The Political Economy of Autocratic Transitions: The Role of Delegation of Violence*

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PRELIMINARY DRAFT - DO NOT CIRCULATE

Abstract

In this paper, we introduce a political economy model of institutional change leading to a transition from democracy to autocracy. A powerful political center lacks commitment to repress surges of instability which threaten the economic elites it represents. A process of institutional delegation of political power to a violent group allows the center to gain the commitment it lacks and to re-establish political stability. However, the limited forward-looking capacity of the democratic political system induces excessive delegation of power, leading the violent group to establish an autocratic government. The institutional mechanism leading to autocracy delineated in the paper fits well the rise to power of Fascism in Italy after World War I - from 1919 to 1925 - but the main political economy component of the mechanism we identify are common to several historical transition phenomena.

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1 Introduction

Ad Hitler è necessario opporre Hitler.

Technique du coup d'état, Ital. Transl. 1948

Sti quattro delinquenti co' le facce come er sego
portavano la morte e il me ne frego
anche noi ce ne saressimo fregati
se il governo come a lor ci avesse armati...

Canto Popolare di San Lorenzo

The first elections after World War I were held in Italy in 1919. They were marked by a triumph of the *Partito Socialista Italiano* and a resounding defeat of the *Fasci Italiani di Combattimento*, the newly founded party by Benito Mussolini and Tommaso Filippo Marinetti.[1] In 1922 Mussolini was - chosen by the king and voted by Parliament as - Prime Minister. In 1925 he delivered a speech to Parliament that marked the beginning of the Fascist autocratic regime. In particular, he famously took the moral and political blame for the murder of the head of the socialist party, Giacomo Matteotti: "If fascism is a criminal organization, I am its leader!" The most direct consequence of this speech was the definitive repression of press and political freedom.[2] How did a marginal political force like Mussolini's *Fascio* become the most influential party in the Italian Kingdom in such a short time, eventually leading to twenty years of Fascist dictatorhsip?

Several different transition paths from democracy to autocracy are possible and have been realized in history; see Levitsky and Ziblatt (2019). In this paper we aim at identifying the main political economy components of the institutional mechanism responsible for this one specific historical event: the advent of Fascism in Italy after World War I (WWI). It is well recognized, however, that the rise to power of Fascism in Italy contains several main elements common to other historical transition events.

> Some version of this story [the advent of Mussolini's Fascist Party] has repeated itself throughout the world over the last century. A cast of potential outsiders, including Adolf Hitler, Getulio Vargas in Brazil, Alberto Fujjimori in Peru, and Hugo Chavez in Venezuela came to power on the same path [Levitsky and Ziblatt (2019), *How Democracies Die*, p. 13]

To identify conceptually a set of core political-economy components of dramatic institutional and political power changes we rely on the general theory of institutional change proposed by Bisin and Verdier (2024). Abstractly,[3] this

[1] We use Italian when referring to proper nouns.

[2] More generally, January 3 1925 is considered the beginning of the fascist regime; see De Felice (2019a), page 726-8.

[3] The following paragraph is taken almost literally from Bisin and Verdier (2024).

theory of institutions is postulated on a society populated by distinct political groups of agents — say, elites and civil society — characterized by distinct economic resources (e.g., elites have more resources or a different technology to obtain them), political power, preferences and values. Each time period, a policy game is played between individual agents and a socioeconomic policy authority (the government). Institutions represent the relative political power of these groups in civil society to affect policy decisions. The government's choice maximizes a social welfare function that encodes the distribution of political power between the groups (institutions), given their preferences and values. A set of government policies and agents' actions arise as societal equilibrium outcomes. Institutions evolve as the result of a process of optimal political delegation, changing the distribution of political power to internalize externalities, lack of commitment, and other distortions leading to an inefficient societal equilibrium outcomes. As a consequence, residual decision rights over public policy tend to be delegated to those political groups that are better able (or have the highest incentives) to internalize the externalities affecting the policy game. Even though institutional change is designed to respond to the inefficiencies of equilibrium outcomes, the societal equilibrium at the stationary state of the dynamics is not necessarily efficient, nor does it lead to a Pareto improvement in society. In particular, a limited forward-looking capacity of institutional design may lead to excessive delegation of power to a political group.

In the theoretical model, institutional change can then be represented to depend on the following factors: i) the political preferences/incentives of elites and civil society, ii) the structure and the characteristics of their political representation, iii) the societal policy game and its externalities/inefficiencies, and iv) the forward-looking properties of institutional design. In the context of the Italian history of the rise to power of Fascism these factors can be delineated as follows. i) The main socio-economic groups which constitute elites and civil society can be described as aristocracy and bourgeoisie on one side and workers, farmers on the other. The political preferences/incentives of these groups align in that all have an interest in the economic growth of the economy, but conflict over the distribution of the rents of this process. ii) As for political representation, the Socialists represent workers and farmers; the Liberals and the Conservatives in the center represent aristocracy and bourgeoisie - as in part do the Fascists as well. The fundamental characteristic separating the Liberals and the Fascists is the greater attitude towards violence of the Fascists. iii) The fundamental inefficiencies are the consequence of the fight over the distribution of the rents of the industrialization process. iv) Institutional change is very fast and relatively myopic.

We then specialize the general model of institutional change we delineated to account for these factors. More specifically, we consider a political environment characterized by a powerful political center lacking commitment to repress surges of instability which threaten the economic elites it represents. The institutional change process aims at reducing the inefficiencies due to the economic instability by delegating political power to the political groups better able to repress it. Institutional delegation of political power to a violent group allows

the center to gain the commitment it lacks and to re-establish political stability. However, the limited forward-looking capacity of the democratic political system - in the form of myopic expectations - induces excessive delegation of power, possibly leading the violent group to establish an autocratic government.

We interpret the analysis of the model as identifying the following three main political economy components of the institutional change mechanism as responsible for a possible transition to autocracy: i) a socio-economic equilibrium plagued by severe inefficiencies affecting the elites, ii) the option of (alleviating these inefficiencies by) delegating violence e.g., to revolutionary political groups, iii) limited forward-looking capacity of the democratic political system with regards to the process of institutional change.[4]

In the context of post-WWI Italy, these three components are arguably central to the rapid institutional change leading to the rise of the Fascist regime. In the early 1920s, the Fascists were a marginal political force, but they had clearly already demonstrated an attitude for the exercise of violence against political oppositions, especially the socialists. They rose to become prominent and gain autocratic power, in the span of only six years, over the Liberal-Conservative establishment—the traditional political force representing the elites, which had been controlling the Italian political scene since the Unification (1861).[5] Indeed, afraid of the possibility of a socialist revolution, the Liberal-Conservative establishment—under the guidance of Giovanni Giolitti and his successor Ivanoe Bonomi—deliberately delegated to the fascists the violent repression of the strikes and other forms of socialist protests. Delegation of violence required substantial delegation of political power to Mussolini and his party, even though strongly minoritarian in the parliament and in the country. Though the Fascists indeed engaged actively and violently against the Socialist Party, the political representatives of the Liberal-Conservative establishment acted with very limited forward-looking capacity in that they did not expect that their institutional delegation of violence would render the Fascists the dominant political force in Italy for the next twenty years.

In Section 2 we outline the theory of institutional change in Bisin and Verdier (2024), showing how it can be projected onto a political economy of autocratic transition. In Section 3 we present the model of institutional change and derive conditions under which the path of political power converges - away from democracy - into autocracy. In Section 4 we map the narrative of the historical events leading Italy to the autocratic transition to Fascist rule into the main politi-

[4]Limits to the forward-looking aspects of institutional design are also documented in the opposite historical process than the one we concentrate on in this paper: the transition from autocracy to democracy. Treisman (2017) argues that in the majority of the democratization events he classifies (in about 65% of them, in fact) democracy has not been primarily the outcome of deliberate institutional choice but rather of various forms of miscalculation and lack of anticipation of the effects of the process set in motion by institutional change. In particular, in almost in half of these instances, the process inducing democratization is characterized by the fact that the "incumbent initiates a partial reform, [...] but cannot stop" (see Table 2 in the paper), a representation which closely maps our modeling of myopic institutional change.

[5]In fact, these liberal and conservative forces had been central to the politics of the Kingdom of Sardinia since the formation of Parliamentary rule in 1848.

cal economy components of the institutional change mechanism identified in the model. In Section 5 we outline how our theory can be applied to other historical instances of democratic backsliding into autocracy. Section 6 concludes.

1.1 Related Literature

The general theory of institutions and institutional change in Bisin and Verdier (2024) can be interpreted to have roots in the traditional classic approach to political science, from the central role of city factions in Machiavelli's Istorie Fiorentine (1532), to the the concept of classes in marxist thought (see e.g., Balibar (1970) and Poulantzas (1973)). The central role of elites and civil society, in this theory, takes also root in the (Italian) Theory of the Elites, developed at the beginning of the 1900's. Power relationships induce institutional change in society through the Circulation of the Elites - a mechanism which guarantees that elites, as opposed to civic society, hold the most power and maintains it over time.[6]

More recently, institutional change - as a mechanism to partly internalize the externalities and inefficiencies of the political system - has been pioneered by Acemoglu, Johnson, and Robinson (2001), Acemoglu and Robinson (2006) and has been greatly expanded by the recent formal political economy literature.[7] We refer to Bisin and Verdier (2021) for a survey of the theoretical aspects of these contributions. These recent studies on institutional change, however, predominantly center on the transition from autocracy to democracy rather than on the opposite transition from democracy to autocracy as in the present paper.

We do not attempt in this paper to define precisely what construes an autocratic regime in the abstract (Stanley (2018)). In the context of our analysis autocracies are defined to be political institutions whose government policies are designed to favor solely the members or the constituency of the political party in power. In particular, we will not make any distinction between fascism and populism as forms of autocracy (Finchelstein (2019)), nor between majoritarian and technocratic democracy (Gratton and Edenhofer (2025)).

Our main analytical narrative builds upon the historical data on Italian political economy post-WWI - and the advent of the Fascist regime, as extensively expanded in the analyses of De Felice De Felice (2011), De Felice (2019a), De Felice (2019b), De Felice (2020); Tasca Tasca (1938). It is consistent with the empirical results of Acemoglu, De Feo, et al. (2022b), showing a strong link between support for the Socialists after World War I and the support for fascism from the 20's.

[6] Appendix A.1 contains relevant quotes of the main exponents of the group of political scientist which have put forth the (Italian) Theory. It should not be surprising that the (Italian) Theory has anticipated - and, in some interpretations, served a s a theoretical justification - of the transition to Fascism.

[7] see for instance, Acemoglu and Robinson (2010), Acemoglu, Aghion, et al. (2012), Acemoglu, García-Jimeno, and Robinson (2015), Acemoglu, Akcigit, et al. (2018), Besley and Persson (2009), Besley and Persson (2010), Besley and Persson (2011), Bisin and Verdier (2024), Tabellini (2008)

More generally, our work is related to the current and rapidly growing literature in political sciences on *democratic backsliding*. This line of research investigates the determinants and conditions under which democracies erode, focusing on the strategic behavior of various actors in the polity (see Gandhi (2019), Haggard and Kaufman (2021), and Grillo et al. (2024) for a recent survey).

Typically this approach emphasizes the nature of the constraints on the executive power, which may be *vertical* (exercised by citizens and voters) or *horizontal* (enforced by institutional actors such as courts, legislatures, and the media). Democratic erosion unfolds when these constraints are circumvented or weakened, enabling incumbents to entrench their authority.

Common to our setup, this line of research acknowledges the existence of various political distortions—such as imperfect or asymmetric information, and vertical or horizontal externalities— that create opportunities for incumbents to consolidate power. These distortions, depending on their nature and location, enable strategic manipulation of electoral processes (the extensive margin of *backsliding*) or the unchecked expansion of executive policymaking authority (the intensive margin of *backsliding*). Whether through overt institutional changes or more subtle forms of norm erosion, *backsliding* is also portrayed as a gradual process of power change shaped by actors facing difficulties to fully assess the future implications of power dynamics (Grillo et al. (2024)).

Our analysis departs from these approaches in two key respects. First, we adopt a more explicit socioeconomic framework, where fundamentals such as preferences, technologies, and opportunity costs interact with concrete policy instruments, thereby shaping the configuration of political interests in society. Second, we offer a distinct perspective on the dynamics of backsliding. Rather than focusing on how incumbents expand discretionary power by circumventing institutional checks and balances, we conceptualize backsliding more abstractly, as a political delegation problem in public decision-making, whereby an incumbent power structure gradually shifts authority toward an initially marginal but radical vanguard group.

Finally, Becker and Voth 2023 identifies a cultural determinant of Nazism and Fascism in "shallow Christianity." The interaction of cultural and institutional determinants of autocratic transitions is a fascinating topic to which the theoretical construct of Bisin and Verdier 2024 could be proficuously applied.

2 A Theory of Institutional Change

In this section, we outline the theory of institutions and institutional change developed by Bisin and Verdier (2024) which will serve as theoretical underpinning of the analysis of the transition from democracy to autocracy we study in the paper.[8]

[8] While we refer the interested reader to the paper for related literature, formal details, and applications - we discuss some foundational references in the Appendix

Institutions are conceptually defined as a reflection of the relative power of different socio-economic groups in society. Socioeconomic groups are generally intended as specific historical manifestations of elites and civil society and may be represented by distinct political counterparts.

More specifically, institutions are modeled as the mechanisms through which policy choices are delineated and implemented. While political dynamics in a society may involve various context-specific elements, we will take here the general view that power change is essentially a mechanism that tends to reallocate public policy decision rights across political groups in a way to internalize externalities and inefficiencies which plague social choice problems.

In this perspective, consider a simple abstract structure of society composed of different socio-economic groups indexed by i. Each period t, a societal policy game is played between individuals belonging to each group i and a public policy authority (the state) controlling relevant socio-economic policies.

Individuals in each group i are characterized by an objective function $u_i(a_i, r)$ that depends on private actions a_i and a relevant policy vector r. Policies are the outcome of a (collective) decision problem, in accordance with the distribution of political power between the different groups encoded and represented by institutions. Institutions will be abstractly defined and represented by the (Pareto) weights $\beta = (\beta_i)_i$ of the different groups in the policy making problem regarding the policy vector. Specifically, the objective of the public policy authority will be identified by a Social welfare function of the form

$$W(\beta) = \sum_i \beta_i u_i(a_i, r) \tag{1}$$

Given institutions β, a set of policies $r = r(\beta)$ and actions $a = a(\beta)$, characterize an equilibrium of the societal policy game between individuals and the public authority.

The equilibrium outcomes $a(\beta)$, $r(\beta)$ of the societal policy game do not fully internalize their impacts on aggregate social outcomes, and as a result inefficient policies and social allocations may be implemented. Indeed, several economic and political externalities generally inform the policy game. First of all, the individuals' strategic choices might be characterized by coordination issues or tragedies of the commons. More generally, externalities may arise because of various socio-economic frictions, like e.g., asymmetric/incomplete information, matching, limited rationality and cognitive biases, strategic behavior associated with market power. Furthermore, the political economy structure of society might in itself give rise to inefficiencies, for instance because of lack-of-commitment on the part of the (collective) policy decision problem, choosing r simultaneously with respect to the choices a of the individuals in society.

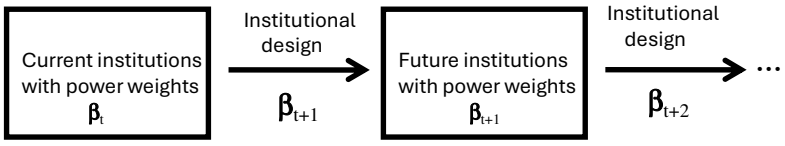
Institutional change is represented by the outcome of an institutional design mechanism driving changes of political power across the different groups. Consider a society characterized by a power structure β_t at any point in time t. Such a society might have an incentive to change the distribution of political power in the future, to internalize the externalities responsible for the inefficiencies at equilibrium. The mechanism for institutional change directs political

power - residual decision rights - towards the group whose private actions at equilibrium lead most effectively to internalize the externalities. More precisely, institutional change follows from institutional design, conceptualized as a mechanism for optimal political delegation.[9] This conceptualization of institutional change is mapped into formal dynamics of political power weights $\beta(t)$. The specific formalization of the dynamics depends on how forward-looking is the institutional design process. In general, the design process at time t will anticipate the fact that by moving to a different structure of decision rights $\beta(t+1)$, may in turn trigger subsequent institutional changes $\beta(t+2)$, $\beta(t+3)$... In the case of one-step-ahead institutional design, a relatively simple dynamic process is obtained,

$$\beta(t+1) \in \arg\max_{\beta'} \sum_i \beta_i(t) u_i\Big(a_i(\beta'), r(\beta')\Big) \tag{2}$$

This process induces a mapping from the political weight structure at t, $\beta(t)$, into the one at $t+1$, $\beta(t+1)$, as schematically illustrated in Figure 1. Formally, the mapping is represented by a differential/difference equation system whose solution describes the path of institutional change in the society.

Figure 1: Institutional Design



Institutional change which satisfies Equation 2 is characterized in detail by Bisin and Verdier (2024). Refererring to the orginal paper for details, we outline here the main intuition for the characterization. The optimal delegation of political power resulting from Equation 2 internalizes - one-step-ahead at the

[9] This is related to the concept of optimal allocation of decision rights in organizational economics and corporate finance; see e.g., Coase (1937), Williamson (1996), Grossman and Hart (1992).

time - inefficiencies due to the direct externalities in the societal game and to the lack of commitment of the policy maker. In particular, the policy r is chosen without commitment. But suppose the policy authority could commit to a policy r^{com}, that is, it could optimally choose r^{com} before and anticipating the Nash equilibrium I, p for any choice r at time t. Policy r^{com} leads to higher social welfare, when social welfare is evaluated with respect to the distribution of power prior to the change, $\beta(t)$. As a consequence, given current set of institutions at t, institutional design will depend on

$$R\Big(\beta(t)\Big) = r^{com}(\beta(t)) - r(\beta(t)),$$

which is an indicator of the extent of the inefficiency - the policy commitment problem - faced by such society.[10] Specifically, the absolute value of $R\Big(\beta(t)\Big)$ indicates the intensity of the commitment problem, reflecting the distance between what can best be achieved under commitment and what is actually achieved at equilibrium. The sign of $R\Big(\beta(t)\Big)$, on the other hand, indicates the direction of institutional change in $\beta(t)$ needed to ameliorate the commitment problem. More precisely and operationally, given a current set of institutions in period t, $\beta(t)$, institutional design will choose a $\beta(t+1)$, so as to decrease $R\Big(\beta(t)\Big)$ in absolute value; that is, so that $r(\beta(t+1))$ will be close to $r^{com}(\beta(t))$. In other words, optimal delegation at time t favors those groups i whose equilibrium actions have the property that an increase in relative political power $\beta_i(t)$ decreases $\mid R\Big(\beta(t)\Big) \mid$; that is, more formally,[11]

$$\frac{\partial r(\beta(t)}{\partial \beta_i(t)} \text{ and } R(\beta(t) \text{ have the same sign} \tag{3}$$

Institutional change, in this conceptualization, tends to drive society toward the efficient internalization of externalities. Nonetheless, it is not generally the case that institutions are efficient in the limit of this process. Most importantly, it is generally not the case that institutional change leads to a Pareto improvement in society. In particular, limitations to the forward-looking aspects of institutional design may lead to excessive delegation of power to a political group. When such limitations are relaxed, in fact, the anticipation of institutional "slippery slopes" may optimally induce a slow-down or even a stop in the the process of delegation.[12]

[10] For simplicity of exposition we suppose here that the policy r is unidimensional.

[11] When the policy r and $R\Big(\beta(t)\Big)$ are vectors, then optimal delegation should favor groups i whose equilibrium actions have the property that an increase in their relative political power $\beta_i(t)$ decreases the vectorial norm $\mid R\Big(\beta(t)\Big) \mid^2$, and condition (3) is equivalent to the vector product $\frac{\partial r(\beta(t)}{\partial \beta_i(t)} \times R\Big(\beta(t)\Big)$ being positive.

[12] The role of forward-looking processes is studied by Acemoglu, Egorov, and Sonin (2015) and Bisin and Verdier (2024). Lagunoff (2009) provides a general theoretical study of political economy equilibria with dynamic endogenous institutions.

2.1 Institutional Transition from Democracy to Autocracy

Consider a society where elites and a civil society share political power through representation by different political organizations in the democratic process. In this context, a transition from democracy to autocracy is a dynamic path of political power weights which converges to the concentration of political power onto one single political organization. Formally, that is, to a stationary state where $\beta_i = 1$, for some i.

> *A transition to an autocratic regime of group i, from an initial institutional state $\beta(t)$, is a dynamic path of political power weights such that, for all $\tau \geq t$,*
>
> $$\beta_i(\tau + 1) > \beta_i(\tau), \text{ and } \lim_{\tau \to \infty} \beta_i(\tau) = 1. \tag{4}$$

The dynamics of power weights $\beta(t)$ is the result of an optimal political delegation mechanism, possibly under limited forward-looking in institutional design, as in Equation (2). Optimal delegation favors those groups i whose actions - at each point in time (when institutional design takes place) - lead to internalizing the externalities at the equilibrium of the societal policy game; that is, formally those groups i such that Condition (3) is satisfied. As a consequence, a transition to an autocratic regime of group i follows when institutional delegation satisfies a monotonicity property; that is, when along the dynamic path of relative political power it is always and only the same group i whose relative political power $\beta_i(t)$ decreases $\mid R\Big(\beta(t)\Big) \mid$. More formally,[13]

> *A transition to an autocratic regime of group i, from an initial institutional state $\beta(t)$, occurs when* institutional delegation is monotonic*, that is when, for all $\beta_i(\tau) \geq \beta_i(t)$ and $j \neq i$,*
>
> $$\frac{\partial r(\beta(\tau)}{\partial \beta_i(\tau)} \quad \text{and} \quad R(\beta(\tau) \text{ have the same sign}$$
> $$\frac{\partial r(\beta(\tau)}{\partial \beta_j(\tau)} \quad \text{and} \quad R(\beta(\tau) \text{ have the opposite sign}$$

Importantly, as we noted, the process of institutional change does not necessarily lead to efficiency nor to Pareto dominance. Indeed, especially with limited forward-looking aspects of institutional design, the delegation process leading to autocracy might produce welfare costs to several of the socio-economic groups in society, by inducing excessive delegation of power along the path.

[13] When r is a multidimensional policy vector, a transition to an autocratic regime of group i occurs when institutional delegation is monotonic and for all $\beta_i(\tau) \geq \beta_i(t)$ and $j \neq i$, the vector products $\frac{\partial r(\beta(\tau)}{\partial \beta_i(\tau)} \times R(\beta(\tau)$ are positive while the vector products $\frac{\partial r(\beta(\tau)}{\partial \beta_j(\tau)} \times R(\beta(\tau) <$ are negative.

3 A Model of the Transition to Fascist Rule

In the context of the theoretical model, institutional change depends then on several factors: i) the political preferences/incentives of elites and civil society, ii) the structure and the characteristics of their political representation, iii) the societal policy game and its externalities/inefficiencies, and iv) the forward-looking properties of institutional design.

3.1 Institutional Change Factors in Italy after WWI

In this section we map the historical conditions in Italy after WWI into these factors. This map will inform our modelling exercise aiming at characterizing the conditions leading to an institutional process of transition from democracy to autocracy.

i) The main socio-economic group which constitute elites and civil society in Italy after WWI can be described as workers (blue collar workers in the industrial sector and farmers), landed aristocracy and the upper-bourgeoisie (industrial and financial capital),[14] and lower-bourgeoisie (shop keepers, artisans, government employeees and land owners). The political preferences/incentives of all these socio-economic groups align in that all have an interest in the economic growth of the economy, which in the historical context translates into investment and industrialization. On the other hand, industrial workers and upper-bourgeoisie, as well as the landed aristocracy and the farmers, have opposing demand/preferences over the distribution of the rents of this process. The preferences of the lower-bourgeosie - at this turn of economic development of the country - align well with those of the upper-bourgeoisie with respect to economic rents.

ii) Three political groups (parties or coalitions) represent elites and civil society:[15] the Socialists representing workers; the Liberals and the Conservatives (the Center from now on) represent the aristocracy and the upper-bourgeoisie; and the Fascists represent the lower-bourgeoisie. The Partito Popolare Italiano - the Catholic party - while on the center, plays an interesting and somewhat distinct role with respect to Liberals and Conservatives: it provides religious legitimacy to the Center in the political process. The fundamental characteristic separating the Center and the Fascists is the greater attitude towards violence of the Fascists.

iii) The fundamental inefficiencies of the societal policy game at equilibrium are a deadweight loss that is a consequence of the fight, between the Socialists on one side and the Center and the Fascists on the other, over the distribution of the rents of the industrialization process. A public policy authority has the

[14] The alignment of the interests of the industrial bourgeoisie (prevalently in the North of Italy) and the landed aristocracy (in the South) represents the main elements of Antonio Gramsci's analysis of the political economy of Risorgimento and Fascism; see Gramsci (1948-51).

[15] This is necessarily a coarse categorization, since political preferences are hardly univocally determined by socio-economic class.

role of pursuing violent repression of the actions of the Socialists which, while intended to appropriate a (larger) share of these rents, reduce the incentives for investment and industrialization.

iv) Institutional change is very fast and relatively myopic, due to the naivetè of the Center, over-estimating their power to stop the process of institutional delegation of violence to the Fascists.

Consider a society where the three political groups are identified with $i \in \{C, S, F\}$: C denotes the members of the Center, S the Socialists, F the Fascists. We denote the structure of political power between the three groups by the political weights (β_i) with $\sum_{i \in \{C,S,F\}} \beta_i = 1$.

3.2 Institutional Delegation towards the Fascist Party

We start with a simple structure in which the relevant public policy $r \geq 0$ is unidimensional and directly related to violent repression against protests by the Socialists.[16] Each group i has an endowment $\omega_i > 0$. Each group's utility and choice problem are next described.[17]

Members of the *Center* have a technology to undertake a productive investment I, at a strictly increasing and convext costs $C(I)$. The return of I is affected by the extent of protests $p \in [0, 1]$ in society. Formally,

$$u_C(I, p) = \omega_C + I(1 - p) - C(I).$$

Socialists have the organizational ability (and the ideological attitude) to undertake protests $p \in [0, 1]$ at strictly increasing and convex costs $V(p, r)$, which increase with the level of state repression r. On the other hand, the take-up of protests on the part of Socialists allows them to extract rents γpI from from the investment of the Center. The parameter $\gamma \in (0, 1]$ represents the deadweight loss in the extraction process. Formally,

$$u_S(I, p, r) = \omega_S + \gamma pI - V(p, r).$$

Fascists also enjoy spillovers $\alpha \in (0, 1]$ from investment I undertaken by members of the Center. Fundamentally, however, they are ideologically opposed to Socialists: while they have no explicit choice problem, they derive a strictly increasing and concave utility $\Phi(r)$ from repression r of the public authority against the Socialists.[18] Their utility function is:

[16] This is, for instance, repression undertaken by the police or the governmental military forces.

[17] In a first approximation we abstract from the role of the catholic component of the Center - the Partito Popolare Italiano - and the Catholic Church. We will study this in detail in the next section.

[18] In order to highlight in the most salient way the political delegation logic of power to the fascist group, we focus exclusively on the centralized government repression of the socialist protests, abstracting away from the violent acts exerted in a decentralized fashion by members of the fascist movement. An extension of the model allowing for the Fascists to exercise violence directly - with a richer set of similar and related results - is studied in the Appendix.

$$u_F(I, r) = \omega_F + \alpha I + \Phi(r).$$

Finally, a *Public policy authority* chooses the (intensity of) repression r to maximize the social welfare function of society with relative political power β of the groups at strictly increasing and convext costs $\Psi(r)$:[19]

$$W(r, I, p; \beta) = \beta_C u_C(I, p) + \beta_S u_S(I, p, r) + \beta_F u_F(I, r) - \Psi(r).$$

Under our assumptions,[20] the outcome of the societal game between the members of the three groups - Center, Socialists, Fascists - and the public choice authority can be characterized by its Nash equilibrium.[21] At a Nash equilibrium the members of the Center solve

$$\max_{I \geq 0} \; u_C(I, p)$$

given p, r; the Socialists solve

$$\max_{0 \leq p \leq 1} \; u_S(I, p, r)$$

given I, r; and the public policy authority (state) solves

$$\max_{r \geq 0} W(\beta, I, p, r)$$

given I, p.

We first analyze the equilibrium investment $I^O(r)$ and protests $p^O(r)$ for given repression r. Under our assumptions they are determined by the first order conditions of the problem of the members of the Center and the Socialists, respectively:

$$1 - p = \frac{dC}{dI}(I)$$

$$\gamma I = \frac{\partial V}{\partial p}(p, r)$$

The simple proof of the following Lemma is detailed in the Appendix as a consequence of the Maximum and the Implicit Function theorems under the assumed convexity.

[19] We assume that these costs are financed by lump sum taxes. This does not affect the equilibrium, as long as these taxes are lower that the initial resource endowment ω_i for each group. We avoid specifying lump-sum taxes in the notation for simplicitly.

[20] Besides convexity, we shall impose various regularity conditions on the costs $C(I), V(p, r), \Psi(r)$ and utility $\Phi(r)$; see the Appendix for details.

[21] To be formally precise, the maximization problems are solved by each (small, infinitesimal) agent in society, not by the (large) groups. This is immaterial in the specifics of our analysis in the paper, however.

Lemma 1 *Given r, $(I^O(r), p^O(r))$ are uniquely determined at equilibrium. Furthermore, $p^O(r)$ is decreasing and $I^O(r)$ is increasing in r.*[22]

The public policy problem is determined by the following first order condition,

$$\beta_F \frac{d\Phi}{dr}(r) = (1 - \beta_F - \beta_C(0)) \frac{\partial V}{\partial r}(p, r) + \frac{d\Psi}{dr}(r). \tag{5}$$

We can now produce the main result of this section:

Proposition 1 *The Nash equilibrium (I, p, r) is unique, for any $\beta_F, \beta_C, \beta_S = 1 - \beta_F - \beta_C$. Furthermore, at the Nash equilibrium,*

r is increasing in both β_F and β_C;

I is increasing in both β_F and β_C;

p is decreasing in both β_F and β_C.

This result indicates that in the society we have modeled, the incentives of the Center and of the Fascists are aligned. We shall see that this alignment has important implications with respect to the institutional dynamics of the society, by contributing to its transition away from democracy.

Let us now analyze the institutional dynamics. We will show that these dynamics lead to the consolidation of the political power of the Fascists. To capture in the starkest way the essence of these dynamics, we assume that the relative political power of the Center is constant: $\beta_C = \beta_C(0)$. We denote then the Nash equilibrium as just a function of the political weight β_F of the Fascists: $(I, p, r) = (I(\beta_F), p(\beta_F), r(\beta_F))$.[23] We also assume that the forward-looking aspects of institutional design are severely limited, that is, we assume one-step-ahead institutional design.

The general mechanism of delegation of political power described in Section

[22] While the intensity of repression r has a positive welfare effect on the Center, its effect on the Socialists is in principle ambiguous. Indeed, r affects the Socialists' utility through two channels. First, repression increases directly the cost taking-up protests. But repression, reducing protest p, stimulates the Center's investment I. This in turn increases the rent γI Socialist extract from investment. It can be shown that the negative effect dominates the positive under additional strong convexity assumptions on $C(I)$ and $V(p, r)$ which we require (see the Appendix).

[23] This is of course a dramatic first approximation of the real political dynamics. In this context, the Nash equilibrium only depends on β_F. See Section 5 where we also allow β_C to change overtime.

2 takes a form analogous to Equation 2:

$$\begin{aligned}\beta_F(t+1) \quad \in \quad & \arg \max_{\beta'_F \in [0,1-\beta_C(0)]} W(\beta_F(t), I', p', r') \\ & \text{s.t} \\ & 1-p' = \frac{dC}{dI}(I') \\ & \gamma I' = \frac{\partial V}{\partial p}(p', r') \\ & \beta'_F \frac{d\Phi}{dr}(r') = (1-\beta'_F - \beta_C(0)) \frac{\partial V}{\partial r}(p', r') + \frac{d\Psi}{dr}(r')\end{aligned} \tag{6}$$

The constraints in the maximization problem are the first order conditions determining the Nash equilibrium at β'_F:

$$I' = I(\beta'_F),\ p' = p(\beta'_F), r' = r(\beta'_F).$$

They determine therefore the Nash Equilibrium after institutional change, at β'_F which represents the relative political power of the Fascists at time $t+1$, $\beta_F(t+1)$. Therefore, according to the institutional change mechanism defined above, $\beta_F(t+1)$ evolves in such a way as to promote the implementation at time $t+1$ of the most efficient Nash equilibrium outcome from the perspective of the political institutional system of time t (associated with the relative power structure $\beta_F(t)$). In other words, the societal equilibrium induced by institutions $\beta_F(t+1)$ at $t+1$ is chosen to maximize the social welfare induced by institutions $\beta_F(t)$.

Given current set of institutions at t, institutional design will depend on indicator of the extent of the inefficiency $R(\beta_F(t)) := r^{com}(\beta_F(t)) - r(\beta_F(t))$. More precisely and operationally, given a current set of institutions in period t, $\beta_F(t)$, institutional design will choose a $\beta_F(t+1)$, so that $r(\beta_F(t+1))$ will be close to $r^{com}(\beta_F(t))$. In fact, in the specific context of our analysis, it can be shown that the institutional dynamics induced by Problem 6 have the property that $\beta_F(t+1)$ satisfies the following condition (when an interior solution exists):

$$r(\beta_F(t+1)) = r^{com}(\beta_F(t)). \tag{7}$$

To characterize the dynamics of $\beta_F(t+1)$, we therefore need to solve for the commitment policy $r^{com}(\beta_F(t))$. This is defined as

$$\begin{aligned} r^{com}(\beta_F(t)) \quad \in \quad & \arg\max_r W(\beta_F(t), I', p', r) \\ & \text{s.t.} \\ & 1-p' = \frac{dC}{dI}(I') \\ & \gamma I' = \frac{\partial V}{\partial p}(p', r)\end{aligned} \tag{8}$$

The constraints are the first order conditions determining the equilibrium investment I and protests p for given repression policy choice r; that is, the

functions $I(r), p(r)$ as characterized in Lemma 1. In the Appendix we show that Equation 7 always has an interior solution exists in our context and hence the dynamics of $\beta_F(t)$ are well defined.

The following result characterizes the extent of the inefficiency at the societal equilibrium, $R(\beta_F) = r^{com}(\beta_F) - r(\beta_F)$.

Lemma 2 *For any β_F such that $\beta_F + \beta_C(0) \leq 1$,*

$$\frac{dr(\beta_F)}{d\beta_F} > 0 \text{ and } R(\beta_F) > 0.$$

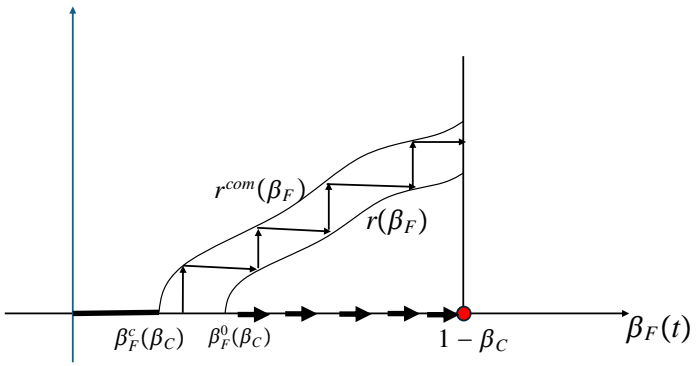
At the Nash equilibrium the intensity of repression r is lower than the one which the public policy authority (state) would choose under commitment r^{com}. This is the case for any institutional structure of society, that is, for any relative political power of the Fascists. In other words, the lack of commitment of the public policy at the Nash equilibrium induces an inefficiently low intensity of repression r. Indeed, three positive externalities plague the Nash equilibrium repression r with respect to r^{com}. First of all, an increase in r induces an increase in the Center's investment I, which in turn produces a positive spillover on the Fascists' utility. Secondly, an increase in r lowers protest p, which in turn produces a positive spillover on the utility of the members of the Center, by reducing the rents extracted by the Socialists from the Center's investment. Thirdly, the increase in I induced by an increase in r represents a positive externality on the Socialists' utility as well, as it increases the total rent they extract the Center's investment, given protests p. These three positive political externalities are at the source of the discrepancy between the Nash equilibrium policy r_t and the committed equilibrium policy $r_t^{com} > r_t$. In order to internalize the externalities that arise in the Nash policy equilibrium, the institutional dynamics reallocates decision rights on public policy between the Fascists and the Socialists in a way to favor an increase in repression r. This is obtained by an increase in the political weight β_F of the Fascists, which is the group favoring a higher level of repression. In other words, power dynamics in favor of the Fascists arises as the result of a political delegation process to a political group that supports repression against the protests undertaken by the Socialists.

In turn, the fact that, in the relevant range of relative political power of the Fascist β_F, $R(\beta_F)$ does not change sign - the statement of Lemma 2 - directly translates into the monotonicity of the dynamics of $\beta_F(t)$ resulting from the institutional design of optimal delegation. Given $r(\beta_F) < r^{com}(\beta_F)$, the institutional dynamics are illustrated in Figure (2). The figure plots the two policy functions $r(\beta_F)$ and $r^{com}(\beta_F)$ for β_F in the interval $[0, \beta_C]$. The dynamics of β_F induced by Equation (7) imply that $\beta_F(t)$ increase over time from $\beta_F(0)$ to the boundary point $\beta_F^* = 1 - \beta_C$.[24]

[24] Accounting for corners, the dynamics of $\beta_F(t)$ are described by the following implicit difference equation:

$$\beta_F(t+1) = \begin{cases} \beta'_F \in (0, 1-\beta_C) \text{ such that } r(\beta'_F) = r^{com}(\beta_F(t)) \\ 1-\beta_C \text{ if } r(\beta'_F) < r^{com}(\beta_F(t)) \text{ for } \beta'_F \in (0, 1-\beta_C) \end{cases}$$

Figure 2: The dynamics of political power of the Fascists



Formally, we can summarize the result as follows.

Proposition 2 *The relative political weight $\beta_F(t)$ of the Fascists increases over time; that is: as long as $\beta_F(t+1) < 1-\beta_C(0)$,*

$$\beta_F(t+1) > \beta_F(t).$$

Naturally, since we have assumed that β_C is constant, we consider the limit of institutional change in which $\beta_F = 1-\beta_C$ and $\beta_S = 0$ as constituting autocratic fascist rule. Conversely, the political weight of the Socialist group $\beta_S(t)$ converges monotonically to $\beta_S = 0$.

Note that under autocratic fascist rule - with $\beta_F = 1-\beta_C(0)$ - the government policy choice r may be higher than optimal for the Center, whose members do not enjoy the ideological gains $\Phi(r)$ the Fascist do.

Indeed, from Equation (5), the long run policy under autocratic fascist rule is obtained as:

$$(1-\beta_C(0))\frac{d\Phi}{dr}(r) = \frac{d\Psi}{dr}(r) \tag{9}$$

At the same time, the policy r^C that maximizes the Center's welfare is the

solution of the following program:

$$
\begin{aligned}
r^C \quad \in \quad & \arg\max_{r\in[0,\infty]} u_C(I,p) - \Psi(r) \\
& \text{s.t} \\
& 1-p = \frac{dC}{dI}(I) \\
& \gamma I = \frac{\partial V}{\partial p}(p,r)
\end{aligned} \tag{10}
$$

Using the envelope theorem and the definition of $p^O(r)$, r^C has to satisfy the following equation:

$$
-I\frac{dp^O}{dr}(r^C) = \frac{d\Psi}{dr}(r^C) \tag{11}
$$

Comparing 9 and 11, it is clear that state repression in the limit with $\beta_F = 1-\beta_C(0)$ is more likely to be larger than what the Center would ideally like, the larger the difference between $(1-\beta_C(0))\frac{d\Phi}{dr} - (-I\frac{dp^O}{dr})$.

This expression itself depends on the initial power of the Center $\beta_C(0)$, the shape of fascist ideology against Socialists $\Phi(r)$, the investment level I of the Center, and how strongly state repression affects the Socialist protest effort (ie. how large is $-\frac{dp}{dr} > 0$). Specifically, when the marginal ideological gain from repression $\frac{d\Phi}{dr}$ is large, it is more likely that the government policy choice r is too large compared to the optimal level for the Center.

3.2.1 Comparative Dynamics

We may also characterize the speed of transition to autocratic fascist rule $[\beta_F(t+1) - \beta_F(t)]$. Indeed, using the fact that along the interior dynamics $r\left(\beta_F(t+1)\right) = r^{com}\left(\beta_F(t)\right) = r_t^{com}$, one can derive the following expression (see Appendix Appendix A.2:):

$$
[\beta_F(t+1) - \beta_F(t)] = \frac{1}{D}\left\{\begin{array}{c} \beta_F(t)\alpha I^{O'}(r_t^{com}) \\ -\beta_C(0)p^{O'}(r_t^{com})I^O(r^{com}) \\ +(1-\beta_F(t)-\beta_C(0))\gamma p^O(r_t^{com})I^{O'}(r_t^{com}) \end{array}\right\} \tag{12}
$$

with

$$
D = \Phi'(r_t^{com}) + V_2'(p^O(r_t^{com}), r_t^{com}) > 0
$$

The intuition for the dynamics of $\beta_F(t)$ may be highlighted by having a closer look at the components affecting the rate of change $\beta_F(t+1) - \beta_F(t)$. Inside the bracket of equation (12), three terms point at the different policy externalities driving the dynamics of power of the fascist group.

The first term $\beta_F(t)\alpha I^{O'}(r_t^{com})$ indicates the political externality on the fascist group that the repression policy r produces through the center's investment spillovers that the Fascists enjoy in the economy. As repression increases such equilibrium investment I, we have $I^{O'}(r_t^{com}) > 0$, and this externality is positive.

The second term $-\beta_C p^{O\prime}(r_t^{com})I^O(r^{com})$ shows the political externality generated on the Center by the repression policy r. Repression reduces the equilibrium level of socialist protest p; i.e., $p^{O\prime}(r_t^{com}) < 0$. This in turn has a positive effect on the payoff $u_C(I,p)$ of members of the Center, and therefore a positive externality on that group.

Finally, the last term $(1-\beta_F(t)-\beta_C)\gamma p^O(r_t^{com})I^{O\prime}(r_t^{com})$ characterizes the political externality on the Socialists. It is also positive as an increase in repression stimulates equilibrium investment I. At constant protest effort, this in turn increases the rents γpI that the socialist group can extract from the center's investment level.

Simple inspection of (24), shows as well that the intensity of a political externality on a given group depends on the political weight of that group. From this and given that $\beta_F(t)$ increases overtime, it follows that during the transition to autocratic fascist rule, the externality of on the Fascists takes predominance in the dynamics of power over the externality on the Socialists.

From a comparative dynamics perspective, we can also conclude that, the more sensitive are the center investment I and socialist protest p to the repression policy of the state (ie. the larger $I^{O\prime}$, and $-p^{O\prime}$), the larger are the political externalities on the different groups; consequently the faster is the transition process to fascist autocratic rule from $\beta_F(0)$ towards $1-\beta_C(0)$.

3.2.2 Decentralized Fascist Violence

In Appendix Appendix A.3: we extend the analysis to allow for decentralized private violent actions of the Fascist against the Socialists, v. In this context the governmental policy choice of the government is modeled as a law enforcement effort w against private violence. Such policy generates a cost for the Fascist and a public benefit $B(w)$ against crime and social disorder. As in the benchmark model, the rate of change $\beta_F(t+1)-\beta_F(t)$ can be decomposed into the externalities that arise in the Nash equilibrium. The first order effect of an increase in the police actions against private violence is decrease in fascist violence, v. This in turn induces an increase in the protests of the Socialists, p, and a decrease in the investment of the Center, I. Therefore the policy w gives rise, first of all, to a negative externality on the Fascists. This is represented by the utility loss associated to their violent actions against the protests of the Socialists and by the economic loss due to the reduction in investment I, which is in turn a consequence of the reduction of fascist violence. This reduction in investment I represents also a negative externality of the policy w on the Center. Finally, w induces a positive externality on the Socialists. A reduction in fascist violence improves the welfare of the Socialists by reducing their cost of protest.[25]

The analysis of this extension in Appendix Appendix A.3: shows that, when the relative political power of the Center is large enough, the negative externalities on the Fascists and the Center overcome the positive externality on the

[25]The negative externality on the Socialist due to the reduction in I can be shown to be dominated.

Socialists. In order to internalize these political externalities, therefore, optimal institutional design consists in delegating political power to obtain a reduction of police enforcement w, which is obtained by increasing of the political weight β_F of the Fascists. In other words, the institutional dynamics in favor of the Fascists - the transition from democracy to autocracy - arises once again as the result of a political delegation process to promote the Fascists' private violence actions against socialist protests.

3.3 The Institutional Decline of the Center

In the previous section we analyzed the institutional dynamics of political delegation of violence to the Fascists, taking as fixed the political power of the Center. In reality, the historical narrative of the transition to Fascism in Italy after WWI can be mapped into a less passive role of the Center, through its legitimation by the Catholic church. In this section we analyze a set-up that allows for a full two-dimensional dynamics of political weights β_F and β_C. The transition to autocratic rule will require in this case that institutional change drive the relative power of the Center, β_C, to 0.

3.3.1 The Historical Role of the Center after WWI

After WWI in Italy the Center - representing the aristocrats and the high-bourgeoisie - is composed of Liberal post-unitary political establishment (the Destra Storica) and various Catholic religious segments of society. The role of the Roman Catholic Church in the political process in Italy is very limited - and behind the scenes - for a few decades after the Italian Unification in 1861, when The Pope's *Non Expedit* (Latin for "It is not opportune") decree established that Italian Catholics shall boycott the polls in parliamentary elections. After WWI, however,[26] the role of the church becomes progressively more manifest and intense until, since 1919, it is exercised openly through the *Partito Popolare Italiano* (PPI), a party founded by Don Luigi Sturzo, which mediates both conservative and progressive political views. The PPI stands then to provide a fundamental contribution the political and social legitimacy of the political process, as the end of non-expedit. It aims at steering the political process to align it with the the dictates of the Roman Catholic Church, acting in the end as a valid support of the liberal and conservative elites: indeed the intransigent anti-clericalism of the Socialists prevented the progressive component of the PPI to ever promoting their political agenda. The Italian government, on its part, is keen to favor the role of the Roman Catholic Church in the political process - hoping that it would finally recognize the legitimacy of the Italian kingdom. Its efforts along this dimension will culminate with the Lateran Accords in 1929, with the key involvement of the Fascist part and Mussolini.

[26] In fact, a 1905 encyclical already allowed for the active political participation in the political process of Catholics to stop "subversive" candidates.

3.4 Religious Legitimacy and the Center

Consider a society where, beyond the repression exerted directly or indirectly by the state on the Socialists, a relevant policy dimension concerns the level of regulation or subsidization of a legitimacy investment of the Church towards the members of the Center. Such a possibility is consistent with the history of relations between the *Santa Sede* and Giolitti. As previously mentioned, one of the main political achievements of the *Età Giolittiana*, during the pre-WWI period, was the gradual realignment between the Liberals and the Church of Rome.[27] This rapprochment, in the mind of the clerical establishment, was arguably due to mounting fears of socialism and that the Italian State would follow the path set by Émile Combes in France. Giolitti was nonetheless a firm believer in the separation of the Church and the State on the basis of 1871 *Guarentigie*.[28]

Consistently with the historical evidence, we thus assume that the Center group is composed of two sub-groups: The Liberals - with weight λ - who decide about productive investment I, and the members close to the Catholic Church (Catholics) - with weight $(1-\lambda)$ - who invest in religious legitimacy m, which benefits both them and the Liberals. Investment in religious legitimacy is promoted by a regulatory policy $s \geq 0$ that is decided by public policy (state).

The utility functions of the three political groups are then as follows:[29]

$$u_F(I,r,m) = \omega_F + \alpha I + \Phi(r) - \delta m - \Psi(r)$$

$$\begin{aligned} u_C(I,p,m,s) &= \omega_C + \lambda\left[I(1-p) - C(I) + m\right] + (1-\lambda)\left[m(1+s) - D(m)\right] \\ &\quad -\Psi(r) \end{aligned}$$

$$u_S(I,p,r,m) = \omega_S + \gamma p I - V(p,r) - \delta m - \Psi(r)$$

There are three differences with the benchmark model. First, there is a legitimacy investment m, which generates a positive legitimacy benefit to the Center coalition members but also conversely a negative status externality $-\delta m$ on both the Fascists and the Socialists, with $\delta > 0$. Second, the level of investment in legitimacy m is undertaken by the Catholics at a convex increasing resource cost $D(m)$. Third, the investment m by the Catholics is promoted by a state regulation s, which is decided by the political economy of the government at an increasing convex resource cost $\Gamma(s)$.[30]

Given the political weight of each group, the government has the following objective function:

$$\begin{aligned} W(\beta,I,p,m,r,s) &= \beta_F \cdot u_F(I,r,m) + \beta_C \cdot u_C(I,p,m,s) + \beta_S \cdot u_S(I,p,r,m) \\ &\quad -\Psi(r) - \Gamma(s) \end{aligned}$$

[27] The famous 1912 *Patto Gentiloni* was the end result of this process.

[28] See Faustini (1960)

[29] Besides convexity, we shall impose various regulatity conditions on the costs $D(m), \Gamma(s)$; see the Appendix for details.

[30] As before these costs can be financed by lump sum taxes on society.

As before we proceed by considering first the Nash equilibrium. Then we consider the committed policy equilibrium vector (r^{com}, s^{com}) that internalize the political externalities generated by the policy interaction problem between the three political groups in society. Finally, we consider the dynamics of the power weight vector $\beta = (\beta_F, \beta_C, \beta_S)$.[31]

Proposition 3 *The Nash equilibrium (I, p, m, r, s) is unique for any β Furthermore, at the Nash equilibrium,*

r is decreasing in β_F and β_C;

s does not depends on β_F and is increasing in β_C;

I is increasing in β_F and β_C;

p is decreasing β_F and β_C;

m does not depends on β_F and is increasing in β_C.

The comparison between the committed equilibrium policies and the Nash equilibrium policies are easily obtained:

Lemma 3 *For all β,*

$$r(\beta_F, \beta_C) < r^{com}(\beta_F, \beta_C).$$

Furthermore, for all $\beta_C \leq \beta_C^* = \frac{\delta}{\lambda+\delta}$

$$s(\beta_C) \geq s^{com}(\beta_C).$$

The intuition behind the comparison between r and r^{com} is as in the previous section. The intuition behind the comparison between s and s^{com} is instead novel and subtle. The policy environment related to the investment in legitimacy m induces a positive externality on the fraction λ of Liberals and at the same time a negative externality of the same size on the Fascists and the Socialists members. Consequently when the political weight of the Center is low enough - i.e., $\beta_C < \beta_C^*$ - the negative externality on the rest of society overcomes the positive one on the Liberals in the Center. Correspondingly, the subsidy with commitment $s^{com}(\beta_C)$ is smaller than at the Nash equilibrium.

Turning now on the dynamics of β (in the interior of the simplex), we show that they take the simple form:

$$r(\beta_F(t+1), \beta_C(t+1)) = r^{com}(\beta_F(t), \beta_C(t)) \quad (13)$$

$$s(\beta_C(t+1)) = s^{com}(\beta_C(t)) \quad (14)$$

[31] We restrict β to the interior of the simplex

$$\Delta = \left\{ \beta = (\beta_F, \beta_C, \beta_S) \in \mathbb{R}^3_+ \middle| \beta_F + \beta_C + \beta_S = 1 \right\}.$$

We will discuss somewhat informally the dynamics of power on the boundary of the simplex Δ, specifically on the face $\beta_S = 0$, $\beta_F + \beta_C = 1$.

Therefore the dynamics of $\beta_C(t)$ is independent from the dynamics of $\beta_F(t)$. This is useful to characterize some features of the two dimensional power dynamics of the system. More specifically, we have the following result

Proposition 4 *The political power of the Center decreases over time,*

$$\beta_C(t+1) \leq \beta_C(t)$$

if and only if $\beta_C(t) \leq \beta_C^*$. *The political power of the Fascists increases over time,*

$$\beta_F(t+1) > \beta_F(t)$$

for all $(\beta_F(t), \beta_C(t))$ *and* $\beta_C(t) \leq \beta_C^*$.

Note that when $\beta_C(t) > \beta_C^*$, the dynamics of $\beta_F(t)$ may not necessarily be monotonic. The reason is the fact that in such a case, the political weight of the Center increases, i.e., $\beta_C(t+1) > \beta_C(t)$). The Nash equilibrium policy r therefore increases even without any change in the political weight of the fascist group, and consequently gets closer to the committed policy level r^{com}. This in turn weakens the motive of political delegation of repression to the fascist group. This in turn may actually have a negative impact on the rate of change of β_F which becomes a priori ambiguous.

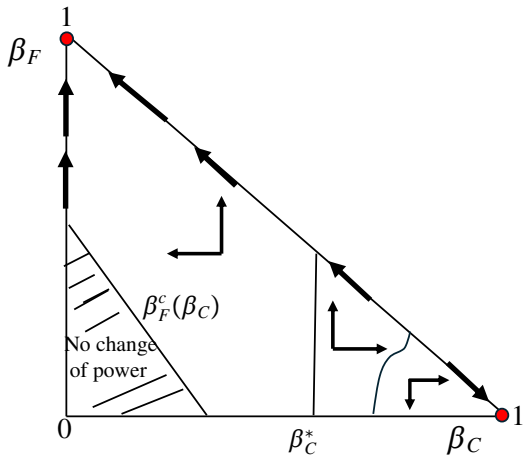
To get a better sense of the two dimensional dynamics of power change, we may again consider the equations that characterize explicitly (13. Tedious but straightforward manipulations show that when $\beta_C(t) > \beta_C^* = \frac{\delta}{\lambda+\delta}$ the rate of change $\beta_F(t+1) - \beta_F(t)$ may be positive or negative depending on the relative size of the externalities.

As for the dynamics of β_C, below the threshold β_C^*, the positive political externality that investment in legitimacy generates on that group is overcome by the negative externality imposed on the rest of society. This puts a downward pressure on the power of center members. This in turn induces the externality of legitimacy investment at the full society to be even more negative, reducing futher down the power weight of the center. The opposite power dynamics are triggered when $\beta_{Ct} > \beta_C^*$. While a full analysis of the system (13-14) is quite involved, a simple phase diagram in the space (β_C, β_F) as shown in Figure (4) illustrates the potential dynamics of power between the fascist and the center groups.

In the region $\beta_F \leq \beta_F^c(\beta_C)$ and $\beta_C \leq \beta_C^*$, we have $r^{com} = r = 0$. There is no change in power and β_F and β_C stay constant. In the region $\beta_F > \beta_F^c(\beta_C)$ and $\beta_C \leq \beta_C^*$, β_F increases while β_C decreases. The system tends to converge towards $\beta_F = 1$ with full power to the fascists. In the region $\beta_F > \beta_F^c(\beta_C)$ and $\beta_C > \beta_C^*$, the weight of the center β_C increases. Initially, for $\beta_C \approx \beta_C^* + \epsilon$, the fascist power weight β_F also increases. The dynamics of β_F could then remain positive in the domain Ξ, and the system therefore leading to no political weight for the socialist group (ie. hitting the line $\beta_F + \beta_C = 1$). At this stage, depending on the relative strength of the positive and negative political externalities, the dynamics of β_F may however stay positive or become negative. If it stays

positive, the fascists reach full power in the limit. Otherwise, the system leads to full power to the center group.[32]

Figure 3: The dynamics of political power of the Fascists and the Center



4 The Transition to Fascist Rule in Italy

The analyis of the model in Section 3.2 can be interpreted as identifying the following main political economy components of the institutional change mechanism responsible for a possible transition to autocracy: i) a socio-economic equilibrium plagued by severe inefficiencies affecting the elites, ii) the option of (alleviating these inefficiencies by) delegating violence e.g., to revolutionary political groups, iii) limited forward-looking capacity of the democratic political system with regards to the process of institutional change. In Section 3.3 a new political economy component of the mechanism responsible for a possible transition to autocracy is identified: iv) the role of cultural institution - formally external to the political process - which however has the power to provide legitimacy to the political process itself, to the Center in particular and eventually to the fascists.

In this section we develop an historical narrative of institutional change in Italy from 1919 and 1925 with the strong implication that the four components identified in the model are arguably central to the turn of events leading to the rise of the Fascist regime.

[32]The dynamics on the boundaries of the simplex of political weights Δ are more delicate to characterize. Indeed, on the bounday, two policy instruments drive a dynamic equation depending on a single weight.

Since the foundation of the Kingdom of Italy in 1861, only Liberals and Conservatives,[33] were able to form governments. Liberals and Conservatives governed the kingdom by means of the skillfull application of the art of *Trasformismo*. At the core of *Trasformismo* laid the idea that the Liberal-Conservative center was supposed to include political actors from, depending on the context, the right or the left, with the ultimate goal of maintaining the center's grip on power and avoiding the rise of extreme political movements. As an example, in 1911 the Liberals - lead by Giovanni Giolitti - obtained the endorsement of the Socialist party to promote a bill introducing universal male suffrage.[34]

In the aftermath of WWI, thanks in part to the success of the October Revolution and the concurrent shock of the world conflict, the Italian Socialist Party (PSI) was both growing in political suffrage and turning maximalist; that is, aiming at the realization of the maximal programme of the Marxist Revolution.[35] This movement of the PSI towards the extreme left of the political spectrum had important political repercussions, not only alienating the support of a large part of the population for a modernising socialist agenda, but also increasing the fears of a Marxist Revolution in the attitudes of the elites. Both the Liberal-Conservative establishment as well as the catholic party - the Partito Popolare Italiano (PPI) - which traditionally representated the elites, operated within the constitutional limits to keep the Socialists from the government. In the 1919 elections PPI and PSI had gathered the majority of parliament seats in the 1919 elections, but never managed to form a coalition.[36]

The *Fasci Italiani di Combattimento* - the group that later constituted the Partito Nazionale Fascista - was founded in early 1919 by Benito Mussolini. Its ideology was vaguely rooted in the interventionist pre-WWI movement: "Revolutionary, because antidogmatic and antidemagogic; extremely innovating because against prejudice of any sort," in its own definition.[37]

[33] The former often labelled as *Sinistra Storica*, the latter as *Destra Storica*

[34] Regarding this historical agreement with the left, Giolitti famously stated "Karl Marx had been stored in the attic;" see Fumagalli (1921) p. 202.

[35] The PSI was the most voted party in both of the 1919 administrative and general elections. The fundamental tension between maximalists and reformists will be a constant throughout the interwar period.

[36] Claudio Treves famously compared the *PPI* to a tree (De Felice (2019b) p. 431) "which is rooted in the hummus of proletariat, whose stems and branches represent the bourgeoisie and whose flowers represent aristocracy. This party sways between the extremes of conservatism and unionism."

[37] Notably, Benito Mussolini's ideological roots where planted in the revolutionary left. As chief editor of the newspaper of the PSI, *Avanti!*, e.g., he supported vehemently the revolts, strikes and demonstrations that - in 1914 - would go under the name of *Settimana Rossa*; see De Felice (2019b) p. 205, 217-220. The outburst of WWI in 1914 marked the definitive fracture between Mussolini and *PSI*. The PSI took an absolute neutralist position (De Felice (2019b) p. 222), while Mussolini - with Filippo Tommaso Marinetti, future co-founder of the *Fasci di Combattimento* - was behind the first rallies in favour of interventionism, through the *Fascio rivoluzionario d'azione internazionalista*, which will form the backbone of the *Fasci di Combattimento*. Mussolini was forced to resign from l'Avanti and founded *Il Popolo d'Italia*, supported by the industrial bourgeoisie (in particular by the *Ansaldo* group, at the times the largest italian industrial group), which supported interventionist ideals; see De Felice (2019b) p. 275; 417.

Fascist violence took a short time to take off, directed towards the Socialists. Just after a few weeks after the foundation of the *Fasci*, in April 1919 a group of *Arditi*, *Fascisti* and other antisocialist forces stormed the headquarters of *Avanti!* in Milano. Most of the reactions from the bourgeoisie were mild and the ones from the police were even benign (De Felice (2019b), p. 522). Similarly, when an international socialist strike - the *scioperissimo* - was set to take place, the secretary of the *Fasci* assured the Prefect of Milano that they would be available to help maintaining law and order during the strike (De Felice (2019b) p. 538).

On June 15th, 1920 Giolitti was appointed again Prime Minister. The seasoned statesman and his enlightened conservative program (De Felice (2019b) p. 600-601) were supported by the elites - aristocrats and high bourgeoisie - which confided in Giolitti's ability to tame the Socialists (Missiroli (1924) p. 157). Mussolini took this opportunity to legitimize the *Fascisti* in the political landscape in support of the Liberal-Conservatives. Indeed, he stalled a coup d'etat planned by Gabriele D'Annunzio, waiting for Giolitti's government to sign the Rapallo Treaty, marked the beginning of the end for D'Annunzio and the *Arditi* (De Felice (2019b), p. 649; 662). At the same time, he turned the *Fasci* into a a reactionary anti-socialist movement (the so-called *Guardia Bianca*; see De Felice (2019b) p. 658-690). The violent attacks of the Fasci against the Socialists were characterized by the bourgeoisie's approval of the initiative and the complicity from the police forces.(Tasca (1938) p.146-7).[38] The *Fasci Agrari* were born, also with a clear anti-socialist scope (De Felice (2019b) p. 658, Tasca (1938) p. 118,144). The combined effects of Mussolini's ability in the eyes of Giolitti and the effectiveness of fascist violence led to a *de facto* alliance that would arguably pave the way to the rise of fascism. Giolitti was persuaded that forming the *Blocchi Nazionali* with Mussolini, which involved including a modest group of fascists in his electoral coalition, would not be problematic.[39] Politicians and intellectuals across different parties - including e.g., Antonio Gramsci - were persuaded that the *Fascio Agrario* was lacking political foundations, so that it would naturally vanish once its repressive purpose againt the socialist movement would end (De Felice (2019a) p. 12-13.) The anti-socialist nature of this alliance will be apparent from the beginning and will turn out to be the reason for which the elites and the politicians representing them would support, even indirectly, the *Blocchi* (De Felice (2019a) p. 80). The May 1921 elections marked the beginning of institutionalisation of fascism. The definitive transformation of *Fasci* in *Partito Nazionale Fascista* substantiated Mussolini's leadership over the party and, most importantly, would definitively show that the syndicalist revolutionary spirit of 1919 was practically dismissed

[38] See Dunnage (1992) p. 92 on one of the most dramatic instances of this violence, the attack of Palazzo D'Accursio in Bologna, November 1920.

[39] His reasoning was that the fascist movement was deeply divided between the original *Fascio urbano* and the more recent *Fascio Agrario* to the point that the whole movement was bound to collapse on its own, after fulfilling their antisocialist duties; see De Felice (2019a) p. 48. Furthermore, he believed that once having tamed the maximalist wing of the socialists, the reformist wing would have come to terms and formed a government with him. See Tasca (1938) p. 153.

in favour of framing fascism into the parliamentary system and a reactionary movement (De Felice (2019a) p. 190).[40] The internal division of the PPI, the prevalence of conservative instances in the party - the most powerful members of the party being represented by its aristocratic and high-level clerical part members - basically made PPI a passive but decisively enabling actor in the process of institutionalisation of fascism.[41]

In the summer of 1922 the role of the PNF as the delegate of the Liberals and, ultimately, of the bourgeoisie, in taming the *red scare*, the fear of the socialist movement, became even clearer.[42] The Socialists proclaimed the *Sciopero Legalitario*, a nationwide, indefinite strike in protest of fascist violence. As a response, the secretary of *Partito Nazionale Fascista* gave the State an ultimatum of 48 hours after which the fascists themselves would have taken care of the strike. The socialist initiative paradoxically became a consecrating moment for Mussolini and his acolytes, who gained the status of saviours from a civil war and a Bolshevik revolution while destroying the remaining socialist resources (Tasca (1938) p. 237-243, 252). In fact, the fascists used the strike as a pretext to occupy several important administrative buildings and city halls. The Conservatives were expecting the *Camicie Nere* to fall back into ranks; but this would not be the case (Malaparte (2011) p. 226; De Felice (2019a) p. 281). At this turn, the PNF had in fact become an unavoidable member of any government alliance, had bolstered by the expansion of the party's reach to the south of Italy and the strengthening of the fascist unions (De Felice (2019a) p. 296). The *Marcia su Roma*, at the end of October 2022, was thus a natural consequence of the favourable moment for the fascists. King Vittorio Emanuele III was not an unconditional supporter of fascism, despite appreciating the anti-socialist nature of the movement. he was also scared by the republican ambitions of many fascists (De Felice (2019a) p. 314). Nevertheless he did not sign the emergency decree that would have allowed stopping the Marcia su Roma.[43] On the contrary, he invited the fascists and Mussolini himself to enter Rome escorted by the army. The fascists were given the opportunity to form a government. After negotiations with several political forces - including the socialists (De Felice (2019a) p. 384)- and the decisive support of the PPI, Mussolini was appointed

[40] Ivanoe Bonomi - Giolitti successor as Prime Minister - had an important role favouring the fascist movement, by recognising in a famous speech the "legitimacy of political fascism" (De Felice (2019a) p. 203) and by strengthening the power of the fascists' squads; see De Felice (2019a) p. 204 for his explicit admission of collaboration with the fascists against Socialists in the region of Friuli Venezia Giulia.

[41] Furthermore, there are instances in which the Popular Party's official newspaper even endorsed and analysed the government's benev- olent approach towards fascism, see De Felice (2019b) page 22.

[42] This so-called *red scare hypothesis* has been recently examined by Acemoglu, De Feo, et al. (2022a). Their findings show that war-induced trauma lead to stronger support for the socialists in 1919 and that this socialists support is locally associated with more intense political and violent fascist activity in the subsequent years. This evidence is consistent with the argument that Fascism rose to prominence as an anti-socialist movement.

[43] This decision was made following the suggestions of several leadership figures of the Conservatives - notably Antonio Salandra, Pietro Badoglio, and Alfredo De Stefani - who worried that the army would not engage against the Fasicsts.

Prime Minister with a ministerial cabinet made of fascists, populars, Conservatives and nationalists. In particular, the support of the PPI proved decisive in appointing Mussolini as Prime Minister.

Whereas the PNF did not have a stable majority in the Parliament, Mussolini managed to obtain a merger of the nationalists into the *PNF* and - especially - the passing of a new strongly majoritarian electoral law (*Legge Acerbo*). The law established that the list gaining 25% of the votes would gain 75% of the seats in the House. The law was delineated by a parliamentary commission whose president was Giovanni Giolitti and was voted in with the approval of the large majority of the Liberals and of the Conservatives. New elections were held in April 1924, with an unsurprising triumph of the *Lista Nazionale*, a cartel of conservatives and fascists. In the end, however, the fascists had to resort again to violence to maintain power: Giacomo Matteotti, the head of *PSI*, who had denounced in a famous speech in May 1924 a number of irregular procedures and episodes of coercive violence that favoured the *Lista*, was kidnapped and killed by members of the secret political police of Benito Mussolini. It is not clear if Mussolini gave direct order of the murder, but he proudly ascribed the "moral blame" of the killing to himself.[44]

By the end of 1926, the PNF was able to get promulgated a series of laws - known as *Leggi fascistissime* - which effectively outlawed opposition political parties and political activity.

5 Historical External Validity

As we noted in the Introduction, it is well recognized that the rise to power of Fascism in Italy contains several main elements common to other historical transition events, as noted by Levitsky and Ziblatt (2019). In this section we briefly discuss the case of the rise of Adolf Hitler in Germany and of General Castelo Branco in Brazil, highlighting the core political-economy components of these dramatic institutional and political power changes in relation to the Italian case.[45]

[44] In his speech - which is reported here in its entirety - Mussolini claimed that "all the violence" was his moral responsibility because he had created the climate of violence: "I assume, I alone, the political, moral, historical responsibility for everything that has happened. If sentences, more or less maimed, are enough to hang a man, out with the noose!" He concluded warning that Italy needed stability and only Fascism would be able to assure it.

[45] The political process which brought the election in 2016 and again in 2024 of President Donald Trump in the U.S. - though notably without any recourse to violence - also shares some elements of the political dynamics we discuss in the paper. This is the case, in particular, for the delegation process within the Republican party. In the course of the campaign for the 2016 presidential elections, a large section of the Republican Party saw Trump's great popular support as an opportunity to pursue major policy initiatives that would require both executive leadership and legislative control; see Espinoza (2018). By the 2024 presidential elections, however, the Republican Party appears to be willing but not anymore able to oppose the consolidation of Trump's control of the party, not having anticipated its strength in time.

5.1 Adolf Hitler and the Nationalist-Conservative Establishment

Adolf Hitler's trajectory toward absolute power was slower and less immediate than Mussolini's. However, several crucial moments arose in which Germany's conservative political center-right chose not to decisively condemn Hitler's actions or ambitions. The first instance was the 1923 *Bürgerbräu-Putsch*, the Nazi Party's failed attempt to overthrow the Bavarian government, after which Hitler was arrested and put on trial. Historical evidence shows this trial[46] and the subequent lenient verdict gave the future *Führer* an increasingly relevant spot in German politics.

A decade later, Hitler gained sufficient momentum to seize absolute power. On 30 January 1933, after Franz Von Papen, a leading Catholic nationalist, convinced President Von Hindenburg that Hitler could be politically contained, Hitler was appointed *Reichskanzler*. Shortly thereafter, on 27 February 1933, the *Reichstag* was set ablaze by a Dutch communist, an event immediately exploited by Hitler and conservative nationalists as a pretext for the *Reichstagsbrandverordnung* (Reichstag Fire Decree). This decree, signed into effect by President Von Hindenburg, allowed the repression and physical intimidation of communist opposition leaders through coordinated actions by Nazi Brown Shirts and police, marking the first institutional step in the delegation of political violence.

The subsequent elections on 5 March 1933 saw significant gains for the NSDAP, though not enough to form an outright majority. The critical final step occurred on 24 March 1933, when the Catholic *Zentrum* Party[47] decisively supported the *Gesetz zur Behebung der Not von Volk und Reich*, commonly known as the Enabling Act of 1933. This law empowered Hitler, as *Reichskanzler*, to issue laws without parliamentary approval.[48]

Drawing a parallel, the leniency shown by Bavarian judges after the failed 1923 coup mirrors the ambiguous response of the conservative liberal center in Italy towards early Fascist violence. Similarly, the Reichstag fire decree and the Enabling Act parallel, respectively, the alliance forged between Mussolini's *Camicie Nere* (Blackshirts) and police forces, and the explicit political agreement between Mussolini and Giolitti, which led to the *Legge Acerbo*.

5.2 The 1964 Civil-Military Coup in Brazil

In 1961, the democratically elected Brazilian vice president, João Goulart,assumed power after the resignation of president Jânio Quadros and the *Legality Campaign* that ensured his inauguration despite military opposition. Facing worsening economic and social conditions, Goulart's administration proposed broad "base reforms" that gained support from various popular sectors but faced stiff resistance from conservative forces, including the military, the business elite, the

[46] See Kershaw (1999) for details
[47] See Zeender (1979) for details on internal *Zentrum* dynamics.
[48] See Jones (2011)

Church, and the Media, who accused him of leaning toward communism and disrupting national order. As political instability deepened, a coup unfolded, with the armed forces rebelling, Congress declaring the presidency vacant, and President João Goulart going into exile on April 4. A military junta briefly took control, and shortly after, Congress elected General Humberto de Alencar Castelo Branco, one of the coup's main leaders, as the new president. These events brought the end of the Fourth Brazilian Republic and the beginning of a military dictatorship that lasted until 1985.

While social scientists have discussed various interpretations of the establishment of the Brazilian military dictatorship, we note three elements which are consistent with our framework of authoritarian power dynamics.

First, there was clearly a lack of internalization of potent political externalities across various groups of society. Indeed, associated to the exhaustion of the import substitution industrialization model, local business and financial elite's interests confronted rising popular demands aimed at greater social inclusion.[49] In this political crisis context, Goulart's inability to pass reforms or maintain centrist support led to his reliance on mass mobilization, with consequently several middle class segments of society fearing a communist revolution.[50]

Second, the coup was initially a "civil-military coup",in the sense that a "strong and consistent civilian base" based on business, media and political sectors saw the alliance with the military as a way of conjuring up what they thought was the communist threat.[51]

Third, the civil support clearly did not expect the military power to take roots for a long period.[52] As a matter of fact, moderate and liberal civil segments progressively got alienated from the new regime or broke with it when they perceived a progressive political hardening of the military power. This transition of enhanced political and administrative power to the Military culmulated with the implementation of the Institutional Act "AI-5" in 1968[53], which led to the total repression of opposition and thus started a military dictatorship.[54]

[49] See Dreifuss (1981) for details.

[50] The brazilian historiography disagrees on whether this fear was purely created by anti-communist propaganda of elites and conservative organizations (Alves (1984), Arquidiocese de São Paulo (1985)), or was actually based on a real elements (Gorender Gorender (1998), Gorender (2014); Ridenti (1993)).

[51] See Dreifuss (1981), D'Araújo, Soares, and Castro (1994), Joffily (2018)).

[52] For instance, Ferreira and Gomes (2014) mention that "...those who applauded and celebrated the victory of the 'revolution for order' had no way of knowing what would happen in the following years..."(p.16). Stepan (1971) as well noted that unlike in previous interventions where they had exercised a traditional kind of "moderating" power, The Military after the coup unexpectedly decided to remain in charge of the country's politics.

[53] Instituted on December 13, 1968, under the government of General Artur da Costa e Silva the "AI-5" allowed the removal of elected politicians at federal, state and municipal level, authorized the President of the Republic to intervene in the governments of states and municipalities and allowed the suspension of individual constitutional rights and guarantees such as habeas corpus, among other measures.

[54] This dramatic historical moment has been defined the "coup within the coup", see Fico (2014), Napolitano (2014) for details.

6 Conclusions

In this paper, we develop a political economy model of institutional change that accounts for transitions from democracy to autocracy. Our point of departure is the observation that collective decision-making is inherently imperfect, marked by commitment problems and political externalities across social groups. We propose a conceptual framework in which shifts in political power—understood as a process of institutional delegation and reallocation of decision rights—can partially internalize these externalities and mitigate political inefficiencies.

Our main contribution is to formalize the logic of this mechanism in the context of authoritarian transitions, highlighting two key components. First, moderate political groups often lack the capacity for credible commitment to suppress instability arising from reformist factions or to counteract political externalities that threaten their economic interests. To restore stability, these groups may delegate authority to more radical or violent actors who possess the means to enforce order—thus acquiring, by proxy, the commitment devices they lack. Second, we emphasize the limited intertemporal foresight of democratic institutions, which can lead to excessive delegation of power. This opens the door for radical or violent actors to consolidate authority and establish an autocratic regime. Paradoxically, this outcome may leave the moderate groups worse off than under the original democratic status quo.

A secondary contribution of the paper is to provide a historically grounded analytical narrative illustrating how this mechanism maps onto the "red scare hypothesis" and the rise of Fascism in Italy between 1919 and 1925. We further suggest that similar patterns of institutional delegation and the strategic use of political violence help illuminate other authoritarian transitions, including the ascent of Nazism in Germany and the establishment of military rule in Brazil during the 1960s and 70s. We hope that our model can be adapted and extended to fit other historical narratives that involve the transition to autocracy, as we show in Section 5.[55]

As previously noted, our work relates to the rapidly expanding political science literature on *democratic backsliding*. This strand of research, often focused on specific institutional contexts, emphasizes fundamental factors that are also embedded in our abstract framework of power change: the presence of political externalities and distortions, the difficulty of anticipating the long-term consequences of shifts in power, and the gradual dynamics underlying the concentration of authority.

While we adopt a more explicit socioeconomic modeling framework in which

[55]The debate on the recursive nature of fascism is not a settled dispute among historians. We take a neutral stance, harmonising two opposed views: that of Gentile (2022), who convincingly argues that fascism was a unique historical phenomenon, versus Stanley (2018), who finds strong parallel between italian fascism and other authoritarian/conservative political movements to the point of grouping them all under a broad notion of fascism. In our approach, we wish to highlight the role and consequences of the institutional delegation of violence to a violent avantguard, without assessing whether the underlying transition from democracy to autocracy involves the notion of political fascism.

fundamentals such as preferences and technologies interact with concrete policy instruments, an interesting direction for future research would be to integrate into our abstract model of power dynamics certain institutional contexts explored in the democratic backsliding literature. Such an extension could in particular highlight how specific political actors strategically leverage fears of societal change and underlying socioeconomic structures to legitimize political violence, fuel radicalism, and ultimately undermine democratic institutions.[56].

[56] See Thompson et al. (2025),

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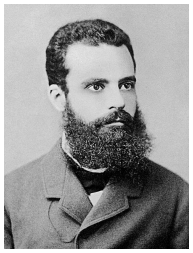
Appendix A.1: Quotes from various exponents of the (Italian) Theory of the Elites


Gaetano Mosca (Elementi di Scienza Politica, 1896):

Small ruling class controls political power and decision-making processes, maintaining power through force, persuasion, and the formation of an ideology or belief system that justifies its dominance; some circulation of elites through new individuals or groups: the stability of a political system depends on the ability of the ruling class to integrate new members and adapt to changing circumstances.


Robert Michaels (*Sociologia del Partito Politico*, 1911)

Iron Law of Oligarchy - all forms of organization will eventually develop oligarchic tendencies, where power is centralized and leadership roles are concentrated in a few individuals.


Vilfredo Pareto (*Trattato di Sociologia Generale* (1916)

Elites are distinguished by their psychological (cultural?) qualities; process of circulation involves the continuous replacement of elites which, while in power become complacent, corrupt, ...; the circulation of elites is necessary for social and political stability, social mobility and renewal.

Appendix A.2: Proofs

In this appendix we provide a detailed analysis of the model and results in Section 3 of the paper, with proofs.

For convenience, we take the following notations for the derivatives and cross-derivatives of given generic functions $f(z)$ and $g(x,y)$:

$$f'(z) = \frac{df}{dz}\,; f''(z) = \frac{d^2 f}{dz^2}$$

$$g'_1(x,y) = \frac{\partial g}{\partial x}; g'_2(x,y) = \frac{\partial g}{\partial y};$$

$$g''_{11}(x,y) = \frac{\partial^2 g}{\partial x^2}\,; g''_{22}(x,y) = \frac{\partial^2 g}{\partial y^2}\,; g''_{12}(x,y) = \frac{\partial^2 g}{\partial x \partial y}$$

Assumption A. 1 *We impose the following assumptions of the various cost and utility functions is society:*

$C(I)$ is strictly increasing and convex in I. Furthermore, $C(0) = C'(0) = 0$;

$V(p,r)$ is strictly increasing and concave in p. Furthermore, $V'_1(p,r) \geq 0, V''_{11}(p,r) > 0$ and $V(0,r) = V'(0,r) = 0$ for all $r \geq 0$, $V'(1,r) = \infty$; also, $V'_2(p,r) \geq 0$, $V''_{12}(p,r) \geq 0$, $V''_{22}(p,r) \geq 0$);[57]

$\Phi(r)$ is strictly increasing and concave. Furthermore, $\Phi'(r) \geq 0$, $\Phi''(r) < 0$) and $\Phi(0) = 0$;

$\Psi(r)$ are strictly increasing and convex.

Lemma 1: Proof. The First Order Conditions for the problem of members of the Center is:

$$1 - p = C'(I)$$

which can be rewritten as:

$$I = C'^{-1}(1-p) \tag{15}$$

Trivially, I is decreasing in p.

The First Order Conditions for the problem of members of the Socialists is:

$$\gamma I = V'_1(p,r) \tag{16}$$

Differentiation of (16) shows that the optimal protest for the Socialists is an increasing function of I and a decreasing function of r as:

$$\frac{\partial p}{\partial I} = \frac{\gamma}{V''_{11}} > 0 \quad \text{and} \quad \frac{\partial p}{\partial r} = -\frac{V'_{12}}{V''_{11}} < 0.$$

■

[57] Although it does not satisfy fully the boundary conditions at $p = 1$, a linear quadratic example that we will sometime use is $V(p,r) = V.\left(p^2/2\right) \cdot (1+r)$ with $V > 0$ a positive constant

For a given value of the repression policy level r, (15) and (16) together determine the civil society Nash equilibrium levels of investment $I^O(r)$ and protest $p^O(r)$. As a higher level of repression increases the cost of protests by socialists, $p^O(r)$ is decreasing in r, while consequently, $I^O(r)$ is increasing in r. Indeed differentiation of (15) and (16) provides

$$I^{O\prime}(r) = \frac{V''_{12}}{C''V''_{11} + \gamma} > 0$$
$$p^{O\prime}(r) = \frac{-C''V''_{12}}{C''V''_{11} + \gamma} < 0$$

Note that at this society equilibrium, the utility of the socialist group as a function of the repression policy r writes as

$$u_S^O(r) = \gamma p^O(r) I^O(r) - V(p^O(r), r)$$

Using the enveloppe theorem for $p^O(r)$, one obtains:

$$\begin{aligned} u_S^{O\prime}(r) &= \gamma p^O(r) I^{O\prime}(r) - V'_2(p^O(r), r) \\ &= \gamma p^O(r) \frac{V''_{12}}{C''V''_{11} + \gamma} - V'_2 \end{aligned}$$

The repression policy level r affects the socialist group welfare through two channels. First, there the negative effect $-V'_2 < 0$ of repression increasing directly the cost of running a protest for the socialist. Second, there is an indirect positive effect $\gamma p^O(r) \frac{V''_{12}}{C''V''_{11}+\gamma} > 0$ associated to the fact that repression by reducing protest p, stimulates center's investment $I^O(r)$. All else equal, this increases the rent that the socialist group can get through investment made by the center group. It is easy to see that the direct effect overcomes the indirect effect when the cost functions $C(I)$ and $V(p, r)$ are sufficiently convex respectively in I and p, something that we assume.

Assumption A. 2 *We assume* $C''V''_{11} > \frac{V''_{12}}{V'_2}$, *which is sufficient to obtain*

$$u_S^{O\prime}(r) < 0.$$

Under Assumption A.2, at the civil society Nash equilibrium, the socialist group' welfare $u_S^O(r)$ is decreasing in the repression policy level r.

Proposition 2: Proof. The First Order Condition for an interior solution of the Public policy choice program is: as:

$$\beta_F \cdot \Phi'(r) - (1 - \beta_F - \beta_C) \cdot V'_2(p, r) - \Psi'(r) = 0 \tag{17}$$

the solution of which provides a best reply policy function $\tilde{r}(p, \beta_F, \beta_C) \geq 0$.

Simple inspection of (17) show that the best reply policy function $\tilde{r}(p, \beta_F, \beta_S) > 0$ if and only if:

$$\beta_F > \beta_F^0(\beta_C) = \frac{V_2'(p^O(0), 0)}{\Phi'(0) + V_2'(p^O(0), 0)}(1 - \beta_C) \tag{18}$$

Moreover differentiation of (17) gives immediately:

$$\begin{aligned}
\frac{\partial \tilde{r}}{\partial p} &= \frac{\beta_S \cdot V_{12}''(p, r)}{\beta_F \cdot \Phi''(r) - \beta_S \cdot V_{22}''(p, r) - \Psi''(r)} \leq 0 \\
\frac{\partial \tilde{r}}{\partial \beta_F} &= -\frac{\Phi'(r)}{\beta_F \cdot \Phi''(r) - \beta_S \cdot V_{22}''(p, r) - \Psi''(r)} > 0 \\
\frac{\partial \tilde{r}}{\partial \beta_C} &= \frac{-V_2'(p, r)}{\beta_F \cdot \Phi''(r) - \beta_S \cdot V_{22}''(p, r) - \Psi''(r)} > 0
\end{aligned}$$

The Nash equilibrium repression is the

$$r = \tilde{r}(p^O(r), \beta_F, \beta_C)$$

or equivalently a solution of:

$$\beta_F \cdot \Phi'(r) - (1 - \beta_F - \beta_C) \cdot V_2'(p^O(r), r) - \Psi'(r) = 0 \tag{19}$$

We now show that a unique solution of (19) exists when the cost functions $\Psi(r)$ and $V(p, r)$ are convex enough. It is represented by a map $r(\beta_F, \beta_C)$.

Indeed consider the function $\Omega(r) = \beta_F \cdot \Phi'(r) - \beta_S \cdot V_2'(p^O(r), r) - \Psi'(r)$. It is clear that given that $\Phi'(0) = \Psi'(\infty) = \infty$, we obtain $\Omega(0) = +\infty$ and $\lim_{r \to \infty} \Omega(r) = -\infty$. The continuity of $\Omega(r)$ ensures the existence of $r(\beta_F, \beta_C) \in]0, +\infty[$ satisfying $\Omega(r(\beta_F, \beta_C)) = 0$, or equivalently a fixed point $r(\beta_F, \beta_C)$ satisfying (19). Furthermore, uniqueness is ensured when $\Omega(r)$ is decreasing in r, which is the case when

$$\Omega'(r) = \beta_F \cdot \Phi''(r) - \beta_S \cdot \left[V_{12}'' p^{O\prime}(r) + V_{22}''\right] - \Psi''(r) < 0$$

Using

$$p^{O\prime}(r) = \frac{-C'' V_{12}'}{C'' V_{11}'' + \gamma} < 0$$

it is easy to see that a sufficient condition for $\Omega'(r) < 0$ is obtained when:

$$\frac{(V_{12}'')^2}{V_{11}''} < \Psi''(r);$$

that is when the cost functions $\Psi(r)$ and $V(p, r)$ are convex enough. Finally

simple differentiation of equation19 provides that $r = r(\beta_F, \beta_C)$ is increasing in both β_F and β_C as

$$\frac{\partial r}{\partial \beta_F} = \frac{\Phi'(r) + V_2'(p^O(r), r)}{-\Omega'(r)} > 0 \tag{20}$$

$$\frac{\partial r}{\partial \beta_C} = \frac{V_2'(p^O(r), r)}{-\Omega'(r)} > 0 \tag{21}$$

Consequently $I = I(\beta_F, \beta_C)$ is also increasing in both β_F and β_C and $p = p(\beta_F, \beta_C)$ is decreasing in both β_F and β_C. ■

Lemma 3: **Proof.** We start by characterizing the commitment policy $r^{com}(\beta_F(t))$; that is, the solution of the following program:

$$\max_{r \geq 0} W^c(\beta, r) = W(\beta, I^O(r), p^O(r), r)$$

with

$$\begin{aligned} W(\beta, I, p, r) &= \beta_F \cdot u_{F'}(I, r) + \beta_C \cdot u_C(I, p) \\ &\quad + (1 - \beta_F - \beta_C) \cdot u_S(I, p, r) - \Psi(r) \end{aligned}$$

Using the conditions (15) and (16) characterizing $I^O(r)$ and $p^O(r)$, one obtains the first order condition:

$$\begin{aligned} \frac{dW^c}{dr} &= \beta_F \cdot \left[\alpha I^{O\prime}(r) + \Phi'(r)\right] - \beta_C \cdot p^{O\prime}(r) I^O(r) \\ &\quad + (1 - \beta_F - \beta_C) \cdot \left[\gamma p^O(r) I^{O\prime}(r) - V_2'(p^O(r), r)\right] - \Psi'(r) = 0 \end{aligned} \tag{22}$$

Assumption A. 3 *We assume that the policy resource costs $\Psi(r)$ are sufficiently convex to guarantee that*

$$W^c(r) \text{ is a concave function of } r$$

Under Assumption A.3, Equation (22) then solves for $r^{com}(\beta_F, \beta_C) \geq 0$. Given Assumption A.3, it is easy to see that $r^{com}(\beta_F, \beta_C) > 0$ if and only if

$$\beta_F > \beta_F^c = \frac{\beta_C \cdot p^{O\prime}(0) I^O(0) - (1 - \beta_C) \cdot \left[\gamma p^O(0) I^{O\prime}(0) - V_2'(p^O(0), 0)\right]}{\alpha I^{O\prime}(0) + \Phi'(0) - \left[\gamma p^O(0) I^{O\prime}(0) - V_2'(p^O(0), 0)\right]} \tag{23}$$

Moreover simple differentiation of (22)provides

$$\begin{aligned} \frac{\partial r^{com}}{\partial \beta_F} &= \frac{\alpha I^{O\prime}(r) + \Phi'(r) - \left[\gamma p^O(r) I^{O\prime}(r) - V_2'(p^O(r), r)\right]}{-\frac{d^2 W^c}{dr^2}} \geq 0 \\ \frac{\partial r^{com}}{\partial \beta_C} &= \frac{-p^{O\prime}(r) I^O(r) - \left[\gamma p^O(r) I^{O\prime}(r) - V_2'(p^O(r), r)\right]}{-\frac{d^2 W^c}{dr^2}} \geq 0 \end{aligned}$$

Therefore, under assumptions A.2 and A.3, r^{com} is increasing in β_F and in β_C. We can now compute

$$\left.\frac{dW^c}{dr}\right|_{r = r(\beta_F, \beta_C)} = \beta_F \cdot \left[\alpha I^{O\prime}(r)\right] - \beta_C \cdot p^{O\prime}(r) I^O(r) + (1 - \beta_F - \beta_C) \cdot \gamma p^O(r) I^{O\prime}(r) > 0$$

Given the concavity of the function $W^c(r)$, and the fact that $\frac{dW^c}{dr}\big|_{r^{com}} = 0$, we have that $r^{com}(\beta_F, \beta_C) \geq r(\beta_F, \beta_C)$. Given that from proposition 2, r(β_F,β_C) is increasing in β_F, the statement of the Lemma follows. ■

In the sequel we denote for convenience $r\left(\beta_F(t), \beta_C(0)\right) = r\left(\beta_F(t)\right)$.

Proposition 4: Proof. By definition when $\beta_F(t+1) < 1 - \beta_C$, we have $r\left(\beta_F(t+1)\right) = r^{com}\left(\beta_F(t)\right) \geq r\left(\beta_F(t)\right)$. Given that the function $r\left(\beta_F\right)$ is increasing in β_F, it follows that $\beta_F(t+1) > \beta_F(t)$. This concludes the proof of the Proposition. ■

Comparative dynamics:

We now characterize $[\beta_F(t+1) - \beta_F(t)]$. Using the fact that along the interior dynamics $r\left(\beta_F(t+1)\right) = r^{com}\left(\beta_F(t)\right) = r_t^{com}$, one obtains the following two conditions:

$$\beta_F(t+1)\Phi'(r_t^{com}) - ((1 - \beta_F(t+1) - \beta_C))V_2'(p^O(r_t^{com}), r_t^{com}) = \Psi'(r_t^{com})$$

and

$$\begin{aligned}&\beta_F(t)\left[\alpha I^{O\prime}(r_t^{com}) + \Phi'(r_t^{com})\right] - \beta_C p^{O\prime}(r_t^{com})I^O(r_t^{com}) \\ &+(1 - \beta_F(t) - \beta_C)\left[\gamma p^O(r_t^{com})I^{O\prime}(r_t^{com}) - V_2'(p^O(r_t^{com}), r_t^{com})\right] = \Psi'(r_t^{com})\end{aligned}$$

Substraction of these two equations and rearranging terms provides:

$$[\beta_F(t+1) - \beta_F(t)] = \frac{1}{D}\left\{\begin{array}{c}\beta_F(t)\alpha I^{O\prime}(r_t^{com}) \\ -\beta_C p^{O\prime}(r_t^{com})I^O(r^{com}) \\ +(1 - \beta_F(t) - \beta_C)\gamma p^O(r_t^{com})I^{O\prime}(r_t^{com})\end{array}\right\} \quad (24)$$

with

$$D = \Phi'(r_t^{com}) + V_2'(p^O(r_t^{com}), r_t^{com}) > 0$$

Simple inspection of (24), shows that the intensity of a political externality on a given group depends on the political weight of that group. From this and given that $\beta_F(t)$ increases overtime, it follows that the externality on the Fascists takes predominance in the dynamics of power over the externality on the Socialists. As well, the more sensitive are the center investment I^O, and socialist protest p^O to the repression policy of the state (ie. $I^{O\prime}$ and $-p^{O\prime}$ large), the larger are the political externalities on the different groups; consequently the faster is the transition process of power towards the Fascists from $\beta_F(0)$ towards $1 - \beta_C$.

Linear quadratic example:

The Nash equilibrium and the dynamics of $\beta_F(t)$ can be completely characterized in a linear-quadratic example with the following functional form assumptions:

$$\Phi(r) = \phi r,\ C(I) = c\frac{I^2}{2},\ V(p,r) = V\left(p^2/2\right)(1+r),\ \text{and}\ \Psi(r) = \psi\frac{r^2}{2}.$$

Thus

$$\begin{aligned} I &= \frac{1-p}{c} \\ p &= \frac{\gamma}{V}\frac{I}{1+r} \end{aligned}$$

$$\begin{aligned} p^O(r) &= \frac{\frac{\gamma}{Vc}}{1+\frac{\gamma}{Vc}+r} \\ I^O(r) &= \frac{1}{c}\frac{1+r}{1+\frac{\gamma}{Vc}+r}; \end{aligned}$$

and $\tilde{r}(p, \beta_F, \beta_C)$ is determined by the condition:

$$\beta_F\phi - (1-\beta_F-\beta_C)\cdot V\frac{p^2}{2} - \psi r = 0. \tag{25}$$

The Nash equilibrium policy $r(\beta_F, \beta_C)$ is characterized by:

$$r(\beta_F,\beta_C) = \begin{cases} 0 \quad \text{when} \quad \beta_F\phi \leq \frac{1-\beta_F-\beta_C}{2V}\left(\frac{\gamma}{c}\right)^2\frac{1}{\left(1+\frac{\gamma}{Vc}\right)^2} \\ r(\beta_F,\beta_C) > 0 \quad \text{when} \quad \beta_F\phi > \frac{1-\beta_F-\beta_C}{2V}\left(\frac{\gamma}{c}\right)^2\frac{1}{\left(1+\frac{\gamma}{Vc}\right)^2} \end{cases},$$

with $r(\beta_F, \beta_C)$ satisfying the equation:

$$\beta_F\phi - (1-\beta_F-\beta_C)\cdot V\frac{\left(\frac{\gamma}{Vc}\right)^2}{2\left(1+\frac{\gamma}{Vc}+r\right)^2} = \psi r.$$

This solution is uniquely defined as long as

$$\frac{V^2 c}{\gamma} < \psi. \tag{26}$$

The commitment policy problem writes as:

$$\begin{aligned} \max_r W(I^O(r), p^O(r), r) &= \beta_F\left[\alpha I^O(r) + \Phi(r)\right] + \beta_C\left[I^O(r)(1-p^O(r)) - C(I^O(r))\right] \\ &\quad +(1-\beta_C-\beta_F)\left[\gamma p^O(r) I^O(r) - V(p^O(r), r)\right] - \Psi(r) \\ s.c. \quad I^O(r) &= \frac{1}{c}\frac{1+r}{1+\frac{\gamma}{Vc}+r} \\ p^O(r) &= \frac{\frac{\gamma}{Vc}}{1+\frac{\gamma}{Vc}+r}; \end{aligned}$$

and thus:

$$\begin{aligned}\frac{dW^c}{dr} &= \beta_F\left[\frac{\frac{\gamma}{Vc}\frac{\alpha}{c}}{\left(1+\frac{\gamma}{Vc}+r\right)^2}+\phi\right]+\beta_C\frac{\gamma}{Vc^2}\frac{1+r}{\left(1+\frac{\gamma}{Vc}+r\right)^3}\\ &\quad -(1-\beta_F-\beta_C)\frac{\gamma^2}{Vc^2}\frac{1+r-\frac{\gamma}{Vc}}{2\left(1+\frac{\gamma}{Vc}+r\right)^3}-\psi r;\end{aligned}$$

and

$$\begin{aligned}\frac{d^2W^c}{dr^2} &= -2\beta_F\cdot\frac{\frac{\gamma}{Vc}\frac{\alpha}{c}}{\left(1+\frac{\gamma}{Vc}+r\right)^3}+(\beta_C-\gamma(1-\beta_F-\beta_C))\frac{\gamma}{Vc^2}\frac{\frac{\gamma}{Vc}+r-2}{\left(1+\frac{\gamma}{Vc}+r\right)^4}\\ &\quad -3(1-\beta_F-\beta_C)\frac{\gamma^3}{V^2c^3}\frac{1}{2\left(1+\frac{\gamma}{Vc}+r\right)^4}-\psi.\end{aligned}$$

A sufficient condition for $\frac{d^2W^c}{dr^2}<0$ - and consequently for Assumption A.3 to hold - is then

$$\frac{\gamma}{Vc^2}<\psi. \tag{27}$$

Consequently, the condition for existence of a unique Nash equilibrium policy $r(\beta_F,\beta_C)$ and assumption A.2 is

$$\max\left(\frac{V^2c}{\gamma},\frac{\gamma}{Vc^2}\right)<\psi,$$

which requires that resource costs $\Psi(r)=\psi\frac{r^2}{2}$ be convex enough.

The Institutional dynamics of the Center

Assumption A. 4 *We impose the following assumption on cost function.*

> $D(m)$ *is strictly increasing and convex. Furthermore,* $D(0)=D'(0)=0$, $D(+\infty)=D'(+\infty)=+\infty$.
>
> $\Gamma(s)$ *is strictly increasing and convex.*

Proposition 5: Proof.

As before we proceed by considering first the Nash policy equilibrium (r,s) (itself the result of the Nash equilibrium within the civil society and between the civil society and the centralized authority reflecting the political forces of society). Then we consider the committed policy equilibrium vector (r^{com},s^{com}) that tends to internalize the political externalities generated by the policy interaction problem between the three political groups in society. Finally, we consider the dynamics of the power weight vector $\beta=(\beta_F,\beta_C,\beta_S)$ belonging to the interior of the simplex set:
$\Delta=\left\{\beta=(\beta_F,\beta_C,\beta_S)\in\mathbb{R}^3_+\,\middle|\,\beta_F+\beta_C+\beta_S=1\right\}$

a) The Civil society Nash equilibrium:

For a given police vector (r, s), the civil society Nash equilibrium is characterized by the following conditions

$$\begin{aligned} 1-p &= C'(I) \\ 1+s &= D'(m) \\ \gamma I &= V_1'(p,r) \end{aligned}$$

the first equation describes as before the optimal investment i of the Center given the level of socialist protest p, the second equation indicates the optimal level of legitimacy m undertaken by the the Catholic part of the Center group given the subsidy regulation s. Finally the last equation shows the optimal protest effort of the socialist group, given the level of state repression r and the level of Center investment I.

Our separability assumptions allow a very simple solution for the civil society Nash equilibrium. As before the first and the last equation provide the equilibrium Center investment $I^O(r)$ and equilibrium Socialist protest level $p^O(r)$ as function of the repression level r, while the second equation gives the optimal level of legitimacy investment $m^O(s) = D'^{-1}(1+s)$. as a function of the subsidy regulation s.

b) Nash Policy Equilibrium:

In the Nash Policy equilibrium, the government decides on the policy vector (r, s) given investments I ,m and protest p:

$$\max_{r,s\geq 0} W(\beta, I, p, m, r, s)$$

The first order conditions for an interior solution of this program write as:

$$\begin{aligned} \beta_F \cdot \Phi'(r) - (1-\beta_F-\beta_C)\cdot V_2'(p,r) - \Psi'(r) &= 0 \\ \beta_C\,(1-\lambda)\,m - \Gamma'(s) &= 0 \end{aligned}$$

from which we obtain a best reply policy vector $(\tilde{r}(p,\beta_F,\beta_C); \tilde{s}(m,\beta_C)) \in [0,\infty[\times [0,\infty[$ where the optimal state repression policy response $\tilde{r}(p,\beta_F,\beta_C)$ is decreasing in protest p in so far that socialist group has some political power (ie. $\beta_F + \beta_C < 1$), increasing in the political weight of the fascist group β_F and of the Center group β_C. The optimal state legitimacy subsidization response $\tilde{s}(m,\beta_C)$ is in turn increasing in the legitimacy investment level m and depends solely and positively on the political weight β_C of the Center group..

At the Nash policy equilibrium, the civil society variables I, p, m have to be consistent with a Civil society Nash equilibrium, namely (I, p, m) equal to $I^O(r), p^O(r), m^O(s)$. From this, we obtain that the characterization of the Nash policy equilibrium levels $r\,(\beta_F, \beta_C)$ and $s(\beta_C)$ as satisfying the two conditions:

$$\beta_F \cdot \Phi'(r) - (1-\beta_F-\beta_C)\cdot V_2'(p^O(r),r) - \Psi'(r) = 0 \tag{28}$$

$$\beta_C\,(1-\lambda)\,m^O(s) - \Gamma'(s) = 0 \tag{29}$$

This system determines a unique equilibrium vector $(r(\beta_F, \beta_C), s(\beta_C)) \in \mathbb{R}^2_+$ when $\Psi(.)$ and $\Gamma(.)$ are sufficiently convex.[58]

[59] While $s(\beta_C) \geq 0$ is increasing in β_C. Because $\Gamma'(0) = 0$, simple inspection of (29) indicates that $s(\beta_C) > 0$ if and only if $\beta_C > 0$.

Finally simple differentiation of equations (28) and(29) provide the comparative statics of proposition 5. ■

Lemma 6: Proof. Consider first the determination of the Committed Policy Equilibrium.

Committed Policy Equilibrium:

Again, we assume that $\Psi(\cdot)$ and $\Gamma(\cdot)$ are sufficiently convex so that $W^c(r, s)$ is a strict concave function of (r, s):

Assumption S5: $W^c(r, s)$ is a strict concave function of (r, s).

■

The *committed policy equilibrium* is the solution of the following program

$$\max_{r,s\geq 0} W^c(\beta, r, s) = W(\beta, I^O(r), p^O(r), m^O(s), r, s)$$

Under assumption S5, the first order conditions:

$$\frac{\partial W^c}{\partial r} = 0 \tag{30}$$

$$\frac{\partial W^c}{\partial s} = 0 \tag{31}$$

characterize the committed policy equilibrium vector when it is interior with

$$\begin{aligned}
\frac{\partial W^c}{\partial r} &= \beta_F \left[\alpha I^{O\prime} + \Phi'\right] + \beta_C(-I^O p^{O\prime}) + (1 - \beta_F - \beta_C)\left[\gamma p^O I^{O\prime} - V_2'(p^O, r)\right] - \Psi'(r) \\
\frac{\partial W^c}{\partial s} &= \beta_F \left(-\delta m^{O\prime}\right) + \beta_C \left[(1-\lambda)m^O(s) + \lambda m^{O\prime}\right] + (1 - \beta_F - \beta_C)\left(-\delta m^{O\prime}\right) - \Gamma'(s)
\end{aligned}$$

The separability between the two policy instruments, implies that the first equation of (30) provides the optimal committed state repression level $r^{com}(\beta_F, \beta_C) \geq 0$, which under assumption $S5$ is increasing in β_F and β_C. Again $r^{com}(\beta_F, \beta_C) > 0$ if and only if

$$\beta_F > \beta_F^c(\beta_C) = \frac{\beta_C \cdot p^{O\prime}(0) I^O(0) - (1 - \beta_C) \cdot \left[\gamma p^O(0) I^{O\prime}(0) - V_2'(p^O(0), 0)\right]}{\alpha I^{O\prime}(0) + \Phi'(0) - \left[\gamma p^O(0) I^{O\prime}(0) - V_2'(p^O(0), 0)\right]}$$

[58] This will be the case when $\frac{(V_{12}'')^2}{V_{11}''} < \Psi''(r)$, and $1 < \Gamma''(s)D''(m)$. Moreover, under assumption $S1$, it is easy to see that $r(\beta_F, \beta_C) \geq 0$ is an increasing function of β_F and β_C.

[59] As in the benchmark model, $r(\beta_F, \beta_C) > 0$ if and only if $\beta_F > \beta_F^0(\beta_C)$.

The second equation (31) provides the optimal committed regulatory subsidy $s^{com}(\beta_C) \geq 0$, which is a positive function of β_C.[60] Inspection then shows that $s^{com}(\beta_C) > 0$ if and only if

$$\beta_C > \beta_C^0 = \frac{\delta \frac{m^{O\prime}(0)}{m^O(0)}}{(1-\lambda) + (\lambda+\delta)\frac{m^{O\prime}(0)}{m^O(0)}}$$

Next we have:

a) For the repression instrument, the comparison is identical to that of the benchmark model.

b) Comparison of (29) with (31) implies that $s^{com}(\beta_C) \leq s(\beta_C)$ if and only if $\frac{\partial W^c}{\partial s}\big|_{s=s(\beta_C)} < 0$, or $[\beta_C\lambda - (1-\beta_C)\delta]\, m^{O\prime}(s) < 0$. After rearranging and given $m^{O\prime}(s) > 0$, we get the condition:

$$\beta_C < \beta_C^* = \frac{\delta}{\lambda+\delta}$$

Proposition 7: Proof.

a) From result 6b), we know that $s^{com}(\beta_C) \leq s(\beta_C)$ if and only if $\beta_C \leq \beta_C^*$. Moreover for power dynamics in the domain Ξ, we have $s(\beta_C(t+1)) = s^{com}(\beta_C(t)) \leq s(\beta_C(t))$ if and only if $\beta_C(t) \leq \beta_C^*$. The result follows from the fact that $s(\beta_C)$ is increasing in β_C.

b) for power dynamics in the domain Ξ, we have:

$$r(\beta_F(t+1), \beta_C(t+1)) = r^{com}(\beta_F(t), \beta_C(t))$$

From result 6a), and the fact that $r(\beta_F, \beta_C)$ is increasing in β_C, and result 7a), for $\beta_C(t) \leq \beta_C^*$ we get

$$\begin{aligned} r^{com}(\beta_F(t), \beta_C(t)) &> r(\beta_F(t), \beta_C(t)) \\ &\geq r(\beta_F(t), \beta_C(t+1)) \end{aligned}$$

Therefore,

$$r(\beta_F(t+1), \beta_C(t+1)) > r(\beta_F(t), \beta_C(t+1))$$

The result follows from the fact that $r(\beta_F, \beta_C)$ is increasing in β_F.

■

[60] s^{com} depends only on the political weight of the Center comes from the fact there is a symmetric negative externality on the fascist and the socialist groups.

Comparative Dynamics:

The two dimensional dynamics of power change may be characterized explicitly using the the system of equations (28) and (29) and noting that $r(\beta_F(t+1), \beta_C(t+1))$ and $s(\beta_C(t+1))$ are equal respectively to $r^{com}(\beta_F(t), \beta_C(t)) = r_t^{com}$ and $s^{com}(\beta_F(t), \beta_C(t)) = s_t^{com}$:

$$\begin{aligned} \beta_F(t+1)\cdot\Phi' - (1-\beta_F(t+1)-\beta_C(t+1))\cdot V_2'(p^O, r_t^{com}) - \Psi'(r_t^{com}) &= 0 \\ \beta_C(t+1)(1-\lambda)m^O - \Gamma'(s_t^{com}) &= 0 \end{aligned} \tag{32}$$

As well rewrite (30) and (31) for r_t^{com} and s_t^{com} as:

$$\begin{aligned} \beta_F(t)\left[\alpha I^{O\prime} + \Phi'\right] + \beta_C(t)\left(-I^O p^{O\prime}\right) \\ +(1-\beta_F(t)-\beta_C(t))\left[\gamma p^O I^{O\prime} - V_2'(p^O, r)\right] - \Psi'(r_t^{com}) &= 0 \\ \beta_F(t)\left(-\delta m^{O\prime}\right) + \beta_C(t)\left[(1-\lambda)m^O + \lambda m^{O\prime}\right] \\ +(1-\beta_F(t)-\beta_C(t))\left(-\delta m^{O\prime}\right) - \Gamma'(s_t^{com}) &= 0 \end{aligned} \tag{33}$$

where all the functions I^O, p^O, m^O and the derivatives $I^{O\prime}$, $p^{O\prime}$, $m^{O\prime}, \Phi'$ are evaluated at r_t^{com} and s_t^{com}.

Using (32) and (33), tedious but straightforward manipulations provide then the following dynamic system for β_F and β_C in the domain Ξ:

$$\begin{aligned} \beta_F(t+1) - \beta_F(t) &= -\frac{[\beta_C(t)\lambda - (1-\beta_C(t))\delta]\cdot V_2'(p^O, r)\frac{m^{O\prime}}{(1-\lambda)m^O}}{[\Phi' + V_2'(p^O, r)]} \\ &\quad + \frac{\beta_F(t)\left[\alpha I^{O\prime}\right] - \beta_C(t) I^O p^{O\prime} + \gamma p^O I^{O\prime}\left[1-\beta_F(t)-\beta_C(t)\right]}{[\Phi' + V_2'(p^O, r)]} \end{aligned} \tag{34}$$

and

$$\beta_{Ct+1} - \beta_{Ct} = [\beta_C(t)\lambda - (1-\beta_C(t))\delta]\frac{m^{O\prime}}{(1-\lambda)m^O} \tag{35}$$

Rearrangement provides

$$\begin{aligned} \beta_F(t+1) - \beta_F(t) &= \frac{1}{D}\cdot\begin{bmatrix} \beta_F(t)\left[\alpha I^{O\prime}\right] \\ -\beta_C(t) I^O p^{O\prime} \\ +\gamma p^O I^{O\prime}\left[1-\beta_F(t)-\beta_C(t)\right] \end{bmatrix} \\ &\quad -\frac{1}{D}\cdot[\beta_C(t) - \beta_C^*]\cdot V_2'(p^O, r)\frac{(\lambda+\delta)m^{O\prime}}{(1-\lambda)m^O} \end{aligned} \tag{36}$$

$$\beta_C(t+1) - \beta_C(t) = [\beta_C(t) - \beta_C^*]\frac{(\lambda+\delta)m^{O\prime}}{(1-\lambda)m^O} \tag{37}$$

with $D = \Phi'(r_t^{com}) + V_2'(p^O(r_t^{com}), r_t^{com}) > 0$.

Appendix A.3: Decentralized Fascist Violence

We assume that the Fascists have the following utility function:

$$U_F(I, v, p, w) = \omega_F + \alpha I - Fp(1 - v) - (1 + w)G(v) + B(w)$$

More precisely, the Fascists choose violent actions v targeted against the Socialist protests, which mitigate the negative impact of the protests at a rate $Fv \cdot p$. Such violent actions v have strictly increasing and convext resource costs $(1 + w)G(v)$ increasing convex in v. These costs are multiplicative in $1 + w$, the government police effort against decentralized violence. The last term finally reflects the net public benefit $B(w) - \Psi(w)$ of law enforcement.

Compared to the benchmark case in the previous section, the utility function of the center includes additionally the net public benefit of police:

$$U_C(I, p, w) = \omega_C + I(1 - p) - C(I) + B(w)$$

As for the Socialists, their utility function is now

$$U_S(I, p, v) = \omega_S + \gamma pI - V(p)(1 + v) + B(w)$$

where the cost of protest $V(p)(1 + v)$ depends now positively on the violent actions v of the Fascistss.[61]

Finally, the objective function of the government is written as:

$$W(\beta, I, p, v, w) = \beta_F \cdot U_F(I, v, p, w) + \beta_C \cdot U_C(I, p) + \beta_S \cdot U_S(I, p, v) - \Psi(w)$$

We make the following assumption:

Assumption A. 5 *We assume that the net public benefit $B(w) - \Psi(w)$ is concave in w with $B'(0) = +\infty$, $B'(\infty) = B(0) = 0$, $\Psi(0) = \Psi'(0) = 0$, $\Psi'(\infty) = \infty$. We assume also that there is a unique $w_0 > 0$ such that $w_0 = \arg\max_w [B(w) - \Psi(w)]$.*

w_0 is therefore the police level that maximizes the net public benefit of police forces, independently from political considerations.

We then have the following results (proofs are gathered below):

Lemma 4 *Given w, (I, P, v) are uniquely determined at equilibrium. Furthermore, I and v are decreasing in w, while p is increasing.*

Proposition 5 *The Nash equilibrium (I, P, v, w) is unique for any $\beta_F \leq 1 - \beta_C(0)$ (with $\beta_F + \beta_S + \beta_C(0) = 1$). Furthermore, at the Nash equilibrium,*

[61] We could also include as in the benchmark model that the police effort w has a state repression component $r = r(w)$ towards leftists protest efforts, and so include a cost term of protest of the form $V(p, r(w)).(1 + v)$. For simplicity, we abstract from this possibility as $r(w)$ is a substitute to fascist violence.

w is $\leq w_0$ and decreasing in β_F;

I is increasing in β_F;

v is increasing in β_F;

p is decreasing in β_F.

Lemma 5 *There exists a $\beta_C^m \in (0,1)$ such that, for $\beta_C(0) > \beta_C^m$ and any $\beta_F \in [0, 1-\beta_C(0)]$,*

$$w(\beta_F) > w^{com}(\beta_F).$$

The Nash equilibrium police effort $w(\beta_F)$ is larger than the effort with commitment $w^{com}(\beta_F, \beta_C)$ when the political weight of the Center β_C is above a certain threshold level β_C^m.[62]

Assuming that $\beta_C(0) \geq \beta_C^m$, we obtain the same result about the institutional dynamics as in the previous section:[63]

Proposition 6 *The political weight $\beta_F(t)$ of the Fascists increases over time; that is: as long as $\beta_F(t+1) < 1-\beta_C(0)$,*

$$\beta_F(t+1) > \beta_F(t).$$

Again the institutional weight of the Fascists will progressively increase from $\beta_F(0)$ to $1-\beta_C(0)$, and conversely the weight of the socialists will be reduced from $\beta_S(0)$ to 0. This is illustrated in Figure (3).

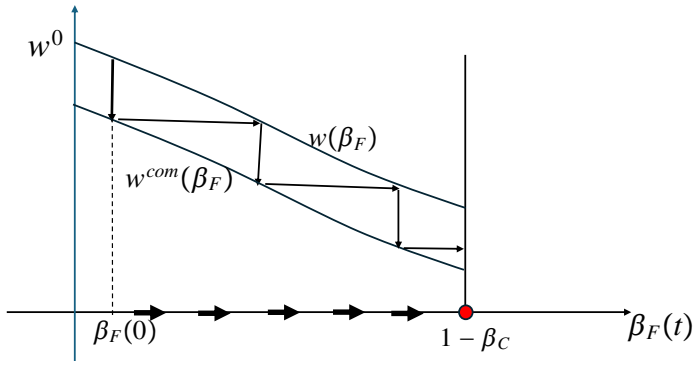
As in the benchmark model, the rate of change $\beta_F(t+1) - \beta_F(t)$ can be decomposed into the political externalities that arise in the Nash policy equilibrium. As already discussed, there are two negative political externalities affecting respectively the Center and the Fascists, as well as a positive externality to the Socialists. When the relative political power of the Center is large enough, i.e., $\beta_C(0) \geq \beta_C^m$, the negative externalities overcome the positive one, and $w(\beta_F(t)) > w^{com}(\beta_F(t))$. In order to internalize these political externalities, the institutional system consequently reallocates decision rights on public policy between the Fascists and the Socialists so as to favor a reduction of police enforcement against the fascist violence targeted towards the protests of the Socialists. This leads to an increase of the political weight β_F of the Fascists, which is the group favoring such reduction in police effort in society. In other words, the institutional dynamics in favor of the Fascists arises as the result of a political delegation process to promote more violence against socialist protests.

[62] The characterization of the threshold β_C^m suggests that it can be relatively small if protests are limited at the optimal police level w_0 or if fascist violence is quite insensitive to the police effort by the government.

[63] Formally, the dynamics of $\beta_F(t)$ are described by the following implicit difference equation:

$$\beta_F(t+1) = \begin{cases} \beta'_F \in (0, 1-\beta_C) \text{ such that } w(\beta'_F) = w^{com}(\beta_F(t)) \\ 1-\beta_C \quad \text{if } w(\beta'_F) > w^{com}(\beta_F(t)) \text{ for } \beta'_F \in (0, 1-\beta_C) \end{cases}$$

Figure 4: The dynamics of political power of the Fascists with decentralized violence



Proofs:

Lemma 8: Proof. Given w, at a Nash Civil society equilibrium, the following conditions are satisfied:

$$\begin{aligned} 1-p &= C'(I) \\ Fp &= (1+w)G'(v) \\ \gamma I &= V'(p)(1+v) \end{aligned}$$

Eliminating the protest variable p between the first and the third equation, provides a condition:

$$\gamma I = V'(1-C'(I))(1+v) \tag{38}$$

that relates the level of investment $I = I(v)$ to the level of fascist violence v. It is easy to see that the relationship is positive, as by the Implicit Function theorem:

$$\frac{dI}{dv} = \frac{V'(1-C'(I))}{\gamma + V''(1-C'(I)) \cdot C''(I)(1+v)} > 0.$$

Similarly, eliminating p between the first and the second equation:

$$F\left[1-C'(I)\right] = (1+w)G'(v) \tag{39}$$

which pins down the level of violence $v = v(I, w)$ as a function of I and w. Again by the Implicit Function theorem:

$$\begin{aligned} \frac{\partial v}{\partial I} &= \frac{-FC''(I)}{(1+w)G''(v)} < 0 \\ \frac{\partial v}{\partial w} &= \frac{-G'(v)}{(1+w)G''(v)} < 0. \end{aligned}$$

The level of violence $v(I, w)$ is decreasing in I and w.

It is easy to see that equations 38 and 39 jointly determine a unique civil society equilibrium $I^O(w)$ and $v^O(w)$ as functions of law enforcement w. Correspondingly, we also obtain the level of Socialist protest $p^O(w)$. ■

The utility of the Socialists for a given value of w, net of the public benefit $B(w)$, is then given by:

$$u_S(w) = U_S(I^O, p^O, v^O, w) - B(w) = \gamma p^O(w) I^O(w) - V(p^O(w))(1 + v^O(w))$$

Differentiation provides:

$$u'_S(w) = \underbrace{\gamma p^O I^{O\prime}}_{-} \underbrace{- V(p^O(w)) v^{O\prime}}_{+} > 0$$

the sign of which is ambiguous. First, police effort tends to reduce the equilibrium level of fascist violence targeted against the socialists. This has a positive effect on the latter, as it reduces their cost of protest. This is reflected by the term $-V(p^O(w))v^{O\prime} > 0$. But also there is the equilibrium effect that an increase in police force stimulates more socialist protests $p^O(w)$ and therefore a lower equilibrium investment $I^O(w)$. This in turn reduces the gains that the socialists group can extract from the center through the protests, as reflected by the term $\gamma p^O I^{O\prime} < 0$.

Proposition 9: Proof. The characterization of the Nash equilibrium policy $w(\beta_F)$ is given by the condition:

$$-\beta_F G(v^O(w)) + B'(w) - \Psi'(w) = 0 \tag{40}$$

Define the function $\Theta(w) = -\beta_F G(v^O(w)) + B'(w) - \Psi'(w)$. When $B(w) - \Psi(w)$ is concave enough in w (ie. $-B''(w) + \Psi''(w) > -G'(v^O(w))\frac{dv^O}{dw}$), $\Theta(w)$ is a decreasing decreasing in w and such that $\Theta(0) = +\infty$, while $\Theta(\infty) = -\infty$. Therefore there exists a unique $w(\beta_F) > 0$ such that $\Theta(w(\beta_F)) = 0$. Moreover as $\Theta(w_0) = -\beta_F G(v^O(w_0)) < 0 = \Theta(w^N(\beta_F))$, it follows that $w(\beta_F) < w^0$ for all $\beta_F \in [0, 1 - \beta_C]$. Simple differentiation of (40) provides that $w(\beta_F)$ is decreasing in β_F. ■

Lemma 10: Proof. The commitment policy is the solution of the following maximization program:

$$\max_w W^c(w) = W(\beta, I^O(w), p^O(w), v^O(w), w)$$

with

$$\begin{aligned} W(\beta,I^O(w),p^O(w),v^O(w),w) &= \beta_F U_F(I^O(w),v^O(w),p^O(w),w) \\ &\quad +\beta_C U_C(I^O(w),p^O(w),w) \\ &\quad +\beta_S U_S(I^O(w),p^O(w),v^O(w),w) \end{aligned}$$

The first order condition gives:

$$\begin{aligned} & \beta_F \left(\alpha I^{O\prime} - F(1-v^O)p^{O\prime} - G(v^O)\right) \\ & -\beta_C(I^O p^{O\prime}) + \beta_S \left[\gamma p^O I^{O\prime} - V(p^O)v^{O\prime}\right] \\ & +B'(w) - \Psi'(w) \\ = \;& 0 \end{aligned} \tag{41}$$

Then, under assumptions A.3 and A.4, these conditions characterizes a unique $w^{com}(\beta_F,\beta_C)$. Differentiation provides that

$$\frac{\partial w^{com}(\beta_F,\beta_C)}{\partial \beta_F} = -\frac{\alpha I^{O\prime} - F(1-v^O)p^{O\prime} - G(v^O) - \left[\gamma p^O I^{O\prime} - V(p^O)v^{O\prime}\right]}{W^{c\prime\prime}(w)}$$

the concavity of $W^c(w)$ ensures that the denominator is negative. Moreover $\alpha I^{O\prime} - F(1-v^O)p^{O\prime} - G(v^O) < 0$ and under assumption A.3 $\left[\gamma p^O I^{O\prime} - V(p^O)v^{O\prime}\right] > 0$. Therefore it follows that

$$\frac{\partial w^{com}(\beta_F,\beta_C)}{\partial \beta_F} < 0$$

Similarly

$$\frac{\partial w^{com}(\beta_F,\beta_C)}{\partial \beta_C} = -\frac{-\beta_C(I^O p^{O\prime}) - \left[\gamma p^O I^{O\prime} - V(p^O)v^{O\prime}\right]}{W^{c\prime\prime}(w)}$$

given that $-\beta_C(I^O p^{O\prime}) < 0$ and with Assumption A.4 $\left[\gamma p^O I^{O\prime} - V(p^O)v^{O\prime}\right] > 0$, we obtain as well that

$$\frac{\partial w^{com}(\beta_F,\beta_C)}{\partial \beta_C} < 0.$$

Given the concavity of $W^c(w)$, the comparison between $w(\beta_F)$ and $w^{com}(\beta_F,\beta_C)$ rests then on the sign of $\frac{dW^c}{dr}\Big|_{w(\beta_F)}$. Differentiation provides immediately that

$$\left.\frac{dW^c}{dr}\right|_{w(\beta_F)} = \beta_F \left(\underbrace{\alpha I^{O\prime} - F(1-v^O)p^{O\prime}}_{-}\right) \tag{42}$$

$$-\beta_C(\underbrace{I^O p^{O\prime}}_{+}) \tag{43}$$

$$+\beta_S \left[\underbrace{\gamma p^O I^{O\prime} - V(p^O)v^{O\prime}}_{+}\right] \tag{44}$$

Typically

$$\left.\frac{dW^c}{dr}\right|_{w(\beta_F)} \gtrless 0$$

The three terms in (42) reflect the political externalities of police force effort at the Nash policy equilibrium on the different groups. The first term shows the negative externality of higher w on the fascist group. It is composed of two elements. First, there is the decrease of the economic spillovers that the fascists enjoy because of lower Center investment (ie. $\alpha I^{O\prime} < 0$). Second, there is the utility loss of facing socialist protests stimulated by higher police action against fascist group violence towards them (ie. $-F(1-v^O)p^{O\prime} < 0$). The second term reflects the negative externality $-I^O p^{O\prime} < 0$ on the Center group due to the fact that higher police repression on fascist violence tends to stimulate socialists protests, and therefore to reduce the share of investment that the Center can keep for itself. Finally, the last term describes the externality associated to the socialist group. Under assumption S2, this externality is positive. A higher level of police enforcement w reduces the Fascist violence. This in turn improves the welfare of socialists by reducing their cost of protest by more than the rent reduction that they can extract on a lower level of Center's investment (ie. $\gamma p^O I^{O\prime} - V(p^O)v^{O\prime} > 0$).

We have $w(\beta_F) > w^{com}(\beta_F, \beta_C)$ if and only if $\frac{dW^c}{dr}\big|_{w(\beta_F)} < 0$, that is when the sum of the three externalities is negative. Examination of (42) shows that this is the case when the weight of the Center β_C, is large enough. More precisely, Using the fact that $\beta_S = 1 - \beta_F - \beta_C$,, one gets a negative sign of the RHS of (42) when

$$\beta_F > \frac{\left[\gamma p^O I^{O\prime} - V(p^O)v^{O\prime}\right] - \beta_C\left\{I^O p^{O\prime} + \left[\gamma p^O I^{O\prime} - V(p^O)v^{O\prime}\right]\right\}}{\left[\gamma p^O I^{O\prime} - V(p^O)v^{O\prime}\right] + F(1-v^O)p^{O\prime} - \alpha I^{O\prime}} \tag{45}$$

Now the RHS of (45) is negative when

$$\beta_C > \beta_C^m = \max_w \frac{-V(p_0)v^{O\prime}(w)}{-V(p_0)v^{O\prime}(w) + I_0^O p^{O\prime}(w)}$$

where $p_0 = p^O(w_0)$. Indeed, given that $w(\beta_F) < w_0$, $p^O(w)$ is increasing in w, $I^O(w)$ is decreasing in w, and $v^O(w)$ is decreasing in w, one has that $V(p^O) = V(p^O\left(w^N(\beta_F)\right)) < V(p^O(w_0)) = V(p_0)$, and $I^O p^{O\prime} > I^O(w_0)p^{O\prime}(w) = I_0^O p^{O\prime}(w)$. thus

$$\frac{\left[\gamma p^O I^{O\prime} - V(p^O)v^{O\prime}\right]}{I^O p^{O\prime} + \left[\gamma p^O I^{O\prime} - V(p^O)v^{O\prime}\right]} < \frac{-V(p_0)v^{O\prime}}{-V(p_0)v^{O\prime} + I_0^O p^{O\prime}(w)} \leq \beta_C^m \tag{46}$$

The RHS of (45) is then negative when

$$\frac{\left[\gamma p^O I^{O\prime} - V(p^O)v^{O\prime}\right]}{\left\{I^O p^{O\prime} + \left[\gamma p^O I^{O\prime} - V(p^O)v^{O\prime}\right]\right\}} - \beta_C < 0$$

which given (46) is ensured when $\beta_C > \beta_C^m$. ■

Proposition 11: Proof. By definition when $\beta_F(t+1) < 1 - \beta_C$,

$$w\left(\beta_F(t+1)\right) = w^{com}\left(\beta_F(t), \beta_C\right) < w\left(\beta_F(t)\right)$$

Given that the function $w\left(\beta_F\right)$ is decreasing in β_F, it follows that $\beta_F(t+1) > \beta_F(t)$. ■

Comparative dynamics

We can now decomposition of the rate of change $\beta_F(t+1) - \beta_F(t)$. We follow the same logic as in the benchmark model. Consider equation (40) evaluated at $w^{com}(\beta_F(t), \beta_C) = w_t^{com}$, and condition (41), which together rewrite as :

$$-\beta_F(t+1)G(v^O(w_t^{com})) + B'(w_t^{com}) - \Psi'(w_t^{com}) = 0$$

$$\begin{aligned}
&\beta_F(t)\left(\alpha I^{O\prime}\left(w_t^{com}\right) - F(1 - v^O\left(w_t^{com}\right))p^{O\prime}\left(w_t^{com}\right) - G(v^O\left(w_t^{com}\right))\right) \\
&-\beta_C(I^O\left(w_t^{com}\right)p^{O\prime}\left(w_t^{com}\right)) + \beta_S\left[\gamma p^O\left(w_t^{com}\right)I^{O\prime}\left(w_t^{com}\right) - V(p^O\left(w_t^{com}\right))v^{O\prime}\left(w_t^{com}\right)\right] \\
&+B'(w_t^{com}) - \Psi'(w_t^{com}) \\
= \;& 0
\end{aligned}$$

Omitting for notational convenience the fact that all the variables are evaluated at w_t^{com}, and combining and rearranging these two conditions, one obtains:

$$\beta_F(t+1) - \beta_F(t) = \frac{1}{G(v_t^O))}\left\{\begin{array}{c} -\beta_F(t)\left[\alpha I_t^{O\prime} - F(1-v_t^O)p_t^{O\prime}\right] \\ -\beta_C I_t^O p_t^{O\prime} \\ +(1-\beta_F(t)-\beta_C)\cdot\left[\gamma p_t^O I_t^{O\prime} - V(p_t^O)v_t^{O\prime}\right] \end{array}\right\} \tag{47}$$

Inside the bracket of equation (47), there are three terms which highlight the different policy externalities driving the dynamics of power of the fascist group. The first term $-\beta_F(t)\left[\alpha I_t^{O\prime} - F(1-v_t^O)p_t^{O\prime}\right]$ indicates the political externality generated on the fascist group by a reduced police enforcement effort w. It has two components. First, there is the positive investment spillovers effect $-\beta_F(t)\alpha I_t^{O\prime} > 0$ that a reduction of police enforcement w on fascist violence has on the economic benefits of the fascist members. Second, there is a positive effet $\beta_F(t)F(1-v_t^O)p_t^{O\prime} > 0$ related to the fact that less police enforcement reduces the equilibrium level of leftist protests, which in turn has a positive effect on the payoff of the fascists.

The second term $-\beta_C I_t^O p_t^{O\prime} > 0$ shows the positive political externality generated on the Center group by a reduced protest level of the socialists as induced by less police enforcement against fasression policy r.

Finally,associated to a reduced police effort the last term $(1-\beta_F(t)-\beta_C)\cdot\left[\gamma p_t^O I_t^{O\prime} - V(p_t^O)v_t^{O\prime}\right]$ characterizes the political externality of reduced police enforcement on the Socialist members. Under assumption S4, this term is negative: the increased level of fascist violence induced by a more laxist police overcomes the positive effect on the investment rents that they extract on the Center group.