Cultural Evolution and Music

Models and Applications in Python

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Table of contents

Welcome **5**

1. Introduction **7**
1.1. The Cultural Evolution of Music . . . 7
1.2. Music in human evolution . . . 7
1.3. History and cultural evolution . . . 8
1.4. Models . . . 8
1.5. Basic cultural inheritance mechanisms . . . 9

2. A Python primer **11**
2.1. Variables and types . . . 11
2.2. On repeat . . . 12
2.3. Functions . . . 13
2.4. Libraries you'll love . . . 15

I. Cultural inheritence mechanisms **17**

3. Unbiased transmission **19**
3.1. Simulating a population . . . 20
3.2. Tracing cultural change . . . 21
3.3. Iterating over generations . . . 23

4. Unbiased and biased mutation **31**

5. Biased transmission: direct bias **37**

6. Biased transmission: frequency-dependent indirect bias **41**

7. Biased transmission: demonstrator-based indirect bias **47**

8. Vertical and horizontal transmission **53**
8.1. Horizontal cultural transmission . . . 56

9. The multiple traits model **77**
9.1. Introducing innovation 79

II. Advanced topics **81**

10. Advanced topics **83**
10.1. Are more complex songs more popular? 83
10.2. Evolution of pitches and intervals 83
10.3. Stylistic change 83
10.4. Cultural evolution of electronic music 83

11. Conclusion **85**
11.1. What can cultural evolution tell us about music 85
11.2. What is the role of models for musicology 85
11.3. Avenues for future research 85

References **87**

Welcome

On these pages you will learn about cultural evolution and music. The overall aim is to attain a basic understanding of formal models in cultural evolution and learn about several recent approaches that apply them to the domain of music.

We start with a minimal introduction to the Python programming language that covers the necessary basic skills in order to follow the remainder of the book. Then, we summarize some general ideas about music and cultural evolution.

Subsequently, we follow the excellent learning path for computational models in cultural evolution provided by the book *Individual-based models of cultural evolution: A step-by-step guide using R* (Acerbi et al., 2022). These pages comprise a translation of this resource to Python. Finally, we will review and discuss a number of recent publications on music and cultural evolution in the advanced topics section at the end.

> If you want to refer to this resource, please cite it as appropriately, e.g.: Moss, F. C. (2022). "Cultural Evolution and Music: Models and Applications in Python". https://fabianmoss.github.io/cultevopy

> **!** Important
>
> The material on this page has been adapted and designed for a research seminar in musicology at Würzburg University, Germany.
> Note that the materials here are still under development. Please inform me if you notice any errors or other issues.

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1. Introduction

1.1. The Cultural Evolution of Music

The field of cultural evolution emerged in the 1980's (e.g., Boyd & Richerson, 1985; Cavalli-Sforza & Feldman, 1981), and has, in parallel with the advancement of computational facilities, gained momentum. Theories on cultural evolution share many facets with approaches on memetics (Aunger, 2001; Blackmore, 2000; Dawkins, 1976), a field that has also been applied to the case of music (Jan, 2016).

In recent years, several approaches have attempted to apply formal models from cultural evolution to the domain of music.

In the present context, we first introduce some central ideas of cultural evolution and review a few major publications for the domain of music.

A few selected important contributions are:

- "Cultural Transmission and Evolution: A Quantitative Approach" (Cavalli-Sforza & Feldman, 1981)
- "Culture and the Evolutionary Process" (Boyd & Richerson, 1985)
- "The Memetics of Music" (Jan, 2016)
- "Cultural Evolution and Music" (Youngblood et al., forthcoming)
- "Cultural Evolution of Music" (Savage, 2019)

1.2. Music in human evolution

This book is about the cultural evolution of music. It has to be mentioned, however, that there is a large body of research on the biological evolution of music. This research asks questions about with which evolutionary advantages music endowed early humans, whether it is something that we share with other animals or whether it makes us unique. Some hold the view that music has no particularly relevant evolutionary function at all (Pinker, 1997), while others see in it a key to our success as a species (Cross, 2016).

Whatever the true role of music in the evolutionary history of humanity may have been, it is most certainly a fascinating topic to reflect upon. After all, human musical activity with dedicated instruments can be traced back at least 20,000 years, although it seems more than likely that human 'musicking' dates back much further, since our own bodies and voices already provided us with excellent musical instruments long before the first instruments have been devised.

If you are interested in learning more about biological-evolutionary aspects of music, I highly recommend to read, e.g. Wallin et al. (2001), Morley (2013), Tomlinson (2018), and Honing (2018).

1.3. History and cultural evolution

Documenting, describing, and interpreting changes in human culture is what historians do. Accordingly, changes in music belong to the field of music history, or historical musicology. However, most historians would probably be hesitant to employ models, or worse: formal models, in order to describe historical processes. The dictum "history doesn't repeat itself" seems to raise a fundamental argument against such endeavors that aim at explaining cultural or historical changes by means of underlying latent 'forces'. Modeling, in that view, seems to erroneously assume that history is teleological – directed towards a predetermined goal.

On the other hand, it is undeniable that there are many aspects of culture exhibit regularities and 'progresses' that are hard to explain if there are no guiding processes. Defining "culture" is, of course, yet another difficult enterprise. Here, I follow more or less the definition of Boyd & Richerson (1985, p. 33): "Culture is information capable of affecting individuals' phenotypes which they acquire from other conspecifics *by teaching or imitation* [emphasis mine]".[1] The part important to us here is "by teaching or imitation", which is meant to imply: *not* by genetic inheritance. If humans can transmit information by other means than genetic inheritance, and if these transmission processes continue over many generations, then they are worth studying. Rest assured, the assumption of a hidden goal towards which all cultural processes are directed is not needed at all! I hope that you will share this conviction after working through this material.

1.4. Models

Throughout this book we will use on quantitative models to describe evolutionary processes. A model is a simplyfied description that captures what we know about a phenomenon of interest in terms of variables and their relations to one another. While models in this sense are always reductions of the real-world process being modeled, they have the benefit that all of their parts can be understood.

[1] I encourage you to read Chapter 3 of their seminal book in order to fully capture the meaning of this quote.

Moreover, formal models usually incorporate one or several *parameters*, that is, variables the value of which is mutable. Changing parameter values thus renders different instances of a model that, in turn, allow one to ask the question "What if?" and to abserve the behavior of simulated processes in the long run. Answering this question under different conditions (different parameter settings) then will allow us to draw inferences from the modeling outcomes to interpret our phenomenon of interest.

It is precisely the ability of formal models to be adapted at will why they form the core part of this book. Starting with simple models with only very few variables, we will gradually make them more complex by including additional variables. However, it will remain our attempt to keep our models as simple as possible. While it is theoretically possible to include any number of variables in modeling, more complex models become harder to understand, and the interactions between the numerous variables more difficult to interpret.

As mentioned above, the goal of modeling is often to approximate rather than to recreate a process in the world. Sometimes, the goal is rather to show that a real process does *not* conform to the outcomes of certain modeling assumptions, from which can be deduced that other factors—possibly yet unknown ones—must be taken into account. One of the most common baseline comparison models is the so-called 'neutral model (see, e.g., Bentley et al., 2004), where cultural traits are randomly copied from previous generations. We will see in Chapter 3 what the consequences of this assumption are.

Questions concerning model comparison—chosing the 'best' model for a problem at hand—or dealing with the trade-off between model complexity and interpretability are central issues in many areas of empirical science, such as machine learning, computational sociology, or cognitive psychology, and many textbooks dedicate some discussion to the topic (e.g. Bishop, 2012; Farrell & Lewandowsky, 2018; McElreath, 2020). For excellent discussions and examples of modeling in music research, see Honing (2006) and Finkensiep et al. (forthcoming).

1.5. Basic cultural inheritance mechanisms

Following this introduction, we introduce some minimal requirements to use Python for this course (Chapter 2).

Subsequently, we introduce with six central mechanisms for cultural inheritance: unbiased transmission (Chapter 3), unbiased and biased mutation (Chapter 4), directly biased transmission (Chapter 5), frequency-dependent indirectly biased transmission (Chapter 6), and demonstrator-based indirectly biased transmission (Chapter 7). We follow up with a chapter on vertical and horizontal transmission (**?@sec-vertical**), and finally introduce the multiple traits model (Chapter 9). The following diagram gives an overview of these processes:

After having a firm grasp on how these processes can be modeled in Python and how modeling results can be interpreted, we move to more advanced topics (Chapter 10), and more specifically into a number of recent exciting results about cultural evolution and music. We conclude our journey (Chapter 11) with a more general discussion of the implications of cultural evolution for how we think about music, on the relevance of modeling in music research and the humanities more generally, as well as on the great potential of this approach for future research.

2. A Python primer

> **Note**
>
> From now on, we will assume that you have a working Python installation running on your computer. You can check this by typing the following into a terminal/console/command line:
>
> ```
> python --version
> ```
>
> If the version number starts with a 3, you're all set. If not, please consider one of the many tutorials online on how to install Python.

2.1. Variables and types

Variable assignment in Python is straight-forward. You choose a name for the variable, and assign a value to it using the = operator, for example:

```
x = 100
```

assigns the value `100` to the variable `x`. If we call the variable now, we can see that it has the value we assigned to it:

```
x
```

```
100
```

Of course, we can also assign things other than numbers, for example:

```
name = "Fabian"
```

What we assigned to the variable `name` is called a *string*, it has the value `"Fabian"`. Strings are sequences of characters.

> **Tip**
>
> Note that `"Fabian"` is enclosed by double-quotes. Why is this the case? Why could we not just type `name = Fabian`?

We can also assign a list of things to a variable:

```
mylist = [1, 2, 3, "A", "B", "C"]
```

Lists are enclosed by square brackets. As you can see, Python allows you to store any kind of data in lists (here: integer numbers and character strings). However, it is good practice to include only—you'll understand later why.

Another structured data type in python are dictionaries. Dictionaries are collections of key-value pairs. For example, a dictionary `addresses` could store the email addresses of certain people:

```
addresses = {
    "Andrew" : "andrew@example.com",
    "Zoe" : "zoe@example.com"
}
```

Now, if we wanted to look up Zoe's email address, we could to so with:

```
addresses["Zoe"]
```

```
'zoe@example.com'
```

2.2. On repeat

Coding something is only useful if you can't do the job as fast or as efficient by yourself. Especially when it comes to repeating the same little thing many, many times, knowing how to code comes in handy.

As a simple example, imagine you want to write down all numbers from 1 to 10, or from 1 to 100, or… you get the idea. In Python, you would do it as follows:

```
for i in range(10):
    print(i, end=" ")
```

```
0 1 2 3 4 5 6 7 8 9
```

You see that this is not exactly what we wanted. We're seeing numbers from 0 to 9, but we wanted everything from 1 to 10. Before we fix the code to produce the desired result, let's explain the bits and pieces of the code above. What we just did was to use a so-called *for-loop*, probably the most common way to repeat things in Python. First we create an *iterator variable* `i` (we could have named any other variable name as well), which takes its value from the list of numbers specified by `range(10)`. If only one number `n` is provided to `range(n)`, it will enumerate all numbers from `0` to `n-1`. If two arguments are provided, the first one determines the starting number, and the second one stands for the terminating number minus one—confusing, right? So, in order to enumerate all numbers from 1 to 10, we have to write `range(1,11)`. Finally, the `print` function outputs the value of `i` for each iteration. The `end` keyword in the `print` function makes the numbers being printed in one single line, separated by a single white space.[1]

```
for i in range(1,11):
    print(i, end=" ")
```

```
1 2 3 4 5 6 7 8 9 10
```

Voilà!

2.3. Functions

With more and more experience in programming, it is likely that your code will become more and more complex. That means that it will become harder to keep track of what every piece of it is supposed to do. A good strategy to deal with this is to aim for writing code that is *modular*: it can be broken down into smaller units (modules) that are easier to understand. Moreover, it is sometimes necessary to reuse the same code several times. It would, however, be inefficient to write the same lines over and over again. With your code being modular you can wrap the pieces that you need in several places into a *function*.

[1] The default value is `end="\n"`, so that each iteration gets printed onto a new line.

Let's look at an example! Assume, your (fairly) complex code involves calculating the sum of the products of two numbers. In Python, we use the + operator to calculate sums and the `**` operator to raise a number to a certain power (**2 for the square of a number).

```
x = 3
y = 5

sum_of_squares = x**2 + y**2
```

The variable `sum_of_squares` now contains the sum of squares of `x=3` and `y=5`. We can inspect the result by calling the variable:

```
sum_of_squares
```

```
34
```

Now, imagine that you would have to do the same calculation several times for different combinations of values for x and y (and always keeping in mind that this stands in for much more complex examples with several lines of code). We can this code in a function:

```
def func_sum_of_squares(x, y):
    return x**2 + y**2
```

Now, each time we want to calculate a sum of squares, we can do so by simply invoking

```
func_sum_of_squares(5,4)
```

```
41
```

And, of course, we could chose a shorter name for the function as well:

```
f = func_sum_of_squares

f(5,4)
```

```
41
```

2.4. Libraries you'll love

Luckily, you don't have to programm all functions by yourself. There is a huge community of Python programmers out there that works and collaborates on several *libraries*. A library is (more or less) simply a collection of certain functions (and some more, but we don't get into this here). This means, instead of writing a function yourself, you can rely on functions that someone else has programmed.

> **Danger**
>
> Whether a Python library or function does actually do what it promises is another story. Popular libraries with tens of thousands of users are very trust-worthy because you can be almost sure that someone would have noticed erroneous behavior. But it is certainly possible that badly-maintained libraries contain errors. So be prudent when using the code of others.

One of the most popular Python libaries is NumPy for numerical computations. We will rely a lot on the functions in this library, especially in order to draw random samples—more on this later! To use the functions or variables from this library, they have to be *imported* so that you can use them. There are several ways to do this. For example, you can import the libary entirely:

```
import numpy
```

Now, you can use the (approximated) value of π stored in this library by typing

```
numpy.pi
```

```
3.141592653589793
```

A different way is to just import everything from the library by writing

```
from numpy import *
```

Here, the `*` stands for 'everything'. Now, to use the value of π we could simply type

```
pi
```

```
3.141592653589793
```

This is, however discouraged for the following reason: imagine we had another library, `numpy2` that also stores the value of π, but less precisely (e.g. only as `3.14`). If we write

```
from numpy import *
from numpy2 import *
```

We would have imported the variables holding the value of π from both libraries. But, because they have the same name `pi`. In this case, `pi` would equal `3.14` because we imported `numpy2` last. This is confusing and shouldn't be so! To avoid this, it is better to keep references to imported libraries explicit. In order not to have to type too much (we're all lazy, after all), we can define an alias for the library.

```
import numpy as np
np.pi
```

```
3.141592653589793
```

All functions of NumPy are now accessible with the prefix `np.`. You can choose any alias when importing a library (it can even by longer than the library name) but certain conventions have emerged that you're encouraged to follow. Importing the most commonly used Python libraries for data-science tasks ("The data science triad"), use the following:

```
import numpy as np # for numerical computations
import pandas as pd # for tabular data
import matplotlib.pyplot as plt # for data visualization
```

We will use all three of them in the following chapters and you'll learn to love them.

Part I.

Cultural inheritence mechanisms

3. Unbiased transmission

What happens if people just randomly copy?

> **Note**
>
> This chapter is based on "Chapter 1: Unbiased transmission" in Acerbi et al. (2022).

In this chapter, we introduce the most basic model for cultural inheritance: unbiased transmission. This process quite literally corresponds to randomly copying traits from previous generations, without any further distinctions and constraints. While this is obviously a too reductive model for how cultural transmission works, it is ideally suited to get us started with the enterprise of modeling evolutionary processes involving random variation.

First we import some modules.

```
import numpy as np
import pandas as pd
```

Because we will model evolutionary processes that are not strictly deterministic, we need to simulate variations due to random change. For this, we can use the *default random number generator* from the NumPy library and store it in the variable `rng`.

```
rng = np.random.default_rng()
```

Next, we define some basic variables that we take into account for our first model. We consider a population of $N = 100$ individuals as well as a time-frame of $t_{max} = 100$ generations.

3.1. Simulating a population

```
N = 100
t_max = 100
```

> **Note**
>
> In general, we use the variable `t` to designate generation counts.

Now we create a variable `population` that will store the data about our simulated population. This population has either of two traits `"A"` and `"B"`, with a certain probability. We store all of this in a so-called 'data frame', which is a somewhat fancy, Pandas-specific term for a table.

```
population = pd.DataFrame(
    {"trait": rng.choice(["A", "B"], size=N, replace=True)}
)
```

Let's take this code apart to understand it better. From the Pandas library, which we imported as the alias pd, we create a `DataFrame` object. The data contained in this the data frame `population` is specified via a dictionary that has `"trait"` as its key and a fairly complex expression starting with the random number generator `rng` as its value. What this value says is, from the list `["A", "B"]` choose randomly N instances with replacement (if `replace` were set to `False`, we could at most sample 2 values from the list). So, the data frame `population` should contain 100 randomly sampled values of A's and B's. Let's confirm this:

```
population.head()
```

	trait
0	B
1	A
2	A
3	B
4	A

As you can see, `population` stores a table (many of the 100 rows are omitted here for display reasons) and a single column called 'trait'. The `.head()` method appended to the `population` data frame shows restricts the output to only the first 5 rows (0 through 4). Each row in the 'trait' column

contains either the value A or B. To the left of the data frame you can see the numbers of rows explicitly spelled out. This is called the data frame's *index*.

> **Note**
>
> A and B are just placeholder names for any of two mutually exclusive cultural traits. These could be, for example, preference for red over white whine (ignoring people who like rosé as well as people who have no preference). You see already here that this is a massive oversimplification of actual taste preferences. The point here is not to construct a plausible model but rather to gradually build up a simple one in order to understand well its inner workings.
> It will help to pause for a moment and to think of other examples that "A" and "B" could stand for. Can you come up with a music-related one?

We can count the number of A's and B's as follows:

```
population["trait"].value_counts()
```

	trait
A	53
B	47

You can read the above code as "From the population table, select the 'trait' colum and count its values.". Since there were only two values to sample from and they were randomly (uniformly) sampled, the number of A's and the number of B's should be approximately equal. We can obtain their relative frequencies by adding setting the `normalize` keyword to `True`:

```
population["trait"].value_counts(normalize=True)
```

	trait
A	0.53
B	0.47

3.2. Tracing cultural change

We now create a second data frame `output` in which we will store the output of our model. This data frame has two columns: `generation`, which is the number of the simulated generation, and p which stands for "the probability of an individual of having trait A".

```
output = pd.DataFrame(
    {
        "generation": np.arange(t_max, dtype=int),
        "p": [np.nan] * t_max
    }
)
```

The `generation` column contains all numbers from `0` to `t_max - 1`. Because we count the numbers of generations (rather than assuming a time-continuous process), we specified that numbers in this column have to be intergers (`dtype=int`). The values for the p column must look cryptic. It literally says: put the `np.nan` value `t_max` times into the p colum. `np.nan` stands for "not a number" (from the NumPy library), since we haven't assigned any values to this probability yet.

```
output.head()
```

	generation	p
0	0	NaN
1	1	NaN
2	2	NaN
3	3	NaN
4	4	NaN

Don't worry that both the index and the 'generation' column contain all numbers from 0 to 99. We need this later when things become more involved.

As the saying goes, from nothing comes nothing, so we have to start somewhere, meaning that we need to assume that the initial probability of having trait A in our population is an actual number. The most sensible thing is to start with the proportions of A and B in our sampled population as a starting value.

So, we approximate the probability of an individual having trait A with the relative frequency of trait A in the population:

```
population["trait"].value_counts(normalize=True)["A"]
```

```
0.53
```

You already know this code from above, we just added the `["A"]` part at the end to select only the relative frequencies of trait A. We want to set this as the value of p of the first generation. This can be achieved with the `.loc` (location) method:

```
output.loc[0, "p"] =
↪  population["trait"].value_counts(normalize=True)["A"]
```

In words, this reads: "Set location 0 (first row) in the p column of the `output` data frame to the relative frequency of the trait 'A' in the population."

3.3. Iterating over generations

Recall that we are trying to observe cultural change over the course of `t_max = 100` generations. We thus simply repeat what we just did for the first generation: based on the relative frequencies of A's and B's in the previous generation, we sample the traits of 100 new individuals for the next generation.

```
for t in range(1, t_max):
    # Copy the population data frame to `previous_population`
    previous_population = population.copy()

    # Randomly copy from previous generation's individuals
    new_population = previous_population["trait"].sample(N,
↪   replace=True).to_frame()

    # Get p and put it into the output slot for this generation t
    output.loc[t, "p"] = new_population[ new_population["trait"] ==
↪   "A"].shape[0] / N
```

This procedure assignes a probability of having trait "A" for each generation (each row of the p colum is filled now):

```
output.head()
```

	generation	p
0	0	0.53
1	1	0.46
2	2	0.50
3	3	0.52
4	4	0.63

To make things easier, we wrap the above code in a function that we'll call `unbiased_transmission` that can take different values for the population size N and number of generations `t_max` as parameters. The code below is exactly the same as above.

```
def unbiased_transmission_1(N, t_max):
    population = pd.DataFrame({"trait": rng.choice(["A", "B"], size=N,
 ↪  replace=True)})

    output = pd.DataFrame({"generation": np.arange(t_max, dtype=int),
 ↪  "p": [np.nan] * t_max })

    output.loc[0, "p"] = population[ population["trait"] ==
 ↪  "A"].shape[0] / N

    for t in range(1, t_max):
        # Copy the population tibble to previous_population tibble
        previous_population = population.copy()

        # Randomly copy from previous generation's individuals
        new_population = previous_population["trait"].sample(N,
 ↪  replace=True).to_frame()

        # Get p and put it into the output slot for this generation t
        output.loc[t, "p"] = new_population[ new_population["trait"] ==
 ↪  "A"].shape[0] / N

    return output
```

```
data_model = unbiased_transmission_1(N=100, t_max=200)
```

```
def plot_single_run(data_model):
    data_model["p"].plot(ylim=(0,1))
```

```
plot_single_run(data_model)
```

Figure 3.1.: Single run of the unbiased transmission model for a population of $N = 100$ individuals and $t_{max} = 200$ generations.

```
data_model = unbiased_transmission_1(N=10_000, t_max=200)
```

```
plot_single_run(data_model)
```

Figure 3.2.: Single run of the unbiased transmission model for a population of $N = 10,000$ individuals and $t_{max} = 200$ generations.

```
def unbiased_transmission_2(N, t_max, r_max):
    output = pd.DataFrame({
        "generation" : np.tile(np.arange(t_max), r_max),
        "p" : [ np.nan ] * t_max * r_max,
        "run" : np.repeat(np.arange(r_max), t_max)
    })

    for r in range(r_max):
        # Create first generation
        population = pd.DataFrame({"trait": rng.choice(["A", "B"],
↪   size=N, replace=True)})

        # Add first generation's p for run r
        output.loc[ r * t_max, "p"] = population[ population["trait"] ==
↪   "A" ].shape[0] / N

        # For each generation
        for t in range(1,t_max):
            # Copy individuals to previous_population DataFrame
```

```
            previous_population = population.copy()

            # Randomly compy from previous generation
            population = population["trait"].sample(N,
↪   replace=True).to_frame()

            # Get p and put it into output slot for this generation t
             ↪  and run r
            output.loc[r * t_max + t, "p"] = population[
↪   population["trait"] == "A" ].shape[0] / N

    return output
```

```
unbiased_transmission_2(100, 100, 3).head()
```

	generation	p	run
0	0	0.54	0
1	1	0.57	0
2	2	0.55	0
3	3	0.59	0
4	4	0.67	0

Tip

Why could we append `.head()` to the `unbiased_transmission_2` function?

```
data_model = unbiased_transmission_2(N=100, t_max=200, r_max=5)
```

```
def plot_multiple_runs(data_model):
    groups = data_model.groupby("run")
    for _, g in groups:
        g.index = g["generation"]
        g["p"].plot(lw=.5, ylim=(0,1))

    data_model.groupby("generation")["p"].mean().plot(c="k", lw="1")
```

```
plot_multiple_runs(data_model)
```

Figure 3.3.: Multiple runs of the unbiased transmission model for a population of $N = 100$ individuals, with average (black line).

```
data_model = unbiased_transmission_2(N=10_000, t_max=200, r_max=5)
```

```
plot_multiple_runs(data_model)
```

Figure 3.4.: Multiple runs of the unbiased transmission model for a population of $N = 10,000$ individuals, with average (black line).

```
def unbiased_transmission_3(N, p_0, t_max, r_max):
    output = pd.DataFrame({
        "generation" : np.tile(np.arange(t_max), r_max),
        "p" : [ np.nan ] * t_max * r_max,
        "run" : np.repeat(np.arange(r_max), t_max)
    })

    for r in range(r_max):
        # Create first generation
        population = pd.DataFrame({"trait": rng.choice(["A", "B"],
↪   size=N, replace=True, p=[p_0, 1 - p_0])})

        # Add first generation's p for run r
        output.loc[ r * t_max, "p"] = population[ population["trait"] ==
↪   "A" ].shape[0] / N

        # For each generation
        for t in range(1,t_max):
            # Copy individuals to previous_population DataFrame
```

```
            previous_population = population

            # Randomly compy from previous generation
            population = population["trait"].sample(N,
↪   replace=True).to_frame()

            # Get p and put it into output slot for this generation t
             ↪  and run r
            output.loc[r * t_max + t, "p"] = population[
↪   population["trait"] == "A" ].shape[0] / N

    return output
```

```
data_model = unbiased_transmission_3(10_000, p_0=.2, t_max=200,
↪   r_max=5)
plot_multiple_runs(data_model)
```

4. Unbiased and biased mutation

> **Note**
>
> This chapter is based on “Chapter 2: Unbiased and biased mutation” in Acerbi et al. (2022).

```
import numpy as np
rng = np.random.default_rng()

import pandas as pd
```

```
def unbiased_mutation(N, mu, p_0, t_max, r_max):
    # Create an output DataFrame
    output = pd.DataFrame({
        "generation" : np.tile(np.arange(t_max), r_max),
        "p" : [ np.nan ] * t_max * r_max,
        "run" : np.repeat(np.arange(r_max), t_max)
    })

    for r in range(r_max):
        # Create first generation
        population = pd.DataFrame({"trait": rng.choice(["A", "B"],
↪   size=N, replace=True, p=[p_0, 1 - p_0])})

        # Add first generation's p for run r
        output.loc[ r * t_max, "p"] = population[ population["trait"] ==
↪   "A" ].shape[0] / N

        # For each generation
        for t in range(1,t_max):
            # Copy individuals to previous_population DataFrame
            previous_population = population.copy()

            # Determine "mutant" individuals
```

```
            mutate = rng.choice([True, False], size=N, p=[mu, 1-mu],
↪   replace=True)

            # TODO: Something is off here! Changing the order of the
             ↪  conditions affects
            # the result. Should be constant with random noise but
             ↪  converges to either A or B

            # If there are "mutants" from A to B
            conditionA = mutate & (previous_population["trait"] == "A")
            if conditionA.sum() > 0:
                population.loc[conditionA, "trait"] = "B"

            # If there are "mutants" from B to A
            conditionB = mutate & (previous_population["trait"] == "B")
            if conditionB.sum() > 0:
                population.loc[conditionB, "trait"] = "A"

            # Get p and put it into output slot for this generation t
             ↪  and run r
            output.loc[r * t_max + t, "p"] = population[
↪   population["trait"] == "A" ].shape[0] / N

    return output
```

```
def plot_multiple_runs(data_model):
    groups = data_model.groupby("run")
    for _, g in groups:
        g.index = g["generation"]
        g["p"].plot(lw=.5, ylim=(0,1))

    data_model.groupby("generation")["p"].mean().plot(c="k", lw="1")
```

```
data_model = unbiased_mutation(N=100, mu=.05, p_0=0.5, t_max=200,
↪   r_max=5)
plot_multiple_runs(data_model)
```

```
data_model = unbiased_mutation(N=100, mu=.05, p_0=0.1, t_max=200,
 ↪  r_max=5)
plot_multiple_runs(data_model)
```

```
def biased_mutation(N, mu_b, p_0, t_max, r_max):
    # Create the output DataFrame
    output = pd.DataFrame({
        "generation" : np.tile(np.arange(t_max), r_max),
        "p" : [ np.nan ] * t_max * r_max,
        "run" : np.repeat(np.arange(r_max), t_max)
    })

    for r in range(r_max):
        # Create first generation
        population = pd.DataFrame({"trait": rng.choice(["A", "B"],
↪   size=N, replace=True, p=[p_0, 1 - p_0])})

        # Add first generation's p for run r
       output.loc[ r * t_max, "p"] = population[ population["trait"] ==
↪   "A" ].shape[0] / N

        # For each generation
        for t in range(1,t_max):
            # Copy individuals to previous_population DataFrame
            previous_population = population.copy()

            # Determine "mutant" individuals
           mutate = rng.choice([True, False], size=N, p=[mu_b, 1-mu_b],
↪   replace=True)

            # TODO: Something is off here! Changing the order of the
             ↪  conditions affects
            # the result. Should be constant with random noise but
             ↪  converges to either A or B

            # If there are "mutants" from B to A
            conditionB = mutate & (previous_population["trait"] == "B")
            if conditionB.sum() > 0:
                population.loc[conditionB, "trait"] = "A"

            # Get p and put it into output slot for this generation t
             ↪  and run r
            output.loc[r * t_max + t, "p"] = population[
↪   population["trait"] == "A" ].shape[0] / N
```

```
    return output
```

```
data_model = biased_mutation(N = 100, mu_b = 0.05, p_0 = 0, t_max =
 ↪  200, r_max = 5)
plot_multiple_runs(data_model)
```

```
data_model = biased_mutation(N = 10000, mu_b = 0.05, p_0 = 0, t_max =
 ↪  200, r_max = 5)
plot_multiple_runs(data_model)
```

```
data_model <- biased_mutation(N = 10000, mu_b = 0.1, p_0 = 0, t_max =
 ↪  200, r_max = 5)
plot_multiple_runs(data_model)
```

5. Biased transmission: direct bias

> **Note**
>
> This chapter is based on “Chapter 3: Biased transmission: direct bias” in Acerbi et al. (2022).

```
import numpy as np
rng = np.random.default_rng()
import pandas as pd
```

```
def plot_multiple_runs(data_model):
    groups = data_model.groupby("run")
    for _, g in groups:
        g.index = g["generation"]
        g["p"].plot(lw=.5, ylim=(0,1))

    data_model.groupby("generation")["p"].mean().plot(c="k", lw="1")
```

```
def biased_transmission_direct(N, s_a, s_b, p_0, t_max, r_max):
    # Create the output DataFrame
    output = pd.DataFrame({
        "generation" : np.tile(np.arange(t_max), r_max),
        "p" : [ np.nan ] * t_max * r_max,
        "run" : np.repeat(np.arange(r_max), t_max)
    })

    for r in range(r_max):
        # Create first generation
        population = pd.DataFrame({"trait": rng.choice(["A", "B"],
↪   size=N, replace=True, p=[p_0, 1 - p_0])})

        # Add first generation's p for run r
        output.loc[ r * t_max, "p"] = population[ population["trait"] ==
↪   "A" ].shape[0] / N
```

```
        # For each generation
        for t in range(1,t_max):
            # Copy individuals to previous_population DataFrame
            previous_population = population.copy()

            # For each individual, pick a random individual from the
             ↪  previous generation
            demonstrator_trait = previous_population["trait"].sample(N,
↪   replace=True).reset_index()

            # Biased probabilities to copy
            copy_a = rng.choice([True, False], size=N, replace=True,
↪   p=[s_a, 1 - s_a])
            copy_b = rng.choice([True, False], size=N, replace=True,
↪   p=[s_b, 1 - s_b])

            # If the demonstrator has trait A and the individual wants
             ↪  to copy A, then copy A
            condition = copy_a & (demonstrator_trait["trait"] == "A")
            if condition.sum() > 0:
                population.loc[condition, "trait"] = "A"

            # If the demonstrator has trait B and the individual wants
             ↪  to copy B, then copy B
            condition = copy_b & (demonstrator_trait["trait"] == "B")
            if condition.sum() > 0:
                population.loc[condition, "trait"] = "B"

            # Get p and put it into output slot for this generation t
             ↪  and run r
            output.loc[r * t_max + t, "p"] = population[
↪   population["trait"] == "A" ].shape[0] / N

    return output
```

```
data_model = biased_transmission_direct(N=10_000, s_a=.1, s_b=0,
                                        p_0=.01, t_max=200, r_max=5)
plot_multiple_runs(data_model)
```

```
data_model = biased_transmission_direct(N=10_000, s_a=.6, s_b=.5,
                                        p_0=.01, t_max=150, r_max=5)
plot_multiple_runs(data_model)
```

```
data_model = biased_transmission_direct(N=10_000, s_a=.2, s_b=0,
                                        p_0=.01, t_max=200, r_max=5)
plot_multiple_runs(data_model)
```

6. Biased transmission: frequency-dependent indirect bias

> **Note**
>
> This chapter is based on “Chapter 4: Biased transmission: frequency-dependent indirect bias” in Acerbi et al. (2022).

```
import numpy as np
rng = np.random.default_rng()

import pandas as pd
```

```
def plot_multiple_runs(data_model):
    groups = data_model.groupby("run")
    for _, g in groups:
        g.index = g["generation"]
        g["p"].plot(lw=.5, ylim=(0,1))

    data_model.groupby("generation")["p"].mean().plot(c="k", lw="1")
```

```
N = 100
p_0 = .5
D = 1.
```

```
# Create first generation
population = pd.DataFrame({"trait": rng.choice(["A", "B"], size=N,
 ↪  replace=True, p=[p_0, 1-p_0])})
```

```
# Create a DataFrame with a set of 3 randomly-picked demonstrators for
 ↪  each agent
```

```
demonstrators = pd.DataFrame({
    "dem1" : population["trait"].sample(N, replace=True).values,
    "dem2" : population["trait"].sample(N, replace=True).values,
    "dem3" : population["trait"].sample(N, replace=True).values
})
```

```
# Visualize the DataFrame
demonstrators.head()
```

```
/home/fmoss/.local/lib/python3.10/site-packages/IPython/core/formatters.py:343: F
  return method()
```

	dem1	dem2	dem3
0	A	A	A
1	B	B	B
2	A	A	A
3	B	A	A
4	A	B	A

```
# Get the number of A's in each 3-demonstrator combination
num_As = (demonstrators == "A").apply(sum, axis=1)
num_As.head()
```

```
/home/fmoss/.local/lib/python3.10/site-packages/IPython/core/formatters.py:343: F
  return method()
```

	0
0	3
1	0
2	3
3	2
4	2

```
# For 3-demonstrator combinations with all A's, set to A
population[ num_As == 3 ] = "A"
# For 3-demonstrator combinations with all B's, set to B
population[ num_As == 0 ] = "B"
```

```
prob_majority = rng.choice([True, False], p=[(2/3 + D/3), 1-(2/3 +
 ↪  D/3)], size=N, replace=True)
prob_minority = rng.choice([True, False], p=[(1/3 + D/3), 1-(1/3 +
 ↪  D/3)], size=N, replace=True)
```

```
# 3-demonstrator combinations with two As and one B
condition = prob_majority & (num_As == 2)
if condition.sum() > 0:
    population.loc[condition, "trait"] = "A"
condition = ~prob_majority & (num_As == 2)
if condition.sum() > 0:
    population.loc[condition, "trait"] = "B"

# 3-demonstrator combinations with two B's and one A
condition = ~prob_minority & (num_As == 1)
if condition.sum() > 0:
    population.loc[condition, "trait"] = "A"
condition = prob_minority & (num_As == 1)
if condition.sum() > 0:
    population.loc[condition, "trait"] = "B"
```

```
demonstrators["new_trait"] = population["trait"]
demonstrators.head()
```

```
/home/fmoss/.local/lib/python3.10/site-packages/IPython/core/formatters.py:343: F
  return method()
```

	dem1	dem2	dem3	new_trait
0	A	A	A	A
1	B	B	B	B
2	A	A	A	A
3	B	A	A	A
4	A	B	A	A

```
def conformist_transmission(N, p_0, D, t_max, r_max):
    # Create the output DataFrame
    output = pd.DataFrame({
        "generation" : np.tile(np.arange(t_max), r_max),
        "p" : [ np.nan ] * t_max * r_max,
        "run" : np.repeat(np.arange(r_max), t_max)
    })

    for r in range(r_max):
        # Create first generation
        population = pd.DataFrame({"trait": rng.choice(["A", "B"],
↪   size=N, replace=True, p=[p_0, 1 - p_0])})

        # Add first generation's p for run r
        output.loc[ r * t_max, "p"] = population[ population["trait"] ==
↪   "A" ].shape[0] / N

        # For each generation
        for t in range(1,t_max):
            demonstrators = pd.DataFrame({
                "dem1" : population["trait"].sample(N,
                 ↪  replace=True).values,
                "dem2" : population["trait"].sample(N,
                 ↪  replace=True).values,
                "dem3" : population["trait"].sample(N,
                 ↪  replace=True).values
            })

            # Get the number of A's in each 3-demonstrator combination
            num_As = (demonstrators == "A").apply(sum, axis=1)
```

```
            # For 3-demonstrator combinations with all A's, set to A
            population[ num_As == 3 ] = "A"
            # For 3-demonstrator combinations with all A's, set to A
            population[ num_As == 3 ] = "A"
            # For 3-demonstrator combinations with all B's, set to B
            population[ num_As == 0 ] = "B"

            prob_majority = rng.choice([True, False], p=[(2/3 + D/3),
 ↪  1-(2/3 + D/3)], size=N, replace=True)
            prob_minority = rng.choice([True, False], p=[(1/3 + D/3),
 ↪  1-(1/3 + D/3)], size=N, replace=True)

            # 3-demonstrator combinations with two As and one B
            condition = prob_majority & (num_As == 2)
            if condition.sum() > 0:
                population.loc[condition, "trait"] = "A"
            condition = ~prob_majority & (num_As == 2)
            if condition.sum() > 0:
                population.loc[condition, "trait"] = "B"

            # 3-demonstrator combinations with two B's and one A
            condition = prob_minority & (num_As == 1)
            if condition.sum() > 0:
                population.loc[condition, "trait"] = "A"
            condition = ~prob_minority & (num_As == 1)
            if condition.sum() > 0:
                population.loc[condition, "trait"] = "B"

            # Get p and put it into output slot for this generation t
             ↪  and run r
            output.loc[r * t_max + t, "p"] = population[
 ↪  population["trait"] == "A" ].shape[0] / N

    return output
```

```
data_model = conformist_transmission(N=1_000, p_0 = 0.5, D = 1, t_max =
 ↪  50, r_max = 10)
plot_multiple_runs(data_model)
```

1.0
0.8
0.6
0.4
0.2
0.0
0
10
20
30
40
50
generation

7. Biased transmission: demonstrator-based indirect bias

> **Note**
>
> This chapter is based on “Chapter 5: Biased transmission: demonstrator-based indirect bias” in Acerbi et al. (2022).

```
import numpy as np
rng = np.random.default_rng()

import pandas as pd
```

```
def plot_multiple_runs(data_model):
    groups = data_model.groupby("run")
    for _, g in groups:
        g.index = g["generation"]
        g["p"].plot(lw=.5, ylim=(0,1))

    data_model.groupby("generation")["p"].mean().plot(c="k", lw="1")
```

```
N = 100
p_0 = 0.5
p_s = 0.05
```

```
population = pd.DataFrame({
    "trait": rng.choice(["A", "B"], size=N, replace=True, p=[p_0,
     ↪  1-p_0]),
    "status": rng.choice(["high", "low"], size=N, replace=True, p=[p_s,
     ↪  1-p_s])
})
```

```
population.head()
```

```
/home/fmoss/.local/lib/python3.10/site-packages/IPython/core/formatters.py:343: F
  return method()
```

	trait	status
0	A	low
1	B	low
2	B	low
3	A	low
4	A	low

```
p_low = 0.01
p_demonstrator = np.ones(N)
p_demonstrator[ population["status"] == "low" ] = p_low
```

```
if sum(p_demonstrator) > 0:
    ps = p_demonstrator / p_demonstrator.sum()
    demonstrator_index = rng.choice(np.arange(N), size=N, p=ps,
 ↪  replace=True)
    population["trait"] = population.loc[demonstrator_index,
 ↪  "trait"].values
```

```
def biased_transmission_demonstrator(N, p_0, p_s, p_low, t_max, r_max):
    # Create the output DataFrame
    output = pd.DataFrame({
        "generation" : np.tile(np.arange(t_max), r_max),
        "p" : [ np.nan ] * t_max * r_max,
        "run" : np.repeat(np.arange(r_max), t_max)
    })

    for r in range(r_max):
            # Create first generation
            population = pd.DataFrame({
                "trait": rng.choice(["A", "B"], size=N, replace=True,
                 ↪  p=[p_0, 1-p_0]),
```

```
                "status": rng.choice(["high", "low"], size=N,
                 ↪  replace=True, p=[p_s, 1-p_s])
            })

            # Assign copying probabilities based on individuals' status
            p_demonstrator = np.ones(N)
            p_demonstrator[population["status"] == "low"] = p_low

            # Add first generation's p for run r
            output.loc[ r * t_max, "p"] = population[
↪   population["trait"] == "A" ].shape[0] / N

            for t in range(1, t_max):
                # Copy individuals to previous_population DataFrame
                previous_population = population.copy()

                # Copy traits based on status
                if sum(p_demonstrator) > 0:
                    ps = p_demonstrator / p_demonstrator.sum()
                    demonstrator_index = rng.choice(np.arange(N),
↪   size=N, p=ps, replace=True)
                    population["trait"] =
↪   population.loc[demonstrator_index, "trait"].values

                # Get p and put it into output slot for this generation
                 ↪  t and run r
                output.loc[r * t_max + t, "p"] = population[
↪   population["trait"] == "A" ].shape[0] / N

    return output
```

```
data_model = biased_transmission_demonstrator(N=100, p_s=0.05,
↪   p_low=0.0001, p_0=0.5, t_max=50, r_max=10)
```

```
plot_multiple_runs(data_model)
```

```
data_model = biased_transmission_demonstrator(N=10_000, p_s=0.005,
 ↪  p_low=0.0001, p_0=0.5, t_max=200, r_max=10)
plot_multiple_runs(data_model)
```

```
def biased_transmission_demonstrator_2(N, p_0, p_s, p_low, t_max,
↪   r_max):
    # Create the output DataFrame
    output = pd.DataFrame({
        "generation" : np.tile(np.arange(t_max), r_max),
        "p" : [ np.nan ] * t_max * r_max,
        "run" : np.repeat(np.arange(r_max), t_max)
    })

    ...

    return output
```

```
data_model = biased_transmission_demonstrator_2(N=100, p_s=0.1,
↪   p_low=0.0001, p_0=0.5, t_max=50, r_max=50)
```

8. Vertical and horizontal transmission

> **Note**
>
> This chapter is based on “Chapter 6: Vertical and horizontal transmission” in Acerbi et al. (2022).

```
import numpy as np
rng = np.random.default_rng()

import pandas as pd
from tqdm import tqdm
```

```
def plot_multiple_runs(data_model):
    groups = data_model.groupby("run")
    for _, g in groups:
        g.index = g["generation"]
        g["p"].plot(lw=.5, ylim=(0,1))

    data_model.groupby("generation")["p"].mean().plot(c="k", lw="1")
```

```
def vertical_transmission(N, p_0, b, t_max, r_max):
    # Create the output DataFrame
    output = pd.DataFrame({
        "generation" : np.tile(np.arange(t_max), r_max),
        "p" : [ np.nan ] * t_max * r_max,
        "run" : np.repeat(np.arange(r_max), t_max)
    })

    for r in range(r_max):
        # Create first generation
        population = pd.DataFrame({"trait": rng.choice(["A", "B"],
↪   size=N, replace=True, p=[p_0, 1 - p_0])})

        # Add first generation's p for run r
```

```
        output.loc[ r * t_max, "p"] = population[ population["trait"] ==
↪  "A" ].shape[0] / N

        # # For each generation
        for t in range(1, t_max):
            # Copy individuals to previous_population DataFrame
            previous_population = population.copy()

            # randomly pick mothers and fathers
            mother = previous_population["trait"].sample(N,
↪  replace=True).reset_index(drop=True)
            father = previous_population["trait"].sample(N,
↪  replace=True).reset_index(drop=True)

            # prepare next generation
            population = pd.DataFrame({"trait": [np.nan] * N })

            # Both parents are A, thus child adopts A
            both_A = (mother == "A") & (father == "A")
            # if sum(both_A) > 0:
            population.loc[both_A,"trait"] = "A"

            # Both parents are A, thus child adopts A
            both_B = (mother == "B") & (father == "B")
            # if sum(both_B) > 0:
            population.loc[both_B,"trait"] = "B"

            # If any empty NA slots are present (i.e. one A and one B
             ↪  parent) they adopt A with probability b
            remaining = rng.choice(["A", "B"],
↪  size=population["trait"].isna().sum(), replace=True, p=[b, 1 - b])
            population.loc[population["trait"].isna(),"trait"] =
↪  remaining

            # Get p and put it into output slot for this generation t
             ↪  and run r
            output.loc[r * t_max + t, "p"] = population[
↪  population["trait"] == "A" ].shape[0] / N

    return output
```

```
data_model = vertical_transmission(N=10_000, p_0=0.01, b=0.6,t_max=50,
 ↪  r_max=5)

plot_multiple_runs(data_model)
```

```
data_model = vertical_transmission(N=10_000, p_0=0.1, b=0.5,t_max=50,
 ↪  r_max=5)
plot_multiple_runs(data_model)
```

8.1. Horizontal cultural transmission

> **Warning**
>
> The code below is not yet correct and runs very slowly.

```
def vertical_horizontal_transmission(N, p_0, b, n, g, t_max, r_max):
    # Create the output DataFrame
    output = pd.DataFrame({
        "generation" : np.tile(np.arange(t_max), r_max),
        "p" : [ np.nan ] * t_max * r_max,
        "run" : np.repeat(np.arange(r_max), t_max)
    })

    for r in range(r_max):
        # Create first generation
        population = pd.DataFrame({"trait": rng.choice(["A", "B"],
↪   size=N, replace=True, p=[p_0, 1 - p_0])})

        # Add first generation's p for run r
        output.loc[ r * t_max, "p"] = population[ population["trait"] ==
↪   "A" ].shape[0] / N
```

```
        # For each generation
        for t in tqdm(range(t_max)):
            ### Vertical transmission
             ↪  ==========================================================

            # Copy individuals to previous_population DataFrame
            previous_population = population.copy()

            # randomly pick mothers and fathers
            mother = previous_population["trait"].sample(N,
↪   replace=True).reset_index(drop=True)
            father = previous_population["trait"].sample(N,
↪   replace=True).reset_index(drop=True)

            # prepare next generation
            population = pd.DataFrame({"trait": [np.nan] * N })

            # Both parents are A, thus child adopts A
            both_A = (mother == "A") & (father == "A")
            # if sum(both_A) > 0:
            population.loc[both_A,"trait"] = "A"

            # Both parents are A, thus child adopts A
            both_B = (mother == "B") & (father == "B")
            # if sum(both_B) > 0:
            population.loc[both_B,"trait"] = "B"

            # If any empty NA slots are present (i.e. one A and one B
             ↪  parent) they adopt A with probability b
            remaining = rng.choice(["A", "B"],
↪   size=population["trait"].isna().sum(), replace=True, p=[b, 1 - b])
            population.loc[population["trait"].isna(),"trait"] =
↪   remaining

            # Get p and put it into output slot for this generation t
             ↪  and run r
            output.loc[r * t_max + t, "p"] = population[
↪   population["trait"] == "A" ].shape[0] / N
```

```
            # ### Horizontal transmission
             ↪  ==========================================================

            # previous_population = population.copy()
            # # # N_B = number of Bs
            # N_B = previous_population[previous_population["trait"] ==
             ↪  "B"].shape[0]

           # # if there are B individuals to switch, and n is not zero:
            # if (N_B > 0) & (n > 0):
            #     # for each B individual:
            #     for i in range(N_B):
            #         # Pick n demonstrators
            #         demonstrator =
             ↪  previous_population["trait"].sample(n, replace=True)
            #         # Get probability g
           #         copy_ = rng.choice([True, False], n, p=[g, 1 - g],
             ↪  replace=True)
            #         # if any demonstrators with A are to be copied:
            #         if sum((demonstrator == "A") & (copy_)) > 0:
            #           # The B individual switches to A
            #           population[previous_population["trait"] ==
             ↪  "B"].loc[i, "trait"] = "A"

            next_population = population.copy()
            # # N_B = number of Bs
            N_B = next_population[next_population["trait"] ==
↪  "B"].shape[0]

            # if there are B individuals to switch, and n is not zero:
            if (N_B > 0) & (n > 0):
                # for each B individual:
                for i in range(N_B):
                    # Pick n demonstrators
                    demonstrator = population["trait"].sample(n,
↪  replace=True)
                    # Get probability g
                    copy_ = rng.choice([True, False], n, p=[g, 1 - g],
↪  replace=True)
                    # if any demonstrators with A are to be copied:
                    if sum((demonstrator == "A") & (copy_)) > 0:
```

```
                    # The B individual switches to A
                    next_population[next_population["trait"] ==
↪  "B"].loc[i, "trait"] = "A"

            # Get p and put it into output slot for this generation t
             ↪  and run r
            output.loc[r * t_max + t, "p"] = next_population[
↪  next_population["trait"] == "A" ].shape[0] / N

    return output
```

```
vertical_horizontal_transmission(N=1000, p_0=0.01, b=0.5, n=5, g=0.1,
↪  t_max=10, r_max=1)
```

```
  0%|          | 0/10 [00:00<?, ?it/s]

/tmp/ipykernel_119111/3300034538.py:81: SettingWithCopyWarning:
A value is trying to be set on a copy of a slice from a DataFrame

See the caveats in the documentation: https://pandas.pydata.org/pandas-docs/stable
 next_population[next_population["trait"] == "B"].loc[i, "trait"] = "A"

 10%|▉         | 1/10 [00:00<00:03,  2.78it/s]

 20%|▉▉        | 2/10 [00:00<00:02,  2.91it/s]

 30%|▉▉▉       | 3/10 [00:00<00:02,  3.06it/s]

 40%|▉▉▉▉      | 4/10 [00:01<00:02,  2.98it/s]

/tmp/ipykernel_119111/3300034538.py:81: SettingWithCopyWarning:
A value is trying to be set on a copy of a slice from a DataFrame

See the caveats in the documentation: https://pandas.pydata.org/pandas-docs/stable
 next_population[next_population["trait"] == "B"].loc[i, "trait"] = "A"
 50%|▉▉▉▉▉     | 5/10 [00:01<00:01,  2.90it/s]
```

```
/tmp/ipykernel_119111/3300034538.py:81: SettingWithCopyWarning:
A value is trying to be set on a copy of a slice from a DataFrame

See the caveats in the documentation: https://pandas.pydata.org/pandas-docs/stable
 next_population[next_population["trait"] == "B"].loc[i, "trait"] = "A"


 60%|██████    | 6/10 [00:02<00:01,  2.94it/s]


 70%|███████   | 7/10 [00:02<00:00,  3.01it/s]


 80%|████████  | 8/10 [00:02<00:00,  3.10it/s]


 90%|█████████ | 9/10 [00:02<00:00,  3.27it/s]


100%|██████████| 10/10 [00:03<00:00,  3.32it/s]


100%|██████████| 10/10 [00:03<00:00,  3.11it/s]


/home/fmoss/.local/lib/python3.10/site-packages/IPython/core/formatters.py:343: F
  return method()
```

	generation	p	run
0	0	0.010	0
1	1	0.011	0
2	2	0.015	0
3	3	0.022	0
4	4	0.026	0
5	5	0.027	0
6	6	0.027	0
7	7	0.020	0
8	8	0.016	0
9	9	0.012	0

```
data_model = vertical_horizontal_transmission(N=5_000, p_0=0.01, b=0.5,
 ↪  n=5, g=0.1, t_max=50, r_max=2)
plot_multiple_runs(data_model)
```

```
  0%|          | 0/50 [00:00<?, ?it/s]

/tmp/ipykernel_119111/3300034538.py:81: SettingWithCopyWarning:
A value is trying to be set on a copy of a slice from a DataFrame

See the caveats in the documentation: https://pandas.pydata.org/pandas-docs/stable
 next_population[next_population["trait"] == "B"].loc[i, "trait"] = "A"

  2%|          | 1/50 [00:01<01:04,  1.31s/it]

  4%|          | 2/50 [00:02<01:01,  1.28s/it]

  6%|          | 3/50 [00:03<01:02,  1.32s/it]

/tmp/ipykernel_119111/3300034538.py:81: SettingWithCopyWarning:
A value is trying to be set on a copy of a slice from a DataFrame

See the caveats in the documentation: https://pandas.pydata.org/pandas-docs/stable
 next_population[next_population["trait"] == "B"].loc[i, "trait"] = "A"
  8%|          | 4/50 [00:05<01:02,  1.36s/it]

 10%|          | 5/50 [00:06<01:03,  1.41s/it]

 12%|          | 6/50 [00:08<01:05,  1.48s/it]

/tmp/ipykernel_119111/3300034538.py:81: SettingWithCopyWarning:
A value is trying to be set on a copy of a slice from a DataFrame

See the caveats in the documentation: https://pandas.pydata.org/pandas-docs/stable
 next_population[next_population["trait"] == "B"].loc[i, "trait"] = "A"
```

```
/tmp/ipykernel_119111/3300034538.py:81: SettingWithCopyWarning:
A value is trying to be set on a copy of a slice from a DataFrame

See the caveats in the documentation: https://pandas.pydata.org/pandas-docs/stable
 next_population[next_population["trait"] == "B"].loc[i, "trait"] = "A"

/tmp/ipykernel_119111/3300034538.py:81: SettingWithCopyWarning:
A value is trying to be set on a copy of a slice from a DataFrame

See the caveats in the documentation: https://pandas.pydata.org/pandas-docs/stable
 next_population[next_population["trait"] == "B"].loc[i, "trait"] = "A"
 14%|└          | 7/50 [00:10<01:07,  1.56s/it]

 16%|└          | 8/50 [00:11<01:07,  1.60s/it]

/tmp/ipykernel_119111/3300034538.py:81: SettingWithCopyWarning:
A value is trying to be set on a copy of a slice from a DataFrame

See the caveats in the documentation: https://pandas.pydata.org/pandas-docs/stable
 next_population[next_population["trait"] == "B"].loc[i, "trait"] = "A"

 18%|└          | 9/50 [00:13<01:06,  1.63s/it]

 20%|└          | 10/50 [00:15<01:03,  1.59s/it]

 22%|└          | 11/50 [00:16<01:01,  1.57s/it]

 24%|└          | 12/50 [00:18<00:58,  1.55s/it]

/tmp/ipykernel_119111/3300034538.py:81: SettingWithCopyWarning:
A value is trying to be set on a copy of a slice from a DataFrame

See the caveats in the documentation: https://pandas.pydata.org/pandas-docs/stable
 next_population[next_population["trait"] == "B"].loc[i, "trait"] = "A"

 26%|└          | 13/50 [00:19<00:57,  1.54s/it]

 28%|└          | 14/50 [00:21<00:54,  1.53s/it]
```

```
/tmp/ipykernel_119111/3300034538.py:81: SettingWithCopyWarning:
A value is trying to be set on a copy of a slice from a DataFrame

See the caveats in the documentation: https://pandas.pydata.org/pandas-docs/stable
 next_population[next_population["trait"] == "B"].loc[i, "trait"] = "A"

/tmp/ipykernel_119111/3300034538.py:81: SettingWithCopyWarning:
A value is trying to be set on a copy of a slice from a DataFrame

See the caveats in the documentation: https://pandas.pydata.org/pandas-docs/stable
 next_population[next_population["trait"] == "B"].loc[i, "trait"] = "A"

 30%|└└└        | 15/50 [00:22<00:54,  1.57s/it]

 32%|└└└        | 16/50 [00:24<00:54,  1.60s/it]

/tmp/ipykernel_119111/3300034538.py:81: SettingWithCopyWarning:
A value is trying to be set on a copy of a slice from a DataFrame

See the caveats in the documentation: https://pandas.pydata.org/pandas-docs/stable
 next_population[next_population["trait"] == "B"].loc[i, "trait"] = "A"

/tmp/ipykernel_119111/3300034538.py:81: SettingWithCopyWarning:
A value is trying to be set on a copy of a slice from a DataFrame

See the caveats in the documentation: https://pandas.pydata.org/pandas-docs/stable
 next_population[next_population["trait"] == "B"].loc[i, "trait"] = "A"

 34%|└└└└       | 17/50 [00:26<00:55,  1.67s/it]

 36%|└└└└       | 18/50 [00:28<00:56,  1.77s/it]

/tmp/ipykernel_119111/3300034538.py:81: SettingWithCopyWarning:
A value is trying to be set on a copy of a slice from a DataFrame

See the caveats in the documentation: https://pandas.pydata.org/pandas-docs/stable
 next_population[next_population["trait"] == "B"].loc[i, "trait"] = "A"
 38%|└└└└       | 19/50 [00:30<00:56,  1.81s/it]
```

```
/tmp/ipykernel_119111/3300034538.py:81: SettingWithCopyWarning:
A value is trying to be set on a copy of a slice from a DataFrame

See the caveats in the documentation: https://pandas.pydata.org/pandas-docs/stable
 next_population[next_population["trait"] == "B"].loc[i, "trait"] = "A"

/tmp/ipykernel_119111/3300034538.py:81: SettingWithCopyWarning:
A value is trying to be set on a copy of a slice from a DataFrame

See the caveats in the documentation: https://pandas.pydata.org/pandas-docs/stable
 next_population[next_population["trait"] == "B"].loc[i, "trait"] = "A"

 40%|████      | 20/50 [00:32<00:55,  1.86s/it]

/tmp/ipykernel_119111/3300034538.py:81: SettingWithCopyWarning:
A value is trying to be set on a copy of a slice from a DataFrame

See the caveats in the documentation: https://pandas.pydata.org/pandas-docs/stable
 next_population[next_population["trait"] == "B"].loc[i, "trait"] = "A"

/tmp/ipykernel_119111/3300034538.py:81: SettingWithCopyWarning:
A value is trying to be set on a copy of a slice from a DataFrame

See the caveats in the documentation: https://pandas.pydata.org/pandas-docs/stable
 next_population[next_population["trait"] == "B"].loc[i, "trait"] = "A"

 42%|████      | 21/50 [00:34<00:53,  1.85s/it]

 44%|████      | 22/50 [00:35<00:49,  1.76s/it]

 46%|△△△△      | 23/50 [00:38<00:53,  1.96s/it]

/tmp/ipykernel_119111/3300034538.py:81: SettingWithCopyWarning:
A value is trying to be set on a copy of a slice from a DataFrame

See the caveats in the documentation: https://pandas.pydata.org/pandas-docs/stable
 next_population[next_population["trait"] == "B"].loc[i, "trait"] = "A"
```

```
/tmp/ipykernel_119111/3300034538.py:81: SettingWithCopyWarning:
A value is trying to be set on a copy of a slice from a DataFrame

See the caveats in the documentation: https://pandas.pydata.org/pandas-docs/stable
 next_population[next_population["trait"] == "B"].loc[i, "trait"] = "A"

 48%|└└└
```

```
/tmp/ipykernel_119111/3300034538.py:81: SettingWithCopyWarning:
A value is trying to be set on a copy of a slice from a DataFrame

See the caveats in the documentation: https://pandas.pydata.org/pandas-docs/stable
 next_population[next_population["trait"] == "B"].loc[i, "trait"] = "A"
/tmp/ipykernel_119111/3300034538.py:81: SettingWithCopyWarning:
A value is trying to be set on a copy of a slice from a DataFrame

See the caveats in the documentation: https://pandas.pydata.org/pandas-docs/stable
 next_population[next_population["trait"] == "B"].loc[i, "trait"] = "A"

 56%|██████     | 28/50 [00:48<00:42,  1.94s/it]

/tmp/ipykernel_119111/3300034538.py:81: SettingWithCopyWarning:
A value is trying to be set on a copy of a slice from a DataFrame

See the caveats in the documentation: https://pandas.pydata.org/pandas-docs/stable
 next_population[next_population["trait"] == "B"].loc[i, "trait"] = "A"

/tmp/ipykernel_119111/3300034538.py:81: SettingWithCopyWarning:
A value is trying to be set on a copy of a slice from a DataFrame

See the caveats in the documentation: https://pandas.pydata.org/pandas-docs/stable
 next_population[next_population["trait"] == "B"].loc[i, "trait"] = "A"

/tmp/ipykernel_119111/3300034538.py:81: SettingWithCopyWarning:
A value is trying to be set on a copy of a slice from a DataFrame

See the caveats in the documentation: https://pandas.pydata.org/pandas-docs/stable
 next_population[next_population["trait"] == "B"].loc[i, "trait"] = "A"

 58%|██████     | 29/50 [00:50<00:41,  1.98s/it]

/tmp/ipykernel_119111/3300034538.py:81: SettingWithCopyWarning:
A value is trying to be set on a copy of a slice from a DataFrame

See the caveats in the documentation: https://pandas.pydata.org/pandas-docs/stable
 next_population[next_population["trait"] == "B"].loc[i, "trait"] = "A"
```

```
/tmp/ipykernel_119111/3300034538.py:81: SettingWithCopyWarning:
A value is trying to be set on a copy of a slice from a DataFrame

See the caveats in the documentation: https://pandas.pydata.org/pandas-docs/stable
 next_population[next_population["trait"] == "B"].loc[i, "trait"] = "A"

 60%|██████     | 30/50 [00:52<00:38,  1.92s/it]

/tmp/ipykernel_119111/3300034538.py:81: SettingWithCopyWarning:
A value is trying to be set on a copy of a slice from a DataFrame

See the caveats in the documentation: https://pandas.pydata.org/pandas-docs/stable
 next_population[next_population["trait"] == "B"].loc[i, "trait"] = "A"

/tmp/ipykernel_119111/3300034538.py:81: SettingWithCopyWarning:
A value is trying to be set on a copy of a slice from a DataFrame

See the caveats in the documentation: https://pandas.pydata.org/pandas-docs/stable
 next_population[next_population["trait"] == "B"].loc[i, "trait"] = "A"

/tmp/ipykernel_119111/3300034538.py:81: SettingWithCopyWarning:
A value is trying to be set on a copy of a slice from a DataFrame

See the caveats in the documentation: https://pandas.pydata.org/pandas-docs/stable
 next_population[next_population["trait"] == "B"].loc[i, "trait"] = "A"

 62%|██████     | 31/50 [00:54<00:36,  1.94s/it]

/tmp/ipykernel_119111/3300034538.py:81: SettingWithCopyWarning:
A value is trying to be set on a copy of a slice from a DataFrame

See the caveats in the documentation: https://pandas.pydata.org/pandas-docs/stable
 next_population[next_population["trait"] == "B"].loc[i, "trait"] = "A"
/tmp/ipykernel_119111/3300034538.py:81: SettingWithCopyWarning:
A value is trying to be set on a copy of a slice from a DataFrame

See the caveats in the documentation: https://pandas.pydata.org/pandas-docs/stable
 next_population[next_population["trait"] == "B"].loc[i, "trait"] = "A"
```

```
/tmp/ipykernel_119111/3300034538.py:81: SettingWithCopyWarning:
A value is trying to be set on a copy of a slice from a DataFrame

See the caveats in the documentation: https://pandas.pydata.org/pandas-docs/stable
 next_population[next_population["trait"] == "B"].loc[i, "trait"] = "A"

 64%|▋▋▋▋▋▋    | 32/50 [00:56<00:36,  2.02s/it]

/tmp/ipykernel_119111/3300034538.py:81: SettingWithCopyWarning:
A value is trying to be set on a copy of a slice from a DataFrame

See the caveats in the documentation: https://pandas.pydata.org/pandas-docs/stable
 next_population[next_population["trait"] == "B"].loc[i, "trait"] = "A"

/tmp/ipykernel_119111/3300034538.py:81: SettingWithCopyWarning:
A value is trying to be set on a copy of a slice from a DataFrame

See the caveats in the documentation: https://pandas.pydata.org/pandas-docs/stable
 next_population[next_population["trait"] == "B"].loc[i, "trait"] = "A"

 66%|▋▋▋▋▋▋▋   | 33/50 [00:58<00:33,  1.98s/it]

/tmp/ipykernel_119111/3300034538.py:81: SettingWithCopyWarning:
A value is trying to be set on a copy of a slice from a DataFrame

See the caveats in the documentation: https://pandas.pydata.org/pandas-docs/stable
 next_population[next_population["trait"] == "B"].loc[i, "trait"] = "A"

 68%|▋▋▋▋▋▋▋   | 34/50 [01:00<00:31,  1.98s/it]

/tmp/ipykernel_119111/3300034538.py:81: SettingWithCopyWarning:
A value is trying to be set on a copy of a slice from a DataFrame

See the caveats in the documentation: https://pandas.pydata.org/pandas-docs/stable
 next_population[next_population["trait"] == "B"].loc[i, "trait"] = "A"
/tmp/ipykernel_119111/3300034538.py:81: SettingWithCopyWarning:
A value is trying to be set on a copy of a slice from a DataFrame
```

```
See the caveats in the documentation: https://pandas.pydata.org/pandas-docs/stable
 next_population[next_population["trait"] == "B"].loc[i, "trait"] = "A"

 70%|▊▊▊▊▊▊▊   | 35/50 [01:01<00:28,  1.93s/it]

/tmp/ipykernel_119111/3300034538.py:81: SettingWithCopyWarning:
A value is trying to be set on a copy of a slice from a DataFrame

See the caveats in the documentation: https://pandas.pydata.org/pandas-docs/stable
 next_population[next_population["trait"] == "B"].loc[i, "trait"] = "A"

 72%|▊▊▊▊▊▊▊   | 36/50 [01:03<00:27,  1.96s/it]

/tmp/ipykernel_119111/3300034538.py:81: SettingWithCopyWarning:
A value is trying to be set on a copy of a slice from a DataFrame

See the caveats in the documentation: https://pandas.pydata.org/pandas-docs/stable
 next_population[next_population["trait"] == "B"].loc[i, "trait"] = "A"

 74%|▊▊▊▊▊▊▊   | 37/50 [01:05<00:25,  1.93s/it]

/tmp/ipykernel_119111/3300034538.py:81: SettingWithCopyWarning:
A value is trying to be set on a copy of a slice from a DataFrame

See the caveats in the documentation: https://pandas.pydata.org/pandas-docs/stable
 next_population[next_population["trait"] == "B"].loc[i, "trait"] = "A"

/tmp/ipykernel_119111/3300034538.py:81: SettingWithCopyWarning:
A value is trying to be set on a copy of a slice from a DataFrame

See the caveats in the documentation: https://pandas.pydata.org/pandas-docs/stable
 next_population[next_population["trait"] == "B"].loc[i, "trait"] = "A"
/tmp/ipykernel_119111/3300034538.py:81: SettingWithCopyWarning:
A value is trying to be set on a copy of a slice from a DataFrame

See the caveats in the documentation: https://pandas.pydata.org/pandas-docs/stable
 next_population[next_population["trait"] == "B"].loc[i, "trait"] = "A"
/tmp/ipykernel_119111/3300034538.py:81: SettingWithCopyWarning:
```

```
A value is trying to be set on a copy of a slice from a DataFrame

See the caveats in the documentation: https://pandas.pydata.org/pandas-docs/stable
 next_population[next_population["trait"] == "B"].loc[i, "trait"] = "A"

 76%|▊▊▊▊▊▊▊   | 38/50 [01:07<00:22,  1.87s/it]

/tmp/ipykernel_119111/3300034538.py:81: SettingWithCopyWarning:
A value is trying to be set on a copy of a slice from a DataFrame

See the caveats in the documentation: https://pandas.pydata.org/pandas-docs/stable
 next_population[next_population["trait"] == "B"].loc[i, "trait"] = "A"
/tmp/ipykernel_119111/3300034538.py:81: SettingWithCopyWarning:
A value is trying to be set on a copy of a slice from a DataFrame

See the caveats in the documentation: https://pandas.pydata.org/pandas-docs/stable
 next_population[next_population["trait"] == "B"].loc[i, "trait"] = "A"

 78%|▊▊▊▊▊▊▊   | 39/50 [01:09<00:19,  1.82s/it]

/tmp/ipykernel_119111/3300034538.py:81: SettingWithCopyWarning:
A value is trying to be set on a copy of a slice from a DataFrame

See the caveats in the documentation: https://pandas.pydata.org/pandas-docs/stable
 next_population[next_population["trait"] == "B"].loc[i, "trait"] = "A"
/tmp/ipykernel_119111/3300034538.py:81: SettingWithCopyWarning:
A value is trying to be set on a copy of a slice from a DataFrame

See the caveats in the documentation: https://pandas.pydata.org/pandas-docs/stable
 next_population[next_population["trait"] == "B"].loc[i, "trait"] = "A"

/tmp/ipykernel_119111/3300034538.py:81: SettingWithCopyWarning:
A value is trying to be set on a copy of a slice from a DataFrame

See the caveats in the documentation: https://pandas.pydata.org/pandas-docs/stable
 next_population[next_population["trait"] == "B"].loc[i, "trait"] = "A"

 80%|▊▊▊▊▊▊▊▊  | 40/50 [01:10<00:17,  1.77s/it]
```

```
/tmp/ipykernel_119111/3300034538.py:81: SettingWithCopyWarning:
A value is trying to be set on a copy of a slice from a DataFrame

See the caveats in the documentation: https://pandas.pydata.org/pandas-docs/stable
 next_population[next_population["trait"] == "B"].loc[i, "trait"] = "A"

 82%|▊▊▊▊▊▊▊▊  | 41/50 [01:12<00:15,  1.77s/it]

 84%|▊▊▊▊▊▊▊▊▊ | 42/50 [01:14<00:14,  1.76s/it]

 86%|▊▊▊▊▊▊▊▊▊ | 43/50 [01:16<00:12,  1.74s/it]

/tmp/ipykernel_119111/3300034538.py:81: SettingWithCopyWarning:
A value is trying to be set on a copy of a slice from a DataFrame

See the caveats in the documentation: https://pandas.pydata.org/pandas-docs/stable
 next_population[next_population["trait"] == "B"].loc[i, "trait"] = "A"

/tmp/ipykernel_119111/3300034538.py:81: SettingWithCopyWarning:
A value is trying to be set on a copy of a slice from a DataFrame

See the caveats in the documentation: https://pandas.pydata.org/pandas-docs/stable
 next_population[next_population["trait"] == "B"].loc[i, "trait"] = "A"

 88%|▊▊▊▊▊▊▊▊▊ | 44/50 [01:17<00:10,  1.78s/it]

 90%|▊▊▊▊▊▊▊▊▊ | 45/50 [01:19<00:08,  1.77s/it]

/tmp/ipykernel_119111/3300034538.py:81: SettingWithCopyWarning:
A value is trying to be set on a copy of a slice from a DataFrame

See the caveats in the documentation: https://pandas.pydata.org/pandas-docs/stable
 next_population[next_population["trait"] == "B"].loc[i, "trait"] = "A"

 92%|▊▊▊▊▊▊▊▊▊ | 46/50 [01:21<00:07,  1.80s/it]

 94%|▊▊▊▊▊▊▊▊▊ | 47/50 [01:23<00:05,  1.81s/it]
```

```
/tmp/ipykernel_119111/3300034538.py:81: SettingWithCopyWarning:
A value is trying to be set on a copy of a slice from a DataFrame

See the caveats in the documentation: https://pandas.pydata.org/pandas-docs/stable
 next_population[next_population["trait"] == "B"].loc[i, "trait"] = "A"

 96%|██████████| 48/50 [01:25<00:03,  1.84s/it]

/tmp/ipykernel_119111/3300034538.py:81: SettingWithCopyWarning:
A value is trying to be set on a copy of a slice from a DataFrame

See the caveats in the documentation: https://pandas.pydata.org/pandas-docs/stable
 next_population[next_population["trait"] == "B"].loc[i, "trait"] = "A"

/tmp/ipykernel_119111/3300034538.py:81: SettingWithCopyWarning:
A value is trying to be set on a copy of a slice from a DataFrame

See the caveats in the documentation: https://pandas.pydata.org/pandas-docs/stable
 next_population[next_population["trait"] == "B"].loc[i, "trait"] = "A"

 98%|██████████| 49/50 [01:27<00:01,  1.80s/it]

/tmp/ipykernel_119111/3300034538.py:81: SettingWithCopyWarning:
A value is trying to be set on a copy of a slice from a DataFrame

See the caveats in the documentation: https://pandas.pydata.org/pandas-docs/stable
 next_population[next_population["trait"] == "B"].loc[i, "trait"] = "A"

/tmp/ipykernel_119111/3300034538.py:81: SettingWithCopyWarning:
A value is trying to be set on a copy of a slice from a DataFrame

See the caveats in the documentation: https://pandas.pydata.org/pandas-docs/stable
 next_population[next_population["trait"] == "B"].loc[i, "trait"] = "A"
100%|██████████| 50/50 [01:28<00:00,  1.79s/it]

100%|██████████| 50/50 [01:28<00:00,  1.78s/it]
```

```
  0%|          | 0/50 [00:00<?, ?it/s]

/tmp/ipykernel_119111/3300034538.py:81: SettingWithCopyWarning:
A value is trying to be set on a copy of a slice from a DataFrame

See the caveats in the documentation: https://pandas.pydata.org/pandas-docs/stable
 next_population[next_population["trait"] == "B"].loc[i, "trait"] = "A"

  2%|          | 1/50 [00:01<01:05,  1.34s/it]

  4%|          | 2/50 [00:02<01:03,  1.32s/it]

/tmp/ipykernel_119111/3300034538.py:81: SettingWithCopyWarning:
A value is trying to be set on a copy of a slice from a DataFrame

See the caveats in the documentation: https://pandas.pydata.org/pandas-docs/stable
 next_population[next_population["trait"] == "B"].loc[i, "trait"] = "A"

  6%|          | 3/50 [00:03<01:01,  1.31s/it]

  8%|          | 4/50 [00:05<01:03,  1.38s/it]

 10%|          | 5/50 [00:06<01:01,  1.37s/it]

 12%|          | 6/50 [00:08<01:03,  1.43s/it]

 14%|          | 7/50 [00:09<01:01,  1.44s/it]

 16%|          | 8/50 [00:11<01:01,  1.46s/it]

 18%|          | 9/50 [00:12<01:01,  1.50s/it]

 20%|          | 10/50 [00:14<00:59,  1.49s/it]

 22%|          | 11/50 [00:15<00:58,  1.49s/it]

 24%|          | 12/50 [00:17<00:56,  1.50s/it]
```

```
 26%|██        | 13/50 [00:18<00:54,  1.48s/it]

 28%|██        | 14/50 [00:20<00:51,  1.44s/it]

 30%|██        | 15/50 [00:21<00:48,  1.39s/it]

 32%|██        | 16/50 [00:22<00:46,  1.36s/it]

 34%|██        | 17/50 [00:24<00:44,  1.34s/it]

 36%|██        | 18/50 [00:25<00:42,  1.33s/it]

 38%|██        | 19/50 [00:26<00:40,  1.32s/it]

 40%|██        | 20/50 [00:27<00:38,  1.30s/it]

 42%|██        | 21/50 [00:29<00:37,  1.30s/it]

 44%|██        | 22/50 [00:30<00:36,  1.29s/it]

 46%|██        | 23/50 [00:31<00:34,  1.28s/it]

 48%|██        | 24/50 [00:33<00:33,  1.30s/it]

/tmp/ipykernel_119111/3300034538.py:81: SettingWithCopyWarning:
A value is trying to be set on a copy of a slice from a DataFrame

See the caveats in the documentation: https://pandas.pydata.org/pandas-docs/stable
  next_population[next_population["trait"] == "B"].loc[i, "trait"] = "A"

 50%|██        | 25/50 [00:34<00:32,  1.31s/it]

 52%|██        | 26/50 [00:35<00:31,  1.32s/it]

 54%|██        | 27/50 [00:37<00:31,  1.36s/it]
```

```
/tmp/ipykernel_119111/3300034538.py:81: SettingWithCopyWarning:
A value is trying to be set on a copy of a slice from a DataFrame

See the caveats in the documentation: https://pandas.pydata.org/pandas-docs/stable
 next_population[next_population["trait"] == "B"].loc[i, "trait"] = "A"

 56%|      | 28/50 [00:38<00:30,  1.36s/it]

 58%|      | 29/50 [00:39<00:28,  1.36s/it]

 60%|      | 30/50 [00:41<00:27,  1.37s/it]

 62%|      | 31/50 [00:42<00:25,  1.35s/it]

 64%|      | 32/50 [00:44<00:24,  1.37s/it]

 66%|      | 33/50 [00:45<00:23,  1.36s/it]

 68%|      | 34/50 [00:46<00:21,  1.35s/it]

 70%|      | 35/50 [00:47<00:20,  1.34s/it]

 72%|      | 36/50 [00:49<00:18,  1.31s/it]

 74%|      | 37/50 [00:50<00:16,  1.29s/it]

 76%|      | 38/50 [00:51<00:15,  1.28s/it]

 78%|      | 39/50 [00:53<00:14,  1.29s/it]

 80%|      | 40/50 [00:54<00:12,  1.30s/it]

 82%|      | 41/50 [00:55<00:11,  1.30s/it]

 84%|      | 42/50 [00:56<00:10,  1.30s/it]

 86%|      | 43/50 [00:58<00:09,  1.33s/it]
```

```
 88%|▉▉▉▉▉▉▉▉▉▉| 44/50 [00:59<00:07,  1.33s/it]

 90%|▉▉▉▉▉▉▉▉▉▉| 45/50 [01:01<00:06,  1.34s/it]

 92%|▉▉▉▉▉▉▉▉▉▉| 46/50 [01:02<00:05,  1.35s/it]

 94%|▉▉▉▉▉▉▉▉▉▉| 47/50 [01:03<00:04,  1.34s/it]

 96%|▉▉▉▉▉▉▉▉▉▉| 48/50 [01:05<00:02,  1.35s/it]

 98%|▉▉▉▉▉▉▉▉▉▉| 49/50 [01:06<00:01,  1.34s/it]

100%|▉▉▉▉▉▉▉▉▉▉| 50/50 [01:07<00:00,  1.33s/it]

100%|▉▉▉▉▉▉▉▉▉▉| 50/50 [01:07<00:00,  1.35s/it]
```

9. The multiple traits model

```
import pandas as pd
import numpy as np
rng = np.random.default_rng()

import matplotlib.pyplot as plt

N = 100
population = pd.DataFrame(
    {"trait" : rng.integers(N, size=N)}
)
```

```
population.head()
```

```
/home/fmoss/.local/lib/python3.10/site-packages/IPython/core/formatters.py:343: F
  return method()
```

	trait
0	77
1	48
2	22
3	93
4	75

```
def multiple_traits(N, t_max):
    output = pd.DataFrame({
        "trait" : np.repeat(np.arange(N), t_max),
        "generation" : np.tile(np.arange(t_max), N),
        "p" : [ np.nan ] * t_max * N,
```

```
    })

    # Create first generation
    population = pd.DataFrame({"trait" : rng.integers(N, size=N)})

    # Add first generation's p for all traits
    output.loc[output["generation"] == 0, "p"] = popula-
↪   tion["trait"].value_counts(normalize=True).reindex(range(N)).fillna(0.).values

    for t in range(t_max):
        # Copy individuals to previous_population DataFrame
        previous_population = population.copy()

        # Randomly copy from previous generation
        population = pd.DataFrame({"trait" :
↪   previous_population["trait"].sample(N, replace=True)})

        # Get p for all traits and put it into output slot for this
         ↪  generation t
        output.loc[output["generation"] == t, "p"] = popula-
↪   tion["trait"].value_counts(normalize=True).reindex(range(N)).fillna(0.).values

    return output
```

```
def plot_multiple_traits(data_model):
    ps = []
    for _, g in data_model.groupby("trait"):
        x = g["generation"]
        ps.append(g["p"])

    plt.stackplot(x, *ps, cmap="tab20")
    plt.margins(0.)
    plt.show()
```

```
data_model = multiple_traits(N=100, t_max=200)
plot_multiple_traits(data_model)
```

```
data_model = multiple_traits(N=100, t_max=1000)
plot_multiple_traits(data_model)
```

9.1. Introducing innovation

Part II.

Advanced topics

10. Advanced topics

10.1. Are more complex songs more popular?

- “The role of population size in folk tune complexity” (Street et al., 2022)

10.2. Evolution of pitches and intervals

- “Cross-cultural data shows musical scales evolved to maximise imperfect fifths” McBride & Tlusty (2020)
- “The line of fifths and the co-evolution of tonal pitch-classes” (Moss et al., 2022)

10.3. Stylistic change

The question of historical change is intricably related to musical styles. Here, we adopt the framework for style analysis layed out by Meyer (1989). In particular, the adaptation of Jan (2016) and its schematic representation is already almost a description of a statistical graphical model.

- “Statistical Evolutionary Laws in Music Styles” (Nakamura & Kaneko, 2019)
- “Investigating style evolution of Western classical music: A computational approach” (Weiß et al., 2019)

10.4. Cultural evolution of electronic music

- “Phylogenetic reconstruction of the cultural evolution of electronic music via dynamic community detection (1975–1999)” (Youngblood et al., 2021)

11. Conclusion

11.1. What can cultural evolution tell us about music

11.2. What is the role of models for musicology

11.3. Avenues for future research

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