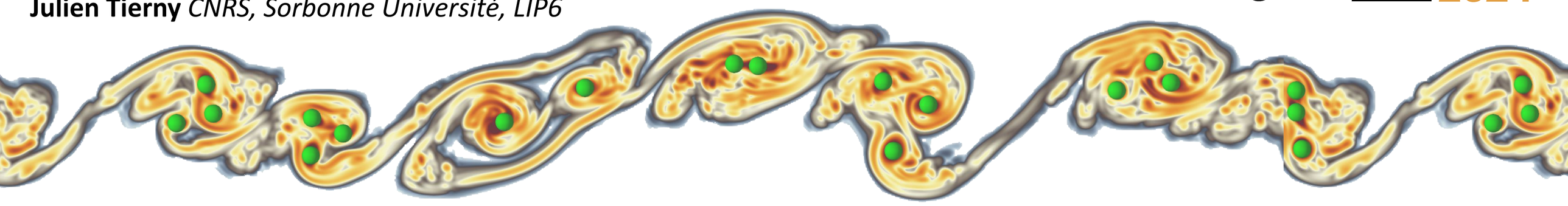
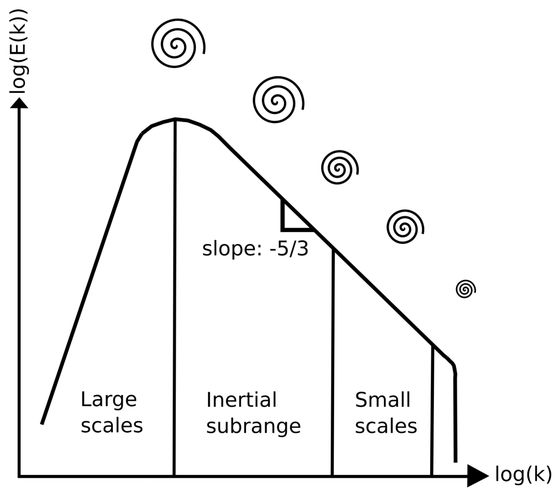
Identifying Locally Turbulent Vortices within Instabilities

Fabien Vivodtzev, Florent Nauleau, Jean-Philippe Braeunig *CEA CESTA*
Julien Tierny *CNRS, Sorbonne Université, LIP6*

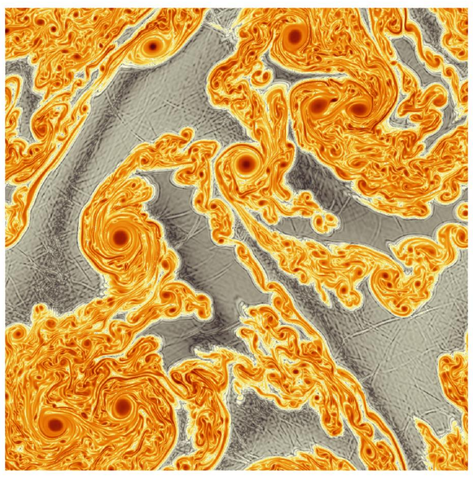


TURBULENCE CHARACTERIZATION

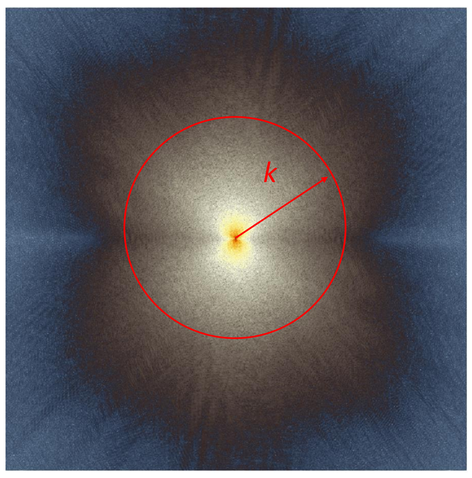
- **Wide range of vortex sizes** that fluctuate at different frequencies in a turbulent flow
- **Laminar flow** varies smoothly and predictably in space and time
- **Turbulent flow** exhibits a chaotic behavior
- **Andrey Kolmogorov** introduced statistical properties of turbulent flows
- **Turbulence** is characterized by energy transfers between these small and large eddies
- **Inertial subrange** describing intermediate range of eddy scales
- **Energy spectrum of turbulence *E(k)*** *:* mean turbulence kinetic energy as a function of the spatial frequency called **wavenumber *k***. *E(k)* has uniform behavior in the inertial subrange.
- **Energy transfer** from low to high wavenumber as the form of $E(k) \sim k^{-5/3}$ [1]
- **Kinetic energy spectrum** computed via a Fast Fourier transform and radial averaging [2]
- **Turbulence characterization** : slope of the linear regression of the energy spectrum



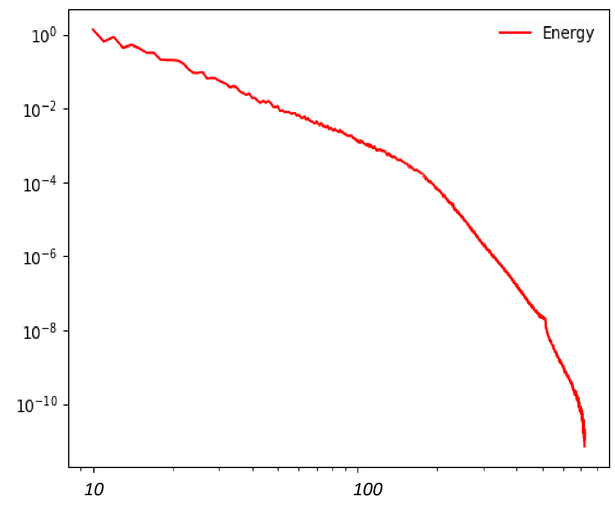
Kolmogorov energy cascade



Enstrophy scalar field



Shifted FFT with radial averaging over wavenumber k



Computed Kolmogorov energy cascade

CFD SIMULATION CODE

- **CFD simulation** code [3] of 2D compressible unsteady Euler equations for inviscid flows
- **Approximate Riemann solver** AUSM+-UP [4] to reconstruct strong discontinuities
- **Reconstruction scheme** : TENO 5 [5]
- **Periodic** boundary conditions
- **Enstrophy** defined locally as the square of the flow vorticity to capture a turbulence in 2D [6]

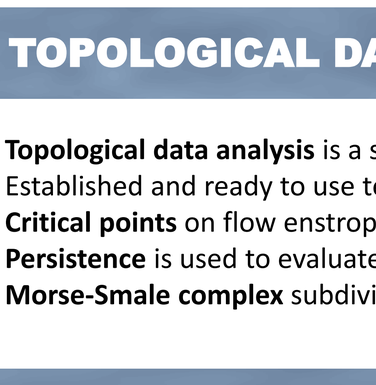
TOPOLOGICAL DATA ANALYSIS

- **Topological data analysis** is a set of techniques [7,8] focus on structural features in data
- Established and ready to use techniques in **Paraview** [9] and **Topology ToolKit** (TTK) [10]
- **Critical points** on flow enstrophy represent local maxima denoting the center of vortices
- **Persistence** is used to evaluate the importance of a critical point with persistence diagrams
- **Morse-Smale complex** subdivides a scalar field into regions of uniform gradient flow

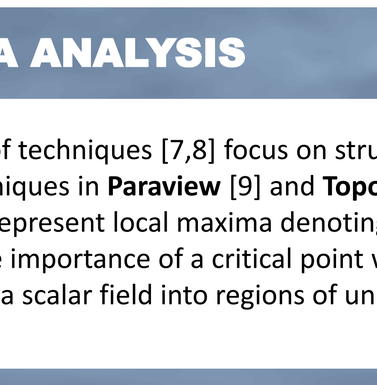
"Enhance the relevance of new indicators extracted with TDA regarding to traditional turbulence descriptors such as the energy cascade introduced in Kolmogorov work."

INTERPRETATIONS OF THE TURBULENCE INDICATORS

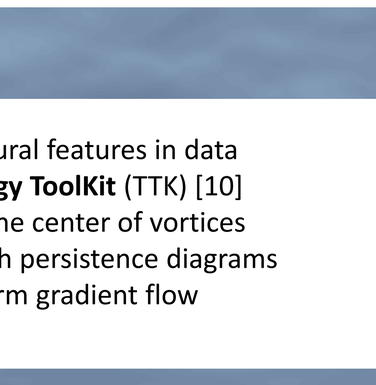
- **Morse-Smale segmentation** using persistence threshold on the enstrophy scalar field
- **Classification** of the segmented vortices performed by CFD Expert
- **Laminar vortices** in blue and **turbulent vortices** in orange
- **Kinetic energy spectrum** calculated for each segmented vortex
- **Turbulence indicator** as the slope of the linear regression on the intertial subrange
- TDA segmented vortices identified as turbulent by CFD experts **respect the property $E(k) \sim k^{-5/3}$**



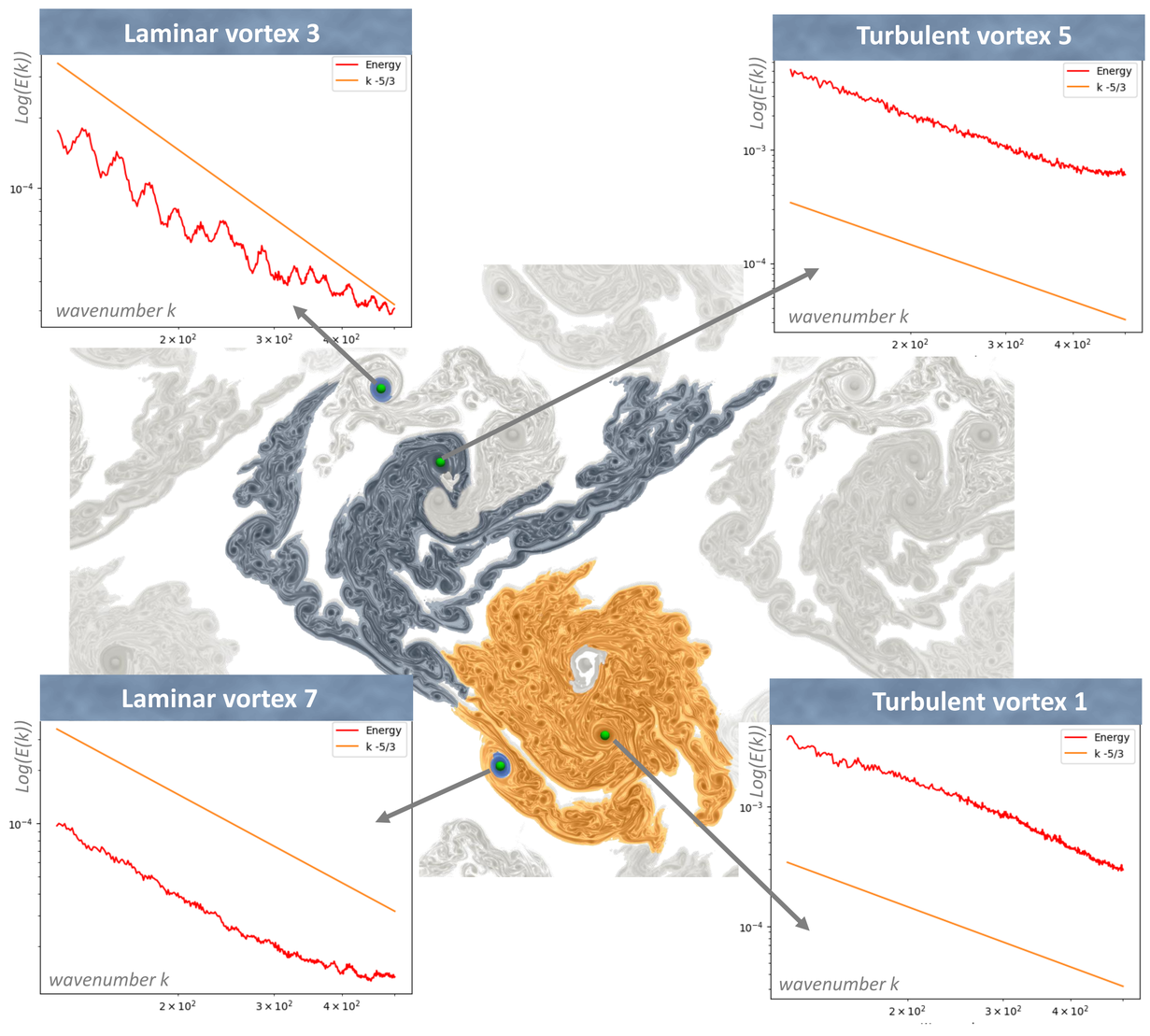
Vortex	*Slope*	*Type*	*diff $k^{-5/3}$*
6	-1,36	**laminar**	19%
14	-1,45	**laminar**	13%



Vortex	*Slope*	*Type*	*diff $k^{-5/3}$*
3	-1,18	**laminar**	29%
7	-1,41	**laminar**	16%
5	-1,51	**turbulent**	9%
1	-1,79	**turbulent**	7%



Vortex	*Slope*	*Type*	*diff $k^{-5/3}$*
0	-1,63	**turbulent**	2%
1	-1,58	**turbulent**	5%



Classification of segmented vortices (laminar and turbulent) with the turbulence indicator. Thresholded Morse-Smale segmentation and kinetic energy spectrum of the vortices

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