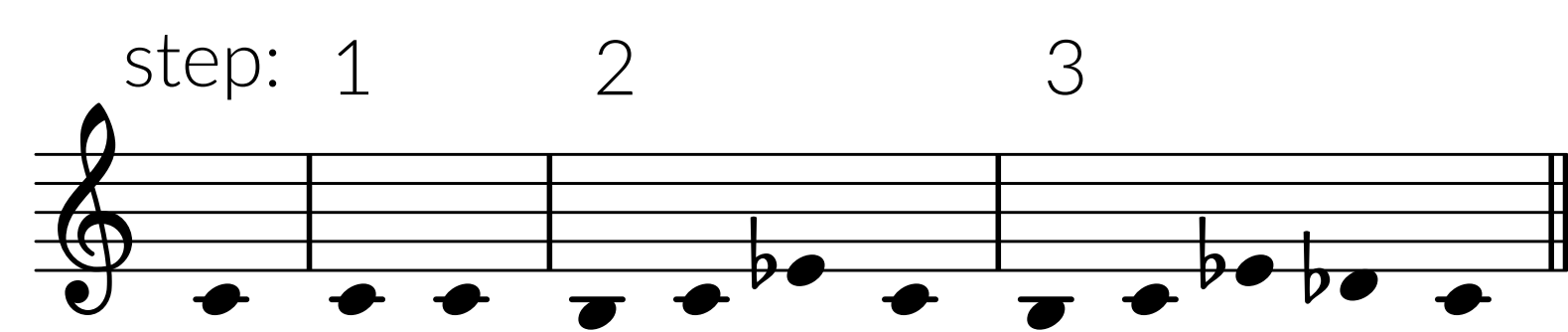


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## Basic Principles

Melodies of ālāp performances can be understood as the result of **recursive elaboration** of **notes** and **intervals**.

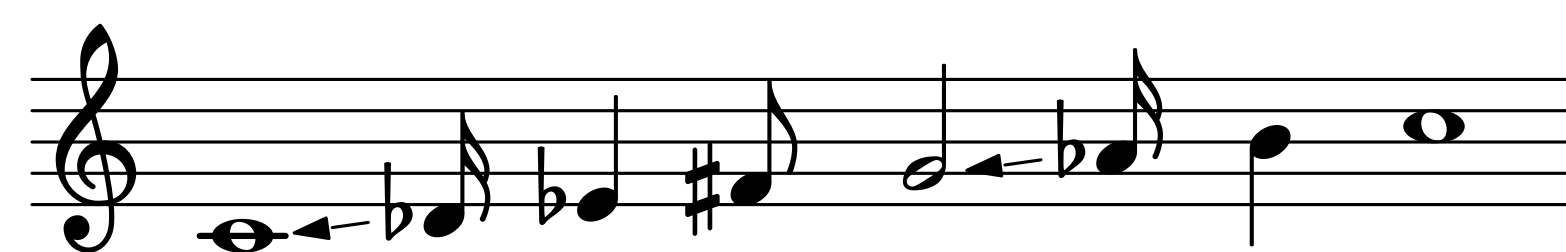


A melody derived in three steps.

Elaborations of notes include **duplication** (1) and left or right **neighbors** (2). Interval elaborations include **passing notes** (3) and **linear fills**.

## Modes and Tonal Hierarchy

A mode is a scale together with a **hierarchy of stability** among the scale degrees and restrictions on their **movement direction**.

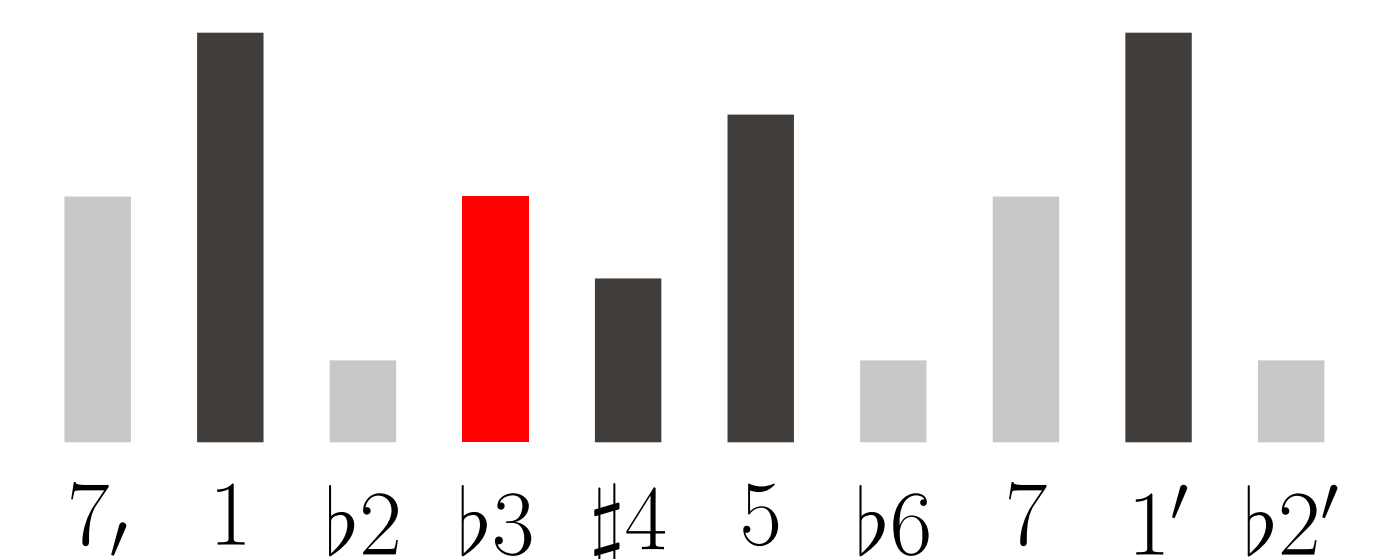


The tonal hierarchy of rāga Multānī.

pitch	1	b2	b3	#4	5	b6	7
direction	↑	↓	↑	↑	↓	↓	↑
stability	4	0	2	1	3	0	2

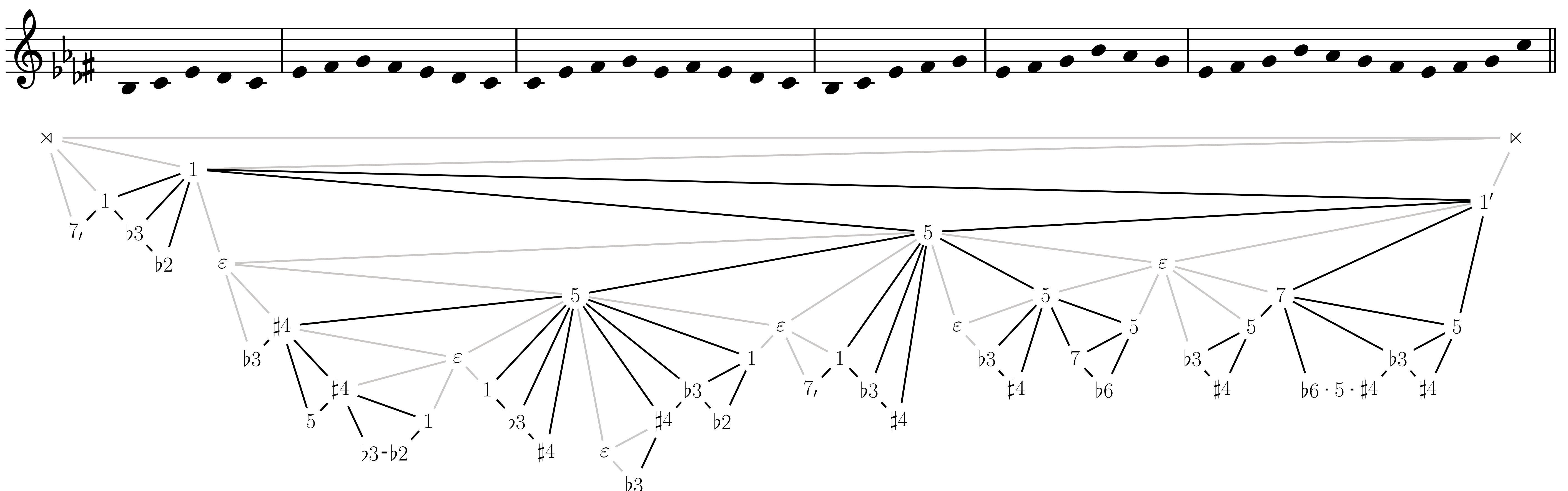
## Distant Neighbors

Neighbors and passing notes can be **non-adjacent** if they respect the tonal hierarchy of the mode.



The generalized neighbors of b3 (dark).

A pitch  $p_n$  is a **generalized neighbor** of some pitch  $p$  if no pitch between  $p$  and  $p_n$  is more stable than  $p$  and  $p_n$ .



Selected phrases, in order of performance, from the ascending part of an ālāp in rāga Multānī, recorded by the sitarist Dharambir Singh.

## Formal Representation

Since both notes and intervals are elaborated, melodies are represented as graphs with **notes as nodes** and **intervals as edges**. The elaboration operations then form an edge-replacement **graph grammar**.

Interval elaborations replace an edge with a new subgraph:

$$(n_1 \rightarrow n_2) \longrightarrow (n_1 \rightarrow n' \rightarrow n_2)$$

Note elaboration also replace edges but consider only one of the nodes:

$$(n_1 \rightarrow *) \longrightarrow (n_1 \rightarrow n' \rightarrow *)$$

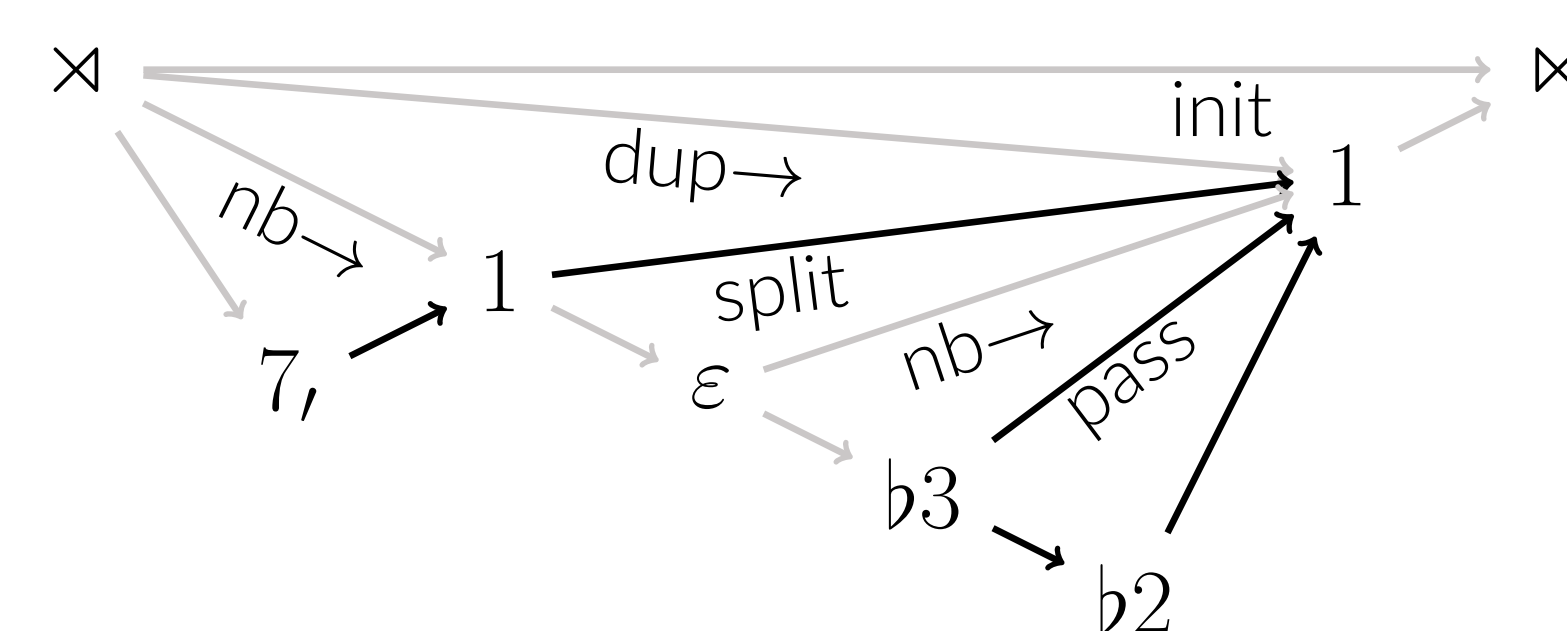
Independent parts of the graph can be separated explicitly by introducing an **empty node**:

$$(n_1 \rightarrow n_2) \longrightarrow (n_1 \rightarrow \varepsilon \rightarrow n_2)$$

Another advantage of graphs is that they can capture more complex structures such as **polyphonic networks**.

## Graphical Notation

Since the graph of a monophonic melody is linear, its derivation can be visualized by an **outerplanar graph** [1], where each polygon represents an edge replacement.



Indepencene is indicated by  $\varepsilon$ -nodes. Adjacent edges are shown in light gray, revealing the differences between note and interval elaborations.

## References

- [1] J. Yust. "The Geometry of Melodic, Harmonic, and Metrical Hierarchy". In: *Mathematics and Computation in Music*. Ed. by E. Chew, A. Childs, and C.-H. Chuan. Communications in Computer and Information Science. 2009, pp. 180–192.

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