

Course: Dynamic Programming and Reinforcement Learning

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### Mid-Term Recap: Flight Ticket Market

#### **Problem Context:**

Time Horizon: finite (take-off)

Action: Price Offer

Demand: Price and Time-Dependent

**Event: Tickets sold** 

Reward: Sales Revenue

#### **Simple Environment:**

State Space: Discrete  $(0, 10) \rightarrow (10, 0)$ 

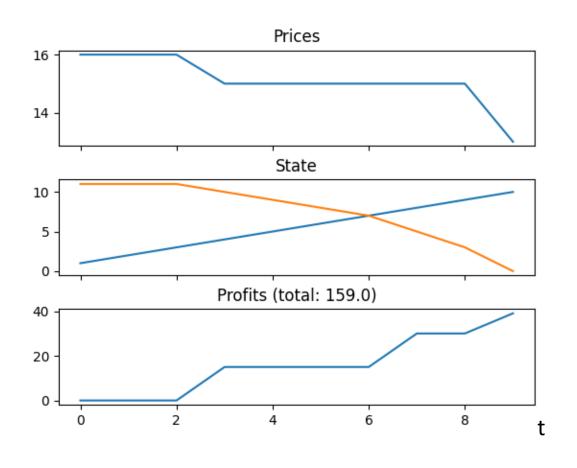
Action Space: Discrete (0, 10, 20, ..., 200)

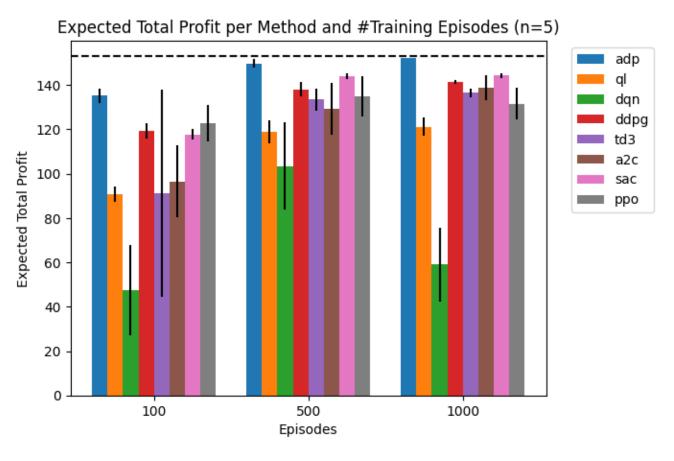
Event Space: Discrete (0, ..., 10)

Reward: Price x Tickets sold

Buying Probability:  $\left(1 - \frac{a}{200}\right) * \left(1 + \frac{t}{10}\right)$ 

# Mid-Term Recap





#### Research Directions

- Realistic Customer Behavior
- Estimating Demand and using Dynamic Programming methods
- Duopoly setup





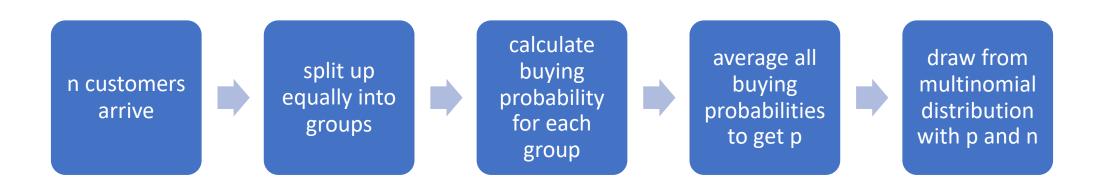






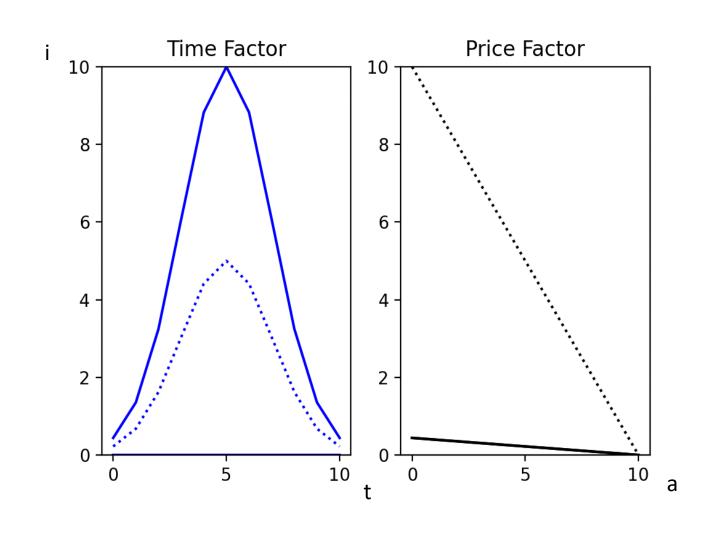
- Five types of customer behaviors implemented
  - Rational, Family, Business, Early Booking, Party
- Every combination of types possible

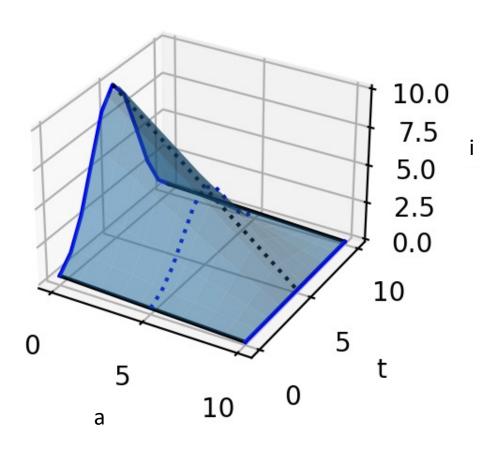
One simulation step:





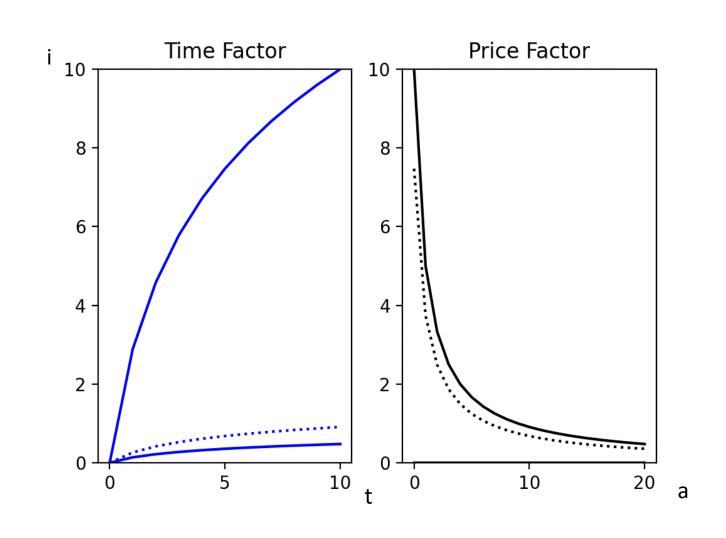
# Family Customer

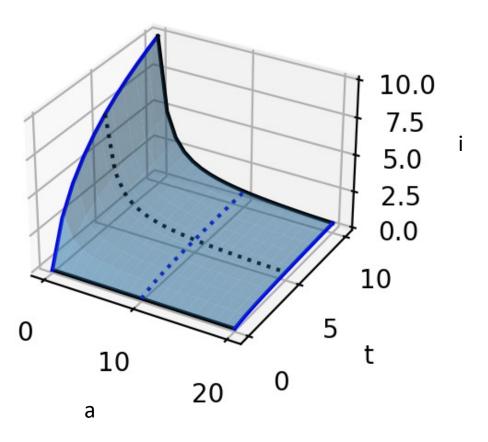






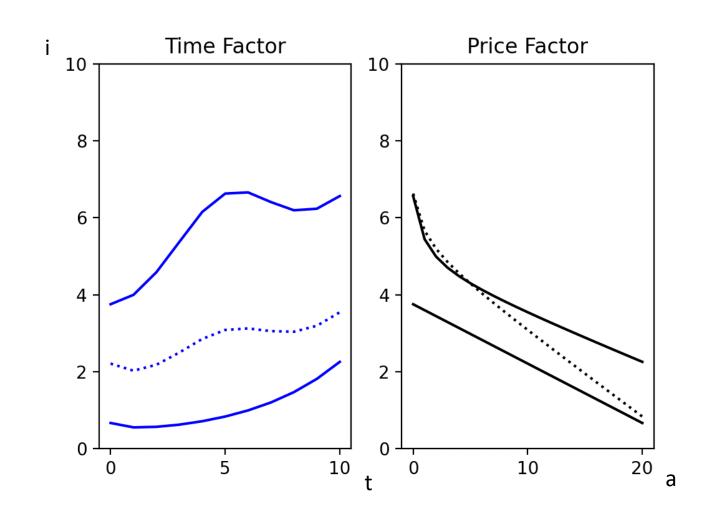
# Party Customer

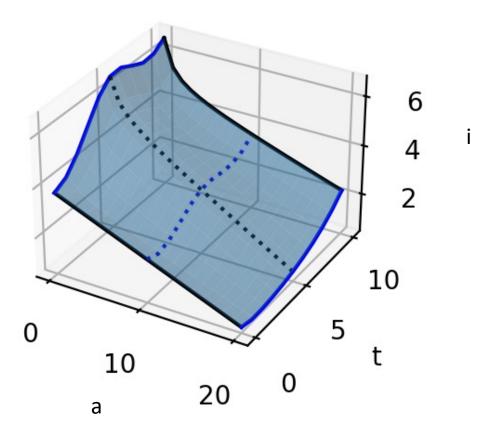






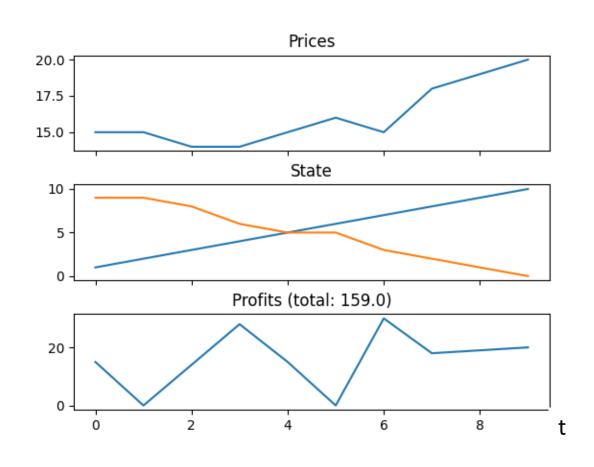


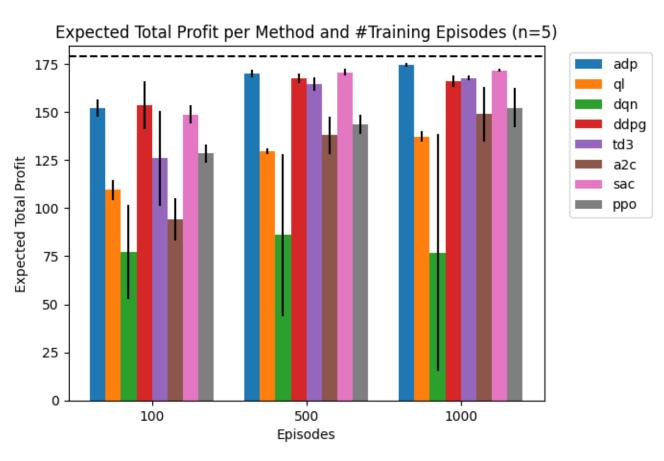


















Airlines cannot tryout prices over 500 Episodes ~ 15 years

DP methods would not require playing random actions





Airlines might have historical data on customer demand



#### Estimating Customer Behavior

$$V_{t}(s) = \max_{a \in A_{t}(s)} \left\{ \sum_{i \in I_{t}} P_{t}(i, a, s) * (r_{t}(i, a, s) + \gamma * V_{t+1}(\Gamma_{t}(i, a, s))) \right\}$$

- Airlines know
  - state transitions, reward function, state, action space
- Airlines do not know
  - Number of arriving customers, probability distribution of customer purchases



Use remaining seats as limit

**Regression Analysis** 

### Regression Analysis



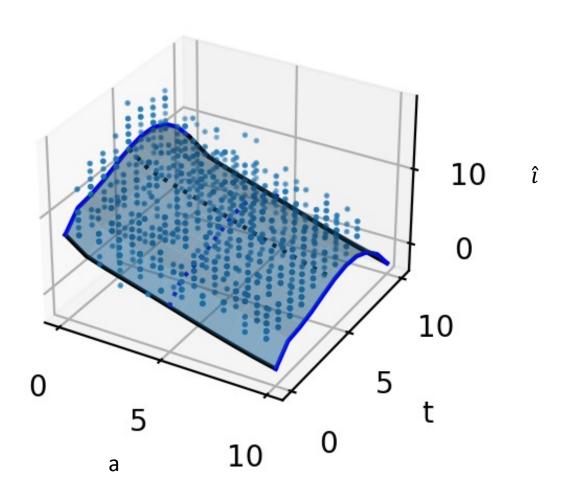
- Example dataset: play random actions over n episodes, store i, a, s
- Estimate expected sales  $(\hat{i})$  with OLS, Ridge or LASSO Regression

#### **Best Performing:**

- OLS & Ridge with at least 1000 datapoints
- Features:  $a, t, a^2, t^2, \sqrt{a+1}, \sqrt{t+1}, \log(a+1), \log(t+1), a * t$
- $\rightarrow$  Without stochastic customers:  $R^2 \sim 0.837$  and  $\emptyset nMSE = 0.021$
- $\rightarrow$  Stochastic customers:  $R^2 \sim 0.35$  and  $\emptyset nMSE = 0.21$





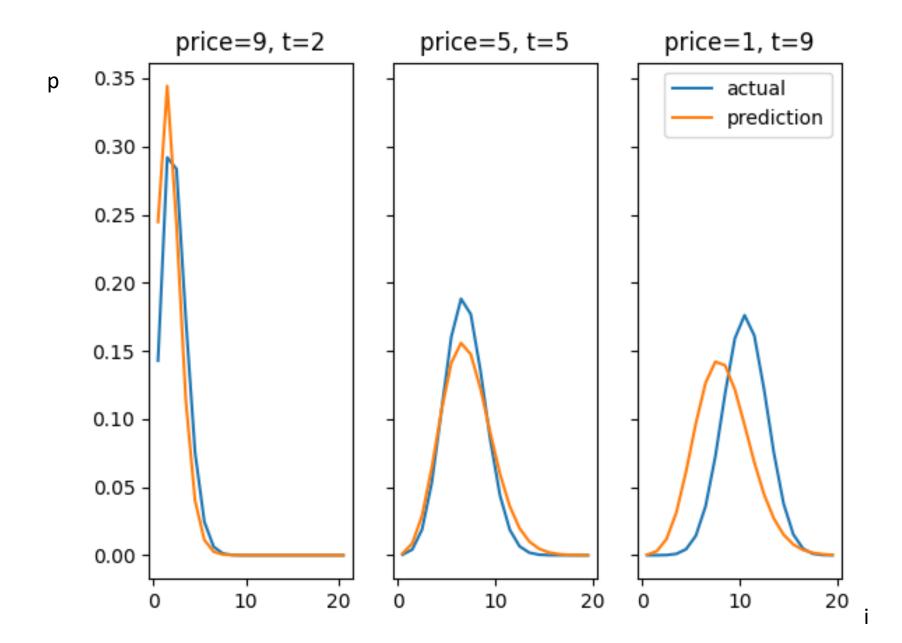


Discrete probability distribution required:

Poisson distribution with our estimation as parameter:

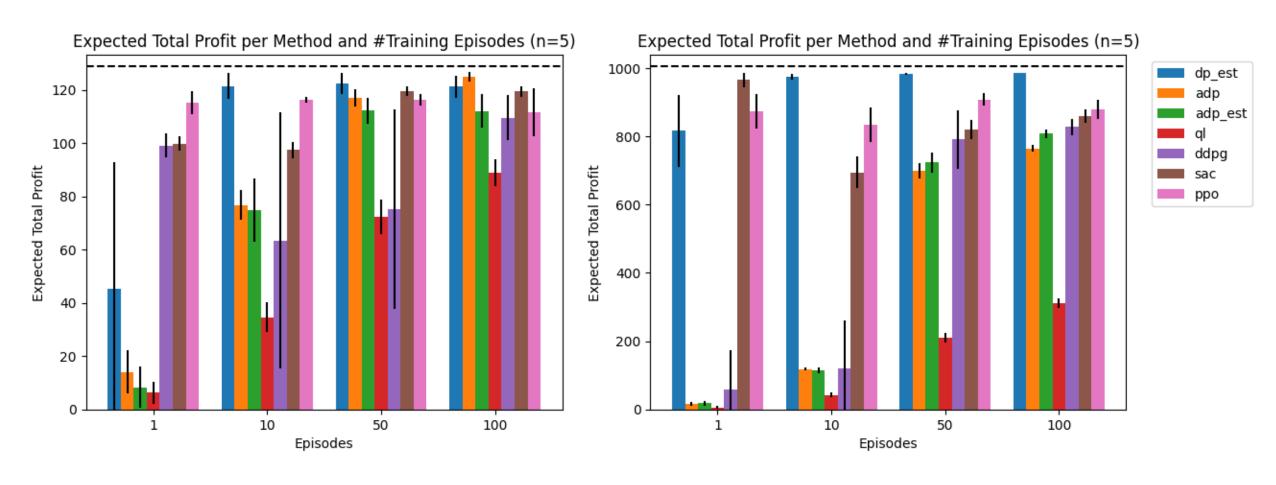
$$P_{\lambda}(k) = \frac{\lambda^k}{k!} * e^{-\lambda} \text{ with } \lambda = \hat{\imath}$$







## DP and ADP using Estimation



Simple Environment

**Complex Environment** 

### Duopoly - Setup



**State Space**: Discrete  $(0, 5, 5, a_{agent}^{t_0}, a_{comp}^{t_0})$ 

 $\rightarrow$  (10,  $s_{agent}^{t_{10}}$ ,  $s_{comp}^{t_{10}}$ ,  $a_{agent}^{t_{10}}$ ,  $a_{comp}^{t_{110}}$ )

**Action Space**: Discrete (0, 1, 2, 3, ..., 100)

Event:  $i_{agent}, i_{comp}$ 

Reward:  $i_{player} \cdot a_{player}$ 

**Demand:**  $\frac{a_{comp}}{a_{agent} + a_{comp}}$ 

 $softmax(D_{agent}, D_{comp}, D_{no\_sell}) \rightarrow$ 

 $P_{agent}, P_{comp}, P_{no\_sell} \rightarrow$ 

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 $i_{agent}, i_{comp}$ 

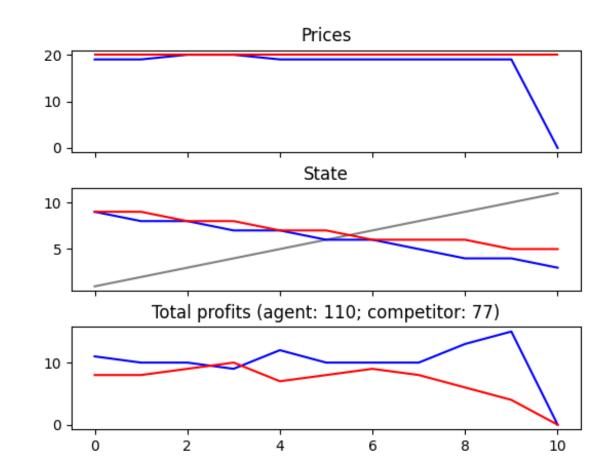


### Duopoly Scenario – Full Information

# agent has **full knowledge** about:

- exact customer behavior
- competitor strategy
  - undercut agents price by 1 (but not go below 20)

 $\rightarrow$ min(20,  $a_{agent\_last} - 1$ )



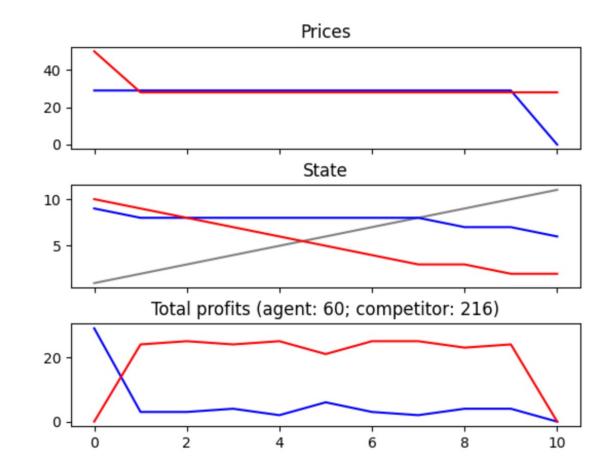


### Duopoly Scenario – Limited Information

#### agent has **knowledge** about:

- own inventory
- own price and competitors price
  - assumes here a time indepedent fix price strategy

 $\rightarrow$  30  $\forall t$ 







- setting duopoly fully up:
  - support all DP and RL methods we used so far
  - provide comprehensive monitoring like in monopoly
  - enable estimation of
    - consumer behavior
    - competitor strategy



