlation

5.4.2 Enhanced Reinforcement Learning-Approach

The chosen network update system is mainly based on a Q-learning algorithm, which receives as input the relative price change of the traded product as well as the correlation coefficient between all products in the network. The algorithm can adjust within its defined discrete action range.
The new price μ of commodity x_1 is determined by

$$\mu_{new} = (1 + \Delta p_{rel_{xt}} c_{x_t - x_1} a_j)\mu_{prev} \tag{39}$$

where

- $\Delta p_{rel_{xt}}$ is the relative price change of the product that has last traded, whose effect on other commodities is now being propagated through the network.
- $c_{x_t - x_1}$ is the used correlation coefficient between product x_t and x_i
- a_j is the action chosen by the Q-learning algo.

Q-Learning Algorithm Details

The algorithm uses a learning rate of ten percent, a discount factor of 0, and an exploration rate of 25 percent for the ϵ-Greedy mechanism.
The chosen reward function is equal to the difference between the log-likelihoods of the chosen action and a reference case. The reference case is defined as "status quo" (or equal of applying a $a_j = 0$). Additional penalties can be set, if the resulting prices μ_{new} are outside of the bid-ask spread boundaries. Rewards are given for all future trades in a ten minute window in the training environment. The rewards are using a time-dependent linear decay of their weight inside the described window.

Possibility of Subsequent Bayes Update with Increased σ_{new}

The framework allows the user to activate the possibility to use the determined μ_{new} and perform another Bayes update. The idea behind this is that with new price information somewhere in the network, the non-traded commodities should also have a somewhat smaller uncertainty in their modelled distribution. But one needs to be careful since the optimal diffusion constant is derived without simulating additional network Bayes updates. The resulting variances of the modeled commodities will become more narrow based on the high number of additional updates. There exist two approaches to address this problem:

- use a significantly larger σ_{new} for the Bayes update
- prevent an update of the modeled fair value standard deviation if the current level is already clearly below the current bid-ask-spread

5.5 Overall Model Working Principle

In the previous sections, individual components of the developed model were presented. The following sections and graphics serve to make the processes of the overall model comprehensible to the reader. newline

The model consists of six main components, which can be distinguished. It starts with new trade information and ends with the overall assessment of the model. The reading direction of the model is from left to right. Throughout the project, the model plans were always recorded on Miro boards. Miro boards (`https://miro.com/de/online-whiteboard/`) are very flexible and offer the possibility to easily place open points or questions in the model diagram by means of notes. The entire model cannot be displayed legibly on an A4 page. Therefore, the following Figures 12 to 14 each show a third of the overall model. Figure 11 explains how the graphics actually belong together.

In the "New Information Fair Value Model" section, both potential information paths discussed previously are shown. Either the trade price and the standard deviation determined via historical settlements are used as parameters for the normal distribution of the new information. The other, more complex path is to use the order book data. Using a logistic regression, it is possible to estimate how large the probabilities are that a next trade will be made on the bid or ask side. Thus, the mean of the new information is not the price, but an average between the trade price and the probability-weighted mid-price between bid and ask. In the more complex case, the standard deviation from the settlements is multiplied by a factor that takes into account the current bid-ask spread.

In Figure 13 the detailed update mechanism of the modeled fair value distributions is explained. A first diffusion step is followed by the closed-form Bayes update. $Sigma_fv$ refers to the standard deviation of the modeled distribution, whereas Mu_fv corresponds to the mean. In the sketch Pt (the product that traded) is differentiated among the other Pis, which are products that currently have not seen a trade.

In the final section the effect of the updated fair value distribution of the traded product on the rest of the modeled network is explained. Which trading product pairs are updated after product Pt has traded is shown in Figure 15. The model only considers first-order effects in the network to prevent cyclic graph loops. Similar to the first section, two possible variants exist:

- In the first version, after the $Mu_NewInfo$ for all the commodities in the network has been determined via Q-learning algorithm, this $Mu_NewInfo$ is directly set to be the Mu_fv of the commodities. In this case the $Sigma_fv$ remains untouched. Meaning that a trade in another commodity can change the price expectation of a particular commodity but not influence the uncertainty that is currently observed for that trading product.
- A second variant uses the gained $MU_NewInfo$ to perform another Bayes update for the particular commodity. The $Mu_NewInfo$ is treated like a new artificial trade. Since it is an artificial trade, the effect in the Bayes update may not be the same as in a real trade. Therefore, a scaling factor for the actual standard deviation of the new-info is used in the update. Since the optimal diffusion constant was determined without this network update step, the resulting standard deviations of the modelled distributions must be prevented from becoming extremely narrow. Therefore, a "sigma check" is introduced into the model. The resulting standard deviation must at least correspond to an order of magnitude of the current bid-ask spread of the commodities.

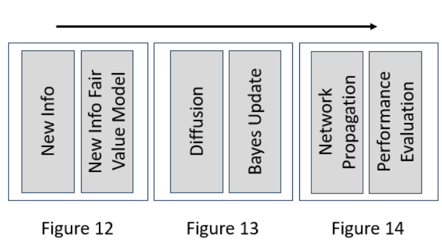

Figure 11: Overall Model - Summary with Reading Direction

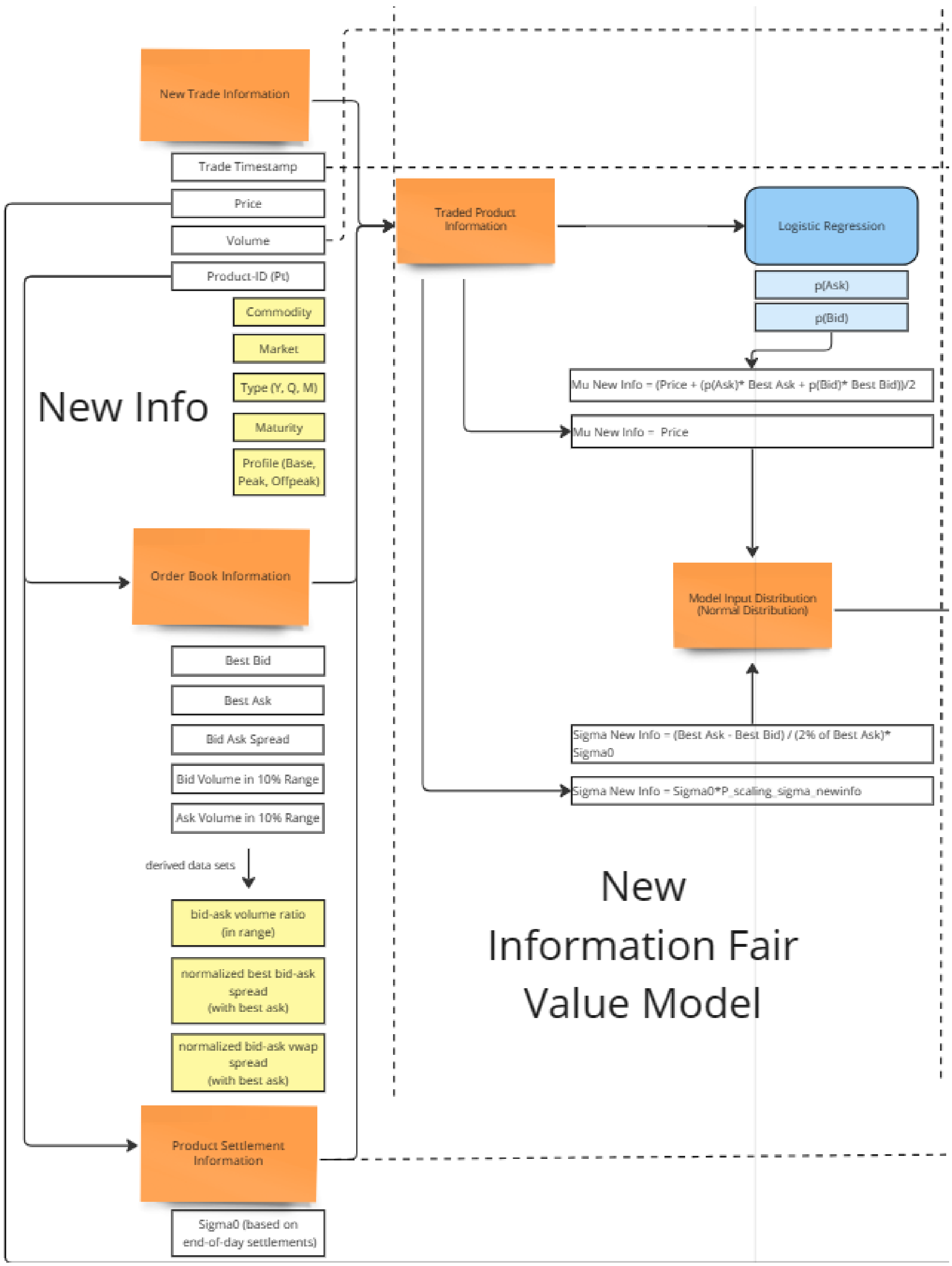

Figure 12: Overall Model Sketch - Part I

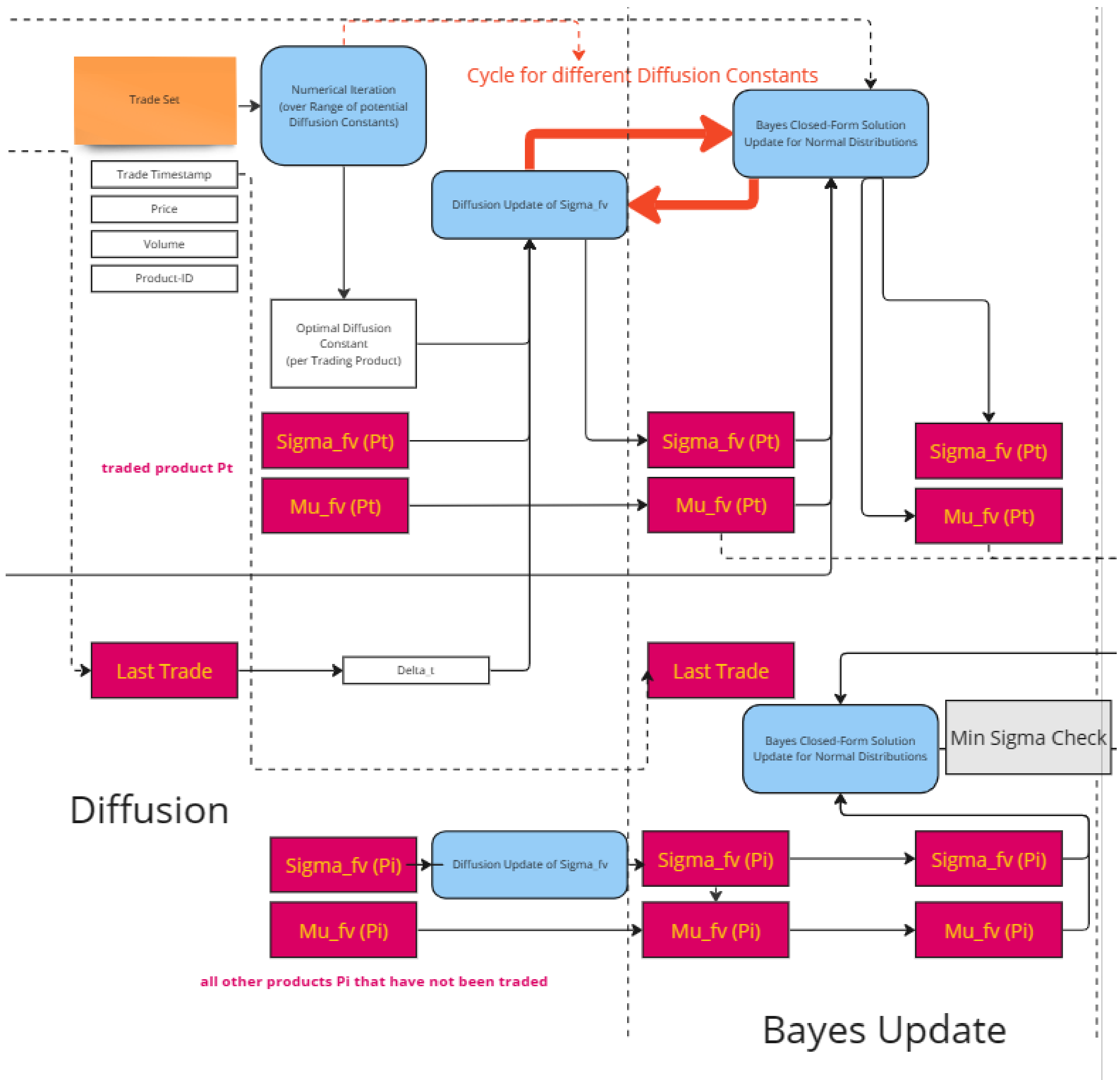

Figure 13: Overall Model Sketch - Part II

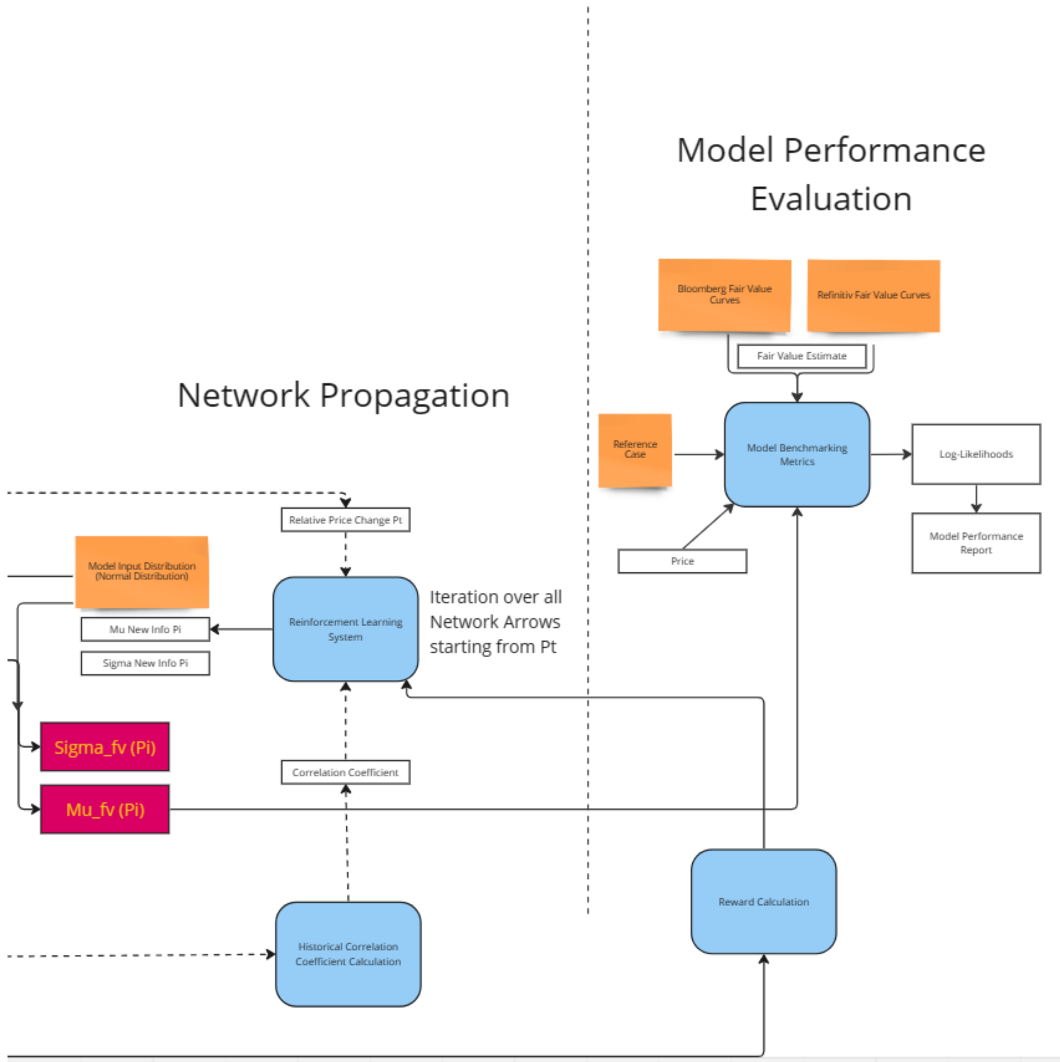

Figure 14: Overall Model Sketch - Part III

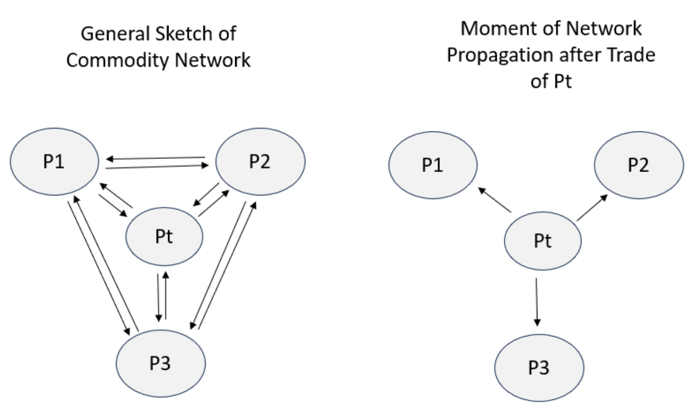

Figure 15: Network at the Moment of Network Propagation

As a final part of the project framework, the settings for the usage in productive environment needs to be discussed. Typical trading hours for the modeled commodities are from 08:00 - 18:00 CET. During these hours, the productive algorithm is active and predicts the fair value distributions at any time. Important to mention here is, that in the productive setup, the Q-learning algorithm only performs the currently known best action updates. Only in the re-training phase overnight, the algorithm is actively challenging the best action by exploring other potential action rewards. The overall productive deployment plan is presented in Figure 16.

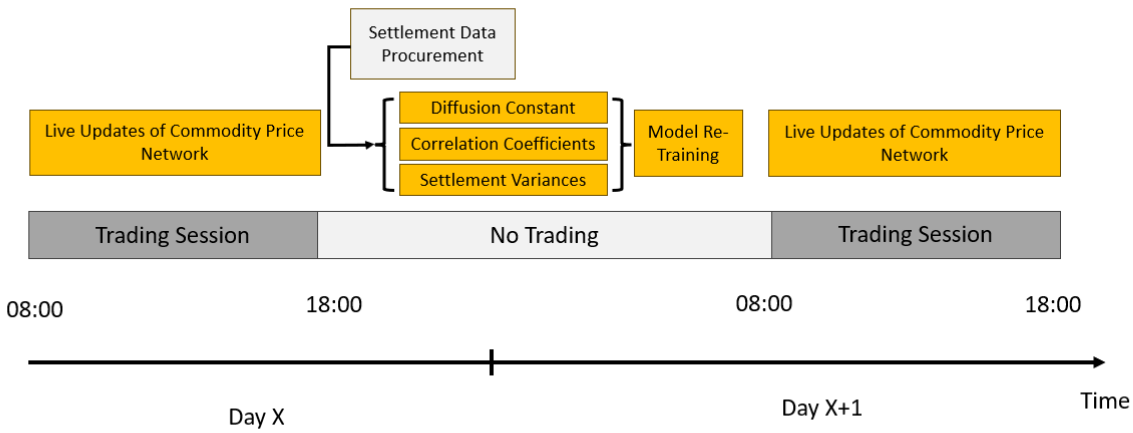

Figure 16: Model Setup in Production Environment

6 Implementation

In the current chapter, the code implementation of the developed modelling framework will be briefly discussed.

6.1 Programming Language

Within the company, Visual Basic .NET is currently used as the main programming language. The reason for this is perfect compatibility with the Microsoft and Windows environment in general and the good modular editor for fast GUI development. However, Microsoft has announced that it will discontinue long-term support for .NET in the medium term, which has led to a decision to switch to C#. For many predictive models and dashboards, Python is widely used in the company. Since the main internal customer of this project prefers .NET/ C# for coding, the main code elements were developed in C#.

6.2 Code Structuring

Throughout the project five main code elements were developed which are quickly presented in this chapter. Figure 17 shows the overall architecture.

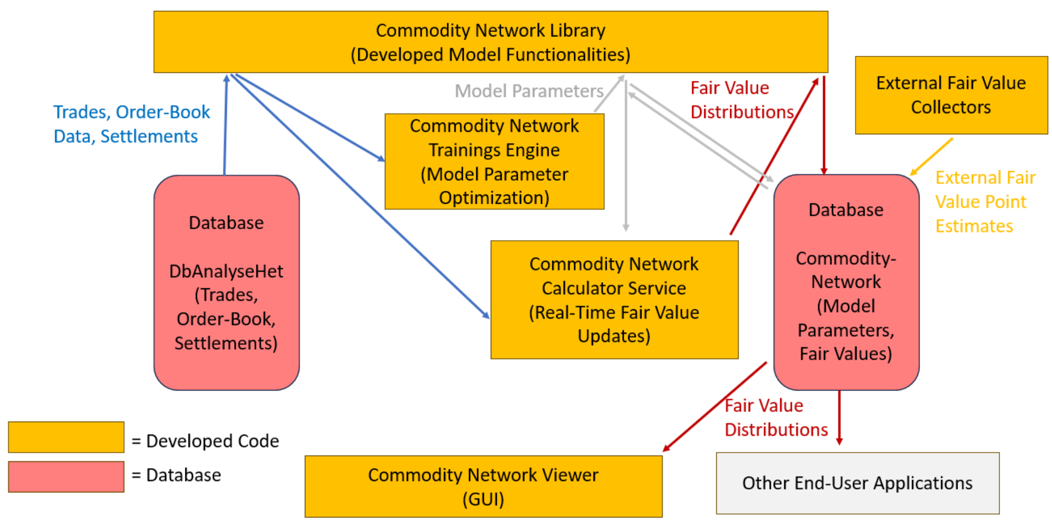

Figure 17: Overall Code Architecture

<u>Commodity Network Library</u>

The main element of the project is the commodity network library project that contains all key functionalities of the different model parts. In addition, the library handles all database accesses. The advantage of this separate library project is, that other users can integrate all the functionalities directly into their applications.

<u>Commodity Network Trainings Engine</u>

The training engine gives the user flexibility in training the developed model with different configurations. The output of the training engine are optimal model parameters (such as q-tables, diffusion constants or correlation coefficients) that can be stored.

Commodity Fair Value Calculator Service

The calculator service is the application that will be run on a local or cloud server machine and uses the optimized model parameters from the training engine, to calculate the real-time fair value distributions of the commodity network. The output is stored in the database and from there available to all interested users / market-data applications.

Commodity Fair Value Viewer

Although the fair values could be directly loaded from the database and integrated into existing applications, a simple GUI app for this project has been developed. Main purpose is to visually track the real-time fair value calculator service. It can be very helpful for being able to track current order book and model distribution parameters mu and sigma of fair value live. This way, potentially problematic model results can be detected much faster during the proof-of-concept phase of the tool. In addition, the tool will in future offer the possibility to simulate model reactions to user-specified inputs. In the benchmarking section, you can compare the fair value point estimates of PointConnect and Bloomberg with our own forecasts. A snapshot from the benchmarking tab of the GUI will be presented in the results section.

External Fair Value Collectors

To enable benchmarking against external data providers and their fair values, data adapters have to be written that take the values and feed them into the database.

7 Results

In this chapter, different model configurations are compared against a reference case. After the best model setup has been found, a comparison is made against the fair values of Refinitiv and Bloomberg.

7.1 Model Evaluation

First, a quick summary of the used training and test time windows is presented in Table 2.

Characteristics	Training	Test
Time Horizon	2023-06-19 - 2023-06-26	2023-06-27
Number of Trades	77'200	15'800

Table 2: Training and Test Data Set

In the following evaluation Table 3 the abbreviation LL is used as synonym for log-likelihood (LL). The selected criteria for the performance evaluation include metrics in the context of the log-likelihood. Since the developed model should explicitly provide high-quality fair value forecasts also for non-liquid markets, not only the average log-likelihood per trade is considered, but also the average log-likelihood per product in the network. Since almost 200 products are modeled, whereof more than half of can be classified as illiquid, the second criterion is a fair measure of performance in this field. The average log-likelihood per trade, on the other hand, corresponds to a more general performance measure.

The reference case in Table 3 corresponds to a "model" run where the fair value of a product is always assumed to be equal to the previous trade and the mean of the distribution is held constant at a σ_0 level (standard deviation of historical settlements).

Model Type	∅ LL per Trade	∅ LL per Prod.
Reference Case Previous Trade with σ_0	-2.01	-2.38
Bayes and Diffusion Bayes and Diffusion $\sigma = \sigma_0$	-0.52	-2.79
Bayes, Diffusion and Network Propagation Q-learning (with 2nd Bayes, γ =0, $\alpha = 0.1$, r= 10min W. lin.)	-0.47	-2.77

Table 3: Model Performance Comparison

Already a model without using network effects improves performance compared to the reference case by a factor of four by means average likelihood per trade. But with the activation of the diffusion model non-liquid products perform worse than in the reference case because the distribution function becomes wide before the next trade comes in and a Bayes update is being performed. Therefore the resulting average log-likelihood per product is lower. This shows the clear need of activating the network propagation part.

Although network propagation clearly improves model performance in the vast majority of products, the average overall log-likelihood per trading product cannot yet be significantly improved. The reason for this is the highly fluctuating forecast for individual gas products, which will be analysed in more detail in the coming weeks. It remains to be said, however, that the average log-likelihood per trade can be significantly improved with the dynamics of the modelled network compared to the already ambitiously selected reference case.

7.2 Model Benchmarking

In a next step, the best-performing model is compared against the fair value predictions of Bloomberg and Refinitiv. Since the two providers do not model a distribution but only point estimates, the mean relative error must be used here instead of the log-likelihood. Due to the limited availability of products and commodity fair value curves of the two external data providers, the number of analysed products in this benchmarking comparison is 59, which is the common intersect of available fair value products of the external providers. Products of the commodities power, gas and coal are included in the subset. For the comparison, again trades and fair values of the trading day 2023-06-27 are used.

Besides mean relative errors (evaluated as mean relative error per trade and as average over all trading products) a third evaluation criterion is added. This performance indicator is the average percentage of correct directional signals of the fair value at the moment of a trade compared to the last traded price. For this purpose an ϵ_t is introduced (0.05 Euro). The price range around $+/-\epsilon_t$ of the previous trade is categorized as neutral region. The model can determine three signals compared to the last trades price:

- +1 (bullish) when the fair value fv is bigger than the last trade + ϵ_t
- 0 (neutral) when the fair value satisfies the condition $p_{last} - \epsilon_t <= fv <= p_{last} + \epsilon_t$
- -1 (bearish) when the fair value satisfies none of the above conditions

This model directional forecast is then compared with the realized directions of the trades. If all price changes would have been correctly predicted, the performance criterion reaches a maximal value of one.

Model Provider	MRE per Trade	MRE per Prod.	∅ Sign. Corr.
BKW	0.003	0.009	0.25
Bloomberg	0.009	0.020	0.22
Refinitiv	0.006	0.011	0.24

Table 4: Model Benchmarking Comparison

The results indicate a general better performance of Refinitiv's fair value model compared to Bloomberg in terms of relative erors, especially in the less liquid markets (MRE per Prod. key indicator). The predictive power of the two external models is lower when compared to the developed models in both dimensions (relative errors and directional signal).

7.3 Developed GUI Elements

Three excerpts from the live tracking tab as well as from the benchmarking section from the CommodityNetworkViewer tool are briefly presented which is a click-once-application.

Figure 18 compares the fair value estimation of the two external data providers and the developed model. The presented trading product (Q4 2023 Power in Germany) is a rather liquid one. The developed model (labeled as "BKW" in the legend and marked with red) captures well the low-price phase around the afternoon. This price dip shows the big benefit of using individual trade information inputs compared to inputs over a rolling time window. Additionally, the prediction is less shaky compared to Bloomberg.

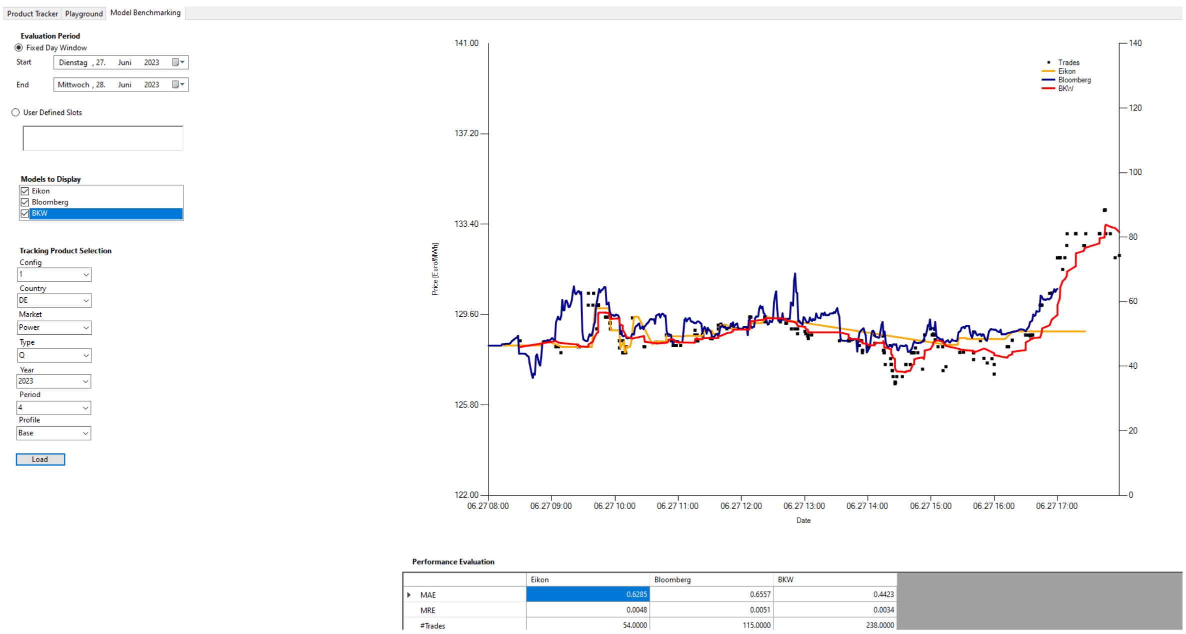

Figure 18: Fair Value Benchmarking Tab - For Trading Product Power DE Q4 2023

In order to show the reader the behavior of the model in a less liquid market, Figure 19 is added. The developed model seems to have a good trade-off between overshooting and lagging.

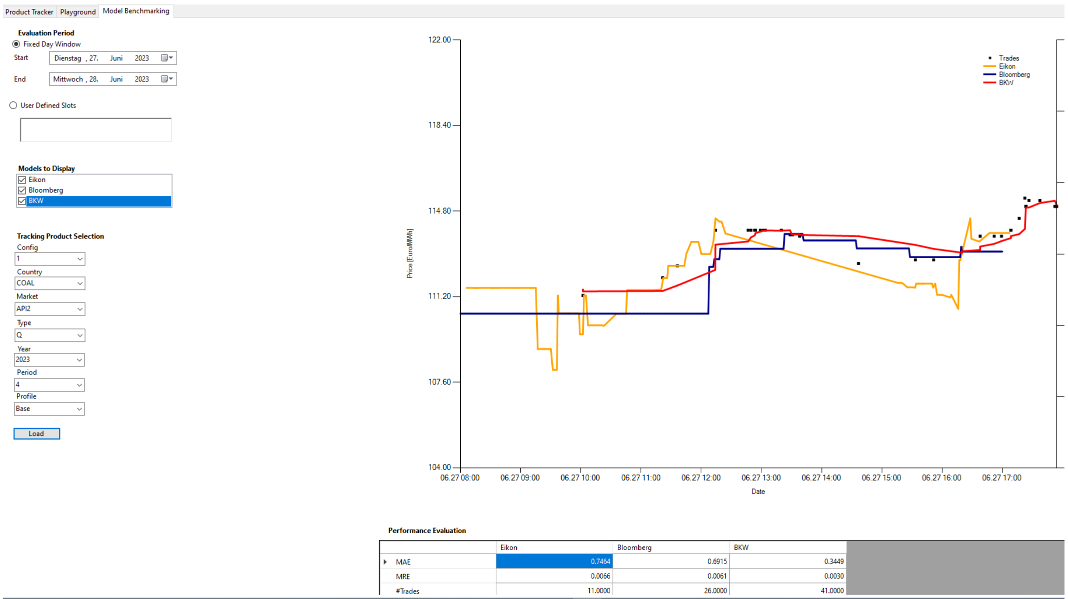

Figure 19: Fair Value Benchmarking Tab - For Trading Product Power Coal Q4 2023

Figure 20 illustrates the live tracking section where the live distribution development of the model is visualized. The current model shape is plotted against the live order book and the last trades. The graph suggests that in the specific case, the standard deviation would optimally be somewhat narrower. This indicates that as a future improvement the more complex modelling of the input standard deviations should be used for the Bayes updates. Because only by using the order book data the influence of the current trade can be well dosed.

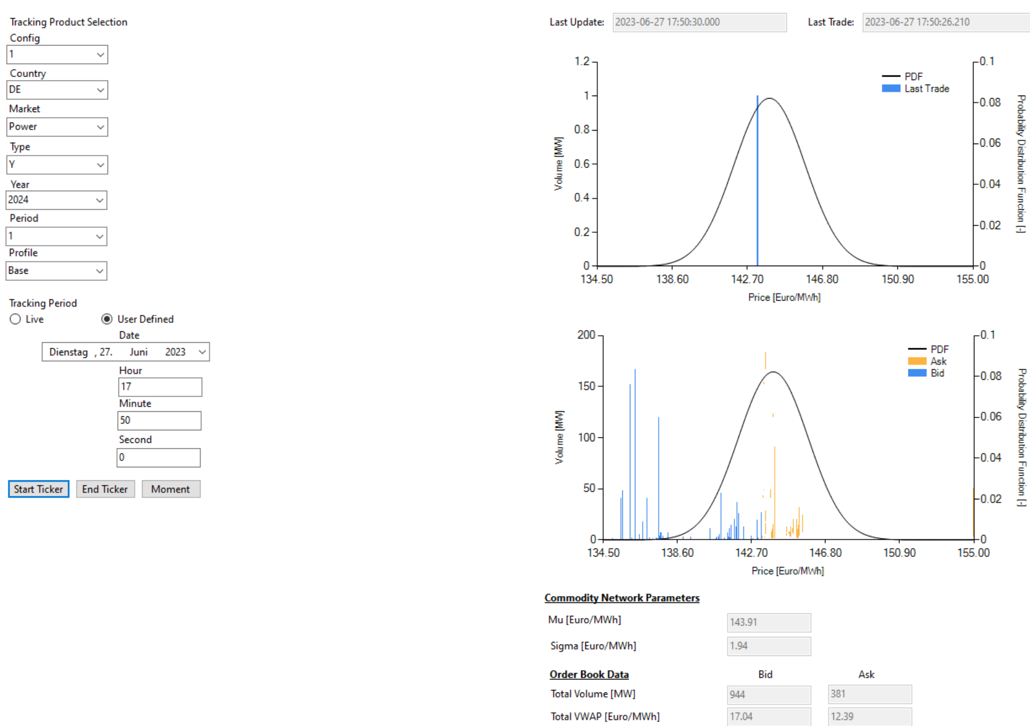

Figure 20: Live Tracking Tab - Snapshot for Power DE Y+1 on 2023-06-27

8 Discussion

The results from the previous chapter clearly show that a commodity network which models cross-commodity effects generates measurable added value to fair value point estimates from the big data providers on the market (PointConnect and Bloomberg). It comes as little surprise that the developed model performed better, especially in the area of illiquid products compared to the benchmark providers. Moreover, the predictive power of the own fair value distributions is slightly higher. That makes total sense. Since Bloomberg and Refinitv do not widely model causal price effects between two commodities such as electricity and gas, the fair value for an electricity product is only adjusted after a next trade in the corresponding product. The fact that one could anticipate the price movement by taking into account the correlation between electricity and gas prices and the last trade in gas, cannot be used in the external approaches.

Although only the simplest possible model variant could be implemented in the short time span given for this thesis, the potential shown is promising and further deepening of the individual sub-models is worthwhile.

With the results achieved and the GUI elements developed, the proof-of-concept phase can be successfully concluded. In the next chapter, the reader is presented with a comprehensive outlook on how the approach can be improved and transferred into a productive system.

9 Outlook

Given the successful proof-of-concept of the developed fair value model, there is still potential for future improvement in each individual sub-section of the project. In many chapters, more complex model extensions were identified, which now need to be discovered further. Some of the most exciting further expansion steps are summarised in this chapter.

- Network scope: It would be interesting to see how well the model can handle oil products such as Brent or WTI. In addition, a geographical expansion in the power sector (towards the Nordics region or UK) as well as in the gas space (NBP, PEG, PSV, THE) could be of interest for the traders. It will be exciting to see how the performance of the NetworkCalculatorService scales in live operation as the network grows.
- Correlation: In the area of correlations, it would be interesting to see how the results change when the coefficients are based on trades and not on settlement data.
- Distribution: Since it has not been done in the previous work environment, it would be exciting to look more closely at bi-modal probability density functions. Research in the field also helps to answer the question of whether the normal distribution assumption is justifiable for the project.
- Fair value input model: As already mentioned, one would like to use a more complex model for the modelled input prices. The aim here would be to use a logistic regression to determine the probabilities that a next trade will take place at bid or ask level. One can hope that a more detailed input model will result in somewhat smoother fair value developments.
- Diffusion: It would be exciting to compare the iteratively determined optimal diffusion constant (based on Bayes-diffusion sequences) with a diffusion constant based on close-to-open settlement data. The difference between the two variants then shows how strongly the diffusion constant used so far is optimised for the model and does not necessarily correspond to the "true" diffusion constant of the financial product.
- Order book: So far, the focus of the model is to process new trade information using the latest order book data. In a next step, the question can be asked at what point an order book change becomes significant enough to trigger a network update on its own.
- Network propagation: It would be desirable to compare the current Q-learning algorithm, which can only select discrete action steps, with more complex continuous reinforcement learning systems. Other modelling approaches such as neural networks or Kalman filters could also be incorporated and tested in the test environment.
- Overall parameter optimization: Some model parameters from the reinforcement learning system (learning rate, discount factor) or from the Bayes model (sigma scaling factors, exponential weighting of the historical time horizon) should be optimised globally in the overall model context. This parameter optimisation is likely to involve a very high computational effort.
- Go-Live of productive system: Apart from the possible improvements in content mentioned above, one of the most important points should not be forgotten. After a short feedback round with the customers as well as data processing coordination within the team, the system should be put into operation as soon as possible. It is certainly important to integrate the fair value forecasts into the most popular tools of the clients. A second important project path will include to publish a transparent quality monitoring report for the daily model performance.
- Extended functionalities: After the successful go-live further functionalities like live profit and loss statement calculation, real-time value at risk tracking or trade anomaly detection could easily be added to the system.

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A Appendix

A.1 Required Mathematical Concepts

A.1.1 Derivation Rules

Chain rule

$$\frac{d}{dx}f(g(x)) = f'(g(x)) \cdot g'(x) \tag{40}$$

Quotient rule

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x)f'(x) - f(x)g'(x)}{(g(x))^2} \tag{41}$$

Product rule

$$\frac{d}{dx}(f(x)g(x)) = f(x)g'(x) + g(x)f'(x) \tag{42}$$

Derivation of exponential function

$$\frac{d}{dx}e^x = e^x \tag{43}$$

Derivation of square-root function

$$\frac{d}{dx}\sqrt{x} = \frac{d}{dx}x^{\frac{1}{2}} = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}} \tag{44}$$

Derivation of natural logarithm

$$\frac{d}{dx}\ln(x) = \frac{1}{x} \tag{45}$$

Derivation of power terms

$$\frac{d}{dx}x^n = nx^{n-1} \tag{46}$$

A.1.2 Logarithm Rules

Logarithm of a Product

$$\ln(a * b) = \ln(a) + \ln(b) \tag{47}$$

Logarithm of a Quotient

$$\ln(\frac{a}{b}) = \ln(a) - \ln(b) \tag{48}$$

A.1.3 Exponential Function Rules

$$\exp{(\frac{a}{2})}^2 = \exp{(\frac{a}{2})}\exp{(\frac{a}{2})} = \exp{(\frac{a}{2} + \frac{a}{2})} = \exp{(a)} \tag{49}$$

Selbständigkeitserklärung

Mit der Abgabe dieser Arbeit versichert der/die Studierende, dass er/sie die Arbeit selbständig und ohne fremde Hilfe verfasst hat (Bei Teamarbeiten gelten die Leistungen der übrigen Teammitglieder nicht als fremde Hilfe).

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