7- Small-Signal Amplifier Theory

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References

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- [2] R.E. Collin, "Foundations for microwave engineering", 2nd Edition, 1992 McGraw-Hill.
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*Recommended

1.0 Basic Amplifier Concepts

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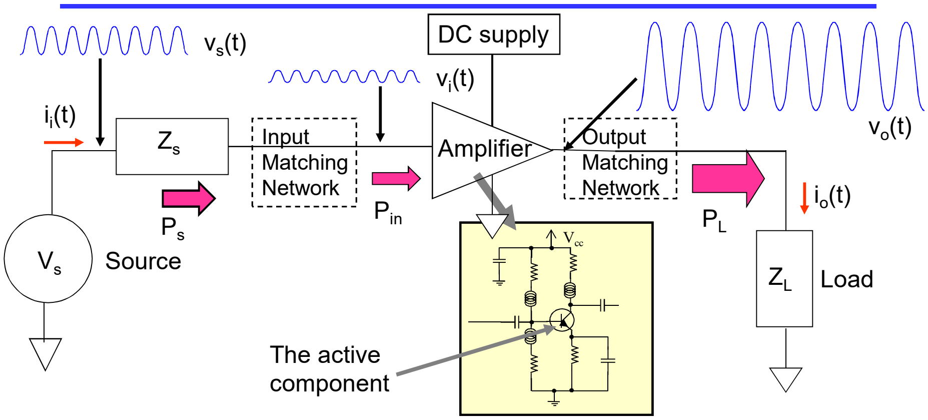
General Amplifier Block Diagram



Input and output voltage relation of the amplifier can be modeled simply as:

$$v_o(t) = a_1 v_i(t) + a_2 v_i^2(t) + a_3 v_i^3(t) + H.O.T.$$

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Amplifier Classification

- Amplifier can be categorized in 2 manners.
- According to signal level:
  - Small-signal/Linear Amplifier (whenever hybrid-π model is accurate).
  - Power/Large-signal Amplifier.

Our approach in this chapter

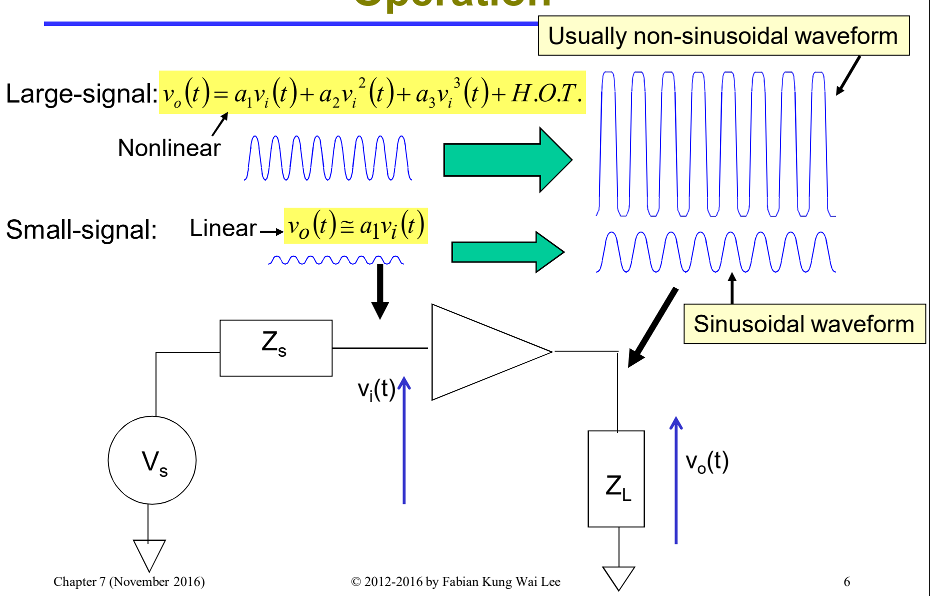
- According to D.C. biasing scheme of the active component and the large-signal voltage current waveforms:
  - Class A.
  - Class B.
  - Class AB.
  - Class C.

Class B has a number of variants apart from the classical Class B amplifier. Each of these variants varies in the shape of the voltage current waveforms, such as Class D (D stands for digital), Class E and Class F.

Small-Signal Versus Large-Signal Operation

Large-signal: $v_o(t) = a_1 v_i(t) + a_2 v_i^2(t) + a_3 v_i^3(t) + H.O.T.$

Small-signal: Linear $v_o(t) \cong a_1 v_i(t)$



Small-Signal Amplifier (SSA)

- All amplifiers are inherently nonlinear.
- However when the input signal is small, the input and output relationship of the amplifier is approximately linear.

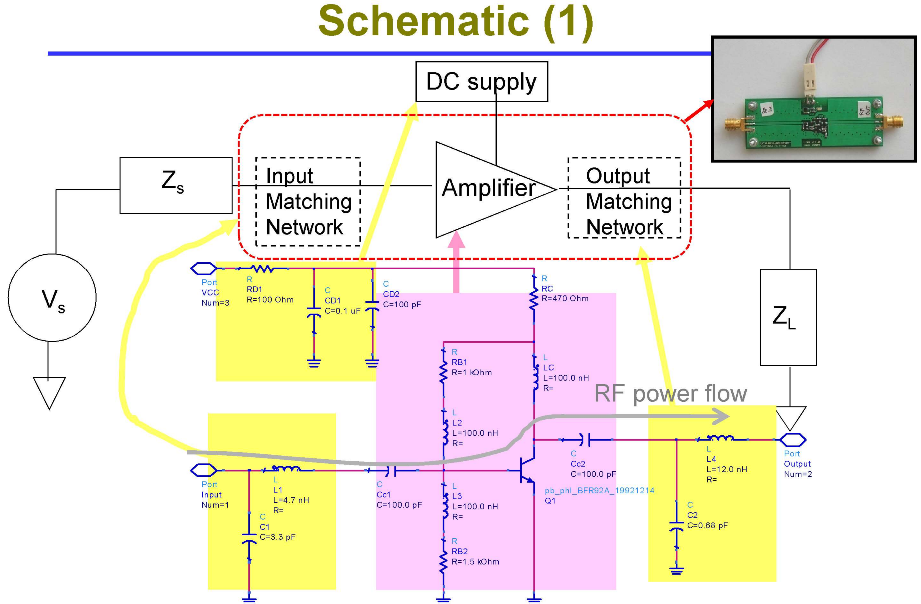
$$v_o(t) = a_1 v_i(t) + a_2 v_i^2(t) + a_3 v_i^3(t) + H.O.T. \cong a_1 v_i(t)$$

Linear relation

When $v_i(t) \to 0$ (< 2.6mV) $\longrightarrow$ $v_o(t) \cong a_1 v_i(t)$ ***(1.1)***

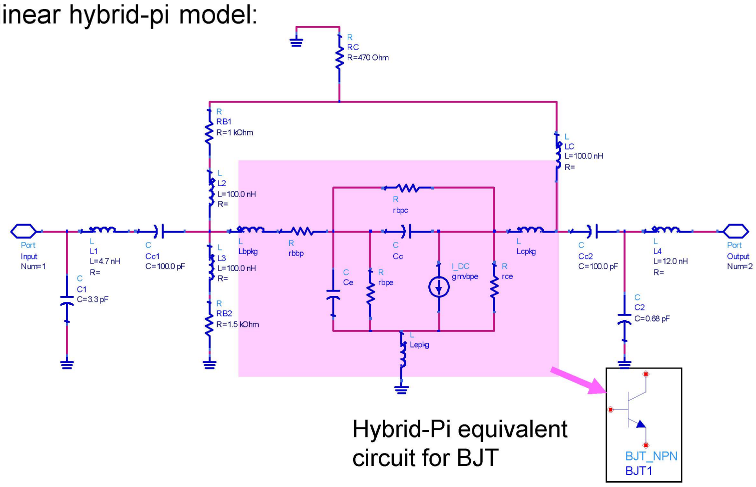
- This linear relationship also applies to current and power.
- An amplifier that fulfills these conditions: (1) small-signal operation (2) linear, is called Small-Signal Amplifier (SSA). SSA will be our focus.
- If a SSA amplifier contains BJT and FET, these components can be replaced by their respective linear small-signal model, for instance the hybrid-Pi model for BJT.

Example 1.1 - An RF Amplifier Schematic (1)



Example 1.1 Cont…

- Under AC and small-signal conditions, the BJT can be replaced with linear hybrid-pi model:



Hybrid-Pi equivalent circuit for BJT

Typical Amplifier Characteristics

- To determine the performance of an amplifier, the following characteristics are typically observed.
- 1. Power Gain.
- 2. Bandwidth (operating frequency range).
- 3. Noise Figure.
- 4. Phase response.
- 5. Gain compression.
- 6. Dynamic range.
- 7. Harmonic distortion.
- 8. Intermodulation distortion.
- 9. Third order intercept point (TOI).

Important to small-signal amplifier

Important parameters of large-signal amplifier (Related to Linearity)

Will elaborate in "High-Power Circuits"

Power Gain

- For amplifiers functioning at RF and microwave frequencies, it is the input and output power relation that is off interest, instead of voltage and current gain.
- The ratio of output power over input power is called the **Power Gain (G)**, and is usually expressed in logarithmic scale (dB).

Power Gain $$G = 10\log_{10}\left(\frac{\text{Output Power}}{\text{Input Power}}\right) \text{ dB} \qquad \textit{(1.2)}$$

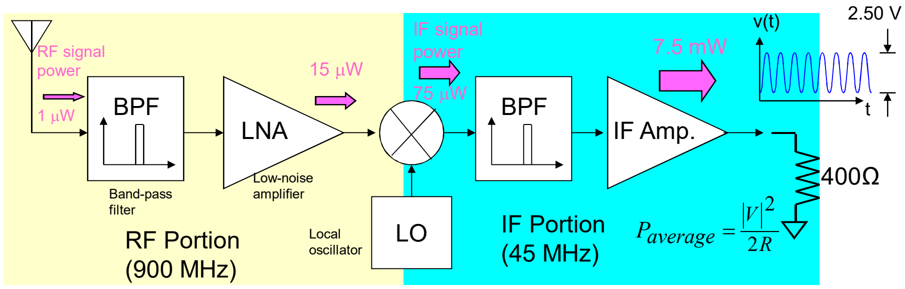
- There are a number of definitions for power gain as we will see shortly.
- Furthermore G is a function of **frequency** and the **input signal level**.

Why Power Gain for RF and Microwave Circuits? (1)

- For high frequency amplifiers the impedance encountered is usually low (due to the presence of parasitic capacitance).
- For instance an amplifier is required to drive a 50Ω load, the voltage across the load may be small, although the corresponding current may be large. Thus the voltage gain is small and the current gain is high. Overall we have power gain because: **Power = Voltage x Current**
- If the amplifier is now functioning at low frequency (< 50 MHz), it is the voltage gain that is of interest, since impedance encountered is usually higher (less parasitic capacitance). Thus the voltage gain is large and the current gain is low.
- Finally at mid-range frequency, the amplifier may have both voltage and current gain.
- In all cases*, we find that it is more convenient to state the amplification ability of the amplifier in terms of power gain, as this encompasses both voltage and current gains, and can be applied to all frequency bands and impedance.*

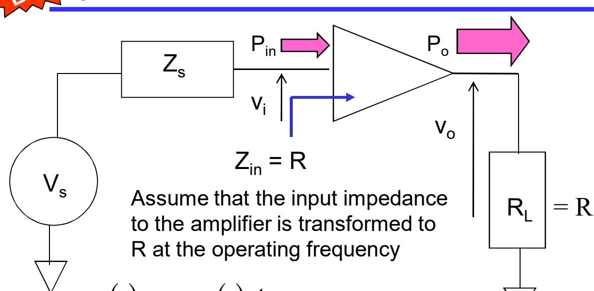
Why Power Gain for RF and Microwave Circuits? (2)

- RF engineers focus on power gain. By working with power gain, the RF designer is free from the constraint of system impedance.
- For instance in the simple receiver block diagram below, each block contribute some power gain. If the final output power is high, a large voltage signal can be obtained from the output of the final block by attaching a high impedance load to it's output.



Derivation of Input and Output Power Relationship for Small-Signal Operation

Extra



- Usually we express power in logarithmic scale (i.e. dBm) so that we can plot very large and very small power on the same axis.
- Here the relation between input and output power is in dB.

$v_o(t) \cong a_1 v_i(t)$ For small-signal operation

$$\frac{1}{2R} v_o^2 \cong a_1^2 \frac{1}{2R} v_i^2$$

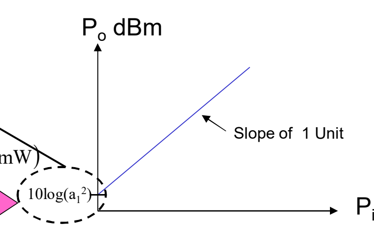
$$\Rightarrow P_o \cong a_1^2 P_{in}$$

$$\Rightarrow P_o / 1\text{mW} \cong a_1^2 (P_{in} / 1\text{mW})$$

$$\Rightarrow 10 \cdot \log(P_o / 1\text{mW}) \cong 10 \cdot \log(a_1^2) + 10 \cdot \log(P_{in} / 1\text{mW})$$

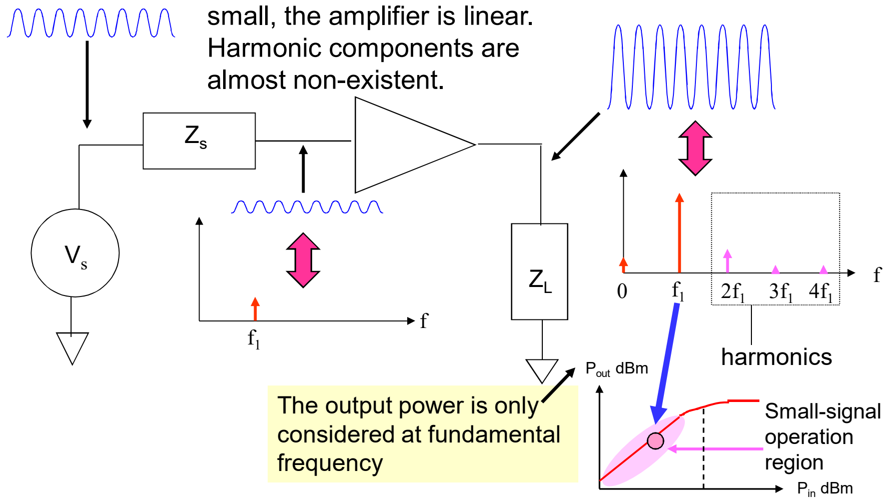
$$\Rightarrow \boxed{P_o dBm \cong 10 \cdot \log(a_1^2) + P_{in} dBm}$$

Power gain in dB



Harmonic Distortion (1)

When the input driving signal is small, the amplifier is linear. Harmonic components are almost non-existent.



The output power is only considered at fundamental frequency

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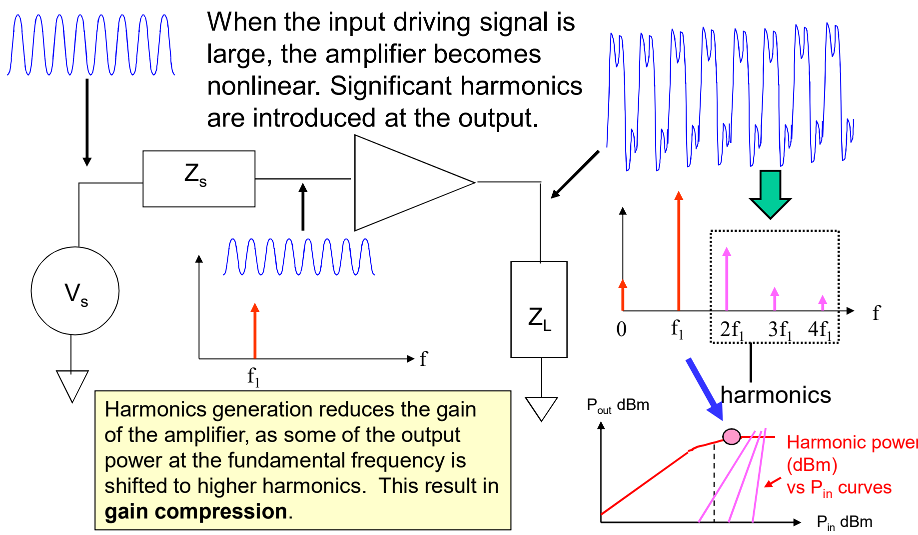
Harmonic Distortion (2)

When the input driving signal is large, the amplifier becomes nonlinear. Significant harmonics are introduced at the output.



Harmonics generation reduces the gain of the amplifier, as some of the output power at the fundamental frequency is shifted to higher harmonics. This result in **gain compression**.

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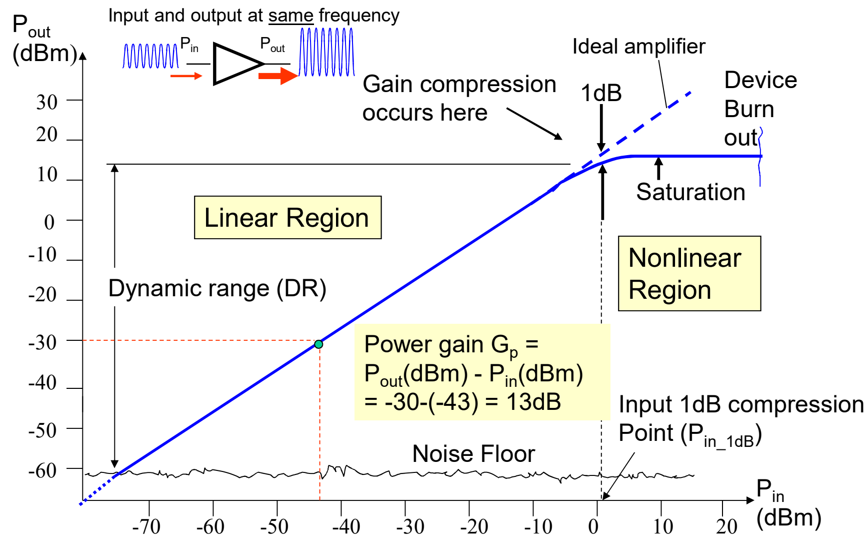
Power Gain, Dynamic Range and Gain Compression

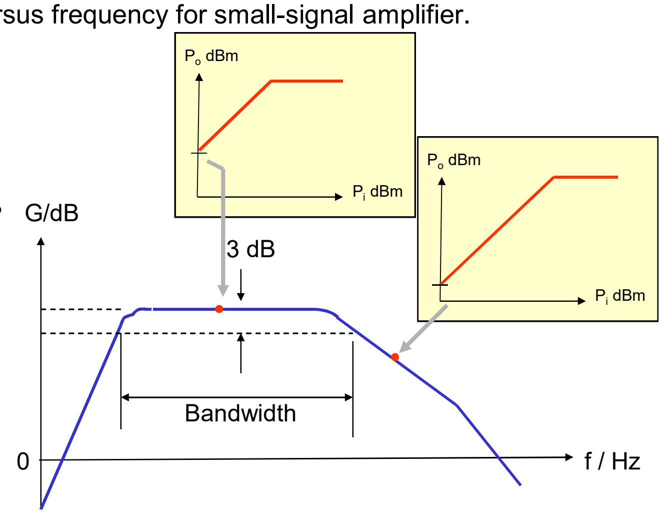


Extra

Bandwidth

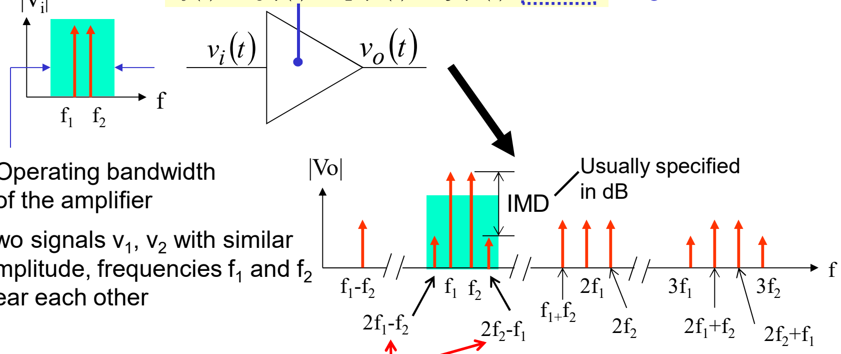
- Power gain G versus frequency for small-signal amplifier.

Can you explain why the power gain changes with frequency? What's the physical reasons for the positive slope at lower frequency and negative slope at higher frequency?



Intermodulation Distortion (IMD)

$$v_o(t) = a_1 v_i(t) + a_2 v_i^2(t) + a_3 v_i^3(t) + \boxed{H.O.T.}$$ — ignored



Operating bandwidth of the amplifier

Two signals v_1, v_2 with similar amplitude, frequencies f_1 and f_2 near each other

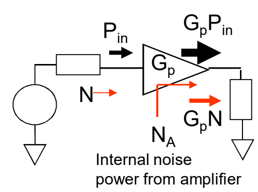
More will be said about this later in large-signal amplifier design.

These are intermodulation components, caused by the term $\alpha_3 v_i^3(t)$, which falls in the operating bandwidth of the amplifier. The difference between IM power and signal power is called IMD.

Noise Factor (NF) (1)

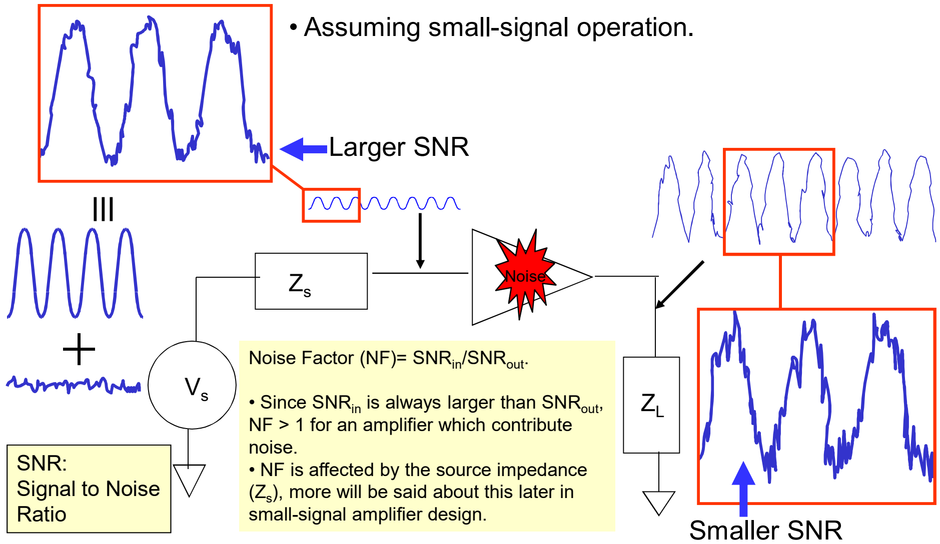
- The output of a practical amplifier can be decomposed into the required signal power and noise power.
- The noise power consist of amplified input noise, and internal noise sources due to the device physics.
- As the quality of a signal in the presence of noise can be measured by the signal-to-noise ratio (SNR), we would expect the SNR at the amplifier output to be worse than the SNR at the input due to addition of the internal noise.
- The degree of SNR degradation is measured by the ratio known as Noise Factor (this ratio is also expressed in dB, called Noise Figure, F).

$$NF = \frac{SNR_{in}}{SNR_{out}}$$



Noise Factor (2)

• Assuming small-signal operation.



Noise Factor (NF)= SNR_{in}/SNR_{out}.

• Since SNR_{in} is always larger than SNR_{out}, NF > 1 for an amplifier which contribute noise.
• NF is affected by the source impedance (Z_s), more will be said about this later in small-signal amplifier design.

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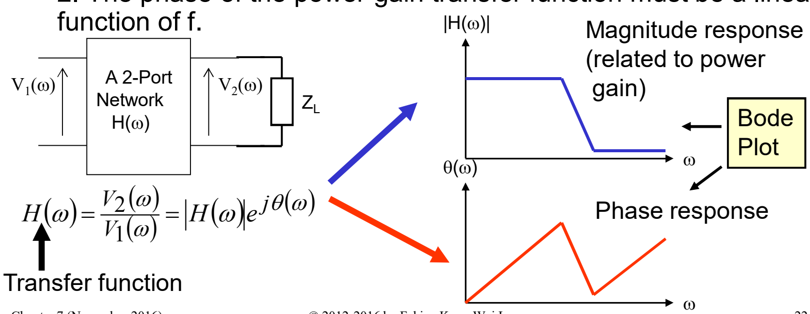
Extra

Phase Response (1)

- Phase consideration is important for amplifier working with wideband signals (for instance digital pulses).
- For a signal to be amplified with no distortion, 2 requirements are needed (from linear systems theory).
  - 1. The magnitude of the power gain transfer function must be a constant with respect to frequency f.
  - 2. The phase of the power gain transfer function must be a linear function of f.



$$H(\omega)=\frac{V_2(\omega)}{V_1(\omega)}=|H(\omega)|e^{j\theta(\omega)}$$

Transfer function

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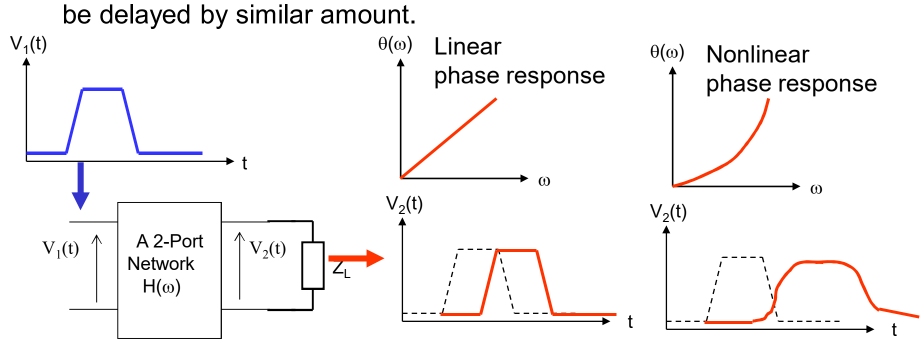
Phase Response (2)

- A linear phase produces a constant time delay for all signal frequencies, and a nonlinear phase shift produces different time delay for different frequencies.
- Property (1) means that all frequency components will be amplified by similar amount, property (2) implies that all frequency components will be delayed by similar amount.



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2.0 Small-Signal Amplifier Power Gain Expressions

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The Theory of Maximum Power Transfer (1)

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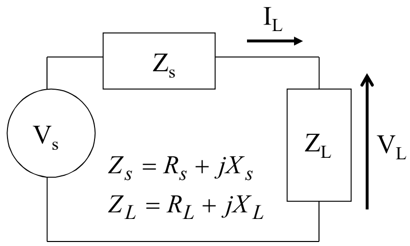

Basic source-load network

Time averaged power dissipated across load Z_L: $P_L = \frac{1}{2}\mathrm{Re}\left\{V_L I_L^*\right\}$

where $V_L = \frac{V_s Z_L}{Z_s + Z_L}$ $\quad I_L = \frac{V_s}{Z_s + Z_L}$

$$P_L = \frac{1}{2}\mathrm{Re}\left\{\frac{V_s Z_L}{Z_s + Z_L}\cdot\left(\frac{V_s}{Z_s + Z_L}\right)^*\right\} = \frac{1}{2}\mathrm{Re}\left\{\frac{|V_s|^2 Z_L}{|Z_s + Z_L|^2}\right\}$$

$$\Rightarrow P_L = \frac{1}{2}\frac{|V_s|^2 R_L}{(R_s + R_L)^2 + (X_s + X_L)^2}$$

$$P_L = P_L(R_L, X_L)$$

Letting $\frac{\partial P_L}{\partial R_L} = \frac{\partial P_L}{\partial X_L} = 0$

We find that the value for R_L and X_L that would maximize P_L is
$R_L = R_s$, $X_L = -X_s$.
In other words: $Z_L = Z_s^*$

To maximize power transfer to the load impedance, Z_L must be the complex conjugate of Z_s, a notion known as Conjugate Matched.

The Theory of Maximum Power Transfer (2)

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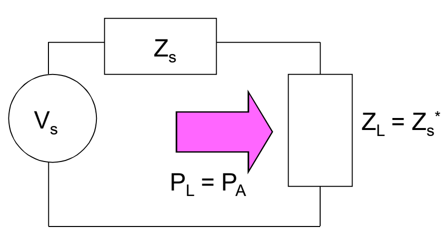
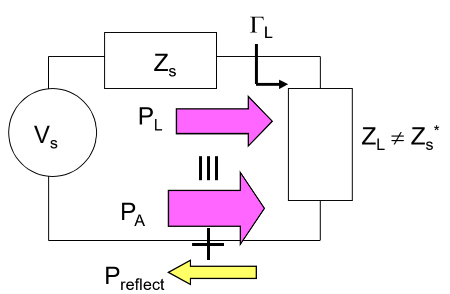


Under conjugate match condition:

$$P_L(\max) = \frac{|V_s|^2}{8R_s} = P_A$$ Available Power



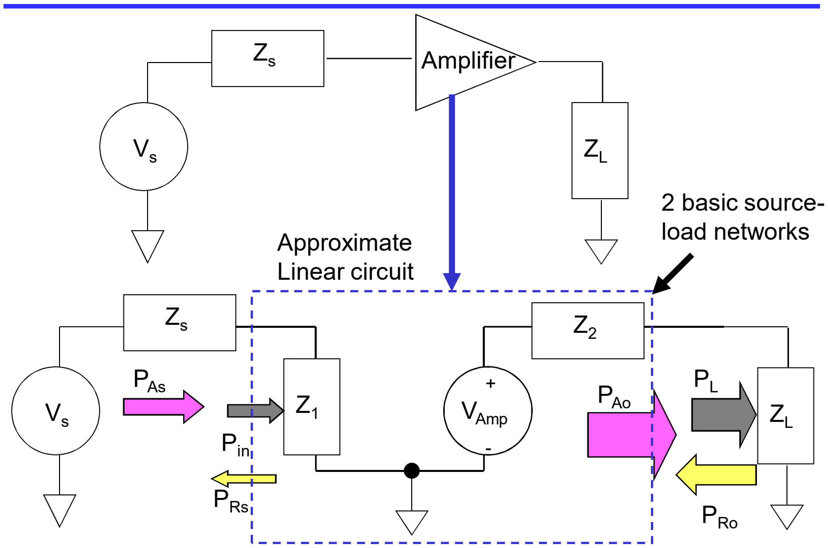
Under non-conjugate match condition:

$$P_L < \frac{|V_s|^2}{8R_s} = P_A - P_{Reflect}$$

or $P_L = P_A\left(1 - |\Gamma_L|^2\right)$

We can consider the load power P_L to consist of the available power P_A minus the reflected power $P_{reflect}$.

Power Components in an Amplifier



Power Gain Definition

- From the power components, 3 types of power gain can be defined.

$$\text{Power Gain } G_p = \frac{\text{Power delivered to load}}{\text{Input power to Amp.}} = \frac{P_L}{P_{in}} \quad \textbf{(2.1a)}$$

$$\text{Available Power Gain } G_A = \frac{\text{Available load Power}}{\text{Available Input power}} = \frac{P_{Ao}}{P_{As}} \quad \textbf{(2.1b)}$$

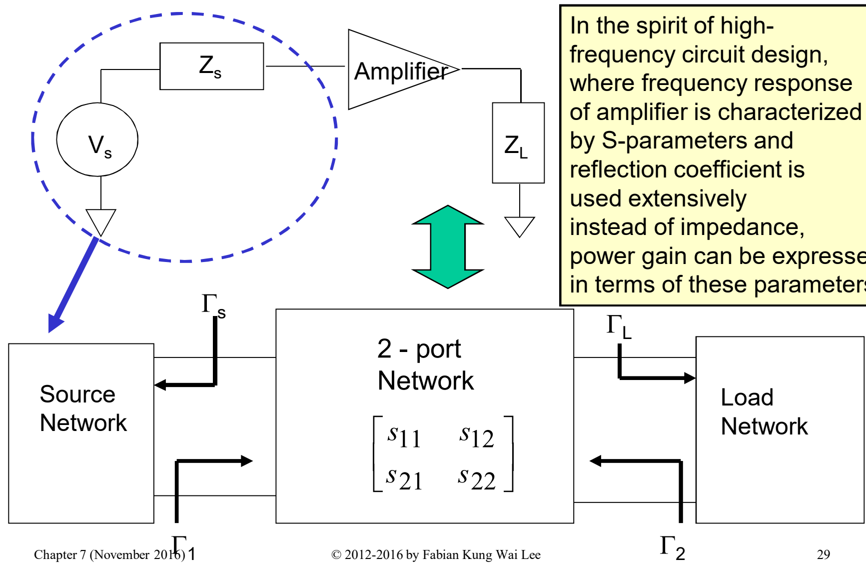
$$\text{Transducer Gain } G_T = \frac{\text{Power delivered to load}}{\text{Available Input power}} = \frac{P_L}{P_{As}} \quad \textbf{(2.1c)}$$

The effective power gain

- G_P, G_A and G_T can be expressed as the S-parameters of the amplifier and the reflection coefficients of the source and load networks. Refer to Appendix 1 for the derivation.

Naming Convention

In the spirit of high-frequency circuit design, where frequency response of amplifier is characterized by S-parameters and reflection coefficient is used extensively instead of impedance, power gain can be expressed in terms of these parameters.



Summary of Important Power Gain Expressions and the Gain Dependency Diagram

$$b_1 = s_{11}a_1 + s_{12}a_2$$
$$b_2 = s_{21}a_1 + s_{22}a_2 \qquad \textit{(2.2a)}$$

$$\Gamma_1 = \frac{s_{11} - D\Gamma_L}{1 - s_{22}\Gamma_L} \qquad \textit{(2.2b)}$$

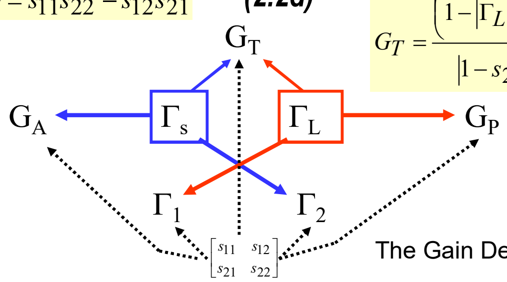
$$\Gamma_2 = \frac{s_{22} - D\Gamma_s}{1 - s_{11}\Gamma_s} \qquad \textit{(2.2c)}$$

$$D = s_{11}s_{22} - s_{12}s_{21} \qquad \textit{(2.2d)}$$

$$G_P = \frac{|s_{21}|^2\left(1 - |\Gamma_L|^2\right)}{|1 - s_{22}\Gamma_L|^2\left(1 - |\Gamma_1|^2\right)} \qquad \textit{(2.2e)}$$

$$G_A = \frac{\left(1 - |\Gamma_s|^2\right)|s_{21}|^2}{|1 - s_{11}\Gamma_s|^2\left(1 - |\Gamma_2|^2\right)} \qquad \textit{(2.2f)}$$

$$G_T = \frac{\left(1 - |\Gamma_L|^2\right)|s_{21}|^2\left(1 - |\Gamma_s|^2\right)}{|1 - s_{22}\Gamma_L|^2|1 - \Gamma_1\Gamma_s|^2} \qquad \textit{(2.2g)}$$



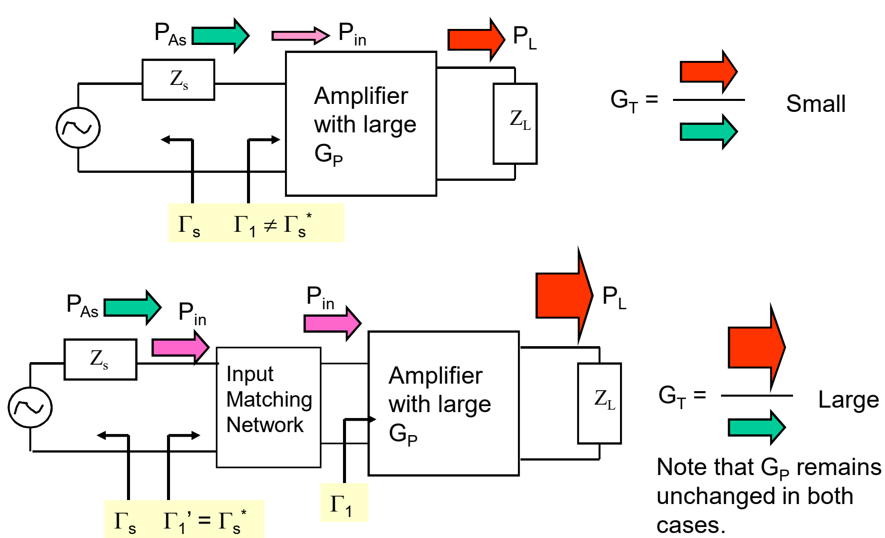
Note:
All G_T, G_P, G_A, Γ_1 and Γ_2 depends on the S-parameters.

The Gain Dependency Diagram

Transducer Power Gain G_T (1)

- G_T is the relevant indicator of the amplifying capability of the amplifier.
- Whenever we design an amplifier to a specific power gain, we refer to the transducer power gain G_T.
- G_P and G_A are usually used in the process of amplifier design or synthesis to meet a certain G_T.
- An amplifier can have a large G_P or G_A and yet small G_T, as illustrated in the next slide.

Transducer Power Gain G_T (2)



Note that G_P remains unchanged in both cases.

The Essence of Small-Signal Amplifier Design

- **Is to find the suitable load and source impedance to be connected to the output and input port,**
- **So that the required transducer power gain G_T, bandwidth, noise figure and other characteristics are obtained.**

This is true whether you are designing a discrete Amplifier or amplifier in IC form

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Extra

Unilateral Condition (1)

- In certain cases, when operating frequency is low, $|s_{12}| \rightarrow 0$. Such condition is known as Unilateral.
- Under unilateral condition: $\Gamma_1 = \dfrac{s_{11} - D\Gamma_L}{1 - s_{22}\Gamma_L} = \dfrac{s_{11} - s_{11}s_{22}\Gamma_L}{1 - s_{22}\Gamma_L} = s_{11}$
- From (2.2d):

$$G_T = \underbrace{\boxed{\frac{1-|\Gamma_s|^2}{|1-s_{11}\Gamma_s|^2}}}_{G_s} \cdot \underbrace{|s_{21}|^2}_{G_o} \cdot \underbrace{\boxed{\frac{1-|\Gamma_L|^2}{|1-s_{22}\Gamma_L|^2}}}_{G_L} = G_s G_o G_L \qquad \textit{(2.3)}$$

- We see that the Transducer Power Gain G_T consists of 3 parts that are independent of each other.

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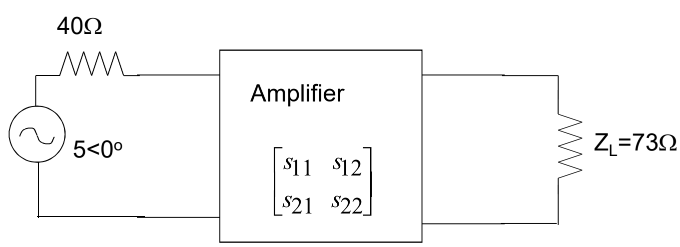
Extra

Unilateral Condition (2)

- In small-signal amplifier design for unilateral condition, we can find suitable source and load impedance for a required G_T by optimizing G_s and G_L independently, and this simplifies the design procedures.
- However in most case, s_{12} is typically not zero especially at frequency above 1 GHz. Thus we will not pursue design techniques for unilateral condition.

Example 2.1 – Familiarization with the Gain Expressions

- An RF amplifier has the following S-parameters at f_o: $s_{11}=0.3<-70^o$, $s_{21}=3.5<85^o$, $s_{12}=0.2<-10^o$, $s_{22}=0.4<-45^o$ at 500 MHz. The system is shown below. Assuming reference impedance (used for measuring the S-parameters) $Z_o=50\Omega$, find, at 500 MHz:
- (a) G_T, G_A, G_P.
- (b) P_L, P_A, P_{inc}.



Example 2.1 Cont...

- Step 1 - Find Γ_s and Γ_L .
- Step 2 - Find Γ_1 and Γ_2 .
- Step 3 - Find G_T, G_A, G_P.
- Step 4 - Find P_L, P_A.

$$\Gamma_s = \frac{Z_s - Z_o}{Z_s + Z_o} = -0.111 \qquad \Gamma_L = \frac{Z_L - Z_o}{Z_L + Z_o} = 0.187$$

$$\Gamma_1 = \frac{s_{11} - D\Gamma_L}{1 - s_{22}\Gamma_L} = 0.146 - j0.151$$

$$\Gamma_2 = \frac{s_{22} - D\Gamma_s}{1 - s_{11}\Gamma_s} = 0.265 - j0.358$$

$$G_P = \frac{|s_{21}|^2\left(1 - |\Gamma_L|^2\right)}{|1 - s_{22}\Gamma_L|^2\left(1 - |\Gamma_1|^2\right)} = 13.742$$

$$P_A = \frac{|V_s|^2}{8 \cdot \mathrm{Re}[Z_s]} = 0.078W$$

Try to derive
These 2 relations

$$P_{in} = P_A\left(1 - \left|\frac{Z_1 - Z_s}{Z_1 + Z_s}\right|^2\right) = 0.0714W$$

$$G_A = \frac{\left(1 - |\Gamma_s|^2\right)|s_{21}|^2}{|1 - s_{11}\Gamma_s|^2\left(1 - |\Gamma_2|^2\right)} = 14.739$$

$$P_L = G_P P_{in} = 0.9814W$$

$$G_T = \frac{\left(1 - |\Gamma_L|^2\right)|s_{21}|^2\left(1 - |\Gamma_s|^2\right)}{|1 - s_{22}\Gamma_L|^2|1 - \Gamma_1\Gamma_s|^2} = 12.562$$

Again note that this is an analysis problem.

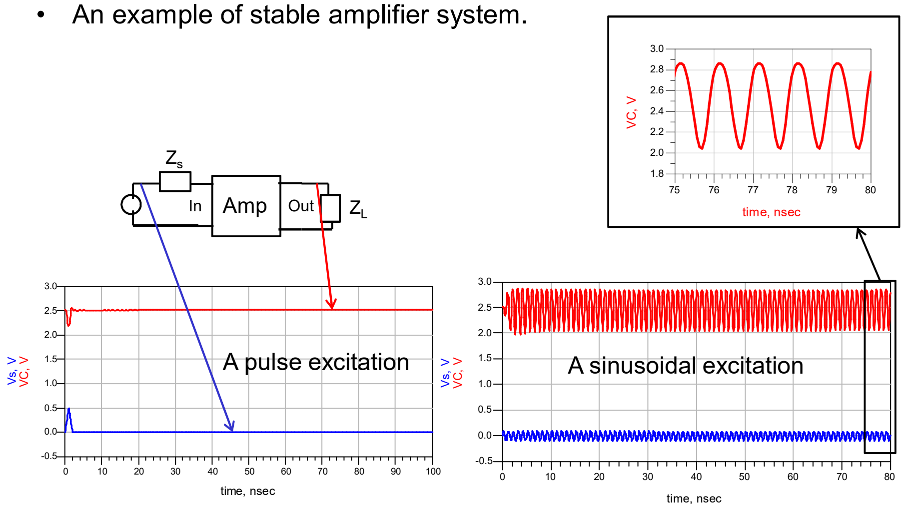
3.0 Stability Analysis of Small-Signal Amplifier

Introduction (1)

- An amplifier is a circuit designed to enlarge electrical signals. For a certain input level (small-signal region), we would expect a certain output level, the ratio of which we call the gain of the amplifier.
- An amplifier, or any two-ports circuits fulfilling the above condition is said to be **stable**, thus a stable system must fulfill the BIBO (Bounded input bounded output) condition.
- On the contrary, an **unstable** circuit will produce an output that gradually increases over time. The output level will increase until the circuit saturates, with the output limited by non-linear effects within the circuit.
- For most unstable circuits, the output will continue to increase even if the initial input is removed. In fact, an unstable amplifier can produce an output when there is no input. The amplifier becomes an oscillator instead!!!
- Stability of an amplifier is affected by the load and source impedance connected to it's two ports, the active device and also by the frequency range.

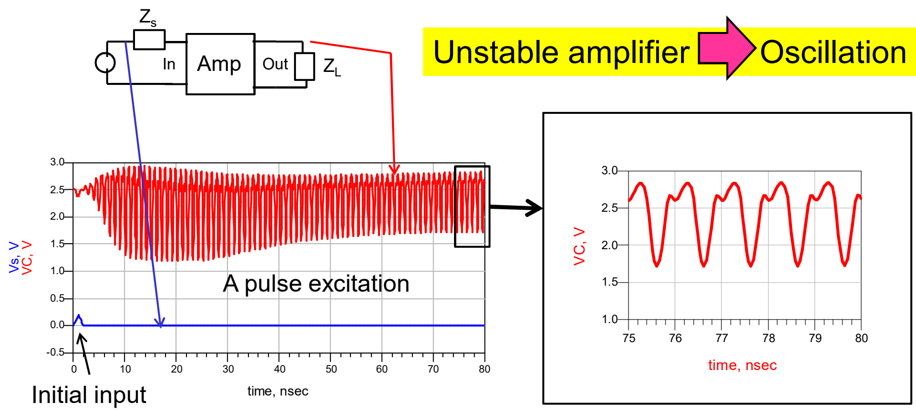
Introduction (2)

- An example of stable amplifier system.



Introduction (3)

- An example of un-stable amplifier system, the input and output voltages as a function of time are shown.



Introduction (4)

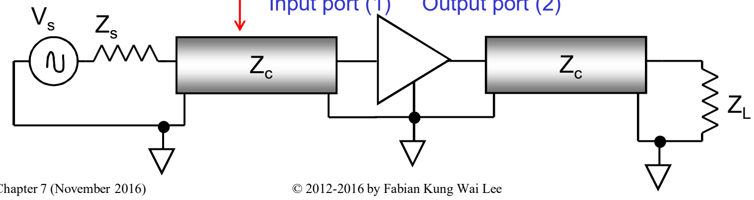
- Stability analysis can be carried out by many ways, for instance using what we learnt from Control Theory.
- Stability analysis using Control Theory assumes the system can be simplified and modeled mathematically.
- Usually under small-signal condition, the system is linear and Laplace Transformation can be applied. The system is thus cast into frequency domain.
- A relationship between the input and output, called the Transfer Function can be obtained.
- By analyzing the "poles" of the Transfer Function, the long-term behavior of the system can be predicted.
- This method is not convenient for high-frequency electronic circuits due to the presence of delay at high-frequency (due to transmission lines). Delay is modeled by exponential function, which can cannot be expressed accurately be rational polynomials.

Introduction (5)

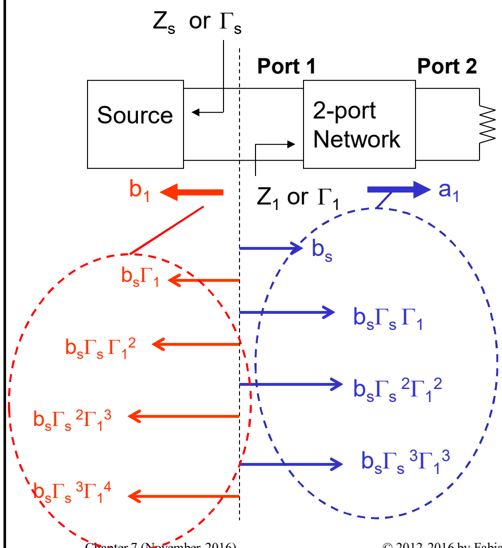
- Stability analysis can also be performed empirically by considering a small-signal amplifier (since the initial signal that causes oscillation is always very small, for instance the noise in the circuit) with a set of source (Z_s) and load (Z_L) impedance. The amplifier circuit is then excited by a small 'seed' signal, and checked for oscillation. The source and load impedance are then varied, and the process is repeated.
- An amplifier that do not oscillate for ALL **passive** Z_s and Z_L (even short and open circuit) is called an **Unconditionally Stable circuit**.
- A **Conditionally Stable** amplifier will oscillate for certain passive Z_s and Z_L. While an **Un-stable** amplifier will oscillate for all impedances.
- An unstable or marginally stable amplifier can be made more stable.

Introduction (6)

- A third approach, since we are considering high frequency electrical systems with sinusoidal signals, is to consider the dynamics of voltage and current waves propagating at the ports of the amplifier system.
- The amplifier is considered as connected to the Source and Load networks via transmission lines. As the system is powered up, the Source network launches an initial sinusoidal voltage/current wave into the input port of the amplifier.
- A series of multiple reflections then follows, leading to steady-state condition. Here we will adopt this approach to show the criteria for AC stability in terms of reflection coefficients.



Stability Concept (1) - Perspective of Instability from Wave Propagation



- Consider the input port.
- An incident wave V^+ propagating towards port 1 will encounter multiple reflections.
- If $|\Gamma_s\Gamma_1|>1$, then the magnitude of the incident wave towards port 1 will increase indefinitely, leading to unstable condition.

A geometric series

$$\boxed{\begin{aligned} a_1 &= b_s + b_s\Gamma_1\Gamma_s + b_s\Gamma_1^{\,2}\Gamma_s^{\,2} + \ldots \\ \Rightarrow a_1 &= \frac{b_s}{1-\Gamma_1\Gamma_s} \end{aligned}}$$

Only if $|\Gamma_1\Gamma_s|<1$

$$\boxed{\begin{aligned} b_1 &= b_s\Gamma_1 + b_s\Gamma_1^{\,2}\Gamma_s + b_s\Gamma_1^{\,3}\Gamma_s^{\,2} + \ldots \\ \Rightarrow b_1 &= \frac{b_s\Gamma_1}{1-\Gamma_1\Gamma_s} \end{aligned}}$$

Stability Concept (2)

- Thus a system is unstable when $|\Gamma_s\Gamma_1| > 1$ at the input port.
- Since the source network is usually passive, $|\Gamma_s| < 1$. Thus for instability to arise, the requirement boils down to making $|\Gamma_1| > 1$, this condition represents the potential for oscillation.
- Similar argument can be applied to port 2, and we see that the condition for instability at Port 2 is $|\Gamma_2| > 1$.
- Since input and output power of a 2-port network are related, when either port is stable, the other will also be stable.

Port 1:
$$\begin{aligned} &|\Gamma_s\Gamma_1| = |\Gamma_s||\Gamma_1| > 1 \\ &\Rightarrow |\Gamma_1| > 1 \;\; \text{since } |\Gamma_s| \text{ always} < 1 \end{aligned}$$

Port 2:
$$\begin{aligned} &|\Gamma_L\Gamma_2| = |\Gamma_L||\Gamma_2| > 1 \\ &\Rightarrow |\Gamma_2| > 1 \;\; \text{since } |\Gamma_L| \text{ always} < 1 \end{aligned}$$

Extra How to Make |Γ| Greater Than One?

- Consider the expression for reflection coefficient, with reference impedance Z_o , which is real.

$$\Gamma = \frac{Z - Z_o}{Z + Z_o} = \frac{(R - Z_o) + jX}{(R + Z_o) + jX}$$

$$\Rightarrow |\Gamma| = \sqrt{\frac{(R - Z_o)^2 + X^2}{(R + Z_o)^2 + X^2}} \qquad \textit{(3.1)}$$

- You can see for yourself that, provided R < 0 (i.e. negative resistance), the magnitude of Γ is always greater than or equal to 1.
- The same is true if we consider admittance (Y) instead of impedance (Z), a system with negative conductance (G) will results in |Γ
- Only active circuits, which can provide gain, can give a negative resistance, which physically means the system is giving out energy instead of absorbing energy (positive resistance).
- We will explore this concept more in discussion of oscillator.

Important Summary on Oscillation Requirements

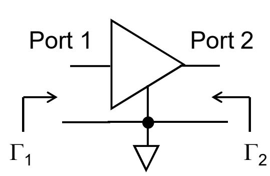
- Assuming that $|\Gamma_s|$ and $|\Gamma_L|$ always < 1 for passive components, we conclude that:
- For a 2-ports network to be stable, it is necessary that the load and source impedance are chosen in such a way that $|\Gamma_1| < 1$ and $|\Gamma_2| < 1$.

To prevent oscillation:

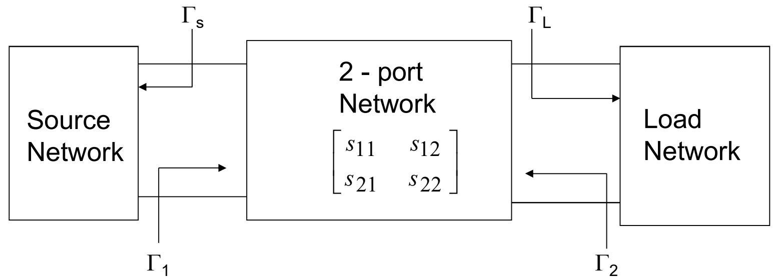
$$|\Gamma_1(\omega)| < 1 \qquad \textit{(3.2a)}$$

$$|\Gamma_2(\omega)| < 1 \qquad \textit{(3.2b)}$$

The range of frequency of interest



Stability Criteria for Amplifier



- As seen previously, for stability $|\Gamma_1| < 1$ and $|\Gamma_2| < 1$.
- Using (2.2a) this can be expressed as:

$$|\Gamma_1| = \left| s_{11} + \frac{s_{12}s_{21}\Gamma_L}{1 - s_{22}\Gamma_L} \right| < 1 \qquad \textbf{\textit{(3.3a)}}$$

$$|\Gamma_2| = \left| s_{22} + \frac{s_{12}s_{21}\Gamma_s}{1 - s_{11}\Gamma_s} \right| < 1 \qquad \textbf{\textit{(3.3b)}}$$

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3.1 Stability Circles and Stable Regions for Load and Source Impedance

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Load Stability Circle (LSC)

- Setting $|\Gamma_1| = 1$, we can determine all the corresponding values of Γ_L for which $|\Gamma_1| = 1$.
- The Γ_L values fall on the locus of a circle on Smith Chart, which is called **Load Stability Circle**.

$$\left| s_{11} + \frac{s_{12}s_{21}\Gamma_L}{1 - s_{22}\Gamma_L} \right| = 1$$

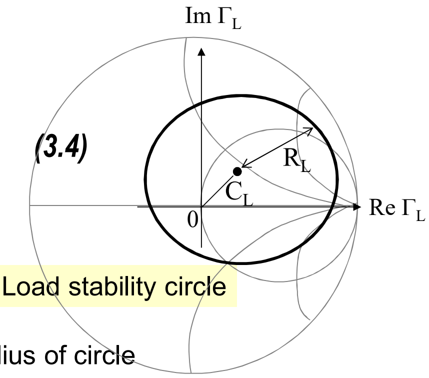
$$\Rightarrow \left| \Gamma_L - \frac{\left(s_{22} - D s_{11}^{\;*}\right)^*}{|s_{22}|^2 - |D|^2} \right| = \left| \frac{s_{12}s_{21}}{|s_{22}|^2 - |D|^2} \right| \quad \textit{(3.4)}$$

$$\Rightarrow \left| \Gamma_L - C_L \right| = R_L$$

Load stability circle

Center of circle

Radius of circle



- For detailed derivation, See [2], Chapter 10 or [1], Chapter 11.

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Source Stability Circle (SSC)

- Similarly we find that all the values of Γ_s for which $|\Gamma_2| = 1$, falls on the locus of a circle on Smith Chart.
- We call this locus the **Source Stability Circle**.

$$\left| s_{22} + \frac{s_{12}s_{21}\Gamma_s}{1 - s_{22}\Gamma_s} \right| = 1$$

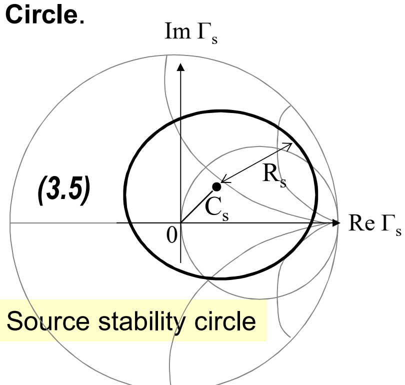
$$\Rightarrow \left| \Gamma_s - \frac{\left(s_{11} - D s_{22}^{\;*}\right)^*}{|s_{11}|^2 - |D|^2} \right| = \left| \frac{s_{12}s_{21}}{|s_{11}|^2 - |D|^2} \right| \quad \textit{(3.5)}$$

$$\Rightarrow \left| \Gamma_s - C_s \right| = R_s$$

Source stability circle

Radius of circle

Center of circle



- For detailed derivation, See [2], chapter 10 or [1], Chapter 11.

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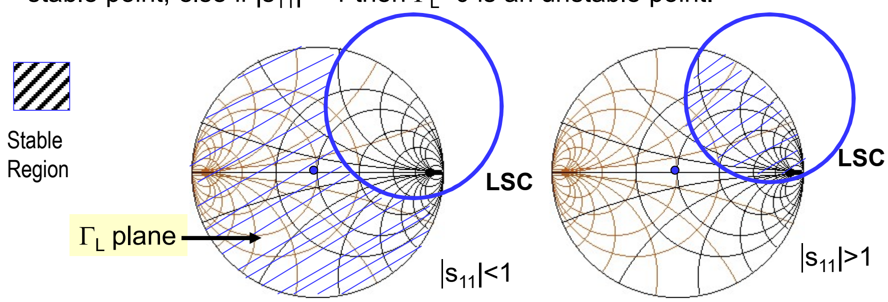
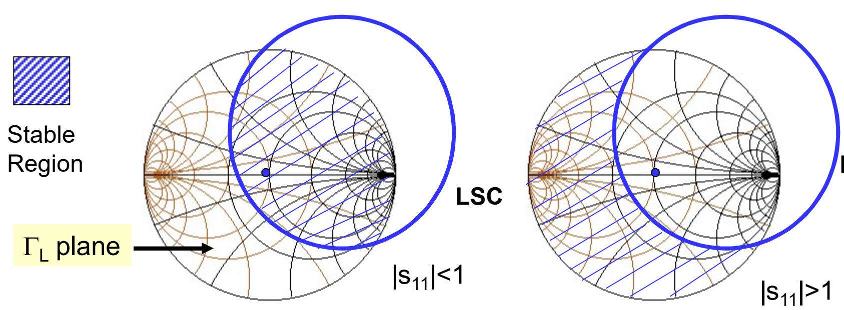
Stable Regions (1)

- The source and load stability circles only indicate the value of Γ_s and Γ_L where $|\Gamma_2| = 1$ and $|\Gamma_1| = 1$. We need more information to show the stable regions for Γ_s and Γ_L plane on the Smith Chart.
- For example for LSC, when $\Gamma_L = 0$, $|\Gamma_1| = |s_{11}|$ (See (2.2a)).
- Assume LSC does not encircle $s_{11} = 0$ point. If $|s_{11}| < 1$ then $\Gamma_L = 0$ is a stable point, else if $|s_{11}| > 1$ then $\Gamma_L = 0$ is an unstable point.



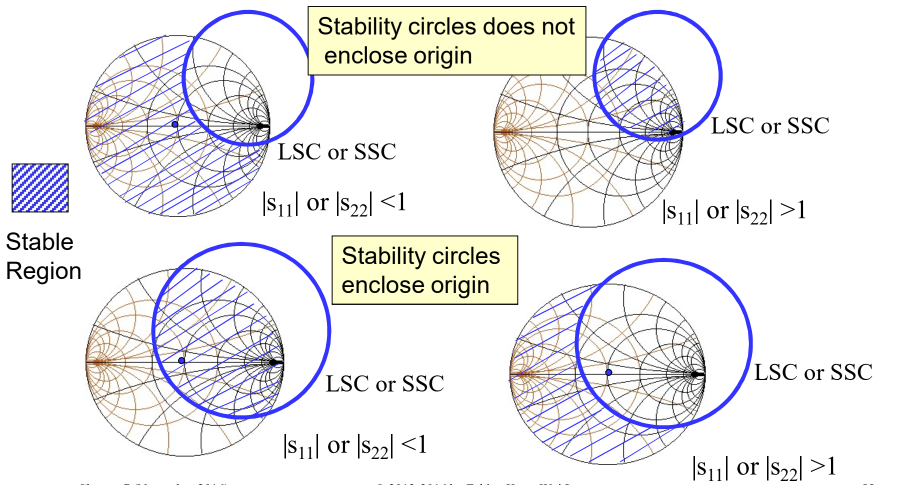
Stable Region (2)

- Now let the LSC encircles $s_{11} = 0$ point. Similarly if $|s_{11}| < 1$ then $\Gamma_L = 0$ is an stable point, else if $|s_{11}| > 1$ then $\Gamma_L = 0$ is an unstable point.
- This argument can also be applied for SSC, where we consider $|s_{22}|$ instead and the Smith Chart corresponds to Γ_s plane.



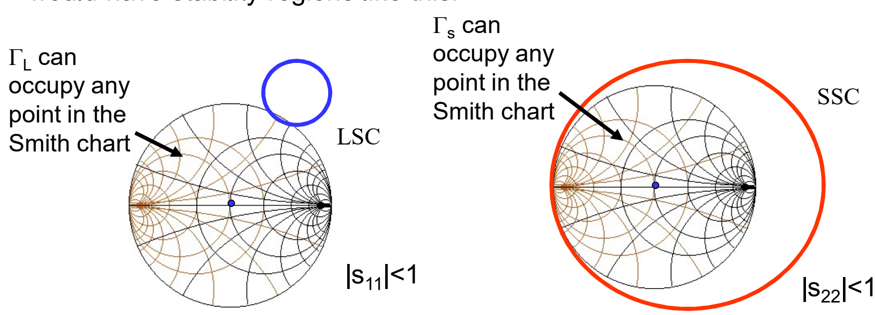
Summary for Stability Regions

- For both Source and Load reflection coefficients (Γ_s and Γ_L) :



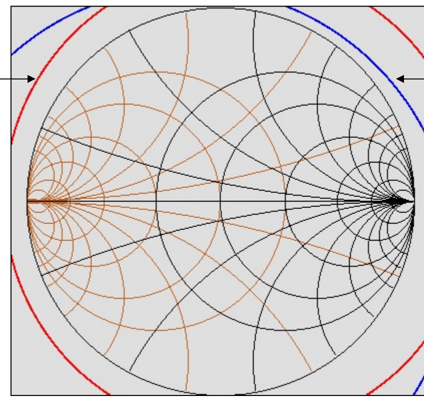
Unconditionally Stable Amplifier

- There are times when the amplifier is stable for all passive source and load impedance.
- In this case the amplifier is said to be unconditionally stable.
- Assuming $|s_{11}| < 1$ and $|s_{22}| < 1$, an unconditionally stable amplifier would have stability regions like this:



Example 3.1

- Use the S-parameters of the amplifier in Example 2.1, draw the load and source stability circles and find the stability region.



Hint:
Apply equations (3.4) and (3.5) to find the center and radius of the circles.

Example 3.1 Cont…

Macro to convert complex number in polar form to rectangular form

$$\text{Polar}(R, \text{theta}) := R\cdot\cos\left(\frac{\text{theta}}{180}\cdot\pi\right) + i\cdot R\cdot\sin\left(\frac{\text{theta}}{180}\cdot\pi\right)$$

Definition of S-parameters:

$S11 := \text{Polar}(0.3, -70)$

$S12 := \text{Polar}(0.2, -10)$

$S21 := \text{Polar}(3.5, 85)$

$S22 := \text{Polar}(0.4, -45)$

$S11 = 0.1026 - 0.2819i$ $\quad$ $S12 = 0.197 - 0.0347i$

$S21 = 0.305 + 3.4867i$ $\quad$ $S22 = 0.2828 - 0.2828i$

$D := S11\cdot S22 - S12\cdot S21$

$D = -0.2319 - 0.7849i$

Computation Using the Software MATHCAD®

Example 3.1 Cont…

Finding the Load Stability Circle parameters

$$CL := \frac{\overline{\left(S22 - D\cdot\overline{S11}\right)}}{\left(|S22|\right)^2 - \left(|D|\right)^2} \qquad RL := \left|\frac{S12\cdot S21}{\left(|S22|\right)^2 - \left(|D|\right)^2}\right|$$

$$CL = -0.1674 - 0.2686i \qquad RL = 1.373$$

Finding the Source Stability Circle parameters

$$CS := \frac{\overline{\left(S11 - D\cdot\overline{S22}\right)}}{\left(|S11|\right)^2 - \left(|D|\right)^2} \qquad RS := \frac{S12\cdot S21}{\left|\left(|S11|\right)^2 - \left(|D|\right)^2\right|}$$

$$CS = 0.0928 + 9.8036i \times 10^{-3} \qquad RS = 0.3124 + 1.1661i$$

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3.2 Test for Unconditional Stability – The Stability Factor

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Stability Factor (1)

- Sometimes it is not convenient to plot the stability circles, or we just want a quick check whether an amplifier is unconditionally stable or not.
- In such condition we can compute the Stability Factor of the amplifier.
- Rollett* has come up with a factor, the K factor that tell us whether or not an amplifier (or any linear 2-port network) is unconditionally stable based on its S-parameters at a certain frequency.
- A complete derivation can be found in reference [1], [4], [5], here only the result is shown.

See:
1. Rollett J., "Stability and power gain invariants of linear two-ports", IRE Transactions on Circuit Theory, 1962, CT-9, p. 29-32.
2. Jackson R. W., "Rollett proviso in the stability of linear microwave Circuits – a tutorial", IEEE Trans. Microwave Theory and Techniques, 2006, Vol. 54, No. 3, p. 993-1000.

Stability Factor (2)

- The condition for a 2-port network to be unconditionally stable is:

If the S-parameters of the 2-port network have no poles on the Right-half-plane (RHP) (This is usually called the Rollett Proviso), then the network is unconditionally stable if

This means the unterminated 2-port network must be stable initially.

$$K = \frac{1-|s_{11}|^2 - |s_{22}|^2 + |D|^2}{2|s_{12}s_{21}|} > 1 \qquad (3.6)$$

$$|D| < 1$$

- Otherwise the amplifier is conditional stable or unstable at all (it is an oscillator !).
- K is also known as the Rollett Stability Factor.

Note that the K factor only tells us if an amplifier (or any linear 2-port network) is unconditionally stable. It doesn't indicate the relative stability of 2 amplifiers which fail the test. A newer test, called the μ factor allows comparison of 2 conditionally stable amplifiers (See Appendix 2).

What if Amplifier is Unstable, or Stable Region is too Small?

- Use negative feedback to reduce amplifier gain.
- Use ‘neutralization’ network on the amplifier.
- Redesign d.c. biasing, find new operating point (or Q point) that will result in more stable amplifier.
- Add some resistive loss to the circuit to improve stability.
- Use a new component with better stability.

Comparison of High-Frequency Stability of Amplifier Topology

	CE	CB	CC	Cascode
Stability Factor	Moderate (Often requires compensation)	Good	Good	Good

Due to Miller’s Effect

3.3 Stabilization of Amplifier

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Introduction

- The main reason why amplifiers become unstable is feedback (e.g. part of the output energy is put back into the input).
- One possible feedback path is the parasitic capacitance $C_{b'c}$ between the Base and Collector terminals of the transistor (the same can be said for FET). This is compounded by the Miller Effect ([4], [5]) if the transistor is used in CE configuration.
- This usually results in elevated magnitude of s_{12} parameter as frequency increases (as seen in equation (3.6) the K value decreases if $|s_{12}|$ increases).
- Classically the effect $C_{b'c}$ can be cancelled by using a suitable inductor across B and C terminals, in a process called **Neutralization***.
- Another approach is to add dissipation loss at the input and output of the amplifier, this approach that will be elaborated in this section.

• For instance see J.R. Smith, "Modern communication circuits", 1998, McGraw-Hill, or P.H. Young, "Electronic communication techniques", 2004, Prentice Hall.

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Stabilization Methods (1)

- $|\Gamma_1| > 1$ and $|\Gamma_2| > 1$ can be written in terms of input and output impedances:

$$|\Gamma_1| = \left|\frac{Z_1 - Z_o}{Z_1 + Z_o}\right| > 1 \quad \text{and} \quad |\Gamma_2| = \left|\frac{Z_2 - Z_o}{Z_2 + Z_o}\right| > 1$$

- As we have seen, this implies that $\mathrm{Re}[Z_1] < 0$ or $\mathrm{Re}[Z_2] < 0$.

$$\Gamma_1 = \frac{(R_1 - Z_o) + jX_1}{(R_1 + Z_o) + jX_1} \Rightarrow |\Gamma_1| = \sqrt{\frac{(R_1 - Z_o)^2 + X_1^2}{(R_1 + Z_o)^2 + X_1^2}}$$

- Thus one way to stabilize an amplifier is to add a series resistance or shunt conductance to the port. This should made the real part of the impedance become positive.
- In other words we deliberately add loss to the network.

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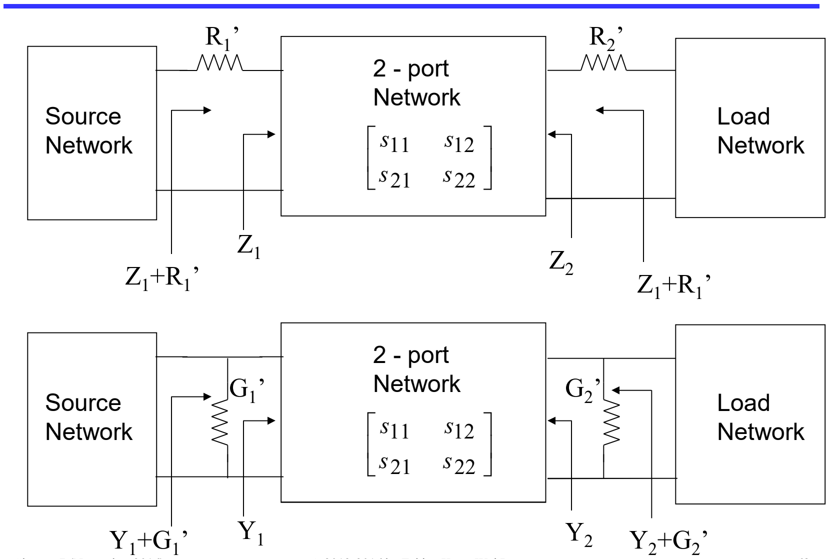
Stabilization Methods (2)



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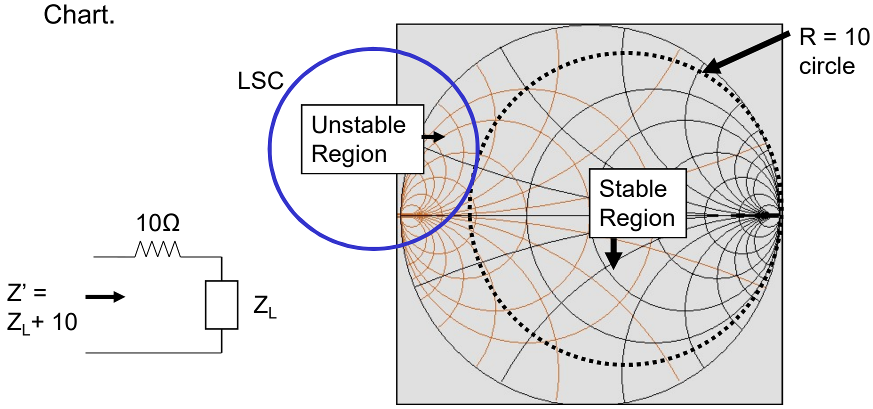
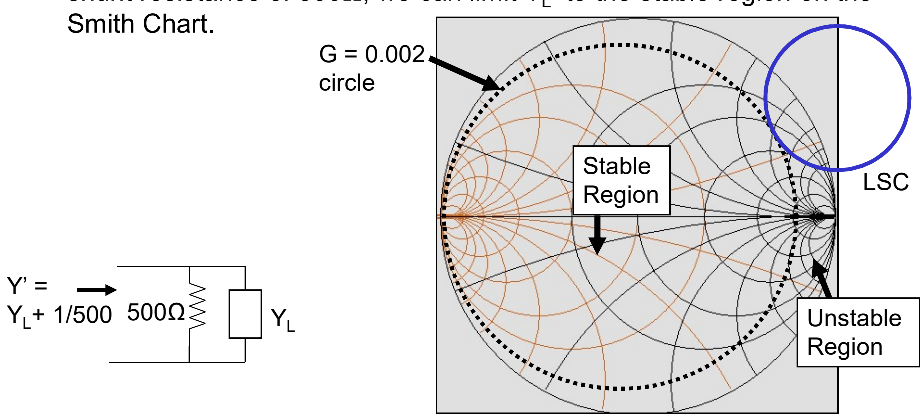
Effect of Adding Series Resistance on Smith Chart

- Suppose we have an impedance Z_L and a load stability circle (LSC). Assuming the LSC touches the R=10 circle. Thus by inserting a series resistance of 10Ω, we can limit Z_L' to the stable region on the Smith Chart.

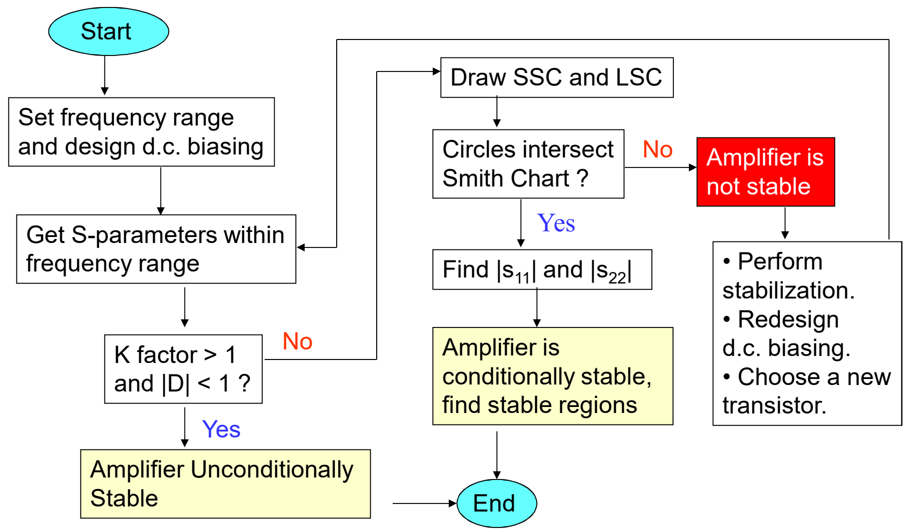


Effect of Adding Shunt Resistance on Smith Chart

- Suppose we have an admittance Y_L and a load stability circle (LSC). Assuming the LSC touches the G=0.002 circle. Thus by inserting a shunt resistance of 500Ω, we can limit Y_L' to the stable region on the Smith Chart.



Summary for Stability Check of Single-Stage Amplifier



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Chapter Summary

- Here we have learnt the important concepts of small-signal amplifier and amplifier characteristics.
- Here we have derived the three types of power gain expressions for an amplifier using S-parameters.
- We also studied how the various gains depend on either Γ_s and Γ_L (the dependency diagram).
- We have looked at the concept of oscillation and how it applies to stability analysis.
- Learnt about stability circles and how to find the stable region for source and load impedance.
- Learnt about the Rolette Stability Factor test (K) for unconditionally stable amplifier.
- Learnt about elementary stabilization methods.

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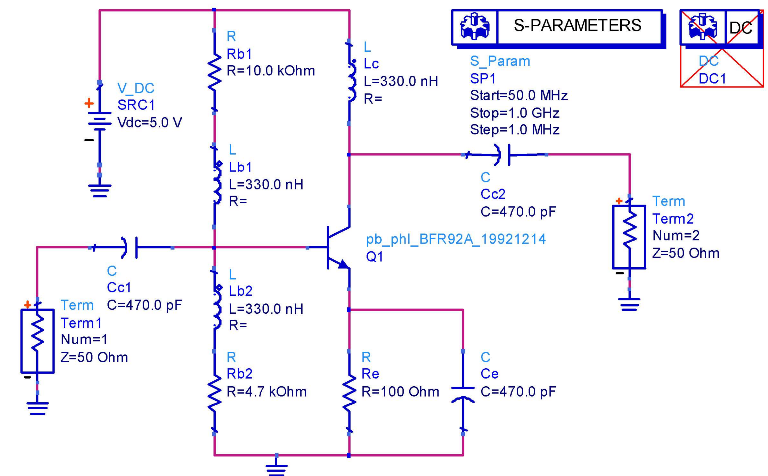
Example 3.2 - S-Parameters Measurement and Stability Analysis Using ADS Software

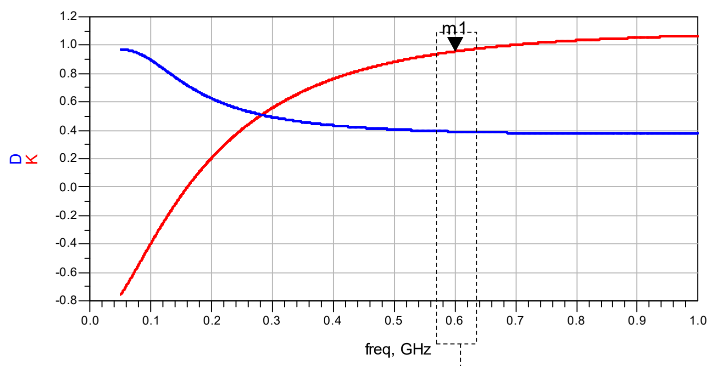


Example 3.2 – Stability Test

Plotting K and |D| versus frequency (from 50MHz to 1.0GHz):

m1
freq=600.0MHz
K=0.956

Amplifier is conditionally stable

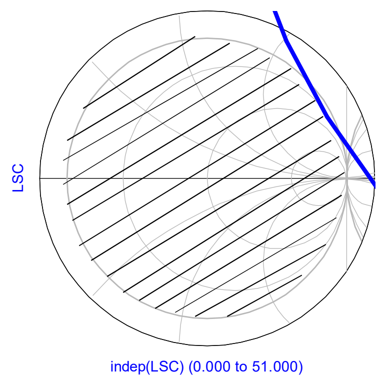


This is the frequency we are interested in

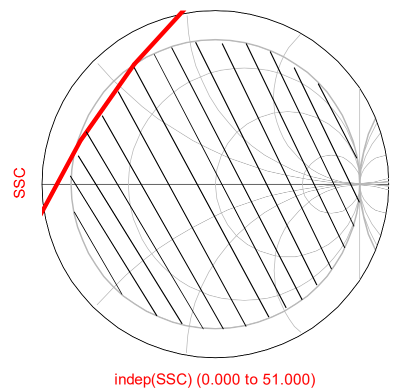
Example 3.2 - Viewing S_{11} and S_{22} at 600 MHz and Plotting Stability Circles

freq	S(1,1)	S(2,2)
600.0MHz	0.263 / -114.092	0.491 / -20.095

Since $|s_{11}| < 1$ @ 600MHz



Since $|s_{22}| < 1$ @ 600MHz



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Appendix 1 – Derivation of Small-Signal Amplifier Power Gain Expressions

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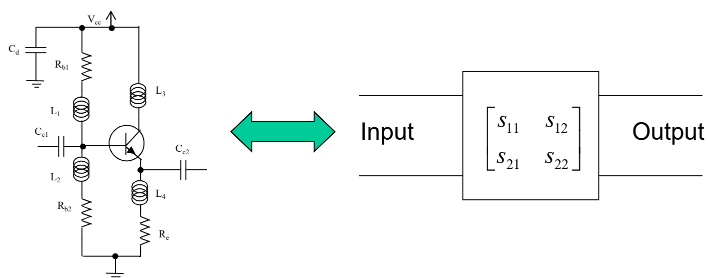
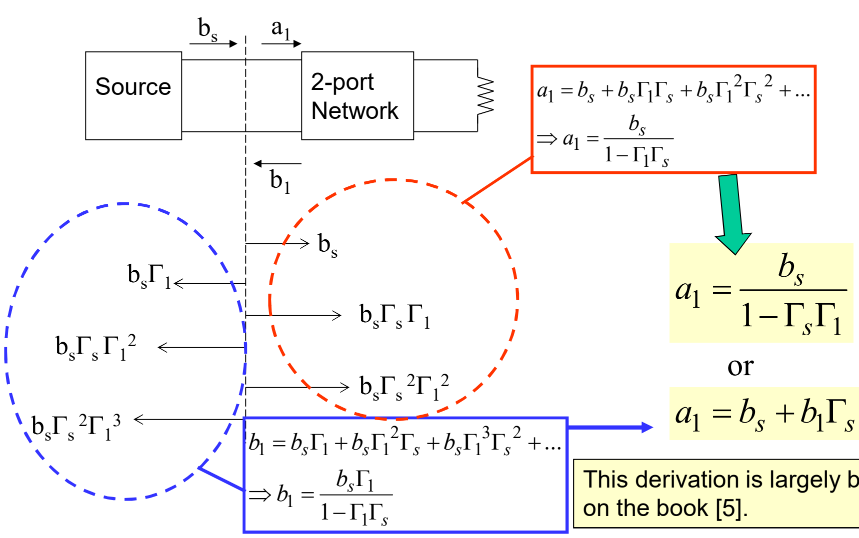
Derivation of Amplifier Gain Expressions in terms of S-parameters

- When the electrical signals in the amplifier is small, the active component (BJT) can be considered as linear.
- Thus the amplifier is a linear 2-port network, S-parameters can be obtained and it is modeled by an S matrix.
- The preceding gains G_P , G_A , G_T can be expressed in terms of the Γ_s , Γ_L and s_{11}, s_{12}, s_{21}, s_{22}.

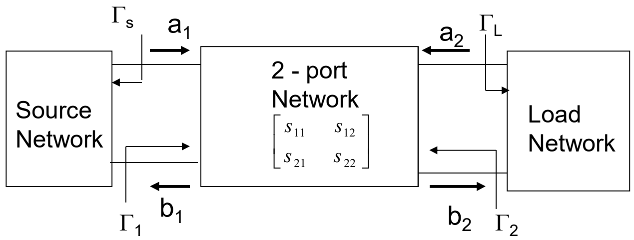


Derivation of Gain Expression - 1



This derivation is largely based on the book [5].

Derivation of Gain Expression - 2



From:

$$b_1 = s_{11}a_1 + s_{12}a_2$$

$$b_2 = s_{21}a_1 + s_{22}a_2$$

and

$$a_2 = \Gamma_L b_2$$

$$a_1 = \Gamma_s b_1$$

One obtains:

$$b_1 = \frac{s_{12}a_2}{1 - s_{11}\Gamma_s}$$

$$\Gamma_1 = \frac{b_1}{a_1} = \frac{s_{11} - D\Gamma_L}{1 - s_{22}\Gamma_L}$$

$$D = s_{11}s_{22} - s_{12}s_{21}$$

In a similar manner we can also obtains:

$$b_2 = \frac{s_{21}a_1}{1 - s_{22}\Gamma_L}$$

$$\Gamma_2 = \frac{b_2}{a_2} = \frac{s_{22} - D\Gamma_s}{1 - s_{11}\Gamma_s}$$

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Derivation of Gain Expression - 3

Finding Transducer Power Gain G_T:

Only for available gain

$$P_L = \tfrac{1}{2}|b_2|^2\left(1 - |\Gamma_L|^2\right) \qquad P_A = \tfrac{1}{2}|a_1|^2\left(1 - |\Gamma_1|^2\right)\Big|_{\Gamma_s = \Gamma_1^*}$$

$$a_1 = \frac{b_s}{1 - \Gamma_s\Gamma_1}\bigg|_{\Gamma_s = \Gamma_1^*}$$

From slide DGE-1

$$P_A = \frac{1}{2}\frac{|b_s|^2}{1 - |\Gamma_1|^2}$$

$$G_T = \frac{\text{Load Power}}{\text{Available Soure Power}} = \frac{P_L}{P_A} = \frac{|b_2|^2}{|b_s|^2}\left(1 - |\Gamma_L|^2\right)\left(1 - |\Gamma_1|^2\right)\Big|_{\Gamma_s = \Gamma_1^*}$$

$$\Rightarrow G_T = \frac{|b_2|^2}{|b_s|^2}\left(1 - |\Gamma_L|^2\right)\left(1 - |\Gamma_s|^2\right)$$

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Derivation of Gain Expression - 4

Finding Transducer Power Gain G_T Cont… :

Using $\boxed{\dfrac{b_2}{a_1} = \dfrac{s_{21}}{1 - s_{22}\Gamma_L}}$ From slide DGE-2 $\quad \boxed{b_s = a_1\left(1 - \Gamma_1\Gamma_s\right)}$ From slide DGE-1

$$\frac{b_2}{b_s} = \frac{s_{21}}{\left(1 - s_{22}\Gamma_L\right)\left(1 - \Gamma_1\Gamma_s\right)}$$

$$G_T = \frac{|b_2|^2}{|b_s|^2}\left(1 - |\Gamma_L|^2\right)\left(1 - |\Gamma_s|^2\right)$$

$$G_T = \frac{\left(1 - |\Gamma_L|^2\right)|s_{21}|^2\left(1 - |\Gamma_s|^2\right)}{|1 - s_{22}\Gamma_L|^2\,|1 - \Gamma_1\Gamma_s|^2}$$

$$\left(1 - \Gamma_1\Gamma_s\right)\left(1 - \Gamma_L s_{22}\right) = \left(1 - \Gamma_2\Gamma_L\right)\left(1 - \Gamma_s s_{11}\right)$$

$$G_T = \frac{\left(1 - |\Gamma_L|^2\right)|s_{21}|^2\left(1 - |\Gamma_s|^2\right)}{|1 - s_{11}\Gamma_s|^2\,|1 - \Gamma_2\Gamma_L|^2}$$

$$\Gamma_1 = \frac{s_{11} - D\Gamma_L}{1 - s_{22}\Gamma_L}$$

$$G_T = \frac{\left(1 - |\Gamma_L|^2\right)|s_{21}|^2\left(1 - |\Gamma_s|^2\right)}{\left|\left(1 - s_{22}\Gamma_L\right)\left(1 - s_{11}\Gamma_s\right) - s_{12}s_{21}\Gamma_s\Gamma_L\right|^2}$$

Derivation of Gain Expression - 5

- The Available Power Gain G_A can be obtained from G_T when $\Gamma_L = \Gamma_2^*$
- Since

$$G_A = \frac{\text{Available Load Power (when output is conjugately matched)}}{\text{Available Source Power}}$$

We have…

$$G_A = G_T\big|_{\Gamma_2 = \Gamma_L^*} = \left.\frac{\left(1 - |\Gamma_L|^2\right)|s_{21}|^2\left(1 - |\Gamma_s|^2\right)}{|1 - s_{11}\Gamma_s|^2\,|1 - \Gamma_2\Gamma_L|^2}\right|_{\Gamma_2 = \Gamma_L^*}$$

$$G_A = \frac{\left(1 - |\Gamma_s|^2\right)|s_{21}|^2}{|1 - s_{11}\Gamma_s|^2\left(1 - |\Gamma_2|^2\right)}$$

Derivation of Gain Expression - 6

Finding Power Gain G_P :

$$P_L = \tfrac{1}{2}|b_2|^2\left(1-|\Gamma_L|^2\right) \qquad P_{in} = \tfrac{1}{2}|a_1|^2\left(1-|\Gamma_1|^2\right)$$

$$\boxed{\frac{b_2}{a_1} = \frac{s_{21}}{1-s_{22}\Gamma_L}}$$

From slide DGE-2

$$G_P = \frac{\text{Load Power}}{\text{Source Power}} = \frac{P_L}{P_{in}} = \frac{|s_{21}|^2\left(1-|\Gamma_L|^2\right)}{|1-s_{22}\Gamma_L|^2\left(1-|\Gamma_1|^2\right)}$$

Appendix 2 – The μ Stability Test

The μ Stability Test

- The Roulette Stability Test consist of 2 separate tests (the K and D values).
- This makes it difficult to compare the relative stability of 2 conditionally stable amplifiers.
- A later development combines the 2 tests into one, known as the μ factor. Larger value indicates greater stability.
- For an amplifier to be unconditionally stable, it is necessary that:

$$\mu_1 = \frac{1-|s_{22}|^2}{|s_{11} - D(s_{22}^*)| + |s_{21}s_{12}|} > 1$$

Edwards, M. L., and J. H. Sinsky, "A new criterion for linear two-port stability using A single geometrically derived parameter", IEEE Trans. On Microwave Theory and Techniques, Dec 1992.

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