PASSIVE RADIO-FREQUENCY (RF) CIRCUIT DESIGN

Agenda for 3-Days

- Advanced Transmission Line Theory.
- Transmission Line Circuit and RF Network Theory.
- Impedance Transformation.
- RF Filters.
- Three and Four Ports Components.
- Coaxial and Waveguide Components (Optional).

1. Advanced Transmission Line Theory

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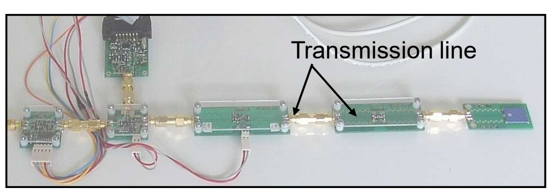
Preface

- Transmission lines and waveguides are the most important elements in microwave or RF circuits and systems.
- Transmission lines and waveguides are used to connect various components together to form a complex circuit. This is similar to low frequency circuit, where we use wires or copper track to connect the various components in an electronic circuit.
- In addition, you will see later that many types of microwave components are fabricated from short sections of transmission lines or waveguides.
- For these reasons, a lot of emphasis is placed on understanding the behavior of electromagnetic fields in transmission lines and waveguides.



References

- [1] R. E. Collin, "Foundation for microwave engineering", 2nd edition, 1992, McGraw-Hill.

★★★ A very advanced and in-depth book on microwave engineering. Difficult to read but the information is very comprehensive. A classic work. Recommended.

- [2] D. M. Pozar, "Microwave engineering", 3rd edition, 2005, John-Wiley & Sons).

★★ Easier to read and understand. Also a good book. Recommended.

- [3] S. Ramo, J.R. Whinnery, T.D. Van Duzer, "Field and waves in communication electronics" 3rd edition, 1993 John-Wiley & Sons.

★★ Good coverage of EM theory with emphasis on applications.

- [4] D. K. Cheng, "Field and waves electromagnetics", 2nd edition, 1989, Addison Wesley.

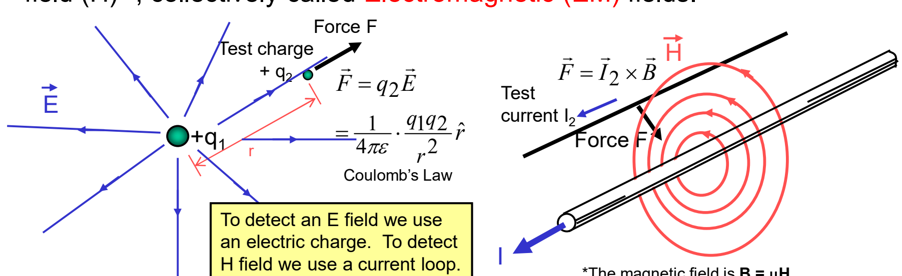
References Cont...

- [5] F. Kung, "Modeling of high-speed printed circuit board." Master degree dissertation, 1997, University Malaya.
  http://pesona.mmu.edu.my/~wlkung/Master/mthesis.htm
- [6] F. Kung unpublished notes and works.

1.0 Review of Electromagnetic (EM) Fields

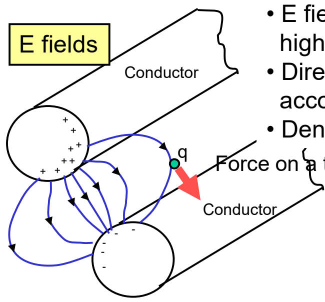
Electric and Magnetic Fields (1)

- In an electronic system, such as on PCB assembly, there are electric charges (q). To make our electronic system works, we essentially control electric charges (the charge density and rate of flow on various point in the circuit).
- Flow of electric charges due to potential difference (V) produces electric current (I).
- Associated with charge is electric field (E) and with current is magnetic field (H) *, collectively called Electromagnetic (EM) fields.



*The magnetic field is **B = μH**, **H** is the magnetization.

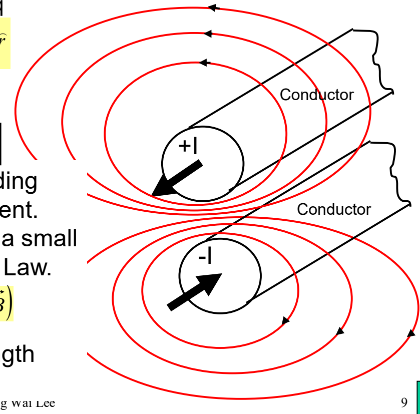
Electric and Magnetic Fields (2)



- E fields, by convention is directed from conductor with higher potential to conductor with less potential.
- Direction indicates force experienced by a small test charge according to Coulomb's Force Law.
- Density of the field lines corresponds to strength of the field.

Force on a test charge q

$$\vec{F} = \frac{1}{4\pi\varepsilon_o} \cdot \frac{Qq}{r^2}\hat{r}$$



H fields

- H fields, by convention is directed according to the right-hand rule with respect to current.
- Direction indicates force experienced by a small test current according to Lorentz's Force Law.

$$\vec{F} = q\left(\vec{v} \times \vec{B}\right)$$

•Density of field lines corresponds to strength of the field.

Maxwell Equations (Linear Medium) - Time-Domain Form (1)

Faraday's law

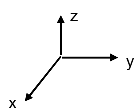
$$\nabla \times \vec{E} = -\frac{\partial}{\partial t}\vec{B}$$

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial}{\partial t}\vec{D}$$

Modified Ampere's law

$$\nabla \cdot \vec{D} = \rho_v$$ Gauss's law

$$\nabla \cdot \vec{B} = 0$$ No name, but can be called Gauss's law for magnetic field



Each parameter depends on 4 independent variables

Where:
$$\vec{E} = E_x(x,y,z,t)\hat{x} + E_y(x,y,z,t)\hat{y} + E_z(x,y,z,t)\hat{z}$$
$$\vec{H} = H_x(x,y,z,t)\hat{x} + H_y(x,y,z,t)\hat{y} + H_z(x,y,z,t)\hat{z}$$
$$\vec{J} = J_x(x,y,z,t)\hat{x} + J_y(x,y,z,t)\hat{y} + J_z(x,y,z,t)\hat{z}$$
$$\rho_v = \rho_v(x,y,z,t)$$

Unit vector in x-direction

x component

Constitutive relations

$$\boxed{\vec{B} = \mu\vec{H} \qquad \vec{D} = \varepsilon\vec{E} \qquad \mu = \mu_o\mu_r \qquad \varepsilon = \varepsilon_o\varepsilon_r}$$

For linear medium

E – Electric field intensity
H – Auxiliary magnetic field
D – Electric flux
B – Magnetic field intensity
J – Current density
ρ_v – Volume charge density
ε_o – permittivity of free space ($\cong 8.85412 \times 10^{-12}$)
μ_o – permeability of free space ($4\pi \times 10^{-7}$)
ε_r – relative permittivity
μ_r – relative permeability

Maxwell Equations (Linear Medium) - Time-Domain Form (2)

Extra

- Maxwell Equations as shown are actually a collection of 4 partial differential equations (PDE) that describe the physical relationship between electromagnetic (EM) fields, electric current and electric charge.
- The Del operator ∇ is a shorthand for three-dimensional (3D) differentiation:

$$\nabla = \left(\frac{\partial}{\partial x}\hat{x} + \frac{\partial}{\partial y}\hat{y} + \frac{\partial}{\partial z}\hat{z}\right)$$

- For instance consider the 1st and 3rd Maxwell Equations:

Curl

$$\nabla \times \tilde{E} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & E_z \end{vmatrix} = \left(\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}\right)\hat{x} + \left(\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x}\right)\hat{y} + \left(\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}\right)\hat{z}$$

$$= -\frac{\partial}{\partial t}\left(B_x x + B_y \hat{y} + B_z \hat{z}\right)$$

Gradient

$$\nabla F = \frac{\partial F}{\partial x}\hat{x} + \frac{\partial F}{\partial y}\hat{y} + \frac{\partial F}{\partial z}\hat{z}$$

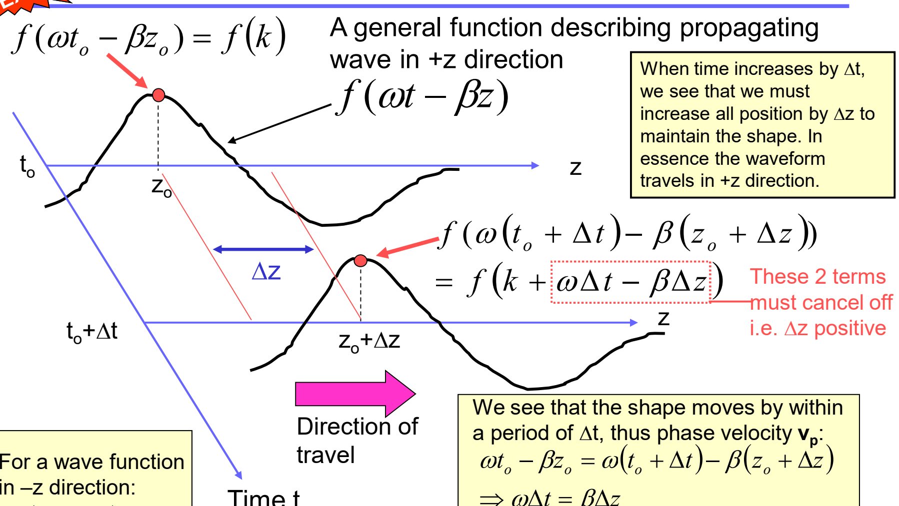
$$\nabla \cdot \tilde{E} = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} = \frac{\rho}{\varepsilon}$$

Divergence

To truly understands this subject, and also RF/Microwave circuit design, one needs to have a strong grasp of Electromagnetism (EM). Read references [1], [3] or any good book on EM.

Extra: Wave Function and Phasor (1)

Extra



When time increases by Δt, we see that we must increase all position by Δz to maintain the shape. In essence the waveform travels in +z direction.

We see that the shape moves by within a period of Δt, thus phase velocity $\mathbf{v_p}$:

$$\omega t_o - \beta z_o = \omega(t_o + \Delta t) - \beta(z_o + \Delta z)$$

$$\Rightarrow \omega \Delta t = \beta \Delta z$$

$$\Rightarrow v_p = \frac{\Delta z}{\Delta t} = \frac{\omega}{\beta}$$

For a wave function in –z direction:

$$f(\omega t + \beta z)$$

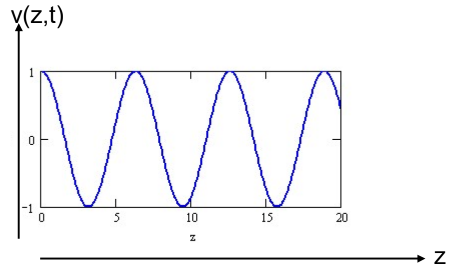
Extra: Wave Function and Phasor (2)

Extra

- An example: $$v(z,t) = V_o \cos(2\pi f t - \beta z)$$

$$f = 1.0\text{MHz}, \ \beta = 1$$

A sinusoidal wave



Phase Velocity: $v_p = \frac{\omega}{\beta} = \frac{2\pi f}{\beta}$

wavelength $\lambda = \frac{2\pi}{\beta}$

Extra: Wave Function and Phasor (3)

Extra

- In many ways the frequency *f* does not carry much information. If a **linear** system is excited by a sinusoidal source with frequency *f*, we know the response at every point in the system will be sinusoidal with frequency *f*.
- It is the phase constant β which carries more information, it determines the velocity and wavelength, of the wave.
- Thus it is more convenient if we convert the expressions for EM fields into phasor or Time-Harmonic form, as shown below:

Euler's formula
$e^{j\alpha} = \cos\alpha + j\sin\alpha$
$j = \sqrt{-1}$

$$\cos(\omega t \mp \beta z) = \mathrm{Re}\left\{e^{j\omega t} e^{\mp j\beta z}\right\}$$

Using Euler's formula

$$\cos(\omega t \mp \beta z) \Leftrightarrow e^{\mp j\beta z}$$

More compact form

$$v(z,t) = V_o \cos(2\pi f t - \beta z) \Leftrightarrow \boxed{\mathrm{V}(z) = V_o e^{-j\beta z}}$$

Phasor for *v*(z,t)

Convention: small letter for time-domain form, capital letter for phasor.

Extra: Wave Function and Phasor (4)

Extra

- Wave function and phasor notation is not only applicable to quantities like voltage, current or charge. It is also applied to vector quantities like **E** and **H** fields.
- For instance for sinusoidal **E** field traveling in +z direction:

Propogating function

$$\vec{E}^{+}(x,y,z,t) = e_x(x,y)\cos(\omega t - \beta z)\hat{x} + e_y(x,y)\cos(\omega t - \beta z)\hat{y} + e_z(x,y)\cos(\omega t - \beta z)\hat{z}$$

$$= \underbrace{\left(e_x\hat{x} + e_y\hat{y} + e_z\hat{z}\right)}_{E_o}\cos(\omega t - \beta z) = \mathrm{Re}\left\{\vec{E}_o^{+} e^{-j\beta z} e^{j\omega t}\right\}$$

Pattern function (x, y dependent)

- The phasor is given by:

$$\vec{\mathrm{E}}^{+}(x,y,z) = e_x(x,y)e^{-j\beta z}\hat{x} + e_y(x,y)e^{-j\beta z}\hat{y} + e_z(x,y)e^{-j\beta z}\hat{z}$$

- Finally if we substitute the phasor form $\vec{E}_o^{+} e^{-j\beta z} e^{j\omega t}$ for **E**, **H**, **J** and ρ into time-domain Maxwell's Equations, we would obtain the Maxwell's Equations in time-harmonic form.

Maxwell Equations (Linear Medium) - Time-Harmonic Form (1)

Extra

- For sinusoidal variations with time *t*, we substitute the phasors for **E**, **H**, **J** and ρ into Maxwell's Equations, the result are Maxwell's Equations in time-harmonic form.

$$\frac{\partial}{\partial t} \rightarrow j\omega$$

$$\nabla \times \vec{\mathrm{E}} = -j\omega\vec{\mathrm{B}} \qquad \textit{(1.2a)}$$

$$\nabla \times \vec{\mathrm{H}} = \vec{\mathrm{J}} + j\omega\vec{\mathrm{D}} \qquad \textit{(1.2b)}$$

$$\nabla \cdot \vec{\mathrm{D}} = \rho_V \qquad \textit{(1.2c)}$$

$$\nabla \cdot \vec{\mathrm{B}} = 0 \qquad \textit{(1.2d)}$$

Each parameter depends on 3 independent variables

Where: $\vec{\mathrm{E}} = \mathrm{E}_x(x,y,z)\hat{x} + \mathrm{E}_y(x,y,z)\hat{y} + \mathrm{E}_z(x,y,z)\hat{z}$

$\vec{\mathrm{H}} = \mathrm{H}_x(x,y,z)\hat{x} + \mathrm{H}_y(x,y,z)\hat{y} + \mathrm{H}_z(x,y,z)\hat{z}$

$\vec{\mathrm{J}} = \mathrm{J}_x(x,y,z)\hat{x} + \mathrm{J}_y(x,y,z)\hat{y} + \mathrm{J}_z(x,y,z)\hat{z}$

$\rho_V = \rho_V(x,y,z)$

E – Electric field intensity
H – Auxiliary magnetic field
D – Electric flux
B – Magnetic field intensity
J – Current density
ρ_v- Volume charge density
ε_o – permittivity of free space ($\cong 8.85412\times10^{-12}$)
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ε_r – relative permittivity
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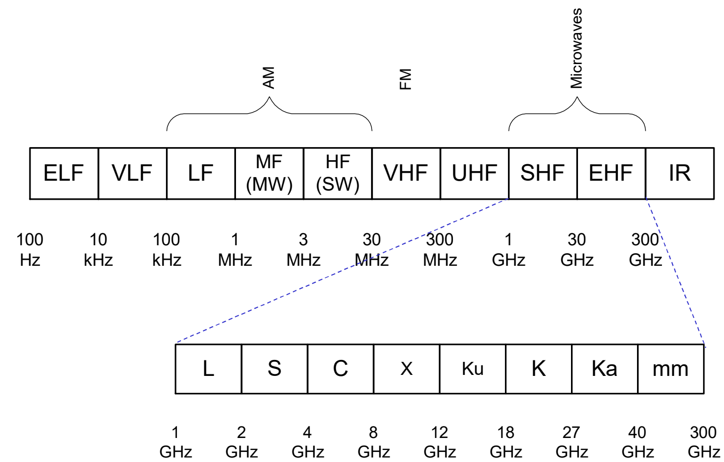
Constitutive relations

$$\boxed{\vec{\mathrm{B}} = \mu\vec{\mathrm{H}} \quad \vec{\mathrm{D}} = \varepsilon\vec{\mathrm{E}} \quad \mu = \mu_o\mu_r \quad \varepsilon = \varepsilon_o\varepsilon_r}$$

For linear medium

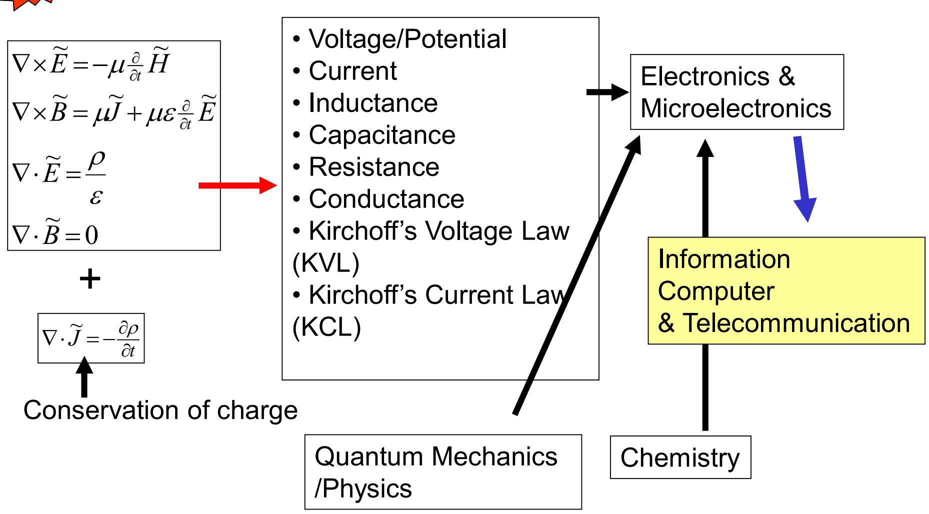
Extra

Electromagnetic Spectrum



Extra

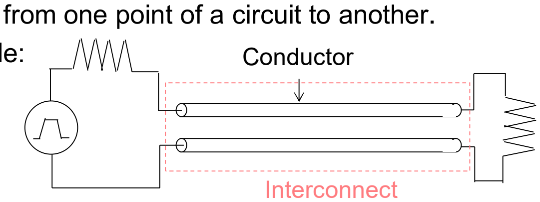
A Little Perspective…

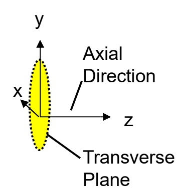


2.0 Introduction – Transmission Line Concepts

Definition of Electrical Interconnect

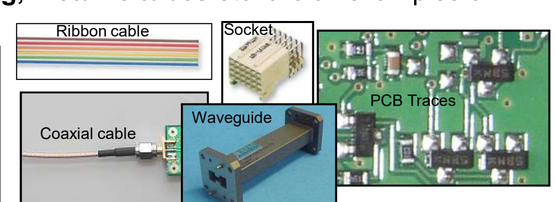
- Interconnect - metallic conductors that is used to transport electrical energy from one point of a circuit to another.
- Example:





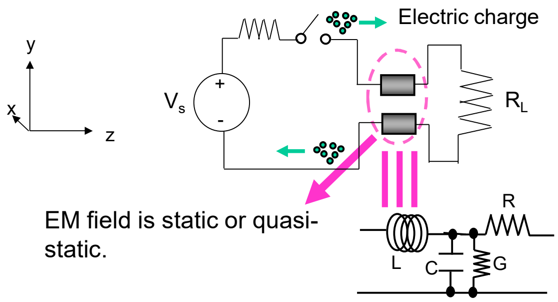
- Thus **cables**, **wires**, **conductive tracks** on printed circuit board (PCB), **sockets, packaging, metallic tubes** etc. are all examples of interconnect.

1. Usually contains 2 or more conductors, to form a closed circuit.
2. Conductors assumed to be perfect electric conductors (PEC)



Short Interconnect – Lumped Circuit

- For short interconnect, the moment the switch is closed, a voltage will appear across R_L as current flows through it. The effect is almost instantaneous.
- Voltage and current are due to electric charge movement along the conductors.
- Associated with the electric charges are static electromagnetic (EM) field in the space surrounding the short interconnect.
- The short interconnect system can be modeled by lumped RLC circuit.



Static EM field changes uniformly, i.e. when field at one point increases, field at the other locations also increases.

EM field is static or quasi-static.

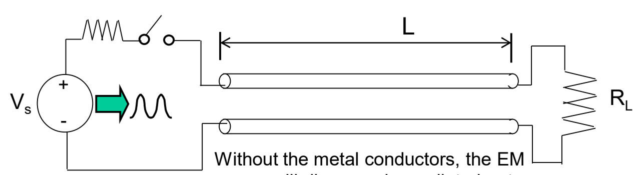
$$\nabla \times \tilde{E} \cong 0$$
$$\nabla \times \tilde{B} \cong \mu \tilde{J}$$
$$\nabla \cdot \tilde{E} = \frac{\rho}{\varepsilon}$$
$$\nabla \cdot \tilde{B} = 0$$

Again we need to stress that typically the values of RLCG are very small, at low frequency their effect can be simply ignored.

Long Interconnect (1)

- If the interconnection is long, it takes some time for voltage and current to appear on the resistor R_L after the switch is closed.
- Electric charges move from V_s to the resistor R_L. As the charges move, there is an associated EM field which travels along with the charges.
- In effect there is a propagating EM field along the interconnect. The propagating EM field is called a wave and the interconnect is guiding the EM wave, this EM field is dynamic.
- Since any arbitrary waveform can be decomposed into its sinusoidal components, let us consider V_s to be a sinusoidal source.

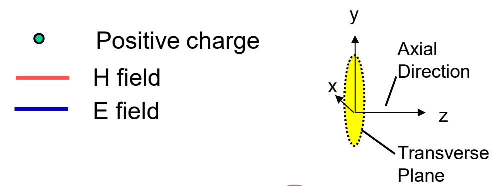
Long interconnect: When there is an appreciable delay between input and output

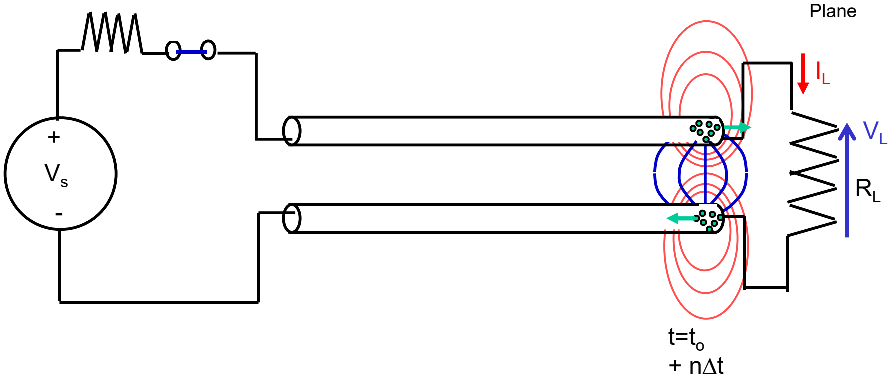


Without the metal conductors, the EM waves will disperse, i.e. radiated out into space.

Long Interconnect (2)

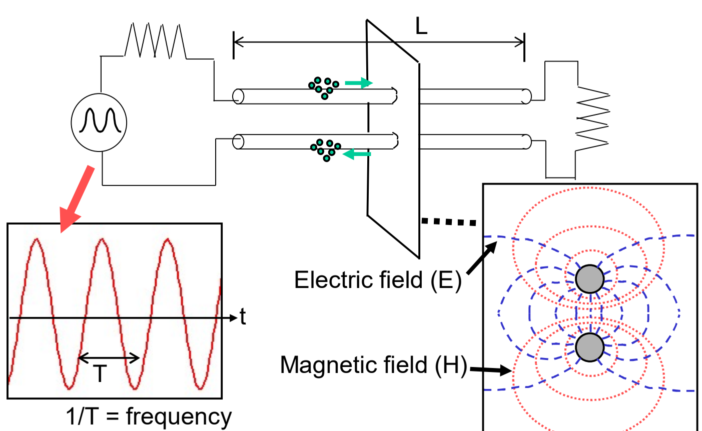
- A simple animation…





Long Interconnect (3)

- The corresponding EM field generated when electric charge flows along the interconnect is also sinusoidal with respect to time and space. The EM fields 'pattern' are dictated by Maxwell's Equations.



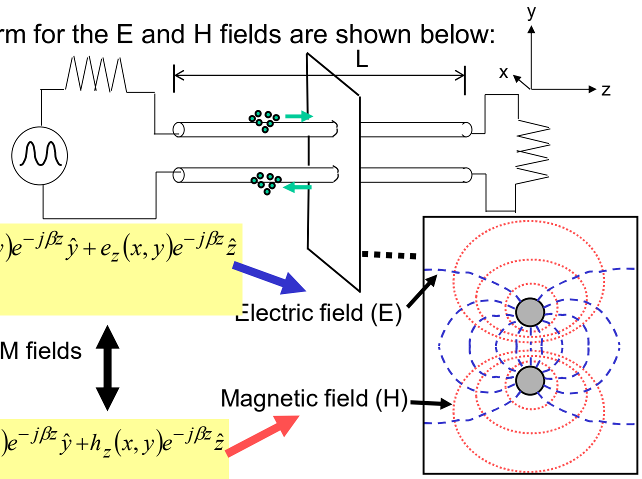
- Remember that current is due to the flow of free electrons.
- **E** and **H** are also sinusoidal.
- Behavior of **E** and **H** are dictated by Maxwell's Equations:

$$\nabla \times \tilde{E} = -\mu \tfrac{\partial}{\partial t} \tilde{H}$$
$$\nabla \times \tilde{B} = \mu \tilde{J} + \mu\varepsilon \tfrac{\partial}{\partial t} \tilde{E}$$
$$\nabla \cdot \tilde{E} = \frac{\rho}{\varepsilon}$$
$$\nabla \cdot \tilde{B} = 0$$

Long Interconnect (4)

- A valid solution form for the E and H fields are shown below:

By analyzing Maxwell's Equations or Wave Equations (Appendix 1)



$$\vec{E}^{+} = e_x(x,y)e^{-j\beta z}\hat{x} + e_y(x,y)e^{-j\beta z}\hat{y} + e_z(x,y)e^{-j\beta z}\hat{z}$$
$$= \left(\vec{e}_t(x,y) + e_z(x,y)\hat{z}\right)e^{-j\beta z}$$

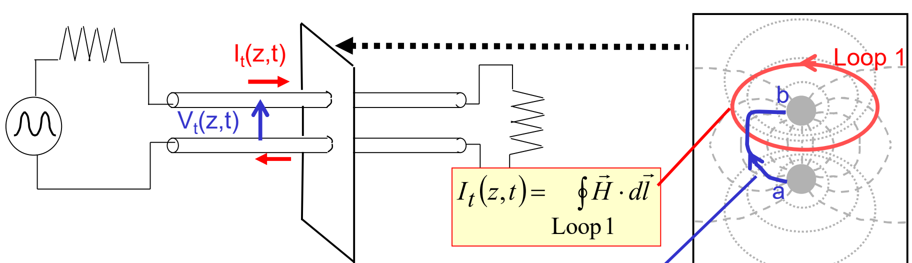
Propagating EM fields

$$\vec{H}^{+} = h_x(x,y)e^{-j\beta z}\hat{x} + h_y(x,y)e^{-j\beta z}\hat{y} + h_z(x,y)e^{-j\beta z}\hat{z}$$
$$= \left(\vec{h}_t(x,y) + h_z(x,y)\hat{z}\right)e^{-j\beta z}$$

Snapshot of EM fields at a certain instant in time on the transverse plane

Voltage and Current on Interconnect (1)

- The **E** field gives rise to voltage or potential difference while the **H** field is related to the current flowing along the conductors.
- Here we are interested in the transverse voltage V_t and current $\mathbf{I}_t$ as shown.

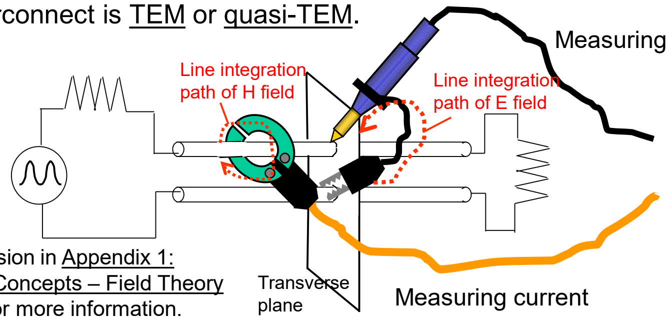


$$I_t(z,t) = \oint_{\text{Loop 1}} \vec{H}\cdot d\vec{l}$$

$$V_t(z,t) = -\int_a^b \vec{E}\cdot d\vec{l}$$

Voltage and Current on Interconnect (2)

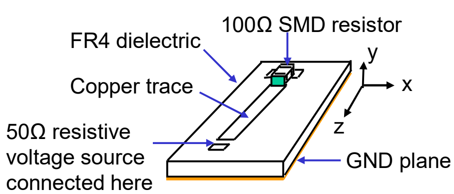
- V_t = Potential difference between two points on transverse plane and I_t = Rate of flow of electric charge across a conductor surface.
- V_t and I_t depends on instantaneous **E** and **H** fields on the interconnect, and correspond to how we would measure them physically with probes.
- V_t and I_t will be unique (e.g. do not depend on measurement setup, but only on the location) if and only if the EM field propagation mode in the interconnect is TEM or quasi-TEM.



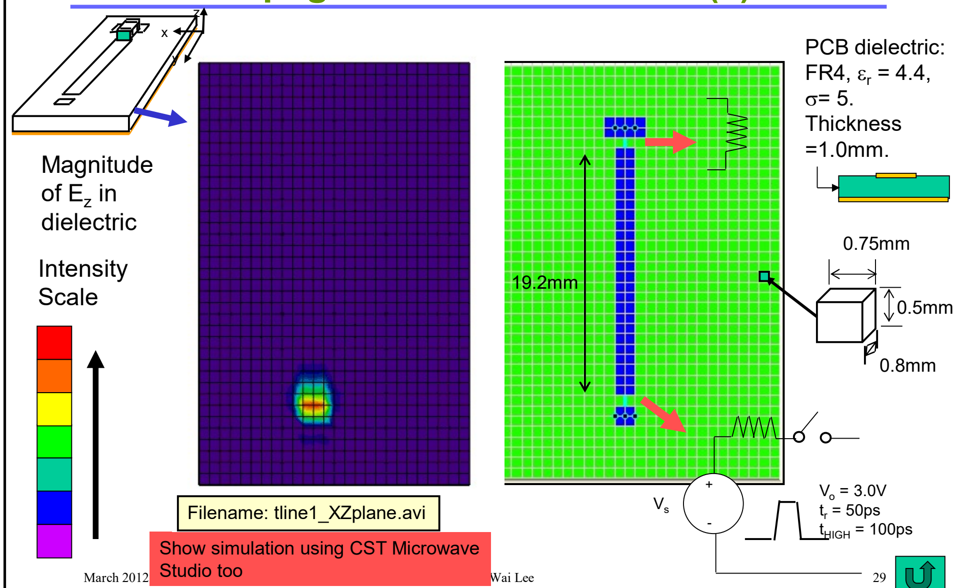
See discussion in Appendix 1: Advanced Concepts – Field Theory Solutions for more information.

Demonstration - Electromagnetic Field Propagation in Interconnect (1)

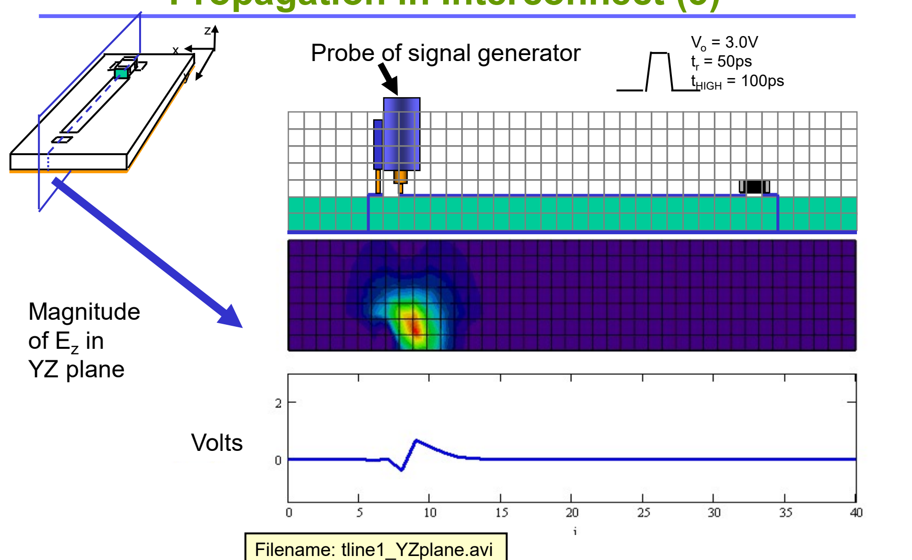
- The following example simulate the behavior of EM field in a simple interconnection system.
- The system is a 3D model of a copper trace with a plane on the bottom.
- A numerical method, known as Finite-Difference Time-Domain (FDTD) is applied to Maxwell's Equations, to provide the approximate value of E and H fields at selected points on the model at every 1.0 picosecond interval. (Search WWW or see http://pesona.mmu.edu.my/~wlkung/Phd/phdthesis.htm)
- Field values are displayed at an interval of 25.0 picoseconds.



Demonstration - Electromagnetic Field Propagation in Interconnect (2)



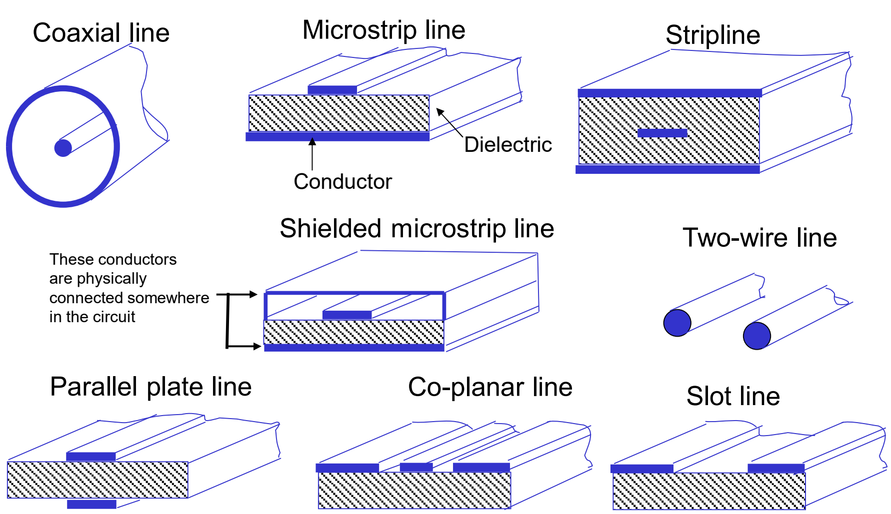
Demonstration - Electromagnetic Field Propagation in Interconnect (3)



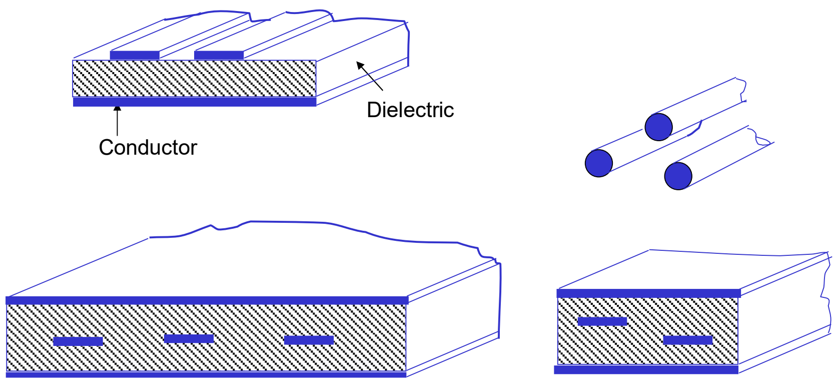
Definition of Transmission Line

- A transmission line is a <u>long interconnect</u> with 2 conductors – the signal conductor and ground conductor for returning current.
- Multiconductor transmission line has more than 2 conductors, usually a few signal conductors and one ground conductor.
- Transmission lines are a subset of a broader class of devices, known as waveguide. Transmission line has at least 2 or more conductors, while waveguides refer collectively to any structures that can allow EM waves to propagate along the structure. This includes structures with only 1 conductor or no conductor at all.
- Widely known waveguides include the rectangular and circular waveguides for high power microwave system, and the optical fiber. Waveguide is used for system requiring (1) high power, (2) very low loss interconnect (3) high isolation between interconnects.
- Transmission line is more popular and is widely used in PCB. From now on we will be concentrating on transmission line, or **Tline** for short.

Typical Transmission Line Configurations

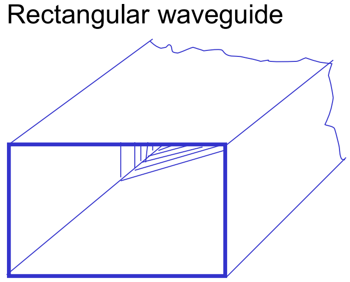


Examples of Multi-conductor Transmission Line Configurations

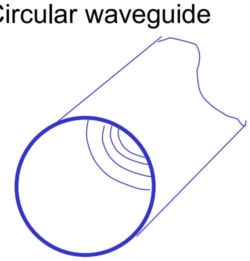


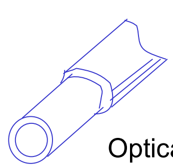
Examples Waveguide Configurations (1)

Rectangular waveguide

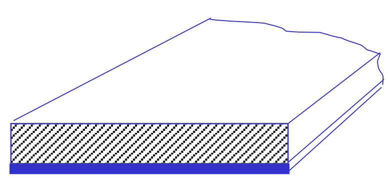


Circular waveguide





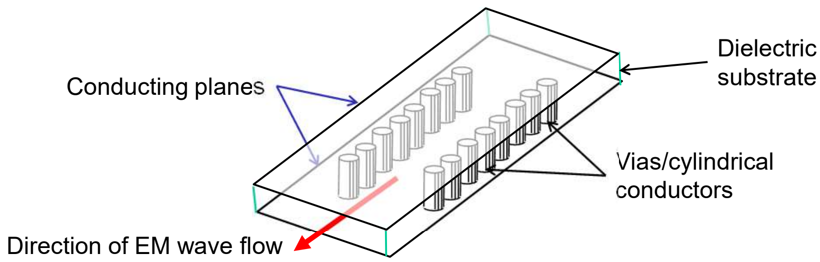
Optical Fiber



Dielectric waveguide

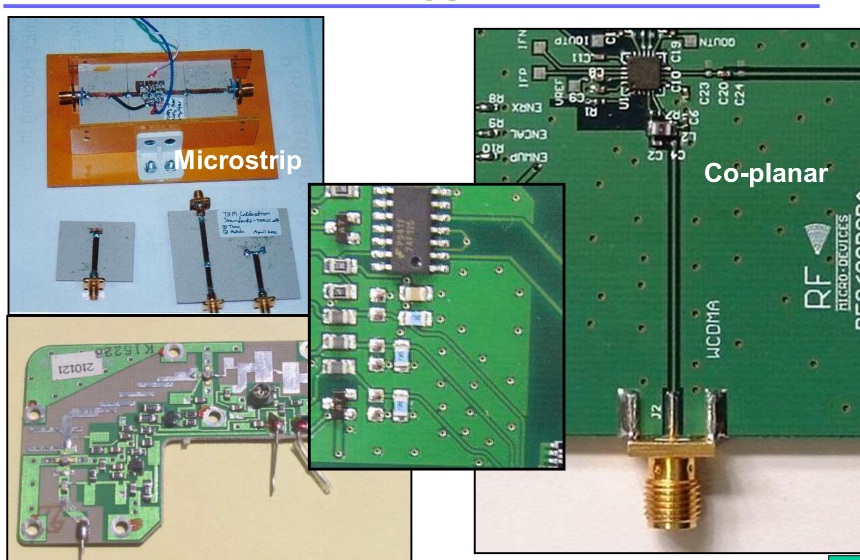
Examples of Waveguide Configurations (2)

- Substrate Integrated Waveguide (SIW), a structure similar to rectangular waveguide but implemented on printed circuit board or Integrated Circuit. Beginning to gain popularity since 2006.



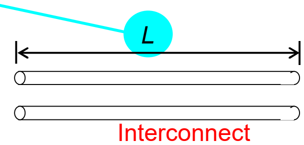
For example see:
1. Ke W., Deslandes D., Cassivi Y., "The substrate integrated circuits – a new concepts for high-frequency electronics and optoelectronics", 6th IEEE International conference on Telecommunications in modern satellite, cable and broadcasting service (TELSIKS 2003), Oct 2003, Vol 1.
2. Xu F., Zhang Y. et al, "Finite-difference frequency-domain algorithm for modeling guided-wave properties of substrate integrated waveguide", IEEE Trans. Microwave Theory and Techniques, Vol. 51, No. 11, pp. 2221-2227, Nov 2003.

Examples of Microstrip and Co-planar Lines



Long or Short Interconnect? The Wavelength Rule-of-Thumb

- How do we determine if the interconnect is long or short, i.e. the EM fields between the conductors is propagating or static?
- We consider the physical length in relation to the wavelength for sinusoidal signals.
- Rule-of-Thumb: If $L < 0.05\lambda$, it is a short interconnect, otherwise it is considered a long interconnect. An example at the end of this section will illustrate this procedure clearly.



We call this the **5% rule**. Less conservative estimate will use 1/10=0.10 (the 10% Rule)

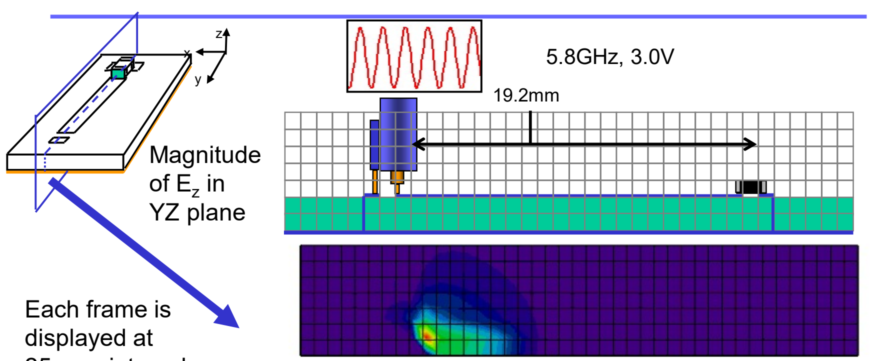
$$v_p = f\lambda \qquad (2.1)$$

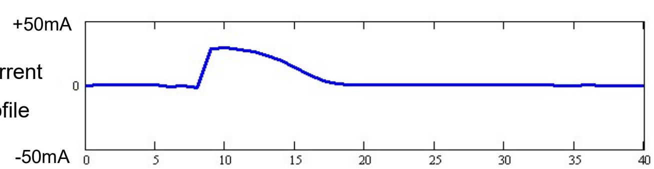
v_p: Phase velocity or propagation velocity; f: frequency; λ: wavelength

$f\uparrow \quad \lambda\downarrow$

$f\downarrow \quad \lambda\uparrow$

Demonstration – Long Interconnect





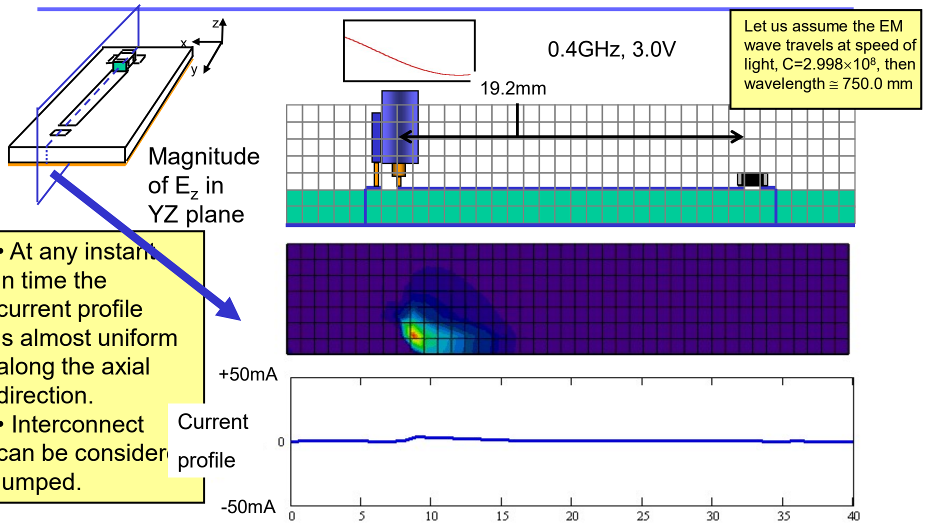
Filename: tline1_YZplane_5_8GHz.avi

Let us assume the EM wave travels at speed of light, C=2.998×10⁸, then wavelength ≅ 52.0 mm

$$\lambda = \frac{C}{f}$$

19.2 mm is greater than 5% of 52 mm

Demonstration – Short Interconnect



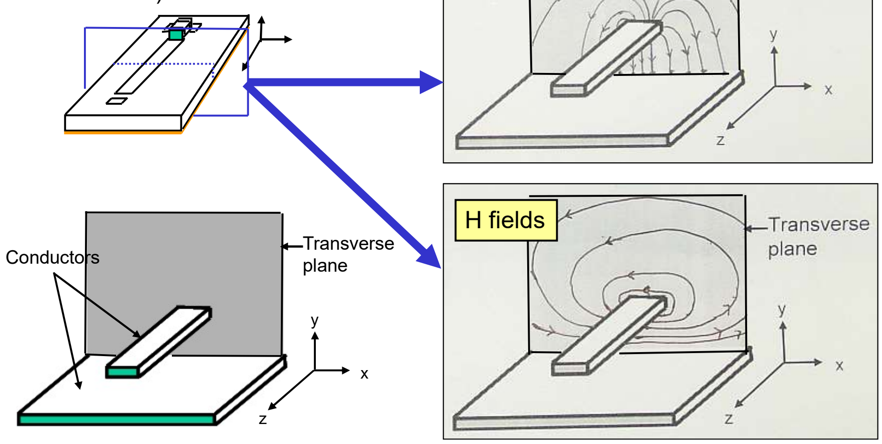
- At any instant in time the current profile is almost uniform along the axial direction.
- Interconnect can be considered lumped.

Filename: tline1_YZplane_0_4GHz.avi

3.0 Propagation Modes

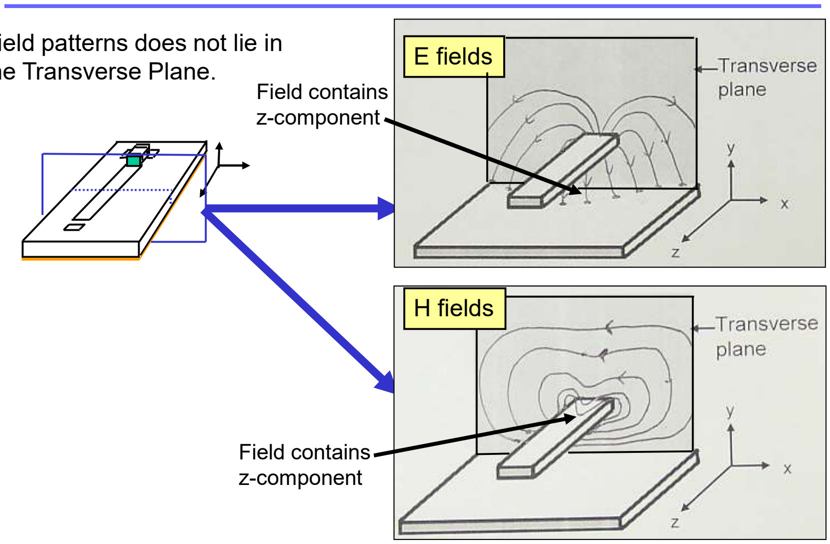
Transverse E and H Field Patterns

Field patterns lie in the Transverse Plane (A plane that is at 90º to the conductors).



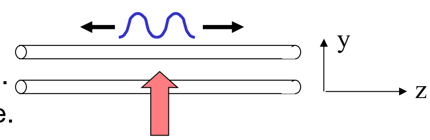
Non-transverse E and H Field Patterns

Field patterns does not lie in the Transverse Plane.



Propagation Modes (1)

- Assuming the transmission line is parallel to z direction. The propagation of **E** and **H** fields along the line can be classified into 4 type of patterns or Modes:
  - TE mode - where $E_z = 0$.
  - TM mode - where $H_z = 0$.
  - TEM mode - where E_z and H_z are 0.
  - Mix mode, any mixture of the above.

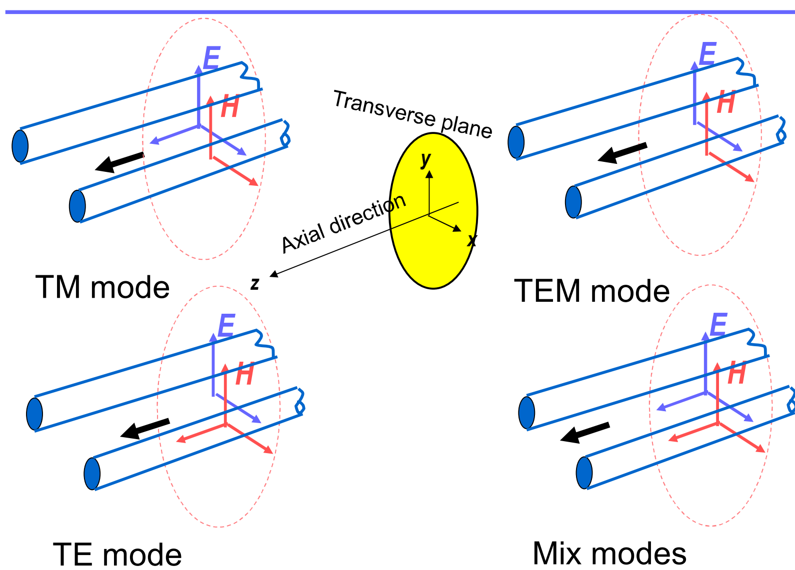


$$\vec{H}^{\pm} = \left(\vec{h}_t(x,y) + h_z(x,y)\hat{z}\right)e^{\mp j\beta z}$$

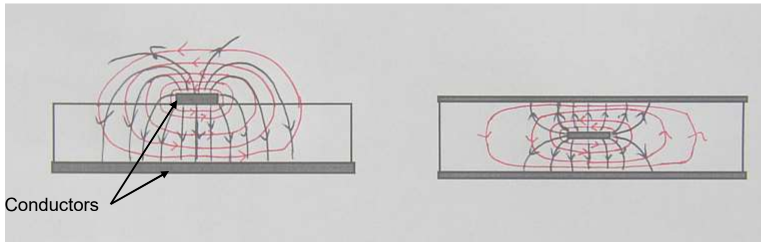
$$\vec{E}^{\pm} = \left(\vec{e}_t(x,y) + e_z(x,y)\hat{z}\right)e^{\mp j\beta z}$$

- A Tline can support a number of modes at any instance, however TE, TM or mix mode usually occur at very high frequency.
- There is another mode, known as quasi-TEM mode, which is supported by stripline structures with non-uniform dielectric. See discussion in Appendix 1.

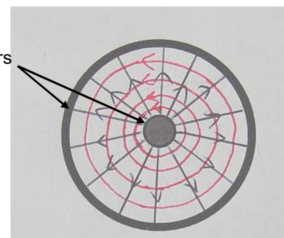
Propagation Modes (2)



Examples of Field Patterns or Modes



Conductors



TEM or quasi-TEM mode

E field

H field

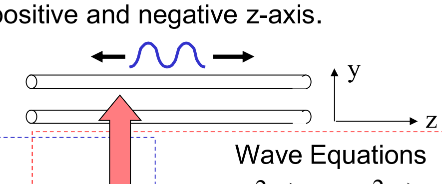
Why Propagation Modes are Important?

- Classification of the EM field patterns guided by conductors is important as properties like how fast the fields are traveling (the phase velocity), the energy stored in the fields, relationship between **E** and **H** fields, dispersion, cut-off etc are all dependent on the propagation modes.
- Again refer to Appendix 1 for more in-depth mathematical treatment.

Appendix 1
Advanced Concepts – Field Theory Solutions for Transmission Lines

Field Theory Solution

- The nature of E and H fields in the space between conductors can be studied by solving the Maxwell's Equations or Wave Equations (which can be derived from Maxwell's Equations) (See [1], [2], [3]). Assuming the condition of long interconnection, the solutions of E and H fields are propagating fields or waves.
- We assume time-harmonic EM fields with $e^{j\omega t}$ dependence and wave propagation along the positive and negative z-axis.



Maxwell Equations

$$\nabla \times \vec{E} = -j\omega\mu\vec{H}$$
$$\nabla \times \vec{H} = \vec{J} + j\omega\varepsilon\vec{E}$$
$$\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon}$$
$$\nabla \cdot \vec{H} = 0$$

\+ Boundary conditions +

For instance tangential E field component on PEC must be zero, continuity of E and H field components across different dielectric material, etc.

Wave Equations

$$\nabla^2\vec{E} + k_o^2\vec{E} = 0$$
$$\nabla^2\vec{H} + k_o^2\vec{H} = 0 \quad \textit{(A.1)}$$
$$k_o = \omega\sqrt{\varepsilon\mu}$$

In free space

Extra: Deriving the Hemholtz Wave Equations From Maxwell Equations

Extra

Performing curl operation on Faraday's Law $\nabla \times \vec{E} = -j\omega\mu\vec{H}$:

$$\nabla \times \left(\nabla \times \vec{E}\right) = \nabla\left(\nabla \cdot \vec{E}\right) - \nabla^2 \vec{E} = -j\omega\mu\left(\nabla \times \vec{H}\right)$$

$$\Rightarrow \nabla^2 \vec{E} + \omega^2 \mu\varepsilon\vec{E} = \boxed{j\omega\mu\vec{J} + \nabla\left(\tfrac{\rho}{\varepsilon}\right)}$$

These are the sources for the E field

Note: use the well-known vector calculus identity

$$\nabla \times \nabla \times \vec{A} = \nabla\left(\nabla \cdot \vec{A}\right) - \nabla^2 \vec{A}$$

$$\nabla^2 = \tfrac{\partial^2}{\partial x^2} + \tfrac{\partial^2}{\partial y^2} + \tfrac{\partial^2}{\partial z^2}$$

In free space there is no electric charge and current:

$$\nabla^2 \vec{E} + \omega^2 \mu\varepsilon\vec{E} = 0$$

Similar procedure can be used to obtain

$$\nabla^2 \vec{H} + \omega^2 \mu\varepsilon\vec{H} = -\nabla \times \vec{J}$$

Or in free space

$$\nabla^2 \vec{H} + \omega^2 \mu\varepsilon\vec{H} = 0$$

Note: This derivation is valid for time-harmonic case under linear medium only. See more advanced text for general wave equation. For Example:
C. A. Balanis, "Advanced engineering Electromagnetics", John-Wiley, 1989.

Obtaining the Expressions for E and H (1)

Extra

- Assuming an ordinary differential equation (ODE) system as shown:

ODE → $\frac{d^2y}{dx^2} + k^2 y = 0, \quad y = f(x), \quad x \in [0,b]$ ← The Domain

$y(0) = C_1 \text{ and } y(b) = C_2$ ← Boundary conditions

- To obtain a solution to the above system (a solution means a function that when substituted into the ODE, will cause left and right hand side to be equal), many approaches can be used (for instance see *E. Kreyszig, "Advance engineering mathematics", 1998, John Wiley*).
- One popular approach is the Trial-and-Error/substitution method, where we guess a functional form for y(x) as follows: $y(x) = e^{\beta x} \Rightarrow \frac{dy}{dx} = \beta e^{\beta x}, \; \frac{d^2y}{dx^2} = \beta^2 e^{\beta x}$
- Substituting this into the ODE:

$$\left(\beta^2 + k^2\right)e^{\beta x} = 0$$
$$\Rightarrow \beta^2 + k^2 = 0$$
$$\Rightarrow \beta = \pm jk \quad \text{where } j = \sqrt{-1}$$

That the trial-and-error method works is attributed to the Uniqueness Theorem for linear ODE.

- Since this is a 2nd order ODE, we need to introduce 2 unknown constants, A and B, and a general solution is:

$$y(x) = Ae^{jkx} + Be^{-jkx} \qquad (1)$$

Obtaining the Expressions for E and H (2)

Extra

- To find A and B, we need to use the boundary conditions.

$$y(0) = C_1 \Rightarrow A + B = C_1 \qquad \text{(2a)}$$

$$y(b) = C_2 \Rightarrow Ae^{jkb} + Be^{-jkb} = C_2 \qquad \text{(2b)}$$

- Solving (2a) and (2b) for A and B:

$$(C_1 - B)e^{jkb} + Be^{-jkb} = C_2$$

$$\Rightarrow B = \frac{C_1 e^{jkb} - C_2}{2j\sin(kb)} \qquad \text{(3a)}$$

$$A = C_1 - B = \frac{C_1 e^{-jkb} - C_2}{2j\sin(kb)} \qquad \text{(3b)}$$

- So the unique solution is:

$$\boxed{y(x) = \left(\frac{C_1 e^{-jkb} - C_2}{2j\sin(kb)}\right)e^{jkx} + \left(\frac{C_1 e^{jkb} - C_2}{2j\sin(kb)}\right)e^{-jkx}} \qquad \text{(q.e.d.)}$$

Obtaining the Expressions for E and H (3)

Extra

- The same approach can be applied to Wave Equations or Maxwell Equations for Tline. Consider the Wave Equations (A.1) in time-harmonic form.
- The unknown functions are vector phasors **E**(x,y,z) and **H**(x,y,z). The differential equation for **E** in Cartesian coordinate is:

$$\left(\nabla^2 + k_o^2\right)\vec{E} = 0$$

$$\Rightarrow \boxed{\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} + k_o^2\right)E_x\hat{x}} + \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} + k_o^2\right)E_y\hat{y} + \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} + k_o^2\right)E_z\hat{z} = 0$$

- This is called a Partial Differential Equation (PDE) as each E_x, E_y and E_z depends on 3 variables, with the differentiation substituted by partial differential. There are 3 PDEs if you observed carefully. For x-component this is:

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} + k_o^2\right)E_x = 0$$

- Based on the previous ODE example, and also the fact that we expect the E field to travel along the z-axis, the following form is suggested:

$$\boxed{E_x(x,y,z) = e_x(x,y)e^{-j\beta z} \text{ or } e_x(x,y)e^{j\beta z}}$$

A function of x and y

The exponent e !!!

Obtaining the Expressions for E and H (4)

Extra

- Carrying on in this manner for y and z-components, we arrived at the following form for E field.

$$\vec{E}^+ = e_x(x,y)e^{-j\beta z}\hat{x} + e_y(x,y)e^{-j\beta z}\hat{y} + e_z(x,y)e^{-j\beta z}\hat{z}$$
$$= \left(\vec{e}_t(x,y) + e_z(x,y)\hat{z}\right)e^{-j\beta z} \quad \textit{(A.1a)}$$

- Notice that up to now we have not solve the Wave Equations, but merely determine the functional form of its solution.
- We still need to find out what is $e_x(x,y)$, $e_y(x,y)$, $e_z(x,y)$ and β.
- Using similar approach on $\left(\nabla^2 + k_o^2\right)\vec{H} = 0$ will yield similar expression for H field.

$$\vec{H}^+ = h_x(x,y)e^{-j\beta z}\hat{x} + h_y(x,y)e^{-j\beta z}\hat{y} + h_z(x,y)e^{-j\beta z}\hat{z}$$
$$= \left(\vec{h}_t(x,y) + h_z(x,y)\hat{z}\right)e^{-j\beta z} \quad \textit{(A.1b)}$$

E and H fields Expressions (1)

- Thus the propagating EM fields guided by Tline can be written as:

Transverse component

$$\vec{E}^+ = e_x(x,y)e^{-j\beta z}\hat{x} + e_y(x,y)e^{-j\beta z}\hat{y} + e_z(x,y)e^{-j\beta z}\hat{z}$$
$$= \left(\vec{e}_t(x,y) + e_z(x,y)\hat{z}\right)e^{-j\beta z} \quad \textit{(A.2a)}$$

Axial component

$$\vec{H}^+ = h_x(x,y)e^{-j\beta z}\hat{x} + h_y(x,y)e^{-j\beta z}\hat{y} + h_z(x,y)e^{-j\beta z}\hat{z}$$
$$= \left(\vec{h}_t(x,y) + h_z(x,y)\hat{z}\right)e^{-j\beta z} \quad \textit{(A.2b)}$$

superscript indicates propagation direction

EM fields Propagating In +z direction

$$\vec{E}^- = e_x(x,y)e^{+j\beta z}\hat{x} + e_y(x,y)e^{+j\beta z}\hat{y} - e_z(x,y)e^{+j\beta z}\hat{z}$$
$$= \left(\vec{e}_t(x,y) - e_z(x,y)\hat{z}\right)e^{+j\beta z} \quad \textit{(A.3a)}$$

$$\vec{H}^- = -h_x(x,y)e^{+j\beta z}\hat{x} - h_y(x,y)e^{+j\beta z}\hat{y} + h_z(x,y)e^{+j\beta z}\hat{z}$$
$$= \left(-\vec{h}_t(x,y) + h_z(x,y)\hat{z}\right)e^{+j\beta z} \quad \textit{(A.3b)}$$

EM fields Propagating In -z direction

E and H fields Expressions (2)

- We can convert the phasor form into time-domain form, for instance for E field propagating in +z direction:

$$\vec{E}^{+}(x,y,z,t) = \mathrm{Re}\left\{\vec{E}^{+}(x,y,z)e^{j\omega t}\right\}$$
$$= e_x(x,y)\cos(\omega t - \beta z)\hat{x} + e_y(x,y)\cos(\omega t - \beta z)\hat{y} + e_z(x,y)\cos(\omega t - \beta z)\hat{z} \quad \textit{(A.4)}$$

- Where

$$\vec{E}^{+}(x,y,z,t) = E_x^{+}(x,y,z,t)\hat{x} + E_y^{+}(x,y,z,t)\hat{y} + E_z^{+}(x,y,z,t)\hat{z} \quad \textit{(A.5a)}$$

Unit vector

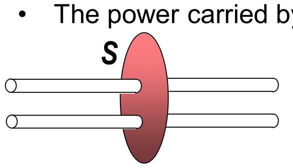
$$E_x^{+}(x,y,z,t) = e_x(x,y)\cos(\omega t - \beta z)$$
$$E_y^{+}(x,y,z,t) = e_y(x,y)\cos(\omega t - \beta z) \quad \textit{(A.5b)}$$
$$E_z^{+}(x,y,z,t) = e_z(x,y)\cos(\omega t - \beta z)$$

E and H fields Expressions (3)

- Usually one only solves for E field, the corresponding H field phasor can be obtained from:

$$\nabla \times \vec{E} = -j\omega\mu\vec{H}$$
$$\Rightarrow \vec{H} = \frac{1}{j\omega\mu}\nabla \times \vec{E} \quad \textit{(A.6)}$$

- The power carried by the EM fields is given by Poynting Theorem:



Positive Z direction…

$$P = \frac{1}{2}\mathrm{Re}\iint_S \vec{E}^{+} \times \left(\vec{H}^{+}\right)^{*} \cdot d\vec{s} = \frac{1}{2}\mathrm{Re}\iint_{S_z} \vec{e}_t \times \vec{h}_t^{\,*} \cdot \vec{ds}$$

Negative Z direction…

$$P = \frac{1}{2}\mathrm{Re}\iint_S \vec{E}^{-} \times \left(\vec{H}^{-}\right)^{*} \cdot d\vec{s} = \frac{1}{2}\mathrm{Re}\iint_{S_z} -\vec{e}_t \times \vec{h}_t^{\,*} \cdot \left(-\vec{ds}\right)$$
$$= \frac{1}{2}\mathrm{Re}\iint_{S_z} \vec{e}_t \times \vec{h}_t^{\,*} \cdot \vec{ds}$$

Positive value means that power is carried along the propagation direction.

E and H fields Expressions (4)

- There are 2 reasons for choosing the sign conventions for +ve and -ve propagating waves as in (A.2) and (A.3).
  - So that $\nabla \cdot \vec{E} = 0$ for both +ve and -ve propagating E field (consistency with Maxwell's Equations).
  - The transverse magnetic field must change sign upon reversal of the direction of propagation to obtain a change in the direction of energy flow.

$$\nabla \cdot \vec{E}^{+} = 0$$
$$\Rightarrow \nabla_t \cdot \vec{e}_t - j\beta e_z = 0$$

For +ve direction

$$\nabla \cdot \vec{E}^{-} = 0$$
$$\Rightarrow \nabla_t \cdot \vec{e}_t + j\beta(-e_z) = 0$$

For -ve direction

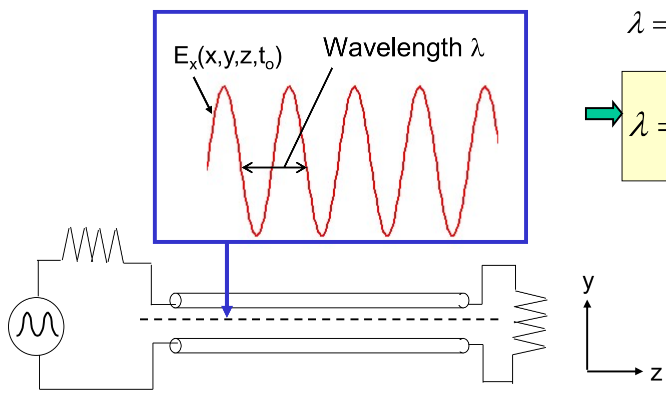
Phase Velocity

- It is easy to show that equation (A.2a) and (A.2b) describes traveling E field waves (also for H).
- The speed where the E and H fields travel is called the Phase Velocity, v_p.
- Phase Velocity depends on the propagation mode (to be discussed later), the frequency and the physical properties of the interconnect.

$$v_p = \frac{\omega}{\beta} \qquad \textbf{\textit{(A.7)}}$$

Wavelength

- For interconnect excited by sinusoidal source, if we freeze the time at a certain instant, say t = t_o, the **E** and **H** fields profile will vary in a sinusoidal manner along z-axis.



$$\lambda = \frac{v_p}{f}$$

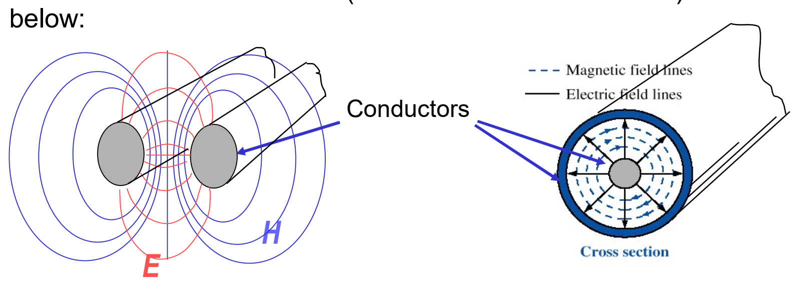
$$\lambda = \frac{\frac{\omega}{\beta}}{\frac{\omega}{2\pi}} = \frac{2\pi}{\beta} \qquad \textbf{(A.8)}$$

Superposition Theorem

- At any instant of time, there are E and H fields propagating in the positive and negative direction along the transmission line. The total fields are a superposition of positive and negative directed fields:

$$\vec{E} = \vec{E}^{+} + \vec{E}^{-} \qquad \vec{H} = \vec{H}^{+} + \vec{H}^{-}$$

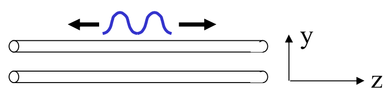
- A typical field distribution at a certain instant of time for the cross section of two interconnects (two-wire and co-axial cable) is shown below:



Field Solution (1)

- To find the value of β and the functions e_x, e_y, e_z, h_x, h_y, h_z, we substitute equations (A.2a) and (A.2b) into Maxwell or Wave equations.

$$\vec{E}^{+} = e_x(x,y)e^{-j\beta z}\hat{x} + e_y(x,y)e^{-j\beta z}\hat{y} + e_z(x,y)e^{-j\beta z}\hat{z}$$
$$= \left(\vec{e}_t(x,y) + e_z(x,y)\hat{z}\right)e^{-j\beta z}$$
$$\vec{H}^{+} = h_x(x,y)e^{-j\beta z}\hat{x} + h_y(x,y)e^{-j\beta z}\hat{y} + h_z(x,y)e^{-j\beta z}\hat{z}$$
$$= \left(\vec{h}_t(x,y) + h_z(x,y)\hat{z}\right)e^{-j\beta z}$$



Maxwell Equations

$$\nabla \times \vec{E} = -j\omega\mu\vec{H}$$
$$\nabla \times \vec{H} = \vec{J} + j\omega\varepsilon\vec{E}$$
$$\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon}$$
$$\nabla \cdot \vec{H} = 0$$

\+ Boundary conditions +

Wave Equations

$$\nabla^2\vec{E} + k_o{}^2\vec{E} = 0$$
$$\nabla^2\vec{H} + k_o{}^2\vec{H} = 0$$
$$k_o = \omega\sqrt{\varepsilon\mu}$$

$e_x(x,y)$
$e_y(x,y)$
$e_z(x,y)$
$h_x(x,y)$
$h_y(x,y)$
$h_z(x,y)$
β

Field Solution (2)

- The procedure outlined here follows those from Pozar [2]. Assume the Tline or waveguide dielectric region is source free. From Maxwell's Equations:

$$\nabla \times \vec{E} = -j\omega\mu\vec{H} \qquad \nabla \times \vec{H} = j\omega\varepsilon\vec{E} \qquad \textbf{(A.9)}$$

$$\vec{J} = 0$$
$$\nabla \times \vec{B} = \mu\vec{J} + j\omega\mu\varepsilon\vec{E}$$

- Substituting the suggested solution for **E**$^+(x,y,z,\beta)$ of (A.2a) into (A.9), and expanding the differential equations into *x*, *y* and *z* components:

$$\frac{\partial e_z}{\partial y} + j\beta e_y = -j\omega\mu h_x \quad \textbf{(A.10a)} \qquad \frac{\partial h_z}{\partial y} + j\beta h_y = j\omega\varepsilon e_x \quad \textbf{(A.10d)}$$

$$-j\beta e_x - \frac{\partial e_z}{\partial x} = -j\omega\mu h_y \quad \textbf{(A.10b)} \qquad -j\beta h_x - \frac{\partial h_z}{\partial x} = j\omega\varepsilon e_y \quad \textbf{(A.10e)}$$

$$\frac{\partial e_y}{\partial x} - \frac{\partial e_x}{\partial y} = -j\omega\mu h_z \quad \textbf{(A.10c)} \qquad \frac{\partial h_y}{\partial x} - \frac{\partial h_x}{\partial y} = j\omega\varepsilon e_z \quad \textbf{(A.10f)}$$

Field Solution (3)

- From (A.10a)-(A.10f), we can express e_x, e_y, h_x, h_y in terms of e_z and h_z:

These equations describe the *x*,*y* components of general EM wave propagation in a waveguiding system. The unknowns are $e_z(x,y)$ and $h_z(x,y)$, called the **Potential** in the literature.

See the book by Collin [1], Chapter 3 for alternative derivation

$$h_x = \frac{j}{k_c^2}\left(\omega\varepsilon\frac{\partial e_z}{\partial y} - \beta\frac{\partial h_z}{\partial x}\right) \qquad \textit{(A.11a)}$$

$$h_y = \frac{-j}{k_c^2}\left(\omega\varepsilon\frac{\partial e_z}{\partial x} + \beta\frac{\partial h_z}{\partial y}\right) \qquad \textit{(A.11b)}$$

$$e_x = \frac{-j}{k_c^2}\left(\beta\frac{\partial e_z}{\partial x} + \omega\mu\frac{\partial h_z}{\partial y}\right) \qquad \textit{(A.11c)}$$

$$e_y = \frac{j}{k_c^2}\left(-\beta\frac{\partial e_z}{\partial y} + \omega\mu\frac{\partial h_z}{\partial x}\right) \qquad \textit{(A.11d)}$$

$$k_c^2 = k_o{}^2 - \beta^2 \;\;,\; k_o = \omega\sqrt{\mu\varepsilon} = \frac{2\pi}{\lambda} \qquad \textit{(A.11e)}$$

TE Mode Summary (1)

- For TE mode, $e_z = 0$ (Sometimes this is called the H mode).
- We could characterize the Tline in TE mode, by EM fields:

$$\vec{H}^{\pm} = \left(\pm\vec{h}_t + h_z\hat{z}\right)e^{\mp j\beta z} \qquad \vec{E}^{\pm} = \vec{e}_t e^{\mp j\beta z}$$

- From wave equation for H field:

$$\nabla^2\vec{H} + k_o{}^2\vec{H} = 0$$

$$k_o = \omega\sqrt{\varepsilon\mu}$$

Transverse Laplacian operator

Note
$\nabla^2 = \nabla_t^2 - \beta^2$

$$\left(\nabla_t{}^2 + \frac{\partial^2}{\partial z^2} + k_o^2\right)\left(\left(\vec{h}_t + h_z\hat{z}\right)e^{-j\beta z}\right) = 0$$

Using the fact that $\frac{\partial^2}{\partial z^2}\left(e^{\mp j\beta z}\right) = -\beta^2 e^{\mp j\beta z}$

From (A.11e)

$$\boxed{\nabla_t{}^2 h_z + k_c{}^2 h_z = 0}$$

$$\nabla_t{}^2\vec{h}_t + k_c{}^2\vec{h}_t = 0$$

$$\boxed{k_c{}^2 = k_o^2 - \beta^2}$$

Only these are needed. The other transverse field components can be derived from h_z

TE Mode Summary (2)

- Setting $e_z = 0$ in (A.11a)-(A.11d):

$$h_x = \frac{-j\beta}{k_c^2}\frac{\partial h_z}{\partial x} \qquad h_y = \frac{-j\beta}{k_c^2}\frac{\partial h_z}{\partial y}$$

$$e_x = \frac{-j\omega\mu}{k_c^2}\frac{\partial h_z}{\partial y} \qquad e_y = \frac{j\omega\mu}{k_c^2}\frac{\partial h_z}{\partial x} \qquad \textbf{\textit{(A.12a)}}$$

- These equations plus the previous wave equation for h_z enable us to find the complete field pattern for TE mode.

$$\boxed{\begin{gathered}\nabla_t{}^2 h_z + k_c^2 h_z = 0 \\ k_c{}^2 = k_o{}^2 - \beta^2\end{gathered}}$$

+ boundary conditions for E and H fields

(A.12b)

From previous slide

TE Mode Summary (3)

- From (A.12a) and (A.12b), we can show that:

$$\nabla_t \times \vec{e}_t = \left[\frac{\partial e_y}{\partial x} - \frac{\partial e_x}{\partial y}\right]\hat{z} = \frac{j\omega\mu}{k_c^2}\left[\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2}\right]\hat{z}$$

$$= \frac{j\omega\mu}{k_c^2}\left(-k_c^2 h_z\right)\hat{z} = -j\omega\mu h_z\hat{z}$$

$$\nabla_t \times \vec{e}_t = -j\omega\mu h_z\hat{z} \neq 0$$

$$\nabla_t \times \vec{h}_t = 0$$

- Therefore we cannot define a unique voltage by (but we can define a unique current):

$$V_t = -\int_{C_1}^{C_2} \vec{e}_t \cdot d\vec{l}$$

- Also from (A.12a) we can define a wave impedance for the TE mode.

$$Z_{TE} = \frac{e_x}{h_y} = \frac{k_o Z_o}{\beta} = \frac{-e_y}{h_x} \qquad \textbf{\textit{(A.13)}}$$

TM Mode Summary (1)

- For TM mode, $h_z = 0$ (Sometimes this is called the E mode).
- We could characterize the Tline in TM mode, by EM fields:

$$\vec{H}^{\pm} = \pm \vec{h}_t e^{\mp j\beta z} \qquad \vec{E}^{\pm} = \left(\vec{e}_t \pm e_z \hat{z}\right) e^{\mp j\beta z}$$

- From wave equation for E field:

$$\nabla^2 \vec{E} + k_o^2 \vec{E} = 0$$

$$k_o = \omega\sqrt{\varepsilon\mu}$$

$$\left(\nabla_t^2 + \frac{\partial^2}{\partial Z^2} + k_o^2\right)\left(\left(\vec{e}_t + e_z \hat{z}\right)e^{-j\beta z}\right) = 0$$

$$\boxed{\nabla_t^2 e_z + k_c^2 e_z = 0}$$

$$\nabla_t^2 \vec{e}_t + k_c^2 \vec{e}_t = 0$$

$$\boxed{k_c^2 = k_o^2 - \beta^2}$$

Only these are needed. The other transverse field components can be derived from e_z

TM Mode Summary (2)

- Setting $h_z = 0$ in (A.11a)-(A.11d):

$$h_x = \frac{j\omega\varepsilon}{k_c^2}\frac{\partial e_z}{\partial y} \qquad h_y = \frac{-j\omega\varepsilon}{k_c^2}\frac{\partial e_z}{\partial x}$$

$$e_x = \frac{-j\beta}{k_c^2}\frac{\partial e_z}{\partial x} \qquad e_y = \frac{-j\beta}{k_c^2}\frac{\partial e_z}{\partial y} \qquad \textit{(A.14a)}$$

- These equations plus the previous wave equation for e_z enable us to find the complete field pattern for TM mode.

$$\boxed{\begin{aligned}&\nabla_t^2 e_z + k_c^2 e_z = 0\\ &k_c^2 = k_o^2 - \beta^2\end{aligned}}$$

+ boundary conditions for E and H fields

(A.14b)

From previous slide

TM Mode Summary (3)

- Similarly from (A.14a) and (A.14b), we can show that

$$\nabla_t \times \vec{h}_t = j\omega\varepsilon e_z \hat{z} \neq 0$$
$$\nabla_t \times \vec{e}_t = 0$$

- We cannot define a unique current by (but we can define a unique voltage):

$$I_t = \oint_C \vec{h}_t \cdot d\vec{l}$$

- Also from (A.14a) we can define a wave impedance for the TM mode.

$$Z_{TM} = \frac{e_x}{h_y} = \frac{\beta}{\omega\varepsilon} = \frac{-e_y}{h_x} \qquad \textbf{(A.15)}$$

TEM Mode (1)

- TEM mode is particularly important, characterized by $e_z = h_z = 0$. For +ve propagating waves:

$$\vec{E}^{\pm} = \vec{e}_t e^{\mp j\beta z} \qquad \vec{H}^{\pm} = \pm\vec{h}_t e^{\mp j\beta z} \qquad \textbf{(A.16a)}$$

- Setting $e_z = h_z = 0$ in (A.11a)-(A.11d), we observe that $k_c = 0$ in order for a non-zero solution to exist. This implies:

$$\beta = k_o = \omega\sqrt{\mu\varepsilon} \qquad \textbf{(A.16b)}$$

- Applying Hemholtz Wave Equation to E field:

$$\nabla_t = \frac{\partial}{\partial x}\hat{x} + \frac{\partial}{\partial y}\hat{y}$$

$$\left(\nabla^2 + k_o^2\right)\vec{e}_t e^{-j\beta z} = \left(\nabla_t^2 + \frac{\partial^2}{\partial z^2}\hat{z} + k_o^2\right)\vec{e}_t e^{-j\beta z} = 0$$

$$\Rightarrow \boxed{\nabla_t^2\left(\vec{e}_t e^{-j\beta z}\right)} + \left(\frac{\partial^2}{\partial z^2}e^{-j\beta z} + k_o^2 e^{-j\beta z}\right)\vec{e}_t = 0$$

0

$$\Rightarrow \nabla_t^2 \vec{e}_t = 0 \qquad \textbf{(A.17a)}$$

TEM Mode (2)

- The same can be shown for h_t:

$$\nabla_t^2 \vec{h}_t = -\nabla \times \vec{J} \qquad \textbf{(A.17b)}$$

- Equation (A.17a) is similar to Laplace equations in 2D. This implies the transverse fields e_t is similar to the static electric fields that can exist between conductors, so we could define a transverse scalar potential Φ:

$$\vec{e}_t = -\nabla_t \boxed{\Phi(x,y)}$$

Transverse potential

$$\Rightarrow \nabla_t{}^2 \Phi(x,y) = 0 \qquad \textbf{(A.18)}$$

- Also note that: $\nabla_t \times \vec{e}_t = \nabla_t \times \left(-\nabla_t \Phi\right) = 0$ **(A.19)**

See [1], Chapter 3 for alternative derivation

Using an important identity in vector calculus
$\nabla \times (\nabla F) = 0$ Where F is arbitrary function of position, i.e. F = F(x,y,z).

Alternative View (TEM Mode)

Extra

- Alternatively from (A.10c), and knowing that h_z = 0 in TEM mode:

$$\frac{\partial e_y}{\partial x} - \frac{\partial e_x}{\partial y} = -j\omega\mu h_z = 0$$

Divergence in 2D (in XY plane)

$$\Rightarrow \frac{\partial e_y}{\partial x} - \frac{\partial e_x}{\partial y} = 0 = \begin{vmatrix} x & y & z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & 0 \\ e_x & e_y & 0 \end{vmatrix} = \nabla_t \times \vec{e}_t = 0$$

- From the well known Vector Calculus identity $\nabla \times (\nabla F) = 0$, we then postulate the existence of a scalar function Φ(x,y) where

$$\vec{e}_t = -\nabla_t \Phi(x,y)$$

- From (A.10f) we can also show that (in free space):

$$\frac{\partial h_y}{\partial x} - \frac{\partial h_x}{\partial y} = j\omega\varepsilon e_z = \nabla_t \times \vec{h}_t = 0$$

TEM Mode (3)

- Normally we would find e_t from (A.17a), then we derive h_t from e_t :

In free space $\Longrightarrow \nabla^2 \vec{h}_t = 0 \ , \ \nabla \times \vec{h}_t = 0$

$$\vec{H} = \frac{-1}{j\omega\mu} \nabla \times \vec{E}$$

$$\vec{H}_t = \frac{-1}{j\omega\mu}\left[-\frac{\partial E_y}{\partial z}\hat{x} + \frac{\partial E_x}{\partial z}\hat{y} + \left(\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} \right)\hat{z} \right]$$

$$= \frac{-1}{j\omega\mu}\left[j\beta E_y \hat{x} - j\beta E_x \hat{y} + \left(\frac{\partial e_y}{\partial x} - \frac{\partial e_x}{\partial y} \right) e^{-j\beta z}\hat{z} \right]$$

$$\nabla_t \times \vec{e}_t = 0$$

Exercise: see if you can derive this equation

$$\Rightarrow \vec{h}_t = \sqrt{\frac{\mu}{\varepsilon}}^{-1} \left(e_y \hat{x} - e_x \hat{y} \right)$$

Z_o = Intrinsic impedance of free space
Z_{TEM} = Wave impedance of TEM mode

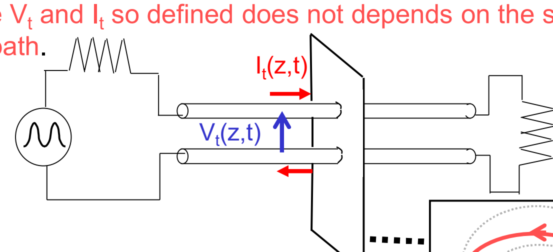
$$\Rightarrow \vec{h}_t = \frac{1}{Z_o}\left(\hat{z} \times \vec{e}_t \right) \quad \textbf{(A.20a)}$$

$$Z_o = \sqrt{\frac{\mu}{\varepsilon}} = \frac{e_x}{h_y} = \frac{-e_y}{h_x} = Z_{TEM} \quad \textbf{(A.20b)}$$

- An important observation is that under TEM mode the transverse field components e_t and h_t fulfill similar equations as in electrostatic.

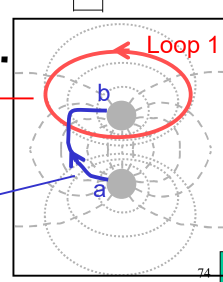
Voltage and Current under TEM Mode

- Due to $\nabla_t \times \vec{e}_t = 0$ and $\nabla_t \times \vec{h}_t = 0$ in the space surrounding the conductors, we could define unique transverse voltage (V_t) and transverse current (I_t) for the system following the standard definitions for V and I. The V_t and I_t so defined does not depends on the shape of the integration path.



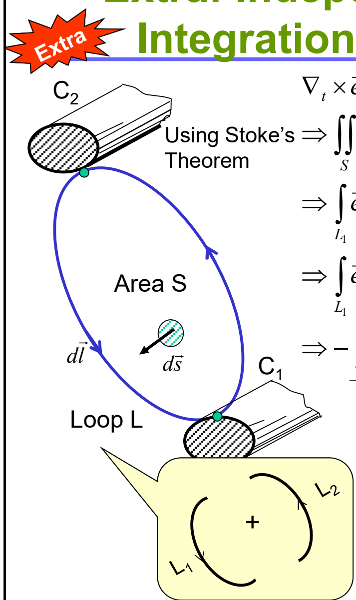
$$I_t(z,t) = \oint_{\text{Loop 1}} \vec{H} \cdot d\vec{l}$$

$$V_t(z,t) = -\int_a^b \vec{E} \cdot d\vec{l}$$



See the more detailed version of this note or see references [1] & [2].

Extra: Independence of V_t and I_t from Integration Path under TEM Mode



$$\nabla_t \times \vec{e}_t = 0$$

Using Stoke's Theorem $\Rightarrow \iint_S \nabla_t \times \vec{e}_t \cdot d\vec{s} = \int_L \vec{e}_t \cdot d\vec{l} = 0$

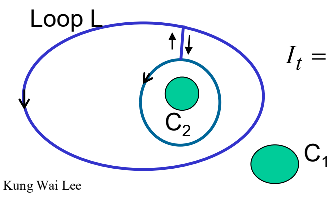
$$\Rightarrow \int_{L_1} \vec{e}_t \cdot d\vec{l} + \int_{L_2} \vec{e}_t \cdot d\vec{l} = 0$$

$$\Rightarrow \int_{L_1} \vec{e}_t \cdot d\vec{l} = -\int_{L_2} \vec{e}_t \cdot d\vec{l}$$

$$\Rightarrow -\int_{-L_1} \vec{e}_t \cdot d\vec{l} = -\int_{L_2} \vec{e}_t \cdot d\vec{l} = V_t$$

Since the shape of loop L is arbitrary, as long as it stays in the transverse plane, paths L_1, L_2 and hence the integration path for V_t is arbitrary.

Similar proof can be carried out for I_t, using the loop as shown and $\nabla_t \times \vec{h}_t = 0$



$$I_t = \oint_{C_1 \text{ or } C_2} \vec{h}_t \cdot d\vec{l}$$

TEM Mode Summary (1)

- For TEM mode, $h_z = e_z = 0$.
- To find the EM fields for TEM mode:
  - Solve $\nabla_t^2 \Phi(x,y) = 0$ with boundary conditions for the transverse potential.
  - Find E from $\vec{E}_t^{\pm} = [-\nabla_t \Phi(x,y)] e^{\mp j\beta z} = \vec{e}_t e^{\mp j\beta z}$
  - Find H from $\vec{H}_t^{\pm} = \frac{1}{Z_o}(\hat{z} \times \vec{e}_t) e^{\pm j\beta z}$
- We could characterize the Tline in TEM mode, by EM fields:

$$\vec{E}^{\pm} = \vec{e}_t e^{\mp j\beta z} \qquad \vec{H}^{\pm} = \pm \vec{h}_t e^{\mp j\beta z} \qquad \beta = k_o = \omega\sqrt{\mu\varepsilon}$$

TEM Mode Summary (2)

- Or through auxiliary circuit theory quantities:

$$V^{\pm} = V_t e^{\mp j\beta z} \qquad I^{\pm} = \pm I_t e^{\mp j\beta z}$$

- The power carried by the EM wave along the Tline is given by Poynting Theorem:

$$P = \frac{1}{2}\mathrm{Re}\iint_S \left(\vec{E}\times\vec{H}^*\right)\cdot\overrightarrow{ds} = \frac{1}{2}\mathrm{Re}\,VI^* \quad \textbf{(A.21)}$$
$$\Rightarrow P = \frac{1}{2}\mathrm{Re}\left(Z_c II^*\right) = \frac{1}{2}\mathrm{Re}\left(Z_c^{-1}VV^*\right)$$

See extra note by F.Kung for the proof

- Because β is always real or complex (when dielectric is lossy) for all frequencies, the TEM mode always exist from near d.c. to extremely high frequencies.

$$\beta = k_o = \omega\sqrt{\mu\varepsilon}$$

Non-TEM Modes and V_t, I_t (1)

- For non-TEM modes, we cannot define both the auxiliary quantities V_t and I_t uniquely using the standard definition for voltage and current (because $\nabla_t \times \vec{e}_t \neq 0$ or $\nabla_t \times \vec{h}_t \neq 0$).
- For instance in TE mode: $\nabla_t \times \vec{e}_t = -j\omega\mu h_z$
- Thus $V_t = -\int_{C_1}^{C_2} \vec{e}_t \cdot \overrightarrow{dl}$ will not be unique and will depends on the line integration path. This means if we attempt to measure the "voltage" across the Tline using an instrument, the reading will depend on the wires and connection of the probe!
- Furthermore for non-TEM modes: $P = \frac{1}{2}\mathrm{Re}\iint_S \left(\vec{E}\times\vec{H}^*\right)\cdot\overrightarrow{ds} \neq \frac{1}{2}\mathrm{Re}\,V_t I_t^*$
- Thus we cannot characterize a Tline supporting non-TEM modes using auxiliary quantities such as V_t and I_t.

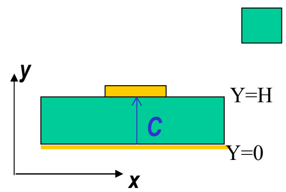
Non-TEM Modes and V_t, I_t (2)

Extra

- As another example consider the TM mode in microstrip line:

$$\vec{e}_t = -\frac{j\beta}{k_c^2}\nabla_t e_z = -\frac{j\beta}{k_c^2}\left(\frac{\partial}{\partial x}\hat{x}+\frac{\partial}{\partial y}\hat{y}\right)e_z(x,y)$$

$$V_t = -\int_C \vec{e}_t\cdot\vec{dl} = \frac{j\beta}{k_c^2}\left(\int_C \frac{\partial e_z}{\partial x}dx + \int_C \frac{\partial e_z}{\partial y}dy\right)$$



- Using path C as shown in figure:

Under quasi-TEM condition, when $e_z \to 0$, k_c also $\to 0$, then V_t will be a non-zero value.

$$V_t = \frac{j\beta}{k_c^2}\left(\int_0^H \frac{\partial e_z}{\partial y}dy\right) = \frac{j\beta}{k_c^2}\left[\cancel{e_z(x,H)} - \cancel{e_z(x,0)}\right] = 0$$

0 because of boundary condition

- In general this is true for arbitrary Tline and waveguide cross section. If we choose integration path other than C, we still obtain $V_t = 0$ due to $\nabla_t \times \vec{e}_t = 0$ in TM mode.

Cut-off Frequency for TE/TM Mode

- Because $\beta = \sqrt{k_o^2 - k_c^2}$ for TE and TM modes:
- There is a possibility that β becomes imaginary when $k_c > k_o$. When this occur the TE or TM mode EM fields will decay exponentially from the source. These modes are known as Evanescent and are non-propagating.
- Thus for TE or TM mode, there is a possibility of a cut-off frequency f_c, where for signal frequency $f < f_c$, no propagating EM field will exist.

Phase Velocity for TEM, TE and TM Modes

Extra

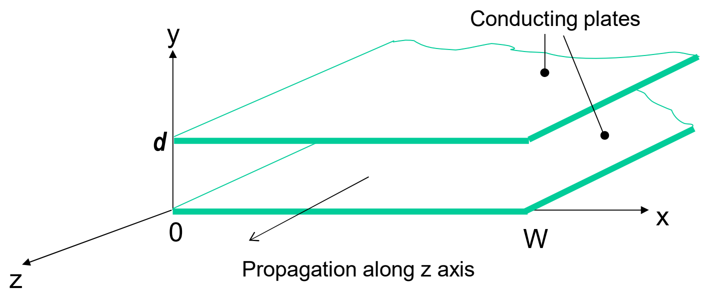
- Phase velocity is the propagation velocity of the EM field supported by the tline. It is given by: $V_p = \frac{\omega}{\beta}$
- For TEM mode: $V_p = \frac{\omega}{\beta} = \frac{1}{\sqrt{\mu\varepsilon}}$ **(A.22)**
- For TE & TM mode: $V_p = \frac{\omega}{\beta} = \frac{1}{\sqrt{k_o^2 - k_c^2}} = \frac{1}{\sqrt{\omega^2\mu\varepsilon - k_c^2}}$ **(A.23)**
- Thus we observe that TEM mode is intrinsically non-dispersive, while TE and TM mode are dispersive.

Final Note on TEM, TE and TM Propagation Modes

- Finally, note that the formulae for TEM, TE and TM modes apply to all waveguide structures, in which transmission line is a subset.

Example A1 - Parallel Plate Waveguide/Tline

- The parallel plate waveguide is the simplest type of waveguide that can support TEM, TE and TM modes. Here we assume that W >> d so that fringing field and variation along *x* can be ignored. ⇒ $\frac{\partial}{\partial x} = 0$



Example A1 Cont...

- Derive the EM fields for TEM, TE and TM modes for parallel plate waveguide.
- Show that TEM mode can exist for all frequencies.
- Show that TE and TM modes possess cut-off frequency f_c , where for operating frequency f less than f_c , the resulting EM field cannot propagate.

Example A1 – Solution for TEM Mode (1)

TEM mode Solution:

$$\nabla_t^2 \Phi(x,y) = 0 \text{ for } 0 \le x \le W \ , \ 0 \le y \le d$$

$$\Phi(x,0) = 0$$

$$\Phi(x,d) = V_o$$

Boundary conditions

Solution for the transverse Laplace PDE:

$$\nabla_t^2 \Phi = \frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial y^2} = 0 \quad \Rightarrow \frac{\partial^2 \Phi}{\partial y^2} \cong 0 \qquad \text{since} \quad \Phi(x,y) = \Phi(y)$$

$$\Rightarrow \Phi(x,y) = A + By$$ General Solution

$$\Phi(x,0) = A = 0 \quad \Rightarrow \quad A = 0$$

$$\Phi(x,d) = Bd = V_o \quad \Rightarrow \quad B = \frac{V_o}{d}$$

Thus $\boxed{\Phi(x,y) = \frac{V_o}{d} y}$ Unique solution

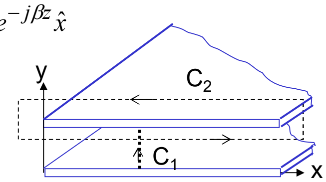
Example A1 – Solution for TEM Mode (2)

Computing the E and H fields:

$$\vec{e}_t = -\nabla_t \Phi = -\left(\frac{\partial}{\partial x}\hat{x} + \frac{\partial}{\partial y}\hat{y}\right)\left(\frac{V_o}{d} y\right) = -\frac{V_o}{d}\hat{y}$$

$$\vec{E}_t = -\frac{V_o}{d} e^{-j\beta z}\hat{y}$$

$$\vec{H}_t = \frac{-1}{j\omega\mu}\nabla \times \vec{E}_t = \frac{1}{\sqrt{\frac{\mu}{\varepsilon}}}\left(\hat{z} \times \left(-\frac{V_o}{d} e^{-j\beta z}\hat{y}\right)\right) = \sqrt{\frac{\varepsilon}{\mu}}\frac{V_o}{d} e^{-j\beta z}\hat{x}$$



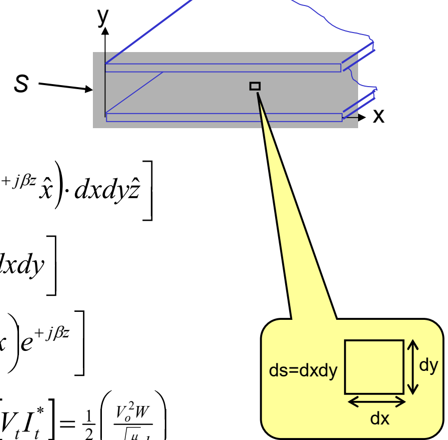
Computing the transverse voltage and current:

$$V_t = -\int_{C_1} \vec{E}_t \cdot d\vec{l} = -\int_0^d \left(-\frac{V_o}{d} e^{-j\beta z}\hat{y}\right) \cdot dy\hat{y} = V_o e^{-j\beta z}$$

$$I_t = \oint_{C_2} \vec{H}_t \cdot d\vec{l} = \int_0^W \left(\sqrt{\frac{\varepsilon}{\mu}}\frac{V_o}{d} e^{-j\beta z}\hat{x}\right) \cdot dx\hat{x} = \sqrt{\frac{\varepsilon}{\mu}}\frac{V_o}{d} W e^{-j\beta z}$$

Example A1 – Solution for TEM Mode (3)

Computing the power flow (power carried by the EM wave guided by the wave -guide):



$$P = \tfrac{1}{2}\mathrm{Re}\left[\iint_S \vec{E}_t \times \vec{H}_t^{\,*} \cdot d\vec{s}\right]$$

$$= \tfrac{1}{2}\mathrm{Re}\left[\int_0^W \int_0^d \left(-\tfrac{V_o}{d} e^{-j\beta z}\hat{y}\right) \times \left(\sqrt{\tfrac{\varepsilon}{\mu}}\tfrac{V_o}{d} e^{+j\beta z}\hat{x}\right) \cdot dxdy\hat{z}\right]$$

$$= \tfrac{1}{2}\mathrm{Re}\left[\int_0^W \int_0^d \left(\tfrac{V_o}{d} e^{-j\beta z}\right)\left(\sqrt{\tfrac{\varepsilon}{\mu}}\tfrac{V_o}{d} e^{+j\beta z}\right) dxdy\right]$$

$$= \tfrac{1}{2}\mathrm{Re}\left[\left(\int_0^d \tfrac{V_o}{d} dy\right) e^{-j\beta z} \left(\int_0^W \sqrt{\tfrac{\varepsilon}{\mu}}\tfrac{V_o}{d} dx\right) e^{+j\beta z}\right]$$

$$= \tfrac{1}{2}\mathrm{Re}\left[V_o e^{-j\beta z} \cdot \sqrt{\tfrac{\varepsilon}{\mu}}\tfrac{V_o W}{d} e^{j\beta z}\right] = \tfrac{1}{2}\mathrm{Re}\left[V_t I_t^*\right] = \tfrac{1}{2}\left(\frac{V_o^2 W}{\sqrt{\tfrac{\mu}{\varepsilon}}\, d}\right)$$

Example A1 – Solution for TEM Mode (4)

Phase velocity v_p for TEM mode:

$$v_p = \frac{\omega}{\beta} = \frac{\omega}{\omega\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu\varepsilon}}$$

The phase velocity is equal to speed-of-light in the dielectric.

Example A1 – Solution for TM Mode (1)

$$\left(\nabla_t^2 + k_c^2\right)e_z(x,y) = 0 \ \text{ for } \ 0 \le x \le W \ , \ 0 \le y \le d$$

$$k_c^2 = k_o^2 - \beta^2$$

$$e_z(x,0) = e_z(x,d) = 0 \longleftarrow \text{Boundary conditions}$$

Solution for e_z:

$$\left(\nabla_t^2 + k_c^2\right)e_z \cong \frac{\partial^2 e_z}{\partial y^2} + k_c^2 e_z = 0 \qquad \text{since} \quad e_z(x,y) = e_z(y)$$

$$\Rightarrow e_z(x,y) = A\sin(k_c y) + B\cos(k_c y) \longleftarrow \text{General solution}$$

Applying boundary conditions:

$$e_z(x,0) = A\cdot 0 + B = 0 \ \Rightarrow \ B = 0$$

$$e_z(x,d) = A\sin(k_c d) = 0$$

$$\Rightarrow A \ne 0 \ \text{ and } \ k_c d = n\pi \ , \ n = 1,2,3\cdots$$

$$\Rightarrow k_c = \frac{n\pi}{d}$$

Thus:

$$\boxed{e_z(x,y) = A_n \sin\left(\frac{n\pi}{d}y\right)}$$

or

$$\boxed{E_z(x,y) = A_n \sin\left(\frac{n\pi}{d}y\right)e^{-j\beta z}}$$

Example A1 – Solution for TM Mode (2)

Computing the transverse EM fields using (A.20a):

$$\vec{e}_t = \frac{-j\beta}{k_c^2}\frac{\partial e_z}{\partial x}\hat{x} + \frac{-j\beta}{k_c^2}\frac{\partial e_z}{\partial y}\hat{y} = -\frac{j\beta}{k_c^2}\frac{\partial}{\partial y}\left(A_n \sin\left(\frac{n\pi}{d}y\right)\right)\hat{y}$$

$$\Rightarrow \vec{e}_t = -\frac{j\beta A_n}{\left(\frac{n\pi}{d}\right)}\cos\left(\frac{n\pi}{d}y\right)\hat{y}$$

$$\text{or} \quad \vec{E}_t = -\frac{j\beta A_n}{\left(\frac{n\pi}{d}\right)}\cos\left(\frac{n\pi}{d}y\right)e^{-j\beta z}\hat{y}$$

Since n is an arbitrary integer, the TM mode is usually called the TM_n mode.

$$\vec{H}_t = \left(\frac{j\omega\varepsilon}{k_c^2}\frac{\partial e_z}{\partial y}\hat{x} - \frac{j\omega\varepsilon}{k_c^2}\frac{\partial e_z}{\partial x}\hat{y}\right)e^{-j\beta z}$$

$$= \frac{jk_o A_n}{\sqrt{\frac{\mu}{\varepsilon}}\left(\frac{n\pi}{d}\right)}\cos\left(\frac{n\pi}{d}y\right)e^{-j\beta z}\hat{x}$$

Example A1 – Solution for TM Mode (3)

We can now determine β knowing k_c:

$$\beta_n = \sqrt{k_o^2 - k_c^2} = \sqrt{\omega^2 \mu\varepsilon - \left(\tfrac{n\pi}{d}\right)^2}$$

Since the TM mode can only propagate if β is real, and the smallest value for β Is 0, then when β=0:

$$\sqrt{\omega^2 \mu\varepsilon - \left(\frac{n\pi}{d}\right)^2} = 0$$

$$\Rightarrow \omega = \frac{n\pi}{d\sqrt{\mu\varepsilon}} = 2\pi f$$

$$\Rightarrow f = \frac{n}{2d\sqrt{\mu\varepsilon}}$$

When n = 1, this represent the cut-off frequency for TM mode.

$$\boxed{f_{cutoff\_TM} = \frac{1}{2d\sqrt{\mu\varepsilon}}}$$

Example A1 – Solution for TM Mode (4)

For arbitrary *n*, phase velocity v_p for TM_n mode:

$$v_p = \frac{\omega}{\beta} = \frac{\omega}{\sqrt{\omega^2 \mu\varepsilon - \left(\frac{n\pi}{d}\right)^2}}$$

For f > f_{cutoff} , we observe that phase velocity v_p is actually greater than the speed of light!!!

NOTE:
The EM fields can travel at speed greater than light, however we can show that the rate of energy flow is less than the speed-of-light. This rate of energy flow corresponds to the speed of the photons if the propagating EM wave is treated as a cluster of photons. See the extra notes for the proof.

Example A1 – Solution for TE Mode

The EM fields for TE mode are shown below:

$$\vec{E}_t(x,y) = \frac{jk_oZ_oB_n}{\left(\frac{n\pi}{d}\right)}\sin\left(\frac{n\pi}{d}y\right)e^{-j\beta z}\hat{x}$$

$$\vec{H}_t(x,y) = \frac{j\beta B_n}{\left(\frac{n\pi}{d}\right)}\sin\left(\frac{n\pi}{d}y\right)e^{-j\beta z}\hat{y}$$

$$H_z(x,y) = B_n\cos\left(\frac{n\pi}{d}y\right)e^{-j\beta z}$$

Since *n* is an arbitrary integer, the TE mode is usually called the TE_n mode.

$$k_o = \omega\sqrt{\mu\varepsilon}$$

$$Z_o = \sqrt{\frac{\mu_o}{\varepsilon_o}}$$

Dominant Propagation Mode

- For the various transmission line topology, there is a dominant mode.
- This dominant mode of propagation is the first mode to exist at the lowest operating frequency. The secondary modes will come into existent at higher frequencies.
- The propagation modes of Tline depends on the dielectric and the cross section of the transmission line.
- For Tline that can support TEM mode, the TEM mode will be the dominant mode as it can exist at all frequencies (there is no cut-off frequency).

Transmission Lines Dominant Propagation Mode

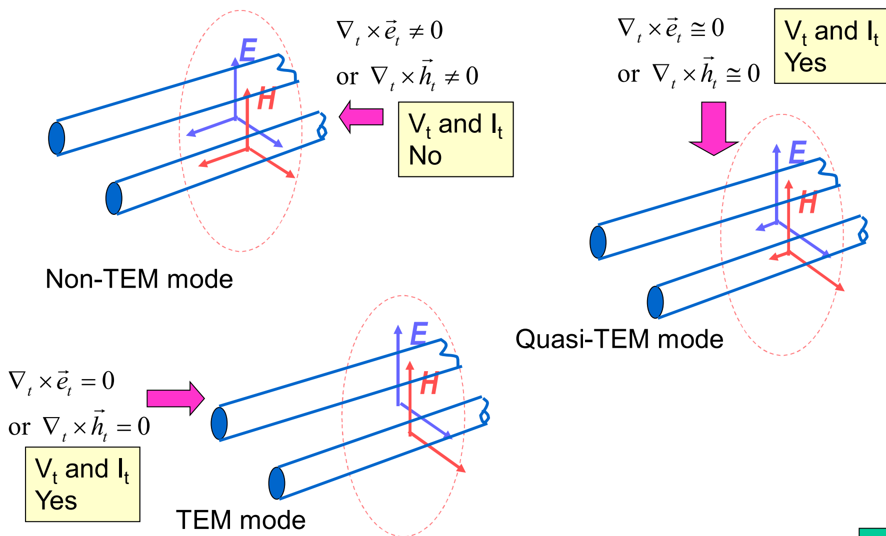
- Coaxial line - TEM.
- Microstrip line - quasi-TEM.
- Stripline - TEM.
- Parallel plate line - TEM or TM (depends on homogenuity of the dielectric).
- Co-planar line - quasi-TEM.
- Note: Generally for Tline with non-homogeneous dielectric, the Tline cannot support TEM propagation mode.

Quasi TEM Mode (1)

- Luckily for planar Tline configuration whose dominant mode is not TEM, the TM or TE dominant modes can be approximated by TEM mode at 'low frequency'.
- For instance microstrip line does not support TEM mode. The actual mode is TM. However at a few GHz, e_z is much smaller than e_t and h_t that it can be ignored. We can assume the mode to be TEM without incurring much error. Thus it is called quasi-TEM mode.
- Low frequency approximation is usually valid when wavelength >> distance between two conductors. For typical microstrip/stripline on PCB, this can means frequency below 20 GHz or lower.
- The E_z and H_z components approach zero at 'low frequency', and the propagation mode approaches TEM, hence known as quasi-TEM. When this happens we can again define unique voltage and current for the system.

See Collin [1], Chapter 3 for more mathematical illustration on this.

Quasi TEM Mode (2)



Extra: Why Inhomogeneous Structures Does Not Support Pure TEM Mode (1)

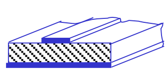
Extra

- We will use Proof by Contradiction. Suppose TEM mode is supported. The propagation factor in air and dielectric would be:

$$\beta_{air} = \omega\sqrt{\mu\varepsilon_o} \qquad \beta_{die} = \omega\sqrt{\mu\varepsilon_o\varepsilon_r}$$

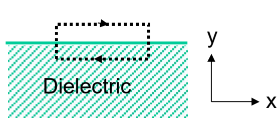
- EM fields in air will travel faster than in the dielectric.

For TEM mode



$$\beta_{air} < \beta_{die}$$
$$\Rightarrow v_{p(air)} = \frac{\omega}{\beta_{air}} > v_{p(die)} = \frac{\omega}{\beta_{die}}$$

- Now consider the boundary condition at the air/dielectric interface. The E field must be continuous across the boundary from Maxwell's equation. Examining the x component of the E field:



$$E_{x(air)}e^{-j\beta_{air}z} = E_{x(die)}e^{-j\beta_{die}z}$$
$$\Rightarrow \frac{E_{x(air)}}{E_{x(die)}} = \frac{e^{-j\beta_{die}z}}{e^{-j\beta_{air}z}} = e^{-j(\beta_{die}-\beta_{air})z}$$

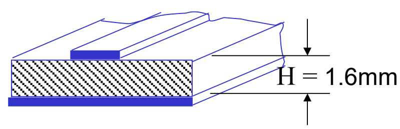
Extra: Why Inhomogeneous Structures Does Not Support Pure TEM Mode (2)

Extra

- Since the left hand side is a constant while the right hand side is not. It depends on distance z, the previous equation cannot be fulfilled.
- What this conclude is that our initial assumption of TEM propagation mode in inhomogeneous structure is wrong. So pure TEM mode cannot be supported in inhomogeneous dielectric Tline.

Example A2 – Minimum Frequency for Quasi-TEM Mode in Microstrip Line

- Estimate the low frequency limit for microstrip line.



$C \approx 3.0\times10^8$

$\lambda = C/f > 20H = 32.0\text{mm}$

$f < C/0.032 = 9.375\text{GHz}$

$f_{critical} = 9.375\text{GHz}$

Here we replace >> sign with the requirement that wavelength > 20H. You can use larger limit, as this is basically a rule of thumb.

Thus beyond $f_{critical}$ quasi-TEM approximation cannot be applied. The propagation mode beyond $f_{critical}$ will be TM. A more conser-vative limit would be to use 30H or 40H.

Summary for TEM, Quasi-TEM, TE and TM Modes

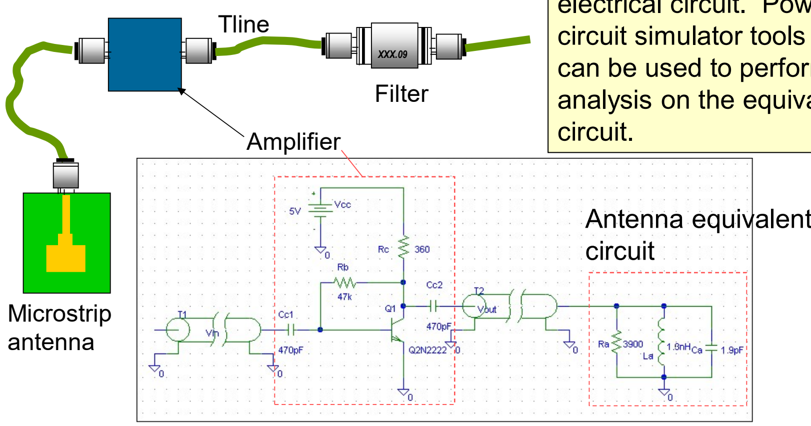
TEM:	**Quasi-TEM:**	**TE:**	**TM:**
$E_z = H_z = 0$	$E_z \approx 0$, $H_z \approx 0$	$E_z = 0$, $H_z \neq 0$	$E_z \neq 0$, $H_z = 0$
$\vec{E}^{\pm} = \vec{e}_t e^{\mp j\beta z}$	$\vec{E}^{\pm} \cong \vec{e}_t e^{\mp j\beta z}$	$\vec{H}^{\pm} = \left(\pm \vec{h}_t + h_z \hat{z}\right) e^{\mp j\beta z}$	$\vec{H}^{\pm} = \pm \vec{h}_t e^{\mp j\beta z}$
$\vec{H}^{\pm} = \pm \vec{h}_t e^{\mp j\beta z}$	$\vec{H}^{\pm} \cong \pm \vec{h}_t e^{\mp j\beta z}$	$\vec{E}^{\pm} = \vec{e}_t e^{\mp j\beta z}$	$\vec{E}^{\pm} = \left(\vec{e}_t \pm e_z \hat{z}\right) e^{\mp j\beta z}$
Can defined unique V_t and I_t.	Can defined unique V_t and I_t.	Cannot defined unique I_t.	Cannot defined unique V_t.
Physical Tline can be Modeled by equivalent Electrical circuit.	Physical Tline can be Modeled by equivalent Electrical circuit.	Physical Tline cannot be modeled by equivalent electrical circuit.	Physical Tline cannot be modeled by equivalent electrical circuit.
Phase velocity. $V_p = \frac{\omega}{\beta} = \frac{1}{\sqrt{\mu\varepsilon}}$	Phase velocity. $V_p = \frac{\omega}{\beta} \cong \frac{1}{\sqrt{\mu\varepsilon_{eff}}}$	Phase velocity. $v_p = \frac{\omega}{\beta} = \frac{1}{\sqrt{\omega^2\mu\varepsilon - k_c^2}}$	Phase velocity. $v_p = \frac{\omega}{\beta} = \frac{1}{\sqrt{\omega^2\mu\varepsilon - k_c^2}}$
No cut-off frequency.	No cut-off frequency.	Cut-off frequency. $f_c = \frac{k_c}{2\pi\sqrt{\mu\varepsilon}}$	Cut-off frequency. $f_c = \frac{k_c}{2\pi\sqrt{\mu\varepsilon}}$
Non-dispersive	Non-dispersive	Dispersive	Dispersive

Why V_t and I_t is so Important ? (1)

- When we can define voltage and current along Tline or high-frequency circuit for that matter, then we can analyze the system using circuit theory instead of field theory.
- Circuit theories such as KVL, KCL, 2-port network theory are much easier to solve than Maxwell equations or wave equations.
- High-frequency circuits usually consist of components which are connected by Tlines. Thus the microwave system can be modeled by an equivalent electrical circuit when dominant mode in the system is TEM or quasi-TEM. For this reason Tline which can support TEM or quasi-TEM is very important.

Why V_t and I_t is so Important ? (2)

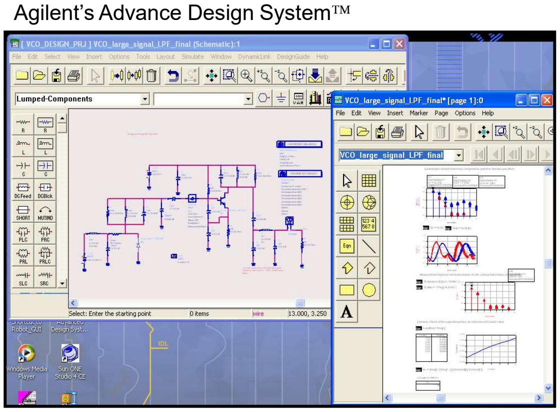
A complex physical system can be cast into equivalent electrical circuit. Powerful circuit simulator tools can be used to perform analysis on the equivalent circuit.



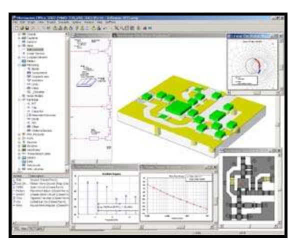
Examples of Circuit Analysis* Based Microwave/RF CAD Software

Extra

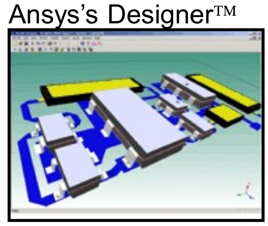
Agilent's Advance Design System™



Applied Wave Research's Microwave Office™



Ansys's Designer™



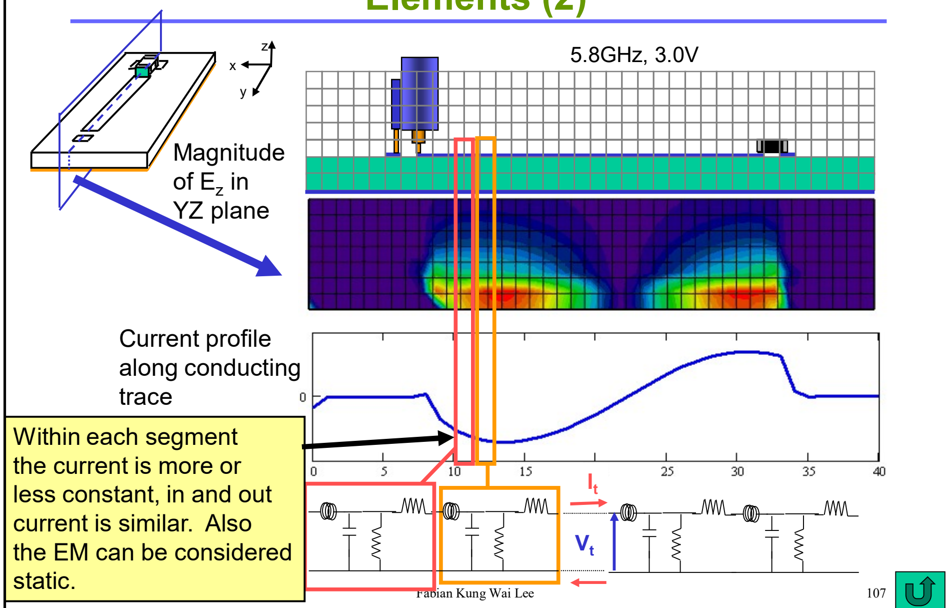
*The software shown here also have numerical EM solver capability, from 2D, 2.5D to full 3D.

4.0 – Transmission Line Characteristics and Electrical Circuit Model

Modeling a Transmission Line With Circuit Elements (1)

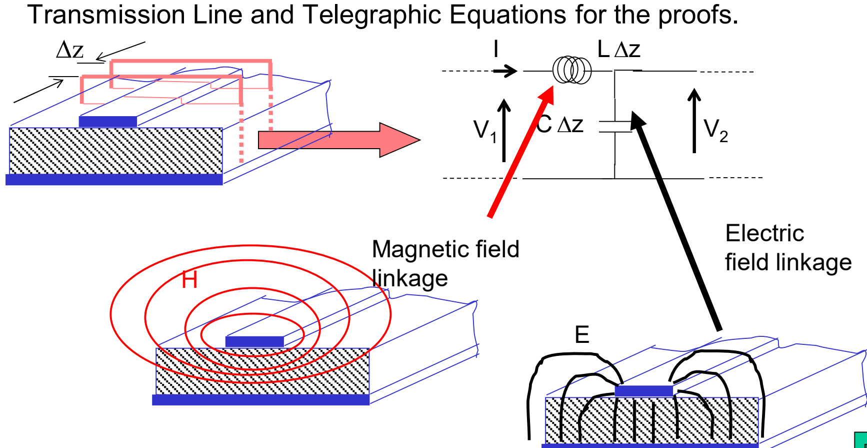
- Since transmission line is a long interconnect, the fields and current profile at any instant in time is not uniform along the line.
- It cannot be modeled by lumped circuit.
- However if we divide the Tline into many short segments (each < 0.1λ), the fields and current profile in each segment is almost uniform.
- Each of these short segments can be modeled as RLCG network.
- This assumption is true when the EM field propagation mode is TEM or quasi-TEM.
- **From now on we will assume the Tline under discussion support the dominant mode of TEM or quasi-TEM.**
- For transmission line, these associated R, L, C and G parameters are distributed, i.e. we use the per unit length values. The propagation of voltage and current on the transmission line can be described in terms of these distributed parameters.

Modeling a Transmission Line With Circuit Elements (2)



Equivalent Electrical Circuit for Single Transmission Line (1)

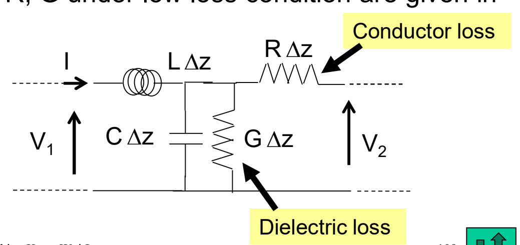
- The L and C elements in the electrical circuit model for Tline is due to magnetic flux linkage and electric field linkage between the conductors.
- See Appendix 2: Advanced Concepts – Distributed RLCG Model for Transmission Line and Telegraphic Equations for the proofs.



Equivalent Electrical Circuit for Single Transmission Line (2)

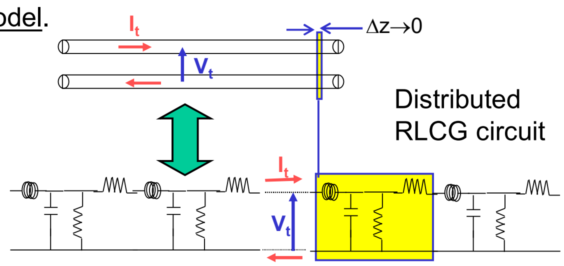
- When the conductor has small conductive loss a series resistance RΔ z can be added to the inductance. This loss is due to a phenomenon known as skin effect, where high frequency current converges on the surface of the conductor.
- When the dielectric has finite conductivity and polarization loss, a shunt conductance GΔ z can be added in parallel to the capacitance.
- The inclusion of R and G in the Tline distributed model is only accurate for small losses. This is true most of the time as Tline is usually made of very good conductive material and good insulator.
- The equations for finding L, C, R, G under low loss condition are given in the following slide.

Under lossy condition, R, L and G are usually function of frequency, hence the Tline is dispersive.



Equivalent Electrical Circuit for Single Transmission Line (3)

- Thus a transmission line can be considered as a cascade of many of these equivalent circuit sections. Working with circuit theory and circuit elements are much easier than working with E and H fields using Maxwell equations.
- In order for this RLCG model for Tline to be valid from low to very high frequency, each segment length must approach zero, and the number of segments needed to accurately model the Tline becomes infinite.
- This electrical circuit model for Tline is commonly known as Distributed RLCG Circuit Model.



Finding the Distributed RLCG Values

$$L = \frac{\mu}{|I_t|^2} \iiint_V \left|\vec{H}_t\right|^2 dv \qquad C = \frac{\varepsilon'}{|V_t|^2} \iiint_V \left|\vec{E}_t\right|^2 dv$$

This indicates the volume enclosing the conductors

(4.1a)

$$R = \frac{1}{\sigma_c \delta_s |I_t|^2} \int_{C_1 + C_2} \left|\vec{H}_t\right|^2 dl \qquad G = \frac{\omega \varepsilon''}{|V_t|^2} \iiint_V \left|\vec{E}_t\right|^2 dv$$

$$\delta_s = \text{skin depth} = \sqrt{\frac{2}{\omega \sigma_c \mu}}$$

(4.1b)

$$\varepsilon' = \varepsilon_r \varepsilon_o \qquad \varepsilon'' = \varepsilon_r \varepsilon_o \boxed{\tan \delta}$$

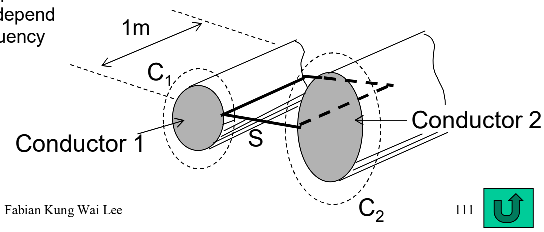
Loss tangent of the dielectric

$$\sigma_c = \text{conductivity of metalic object}$$

Skin depth and G depend on frequency

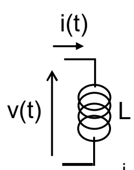
- See Section 3.9 of Collin [1].
- V is the volume surrounding the conductors with length of 1 meter along z axis.
- C_1 and C_2 are the paths surrounding the surface of conductor 1 and 2.

- These formulas are derived from energy consideration.
- Note that conductor loss results in R, while dielectric loss results in G.



Extra

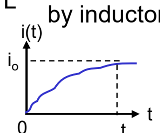
Finding RLCG Parameters From Energy Consideration



The instantaneous power absorbed by an inductor L is:

$$P_{ind}(t) = v(t)i(t)$$

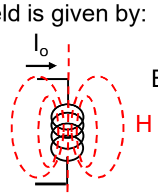
Assuming i(t) increases from 0 at t = 0 to I_o at t = t_o, total energy stored by inductor is:



$$E_{ind} = \int_0^{t_o} P_{ind}(\tau) d\tau = \int_0^{t_o} v(\tau) i(\tau) d\tau = \int_0^{t_o} L \frac{di}{d\tau}(\tau) i(\tau) d\tau$$

$$\Rightarrow E_{ind} = \int_0^{I_o} Li \cdot di = \frac{1}{2} L I_o^2$$

This energy stored by the inductor is contained within the magnetic field created by the current (for instance, see D.J. Griffiths, "Introductory electrodynamics", Prentice Hall, 1999). From EM theory the stored energy in magnetic field is given by: $E_H = \frac{\mu}{2} \iiint_V \left|\vec{H}\right| dxdydz$



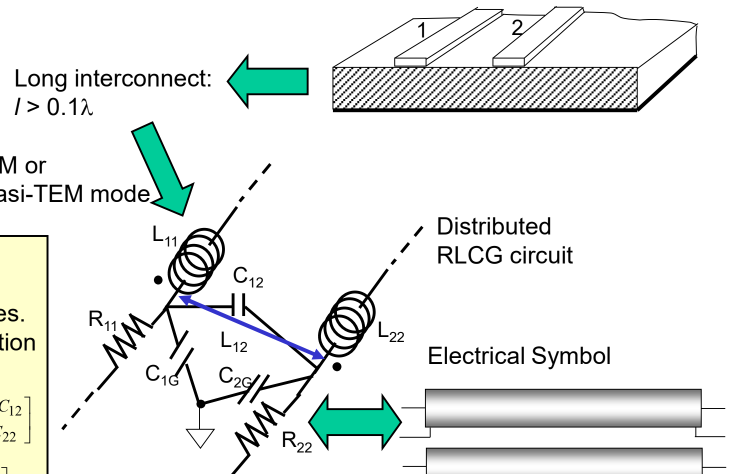
Both energy are the same, hence:

$$E_{ind} = E_H$$

$$\Rightarrow \frac{1}{2} L I_o^2 = \frac{\mu}{2} \iiint_V \left|\vec{H}\right|^2 dxdydz$$

$$\Rightarrow L = \frac{\mu}{I_o^2} \iiint_V \left|\vec{H}\right|^2 dxdydz$$

Multi-Conductor Transmission Line Equivalent Circuit



$C_{1G} = C_{11} - C_{12}$
$C_{2G} = C_{22} - C_{12}$

Parameters:
Per unit length
R, L, C, G matrices.
Usually as a function
Frequency.

$$\begin{bmatrix} L_{11} & L_{12} \\ L_{12} & L_{22} \end{bmatrix} \begin{bmatrix} C_{11} & -C_{12} \\ -C_{12} & C_{22} \end{bmatrix}$$

$$\begin{bmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{bmatrix} \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix}$$

Voltage and Current Along Transmission Line – The Telegraphic Equations

- Much like the EM field in the physical model of the Tline is governed by Maxwell's Equations, we can show that the instantaneous transverse voltage V_t and current I_t on the distributed RLCG model are governed by a set of partial differential equations (PDE) called the Telegraphic Equations (See derivation in Appendix 2).
- For simplicity we will drop the subscript 't' from now.

In time-domain

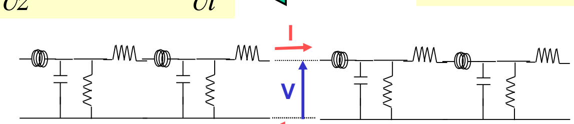
$$\frac{\partial V}{\partial z} = -RI - L\frac{\partial I}{\partial t}$$
$$\frac{\partial I}{\partial z} = -GV - C\frac{\partial V}{\partial t} \quad \textbf{(4.2a)}$$

Fourier Transforms →

← Inverse Fourier Transforms

In time-harmonic form

$$\frac{\partial V}{\partial z} = -(R + j\omega L)I = -ZI$$
$$\frac{\partial I}{\partial z} = -(G + j\omega C)V = -YV \quad \textbf{(4.2b)}$$



Distributed
RLCG circuit

Solutions of Telegraphic Equations (1)

- The expressions for V(z) and I(z) that satisfy the time-harmonic form of Telegraphic Equations (4.2b) are given as:

Wave travelling in -z direction

Wave travelling in +z direction

$$I(z) = I_o^+ e^{-\gamma z} + I_o^- e^{\gamma z} \quad \textbf{(4.3a)}$$

$$V(z) = \boxed{V_o^+ e^{-\gamma z}} + \boxed{V_o^- e^{\gamma z}} \quad \textbf{(4.3b)}$$

Attenuation factor

Phase factor

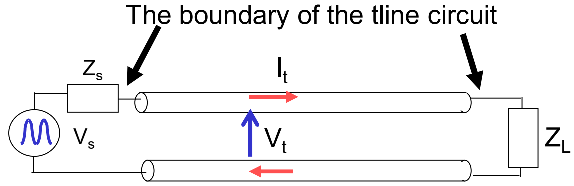
$$\gamma = \alpha(\omega) + j\beta(\omega) = \sqrt{(R + j\omega L)(G + j\omega C)} \quad \textbf{(4.3c)}$$

Propagation coefficient

Solutions of Telegraphic Equations (2)

- V_o^+, V_o^-, I_o^+, I_o^- are unknown constants. When we study transmission line circuit, we will see how V_o^+, V_o^-, I_o^+, I_o^- can be determined from the 'boundary' of the Tline.
- For the rest of this discussions, exact values of these constants are not needed.

The boundary of the tline circuit



Signal Propagation on Transmission Line – General Expressions for V and I

- Considering sinusoidal sources, the expression for V(z) and I(z) can be written in time domain as:

$$V_o^+ = \left|V_o^+\right| e^{j\phi_+} \qquad I_o^+ = \left|I_o^+\right| e^{j\theta_+}$$

$$v(z,t) = \left|V_o^+\right| \cos(\omega t - \beta z + \phi_+) e^{-\alpha z} + \left|V_o^-\right| \cos(\omega t + \beta z + \phi_-) e^{\alpha z} \quad \textbf{(4.4a)}$$

$$i(z,t) = \left|I_o^+\right| \cos(\omega t - \beta z + \theta_+) e^{-\alpha z} + \left|I_o^-\right| \cos(\omega t + \beta z + \theta_-) e^{\alpha z} \quad \textbf{(4.4b)}$$

- From the solution of the Telegraphic Equations, we can deduce a few properties of the equivalent voltage v(z,t) and current i(z,t) on a Tline structure.
  - v(z,t) and i(z,t) propagate, a signal will take finite time to travel from one location to another.
  - One can define an impedance, called the characteristic impedance of the line, it is the ratio of voltage wave over current wave.
  - That the traveling V_t and I_t experience dispersion and attenuation.
  - Other effects such as reflection to be discussed in later part.

Signal Propagation on Transmission Line - Characteristic Impedance (Z_c)

- An important parameter in Tline is the ratio of voltage over current, called the Characteristic Impedance, Z_c.
- Since the voltage and current are waves, this ratio can be only be computed for voltage and current traveling in similar direction.

From Telegraphic Equations →

$$\frac{\partial V}{\partial z} = -ZI$$
$$\Rightarrow -\gamma V_o^+ e^{-\gamma z} = -Z I_o^+ e^{-\gamma z}$$
$$\Rightarrow V_o^+ = \frac{Z}{\gamma} I_o^+$$

$$Z_c = \frac{V_o^+ e^{-\gamma z}}{I_o^+ e^{-\gamma z}} = \frac{V_o^+}{I_o^+} = \frac{Z}{\gamma} = \sqrt{\frac{R + j\omega L}{G + j\omega C}} \quad \textbf{(4.5)}$$

Or $$Z_c = -\frac{V_o^- e^{\gamma z}}{I_o^- e^{\gamma z}} = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$ ← A function of frequency

Signal Propagation on Transmission Line - Propagation Velocity (v_p)

- Compare the expression for v(z,t) of (4.4a) with a general expression for a traveling wave in positive and negative z direction:

$$V(z,t) = \left|V_o^+\right|\cos(\omega t - \beta z + \phi_+)e^{-\alpha z} + \left|V_o^-\right|\cos(\omega t + \beta z + \phi_-)e^{\alpha z}$$

Compare

$$f(\omega t - \beta z)$$

A general function describing propagating wave in +z direction

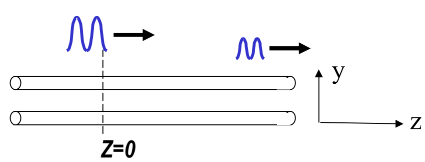
- And recognizing that both v(z,t) and i(z,t) are propagating waves, the phase velocity is given by:

$$v_p = \frac{\omega}{\beta} \qquad \textbf{(4.6)}$$

Signal Propagation on Transmission Line - Attenuation (α)

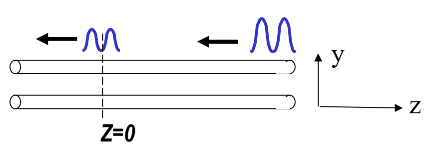
- The attenuation factor α decreases the amplitude of the voltage and current wave along the Tline.
- For +ve traveling wave:

$$\left|V_o^+\right|\cos(\omega t - \beta z + \phi_+)e^{-\alpha z}$$



- For -ve traveling wave:

$$\left|V_o^-\right|\cos(\omega t + \beta z + \phi_-)e^{\alpha z}$$



Signal Propagation on Transmission Line - Dispersion (1)

Since $\gamma = \gamma(\omega) = \alpha(\omega) + j\beta(\omega)$

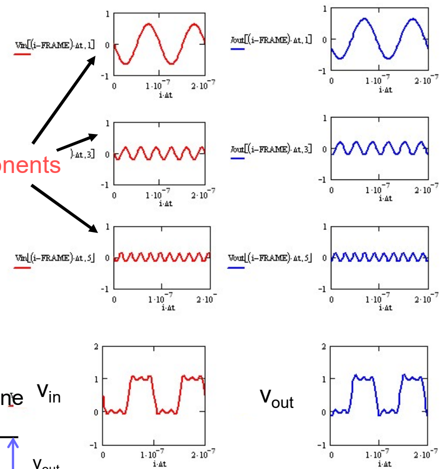
$$v_p = \frac{\omega}{\beta(\omega)}$$

Cause of dispersion

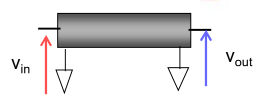
- We observe that the propagation velocity is a function of the wave's frequency.
- Different component of the signal propagates at different velocity (and also attenuate at different rate), resulting in the envelope of the signal being distorted at the output.

Video

Components



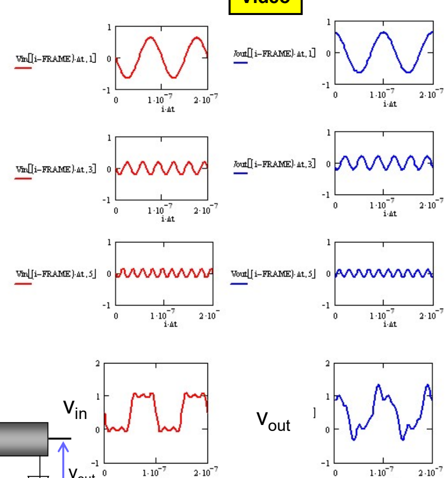
Low dispersion Transmission Line



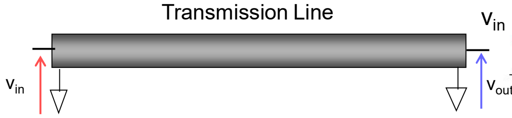
Signal Propagation on Transmission Line - Dispersion (2)

- Dispersion causes distortion of the signal propagating through a transmission line.
- This is particularly evident in a long line.

Video

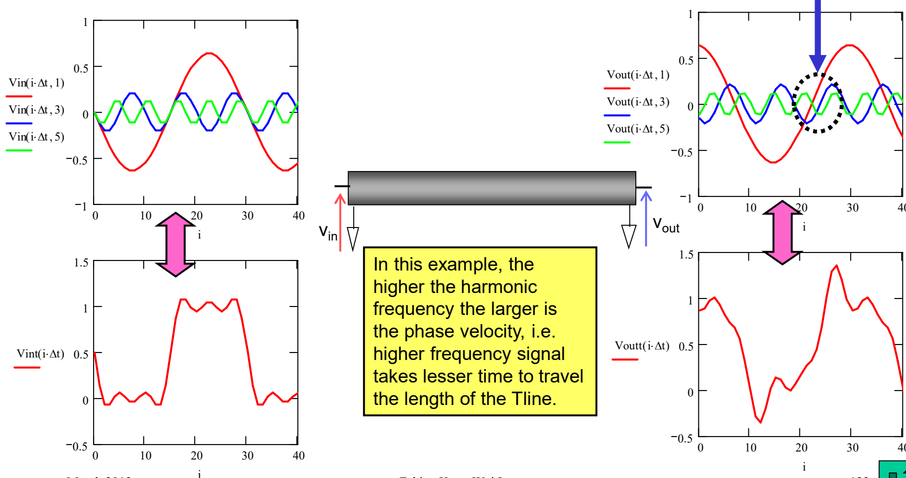


High dispersion Transmission Line



Signal Propagation on Transmission Line - Dispersion (3)

- At the output, the sinusoidal components overlap at the wrong ‘timing’, causing distortion of the pulse.



Ideal Condition - The Lossless Transmission Line

- When the tline is lossless, R= 0 and G = 0.
- We have:

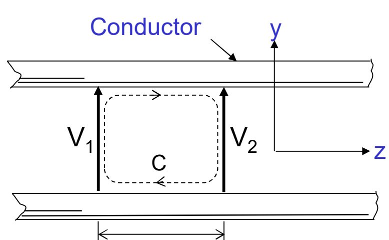
$$\gamma = j\beta = j\omega\sqrt{LC} \qquad Z_c = \sqrt{\frac{L}{C}} \qquad v_p = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\mu\varepsilon}}$$

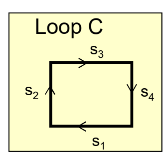
- So the lossless transmission line has no attenuation, no dispersion and the characteristic impedance is real.
- Since lossless Tline is an ideal, in practical situation we try to reduce the loss to as small as possible, by using gold-plated conductor, and using good quality dielectric (low loss tangent).

Appendix 2
Advanced Concepts – Distributed RLCG Model for Transmission Line and Telegraphic Equations

Distributed Parameters Model (1)

- For TEM or quasi-TEM mode propagation along z direction:





Flux linkage (Definition for Inductance)

$$-\oint_C \vec{E}\cdot\vec{dl} = -\iint_S \nabla\times\vec{E}\cdot\vec{ds}$$

Stoke's Theorem

$$= -\int_{s1}\vec{E}\cdot\vec{dl} - \int_{s2}\vec{E}\cdot\vec{dl} - \int_{s3}\vec{E}\cdot\vec{dl} - \int_{s4}\vec{E}\cdot\vec{dl}$$

$$\Rightarrow -\iint_S \nabla\times\vec{E}\cdot\vec{ds} = -\int_{s2}\vec{E}\cdot\vec{dl} - \left(-\int_{-s4}\vec{E}\cdot\vec{dl}\right)$$

$$\Rightarrow -\iint_S\left(-j\mu\omega\vec{H}\right)\cdot\vec{ds} = V_1 - V_2$$

$$-V_1 + V_2 = -j\omega\left(\mu\iint_S \vec{H}\cdot\vec{ds}\right)$$

When Δz is small as compared to wavelength

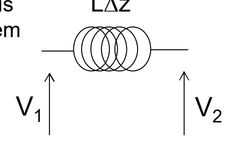
$\vec{H}^+ \cong \vec{h}_t(x,y)+h_z(x,y)\hat{z}$

$$\Rightarrow V_2 - V_1 = -j\omega\left(L\Delta z I\right)$$

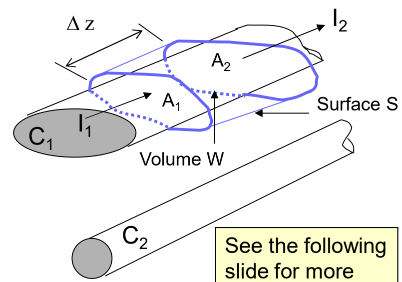
This means the relation between V_1 and V_2 is as if an inductor is between them

L is the inductance per meter

Can be represented in circuit as:



Distributed Parameters Model (2)



See the following slide for more proof.

$$\iiint_W \nabla\cdot\vec{E}\,dW = \iiint_W \frac{\rho}{\varepsilon}dW$$

Using Divergence Theorem

$$\Rightarrow \iint_S \vec{E}\cdot d\vec{S} = \iiint_W \frac{\rho}{\varepsilon}dW$$

$$\Rightarrow \frac{\partial}{\partial t}\iint_S \vec{E}\cdot d\vec{S} = \frac{1}{\varepsilon}\iiint_W \frac{\partial\rho}{\partial t}dW$$

$$\Rightarrow \frac{\partial}{\partial t}\iint_S \vec{E}\cdot d\vec{S} = \frac{1}{\varepsilon}\iiint_W -\nabla\cdot\vec{J}dW$$

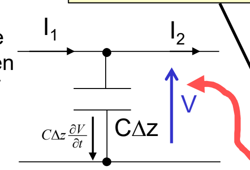
$$\Rightarrow \varepsilon\frac{\partial}{\partial t}\iint_S \vec{E}\cdot d\vec{S} = -\iint_S \vec{J}\cdot d\vec{S} = -\iint_{A_1}\vec{J}\cdot d\vec{S} - \iint_{A_2}\vec{J}\cdot d\vec{S}$$

$$\Rightarrow \varepsilon\frac{\partial}{\partial t}\iint_S \vec{E}\cdot d\vec{S} = -I_1 + I_2$$

$$\Rightarrow \frac{\partial}{\partial t}\left((C\Delta z)V\right) = (C\Delta z)\frac{\partial V}{\partial t} = I_2 - I_1$$

When Δz is small as compared to wavelength

This means the relation between I_1 and I_2 is as if a capacitor is between them



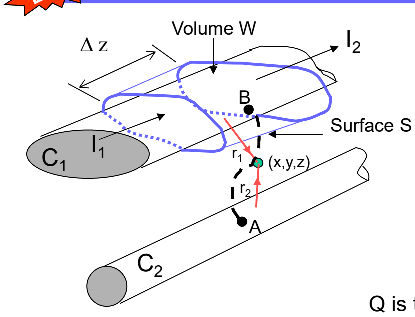
$C\Delta z\frac{\partial V}{\partial t}$

Can be represented in circuit as

- C is the per unit length capacitance between the 2 conductors of the Tline.

Distributed Parameters Model (3)

Extra



Consider the ratio:

$$\frac{\varepsilon\iint_S \vec{E}\cdot\vec{ds}}{-\int_A^B \vec{E}\cdot\vec{dl}} = \frac{Q}{-\int_A^B \vec{E}\cdot\vec{dl}}$$

For static or quasi-static condition, the E field is given by:

$$\vec{E} = \iint_{\text{surface on } C_1}\frac{\sigma_{s1}(x,y,z)}{4\pi\varepsilon}\cdot\frac{\hat{r}_1}{r_1}ds + \iint_{\text{surface on } C_2}\frac{\sigma_{s2}(x,y,z)}{4\pi\varepsilon}\cdot\frac{\hat{r}_2}{r_2}ds =$$

$$= Q\left[\iint_{\text{surface on } C_1}\frac{f_{s1}(x,y,z)}{4\pi\varepsilon}\cdot\frac{\hat{r}_1}{r_1}ds + \iint_{\text{surface on } C_2}\frac{f_{s2}(x,y,z)}{4\pi\varepsilon}\cdot\frac{\hat{r}_2}{r_2}ds\right]$$

Q is the total charge on conductors C_1 or C_2, σ_s is the surface charge density, while f_s is the normalized surface charge density with respect to Q.

Hence the ratio becomes:

$$\frac{\varepsilon\iint_S \vec{E}\cdot\vec{ds}}{-\int_A^B \vec{E}\cdot\vec{dl}} = \frac{Q}{-Q\int_A^B\left[\iint_{\text{surface on } C_1}\frac{f_{s1}(x,y,z)}{4\pi\varepsilon}\cdot\frac{\hat{r}_1}{r_1}ds + \iint_{\text{surface on } C_2}\frac{f_{s2}(x,y,z)}{4\pi\varepsilon}\cdot\frac{\hat{r}_2}{r_2}ds\right]\cdot\vec{dl}}$$

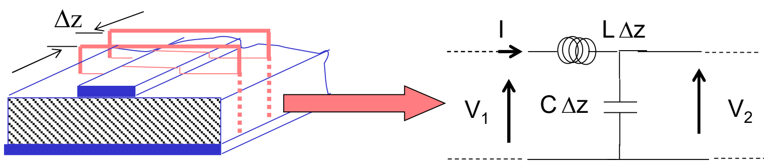
$$= \frac{1}{-\int_A^B\left[\iint_{\text{surface on } C_1}\frac{f_{s1}(x,y,z)}{4\pi\varepsilon}\cdot\frac{\hat{r}_1}{r_1}ds + \iint_{\text{surface on } C_2}\frac{f_{s2}(x,y,z)}{4\pi\varepsilon}\cdot\frac{\hat{r}_2}{r_2}ds\right]\cdot\vec{dl}} = C$$

Potential difference between A and B

C does not depend on the total charge on the conductors, we called this constant the 'Capacitance'. $\varepsilon\iint_S \vec{E}\cdot\vec{ds} = CV$

Distributed Parameters (4)

- Combining the relationship between the L, C and transverse voltages and currents, the equivalent circuit for a short section of transmission line supporting TEM or quasi-TEM propagating EM field can be represented by the equivalent circuit:

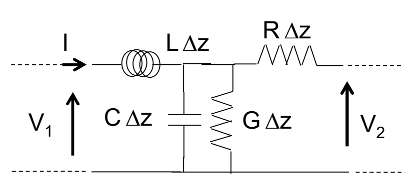


- Thus a long Tline can be considered as a cascade of many of these equivalent circuit sections. Working with circuit theory and circuit elements are much easier than working with E and H fields using Maxwell equations.

Distributed Parameters Model (5)

- When the conductor has small conductive loss a series resistance $R\Delta z$ can be added to the inductance.
- When the dielectric has finite conductivity, a shunt conductance $G\Delta z$ can be added in parallel to the capacitance.
- The inclusion of constant R and G in the Tline's distributed circuit model is only accurate for very small losses*. This is true most of the time as Tline is usually made of very good conductive material.
- The equations for finding L, C, R, G under low loss condition are given in the following slide.

*Under lossy condition, R, L and G are usually function of frequency, hence the Tline is dispersive.



Extra: Lossy Dielectric

Extra

- Assuming the dielectric is non-magnetic, then the dielectric loss is due to leakage (non-zero conductivity) and polarization loss*.
- Polarization loss is due to the vibration of the polarized molecules in the dielectric when an a.c. electric field is imposed.
- Both mechanisms can be modeled by considering an effective conductivity σ_d for the dielectric at the operating frequency. This is usually valid for small electric field.

Loss current density

$$\nabla \times \vec{H} = \vec{J} + j\omega\varepsilon_r\varepsilon_o\vec{E} \Rightarrow \nabla \times \vec{H} = \sigma_d\vec{E} + j\omega\varepsilon_r\varepsilon_o\vec{E}$$

*We should also include hysterisis loss in ferromagnetic material.

$$\Rightarrow \nabla \times \vec{H} = j\omega\varepsilon_r\varepsilon_o\left(1 + \frac{\sigma_d}{j\omega\varepsilon_r\varepsilon_o}\right)\vec{E}$$

$$\Rightarrow \nabla \times \vec{H} = j\omega\varepsilon_r\varepsilon_o(1 - j\tan\delta)\vec{E}$$

$$\boxed{\begin{array}{l}\nabla \times \vec{H} = j\omega\left(\varepsilon' - j\varepsilon''\right)\vec{E} \\ \varepsilon' = \varepsilon_r\varepsilon_o \qquad \varepsilon'' = \varepsilon_r\varepsilon_o\tan\delta\end{array}}$$

$$\tan\delta = \frac{\sigma_d}{\omega\varepsilon_r\varepsilon_o}$$

This is called Loss Tangent

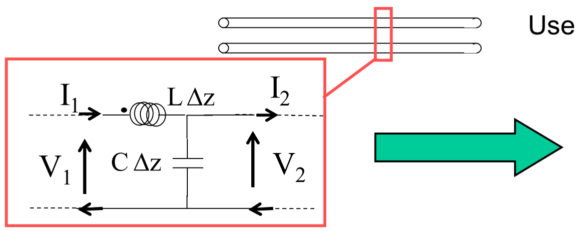
Note that σ_d is a function of frequency

Finding Distributed Parameters for Low Loss Practical Transmission Line

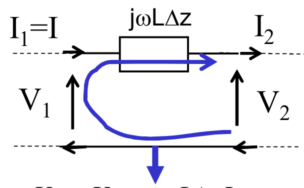
- When loss is present, the propagation mode will not be TEM anymore (Can you explain why this is so?).
- However if the loss is very small, we can assume the propagating EM field to be similar to the EM fields under lossless condition. From the **E** and **H** fields, we could derived the RLCG parameters from equations (4.1a) and (4.1b). Although the RLCG parameters under this condition is only an approximation, the error is usually small.
- This approach is known as perturbation method.

Derivation of Telegraphic Equations (1)

- For Tline supporting TEM and quasi-TEM modes, the V and I on the line is the solution of a hyperbolic partial differential equation (PDE) known as telegraphic equations. Consider first lossless line:



Use Kirchoff's Voltage Law (KVL):



$$V_2 = V_1 - j\omega L\Delta z I$$

$$\Rightarrow \frac{V_2 - V_1}{\Delta z} = -j\omega L I$$

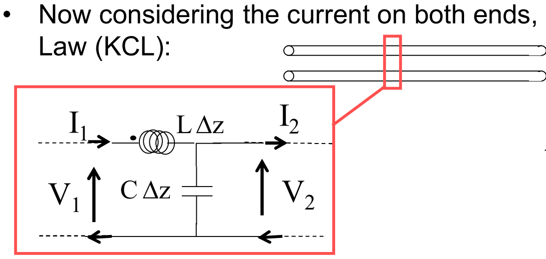
Observing that:

$$\lim_{\Delta z \to 0}\left(\frac{V_2 - V_1}{\Delta z}\right) = \frac{\partial V}{\partial z}$$

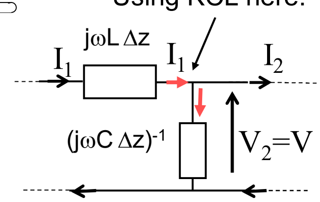
$$\Rightarrow \frac{\partial V}{\partial z} = -j\omega L I$$

Derivation of Telegraphic Equations (2)

- Now considering the current on both ends, and using Kirchoff's Current Law (KCL):



Using KCL here:



$$I_1 - (j\omega C\Delta z)V = I_2$$

$$\Rightarrow \frac{I_2 - I_1}{\Delta z} = -j\omega C V$$

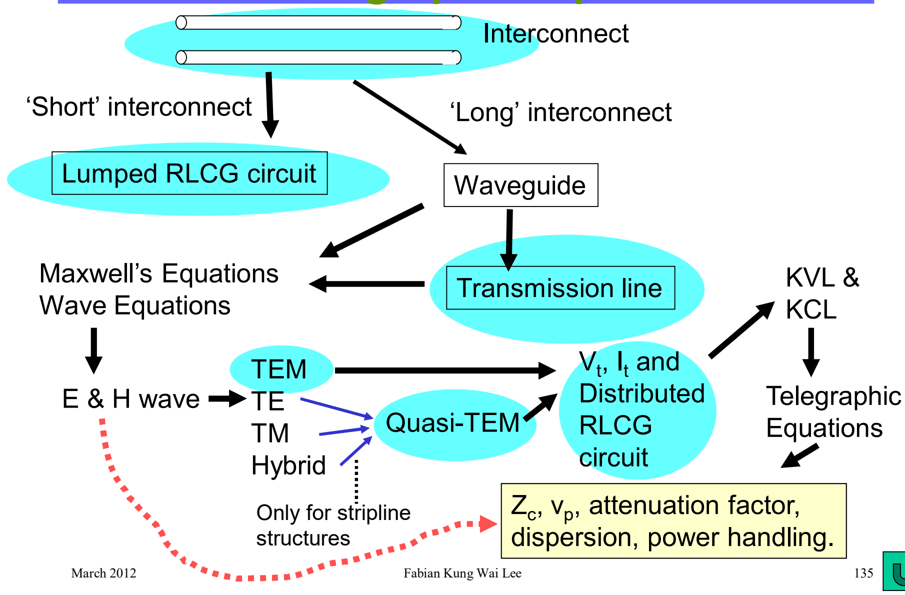
Again observing that:

$$\frac{I_2 - I_1}{\Delta z} = -C\frac{\partial V_2}{\partial t}$$

$$\frac{\partial I}{\partial z} = -j\omega C V$$

Lossless Telegraphic Equations

Relationship Between Field Solutions and Telegraphic Equations



Example A3

- Find the RLCG parameters of the low loss parallel plate waveguide in Example A1. Assuming the conductivity of the conductor is σ and the dielectric between the plates is complex (this means the dielectric is lossy too):

$$\varepsilon = \varepsilon' - j\varepsilon''$$

- Use the expressions for E_t and H_t as derived in Example A1.

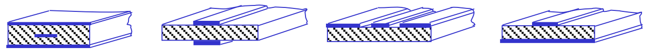
$$L = \frac{\mu d}{W} \ \mathrm{H/m} \qquad C = \frac{\varepsilon' W}{d} \ \mathrm{F/m}$$

$$R = \frac{2}{\sigma_c \delta_s W} \ \Omega/\mathrm{m} \qquad G = \frac{\omega\varepsilon''}{\varepsilon'} \mathrm{C} = \frac{\sigma_d W}{d} \ \Omega^{-1}/\mathrm{m}$$

5.0 - Transmission Line Synthesis On Printed Circuit Board (PCB) And Related Structures

Stripline Technology (1)

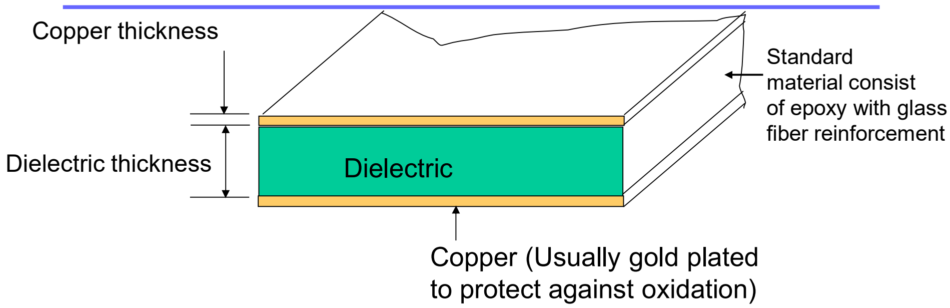
- Stripline is a planar-type Tline that lends itself well to microwave integrated circuit (MIC) and photolithograhpic fabrication.
- Stripline can be easily fabricated on a printed circuit board (PCB) or semiconductor using various dielectric material such as epoxy resin, glass fiber such as FR4, polytetrafluoroethylene (PTFE) or commonly known as Teflon, Polyimide, aluminium oxide, titanium oxide and other ceramic materials or processes, for instance the low-temperature co-fired ceramic (LTCC).
- Three most common Tline configurations using stripline technology are microstrip line, stripline and co-planar stripline.



Stripline Technology (2)

- A variety of substrates, Thin and Thick-Film technologies can be employed.
- For more information on microstrip line circuit design, you can refer to T.C. Edwards, "Foundation for microstrip circuit design", 2nd Edition 1992, John Wiley & Sons. (3rd edition, 2000 is also available).
- For more information on stripline circuit design, you can consult H. Howe, "Stripline circuit design", 1974, Artech House.
- For more information on microwave materials and fabrication techniques, you can refer to T.S. Laverghetta, "Microwave materials and fabrication techniques", 3rd edition 2000, Artech House.

The Substrate or Laminate for Stripline Technology



- Typical dielectric thickness are 32mils (0.80mm), 62mils (1.57mm) for double sided board. For multi-layer board the thickness can be customized from 2 – 62 mils, in 1 mils step.
- Copper thickness is usually expressed in terms of the mass of copper spread over 1 square foot. Standard copper thickness are 0.5, 1.0, 1.5 and 2.0 oz/foot^2.

0.5 oz/foot^2 ≅ 0.7mils thick.
1.0 oz/foot^2 ≅ 1.4mils thick.
2.0 oz/foot^2 ≅ 2.8mils thick.

Factors Affecting Choices of Substrates

- Operating frequency.
- Electrical characteristics - e.g. nominal dielectric constant, anisotropy, loss tangent, dispersion of dielectric constant.
- Copper weight (affect low frequency resistance).
- T_g, glass transition temperature.
- Cost.
- Tolerance.
- Manufacturing Technology - Thin or thick film technology.
- Thermal requirements - e.g. thermal conductivity, coefficient of thermal expansion (CTE) along x,y and z axis.
- Mechanical requirements - flatness, coefficient of thermal expansion, metal-film adhesion (peel strength), flame retardation, chemical and water resistance etc.

Comparison between Various Transmission Lines

Microstrip line	Stripline	Co-planar line
Suffers from dispersion and non-TEM modes	Pure TEM mode	Suffers from dispersion and non-TEM modes
Easy to fabricate	Difficult to fabricate	Fairly difficult to fabricate
High density trace	Mid density trace	Low density trace
Fair for coupled line structures	Good for coupled line structures	Not suitable for coupled line structures
Need through holes to connect to ground	Need through holes to connect to ground	No through hole required to connect to ground

Field Solution for EM Waves on Stripline Structure (1)

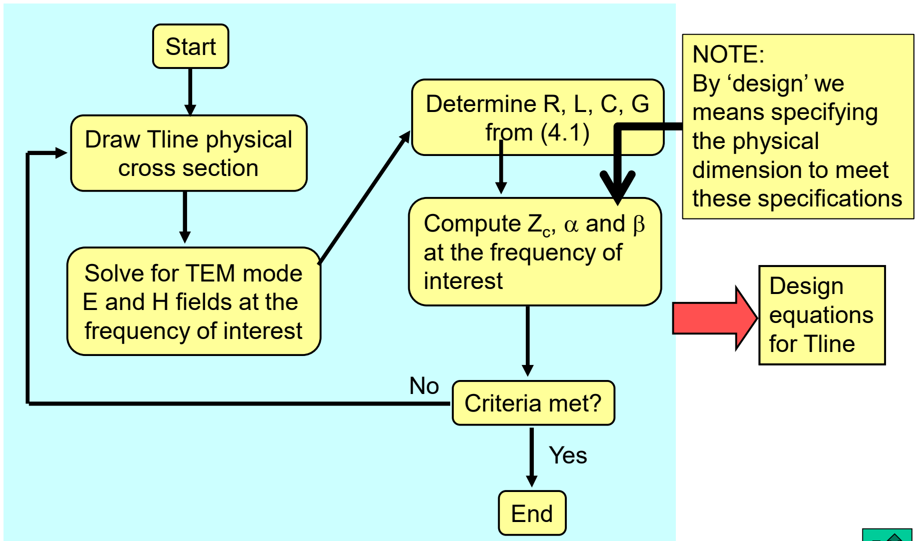
- In microstrip and co-planar Tline the dielectric material does not completely surround the conductor, consequently the fundamental mode of propagation is not a pure TEM mode. However at a frequency below a few GHz (<10GHz at least), the EM field propagation mode is quasi-TEM. The microstrip Tline can be characterized in terms of its approximate distributed RLCG parameters.
- For the stripline, the dominant mode is TEM hence it can be characterized by its distributed RLCG parameters to very high frequencies.
- Unfortunately there is no simple closed-form analytic expressions that for the EM fields or RLCG parameters for a planar Tline.
- A method known as 'Conformal Mapping' is usually used to find the approximate closed-form solution of the Laplace partial differential equation for the TEM/quasi-TEM mode fields. The expression can be very complex.

See Ramo [3], Chapter 7 for more information on Conformal Mapping method. Collin [1], Chapter 3 provides mathematical derivation of the field solutions for parallel plate waveguide with inhomogeneous dielectric and the microstrip line.

Field Solution for EM Waves on Stripline Structure (2)

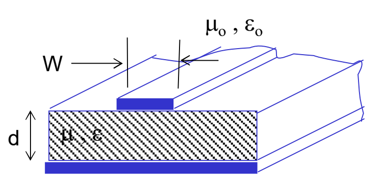
- Another approach is to use numerical methods to solve for the static E and H field along the cross section of the Tline. From the E and H fields, the RLCG parameters can be obtained from equations (4.1a) and (4.1b).
- There are numerous commercial and non-commercial software for performing this analysis.
- Once RLCG parameters are obtained, the propagation constant γ and characteristic impedance Z_c of the Tline can be obtained. Z_c can then be plotted as a function of the Tline dimensions, the dielectric constant and the operating frequency.
- Many authors have solved the static field problem for stripline structures using conformal mapping and other approaches to solve the scalar potential ϕ and vector potential $\boldsymbol{A}$.
- The following slides show some useful results as obtained by researchers in the past for designing planar Tline.
- Some of the equations are obtained by curve-fitting numerically generated results.

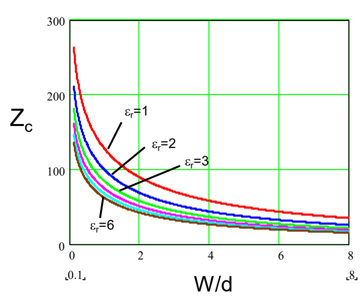
Typical Iterative Flow for Transmission Line Design



Design Equations

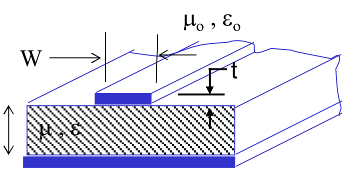
- By varying the physical dimensions and using the flow of the previous slide, one can obtain a collection of results (Z_c, α, β) .
- These results can be plotted as points on a graph.
- Curve-fitting techniques can then be used to derive equations that match the results with the physical parameters of the Tline.





Design Equations for Microstrip Line

Microstrip Line (see reference [3], Chapter 8)



Effective dielectric constant (See Appendix 3)

$$\varepsilon_{eff} = 1 + \frac{\varepsilon_r - 1}{2}\left[1 + \frac{1}{\sqrt{1 + \frac{10d}{w}}}\right] \qquad \textbf{(5.1a)}$$

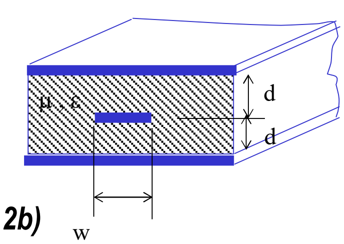
$$Z_c = \frac{377}{\sqrt{\varepsilon_{eff}}}\left[\frac{w}{d} + 1.98\left(\frac{w}{d}\right)^{0.172}\right]^{-1} \qquad \textbf{(5.1b)}$$

$$v_p = \frac{1}{\sqrt{\mu\varepsilon_{eff}\varepsilon_o}} \qquad \textbf{(5.1c)}$$

Only valid when quasi-TEM approximation and very low loss condition applies.

Design Equations for Stripline

Stripline (see reference [3], Chapter 8):



$$Z_c \cong \frac{Z_o}{4}\cdot\frac{K(k)}{K\left(\sqrt{1-k^2}\right)} \qquad \textbf{(5.2a)}$$

$$K(x) = \int_0^{\pi/2}\frac{d\phi}{\sqrt{1 - x^2\sin^2\phi}} \qquad k = \left[\cosh\left(\frac{\pi w}{4d}\right)\right]^{-1} \qquad \textbf{(5.2b)}$$

Complete elliptic integral of the 2nd kind

$$Z_o = \sqrt{\frac{\mu}{\varepsilon}}$$

$$v_p = \frac{1}{\sqrt{\mu\varepsilon}} \qquad \textbf{(5.2c)}$$

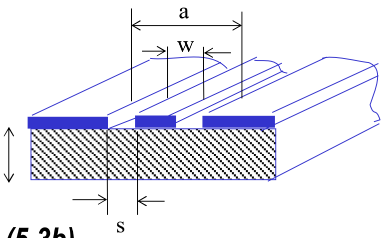
Only valid when TEM, quasi-TEM approximation and very low loss condition applies.

Design Equations for Co-planar Line

Co-planar Line ([3], assume *d* is large compare to *s*):

Effective dielectric constant (See Appendix 3)

$$\varepsilon_{eff} = \frac{\varepsilon_r + 1}{2} \qquad \textbf{(5.3a)}$$



$$Z_c \cong \frac{Z_o}{\pi\sqrt{\varepsilon_{eff}}} \ln\left(2\sqrt{\frac{a}{w}}\right) \quad \text{for } 0 < \frac{w}{a} < 0.173$$

$$Z_c \cong \frac{\pi Z_o}{4\sqrt{\varepsilon_{eff}}}\left[\ln\left(2\frac{1+\sqrt{w/a}}{1-\sqrt{w/a}}\right)\right]^{-1} \quad \text{for } 0.173 < \frac{w}{a} < 1 \qquad \textbf{(5.3b)}$$

$$Z_o = \sqrt{\frac{\mu_o}{\varepsilon_o}} \qquad v_p = \frac{1}{\sqrt{\mu\varepsilon_{eff}\varepsilon_o}} \qquad \textbf{(5.3c)}$$

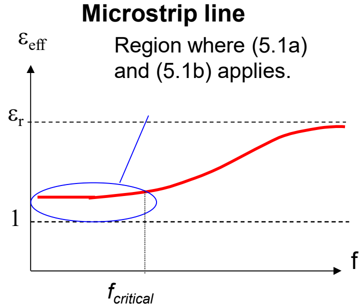
Only valid when quasi-TEM approximation and very low loss condition applies.

Dispersive Property of Microstrip and Co-planar Lines (1)

- The actual propagation mode for microstrip and co-planar lines are a combination TM and TE modes. Both modes are dispersive. The phase velocity of the EM wave is dependent on the frequency (see references [1] and [2], or discussion in Appendix 1).
- This change in phase velocity is reflected by effective dielectric constant that changes with frequency.
- At low frequency ($f < f_{critical}$), when the propagation modes for microstrip and co-planar lines approaches quasi-TEM, the phase velocity is almost constant.
- Appendix 1 shows a simple method to estimate $f_{critical}$ for microstrip and co-planar lines.
- Thus $f_{critical}$ is usually taken as the upper frequency limit for microstrip and co-planar lines. Typical value is 5 – 100 GHz depending on dielectric thickness.
- Stripline does not experience this effect as theoretically it can support pure TEM mode.

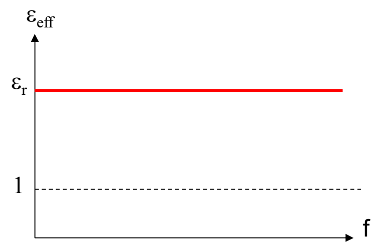
Dispersive Property of Microstrip and Co-planar Line (2)

Microstrip line



Limit for quasi-TEM approximation, see Example A1 and on how to estimate $f_{critical}$

Stripline



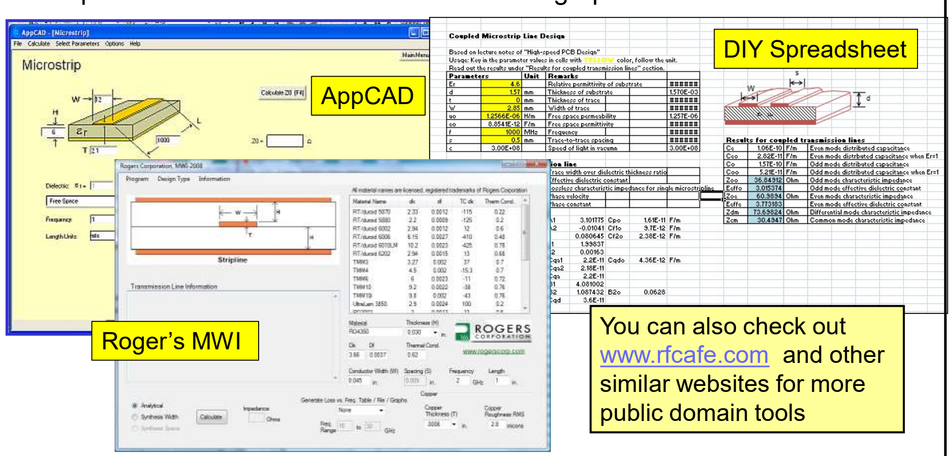
$$v_p = \frac{1}{\sqrt{\mu \varepsilon_{eff} \varepsilon_o}}$$

For microstrip line, the EM field is partly in the air and dielectric. Hence the effective dielectric ε_{eff} constant is between those of air and dielectric.

Note:
Beyond $f_{critical}$ the concept of characteristic impedance becomes meaningless.

Free Tools for Stripline Design (1)

- Here are some examples of software which encapsulate the design equations for transmission line into a nice graphical user interface.



You can also check out www.rfcafe.com and other similar websites for more public domain tools

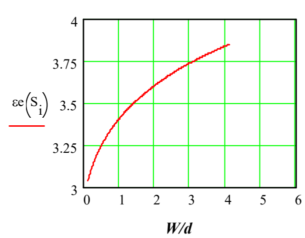
Free Tools for Stripline Design (2)

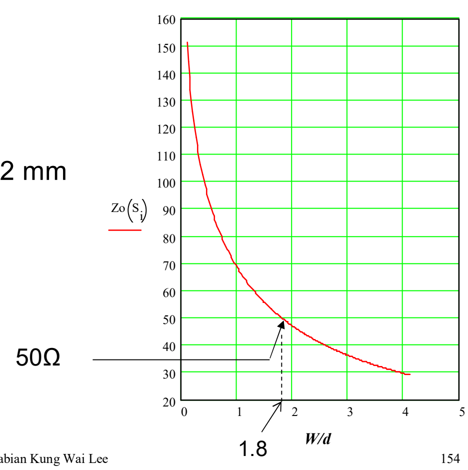
Agilent's PCB Trace Calculator (Apple iPad, iPhone)

Example 5.1 - Microstrip Line Design

- **(a) Design a 50Ω microstrip line, given that d = 1.57 mm and dielectric constant = 4.6 (Here it means find w).**

•Steps...
•Plot out Z_c versus (w/d).
• From the curve, we see that w/d = 1.8 for 50Ω.
• Thus w = 1.8 x 1.57 mm = 2.82 mm





Microstrip Line Design Example Cont…

- **(b) If the length of the Tline is 6.5 cm, find the propagation delay.**
- **(c) Using the *l* < 0.05λ rule, find the frequency range where the microstrip line can be represented by lumped RLCG circuit.**

From ε_{eff} versus w/d, we see that ε_{eff} = 3.55 at w/d = 1.8. Therefore:

$$v_p = \frac{1}{\sqrt{\varepsilon_o \mu_o \cdot 3.51}} = 1.601 \times 10^8 ms^{-1} \Rightarrow t_{delay} = \frac{0.065}{v_p} \cong \underline{406\,ps}$$

To be represented as lumped, the wavelength λ must be > 20 x *Length*:

$$\lambda > 20 \cdot l = 1.30\text{ m}$$

$$\Rightarrow \frac{v_p}{f} > 1.30$$

$$\Rightarrow f < \frac{v_p}{1.30} = 123.2\text{ MHz}$$

$$f_{lumped} = 123.2\text{ MHz}$$

Microstrip Line Design Example Cont…

- **(d) When the low loss microstrip line is considered short, derived its equivalent LC network.**

$$Z_c v_p = \sqrt{\frac{L}{C}} \times \frac{1}{\sqrt{LC}} = \frac{1}{C}$$

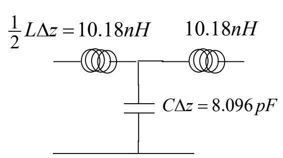
$$L = {Z_c}^2 C = 313.27\text{nH/m}$$

$$C = \frac{1}{Z_c v_p} = \frac{1}{50 \times 1.601 \times 10^8} = 124.6\text{pF/m}$$

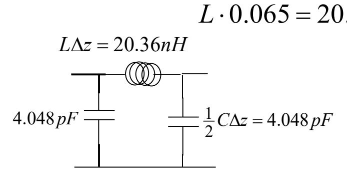
For shot interconnect we could model the Tline as:

$$C \cdot 0.065 = 8.096\,pF$$

$$L \cdot 0.065 = 20.36\,nH$$



or



Microstrip Line Design Example Cont...

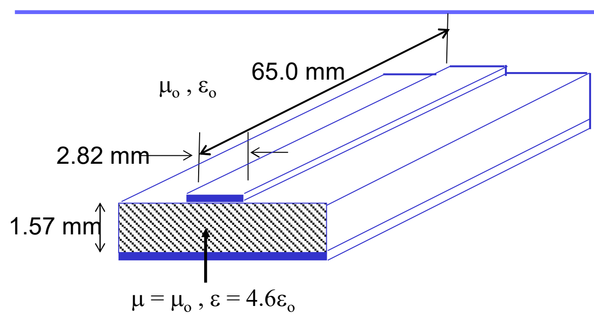
- **(e) Finally estimate $f_{critical}$, the limit where quasi-TEM approximation begins to break down.**

Again using the criteria that wavelength > 20d for quasi-TEM mode to propagate:

$$\lambda = \frac{v_p}{f_{critical}} > 20d = 0.031 \Rightarrow f_{critical} < \frac{v_p}{0.031} \cong 5.10GHz$$

We see that to increase $f_{critical}$, smaller thickness d should be used.

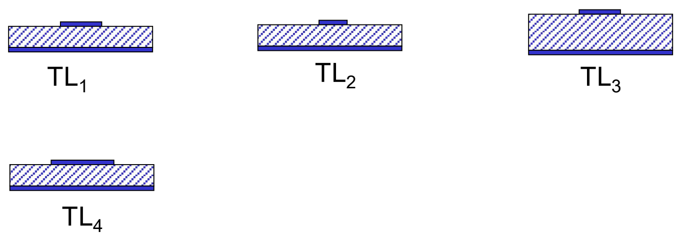
Microstrip Line Design Example Cont...



$Z_c = 50\Omega$
$v_p = 1.601 \times 10^8$ m/s
$t_{delay} = 406$psec
Maximum usable frequency = $f_{critical}$ = 5.10 GHz.
Short interconnect limit = 123.2 MHz

Example 5.2 – Estimating the Effect of Trace Width and Dielectric Thickness on Z_C

- Consider the following microstrip line cross sections, assuming lossless Tline, make a comparison of the characteristic impedance of each line.



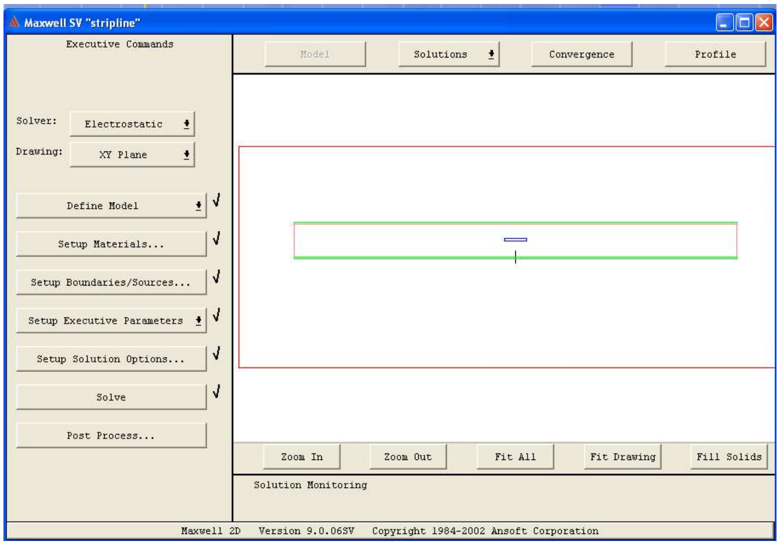
Extra

Example 5.3 - Stripline Design Example Using 2D EM Field Solver Program

- Here we demonstrate the use of a program called Maxwell 2D by Ansoft Inc. www.ansoft.com to design a stripline.
- The version used is called Maxwell SV Ver 9.0, a free version which can be downloaded from the company's website.
- The software uses finite element method (FEM) to compute the two-dimensional (2D) static E and H field of an array of metallic objects.
- It is assumed that the stripline is lossless.
- Two projects are created, one is the Electrostatic problem for calculation of static electric field and distributed capacitance, the other is Magnetostatic problem, for calculation of static magnetic field and distributed inductance.
- Characteristic impedance of the stripline can then be computed from the distributed capacitance and inductance.

Example 5.3 - Screen Shot

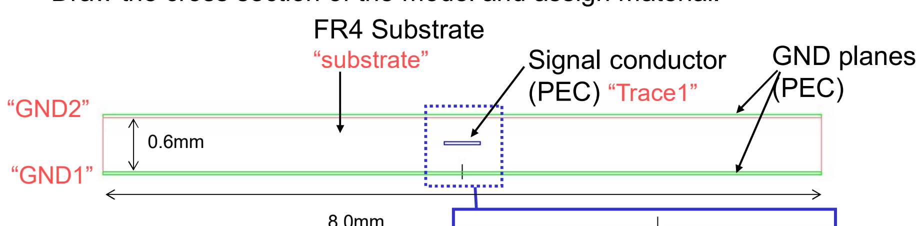
Extra



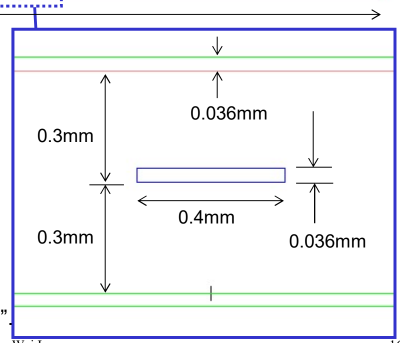
Example 5.3 - Stripline Cross Section

Extra

- Draw the cross section of the model and assign material.



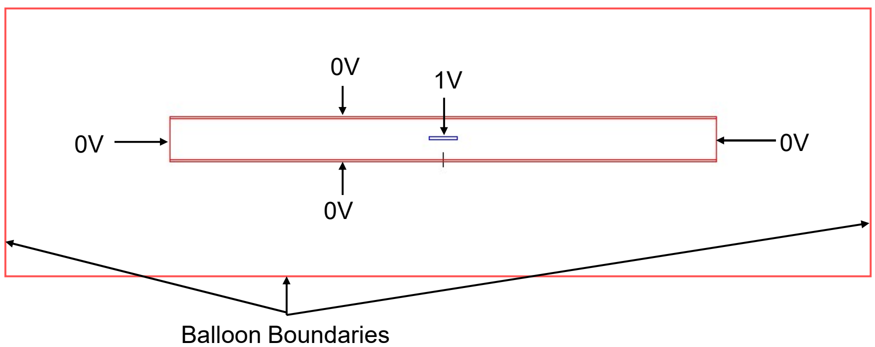
- Set drawing units to micron.
- Set drawing size to 10000um for x and 4000um for y.
- Set grid to dU = dV = 100um.
- Draw model, use direct entry mode for conducting structures like GND planes and signal.
- Name signal conductor "Trace1" and GND planes "GND1" and "GND2".
- Name FR4 as "substrate", then group both "GND1" and "GND2" as one object "GND".



Example 5.3 - Electrostatics: Setup Boundary Conditions

Extra

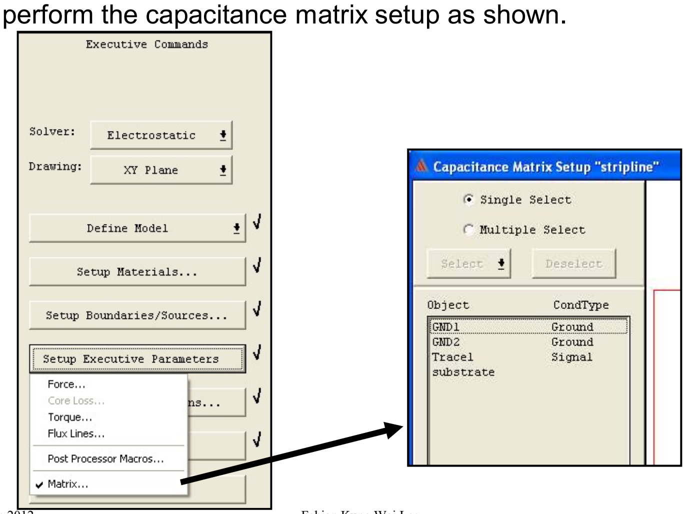
- Set the boundary conditions.
- All boundary are Dirichlet type, i.e. voltages are specified.
- Let the edges of the model domain remain as Balloon Boundary.



Example 5.3 - Electrostatics: Setup Executive Parameters

Extra

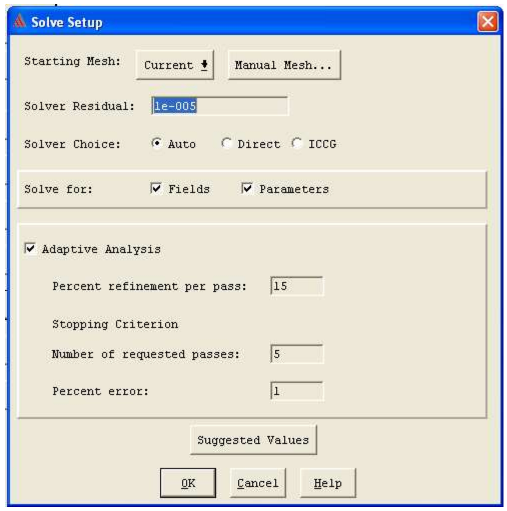
- Under 'Setup Executive Parameters' tab, select 'Matrix…' and proceed to perform the capacitance matrix setup as shown.



Example 5.3 - Electrostatics: Setup Solver and Solve for Scalar Potential ϕ

Extra

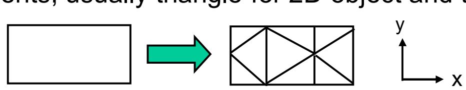
- Setup the solver and solve for the approximate potential solution. Use the 'Suggested Values' if you are not sure.



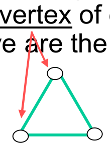
Example 5.3 - Finite Element Method (1)

Extra

- In finite-element method (FEM) an object is thought to consist of many smaller elements, usually triangle for 2D object and tetrahedron for 3D object.



- FEM is used to solve for the approximate scalar potential V (or ϕ) for electrostatic problem and vector potential A for magnetostatic problem at the vertex of each triangle. The partial differential equations (PDE) to solve are the Poisson's equations.



$$\nabla^2 V = \frac{\rho}{\varepsilon} \qquad \nabla^2 \vec{A} = \mu \vec{J}$$

- Potential value inside the triangle can be estimated via interpolation.
- For 2D problem the PDE can be written as:

$$\nabla_t^2 V_t = \frac{\rho}{\varepsilon} \qquad \nabla_t = \frac{\partial}{\partial x}\hat{x} + \frac{\partial}{\partial y}\hat{y}$$
$$V_t = V_t(x,y) \quad \rho = \rho(x,y)$$

$$\nabla_t^2 A_z = \mu J_z$$
$$A_z = A_z(x,y) \quad J_z = J_z(x,y)$$

Example 5.3 - Finite Element Method (2)

Extra

- 2D quasi-static E field can then be obtained by:

$$\vec{E}_t(x,y) = -\nabla_t V_t(x,y)$$

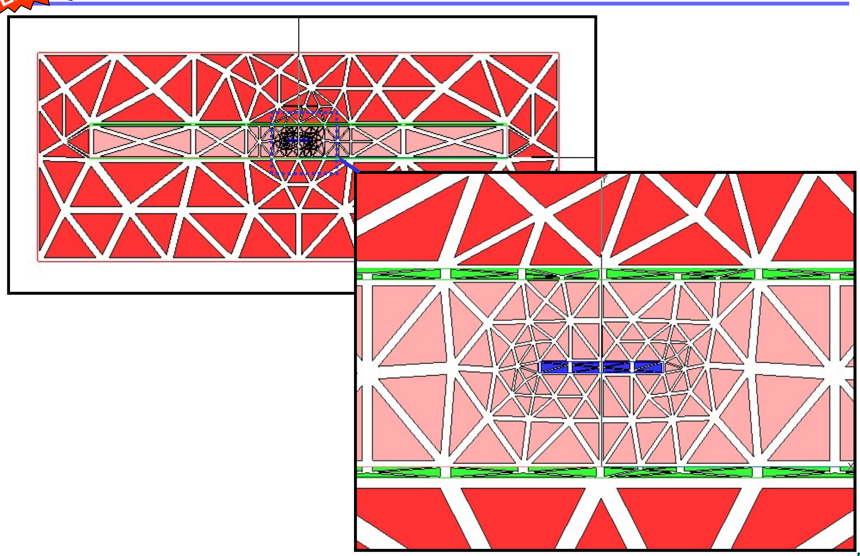
- Similarly magnetic flux intensity H can be obtained from:

$$\vec{H}_t(x,y) = -\frac{1}{\mu}\nabla_t \times [A_z(x,y)\hat{z}]$$

- For more information, refer to
  - T. Itoh (editor), "Numerical techniques for microwave and milimeter-wave passive structures", John-Wiley & Sons, 1989.
  - P. P. Silvester, R. L. Ferrari, "Finite elements for electrical engineers", Cambridge University Press, 1990.
  - Other newer books.

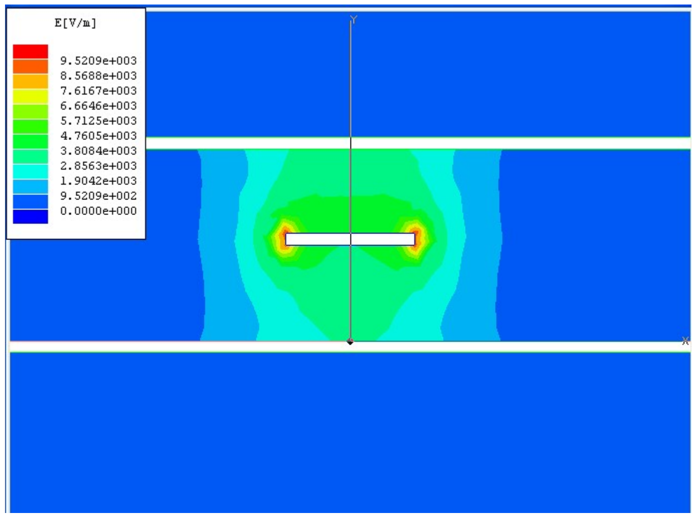
Example 5.3 - Electrostatics: The Triangular Mesh

Extra



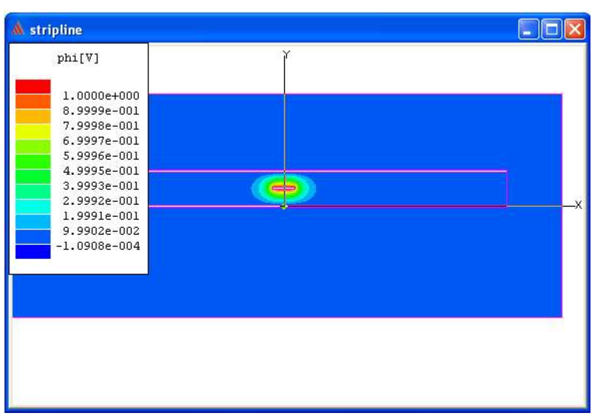
Extra

Example 5.3 - Electrostatics: Plot of E field Magnitude



Extra

Example 5.3 - Electrostatics: Plot of Voltage Contour



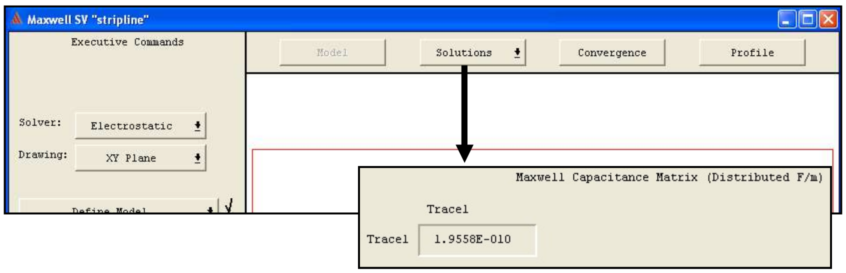
Example 5.3 - Electrostatics: Capacitance

Extra

- The estimated distributed capacitance is then computed using:

$$C = \frac{\varepsilon'}{|V_t|^2} \iiint_V \left|\vec{E}_t\right|^2 dv$$

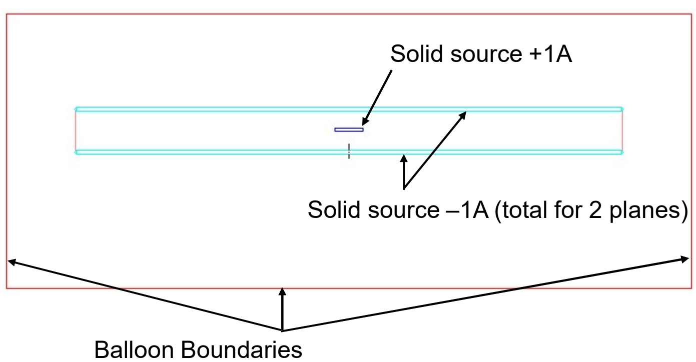
- Approximation to the integration using summation is performed by the software. The result is shown below:



- C ≈ 1.9958×10^{-10} F/m or 199.58 pF/m.

Example 5.3 - Magnetostatics: Setup Boundary Conditions

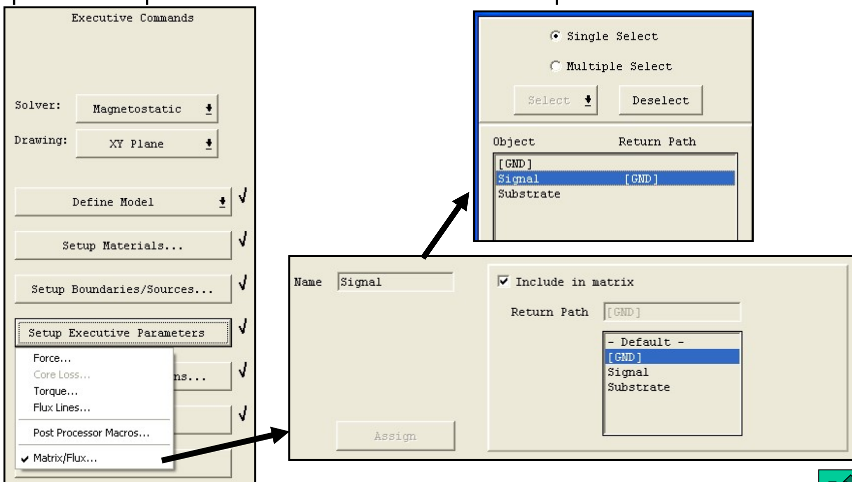
Extra



Example 5.3 - Magnetostatics: Setup Executive Parameters

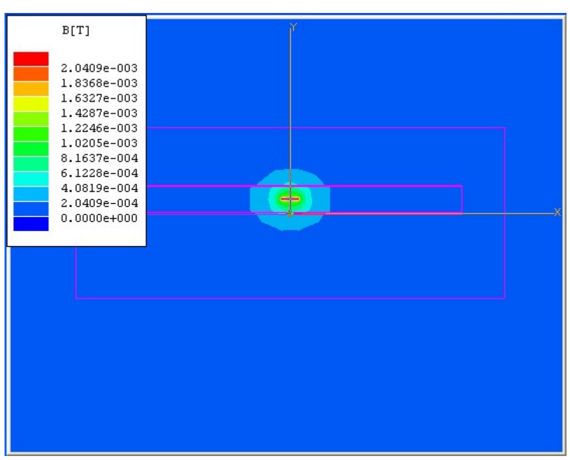
Extra

- Under 'Setup Executive Parameters' tab, select 'Matrix/Flux…' and proceed to perform the inductance matrix setup as shown.



Example 5.3 - Magnetostatics: Plot of B field Magnitude

Extra



Example 5.3 - Magnetostatics: Inductance

Extra

- The estimated distributed capacitance is then computed using:

$$L = \frac{\mu}{|I_t|^2} \iiint_V \left|\vec{H}_t\right|^2 dv$$

Inductance Matrix (Distributed H/m)

	Signal
Signal	2.5093E-007

- L ≈ 2.5093×10^{-7} H/m or 250.93 nH/m.

Example 5.3 - Derivation of Parameters for Stripline

Extra

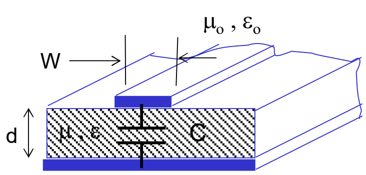
$$Z_c = \sqrt{\frac{L}{C}} = \sqrt{\frac{2.5093\times10^{-7}}{1.9558\times10^{-10}}} = 35.819\Omega$$

$$v_p = \frac{1}{\sqrt{LC}} = 1.427\times10^{8}\,ms^{-1}$$

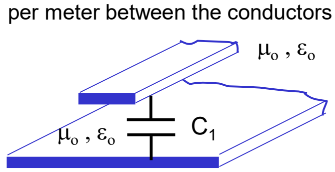
Appendix 3

The Origin of Effective Dielectric Constant (ε_{eff})

- The approach can be traced to a paper by Bryant and Weiss[1]. Assuming low loss and quasi-TEM mode: $v_p = \frac{1}{\sqrt{LC}}$ $\quad Z_c = \sqrt{\frac{L}{C}} = \frac{1}{v_p C}$



With dielectric, C is the capacitance per meter between the conductors



Without dielectric, C_1 is the capacitance per meter between the conductors

Define $\varepsilon_{eff} = \frac{C}{C_1}$

Then $v_p = \frac{1}{\sqrt{LC_1}\cdot\sqrt{\varepsilon_{eff}}} = \frac{c}{\sqrt{\varepsilon_{eff}}}$ ← Speed of light in vacumn

$$Z_c = \frac{1}{\frac{c}{\sqrt{\varepsilon_{eff}}}\cdot C} = \frac{1}{\frac{c}{\sqrt{\varepsilon_{eff}}}\cdot C_1\varepsilon_{eff}}$$

$$\Rightarrow Z_c = \frac{1}{c\sqrt{\varepsilon_{eff}}\cdot C_1}$$ ← C_1 is computed via numerical methods for various W and d

Note 1: T. G. Bryant and J. A. Weiss,"Parameters of microstrip transmission lines and coupled pairs of microstrip lines", IEEE Trans. Microwave Theory Tech., vol.MTT-16, pp.1021-1027,1968.

When Quasi-TEM Approximation is not Valid - Full Wave Analysis

Extra

- When quasi-TEM approximation is no longer valid, the preceeding formulation is not accurate and the telegraphic equations cannot be used.
- Also there is no longer a unique voltage and current, only E and H fields are used. More dispersion will also be observed.
- We can resort to defining equivalent voltage and current as employed for waveguides.
- However this is usually quite cumbersome, another option is to use full wave analysis.

Full-Wave Analysis

Extra

- Full-wave analysis is usually carried out using numerical methods such as Method of Moments (MoM), Finite Element Methods (FEM), Finite-Difference Time-Domain (FDTD) and Transmission Line Matrix (TLM).
- In all these methods, the Maxwell Equations are solved directly or indirectly instead of transforming the equations into circuit theory expressions (e.g. the telegraphic equations).

THE END

Exercise 1

- Suppose we have a parallel-plate transmission line with the following parameters:
  - W = 16.0 mm
  - d = 1.0 mm
  - $\varepsilon_r = 2.5$, $\mu_r = 1.0$, dielectric breakdown at $|E| = 3000$ V/m.
- The length of the line is 10 meter. Find the cut-off frequency of the for TM and TE modes.
- If TEM mode is propagating along the line, find the characteristic impedance Z_c of the line, and the maximum power that can be carried by this line without damaging the dielectric.

Exercise 1 Cont…

- Assuming this transmission line is used in the following system. Estimate the maximum working distance R.