Chapter 5: Coaxial Components and Rectangular Waveguide Components

References

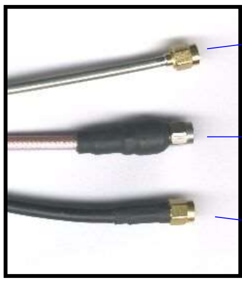
- [1] D.M. Pozar, "Microwave engineering", 2nd edition, 1998 John-Wiley & Sons. (3rd edition, 2005 is also available from John-Wiley & Sons).
- [2] R.E. Collin, "Foundation for microwave engineering", 2nd edition, 1992, McGraw-Hill.
- [3] C.A. Balanis, "Advanced engineering electromagnetics", 1989, John-Wiley & Sons.

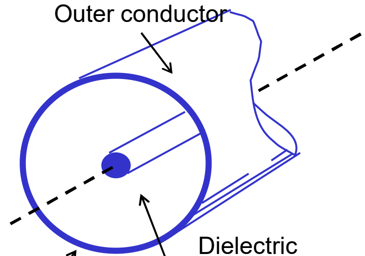
5.1 – Coaxial Components

Introduction

- The microstrip and stripline structures are important for guiding electromagnetic waves on printed circuit board (PCB).
- For system-to-system or board-to-board, a cable is used for guiding electromagnetic waves.
- The most common cable type for this purpose is the coaxial cable, which consists of two circular conductors, one is hollow and the other is usually solid, sharing a similar center axis (hence the name coaxial).
- Although generally used for transporting high-frequency electrical signal, the coaxial cable can also be used for low-frequency signal by virtue of it being a two-conductor interconnection. One conductor would serve as the signal and the other for the return current.
- The coaxial cable can support TEM, TE and TM modes of propagating electromagnetic waves.

Pictures of Coaxial Cables

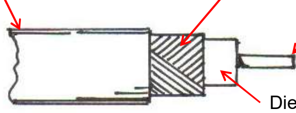




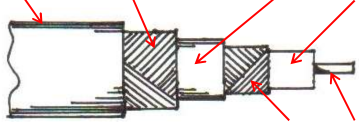
Flexible Coax Cable

Semi-rigid Coax Cable

Flexible Coaxial Cables



Single braid



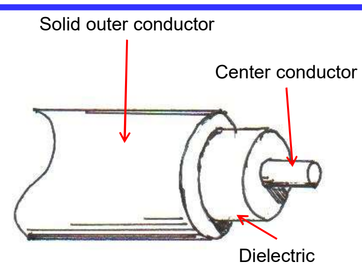
Double braid

Velocity of Propagation:

Solid Dielectric = 66.5 - 69% of the speed of light in vacuum

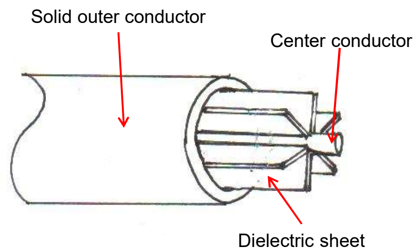
Foamed Dielectric = 72 - 85% of the speed of light

Semi-Rigid Coaxial Cables



Solid Dielectric

Typical dielectric types are PTFE (Polytetraflouro-ethylene), foam and air.



Air Articulated

Coaxial Cable Parameters (1)

- The dominant propagation mode for electromagnetic waves in coaxial cable is TEM mode.
- In this mode the RLCG parameters (under low-loss condition) are given as follows:

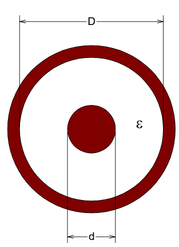
$$\varepsilon = \varepsilon' - j\varepsilon'' \quad (1.1)$$

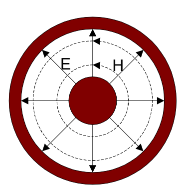
$$R = \frac{1}{\pi\sigma_c\delta_s}\left(\tfrac{1}{D}+\tfrac{1}{d}\right) \quad (1.2a) \qquad \delta_s = \sqrt{\frac{2}{\omega\sigma_c\mu}}$$

$$L = \frac{\mu}{2\pi}\ln(D/d) \quad (1.2b)$$

$$C = \frac{2\pi\varepsilon'}{\ln(D/d)} \quad (1.2c)$$

$$G = \frac{2\pi\omega\varepsilon''}{\ln(D/d)} \quad (1.2d)$$





Coaxial Cable Parameters (2)

- From these basic RLCG parameters under TEM mode, other parameters of interest such as characteristic impedance, attenuation factor, velocity of propagation, maximum power handling, cut-off frequency (when non-TEM modes start to propagate) can be derived.
- Other parameters which are influenced by the mechanical aspects of the coaxial cable are flexibility of the cable, operating temperature range, connector type, cable diameter, cable noise or shielding effectiveness etc.
- Of these, the most importance is the characteristic impedance Z_c. Under lossless approximation, with R = G = 0, the characteristic impedance is given by:

$$Z_c = \sqrt{\frac{L}{C}} = \frac{1}{2\pi}\sqrt{\frac{\mu}{\varepsilon'}}\ln(D/d) \qquad \textit{(1.3)}$$

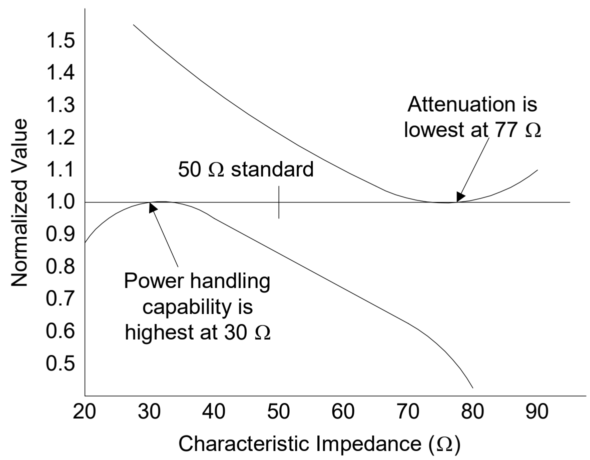
- Typical Z_c values are 50, 75 and 93. Of these Z_c = 50 is the most common.

Why Z_c = 50 Ohms ? (1)

- Most coaxial cables have $Z_c = 50\Omega$ under low loss condition, with the 75Ω being used in television systems. The original motivation behind these choice is that an air-filled coaxial cable has minimum attenuation for $Z_c = 75\Omega$, while maximum power handling occurs for a cable with $Z_c = 30\Omega$.
- A cable with $Z_c = 50\Omega$ thus represents a compromise between minimum attenuation and maximum power capacity.
- Bear in mind this is only true for air-filled coaxial cable, but the tradition prevails for coaxial cable with other type of dielectric.

In the old days coaxial cable with $Z_c = 93\Omega$ is also manufactured, these are mainly used for sending digital signal, between computers. The capacitance per unit length C is minimized for this impedance value.

Why Z_c = 50 Ohms ? (2)



Cable Specifications (1)

- A series of standard types of coaxial cable were specified for military uses in the United States, in the form "RG-#" or "RG-#/U". These are dated back to World War II and were listed in MIL-HDBK-216 (1962).
- These designations are now obsolete. The current US military standard is Military Specifications MIL-C-17. MIL-C-17 numbers, such as "M17/75-RG214," are given for military cables and manufacturer's catalog numbers for civilian applications.
- However, the RG-series designations were so common for generations that they are still used today although the handbook is withdrawn.

Cable Specifications (2)

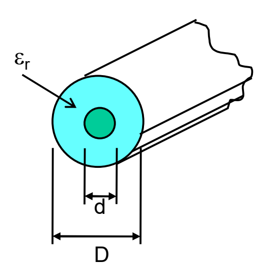
Cable type	Z_o (Ω)	Dielectric	Overall Diameter (inch)	Attenuation (dB/100ft@ 3GHz)	Maximum power (W@3GHz)	Capacitance (pF/ft)
RG-8A	52	Polyethylene	0.405	16	115	29.5
RG-58C	50	Polyethylene	0.195	54	25	30.0
RG-174A	50	Polyethylene	0.100	64	15	30.0
RG-196A	50	Teflon	0.080	85	40	29.4
RG-179B	75	Teflon	0.100	44	100	19.5
RG-401	50	Teflon	0.250 (S)	14	750	28.5
RG-402	50	Teflon	0.141 (S)	21.5	250	28.5
RG-405	50	Teflon	0.086 (S)	34	90	28.5

Upper Usable Frequency

- Since a coaxial cable supports TEM, TE and TM electromagnetic wave propagation modes, the TE and TM modes will come into existent for sufficiently high operating frequency.
- The Upper Usable Frequency (UUF) for coaxial cable refers to the frequency where the first non-TEM mode comes into existent.
- For coaxial structure, the non-TEM mode with the lowest cut-off frequency (f_c) is the TE_{11} mode.
- The UUF can be estimated by [1]:

$$k_c d \cong \frac{4}{1+\frac{D}{d}} \qquad \textbf{(1.4a)}$$

$$f_c = \frac{c k_c}{2\pi\sqrt{\varepsilon_r}} = UUF \qquad \textbf{(1.4b)}$$



where c = speed of light in vacuum

Example

- For instance for RG-142 coaxial cable (Ref [1]), with inner conductor radius = 0.89mm and outer conductor radius = 2.95mm, the estimated UUF is:

$$d = 1.78\text{mm} \quad D = 5.89\text{mm}$$

$$\tfrac{D}{d} = 3.314$$

$$k_c d = 0.927 \quad k_c = 521.51$$

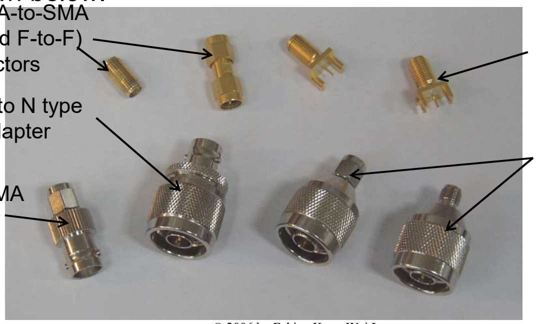
$$f_c \cong 16.78\,\text{GHz}$$

- As a safety precaution we usually include some margin, say 5%, thus the upper usable frequency is rated at:

$$f_{UUF} = f_c \cdot 0.95 \cong 16\,\text{GHz}$$

Connectors and Adapters (1)

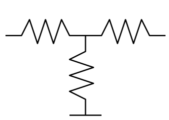
- The ends of a coaxial cable are fixed to connectors.
- Such connectors are cylindrical in shape, thus the connectors also exhibit upper usable frequency limit.
- Some common examples of connectors/adapters for coaxial cable are shown below.



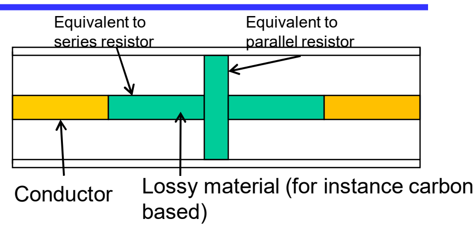
Connectors and Adapters (2)

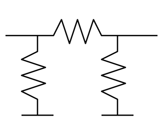
- A comparison of RF/microwave connectors usable frequency range:
  – BNC (baby N connector), for DC to 400 MHz.
  – N connector, for DC to 8 GHz.
  – SMA (Sub-miniature version A) connector (inner diameter, D≅ 4.6mm), for DC to 18 GHz.
  – 3.5 mm connectors, for DC to around 30 GHz.
  – 2.9 mm connectors, for DC to around 40 GHz.
  – 2.4 mm connectors, for DC to around 50 GHz.
  – 1.8 mm connectors, for DC to around 65 GHz.

Attenuators

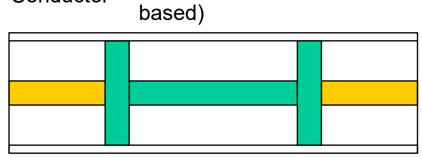


T-section





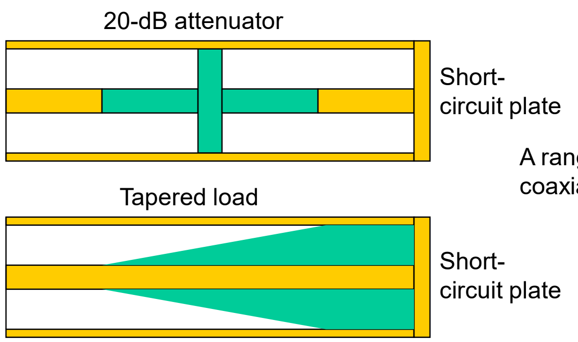
π-section

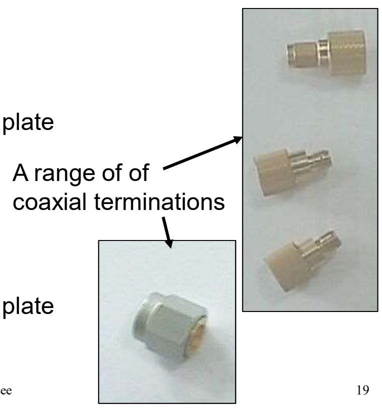


A commercial coaxial attenuator

Terminations

- A termination is a component which absorb all incident wave.
- This can be approximated by having an attenuator in series with a short circuit. For instance in the example below, the reflected wave power will be 40 dB (or 10000 times) smaller than the incident wave power.



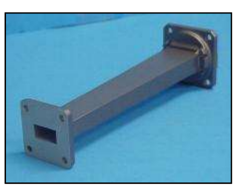


5.2 - Rectangular Waveguide

Introduction

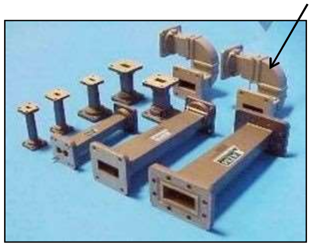
- As we have seen earlier waveguides refer to any structure that can guide electromagnetic (EM) waves along its axial direction, which include transmission line.
- Here we consider waveguide as specifically refers to long metallic structures with only 1 piece of conductor between the source and load end.
- These metallic structures are usually hollow, so that EM fields are confined within the hollow and be guided along the axial direction.
- Applying Maxwell's Equations with the proper boundary conditions (see Appendix) shows that propagating EM waves can be supported by the waveguide.
- Due to the absence of center conductor, only TE and TM mode exist.

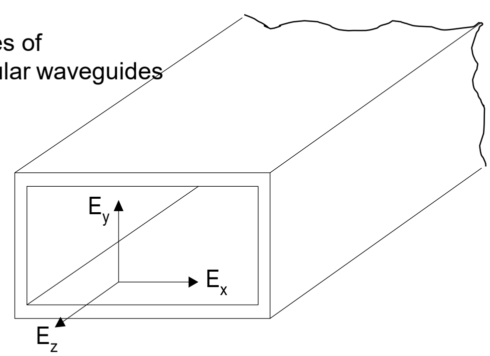
Rectangular Waveguide (1)



Examples of rectangular waveguides sections

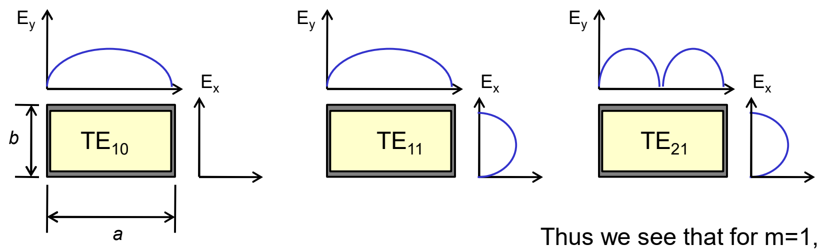
Bend





$$\tilde{E} = \tilde{E}_t + E_z \hat{a}_z$$

EM Waves Propagating Modes in Waveguide



Thus we see that for m=1, n=0, the TE_{10} has the lowest cut-off frequency, and is the dominant mode.

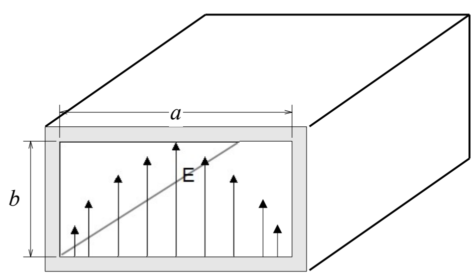
Cutoff frequency for TE_{mn} or TM_{mn} mode

$$f_{c,mn} = \frac{c}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

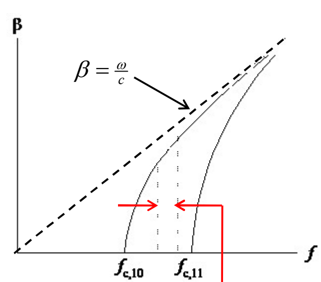
The corresponding cutoff wavelength for TE_{mn} or TM_{mn} mode

$$\lambda_{c,mn} = \frac{2}{\sqrt{(m/a)^2 + (n/b)^2}}$$

Dominant Mode for Rectangular Waveguide



TE_{10} mode



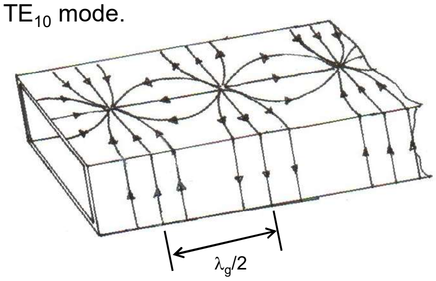
For TE_{10} mode, m=1, n=0, thus the cut-off wavelength is:

$$\lambda_{c,10} = \frac{2}{\sqrt{(m/a)^2 + (n/b)^2}} = 2a$$

Recommended operating frequency range for rectangular waveguide

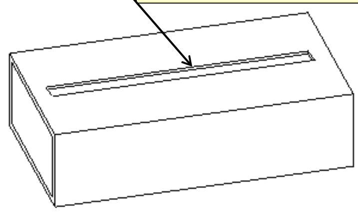
Waveguide Wall Currents

The current on the wall of the rectangular waveguide at a certain instance in time for TE_{10} mode.



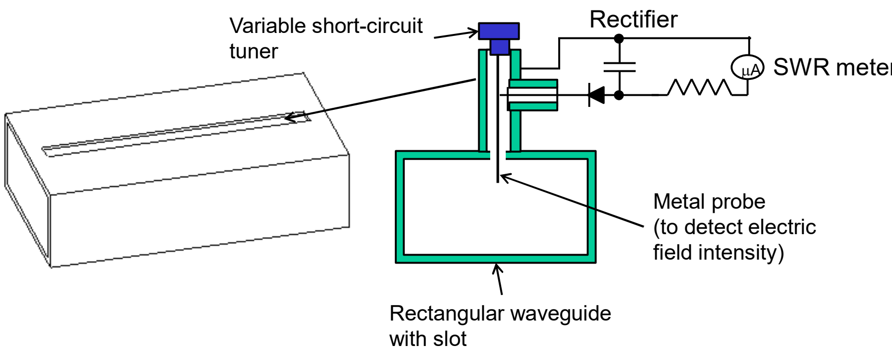
λ_g = Guide wavelength

We can create a narrow slot parallel to the center axis of the waveguide without disturbing the current flow, hence the internal EM fields.

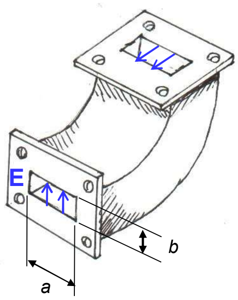


Slotted-Line Probe

A short metal probe can be inserted into the slot with minimal disturbance to the EM fields of the TE_{10} mode. This probe can be used to measure the relative strength of the electric field in the waveguide cavity. Usually the microwave EM fields will be modulated by a low-frequency signal, and the diode/capacitor pair acts as envelope detector to demodulate this low-frequency signal.

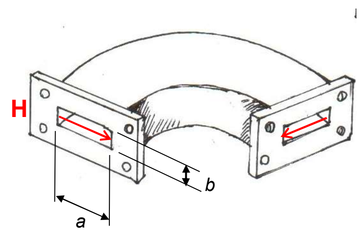


Bends and Twists



E-bend

The orientation of electric (**E**) field is altered.

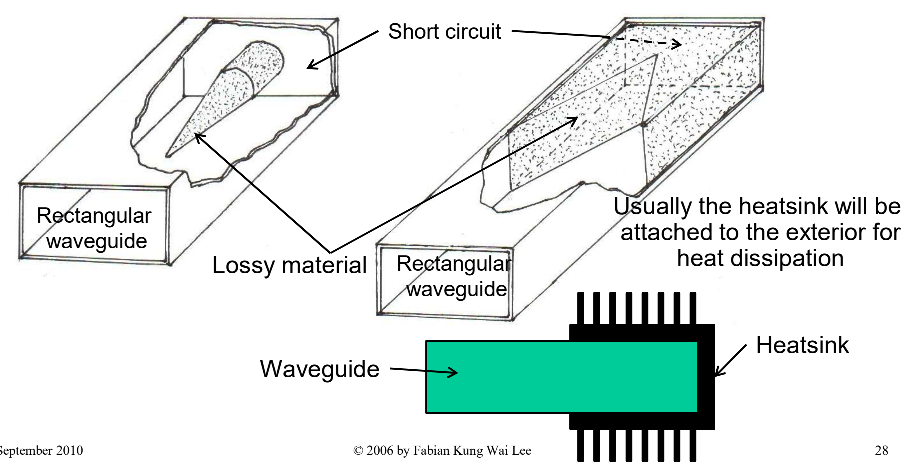


H-bend

The orientation of magnetic (**H**) field is altered.

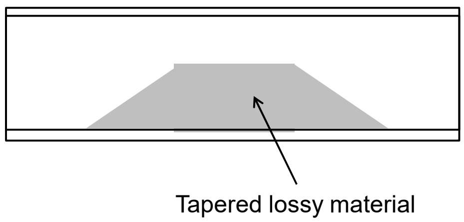
Matched Terminations

The gradual transition from the rectangular waveguide to the lossy material ensures minimal reflection of the electromagnetic wave.

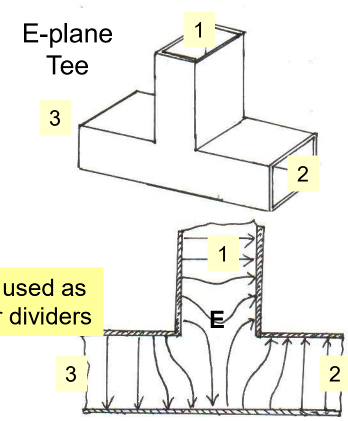


Attenuators

An example of a waveguide attenuator, here the lossy material is shaped so as to provide gradual change in the waveguide internal geometry, resulting in small reflection of incident wave.

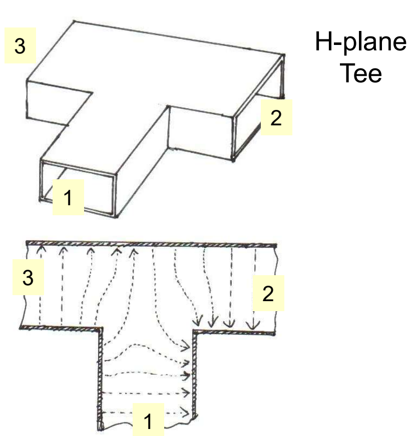


Waveguide Tees



Often used as power dividers

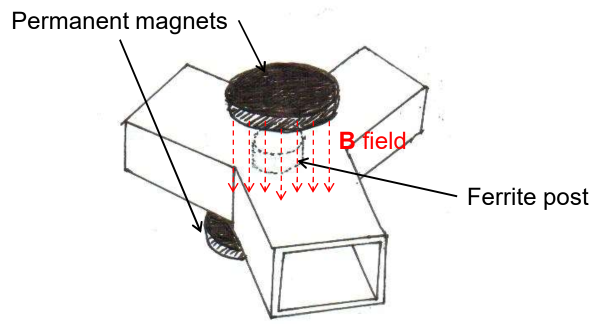
$$\begin{bmatrix} V_1^- \\ V_2^- \\ V_3^- \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & 1 & -1 \\ 1 & 0 & 1 \\ -1 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} V_1^+ \\ V_2^+ \\ V_3^+ \end{bmatrix}$$



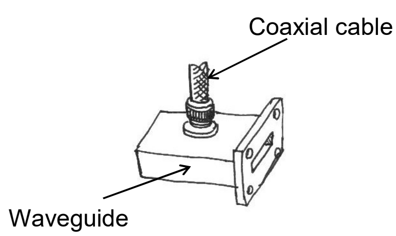
$$\begin{bmatrix} V_1^- \\ V_2^- \\ V_3^- \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} V_1^+ \\ V_2^+ \\ V_3^+ \end{bmatrix}$$

Circulator

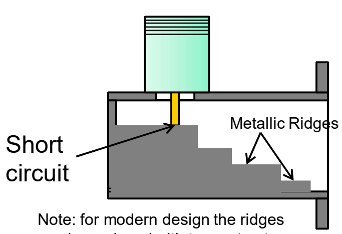
Example of a rectangular waveguide circulator (see Chapter 4):



Waveguide to Co-axial Adapter



A waveguide to coaxial adapter



Note: for modern design the ridges can be replaced with taper structure.

Front view of the adapter

Appendix 1.0 – Solution of Electromagnetic Fields for Rectangular Waveguide

Introduction

- Rectangular waveguides are one of the earliest waveguide structures used to transport microwave energies.
- Because of the lack of a center conductor, the electromagnetic field supported by a waveguide can only be TM or TE modes.
- For rectangular waveguide, the dominant mode is TE, which has the lowest cut-off frequency.

Why No TEM Mode in Waveguide?

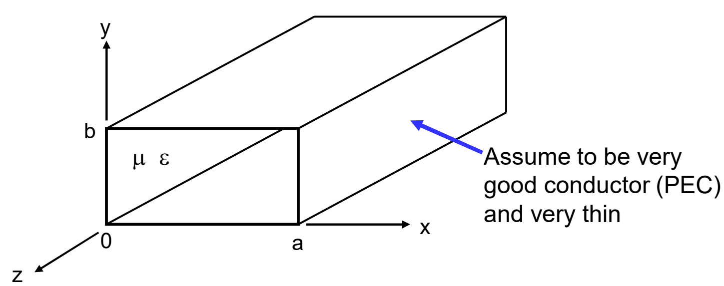
- From Maxwell's Equations, the magnetic flux lines always close upon themselves. Thus if a TEM wave were to exist in a waveguide, the field lines of **B** and **H** would form closed loop in the transverse plane.
- However from the modified Ampere's law:

$$\nabla \times \vec{H} = \vec{J} + \tfrac{\partial}{\partial t}\vec{D} \Rightarrow \oint_C \vec{H} \cdot d\vec{l} = \vec{I} + \tfrac{\partial}{\partial t}\iint_S \vec{D} \cdot d\vec{s}$$

- The line integral of the magnetic field around any closed loop in a transverse plane must equal the sum of the longtitudinal conduction and displacement currents through the loop.
- Without an inner conductor and with TEM mode there is no longtitudinal conduction current and displacement current inside the waveguide. Consequently this leads to the conclusion that there can be no closed loops of magnetic field lines in the transverse plane.

Rectangular Waveguide

- The perspective view of a rectangular waveguide is referred below.
- The following slides shall illustrate the standard procedures of obtaining the electromagnetic (EM) fields guided by this structure.



TE Mode Solution (1)

- To obtain the TE mode electromagnetic (EM) field pattern, we use the systematic procedure developed in Chapter 1 – Advanced Transmission Line Theory.
- We start by solving the pattern function for z-component of the magnetic field: $\nabla_t^2 h_z + k_c^2 h_z = 0 \;,\; k_c^2 = k_o^2 - \beta^2$ and boundary conditions ***(1.1)***
- Problem (1.1) is called Boundary Value Problem (BVP) in mathematics.
- Once we know the function of $h_z(x,y)$, the EM fields are given by:

$$\vec{H} = \left(\frac{-j\beta}{k_c^2}\frac{\partial h_z}{\partial x}\right)e^{-j\beta z}\hat{x} + \left(\frac{-j\beta}{k_c^2}\frac{\partial h_z}{\partial y}\right)e^{-j\beta z}\hat{y} + h_z e^{-j\beta z}\hat{z} \qquad \textbf{\textit{(1.2a)}}$$

$$\vec{E} = \left(\frac{-j\omega\mu}{k_c^2}\frac{\partial h_z}{\partial y}\right)e^{-j\beta z}\hat{x} + \left(\frac{j\omega\mu}{k_c^2}\frac{\partial h_z}{\partial x}\right)e^{-j\beta z}\hat{y} \qquad \textbf{\textit{(1.2b)}}$$

Note: Here we only consider propagation in positive direction, treatment for negative propagation is similar.

TE Mode Solution (2)

- Expanding the partial differential equation (PDE) of (1.1) in cartesian coordinates: $\left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + k_c^2\right]h_z(x,y) = 0$ ***(1.3)***
- Using the Separation of Variables Method, we can decompose $h_z(x,y)$ into the product of 2 functions and k_c^2 to be the sum of 2 constants:

$$h_z(x,y) = X(x)Y(y) \quad \textbf{\textit{(1.4a)}} \qquad k_c^2 = k_x^2 + k_y^2 \quad \textbf{\textit{(1.4b)}}$$

- Putting these into (1.3), and after some manipulation we obtain 2 ordinary differential equations (ODEs):

$$Y\frac{\partial^2 X}{\partial x^2} + X\frac{\partial^2 Y}{\partial x^2} + k_c^2 XY = 0 \Rightarrow \frac{1}{X}\frac{\partial^2 X}{\partial x^2} + \frac{1}{Y}\frac{\partial^2 Y}{\partial x^2} + k_c^2 = 0 \Rightarrow \frac{1}{X}\frac{\partial^2 X}{\partial x^2} + \frac{1}{Y}\frac{\partial^2 Y}{\partial x^2} = -k_x^2 - k_y^2$$

$$\frac{1}{X}\frac{\partial^2 X}{\partial x^2} = -k_x^2 \quad \textbf{\textit{(1.5a)}} \qquad \frac{1}{Y}\frac{\partial^2 Y}{\partial x^2} = -k_y^2 \quad \textbf{\textit{(1.5b)}}$$

TE Mode Solution (3)

- From elementary calculus, we know that the general solution for (1.5a) and (1.5b) are:

$$X(x) = A\cos(k_x x) + B\sin(k_x x) \qquad \textit{(1.6a)}$$

$$Y(y) = C\cos(k_y y) + D\sin(k_y y) \qquad \textit{(1.6b)}$$

- Thus $h_z(x,y)$ is given by:

$$h_z(x,y) = [A\cos(k_x x) + B\sin(k_x x)][C\cos(k_y y) + D\sin(k_y y)] \qquad \textit{(1.7)}$$

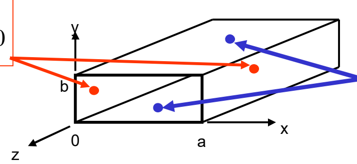
TE Mode Solution (4)

- A, B, C and D in (1.7) are unknown constants, to be determined by applying the boundary conditions that the tangential electric field must vanish on the conductive walls of the waveguide. From (1.2b):

$$\vec{E} = E_x^+\hat{x} + E_y^+\hat{y} = \left(\frac{-j\omega\mu}{k_c^2}\frac{\partial h_z}{\partial y}\right)e^{-j\beta z}\hat{x} + \left(\frac{j\omega\mu}{k_c^2}\frac{\partial h_z}{\partial x}\right)e^{-j\beta z}\hat{y}$$

$$E_y(0,y) = E_y(a,y) = 0$$
$$\Rightarrow \frac{\partial h_z(0,y)}{\partial x} = \frac{\partial h_z(a,y)}{\partial x} = 0$$

(1.8b)



$$E_x(x,0) = E_x(x,b) = 0$$
$$\Rightarrow \frac{\partial h_z(x,0)}{\partial y} = \frac{\partial h_z(x,b)}{\partial y} = 0$$

(1.8a)

TE Mode Solution (5)

- Using (1.6) and applying the boundary condition (1.8a):

$$\frac{\partial h_z}{\partial y} = k_y\left[A\cos(k_x x) + B\sin(k_x x)\right]\left[-C\sin(k_y y) + D\cos(k_y y)\right]$$

$$\frac{\partial h_z(x,0)}{\partial y} = 0 \Rightarrow D = 0 \qquad \frac{\partial h_z(x,b)}{\partial y} = 0 \Rightarrow k_y = \frac{n\pi}{b} \;, n = 0,1,2,3\cdots$$

- Using (1.6) and applying the boundary condition (1.8b):

$$\frac{\partial h_z}{\partial x} = k_x\left[-A\sin(k_x x) + B\cos(k_x x)\right]\left[C\cos(k_y y) + D\sin(k_y y)\right]$$

$$\frac{\partial h_z(0,y)}{\partial x} = 0 \Rightarrow B = 0 \qquad \frac{\partial h_z(a,y)}{\partial x} = 0 \Rightarrow k_x = \frac{m\pi}{a} \;, m = 0,1,2,3\cdots$$

- In the above equations, we can combine the product of A·C, let's call it R. Common sense tells us that R would be different for each pair of integer (m,n), thus we should denote R by: R_{mn}

TE Mode Solution (6)

- From (1.4b), k_c and the propagation constant β are given by:

$$k_c = \sqrt{k_x^2 + k_y^2} = \sqrt{\left(\frac{n\pi}{b}\right)^2 + \left(\frac{m\pi}{a}\right)^2} \qquad \beta = \sqrt{k_o^2 - k_c{}^2} = \sqrt{k_o^2 - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2}$$

- Since k_c and β also depends on the integer pairs (m,n), it is more appropriate to write these as:

$$k_{c_{mn}} = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2} \qquad \textbf{(1.9a)}$$

$$\beta_{mn} = \sqrt{k_o^2 - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2} \;,\quad k_o = \omega\sqrt{\mu\varepsilon} \qquad \textbf{(1.9b)}$$

TE Mode Solution (7)

- With these information, and using (1.2a) and (1.2b), we can write out the complete mathematical expressions for the EM fields under TE propagation mode for a rectangular waveguide:

$$E_x^+ = \left(\frac{j\omega\mu}{k_{c_{mn}}^2}\right)\left(\frac{n\pi}{b}\right)R_{mn}\cos\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right)e^{-j\beta_{mn}z} \qquad \textit{(1.10a)}$$

$$E_y^+ = \left(\frac{j\omega\mu}{k_{c_{mn}}^2}\right)\left(\frac{m\pi}{a}\right)R_{mn}\sin\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right)e^{-j\beta_{mn}z} \qquad \textit{(1.10b)}$$

$$H_x^+ = \left(\frac{j\beta_{mn}}{k_{c_{mn}}^2}\right)\left(\frac{m\pi}{a}\right)R_{mn}\sin\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right)e^{-j\beta_{mn}z} \qquad \textit{(1.10c)}$$

$$H_y^+ = \left(\frac{j\beta_{mn}}{k_{c_{mn}}^2}\right)\left(\frac{n\pi}{b}\right)R_{mn}\cos\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right)e^{-j\beta_{mn}z} \qquad \textit{(1.10d)}$$

$$H_z^+ = R_{mn}\cos\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right)e^{-j\beta_{mn}z} \qquad \textit{(1.10e)}$$

Cut-Off Frequency for TE Mode (1)

- Notice from (1.9b) that the propagation constant β_{mn} is real when:

$$k_o^2 > \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$$

- When β_{mn} is imaginary the EM fields cannot propagate.
- Since $\omega=2\pi f$, we can define a limit for the frequency f as follows:

$$k_o^2 = \omega^2\mu\varepsilon > \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$$

$$f > \frac{1}{2\pi\sqrt{\mu\varepsilon}}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

- The lower limit of this frequency is called the Cut-off Frequency f_c.

$$f_c\big|_{TE_{mn}} = \frac{1}{2\pi\sqrt{\mu\varepsilon}}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2} \qquad \textit{(1.11)}$$

Cut-Off Frequency for TE Mode (2)

- The TE mode electromagnetic field is usually labeled as TE_{mn} since the mathematical function of the field components depend on the integer pair (m,n).
- The pair (m,n) cannot be both zeros, otherwise from (1.10a) to (1.10e), E_x^+, E_y^+, H_x^+, H_y^+, H_z^+ are all zero, no fields at all! This is a trivial solution, although a valid one.
- The smallest combination of (m,n) are (m,n) = (1,0) or (0,1).
- Since a > b (the lateral dimensions of the rectangle), we see that (m,n) = (1,0) produces a smaller f_c, thus lower cutoff frequency. Therefore the TE propagation mode TE_{10} is the dominant mode for TE waves. It's corresponding cut-off frequency is given by: $$f_c\big|_{TE10} = \frac{1}{2\pi\sqrt{\mu\varepsilon}}\sqrt{\left(\frac{\pi}{a}\right)^2} = \frac{1}{2a\sqrt{\mu\varepsilon}}$$
- Only excitation frequency greater than $f_c\big|_{TE10} = \frac{1}{2a\sqrt{\mu\varepsilon}}$ will cause EM waves to propagate within the rectangular waveguide.

Example

- Consider a rectangular waveguide, a = 45.0mm, b = 35.0mm, filled with air ($\varepsilon = \varepsilon_o$, $\mu = \mu_o$).

$$\varepsilon_o = 8.854\times10^{-12}$$
$$\mu_o = 4\pi\times10^{-7}$$
$$f_c\big|_{TE10} = \frac{1}{2\cdot 0.045\cdot\sqrt{\mu_o\varepsilon_o}} = 3.331\times10^9$$

Phase Velocity in Waveguide

- Since phase velocity v_p depends on propagation constant β_{mn}, it too depends on the integer pair (m,n) hence the property of the TE mode fields.

$$v_p = \frac{\omega}{\beta_{mn}} = \frac{\omega}{\sqrt{k_o^2 - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2}} > \frac{\omega}{k_o} \qquad \textbf{(1.12)}$$

Speed of light in dielectric of (μ,ε)

- We thus observe that the phase velocity of TE mode has the peculiar property of traveling faster than the speed of light!

Group Velocity in Waveguide

- The velocity of energy propagation, or the speed that information travel in a waveguide is given by the Group Velocity v_g.

$$v_g = \frac{\partial \omega}{\partial \beta}$$

- Thus from: $\frac{\partial \beta_{mn}}{\partial \omega} = \frac{\mu\varepsilon\omega}{\sqrt{k_o^2 - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2}} = \frac{\mu\varepsilon\omega}{\beta_{mn}}$

$$v_g = \frac{\beta_{mn}}{\mu\varepsilon\omega} = \frac{1}{\left(\frac{\omega}{\beta_{mn}}\right)\left(\sqrt{\mu\varepsilon}\right)^2} = \frac{c^2}{v_p}$$

- Since $v_p > c$, $v_g = \left(\frac{c}{v_p}\right)c < c$

- The group velocity is thus less than speed of light in vacuum, maintaining the assertion of Relativity Theory that no mass/energy can travel faster than speed of light.

TM Mode Solution (1)

- The procedure for obtaining the EM field solution for TM propagation is similar to the TE procedure.
- We start by solving the pattern function for the z-component of the electric field:

$$\nabla_t^2 e_z + k_c^2 e_z = 0 \quad , \quad k_c^2 = k_o^2 - \beta^2 \quad \text{and boundary conditions} \qquad \textbf{\textit{(1.13)}}$$

- As in solving TE mode problem, the Separation of Variables Method is used in solving (1.13), and integer pair (m,n) needs to be introduced in the TM mode solution.
- The mathematical expressions for the EM field components thus depends on the integer pair (m,n), and is denoted by TM_{mn} field.
- The derivation details will be omitted here due to space constraint. You can refer to reference [1] for the procedure.

TM Mode Solution (2)

- The complete expressions for the TM_{mn} field components are shown below:

$$E_x^+ = \left(\frac{-j\beta_{mn}}{k_{c_{mn}}^2}\right)\left(\frac{m\pi}{a}\right)R_{mn}\cos\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right)e^{-j\beta_{mn}z} \qquad \textbf{\textit{(1.14a)}}$$

$$E_y^+ = \left(\frac{-j\beta_{mn}}{k_{c_{mn}}^2}\right)\left(\frac{n\pi}{b}\right)R_{mn}\sin\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right)e^{-j\beta_{mn}z} \qquad \textbf{\textit{(1.14b)}}$$

$$H_x^+ = \left(\frac{j\omega\varepsilon}{k_{c_{mn}}^2}\right)\left(\frac{n\pi}{b}\right)R_{mn}\sin\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right)e^{-j\beta_{mn}z} \qquad \textbf{\textit{(1.14c)}}$$

$$H_y^+ = \left(\frac{j\beta_{mn}}{k_{c_{mn}}^2}\right)\left(\frac{n\pi}{b}\right)R_{mn}\cos\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right)e^{-j\beta_{mn}z} \qquad \textbf{\textit{(1.14d)}}$$

$$E_z^+ = R_{mn}\sin\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right)e^{-j\beta_{mn}z} \qquad \textbf{\textit{(1.14e)}}$$

TM Mode Solution (3)

- Where

$$k_{c_{mn}} = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2} \qquad \textbf{\textit{(1.15a)}}$$

$$\beta_{mn} = \sqrt{k_o^2 - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2} \quad , \quad k_o = \omega\sqrt{\mu\varepsilon} \qquad \textbf{\textit{(1.15b)}}$$

Cut-Off Frequency for TM Mode

- Since the propagation constant β_{mn} is similar for both TE_{mn} and TM_{mn} mode, a cut-off frequency also exists for TM_{mn} :

$$f_c\big|_{TM_{mn}} = \frac{1}{2\pi\sqrt{\mu\varepsilon}}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2} \qquad \textbf{\textit{(1.16)}}$$

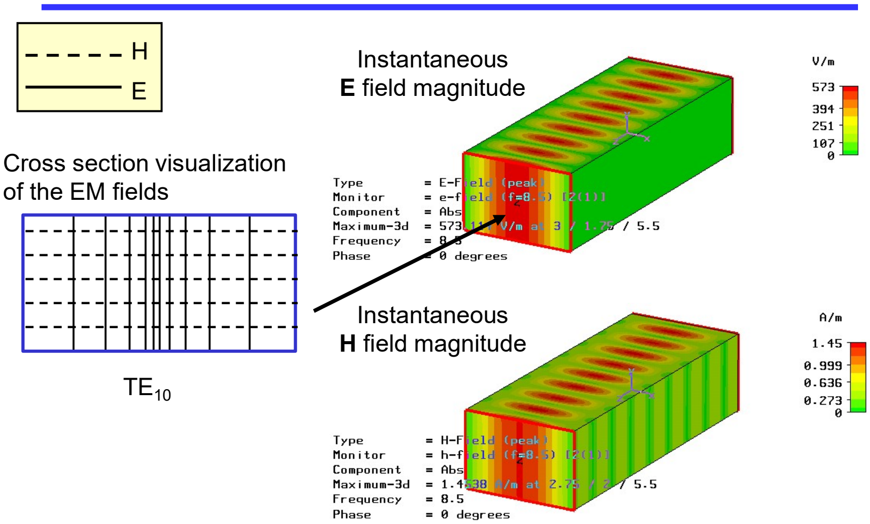
- Observe that from (1.14a) to (1.14e) the EM field components become 0 if either m or n is 0. Thus TM_{00} , TM_{10} and TM_{01} do not exist. The lowest order mode is TM_{11}.

$$f_c\big|_{TM_{11}} = \frac{1}{2\pi\sqrt{\mu\varepsilon}}\sqrt{\left(\frac{\pi}{a}\right)^2 + \left(\frac{\pi}{b}\right)^2} > \frac{1}{2\pi\sqrt{\mu\varepsilon}}\sqrt{\left(\frac{\pi}{a}\right)^2} = f_c\big|_{TE_{10}}$$

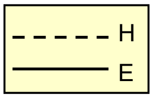
- It is for this reason that we consider TE_{10} to be the dominant mode of rectangular waveguide.

Appendix 2.0 – Plots of Electromagnetic Fields for Rectangular Waveguide

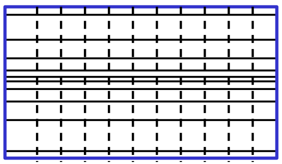
TE$_{10}$ Mode



TE_{01}

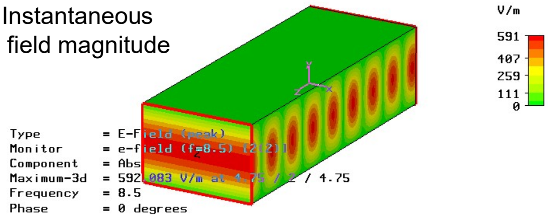


Cross section visualization of the EM fields

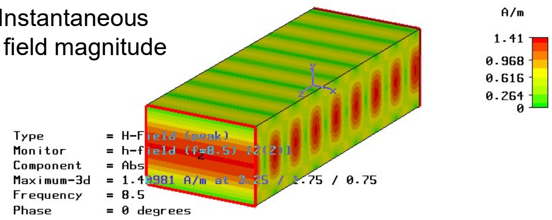


TE_{10}

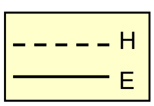
Instantaneous **E** field magnitude

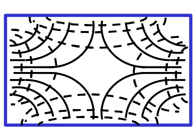


Instantaneous **H** field magnitude



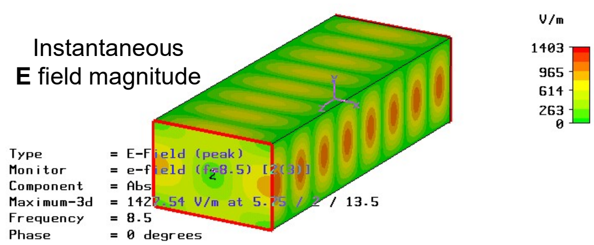
TE_{11}



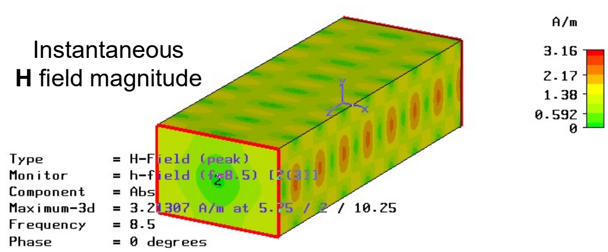


TE_{11}

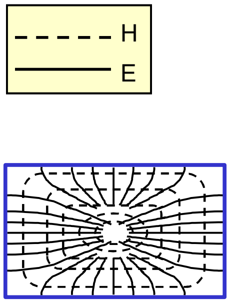
Instantaneous **E** field magnitude



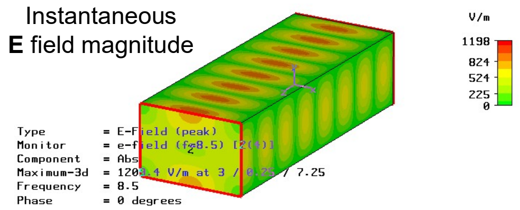
Instantaneous **H** field magnitude

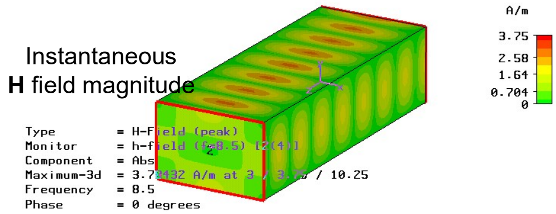


TM$_{11}$



TM$_{11}$





Appendix 3.0 – Solution of Electromagnetic Fields for Coaxial Transmission Line