MATHEMATIQUES

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A \cup B = A + B - A \cap B
                                                                            A \subseteq B B \subseteq A dim(A) = dim(B) A = B A \cap B = A \cdot B | A = B \cdot A | B
 Axiomes d'extensionnalité :
                                              E = |n \in [-10, x] \cap \mathbb{Z} \mid x \in \mathbb{R} ; -3 < x \le 2| = |-2, -1, 0, 1, 2|  (A_1, A_2)|B = (A_1|B) \cdot (A_2|(B, A_1))
Logique:
                                                                                                  \neg (A \land B) \Leftrightarrow \neg A \lor \neg B
                                                (p \Rightarrow q) \Leftrightarrow (\neg p \lor q)
                                                                              x \Re x x \Re y \Leftrightarrow y \Re x (x \Re y \land y \Re x) \Rightarrow x = y (x \Re y \land y \Re z) \Rightarrow x \Re z
Relation binaire:
                                               f:E \rightarrow F | x \mapsto f\left(x\right) = y \quad E \rightarrow E \quad f \circ f^{-1} = e \quad c_{i,j} = \sum_{n=1}^{\infty} a_{i,n} \cdot b_{R,j} \quad dim\left(E,F\right) = dim\left(M_{np}\right) = n \times p
Application:
                                                     (E, *) a*b \in E (a*b)*c = a*(b*c) e*a = a x(y+z) = xy + xz a*b = b*a = e
Structure interne:
                                                                                      \varphi:(G, \star) \rightarrow (H, \star); \varphi(G_1 \star G_2) = \varphi(G_1) * \varphi(G_2) = H_1 * H_2
\underline{\mathbf{Lin\acute{e}arit\acute{e}:}} \qquad f(x,y) = f(a \cdot x + y) = a \cdot f(x) + f(y) \qquad F \neq \emptyset \qquad F \subset E \qquad \sum u_{[a,b]} + u_{[b,c]} = u_{[a,c]}
                                                     \sum_{i=1}^{n} \lambda_i \cdot e_i = 0 \Rightarrow \lambda_i = 0 \quad x = \sum_{i=1}^{n} \lambda_i \cdot e_i \quad L_i \leftarrow \lambda \cdot L_i; \quad L_i \leftarrow L_i + \lambda \cdot L_j; \quad L_i \leftarrow \lambda \cdot L_j \quad (A|I_n) \rightarrow (I_n|A^{-1})
                                                                               (DE)||(BC) \qquad (d') \qquad (AB)\nmid (AC) \qquad \tan(\phi) = \frac{\sin(\phi)}{\cos(\phi)} = \frac{\lfloor AB \rfloor}{\lceil BC \rceil}
Théorème de géométrie :
Conique: \Delta = b^2 - 4ac d = |\det(\overrightarrow{AP}, u, v)|/||u \wedge v|| \quad ||u \wedge v|| = ||u|| \cdot ||v|| \cdot \sin(u, v) \quad (a+b)(a-b) = a^2 - b^2
 \begin{array}{lll} \underline{\textbf{Lieu g\'eom\'etrique}:} & arg(z) = (\vec{u}\,, \overrightarrow{OM}) = \theta & z = \Re\left(z\right) + i\,\Im\left(z\right) = \rho\,\,e^{i\,\theta} & arg\left(Z_1\cdot Z_2\right) = arg\left(Z_1\right) + arg\left(Z_2\right) \\ & i^2 = j^2 = k^2 = ijk = -1 & q = a + bi + cj + dk = a + \vec{v} & q_1\,q_2 = \left(a_1\,a_2 - \vec{v_1}\cdot\vec{v_2}\right) + \left(a_1\,\vec{v_2} + a_2\,\vec{v_1} + \vec{v_1}\wedge\vec{v_2}\right) \end{array} 
Noyau: Ker f = f^{-1} \{e_F\} = \{x \in E | f(x) = e_F\} = \{X \in \mathbb{R}^n | A \cdot X = 0\}
Image:  Img f = f(E) = \{ y \in F | \exists x \in E, f(x) = y \} = vect((v_{colonne})_n) 
                                                                                                                                                                      Img f = F
                                                                      Rg(f) + dim Ker(f) = dim(E)
                                                                                                                                           Rg(f) = dim(Img(f))
Théorème du rang:
\underline{\textbf{Th\'eor\`eme isomorphisme}} \; : \; f: G \rightarrow G ', f(x \cdot H) = f(x \cdot Ker \, f) = f(x) \quad Card(G) = Card(Ker \, (f)) \times Card(Img(f)) = f(x \cdot Ker \, f) = f(x \cdot Ker \, f)
<u>Décomposition PLU:</u> A = P \cdot L \cdot U det(A) = det(P) \cdot det(L) \cdot det(U) P = \delta_{i,\sigma(j)} = 1 i = \sigma(j)
Evaluation polynome: P[X] = a_n X^n + ... + a_0 \quad (1, X, ..., X^n) \quad P \rightarrow u(P) = \sum_i (C_i) \cdot u(X^i)
Composition de transposition: \sigma = \begin{pmatrix} a & b & c \\ b & c & a \end{pmatrix} = (a \ b \ c) = (a \ b) \circ (b \ c)
\sigma \circ \sigma(a) = c \ ; \ \epsilon(\sigma) = (-1)^{N_t}
                                                                                    \forall : (n^2[2]=0 \Rightarrow n[2]=0) \Leftrightarrow (\neg (n[2])=1 \Rightarrow \neg (n^2[2])=1) \\ ((2k+1)[2]=1 \Rightarrow (2k+1)^2[2]=1)
Contraposé: A \Rightarrow B \equiv \neg B \Rightarrow \neg A
                                                                                                          \sqrt{2} = p/q; p[2] = 0, q[2] = 0 \Rightarrow \sqrt{2}[2] = 0
                                (A \Rightarrow B) \land (\neg B \Rightarrow \neg A)
Absurde:
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||x+y|| \le ||x|| + ||y|| P(|X| < a) \le \frac{E(|X|^p)}{a^p} P(A) = Cd(A) / Cd(\Omega)
                                                   |\langle x|y\rangle| \leq ||x|| ||y||
 Inégalité :
                                                  u(n) \sim_{+\infty} v(n) \qquad \qquad \lim_{n \to +\infty} \frac{u(n)}{v(n)} = \lim_{n \to +\infty} \frac{v(n)}{u(n)} = 1 \qquad \qquad \lim_{x \to 0} f(x, x) = \lim_{x \to 0} f(x, ax)
 Limite:
 Exponential: (e^{\pm i\theta})^n = (\cos(\theta) \pm i\sin(\theta))^n = \cos(n\theta) \pm i\sin(n\theta) e^{a+b} = e^a + e^b \ln(a^n) = n\ln(a) \log_p(x) = \frac{\ln(t)}{\ln(n)}
<u>Théorème continuité</u>: f: I \to \mathbb{R}, ([|x-a| < \delta \Rightarrow |f(x)-f(a)| < \epsilon]) C_i: [a^-, a^+]
 Boule : B(a,r) = \{x \in E \mid ||x-a|| < r\}
                                                                                                                               A = \{(x, r) \in \mathbb{R}^2, a \leq f(x, y) \leq b\}
 Théorème point fixe : g: E \rightarrow E
                                                                                                                                                        d(f(x), f(y)) < k \cdot d_E
                                                                                                                                                                                                                                            k \in [0,1]
 <u>Dérivée</u>: f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \frac{df}{dx} (f \circ f^{-1})' = 1 v(u)' = u' \cdot v'(u) |u| = \sqrt{x^2} (u \cdot v)' = u' v + v' u
<u>Théorème encadrement</u>: f \le g \le h \lim_{h \to L} f = \lim_{h \to L} h = L
                                                                                                                                                                         \lim g = L \lim \inf (u_n) = \lim \sup (u_n)
|f_n(x)| \leq a_n
 Règle d'Alembert :
 Régularité: C^1: \lim_{t \to p} f(t) = f'(p) C^2: \frac{\partial^2}{\partial x \partial y} = \frac{\partial^2}{\partial y \partial x}
 Serie de Taylor: a_k = \frac{f^{(n)}(a)}{k!} P(x) = \sum_{k=0}^{n} a_k (x-a)^k (1+x)^{\alpha} = 1 + \sum_{k=0}^{\infty} {\alpha \choose k} x^k
                                                   ||x(n)||_n = (|x_1(n)|^p + (\dots) + |x_n(n)|^p)^{1/p}
 Suite L^p:
                                          J_{F}(M) = \begin{vmatrix} \partial f_{1} & \partial x_{n} \\ \partial x_{1} & \partial f_{m} \end{vmatrix} \qquad \qquad \phi(x,y) \Rightarrow \phi(r,\theta) \; \; ; \; \; J_{\phi} = \begin{vmatrix} \cos \theta & -r \sin \theta \\ \sin \theta & r \cos \theta \end{vmatrix}
 Jacobien:
                                         f(z) = \frac{q(z)}{p_0(Z).(\dots).p_j(z)} \qquad \operatorname{Res}(f(z), p_i(z)) = \lim_{z \to p_i} q(z) / \prod_{i \neq i} p_j(z)
 <u>Résidu :</u>
\underline{\textbf{Crit\`ere d'int\'egration :}} \lim_{t \to [a^*, +\infty]} (t-a)^\alpha f(t) = 0 \quad \int\limits_a^b f(t) \, dt = \frac{b-a}{n} \sum_{n=1}^{N \to +\infty} f\left(a + k(b-a) / n\right) \\ \sum \left(n + m\right) = \sum \left(n(1 + \frac{m}{n})\right) \left(n + \frac{m}{n}\right) = \sum \left(n(1 + \frac{m}{n})\right) \left(n + \frac{m}{n}\right) \\ = \sum \left(n(1 + \frac{m}{n})\right) \left(n + \frac{m}{n}\right) \left(n + \frac{m}{n}\right) \\ = \sum \left(n(1 + \frac{m}{n})\right) \left(n + \frac{m}{n}\right) \left(n + \frac{m}{n}\right) \\ = \sum \left(n(1 + \frac{m}{n})\right) \left(n + \frac{m}{n}\right) \left(n + \frac{m}{n}\right) \\ = \sum \left(n(1 + \frac{m}{n})\right) \left(n + \frac{m}{n}\right) \left(n + \frac{m}{n}\right) \\ = \sum \left(n(1 + \frac{m}{n})\right) \left(n + \frac{m}{n}\right) \left(n + \frac{m}{n}\right) \\ = \sum \left(n(1 + \frac{m}{n})\right) \left(n + \frac{m}{n}\right) \left(n + \frac{m}{n}\right) \\ = \sum \left(n(1 + \frac{m}{n})\right) \left(n + \frac{m}{n}\right) \\ = \sum \left(n(1 + \frac{m}{n})\right) \left(n + \frac{m}{n}\right) \\ = \sum \left(n(1 + \frac{m}{n})\right) \left(n + \frac{m}{n}\right) \\ = \sum \left(n(1 + \frac{m}{n})\right) \left(n + \frac{m}{n}\right) \\ = \sum \left(n(1 + \frac{m}{n})\right) \left(n + \frac{m}{n}\right) \\ = \sum \left(n(1 + \frac{m}{n})\right) \left(n + \frac{m}{n}\right) \\ = \sum \left(n(1 + \frac{m}{n})\right) \left(n + \frac{m}{n}\right) \\ = \sum \left(n(1 + \frac{m}{n})\right) \left(n + \frac{m}{n}\right) \\ = \sum \left(n(1 + \frac{m}{n})\right) \left(n + \frac{m}{n}\right) \\ = \sum \left(n(1 + \frac{m}{n})\right) \left(n + \frac{m}{n}\right) \\ = \sum \left(n(1 + \frac{m}{n})\right) \left(n + \frac{m}{n}\right) \\ = \sum \left(n(1 + \frac{m}{n})\right) \left(n + \frac{m}{n}\right) \\ = \sum \left(n(1 + \frac{m}{n})\right) \left(n + \frac{m}{n}\right) \\ = \sum \left(n(1 + \frac{m}{n})\right) \left(n + \frac{m}{n}\right) \\ = \sum \left(n(1 + \frac{m}{n})\right) \left(n + \frac{m}{n}\right) \\ = \sum \left(n(1 + \frac{m}{n})\right) \left(n + \frac{m}{n}\right) \\ = \sum \left(n(1 + \frac{m}{n})\right) \left(n + \frac{m}{n}\right) \\ = \sum \left(n(1 + \frac{m}{n})\right) \left(n + \frac{m}{n}\right) \\ = \sum \left(n(1 + \frac{m}{n})\right) \left(n + \frac{m}{n}\right) \\ = \sum \left(n(1 + \frac{m}{n})\right) \left(n + \frac{m}{n}\right) \\ = \sum \left(n(1 + \frac{m}{n})\right) \left(n + \frac{m}{n}\right) \\ = \sum \left(n(1 + \frac{m}{n})\right) \left(n + \frac{m}{n}\right) \\ = \sum \left(n(1 + \frac{m}{n})\right) \left(n + \frac{m}{n}\right) \\ = \sum \left(n(1 + \frac{m}{n})\right) \left(n + \frac{m}{n}\right) \\ = \sum \left(n(1 + \frac{m}{n})\right) \left(n + \frac{m}{n}\right) \\ = \sum \left(n(1 + \frac{m}{n})\right) \left(n + \frac{m}{n}\right) \\ = \sum \left(n(1 + \frac{m}{n})\right) \\ = \sum \left(n
 <u>Théorème fondamental d'analyse :</u>
                                                                                                                                    A'(x)=f(x) \int f(x) dx = F(b) - F(a)
 <u>Théorème changement de variable</u>: \int g(y_i) dy_i = \int g(F(x_i)) \cdot |\det J_F(x_i)| dx_i \quad dy = f'(x) dx \quad , \quad \alpha = f'(a)
 \underline{\textbf{Th\'eor\`eme convergence domin\'e}:} \quad (f_n) \in (E,A,\mu) \rightarrow f \qquad \lim_{n \rightarrow +\infty} \int f_n(\mu) \, d\mu = \int \lim_{n \rightarrow +\infty} f_n(\mu) \, d\mu
<u>Théorème de transfert</u>: G = E[g(X)] = \int g(x) f_X(x) dx = \sum g(x_i) . f(x_i) F_X = P(X \le x)
Théorème central limite : \lim_{n \to +\infty} P(Z_n < z) = \Phi_{N(0,1)}(z) \qquad \sigma \to \frac{\sigma}{\sqrt{n}}
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