2. Transmission Line Circuits and RF/Microwave Network Analysis

Agenda

- 1.0 – Terminated transmission line circuit.
- 2.0 – Smith Chart and its applications.
- 3.0 – Practical considerations for stripline implementation.
- 4.0 – Linear RF network analysis – 2-port network parameters.

References

- [1] R.E. Collin, "Foundation for microwave engineering", 2nd edition, 1992, McGraw-Hill.

  ★★★ A very advanced and in-depth book on microwave engineering. Difficult to read but the information is very comprehensive. A classic work. Recommended.

- [2] T. C. Edwards, "Foundations for microstrip circuit design", 2nd edition, 1992 John-Wiley & Sons (3rd Edition is also available).

  ★★ Contains a wealth of practical microstrip design information. A must have for every microwave circuit design engineer.

- [3] D.M. Pozar, "Microwave engineering", 2nd edition, 1998 John-Wiley & Sons (3rd edition, 2005 from John-Wiley & Sons is also available).

  ★★ Good coverage of EM theory with emphasis on applications.

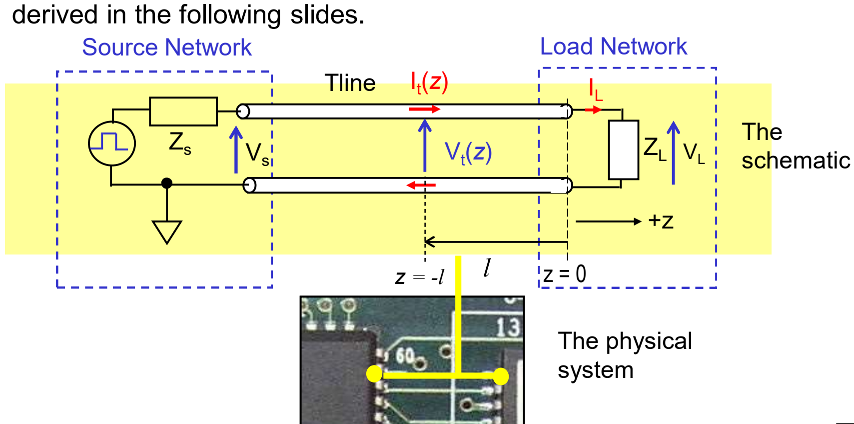
Review of Previous Lecture

- In previous lecture we have studied how a transmission line (Tline) structure can guide a travelling EM wave.
- We covered the various type of propagation modes for EM waves, in particular we are interested in TEM and quasi-TEM mode operation.
- Under these two modes, the Tline can be represented by distributed circuit model consisting of RLCG network, the E field corresponds to the transverse voltage V_t and the H field corresponds to the axial current I_t, V_t and I_t are also propagating waves in the Tline.
- We have also covered how to derived the RLCG parameters under low loss condition.
- Finally, in the last section of previous chapter, we also studied the design procedure of stripline structures on printed circuit board.
- In this chapter, we are going to study the characteristics of Tline terminated with impedance, and how to use Tline in RF/microwave circuits and systems.

1.0 Terminated Transmission Line Circuit

The Lossless Transmission Line Circuit

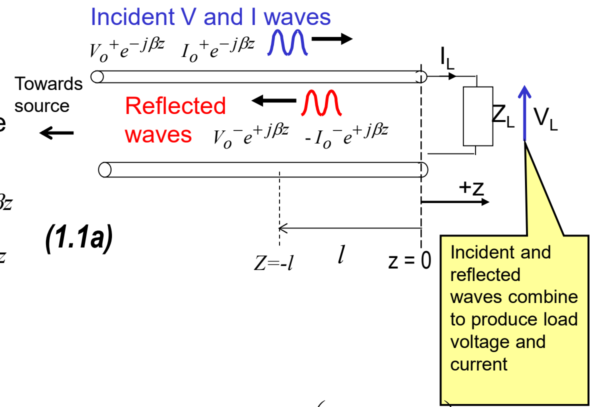
- A transmission line circuit consists of source, load networks and the Tline itself.
- We will use the coordinate as shown. Some basic parameters will be derived in the following slides.



Voltage and Current on Transmission Line Circuit

- Assumption: Tline is lossless (α=0), and supporting TEM mode.
- At a position z along the Tline:

$$V(z) = V_o^+ e^{-j\beta z} + V_o^- e^{+j\beta z}$$
$$I(z) = I_o^+ e^{-j\beta z} - I_o^- e^{+j\beta z} \qquad \textbf{(1.1a)}$$



- At z=0:

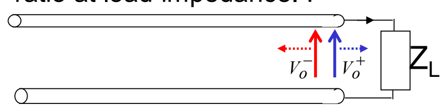
$$V(0) = V_o^+ + V_o^- = V_L$$
$$I(0) = I_o^+ - I_o^- = I_L$$
$$I_L = \frac{1}{Z_c}\left(V_o^+ - V_o^-\right)$$

Using the definition of Z_c

$$Z_L = \frac{V_L}{I_L} = Z_c\left(\frac{V_o^+ + V_o^-}{V_o^+ - V_o^-}\right) \qquad \textbf{(1.1b)}$$

Reflection Coefficient (1)

- The ratio of V_o^- over V_o^+ is described by a voltage reflection coefficient Γ. At the load end a subscript 'L' is inserted to denote that this is the ratio at load impedance. :



$$\Gamma_L = \frac{V_o^-}{V_o^+} \qquad \textbf{(1.2a)}$$

- Using (1.1b): $Z_L = Z_c\left(\frac{1+\Gamma_L}{1-\Gamma_L}\right)$

$$\Gamma_L = \frac{Z_L - Z_c}{Z_L + Z_c} \qquad \textbf{(1.2b)}$$

or

$$\Gamma_L = \frac{\overline{Z}_L - 1}{\overline{Z}_L + 1} \quad \overline{Z}_L = \frac{Z_L}{Z_c} \qquad \textbf{(1.2c)}$$

- Similarly we could also derive the current reflection coefficient:

$$\Gamma_I = \frac{-I_o^-}{I_o^+} = -\Gamma_L \qquad \textbf{(1.2d)}$$

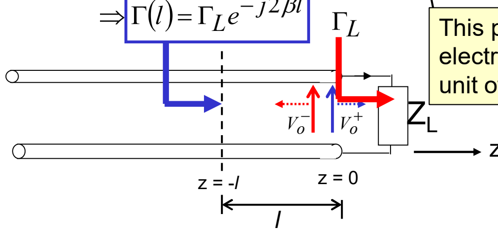
Reflection Coefficient (2)

- At a distance *l* from the load, the voltage reflection coefficient is given by:

$$\Gamma(l)=\frac{V_o^{-}e^{-j\beta l}}{V_o^{+}e^{j\beta l}}=\frac{V_o^{-}}{V_o^{+}}e^{-j2\beta l} \qquad \textbf{(1.2e)}$$

$$\Rightarrow \boxed{\Gamma(l)=\Gamma_L e^{-j2\beta l}}$$

This product is usually called electrical length, is given a unit of radian or degree

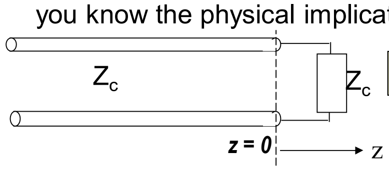


- Note that this equation is only valid when the z=0 reference is at the load impedance, AND *l* is always positive.
- From now on we will deal exclusively with voltage reflection coefficient.

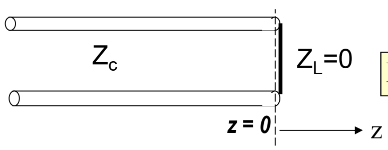
Reflection Coefficient (3)

- Reflection coefficient for various load impedance values. Make sure you know the physical implication of these.

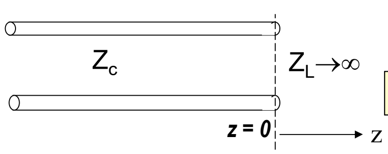
$$\Gamma_L=\frac{Z_L-Z_c}{Z_L+Z_c}$$



$$\boxed{\Gamma_L=0}$$

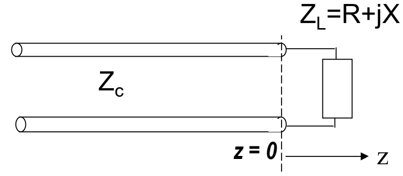


$$\boxed{\Gamma_L=-1}$$



$$\boxed{\Gamma_L=1}$$

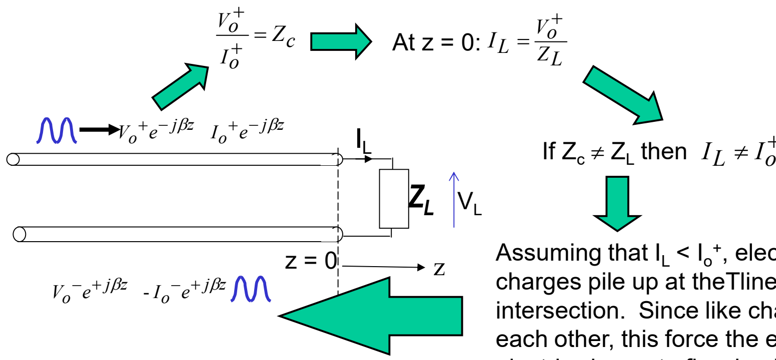
Z_L=R+jX



$$\Gamma_L(\omega)=\frac{(R(\omega)+jX(\omega))-Z_c}{(R(\omega)+jX(\omega))+Z_c}=\frac{(R(\omega)-Z_c)+jX(\omega)}{(R(\omega)+Z_c)+jX(\omega)}$$

Why Do Reflection Occur? (1)

Extra



Note that the total voltage at the Tline-load intersection is $V_o^+ + V_o^- = V_L$. Thus the current drawn by the load Z_L will also increase from the original value V_o^+/Z_L until equilibrium is reached.

Assuming that $I_L < I_o^+$, electric charges pile up at theTline and load intersection. Since like charges repel each other, this force the excess electric charge to flow back to the Tline. This constitutes the reflected current. From our understanding of Tline theory, if there is current, there is also associated voltage, hence a reflected voltage and current wave occur.

Why Do Reflection Occur? (2)

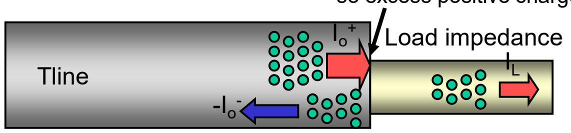
Extra

- We can visualize the current flow as due to positive charges (conventional current).

Positive charge
Negative charge

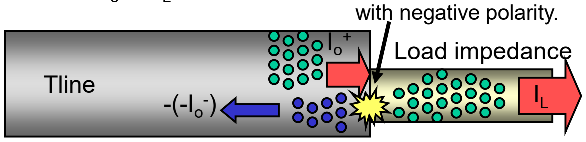
When $I_o^+ > I_L$

The Tline supply more charges per unit time than the load can absorb so excess positive charges reflected back



At the interface positive and negative charge pairs are created. The positive charge flows into the load, while the negative charge flows back to the source, constituting the reflected current with negative polarity.

When $I_o^+ < I_L$



Of course there is only free electron in conductor. When a region is said to contain positive charge, it actually has less free electrons as compare to equilibrium state.

Power Delivered to Load Impedance

- Power to load:

Let Z_c be real

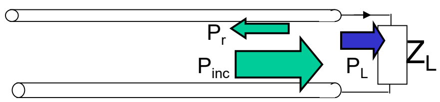
$$P_L = \frac{1}{2}\mathrm{Re}\left(V_L I_L^*\right) = \frac{1}{2}\mathrm{Re}\left(\left(V_o^+ + V_o^-\right)\frac{\left(V_o^+ - V_o^-\right)^*}{Z_c}\right)$$

$$\Rightarrow P_L = \tfrac{1}{2Z_c}\left|V_o^+\right|^2\left(1-\left|\Gamma_L\right|^2\right) = \underbrace{\tfrac{1}{2Z_c}\left|V_o^+\right|^2}_{P_{incident}} - \left|\Gamma_L\right|^2\underbrace{\left[\tfrac{1}{2Z_c}\left|V_o^+\right|^2\right]}_{P_{reflected}} \quad \textit{(1.3)}$$

- Thus when $\Gamma_L = 0$, all incident power is absorbed by Z_L. We say that the load is matched to the Tline. Otherwise there will be reflected power in the form of:

$$P_r = \frac{1}{2}Y_c\left|V_o^-\right|^2 = \frac{1}{2}Y_c\left|V_o^+\right|^2\left|\Gamma_L\right|^2$$

- The maximum power to load is called the incident power P_{inc}:



$$P_{inc} = \frac{1}{2}Y_c\left|V_o^+\right|^2$$

Impedance Matching

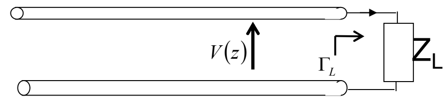
- The purpose of impedance matching is to reduce reflection from both the load and the source.
- We strive to get maximum power from the source and transport this power (the available power) to the load.

Impedance matching - Make $|\Gamma_L| \Rightarrow 0$

- In other words impedance matching provides a 'smooth' flow of EM wave along a system of interconnect.

Voltage Standing Wave Ratio (VSWR) (1)

- At any point z along the Tline:



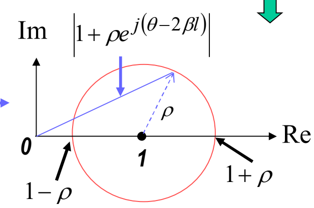
$$V(z) = V_o^{+} e^{-j\beta z} + V_o^{-} e^{j\beta z}$$

$$\Rightarrow V(z) = V_o^{+} e^{-j\beta z}\left(1 + \Gamma_L e^{j2\beta z}\right) = V_o^{+} e^{j\beta l}\left(1 + \Gamma_L e^{-j2\beta l}\right)$$

Argand Diagram

- Expressing Γ_L in polar form $\Gamma_L = \rho e^{j\theta}$

$$\left|V(z)\right| = \left|V_o^{+}\right| \left|1 + \rho e^{j(\theta - 2\beta l)}\right| \qquad \textbf{(1.4)}$$



- The ratio of maximum |V| to mininum |V| is known as VSWR.

$$\text{VSWR} = \frac{\left|V(z)\right|_{\max}}{\left|V(z)\right|_{\min}} = \frac{1+\rho}{1-\rho} \qquad \textbf{(1.5)}$$

Voltage Standing Wave Ratio (2)

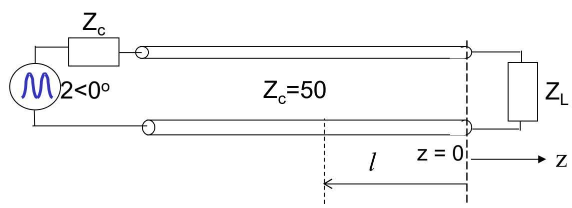
- A similar expression can be obtained for I(z).

$$\left|I(z)\right| = \left|I^{+}\right| \left|1 - \rho e^{j(\theta - 2\beta l)}\right| \qquad \textbf{(1.6)}$$

- VSWR measures how good the load Z_L is matched to the Tline. For good match, ρ = 0 and VSWR=1. For mismatch load, ρ >0 and VSWR >1.
- We can view the incident and reflected wave as interfering with each other, causing standing wave along the Tline.
- A similar phenomenon also exist in waveguide, however it is the E and H field standing wave that are being measured, so generally the alphabet 'V' is dropped when dealing with waveguide.
- Why SWR is a popular (than reflection coefficient)? – (1) VSWR is a scalar or real value. (2) In the early days waveguides are widely used, and a simple way to measure SWR is to use the slotted line waveguide with diode detector.

Example 1.1

- A Tline with 2 conductors and separated by air. A sinusoidal voltage of magnitude 2V and frequency 3.0 GHz is launched into the Tline. The characteristic impedance of the Tline is 50Ω and one end of the Tline is terminated with load impedance Z_L=100+j100 @ 3.0 GHz. Assume phase velocity = *c* (speed of light in vacuum, ≅ 3.0×10⁸ m/s).
  – Find the load reflection coefficient Γ_L.
  – Find the power delivered to the load Z_L.
  – Plot |V| and |I| versus *l*, the distance from Z_L.
  – Determine the VSWR of the system.



Example 1.1 Cont...

- The solution (as calculated using MathCAD™):

Parameters of the system

$fo := 3.0\cdot 10^9$ $\quad Zc := 50$ $\quad ZL := 100 + i\cdot 100$

$Vs := 2$ $\quad Zs := 50$ $\quad vp := 3.0\cdot 10^8$

Finding β: $\lambda := \frac{vp}{fo}$ $\quad \lambda = 0.1$

$\beta := 2\cdot\frac{\pi}{\lambda}$ $\quad \beta = 62.832$

Finding Vo⁺ :

$Vo := \frac{Zc}{Zc + Zs}\cdot Vs$ $\quad Vo = 1$

Reflection coefficient at the load: $\Gamma L := \frac{ZL - Zc}{ZL + Zc}$ $\quad \Gamma L = 0.538 + 0.308i$

VSWR of the system: $VSWR := \frac{1 + |\Gamma L|}{1 - |\Gamma L|}$ $\quad VSWR = 4.266$

Voltage and current phasor along z axis:

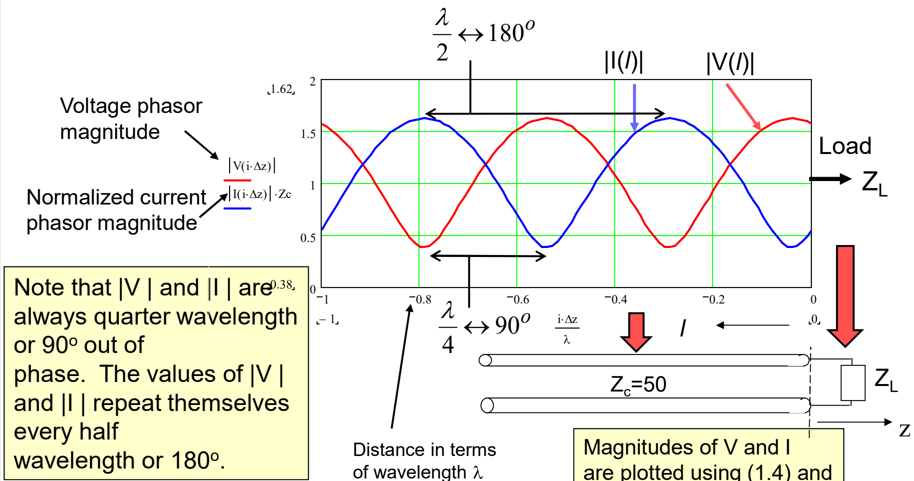
$V(z) := Vo\cdot(\exp(-i\cdot\beta\cdot z) + \Gamma L\cdot\exp(i\cdot\beta\cdot z))$ $\quad I(z) := \frac{Vo}{Zc}\cdot(\exp(-i\cdot\beta\cdot z) - \Gamma L\cdot\exp(i\cdot\beta\cdot z))$

Average power dissipated by the load

$PL := \frac{Vo^2}{2\cdot Zc}\cdot\left[1 - (|\Gamma L|)^2\right]$ $\quad PL = 6.154\times 10^{-3}$

Example 1.1 Cont...

- Plotting out the voltage and current phasor along the transmission line:



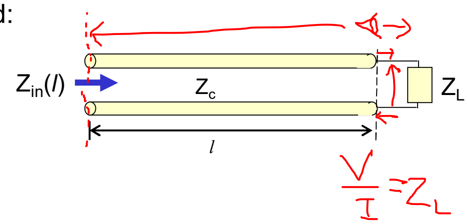
Note that |V | and |I | are always quarter wavelength or 90° out of phase. The values of |V | and |I | repeat themselves every half wavelength or 180°.

Magnitudes of V and I are plotted using (1.4) and (1.6)

Input Impedance of a Terminated Tline

- At any length l from the termination impedance, we can compute the impedance looking towards the load:

$$Z_{in}(l) = \frac{V(l)}{I(l)} = \frac{V_o^+ e^{j\beta l} + V_o^- e^{-j\beta l}}{\frac{1}{Z_c}\left(V_o^+ e^{j\beta l} - V_o^- e^{-j\beta l}\right)}$$



Use (1.2b)

$$Z_{in}(l) = Z_c \frac{e^{j\beta l} + \Gamma_L e^{-j\beta l}}{e^{j\beta l} - \Gamma_L e^{-j\beta l}}$$

$$e^{j\beta l} = \cos(\beta l) + j\sin(\beta l)$$

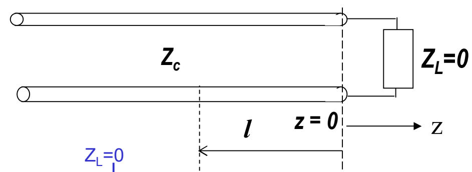
$$\Gamma_L = \frac{Z_L - Z_c}{Z_L + Z_c}$$

$$Z_{in}(l) = Z_c \frac{Z_L + jZ_c \tan(\beta l)}{Z_c + jZ_L \tan(\beta l)} \quad \textbf{(1.7a)}$$

$$Y_{in}(l) = Y_c \frac{Y_L + jY_c \tan(\beta l)}{Y_c + jY_L \tan(\beta l)} \quad \textbf{(1.7b)}$$

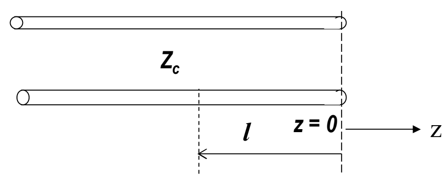
Special Cases of Terminated Lossless Tline (1)

- When Z_L=0:



$$Z_{in}(l) = Z_c\left(\frac{\overset{Z_L=0}{\downarrow}0 + jZ_c\tan(\beta l)}{Z_c + 0}\right) = jZ_c\tan(\beta l) \quad \textbf{(1.8a)}$$

- When $Z_L \to \infty$



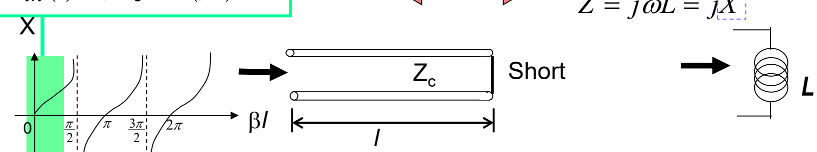
$$Z_{in}(l)\big|_{Z_L\to\infty} \to Z_c\left(\frac{Z_L}{jZ_L\tan(\beta l)}\right) = -jZ_c\cot(\beta l) \quad \textbf{(1.8b)}$$

Special Cases of Terminated Lossless Tline (2)

$$Z_{in}(l) = jZ_c\tan(\beta l) = jX$$

Reactance

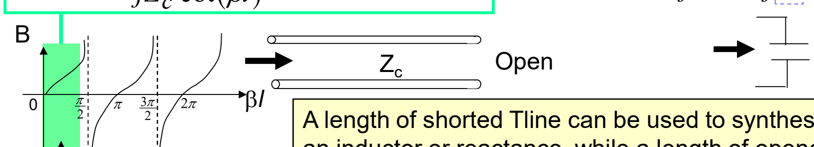
$$Z = j\omega L = jX$$



$$Y_{in}(l) = \frac{1}{-jZ_c\cot(\beta l)} = jY_c\tan(\beta l) = jB$$

Susceptance

$$Y = j\omega C = jB$$

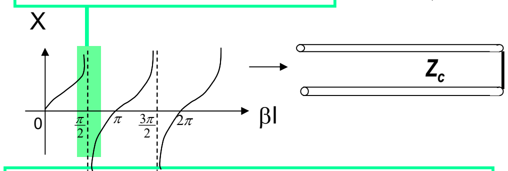


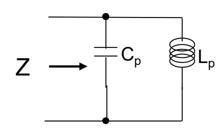
This area corresponds to $\beta l < \pi/2$, or $l < \lambda/4$

A length of shorted Tline can be used to synthesize an inductor or reactance, while a length of opened Tline can be used to synthesize a capacitor or susceptance, within a limited range of frequency ($f < v_p/(4l)$)

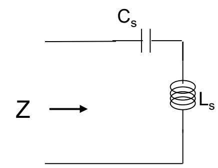
Special Cases of Terminated Lossless Tline (3)

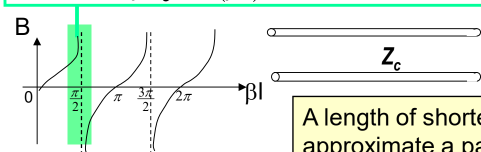
$$Z_{in}(l) = jZ_c \tan(\beta l) = jX$$





$$Y_{in}(l) = \frac{1}{-jZ_c \cot(\beta l)} = jY_c \tan(\beta l) = jB$$

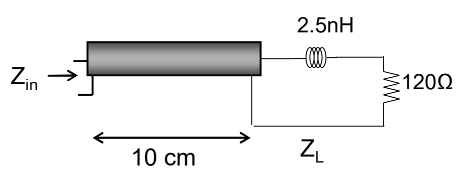




A length of shorted Tline can also be used to approximate a parallel LC resonator, while a length of opened Tline can also be used to approximate series LC resonator, within a limited frequency range.

Example 1.2

- A lossless Tline of length *l* = 10 cm supports TEM propagation mode. The per unit length L and C are given as L = 209.4 nH/m, C = 119.5pF/m. The Tline is terminated with a series RL load impedance:



- Plot the real and imaginary part of Z_{in} from f = 1.0 GHz to 4.0 GHz.

Example 1.2 Cont...

$$Z_c = \sqrt{\frac{L}{C}} = 41.86$$

$$v_p = \frac{1}{\sqrt{LC}} = 1.999\times 10^8$$

$$\beta = \frac{\omega}{v_p} \qquad Z_L = 120 + j\omega\left(2.5\times 10^{-9}\right)$$

$$Z_{in}(l) = Z_c \frac{Z_L + jZ_c \tan(\beta l)}{Z_c + jZ_L \tan(\beta l)}$$

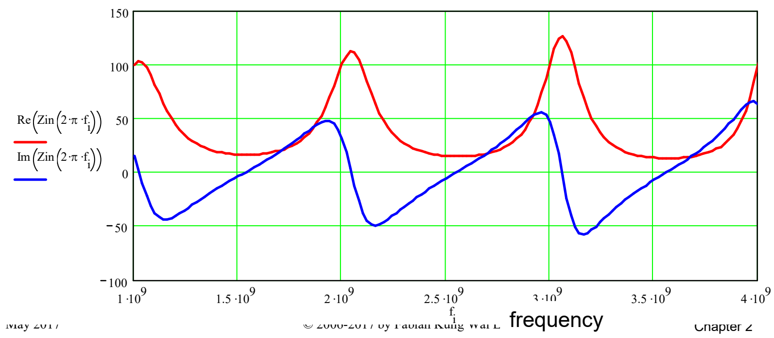
Note that the pattern of the waveform repeat at an interval of close to 1GHz, this is due to the periodic nature of tan(β*l*) function.

$$\beta l = n\pi \qquad \text{n=1,2,3,4...}$$

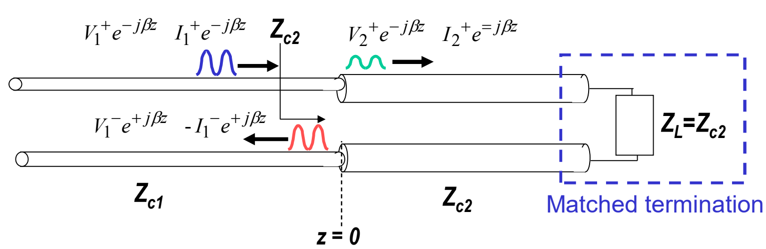
$$\Rightarrow \frac{2\pi f}{v_p} l = n\pi$$

$$\Rightarrow f = n\frac{v_p}{2l}$$

$$f = 0.9995GHz, 1.999GHz, 2.999GHz...$$



Cascading Transmission Lines - Transmission Coefficient



$$T \equiv \frac{\text{transmitted voltage}}{\text{incident voltage}} = \frac{V_2^+}{V_1^+} = \frac{Z_{c2}I_2^+}{Z_{c1}I_1^+}$$

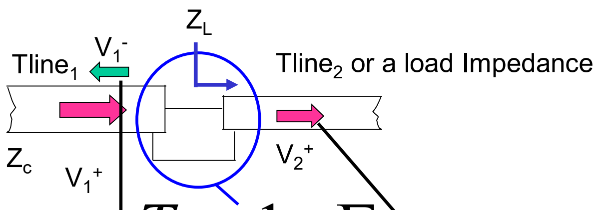
At z = 0:

$$V_1^+ + V_1^- = V_2^+$$

$$\Rightarrow T(0) = \frac{V_2^+}{V_1^+} = 1 + \frac{V_1^-}{V_1^+} = 1 + \Gamma(0)$$

Using $\Gamma(0) = \frac{Z_{c2} - Z_{c1}}{Z_{c2} + Z_{c1}}$ $\Rightarrow$ $T(0) = \frac{2Z_{c2}}{Z_{c2} + Z_{c1}}$ **(1.9)**

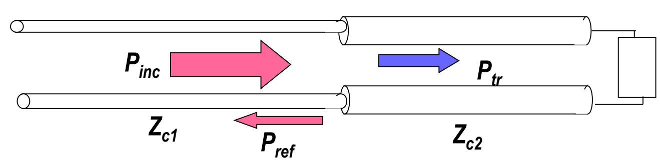
Relationship between Reflection and Transmission Coefficient



$$T_L = 1 + \Gamma_L \qquad (1.10)$$

$$\Gamma_L = \frac{Z_L(\omega) - Z_c}{Z_L(\omega) + Z_c} \qquad T_L = \frac{2Z_L(\omega)}{Z_L(\omega) + Z_c}$$

Power Relations



$$P_{inc} = \frac{1}{2}\frac{\left|V_1^+\right|^2}{Z_{c1}}$$

$$P_{ref} = \frac{1}{2}\frac{\left|V_1^-\right|^2}{Z_{c1}} = \left|\Gamma\right|^2 P_{inc}$$

$$P_{tr} = \frac{1}{2}\frac{\left|V_2^+\right|^2}{Z_{c2}} = \left|T\right|^2 P_{inc}\frac{Z_{c1}}{Z_{c2}}$$

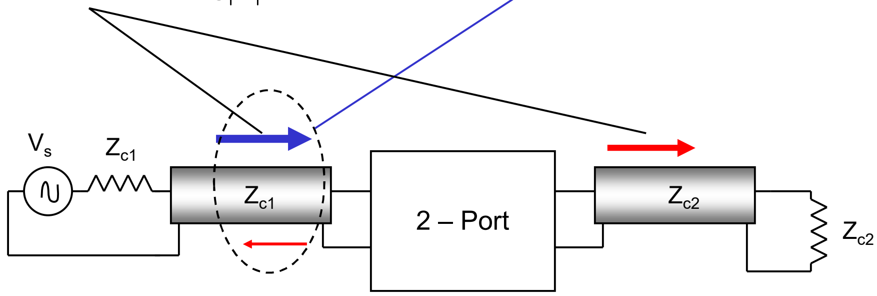
$P_{inc} = P_{ref} + P_{tr}$

Return Loss and Insertion Loss

- Sometimes both voltage reflection and transmission coefficient are expressed in dB, these are then termed return loss (RL) and insertion loss (IL).

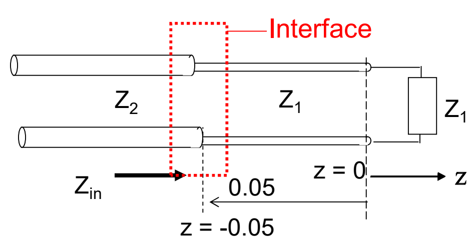
$$IL = -20\log|T| \text{ dB} \quad \textbf{(1.11a)}$$

$$RL = -20\log|\Gamma| \text{ dB} \quad \textbf{(1.11b)}$$

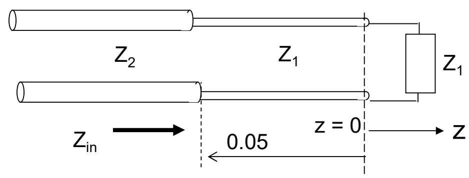


Example 1.3

- Find the return loss (RL) and insertion loss (IL) at the intersection between the Tlines. Assume TEM propagation mode at f = 1.9 GHz for both Tlines, Z_1 =100, Z_2 =50.



Example 1.3 Cont...



$$\Gamma\big|_{z=-0.05} = \frac{Z_1 - Z_2}{Z_1 + Z_2} = 0.3333$$

$$-20\log_{10}\left|\Gamma\big|_{z=-0.05}\right| = 9.542$$

$$T\big|_{z=-0.05} = \frac{2Z_1}{Z_1 + Z_2} = 1.3333$$

$$-20\log_{10}\left|T\big|_{z=-0.05}\right| = -2.499$$

Exercise 1.1

- We would like to use a short length of transmission line to implement a reactance of X = 60 at 2.4GHz. Show how this can be done using the microstrip line of Example 5.1, Chapter 1 – Advanced Transmission Line Theory. Hint: use equation (1.8a).

For a transmission line terminated with short circuit:

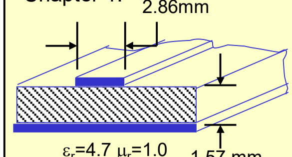
$$Z_{in}(l) = jZ_c \tan(\beta l) = j60$$

From Example 5.1 (Chapter 1):

$$\beta = \frac{\omega}{v_p} = \frac{2\pi \cdot 2.4\times10^9}{1.591\times10^8} = 94.781$$

$$l = \frac{1}{\beta}\tan^{-1}\left(\frac{60}{Z_c}\right) = \frac{1}{94.781}\cdot 0.876 = 0.00924\text{m}$$

From Example 5.1, Chapter 1:

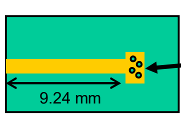


ε_r=4.7 μ_r=1.0

$Z_c \cong 50$

v_p = 1.591×10^8

Top View



A number of plated through hole to reduce the parasitic inductance of the short.

Extra

Terminated Lossy Tline (1)

- In the case of a lossy Tline we replace $j\beta$ with $\gamma = \alpha + j\beta$ in equations (1.2) to (1.9). As seen from equation (2.7) (Chapter 1 - Advanced Transmission Line Theory), the characteristic impedance Z_c becomes a complex value too.
- Furthermore:

$$\bar{Z}_{in}(l) = \frac{1 + j\Gamma_L e^{-2\alpha l - j2\beta l}}{1 - j\Gamma_L e^{-2\alpha l - j2\beta l}} = \frac{Z_L + Z_c \tanh(j\beta l + \alpha l)}{Z_c + Z_L \tanh(j\beta l + \alpha l)} \qquad (1.12)$$

- The losses have the effect of reducing the standing-wave ratio (SWR) towards unity as the point of observation is moved away from the load towards the generator/source.
- Most of the time the losses are so small that for short length of Tline, the neglect of α is justified. However as frequency increases beyond 3 GHz, the skin effect and dielectric loss become important for typical PCB dielectric and conductor and for Tline of more than 20mm, at frequency above 3GHz, loss has to be included to model the effect of the Tline on the electrical signal.

Extra

Terminated Lossy Tline (2)

- At some point $z = -l$ from the load-Tline interface, the power directed towards the load is:

$$P(l) = \frac{1}{2}\mathrm{Re}\left(VI^*\right) = \frac{1}{2}Y_c\left|V^+\right|^2\left(e^{2\alpha l} - \left|\Gamma_L\right|^2 e^{-2\alpha l}\right)$$

$$\Rightarrow P(l) = \tfrac{1}{2}Y_c\left|V^+\right|^2\left(1 - \left|\Gamma(l)\right|^2\right)e^{2\alpha l} \qquad (1.13)$$

Power delivered increases as we proceed towards the generator!

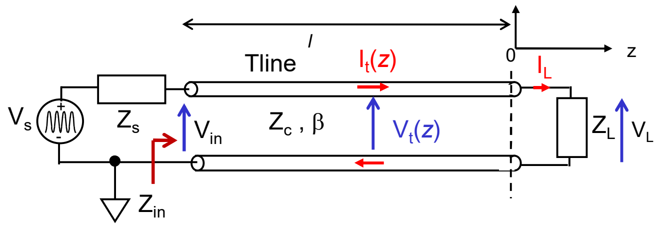
- Of the power given by (1.11), the power dissipated by the load is given by (1.3), the remainder is dissipated by the lossy line.

$$P(l) - P_L = \frac{1}{2}Y_c\left|V^+\right|^2\left(1 - \left|\Gamma(l)\right|^2\right)e^{2\alpha l} - \frac{1}{2}Y_c\left|V^+\right|^2\left(1 - \left|\Gamma_L\right|^2\right)$$

$$= \frac{1}{2}Y_c\left|V^+\right|^2\left[\underbrace{\left(e^{2\alpha l} - 1\right)}_{\text{Loss due to incident wave}} + \underbrace{\left|\Gamma_L\right|^2\left(1 - e^{-2\alpha l}\right)}_{\text{Loss due to reflected wave}}\right]$$

Exercise 1.2 – Finding Voltage Phasors for Transmission Line Circuit

- Consider a transmission line circuit below, determine V_{in} and V_L.



$$Z_{in} = Z_c\left[\frac{Z_L + jZ_c\tan(\beta l)}{Z_c + jZ_L\tan(\beta l)}\right] \qquad \Gamma_L = \frac{Z_L - Z_c}{Z_L + Z_C}$$

Thus $V_{in} = \frac{Z_{in}}{Z_s + Z_{in}} V_s = V_o^+\left(e^{j\beta l} + \Gamma_L e^{-j\beta l}\right)$

Solving for V_o^+: $V_o^+ = \frac{V_s}{\left[\frac{Z_c}{Z_s}\left[\frac{Z_c + jZ_L\tan(\beta l)}{Z_L + jZ_c\tan(\beta l)}\right] + 1\right]} \cdot \frac{1}{\left[e^{j\beta l} + \left(\frac{Z_L - Z_c}{Z_L + Z_c}\right)e^{-j\beta l}\right]}$

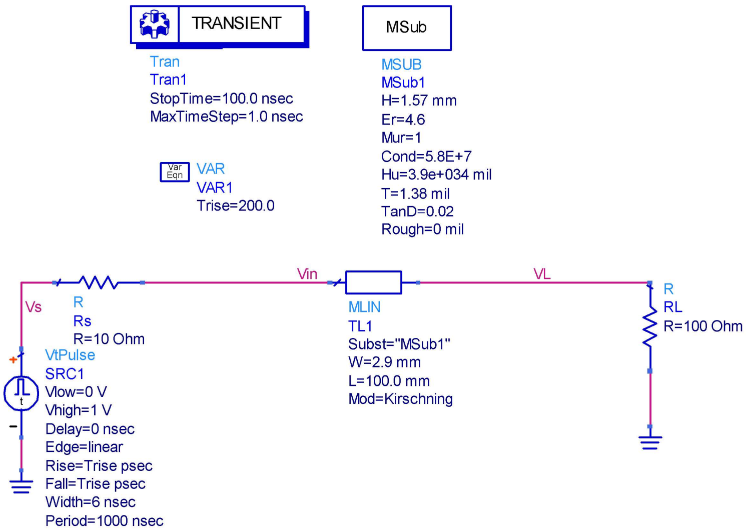
Exercise 1.2 Cont…

- Knowing V_o^+, we can find V_{in} and V_L:

$$V_{in} = V(z = -l) = V_o^+\left(e^{j\beta l} + \Gamma_L e^{-j\beta l}\right)$$
$$V_L = V(z = 0) = V_o^+\left(1 + \Gamma_L\right)$$

Extra

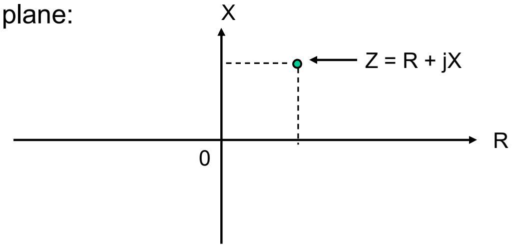
Demo – Transmission Line Circuit Simulation Exercise with Agilent ADS Software



2.0 Smith Chart and Its Applications

Introduction (1)

- In analyzing electrical circuits, one very important parameter is the impedance Z or admittance Y seen at a terminal/port.
- For time-harmonic circuits Z or Y is dependent on frequency and is a complex value.
- To visualize arbitrary Z or Y values graphically, we would need an infinite 2D plane:



Z = V/I = R + jX

Resistance Reactance

Y = I/V = G + jB

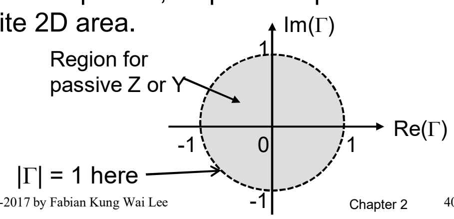
Conductance Susceptance

Introduction (2)

- In RF circuit design, we can represent an impedance Z = R+jX in terms of its reflection coefficient (Γ) with respect to a reference impedance (Z_o):

$$\Gamma = \frac{Z - Z_O}{Z - Z_O} = -\frac{Y - Y_O}{Y + Y_O} \quad Z_O = \frac{1}{Y_O} = \text{reference impedance}$$

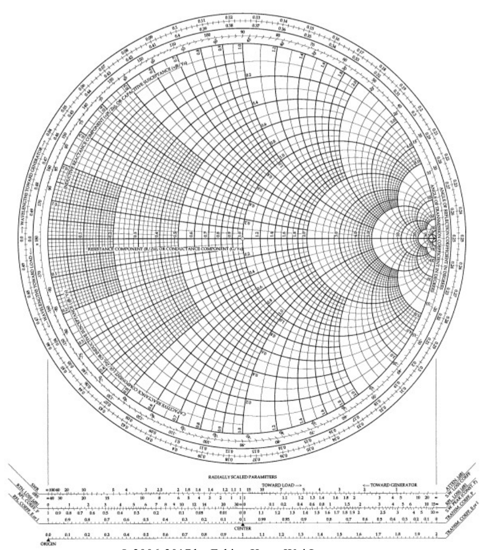
- Usually we would take $Z_o = Z_c$, the characteristic impedance of a Tline in the system.
- Γ is also a complex value, however we have learnt that its magnitude is always < 1 for passive impedance value.
- Effectively if reflection coefficient is plotted, all possible passive Z and Y values can be fitted into a finite 2D area.



Introduction (3)

- To facilitate the evaluation of reflection coefficient, a graphical procedure based on conformal mapping is developed by P.H. Smith in 1939.
- This procedure, now known as the Smith Chart permits easy and intuitive display of reflection coefficient Γ as well as impedance Z and admittance Y in one single graph.

Introduction (4)



Formulation (1)

$$\Gamma = \frac{Z - Z_o}{Z + Z_o} = \frac{\left(\frac{Z}{Z_o}\right) - 1}{\left(\frac{Z}{Z_o}\right) + 1} = \frac{z-1}{z+1} \quad \Rightarrow \quad z = \frac{1+\Gamma}{1-\Gamma} = \text{normalized impedance}$$

Let z = r + jx and Γ = U + jV:

Then $$r + jx = \frac{1 + U + jV}{1 - U - jV}$$

Equating real and imaginary part:

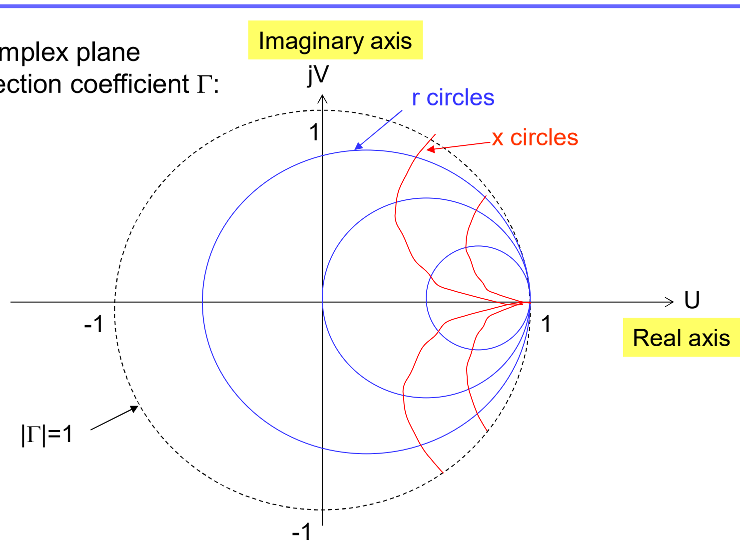
Equation of circles for U and V:

Depends only on r (r circle) $\longrightarrow$ $$\left(U - \frac{r}{1+r}\right)^2 + V^2 = \frac{1}{(1+r)^2}$$

Depends only on x (x circle) $\longrightarrow$ $$(U-1)^2 + \left(V - \frac{1}{x}\right)^2 = \frac{1}{x^2}$$

Formulation (2)

The complex plane
for reflection coefficient Γ:



Formulation (3)

Extra

Note that:

$$y = \frac{1}{z} = \frac{1-\Gamma}{1+\Gamma} = \frac{1+e^{j\pi}\Gamma}{1-e^{j\pi}\Gamma}$$

Let y = g + jb and Γ = U + jV:

$$g + jb = \frac{1-U-jV}{1+U+jV}$$

Again proceeding as before we obtain:

Depends only on g (g circle) $\longrightarrow$

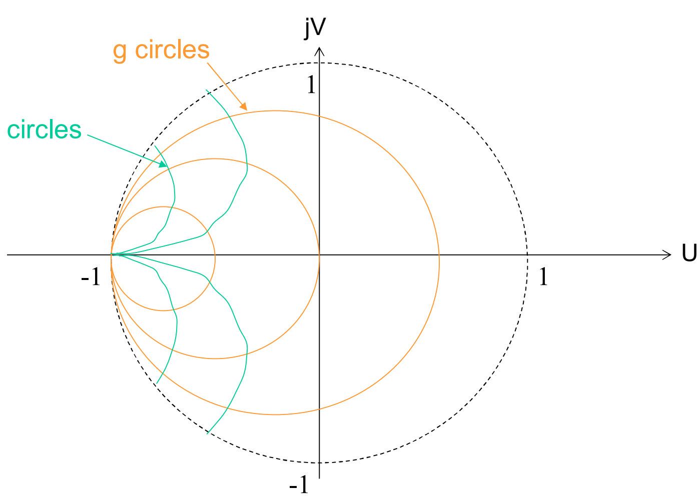
$$\left(U + \frac{g}{1+g}\right)^2 + V^2 = \frac{1}{(1+g)^2}$$

Depends only on b (b circle) $\longrightarrow$

$$(U+1)^2 + \left(V + \frac{1}{b}\right)^2 = \frac{1}{b^2}$$

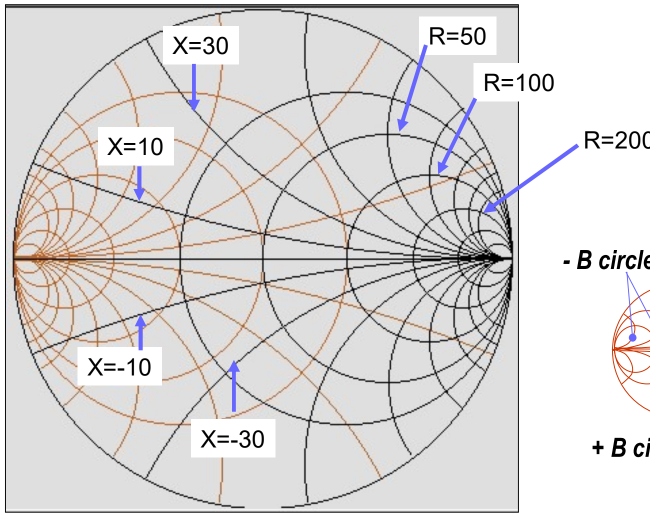
Formulation (4)

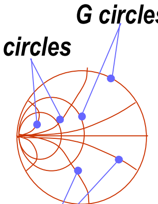
Extra



The Complete Smith Chart with r,x,g and b circles

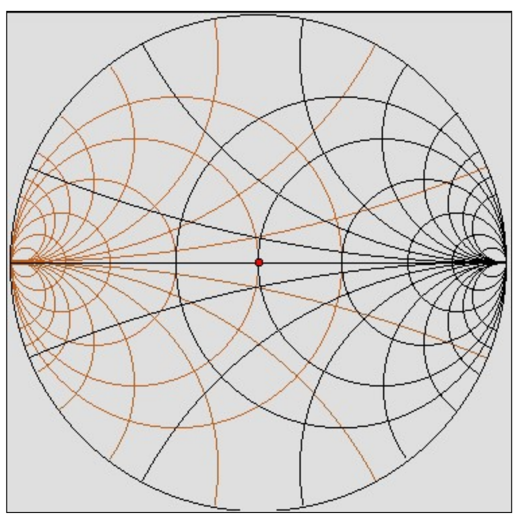
R and X circles





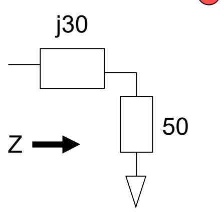
Smith Chart Example 1

- Z_o = 50Ohm.
- Z = 50 + j0

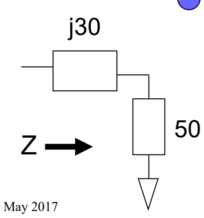


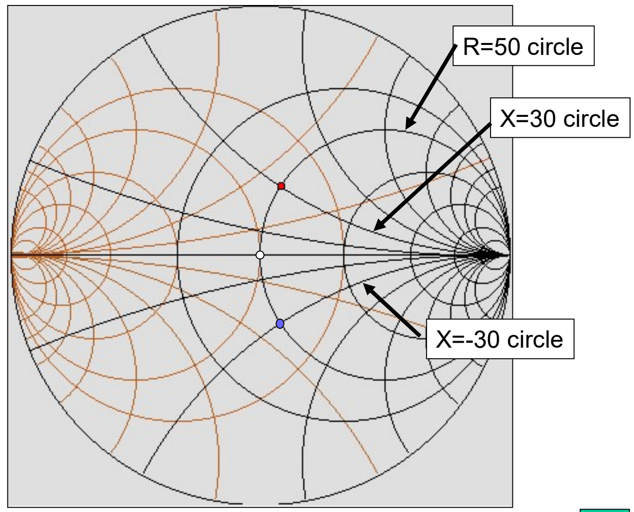
Smith Chart Example 2

- Z_o = 50Ohm.
- Z = 50 + j30



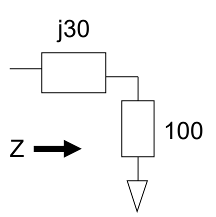
- Z = 50 - j30



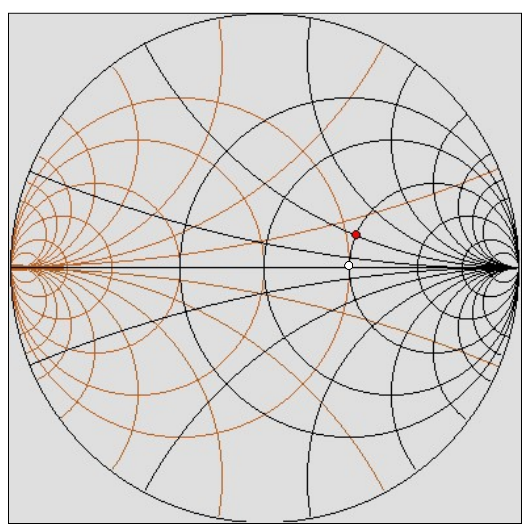


Smith Chart Example 3

- Z_o = 50Ohms
- Z = 100 + j30

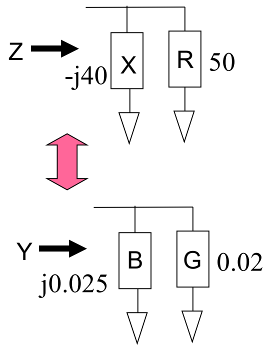


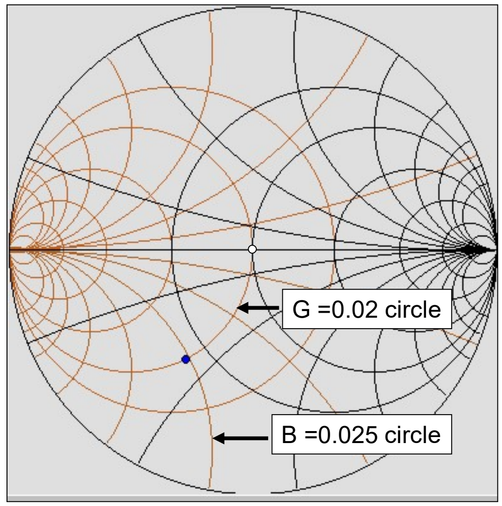
Question: What would you expect if the real part of Z is negative ?



Smith Chart Example 4

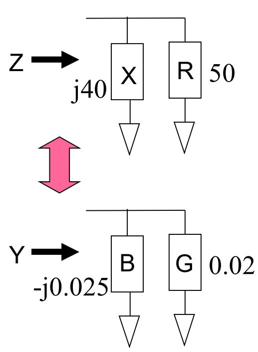
- Z_o = 50Ohm
- Z = 50//(-j40) or
- Y = 0.020 + j0.025

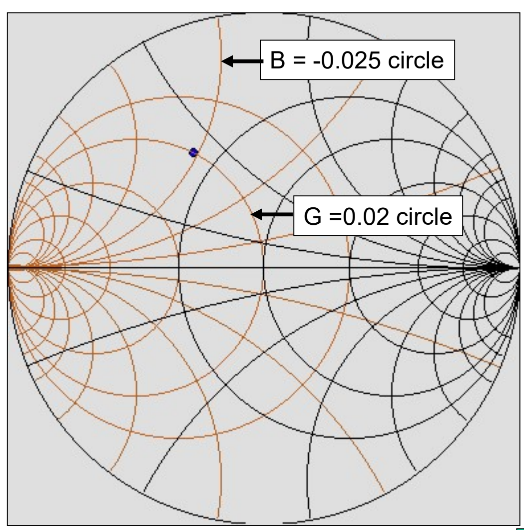




Smith Chart Example 5

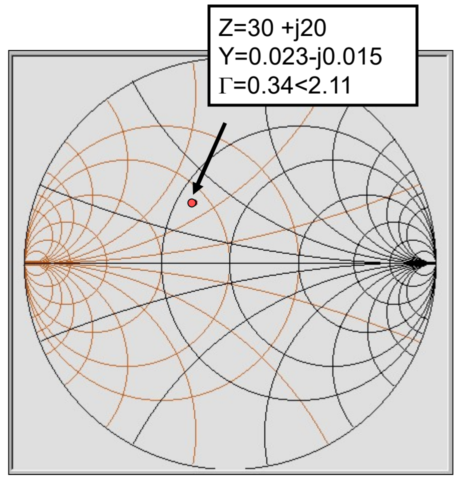
- Z_o = 50Ohm
- Z = 50//(j40) or
- Y = 0.020 - j0.025





Smith Chart Summary (1)

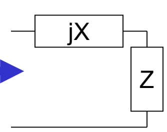
- Thus a point on a Smith Chart can be interpreted as reflection coefficient Γ.
- It can also be read as impedance Z = R + jX.
- It can also be read as admittance Y = G + jB.

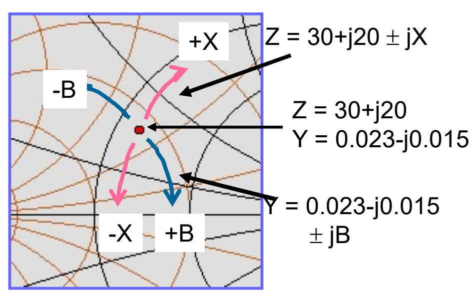


Smith Chart Summary (2)

- When we add a reactance in series with Z, the point on the Smith Chart will move in such a way that it remains on the constant R circle.

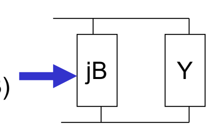
$Z_{in} = Z_L + jX$
$= R_L + j(X_L + X)$





- When we add a susceptance in parallel to Y, the point on the Smith Chart will move in such a way that it remains on the constant G circle.

$Y_{in} = Y_L + jB$
$= G_L + j(B_L + B)$



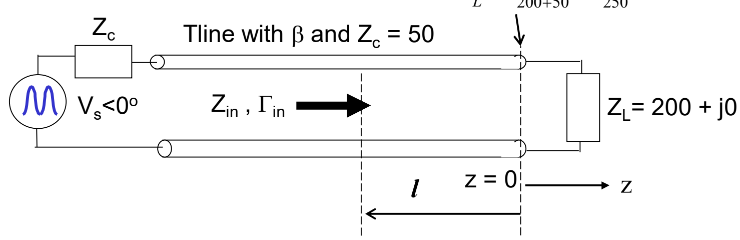
Smith Chart Example 6

- In this example we would like to observe the locus of impedance Z_{in} as l is changed for Z_L = 200 + j0 (Say at a certain operating frequency f_o).
- Recall equations (1.2d) and (1.7) in this chapter:

$$\bar{Z}_{in}(l) = \frac{Z_L + jZ_c \tan(\beta l)}{Z_c + jZ_L \tan(\beta l)} \qquad \Gamma_{in} = \Gamma(l) = \Gamma_L e^{-j2\beta l}$$

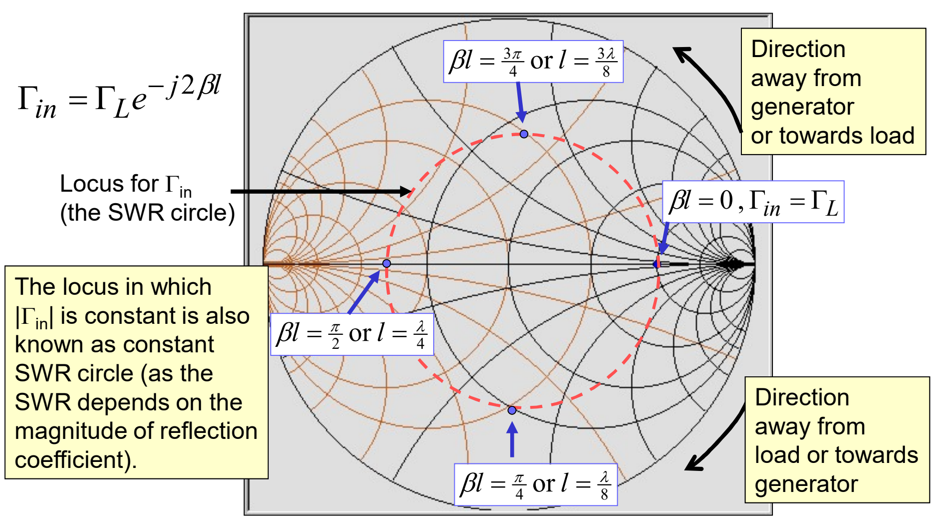
with $\beta = \frac{2\pi}{\lambda}$ where λ is the wavelength

$$\Gamma_L = \frac{200-50}{200+50} = \frac{150}{250} = 0.6$$



Smith Chart Example 6 Cont...

$$\Gamma_{in} = \Gamma_L e^{-j2\beta l}$$

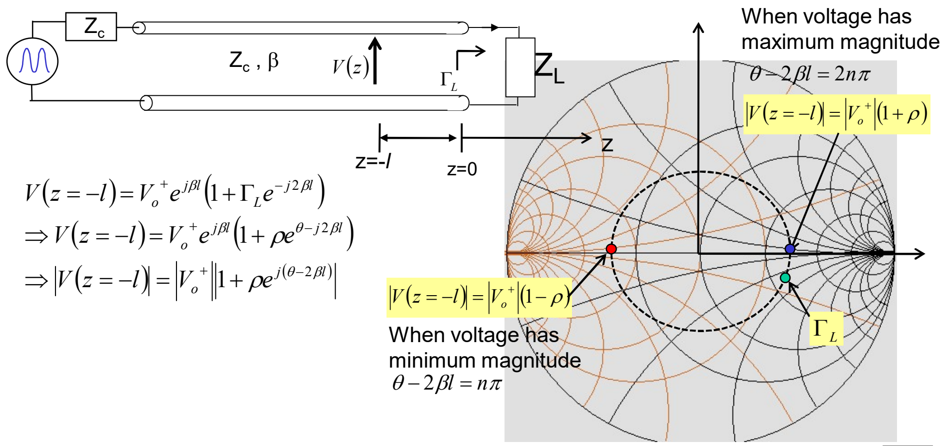


Typical Applications of Smith Chart (1)

- Smith Chart is used in impedance transformation and impedance matching. Although this can be performed using analytical method, using graphical tool such as Smith Chart allows us to visualize the effect of adding a certain element in the network. The effective impedance of a load after adding series, shunt or transmission line section can be read out directly from the coordinate lines of the Smith Chart.
- Smith Chart is used in RF active circuits design, such as when designing amplifiers. Usually certain contours in the form of circles are plotted on the Smith Chart.
- 2-port network parameters such as s_{11} and s_{22} are best viewed in Smith Chart.

Typical Applications of Smith Chart (2)

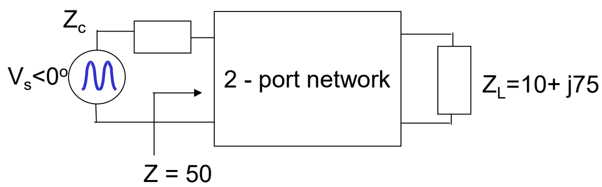
- Classically Smith Chart can also be used to find the position along the transmission line with maximum and minimum voltage or current.



$$V(z=-l) = V_o^+ e^{j\beta l}\left(1+\Gamma_L e^{-j2\beta l}\right)$$
$$\Rightarrow V(z=-l) = V_o^+ e^{j\beta l}\left(1+\rho e^{\theta - j2\beta l}\right)$$
$$\Rightarrow |V(z=-l)| = |V_o^+|\left|1+\rho e^{j(\theta-2\beta l)}\right|$$

Exercise 2.1 - Impedance Transformation

- Employing the software fkSmith (http://pesona.mmu.edu.my/~wlkung/), find a way of transforming a load impedance of $Z_L = 10 + j75$ into $Z = 50$ using either lumped L, C or section of Tline. Assume an operating frequency of 1.8GHz.

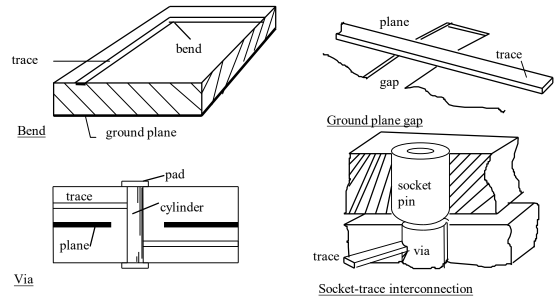


3.0 Practical Considerations for Stripline Implementation

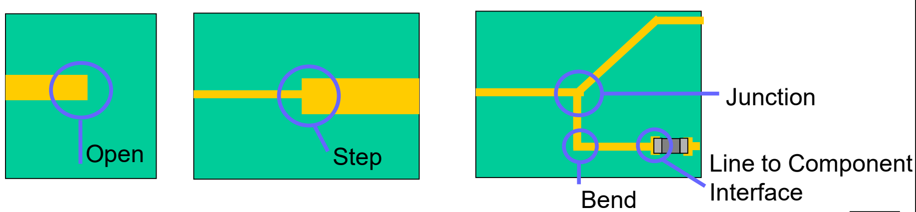
Practical Transmission Line Design and Discontinuities

- Discontinuities in Tline are changes in the Tline geometry to accommodate layout and other requirements on the printed circuit board.
- Virtually all practical distributed circuits, whether in waveguide, coaxial cables, microstrip line etc. must inherently contains discontinuities. A straight uninterrupted length of waveguide or Tline would be of little engineering use.
- The following discussion consider the effect and compensation for discontinuities in PCB layout. This discussion is restricted to TEM or quasi-TEM propagation modes.

Practical Transmission Line Discontinuities Found in PCB (1)

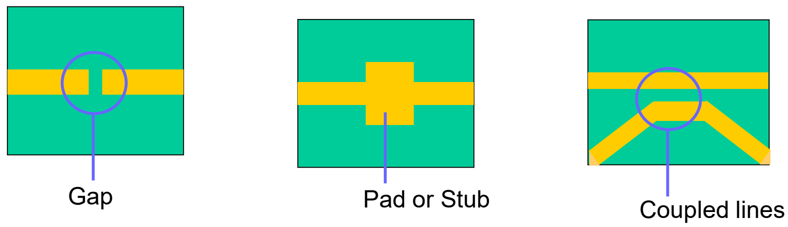


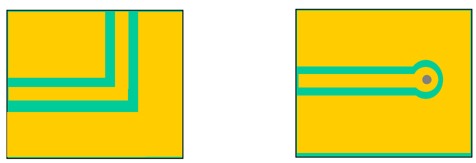
Here we illustrate the discontinuities using microstripline. Similar structures apply to other transmission line configuration as well.



Practical Transmission Line Discontinuities Found in PCB (2)

- Further examples of microstrip and co-planar line discontinuities.





Examples of bend and via on co-planar Tline.

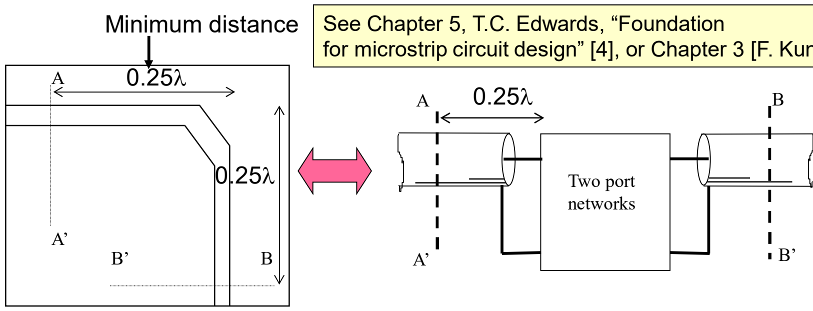
Discontinuities and EM Fields (1)

- Introduction of discontinuities will distort the uniform EM fields present in the infinite length Tline. Assuming the propagation mode is TEM or quasi-TEM, the discontinuity will create a multitude of higher modes (such as TM_{11} , TM_{12} , TE_{11} , …) in its vicinity in order to fulfill the boundary conditions (Note - there is only one type of TEM mode !!).
- Most of these induced higher order modes are evanescent or non-propagating as their cut-off frequencies are higher than the operating frequency of the circuit. Thus the fields of the higher order modes are known as local fields.
- The effect of discontinuity is usually reactive (the energy stored in the local fields is returned back to the system) since loss is negligible.
- The effect of reactive system to the voltage and current can be modeled using LC circuits (which are reactive elements).
- For TEM or quasi-TEM mode, we can consider the discontinuity as a 2-port network containing inductors and capacitors.

Discontinuities and EM Fields (2)

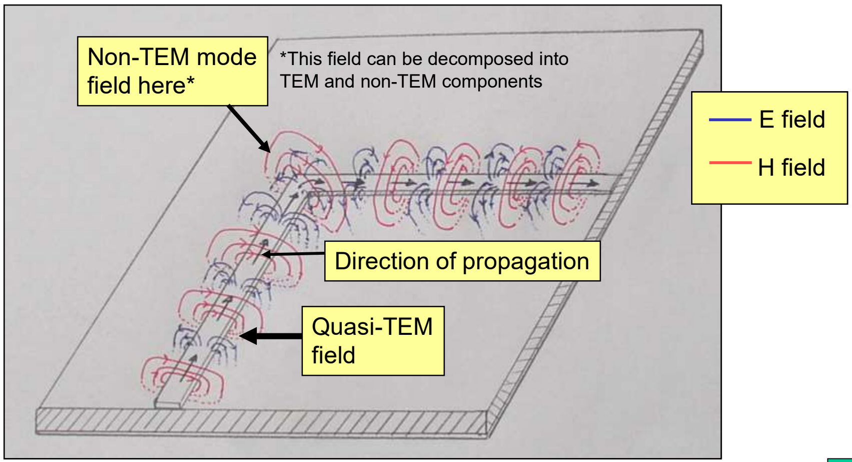
- Modeling a discontinuity using circuit theory element such as RLCG is a good approximation for operating frequency up to 6 – 20 GHz. This upper limit will depends on the size of the discontinuity and dielectric thickness.
- The smaller the dimension of the discontinuity as compared to the wavelength, the higher will be the upper usable frequency.
- As a example, the 2-port model for microstrip bend is usually accurate up to 10GHz.

See Chapter 5, T.C. Edwards, "Foundation for microstrip circuit design" [4], or Chapter 3 [F. Kung]



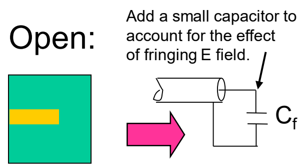
Discontinuities and EM Fields (3)

- For instance for a microstrip bend, a snapshot of the EM fields at a particular instant in time:



Microstrip Line Discontinuity Models (1)

Open:



The open end of a stripline contains fringing E field. The effect of the fringing E field can be accounted for by two methods:
1. Adding a capacitor C_f, at the end of the stripline.
2. Slightly increase the length of the ideal stripline by Δl.

For microstripline, the approximate value of C_f have been derived by Silvester and Benedek from the EM fields of an open-end structure using numerical method and curve fitted, also C_f is related to Δl from the RLCG model of transmission line:

$$\frac{C_f}{W} = \exp\left[2.2036\sum_{i=1}^{5} K_{\varepsilon i}\left(\log\frac{W}{h}\right)^{i-1}\right] \text{ pF/m} \quad \textbf{(3.1a)}$$

$$\Delta l = \frac{C_f h}{\varepsilon_{eff}\varepsilon_o W} \quad \textbf{(3.1b)}$$

ε_r / i	1.0	2.5	4.2	9.6	16.0	51.0
1	1.110	1.295	1.443	1.738	1.938	2.403
2	-0.2892	-0.2817	-0.2535	-0.238	-0.2233	-0.2220
3	0.1815	0.1367	0.1062	0.1308	0.1317	0.2170
4	-0.0033	-0.0133	-0.0260	-0.0087	-0.0267	-0.0240
5	-0.0540	-0.0267	-0.0073	-0.0133	-0.0147	-0.0840

h=dielectric thickness
W=width

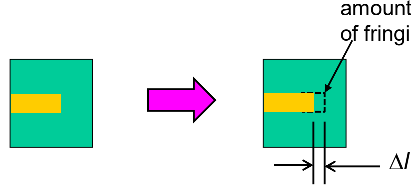
P. Silvester and P. Benedek,"Equivalent capacitances of microstrip open circuits", IEEE Trans. MTT-20, No. 8 August 1972, 511-576.

Microstrip Line Discontinuity Models (2)

- Equation (3.1a) and (3.1b) are fairly difficult to use. A simpler approximation (up to 5% difference) is provided by E. O. Hammerstad, which can be easily incorporated in CAD software:

$$\Delta l = 0.412h\left[\frac{\left(\varepsilon_{eff}+0.3\right)\left(\frac{W}{h}+0.264\right)}{\left(\varepsilon_{eff}-0.258\right)\left(\frac{W}{h}+0.8\right)}\right] \quad \textbf{(3.1c)}$$

Increase the length by a small amount to account for the effect of fringing E field.

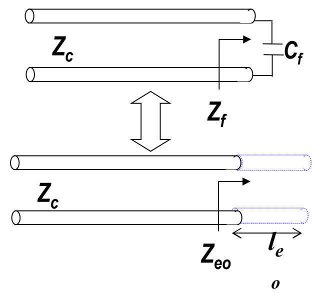


E.O. Hammerstad, "Equations for microstrip circuit design", Proc. 5th European Microwave Conference, pp. 268-272, Sep 1975.

Microstrip Line Discontinuity Models (3)

Extra

- Assuming the effect of C_f can be represented by a short length of Tline:

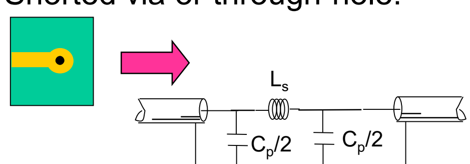


$$l_{eo} \cong \frac{cZ_c C_f}{\sqrt{\varepsilon_{eff}}} \quad (3.2)$$

- Thus in microstrip Tline design, we need to fore-shorten the actual physical length by l_{eo} to compensate for fringing E field effect.

Microstrip Line Discontinuity Models (4)

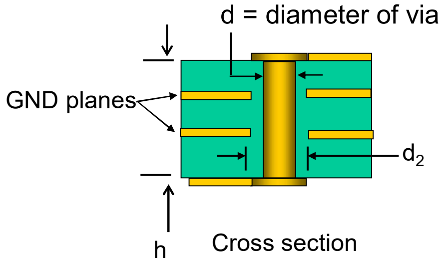
Shorted via or through-hole:



$$L_s \cong 0.2h\left(\ln\left(\frac{4h}{d}\right)+1\right) \quad (3.3a)$$

$$C_p \cong 0.056\frac{\varepsilon_r hd}{d_2 - d}N \quad (3.3b)$$

This is the capacitance between the via and internal plane. If there are multiple internal conducting planes, then there should be one C_p corresponding to each internal plane.

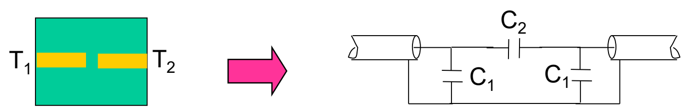


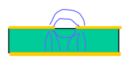
Cross section of a Via

L_s in nH
C_p in pF
h in mm
d and d_2 in mm
ε_r = dielectric constant of PCB
N = number of GND planes

Microstrip Line Discontinuity Models (5)

Gap:

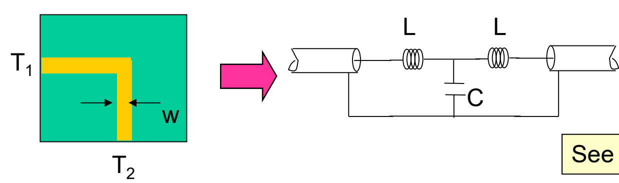




See Chapter 5, T.C. Edwards, "Foundation for microstrip circuit design" [4], or
B. Easter, "The equivalent circuit of some microstrip discontinuities", IEEE Trans. Microwave Theory and Techniques vol. MTT-23 no.8 pp 655-660,1975.

Microstrip Line Discontinuity Models (6)

90^o Bend:



See Edwards [4], chapter 5

Approximate quasi-static expressions for L_1, L_2 and C:

$$\frac{C}{w} = \frac{(14\varepsilon_r + 12.5)\frac{w}{d} - (1.83\varepsilon_r - 2.25)}{\sqrt{w/d}} \text{ pF/m} \quad \text{for} \quad \frac{w}{d} < 1$$

$$\frac{C}{w} = (9.5\varepsilon_r + 1.25)\frac{w}{d} + 5.2\varepsilon_r + 7.0 \text{ pF/m} \quad \text{for} \quad \frac{w}{d} > 1 \qquad \textit{(3.4)}$$

$$\frac{L}{d} = 100\left[4\sqrt{\frac{w}{d}} - 4.21\right] \text{ nH/m}$$

ε_r = dielectric constant of substrate, assume non-magnetic.
d = thickness of dielectric in meter.

Example 3.1 - Microstrip Line Bend

- For a 90° microstrip line bend, with w=2.8mm, d=1.57mm, ε_r = 4.2. Find the value of L and C.

$$\frac{C}{w} = (9.5\times 4.2 + 1.25)1.834 + 5.2\times 4.2 + 7.0$$

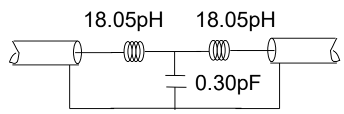
$$= 104.309 \text{ pF/m}$$

$$\frac{w}{d} = 1.834$$

$$\Rightarrow \text{C} = 104.309 \times 0.00288 = 0.30\text{pF}$$

$$\frac{L}{d} = 100\left[4\sqrt{1.834} - 4.21\right] = 120.701 \text{ nH/m}$$

$$\Rightarrow \text{L} = 18.95\text{pH}$$



Typically the effect of bend is not important for frequency below 1 GHz This is also true for discontinuities like step and T-junction.

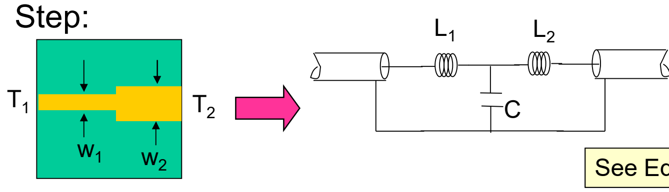
At 1GHz:

Reactance of C $\quad X_c = \frac{1}{2\pi fC} \cong 530.5$

Reactance of L $\quad X_L = 2\pi fL \cong 0.119$

Microstrip Line Discontinuity Models (7)

Step:



See Edwards [4] chapter 5

Approximate quasi-static expressions for L_1, L_2 and C:

$$\frac{C}{\sqrt{w_1 w_2}} = (10.1\log\varepsilon_r + 2.33)\frac{w_1}{w_2} - 12.6\log\varepsilon_r - 3.17 \text{ pF/m} \quad \text{for} \quad \varepsilon_r \le 10;\ 1.5 \le \frac{w_2}{w_1} \le 10$$

$$\frac{C}{\sqrt{w_1 w_2}} = 130\log\left(\frac{w_2}{w_1}\right) - 44 \text{ pF/m} \quad \text{for} \quad \varepsilon_r = 9.6;\ 3.5 \le \frac{w_2}{w_1} \le 10$$

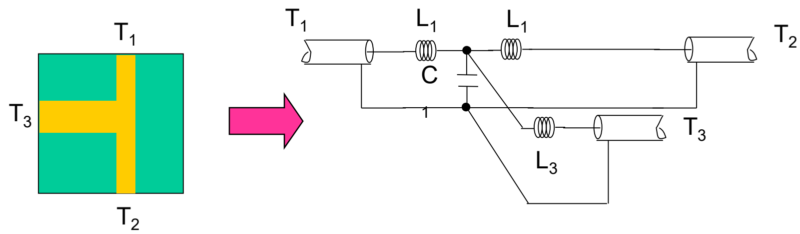
$$\frac{L}{d} = 40.5\left(\frac{w_1}{w_2} - 1.0\right) - 75\frac{w_1}{w_2} + 0.2\left(\frac{w_1}{w_2} - 1.0\right)^2 \text{ nH/m} \qquad \textit{(3.5)}$$

$$L_1 = \frac{L_{m1}}{L_{m1} + L_{m2}}L \qquad L_2 = \frac{L_{m2}}{L_{m1} + L_{m2}}L$$

L_{m1} and L_{m2} are the per unit length inductance of T_1 and T_2 respectively.

Microstrip Line Discontinuity Models (8)

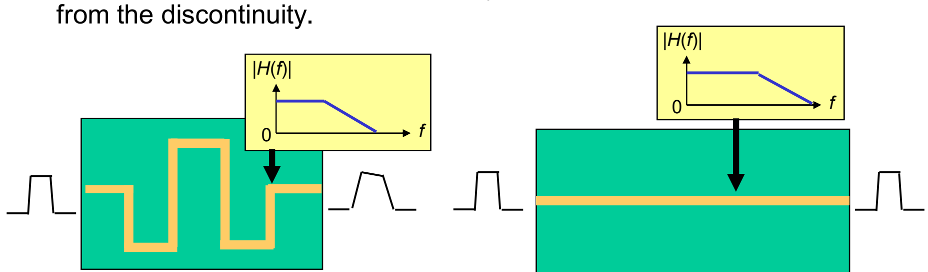
T-Junction:



See Edwards [4], chapter 5 for alternative model and further details.

Effect of Discontinuities

- Looking at the equivalent circuit models for the microstrip discontinuities, the sharp reader will immediately notice that all these networks can be interpreted as Low-Pass Filters. The inductor attenuates electrical signal at high frequency while the capacitor shunts electrical energy at high frequency.
- Thus the effect of having too many discontinuities in a high-frequency circuit reduces the overall bandwidth of the interconnection.
- Another consequence of discontinuity is attenuation due to radiation from the discontinuity.



Some Intuitive Concepts on Discontinuities

- Seeing the equivalent circuit models on the previous slides, one can't help to wonder how does one knows which model to use for which discontinuity?
- The answer can be obtained by understanding the relationship between electric charge, electric field, current, magnetic flux linkage and quantities such as inductance and capacitance.
- A few observations are crucial:
  - As current encounter a bend, the flow is interrupted and the current is reduced. Moreover there will be accumulation of electric charges at the vicinity of the bend because of the constricted flow.
  - As current encounter a change in Tline width, the flow of charge either accelerate or decelerate.

Intuitive Concepts (1)

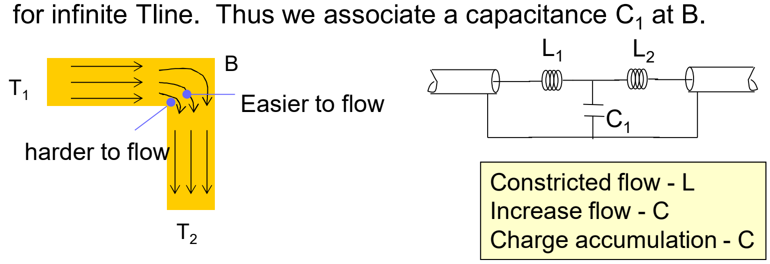
- Excess charge - whenever there is constricted flow or a sudden enlargement of Tline geometry, electric charge will accumulate. The amount of charge greater than the charge distribution on an infinite length of Tline is known as excess charge. The excess charge can be negative. Associated with excess charge is a capacitance.
- Excess flux - similarly constricted flow or ease of flow, change in Tline geometry also result in excess magnetic flux linkage. The amount of flux linkage greater than the flux linkage on an infinite length of Tline is known as excess flux. This excess flux is associated with an inductance, again the excess flux can be negative although it is usually positive (inductance is always corresponds to resistance in current flow, recall V=L(di/dt)).

Intuitive Concepts (2)

- Both excess charge and excess flux can be computed from the higher order mode EM fields in the vicinity of the discontinuity. For instance by subtracting the total E field from the normal E field distribution for infinite Tline, we would obtain the higher order modes E field (or local E field). From the boundary condition of the local E field with the conducting plate, the excess charge can be calculated. Similar procedure is carried out for the excess flux.
- This argument although presented for stripline, is also valid for coaxial line and waveguide in general.
- Usually numerical methods are employed to determine the total E and H field at the discontinuity, and it is assumed the fields are quasi-static.

Intuitive Concepts (3)

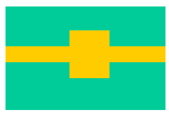
- For instance in a bend. As current approach point B, the current density changes. We can imagine that current flow easily in the middle as compared to near the edges. As a result the flux linkage at B for both T_1 and T_2 increases as compared to the flux linkage when there is no bend. Associated with the excess flux we introduce two series inductors, L_1 and L_2. The inductance are similar if T_1 and T_2 are similar in geometry.
- Also at B, more positive electric charges (we think in terms of conventional charge) accumulate as compared to the charge distribution for infinite Tline. Thus we associate a capacitance C_1 at B.

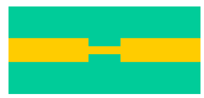


Constricted flow - L
Increase flow - C
Charge accumulation - C

Exercise 3.1

- What do you expect the equivalent circuit of the following discontinuity to be ?





Methods of Obtaining Equivalent Circuit Model for Discontinuities (1)

- **3 Typical approaches…**
- **Method 1**: Analytical solution - see Chapter 4, reference [3] on Modal Analysis for waveguide discontinuities.
- **Method 2**: Numerical methods such as Agilent's Momentum
  - Method of Moments (MOM).
  - Finite Element Method (FEM). Ansoft's HFSS
  - Finite Difference Time Domain Method (FDTD).
  - And many others.

•CST's Microwave Studio
•Sonnet

- Numerical methods are used to find the quasi-static EM fields of a 3D model containing the discontinuity. The EM field in the vicinity of the discontinuity is split into TEM and non-TEM fields. LC elements are then associated with the non-TEM fields using formula similar to (3.1) in Part 3.

Methods of Obtaining Equivalent Circuit Model for Discontinuities (2)

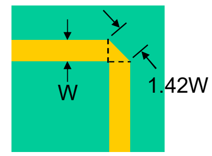
- **Method 3**: Fitting measurement with circuit models. By proposing an equivalent circuit model, we can try to tune the parameters of the circuit elements in the model so that frequency/time domain response from theoretical analysis and measurement match.
- Measurement can be done in time domain using time-domain reflectometry (TDR) and frequency domain measurement using a vector network analyzer (VNA) (see Chapter 3 of Ref [4] for details).

Radiation Loss from Discontinuities

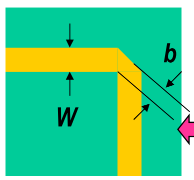
- At higher frequency, say > 5 GHz, the assumption of lossless discontinuity becomes flawed. This is because the higher order mode EM fields can induce surface wave on the printed circuit board, this wave radiates out so energy is loss.
- Furthermore the acceleration or deceleration of electric charge also generates radiation.
- The losses due to radiation can be included in the equivalent circuit model for the discontinuity by adding series resistance or shunt conductance.

Reducing the Effects of Discontinuity (1)

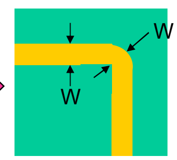
- To reduce the effect of discontinuity, we must reduce the values of the associated inductance and capacitance. This can be achieved by decreasing the abruptness of the discontinuity, so that current flow will not be disrupted and charge will not accumulate.



Chamfering of bends

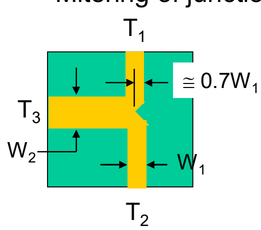


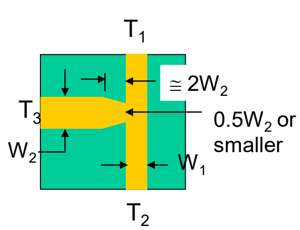
For 90º bend:
It is seen that the optimum chamfering is b=0.57W (see Chapter 5, Edwards [4])
For further examples see Chapter 2, Bahl [7].



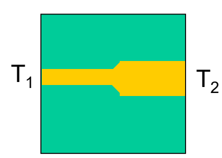
Reducing the Effects of Discontinuity (2)

Mitering of junction





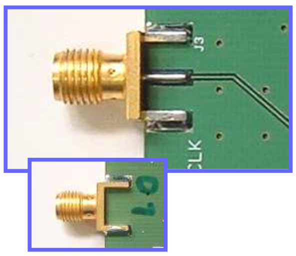
Mitering of step

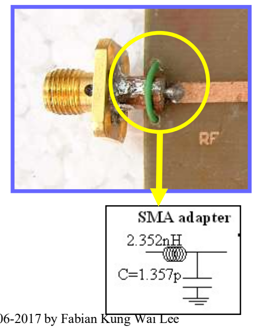


For more details of compensation for discontinuity, please refer to chapter 5 of Edwards [4] and Chapter 2 of Bahl [7].

Connector Discontinuity: Coaxial - Microstrip Line Transition (1)

- Since most microstrip line invariably leads to external connection from the printed circuit board, an interface is needed. Usually the microstrip line is connected to a co-axial cable.
- An adapter usually used for microstrip to co-axial transistion is the SMA to PCB adapter, also called the SMA End-launcher.





Connector Discontinuity: Coaxial - Microstrip Line Transition (2)

Extra

- Again the coaxial-to-microstrip transition is a form of discontinuity, care must be taken to reduce the abruptness of the discontinuity. For a properly designed transition such as shown in the previous slide, the operating frequency could go as high as 6 GHz for the coaxial to microstrip line transition and 9 GHz for the coaxial to co-planar line transition.

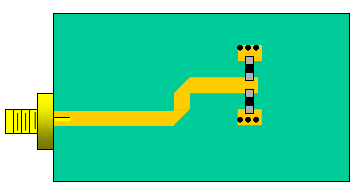
Exercise 3.2

- Explain qualitatively the effect of compensation on the equivalent electrical circuits of the discontinuity in the previous slides.

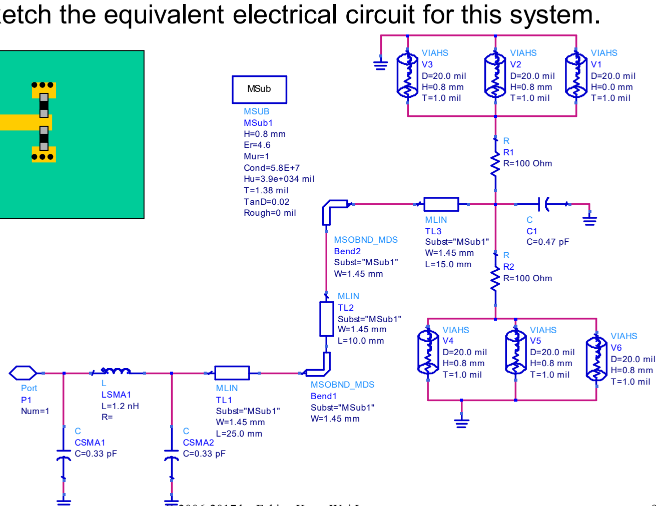
Example 3.2

Extra

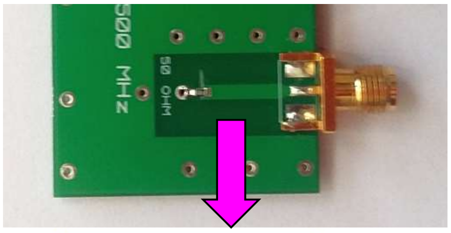
- A Z_c = 50Ω microstrip Tline is used to drive a resistive termination as shown. Sketch the equivalent electrical circuit for this system.

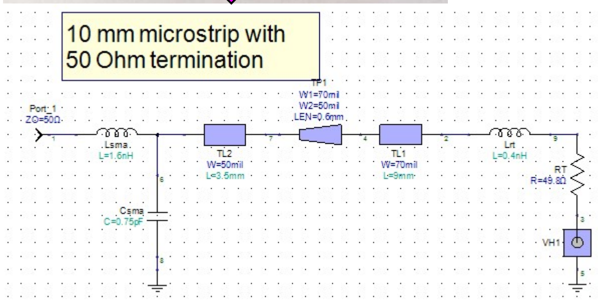


Top view



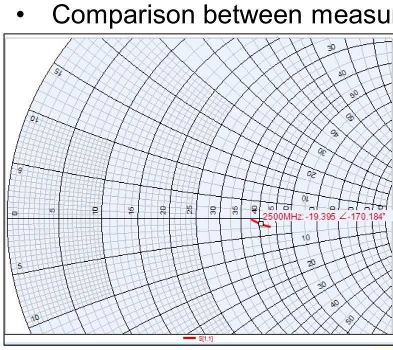
Example 3.3 – Modelling of Simple Microstrip Circuit

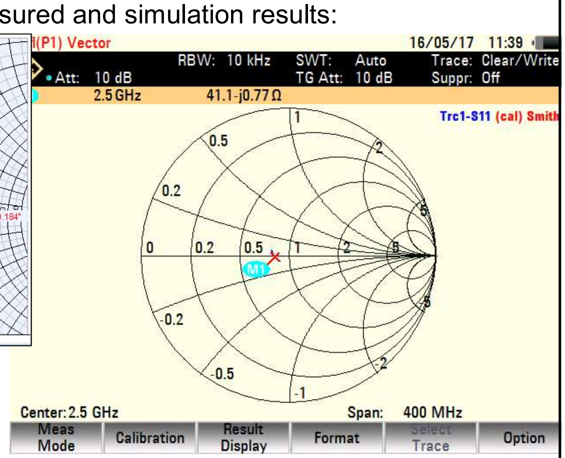




Example 3.3 Cont…

- Comparison between measured and simulation results:

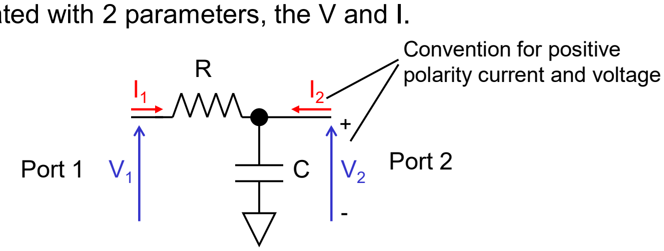




4.0 Linear RF Network Analysis – 2-Port Network Parameters

Network Parameters (1)

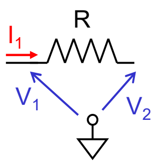
- Many times we are only interested in the voltage (V) and current (I) relationship at the terminals/ports of a complex circuit.
- If mathematical relations can be derived for V and I, the circuit can be considered as a black box.
- For a linear circuit, the I-V relationship is linear and can be written in the form of matrix equations.
- A simple example of linear 2-port circuit is shown below. Each port is associated with 2 parameters, the V and I.



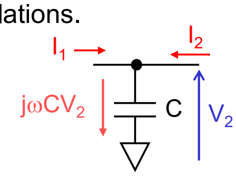
Network Parameters (2)

- For this 2 port circuit we can easily derive the I-V relations.

$$I_1 = \frac{1}{R}(V_1 - V_2)$$



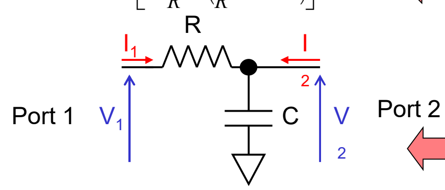
$$I_1 + I_2 = j\omega C V_2$$
$$\Rightarrow I_2 = -\frac{1}{R}V_1 + \left(\frac{1}{R} + j\omega C\right)V_2$$

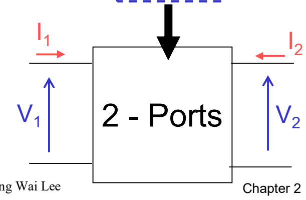


- We can choose V_1 and V_2 as the independent variables, the I-V relation can be expressed in matrix equations.

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{R} & -\frac{1}{R} \\ -\frac{1}{R} & \left(\frac{1}{R} + j\omega C\right) \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \Leftrightarrow \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \boxed{\begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix}} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

Network parameters (Y-parameters)





Network Parameters (3)

- To determine the network parameters, the following relations can be used:

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

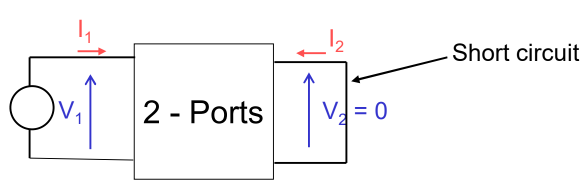
or

$$\overline{I} = \overline{\overline{Y}} \cdot \overline{V}$$

$$y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} \quad y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0}$$
$$y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0} \quad y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0}$$

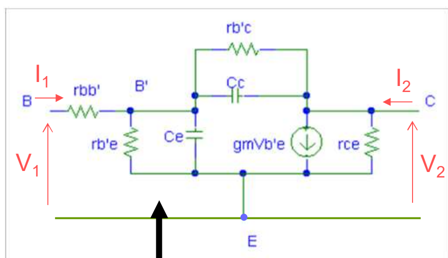
This means we short circuit the port

- For example to measure y_{11}, the following setup can be used:

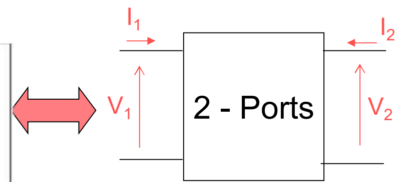


Network Parameters (4)

- By choosing different combination of independent variables, different network parameters can be defined. This applies to all linear circuits no matter how complex.
- Furthermore this concept can be generalized to more than 2 ports, called N - port networks.



Linear circuit, because all elements have linear I-V relation



Z-parameters

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

H-parameters

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

ABCD Parameters (1)

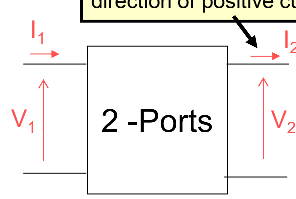
- Of particular interest in RF and microwave systems is ABCD parameters. ABCD parameters are the most useful for representing Tline and other linear microwave components in general.

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix} \quad \textbf{(4.1a)}$$

$$\Rightarrow V_1 = AV_2 + BI_2$$

$$I_1 = CV_2 + DI_2$$

Take note of the direction of positive current!



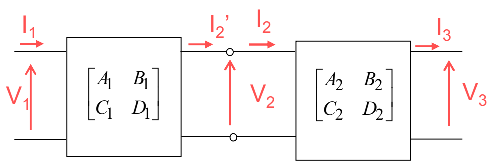
$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0} \quad B = \left.\frac{V_1}{I_2}\right|_{V_2=0} \quad C = \left.\frac{I_1}{V_2}\right|_{I_2=0} \quad D = \left.\frac{I_1}{I_2}\right|_{V_2=0} \quad \textbf{(4.1b)}$$

Open circuit Port 2

Short circuit Port 2

ABCD Parameters (2)

- The ABCD matrix is useful for characterizing the overall response of 2-port networks that are cascaded to each other.

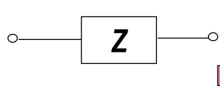


$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \begin{bmatrix} V_3 \\ I_3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A_3 & B_3 \\ C_3 & D_3 \end{bmatrix} \begin{bmatrix} V_3 \\ I_3 \end{bmatrix}$$

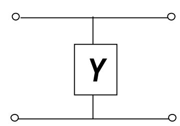
Overall ABCD matrix

ABCD Parameters of Some Useful 2-Port Network



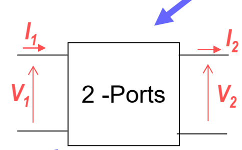
$$A = 1 \quad B = Z \quad C = 0 \quad D = 1$$

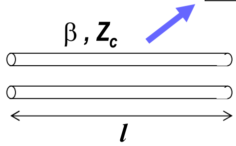
(4.2a)



$$A = 1 \quad B = 0 \quad C = Y \quad D = 1$$

(4.2b)



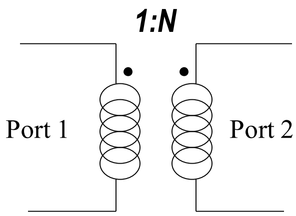


$$A = \cos \beta l$$
$$B = jZ_c \sin \beta l$$
$$C = j\frac{1}{Z_c}\sin \beta l$$
$$D = \cos \beta l$$

(4.2c)

Example 4.1

- Derive the ABCD parameters of an ideal transformer.



Hints: for an ideal transformer, the following relations for terminal voltages and currents apply:

$$V_2 = NV_1$$
$$V_2 I_2 = V_1 I_1$$

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} \frac{1}{N} & 0 \\ 0 & N \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$$

Exercise 4.1

- Derive equations (4.2a), (4.2b) and (4.2c).
- Hint : For the Tline, assume the voltage and current on the Tline to be the superposition of incident and reflected waves. And let the terminals at port 2 corresponds to z = 0 (assuming propagation along z axis).

Partial Solution for Exercise 4.1

For (4.2c)…

First we note that for terminated Tline, voltage and current along z axis are given by:

$$V(z) = V_o^+ e^{-j\beta z} + \Gamma_L V_o^+ e^{+j\beta z} \qquad I(z) = \frac{V_o^+}{Z_c} e^{-j\beta z} - \Gamma_L \frac{V_o^+}{Z_c} e^{+j\beta z}$$

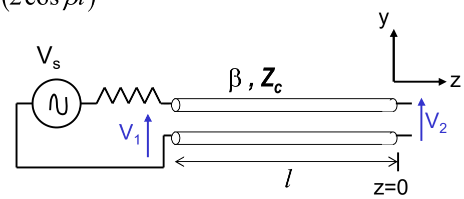
Case1: For I_2= 0 (open circuit port 2), Γ_L= 1, thus:

$$V_1 = V(z=-l) = V_o^+ e^{j\beta l}\left(1 + e^{-j2\beta l}\right) = V_o^+ (2\cos\beta l)$$

$$V_2 = V(z=0) = V_o^+ (1+1) = 2V_o^+$$

$$I_1 = I(z=-l) = \frac{V_o^+}{Z_c} e^{-j\beta z}\left(1 - e^{-j2\beta l}\right)$$

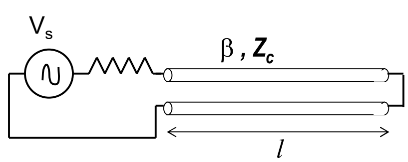
$$= \frac{V_o^+}{Z_c}(j2\sin\beta l)$$



Thus $A = \left.\frac{V_1}{V_2}\right|_{I_2=0} = \frac{V_o^+ 2\cos(\beta l)}{2V_o^+} = \cos(\beta l) \qquad C = \left.\frac{I_1}{V_2}\right|_{I_2=0} = \frac{\frac{V_o^+}{Z_c} j2\sin(\beta l)}{2V_o^+} = jY_c\cos(\beta l)$

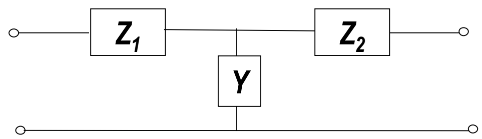
Partial Solution for Exercise 4.1 Cont…

Case 2: For V_2= 0, Γ_L = -1. Proceeding in a similar manner, we would be able to obtain B and D terms.



Exercise 4.2

- Find the ABCD parameters of the following network:



Hint: Consider each element as a 2-port network.

Conversion Between ABCD and Z,Y Parameters for 2-Ports Network

- By employing the definition of Z, Y and ABCD parameters for 2-port networks, we can easily proof the following conversion formula between network parameters.
- From ABCD to Z:

$$z_{11} = \frac{A}{C} \qquad z_{12} = \frac{AD - BC}{C} \qquad z_{21} = \frac{1}{C} \qquad z_{22} = \frac{D}{C} \qquad \textit{(4.3a)}$$

- From Z to ABCD:

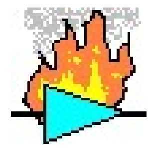
$$y_{11} = \frac{D}{B} \qquad y_{12} = \frac{BC - AD}{B} \qquad y_{21} = \frac{-1}{B} \qquad y_{22} = \frac{A}{B} \qquad \textit{(4.3b)}$$

- Between Z and Y parameters:

$$\bar{\bar{Z}} = \bar{\bar{Y}}^{-1} \qquad \bar{\bar{Y}} = \bar{\bar{Z}}^{-1} \qquad \textit{(4.3c)}$$

S-Parameters - Why Do We Need Them?

- Usually we use Y, Z, H or ABCD parameters to describe a linear two port network.
- These parameters require us to open or short a network to find the parameters.
- At radio frequencies it is difficult to have a proper short or open circuit, there are parasitic inductance and capacitance in most instances.
- Open and short conditions lead to standing wave, which can cause oscillation and destruction of the device.
- For non-TEM propagation mode, it is not possible to measure voltage and current. We can only measure power from E and H fields.



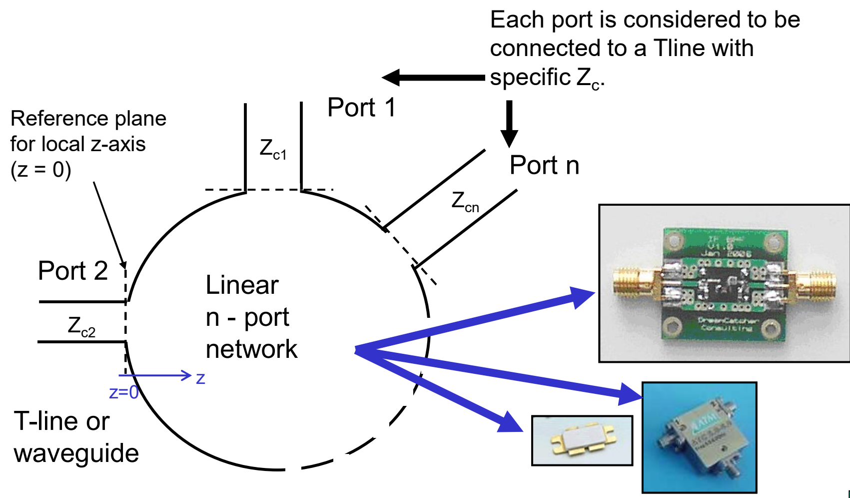
S-parameters

- Hence a new set of parameters (S) is needed which
  - Do not need open/short condition.
  - Do not cause standing wave.
  - Relates to incident and reflected power waves, instead of voltage and current.

• As oppose to V and I, S-parameters relate the reflected and incident voltage waves.
• S-parameters have the following advantages:
1. Relates to familiar measurement such as reflection coefficient, gain, loss etc.
2. Can cascade S-parameters of multiple devices to predict system performance (similar to ABCD parameters).
3. Can compute Z, Y or H parameters from S-parameters if needed.

Normalized Voltage/Current Waves (1)

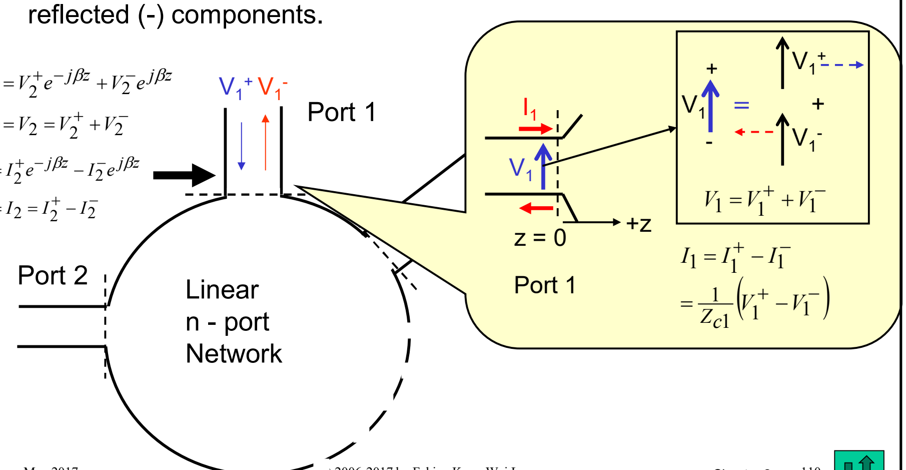
- Consider an n – port network:



Normalized Voltage/Current Waves (2)

- There is a voltage and current on each port.
- This voltage (or current) can be decomposed into the incident (+) and reflected (-) components.

$$V(z)=V_2^+e^{-j\beta z}+V_2^-e^{j\beta z}$$
$$V(0)=V_2=V_2^++V_2^-$$
$$I(z)=I_2^+e^{-j\beta z}-I_2^-e^{j\beta z}$$
$$I(0)=I_2=I_2^+-I_2^-$$



Normalized Voltage/Current Waves (3)

- The port voltage and current can be normalized with respect to the impedance connected to it.
- It is customary to define the normalized voltage waves at each port as:

Normalized incident waves

$$a_i = \frac{V_i^+}{\sqrt{Z_{ci}}} \qquad a_i = I_i^+ \sqrt{Z_{ci}} \qquad \textbf{(4.4a)}$$

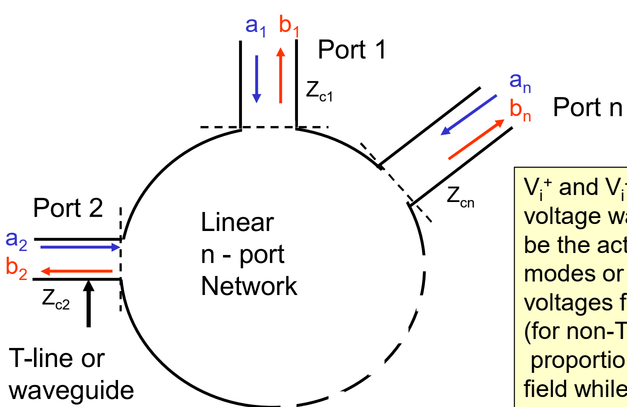
i = 1, 2, 3 … n

$$b_i = \frac{V_i^-}{\sqrt{Z_{ci}}} \qquad b_i = I_i^- \sqrt{Z_{ci}} \qquad \textbf{(4.4b)}$$

Normalized reflected waves

Normalized Voltage/Current Waves (4)

- Thus in general:

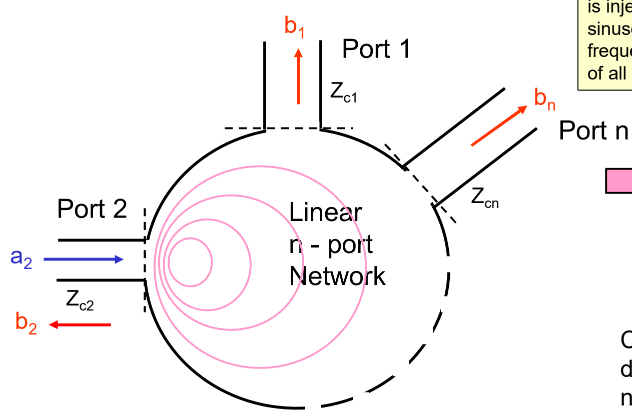


V_i^+ and V_i^- are propagating voltage waves, which can be the actual voltage for TEM modes or the equivalent voltages for non-TEM modes. (for non-TEM, V is defined proportional to transverse E field while I is defined proportional to transverse H field, see [1] for details).

Scattering Parameters (1)

- If the n – port network is linear (make sure you know what this means!), then there is a linear relationship between the normalized waves.
- For instance if we energize port 2:

In a nutshell: Linearity
When a sinusoidal EM wave is injected into one port, sinusoidal EM waves of similar frequency are generated out of all ports.



$$
\begin{aligned}
b_1 &= s_{12}a_2 \\
b_2 &= s_{22}a_2 \\
&\vdots \\
b_n &= s_{n2}a_2
\end{aligned}
$$

Constant that depends on the network construction

Scattering Parameters (2)

- Considering that we can send energy into all ports, this can be generalized to:

$$
\begin{aligned}
b_1 &= s_{11}a_1 + s_{12}a_2 + s_{13}a_3 + \cdots + s_{1n}a_n \\
b_2 &= s_{21}a_1 + s_{22}a_2 + s_{23}a_3 + \cdots + s_{2n}a_n \\
&\vdots \\
b_n &= s_{n1}a_1 + s_{n2}a_2 + s_{n3}a_3 + \cdots + s_{nn}a_n
\end{aligned}
\qquad \textit{(4.5a)}
$$

- Or written in Matrix equation:

$$
\bar{b} = \bar{\bar{S}}\bar{a} \quad \text{or} \quad
\begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix} =
\begin{bmatrix} s_{11} & s_{12} & \cdots & s_{1n} \\ s_{21} & s_{22} & \cdots & s_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ s_{n1} & s_{n2} & \cdots & s_{nn} \end{bmatrix}
\begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix}
\qquad \textit{(4.5b)}
$$

- Where s_{ij} is known as the generalized Scattering (S) parameter, or just S-parameters for short. From (4.4), each port *i* can have different characteristic impedance Z_{ci}.

Extra

Linear Relation Between a_i and b_i

- That a_i and b_i are related by linear relationship can be proved using Green's Function Theory for partial differential equations.
- For a hint on proof of this, you can refer to the advanced text by R.E. Collins, "Field theory of guided waves", IEEE Press, 1991.

S-parameters for 2-port Networks

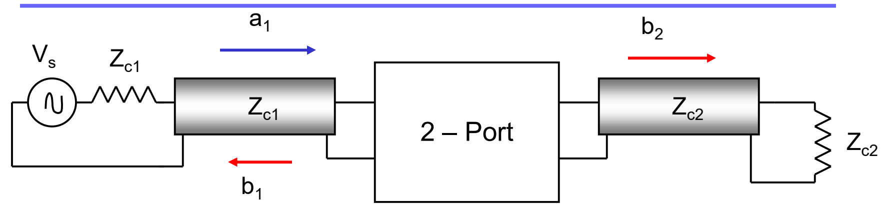
- For a 2-port networks, (4.5) reduces to:

$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} s_{11} & s_{12} \\ s_{21} & s_{22} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = \overline{\overline{S}} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} \qquad \textbf{(4.6a)}$$

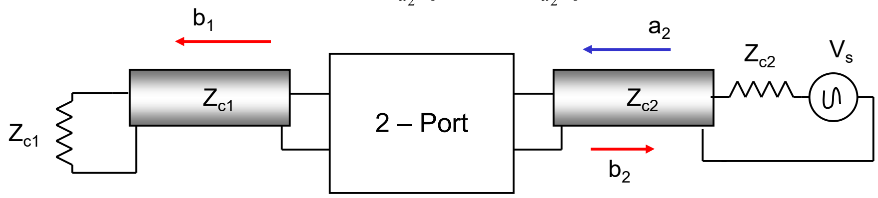
$$s_{11} = \left.\frac{b_1}{a_1}\right|_{a_2=0} \quad s_{21} = \left.\frac{b_2}{a_1}\right|_{a_2=0} \quad s_{22} = \left.\frac{b_2}{a_2}\right|_{a_1=0} \quad s_{12} = \left.\frac{b_1}{a_2}\right|_{a_1=0} \qquad \textbf{(4.6b)}$$

- Note that $a_i = 0$ implies that we terminate the *i*th port with its characteristic impedance.
- Thus zero reflection eliminates standing wave.
- Good termination can be established reliably for RF and Microwave frequencies in a number of transmission line systems. This will be illustrated in the following slides.

Measurement of S-parameter for 2-port Networks



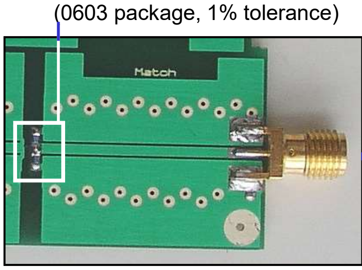
Measurement of s_{11} and s_{21}: $s_{11} = \left.\frac{b_1}{a_1}\right|_{a_2=0}$ $s_{21} = \left.\frac{b_2}{a_1}\right|_{a_2=0}$



Measurement of s_{22} and s_{12}: $s_{22} = \left.\frac{b_2}{a_2}\right|_{a_1=0}$ $s_{12} = \left.\frac{b_1}{a_2}\right|_{a_1=0}$

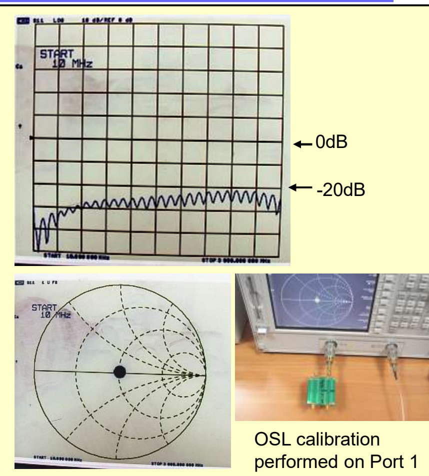
Example of Terminations

2 × 100Ω SMT resistor
(0603 package, 1% tolerance)



DIY 50Ω Termination for grounded co-planar Tline

Another example of broadband 50Ω co-axial termination



OSL calibration performed on Port 1

Measured s_{11} from 10 MHz to 3 GHz using Agilent's 8753ES VNA

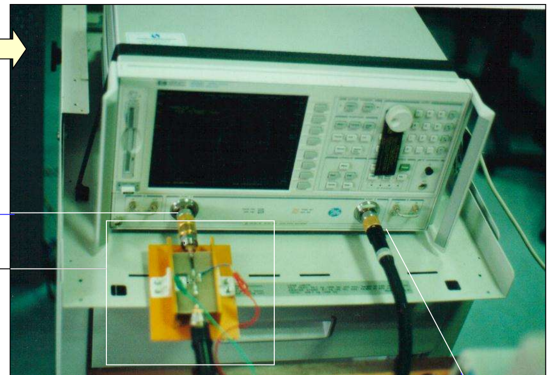
Practical Measurement of S-parameters

- Vector Network Analyzer (VNA) - an instrument that can measure the magnitude and phase of S_{11}, S_{12}, S_{21}, S_{22}.

An example of VNA by Agilent Technologies. Other manufacturers of VNA are Anritsu, Rhode & Schwartz etc.

Port 1

Device under test (DUT)

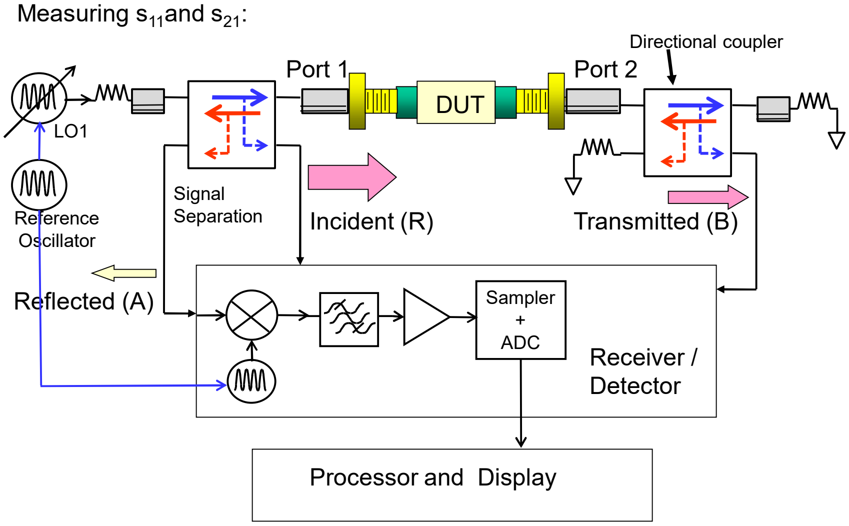


Port 2

General Vector Network Analyzer Block Diagram

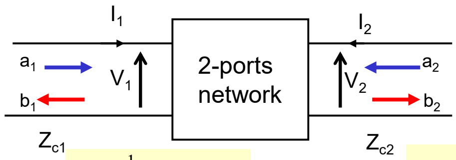
Extra

Measuring $s_{11}$and s_{21}:



Relationship Between Port Voltage/Current and Normalized Waves

- From the relations: $V_1 = V_1^+ + V_1^-$ $\quad I_1 = I_1^+ - I_1^- = \frac{1}{Z_c}\left(V_1^+ - V_1^-\right)$
- One can easily obtain *a* and *b* from the port voltages and currents (for instance a 2-port network):



Usually $Z_{c1} = Z_{c2} = Z_c$

$$a_1 = \frac{1}{2\sqrt{Z_{c1}}}\left(V_1 + Z_{c1}I_1\right) \qquad b_1 = \frac{1}{2\sqrt{Z_{c1}}}\left(V_1 - Z_{c1}I_1\right) \qquad \textbf{(4.7a)}$$

$$a_2 = \frac{1}{2\sqrt{Z_{c2}}}\left(V_2 + Z_{c2}I_2\right) \qquad b_2 = \frac{1}{2\sqrt{Z_{c2}}}\left(V_2 - Z_{c2}I_2\right) \qquad \textbf{(4.7b)}$$

- This shows that S-parameters can be computed if we know the port's voltage and current (take note these are phasors).
- Most RF circuit simulator software uses this approach to derive the S-parameters.

Power Waves

- Consider port i, since (assuming z = 0):

$$V_i = V_i^+ + V_i^- = \sqrt{Z_{ci}}\left(a_i + b_i\right)$$

$$I_i = I_i^+ - I_i^- = \frac{1}{\sqrt{Z_{ci}}}\left(a_i - b_i\right)$$

- Power along a waveguide or transmission line on Port i :

$$P_i = \frac{1}{2}\mathrm{Re}\left(V_i I_i{}^*\right) = \frac{1}{2}\left(\left|a_i\right|^2 - \left|b_i\right|^2\right) \qquad \textbf{(4.8)}$$

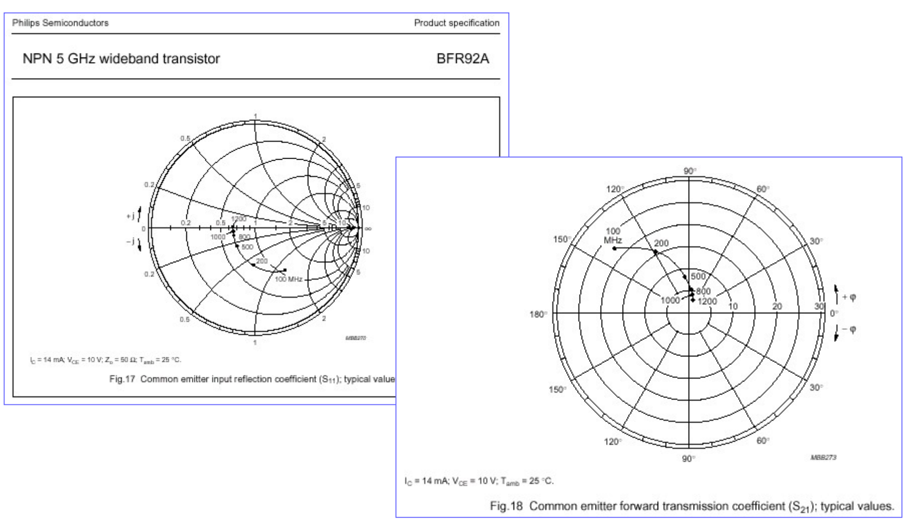
- Because of this, a_i and b_i are sometimes referred to as incident and reflected power waves. S-parameters relate the incident and reflected power of a port.

More on S Parameters

- S parameters are very useful at microwave frequency. Most of the performance parameters of microwave components such as attenuators, microwave FET/Transistors, coupler, isolator etc. are specified with S parameters.
- In fact, theories on the realizability of 3 ports and 4 ports network such as power divider, directional coupler are derived using the S matrix.
- In the subject "RF Transistor Circuits Design or RF Active Circuit Design", we will use S parameters exclusively to design various small signal amplifiers and oscillators.
- At present, the small signal performance of many microwave semiconductor devices is specified using S parameters.

Extra Knowledge 1 - Sample Datasheet of RF BJT

Extra



Extra Knowledge 2

Extra

- We can actually use S matrix to relate reflected voltage waves to incident voltage waves. Call this the S' matrix to distinguish from the S matrix which relate the generalized voltage waves.
- The reason generalized voltage and current are used more often is the ratio of generalized voltage to current at a port n is always 1. This is useful in deriving some properties of the S matrix.

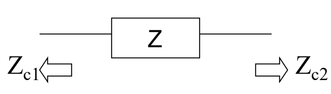
$$\begin{bmatrix} V_1^- \\ V_2^- \\ \vdots \\ V_n^- \end{bmatrix} = \begin{bmatrix} S_{11}' & S_{12}' & \cdots & S_{1n}' \\ S_{21}' & S_{22}' & \cdots & S_{2n}' \\ \vdots & \vdots & \ddots & \vdots \\ S_{n1}' & S_{n2}' & \cdots & S_{nn}' \end{bmatrix} \begin{bmatrix} V_1^+ \\ V_2^+ \\ \vdots \\ V_n^+ \end{bmatrix}$$

$$\left(\frac{V_i^+}{\sqrt{Z_{ci}}}\right) \Big/ \left(\sqrt{Z_{ci}}\, I_i^+\right) = \frac{V_i^+}{I_i^+} \cdot \frac{1}{Z_{ci}} = 1$$

- Of course when the characteristic impedance of all Tlines in the system are similar, then S' = S.

Example 4.2 – S-parameters for Series Impedance

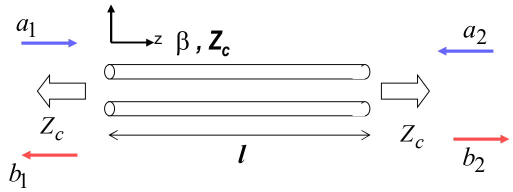
- Find the S matrix of the 2 port network below.



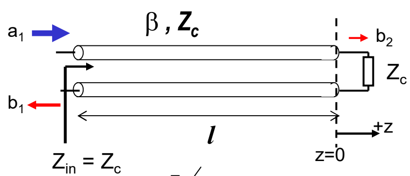
See extra notes for solution.

Example 4.3 – S-parameters for Lossless Tline Section

- Derive the 2-port S-matrix for a Tline as shown below.



Case 1: Terminated Port 2



$$s_{11} = \left.\frac{b_1}{a_1}\right|_{a_2=0} = \frac{V_1^- / \sqrt{Z_c}}{V_1^+ / \sqrt{Z_c}} = \Gamma_1 = \frac{Z_1 - Z_c}{Z_1 + Z_c} = 0$$

Since $V_2^- e^{j\beta l} = V_1^+$

$$\Rightarrow \frac{V_2^-}{V_1^+} = \frac{V_2^- / \sqrt{Z_c}}{V_1^+ / \sqrt{Z_c}} = e^{-j\beta l} = \left.\frac{b_2}{a_1}\right|_{a_2=0} = s_{21}$$

Case 2: from symmetry, $s_{11} = s_{22} = 0$, $s_{12}=s_{21}=e^{j\beta \cdot l}$

$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} 0 & e^{-j\beta l} \\ e^{-j\beta l} & 0 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

Exercise 4.3 – Conversion Between Impedance (Z) and S Matrix

- Show how we can convert from Z matrix to S matrix and vice versa. hint: use equations (4.5) and (4.7) and the fact that:

$$\overline{V}^+ + \overline{V}^- = \overline{V} \qquad \overline{I}^+ - \overline{I}^- = \overline{I}$$

- For a system with $Z_{c1} = Z_{c2} = \ldots = Z_{cn} = Z_c$:

$$\overline{\overline{E}} = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{bmatrix}$$

$$\overline{\overline{S}} = \left(\overline{\overline{Z}} - Z_c\overline{\overline{E}}\right)\left(\overline{\overline{Z}} + Z_c\overline{\overline{E}}\right)^{-1}$$

$$\overline{\overline{Z}} = Z_c\left(\overline{\overline{S}} + \overline{\overline{E}}\right)\left(\overline{\overline{E}} - \overline{\overline{S}}\right)^{-1}$$

Exercise 4.3 Cont…

- Or expanding out …

$$z_{11} = Z_c \frac{(1+s_{11})(1-s_{22})+s_{12}s_{21}}{(1-s_{11})(1-s_{22})-s_{12}s_{21}}$$

$$z_{12} = Z_c \frac{2s_{12}}{(1-s_{11})(1-s_{22})-s_{12}s_{21}}$$

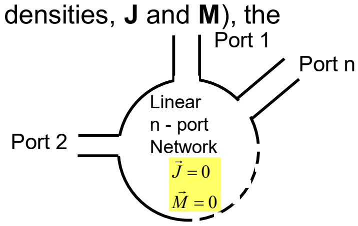
$$z_{21} = Z_c \frac{2s_{21}}{(1-s_{11})(1-s_{22})-s_{12}s_{21}}$$

$$z_{22} = Z_c \frac{(1-s_{11})(1+s_{22})+s_{12}s_{21}}{(1-s_{11})(1-s_{22})-s_{12}s_{21}}$$

S Matrix for Reciprocal Network – Symmetry (1)

- A linear N-port network is made up of materials which are isotropic and linear, the **E** and **H** fields in the network observe Lorentz reciprocity theorem (See Section 2.12, ref [1]).
- A special condition arises when the linear N-port network does not contain any sources (electric and magnetic current densities, **J** and **M**), the network is called reciprocal.

* See for instance the book by D.J. Griffith, "Introductory electrodynamics", Prentice Hall, or any EM book for further discussion on magnetization and μ (permeability)



- Reciprocal network cannot contain active devices, ferrites or plasmas.
- Active devices such as transistor contains equivalent current or voltage source in the model, while in ferro-magnetic material, the bound current due to magnetization* constitutes current source. Similarly the ions moving in a plasma also constitute current source.

S Matrix for Reciprocal Network – Symmetry (2)

- Under reciprocal condition, we can show that the S Matrix is symmetry, i.e. $s_{ij} = s_{ji}$.

$$\overline{\overline{S}} = \overline{\overline{S}}^t \qquad \textbf{(4.9)}$$

This is achieved by using Reciprocity Theorem in Electromagnetism, and using the definition of V and I from EM fields, one can show that the Z matrix is symmetrical. Then using the relationship between S and Z matrices, we can show that S matrix is also symmetry.

- For example for a 3-port reciprocal network:

$$\overline{\overline{S}} = \begin{bmatrix} s_{11} & s_{12} & s_{13} \\ s_{21} & s_{22} & s_{23} \\ s_{31} & s_{32} & s_{33} \end{bmatrix} = \begin{bmatrix} s_{11} & s_{21} & s_{31} \\ s_{12} & s_{22} & s_{32} \\ s_{13} & s_{23} & s_{33} \end{bmatrix} = \overline{\overline{S}}^t$$

- Many types of RF components fulfill reciprocal and linear conditions, for example passive filters, impedance matching networks, power splitter/combiner etc.
- You can refer to Section 4.2 and 4.3 of Ref. [3] or extra notes from F. Kung for the derivation.

S Matrix for Lossless Network - Unitary

- When the network is lossless, then no real power can be delivered to the network. By considering the voltage and current at each port, and equating total incident power to total reflected power, we can show that:

$$|a_1|^2 + |a_2|^2 + \ldots + |a_n|^2 = |b_1|^2 + |b_2|^2 + \ldots + |b_n|^2$$

$$\left[\overline{\overline{S}}^*\right]^t = \left[\overline{\overline{S}}^t\right]^* = \overline{\overline{S}}^{-1}$$

$$\left[\overline{\overline{S}}^t\right]^* \overline{\overline{S}} = \overline{\overline{S}}\left[\overline{\overline{S}}^t\right]^* = \begin{bmatrix} 1 & 0 & .. & 0 \\ 0 & 1 & .. & 0 \\ : & : & \ddots & 0 \\ 0 & 0 & .. & 1 \end{bmatrix} = \overline{\overline{U}} \qquad \textbf{(4.10)}$$

- Again you can refer to Section 4.3 of Ref. [3] or extra note from F.Kung for the derivation.
- Matrix S of this form is known as Unitary.

Reciprocal and Lossless Network

- Thus when the network is both reciprocal and lossless, symmetry and unitary of the S matrix are fulfilled.

$$\overline{\overline{S}} \cdot \left(\overline{\overline{S}}^* \right)^t = \overline{\overline{S}} \cdot \overline{\overline{S}}^* = \overline{\overline{U}}$$

- This is the case for many microwave circuits, for instance those constructed using stripline technology.

Conversion Between ABCD and S-Parameters

- For 2-port networks, with $Z_{c1} = Z_{c2} = Z_o$:

$$A = \frac{(1+S_{11})(1-S_{22}) + S_{12}S_{21}}{2S_{21}}$$
$$B = Z_o \frac{(1+S_{11})(1+S_{22}) - S_{12}S_{21}}{2S_{21}}$$
$$C = \frac{1}{Z_o}\frac{(1-S_{11})(1-S_{22}) - S_{12}S_{21}}{2S_{21}}$$
$$D = \frac{(1-S_{11})(1+S_{22}) - S_{12}S_{21}}{2S_{21}}$$

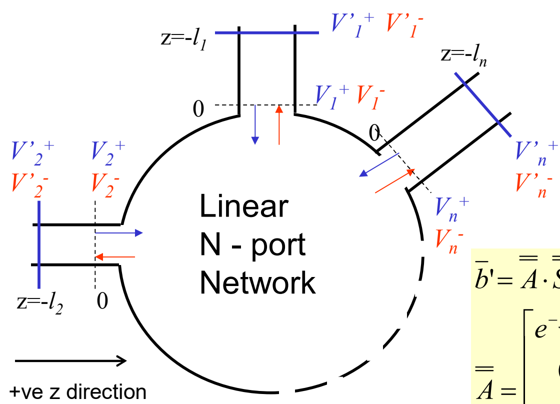
(4.11a)

$$S_{11} = \frac{A + B/Z_o - CZ_o - D}{A + B/Z_o + CZ_o + D}$$
$$S_{12} = \frac{2(AD - BC)}{A + B/Z_o + CZ_o + D}$$
$$S_{21} = \frac{2}{A + B/Z_o + CZ_o + D}$$
$$S_{22} = \frac{-A + B/Z_o - CZ_o + D}{A + B/Z_o + CZ_o + D}$$

(4.11b)

Shift in Reference Plane (1)

Extra



Note: For a wave propagating in +ve z direction, $V_i^+ e^{-j\beta l}$

Let:

$$a_i' = \frac{V'^+_i}{\sqrt{Z_{ci}}} \qquad b_i' = \frac{V'^-_i}{\sqrt{Z_{ci}}}$$

i = 1,2,…n

$$\bar{b}' = \bar{\bar{A}} \cdot \bar{\bar{S}} \cdot \bar{\bar{A}} \cdot \bar{a}' \qquad \textbf{\textit{(4.12)}}$$

$$\bar{\bar{A}} = \begin{bmatrix} e^{-j\beta_1 l_1} & 0 & \dots & 0 \\ 0 & e^{-j\beta_2 l_2} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & e^{-j\beta_n l_n} \end{bmatrix}$$

Shift in Reference Plane (2)

Extra

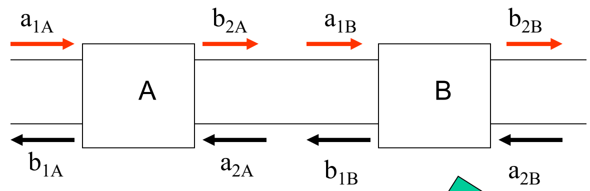
- For 2-port network:

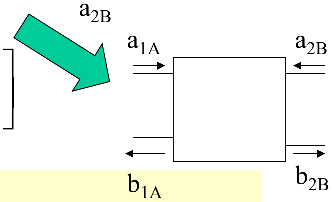
$$\begin{bmatrix} b_1' \\ b_2' \end{bmatrix} = \bar{\bar{S}}' \begin{bmatrix} a_1' \\ a_2' \end{bmatrix} \quad , \quad \bar{\bar{A}} = \begin{bmatrix} e^{-j\beta_1 l_1} & 0 \\ 0 & e^{-j\beta_2 l_2} \end{bmatrix}$$

$$\bar{\bar{S}}' = \bar{\bar{A}}\bar{\bar{S}}\bar{\bar{A}} = \begin{bmatrix} s_{11} e^{-j2\beta_1 l_1} & s_{12} e^{-j(\beta_1 l_1 + \beta_2 l_2)} \\ s_{21} e^{-j(\beta_1 l_1 + \beta_2 l_2)} & s_{22} e^{-j2\beta_2 l_2} \end{bmatrix} \qquad \textbf{\textit{(4.13)}}$$

Extra

Cascading 2-port Networks





Let: $\begin{bmatrix} b_{1B} \\ b_{2B} \end{bmatrix} = \overline{\overline{S}}_B \begin{bmatrix} a_{1B} \\ a_{2B} \end{bmatrix}$ $\quad \begin{bmatrix} b_{1A} \\ b_{2A} \end{bmatrix} = \overline{\overline{S}}_A \begin{bmatrix} a_{1A} \\ a_{2A} \end{bmatrix}$

New S matrix:

$$\begin{bmatrix} b_{1A} \\ b_{2B} \end{bmatrix} = \overline{\overline{S}} \begin{bmatrix} a_{1A} \\ a_{2B} \end{bmatrix}$$

$$\overline{\overline{S}} = \frac{1}{1 - S_{11B}S_{22A}} \begin{bmatrix} S_{11A} - S_{11B}D_A & S_{12A}S_{12B} \\ S_{21B}S_{21A} & S_{22B} - S_{22A}D_B \end{bmatrix} \qquad (4.14)$$

THE END