Spatial Data Structures

CS425: Computer Graphics I

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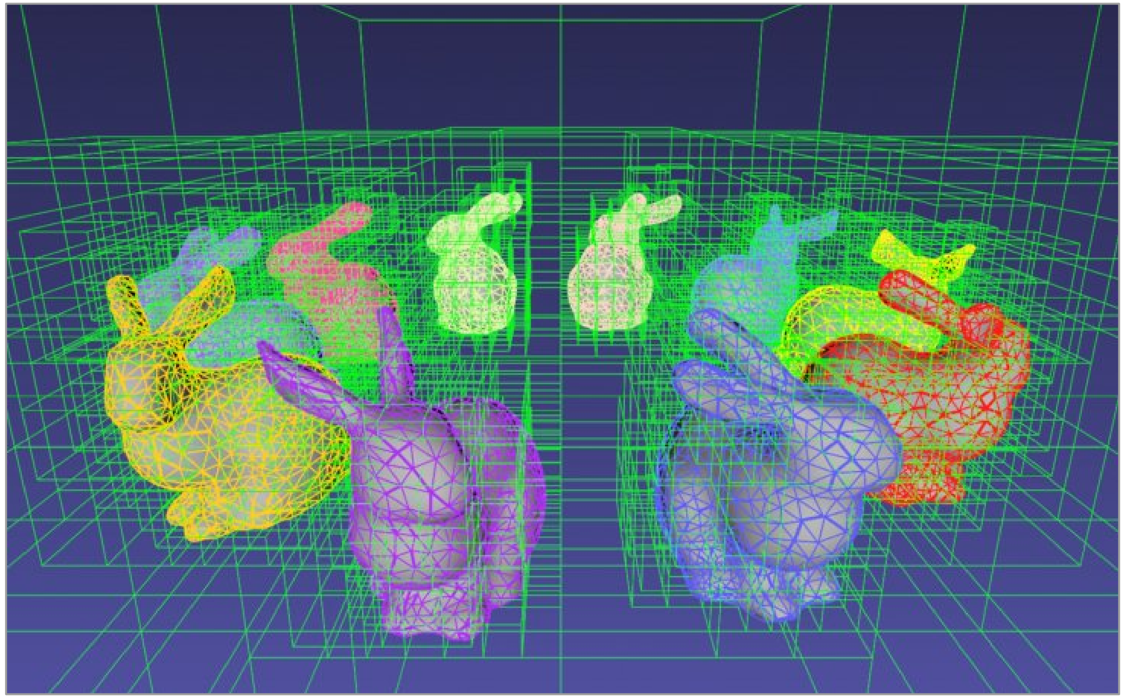
Overview

- Spatial data structures
  - Uniform grid
  - Nested grids
  - Quadtree / octree
  - K-d tree
  - Bounding Volume Hierarchy

How to efficiently organize objects?

- 3D data contains spatial information.
- How to perform queries when there are thousands / millions of objects (points, polygons)?
  - Ray-scene intersection.
  - Proximity queries
  - Point in polygon.
  - Range query.

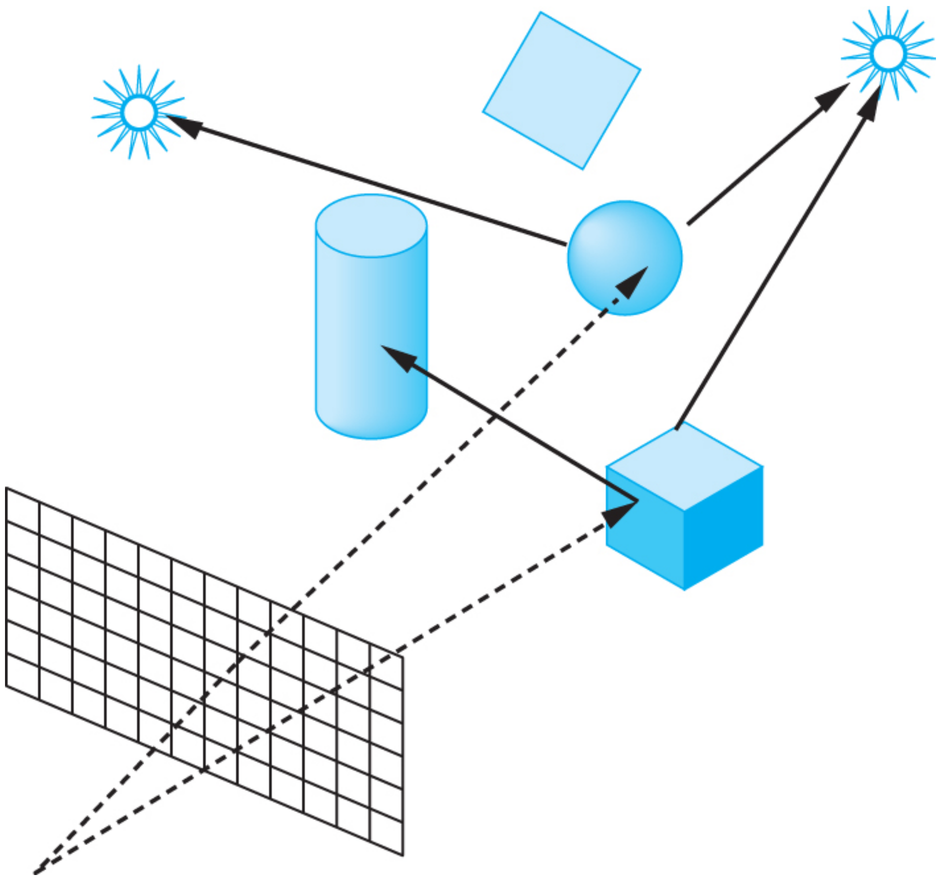


Ray-scene intersection

- Given a scene with n primitives and a ray r, find the closest point of intersection of r with the scene.

```
function intersectObjects(ray, scene) {
    for(var i=0; i < scene.objects.length; i++) {
        var object = scene.objects[i];
        var dist = intersection(ray, object);
        // ...
    }
}
```

- Complexity: $O(n)$
- How to do better?

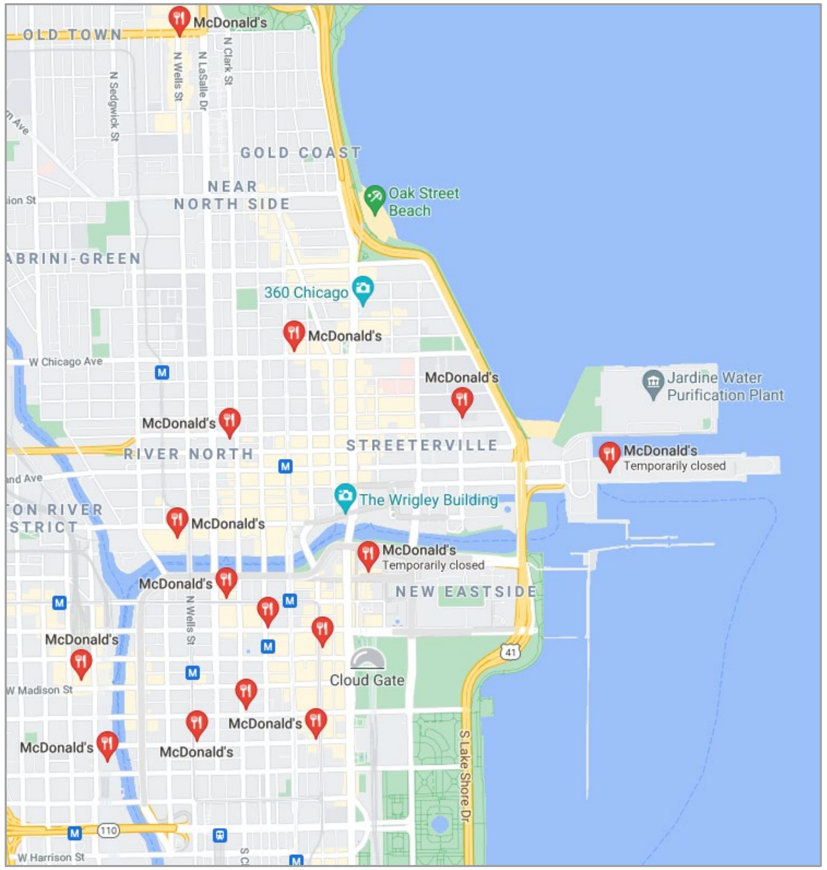


Proximity query

- Query based on proximity.
- “What is the closest McDonald’s?”

```
function findPlaces(query, scene) {
    for(var i=0; i < scene.places.length; i++) {
        var place = scene.places[i];
        var dist = satisfyQuery(query, place);
        // ...
    }
}
```

- Complexity: $O(n)$
- How to do better?



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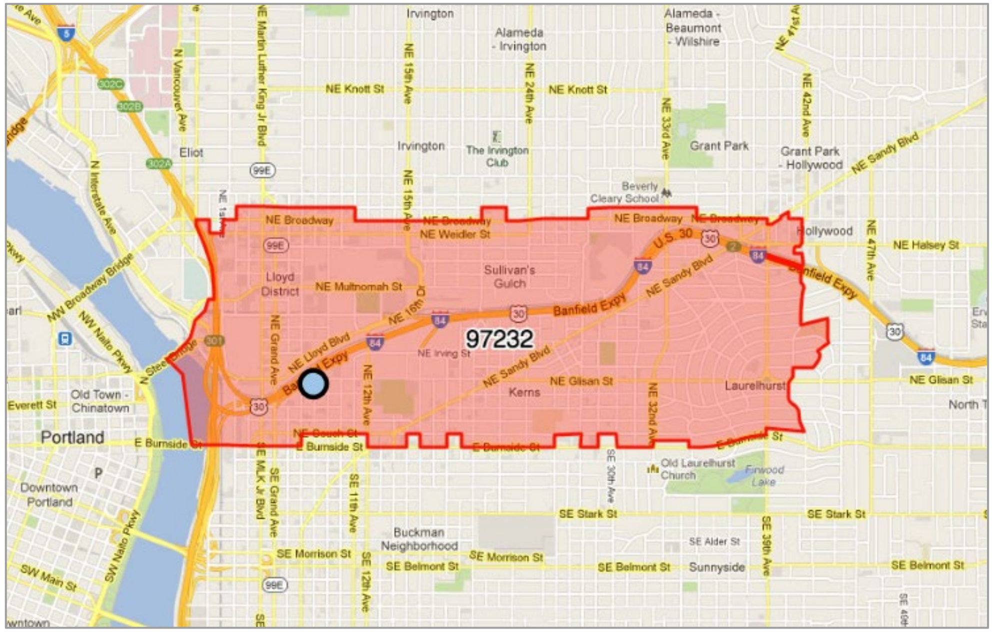
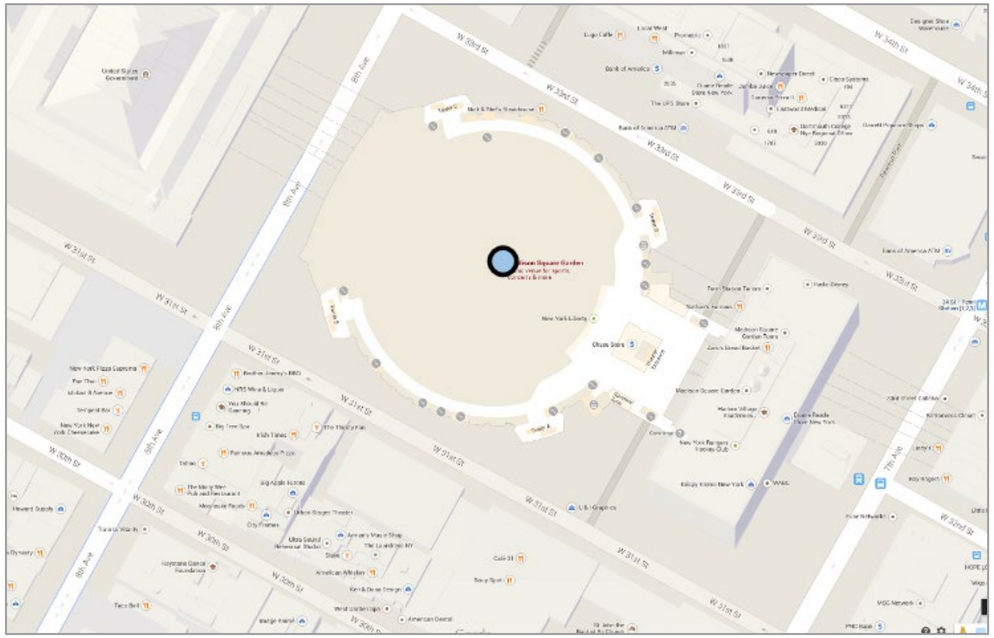
Point in polygon



What is the zip code for this complaint?



Am I inside a specific building?

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Time complexity

- Ray-scene intersection: $O(n)$
- Proximity query: $O(n)$
- Point in polygon: $O(n)$

How to reduce the time complexity?

Motivation

- Expensive operations (ray tracing, query).
  - Complex scenes (millions of objects).
  - Large number of operations (hundreds of millions per second).
- Reduce complexity through pre-processing data.
  - Spatial data structures: structures of objects in space.
  - Eliminate candidates as early as possible.
  - Reduce complexity to $O(log\ n)$ on average.
  - Worst case complexity still $O(n)$.
    - Can you come up with a worst case example?

Spatial data structures

- Data structures to accelerate queries of the kind:

  “I’m here, which object is around me?”

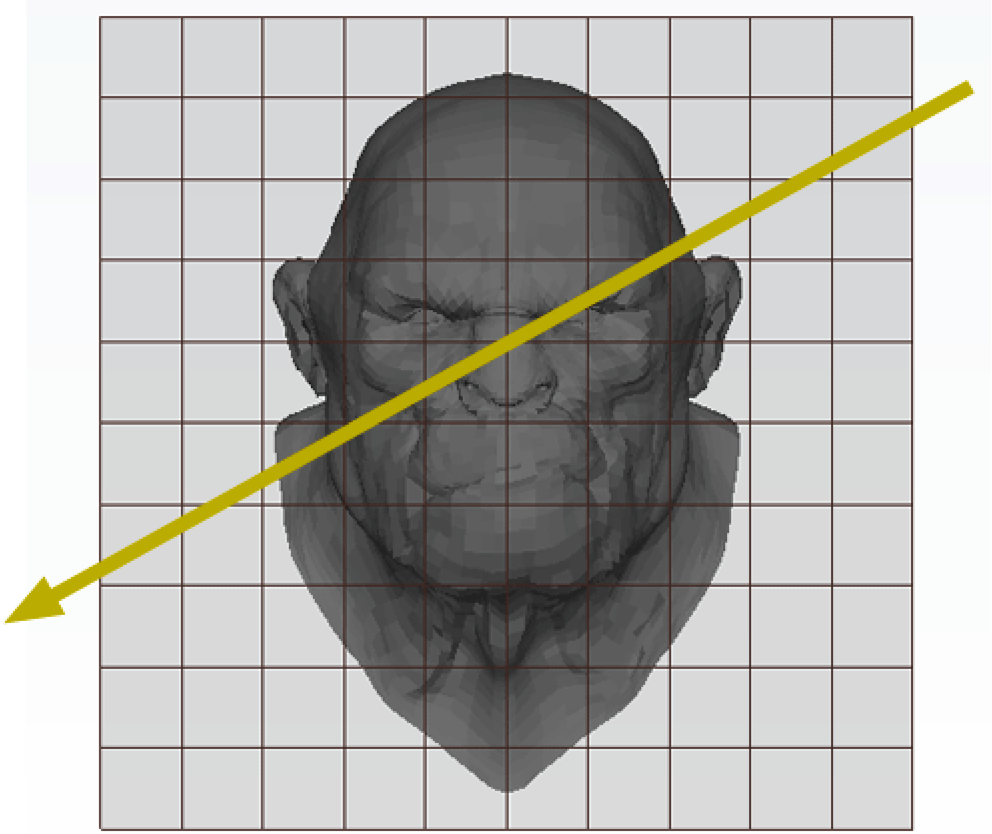
- Partition space or set of objects.
- Tasks:
  1. Construction / update:
     - Pre-processing for static parts of the scene.
     - Update for moving parts of the scene.
  2. Access:
     - Optimize so it is done as fast as possible.

Spatial data structures

- Uniform grid: 2D/3D data, uniform distribution.
- Quadtree: 2D data, non-uniform distribution.
- Octree: 3D data, non-uniform distribution.
- KD-tree: 2D/3D data, avoid empty cells.

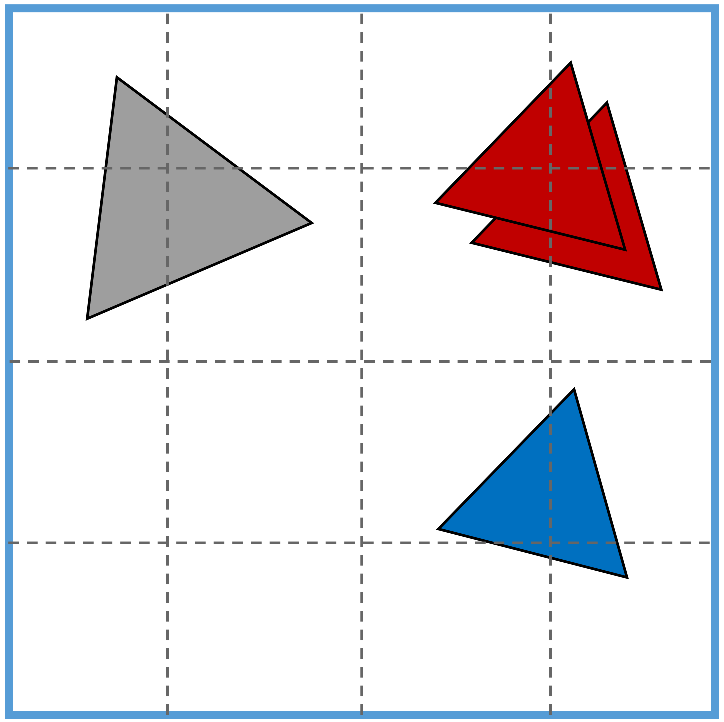
Uniform grid

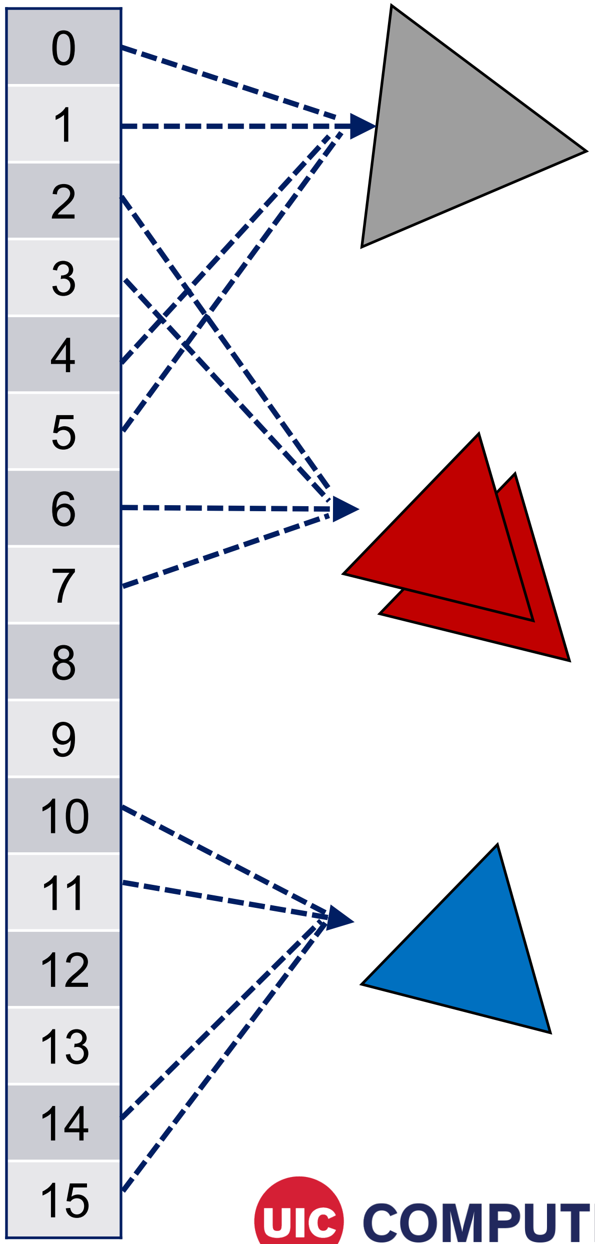
- Partition space into equal-sized volumes (i.e., voxels).
- Each cell will contain objects that overlap the voxel.
- Good for uniform data (points are evenly distributed in space).
- Fast construction and queries.



scratchpixel.com

Uniform grid





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Uniform grid: construction and query

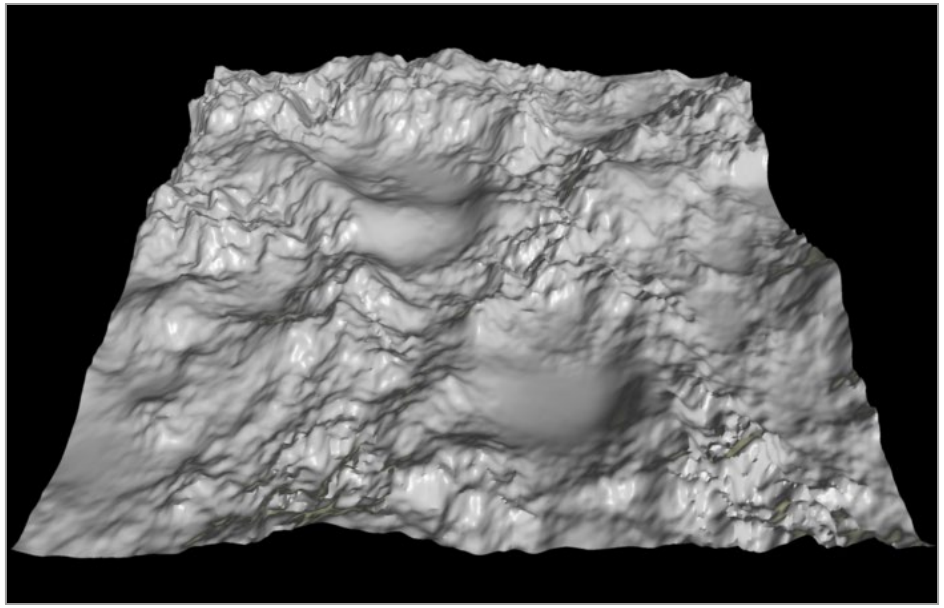
- Array of 3D voxels
  - Each voxel: list of pointers to colliding objects.
- Indexing function:
  - 3D point → cell index (constant time!)
- Construction:
  - Initialize cells for grid with size $w * h$
  - For each object $\mathrm{p}(x, y)$:
    - Compute grid cell using (x, y).
    - Store p in cell.
- Query:
  - For query rectangle $(x_1, y_1) \times (x_2, y_2)$:
    - Compute subgrid for (x_1, y_1) and (x_2, y_2).
    - For all cells inside subgrid, report all objects.
    - For all cells on the border of the subgrid, test objects against rectangle.

Uniform grid: complexity

- Build time: $O(n)$
- Space: $O(w * h) + O(n)$
- Query: $O(k)$

Uniform grid: complexity

- When uniform grids work well? Uniform distribution of objects.

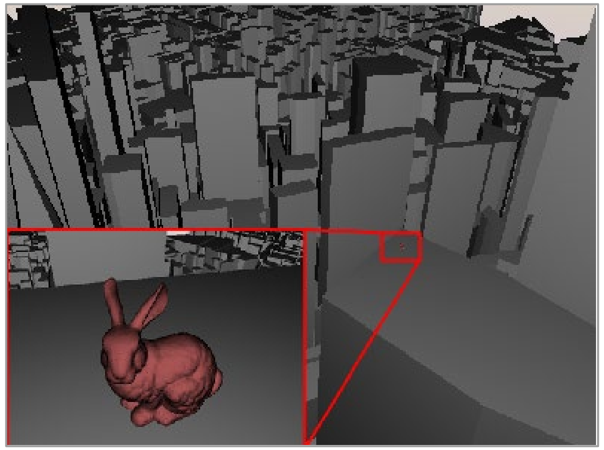
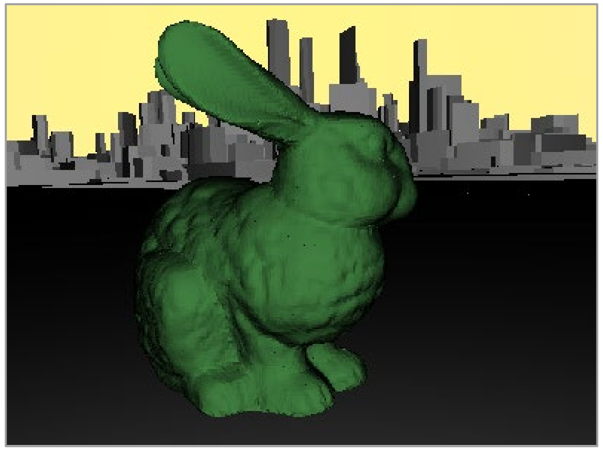
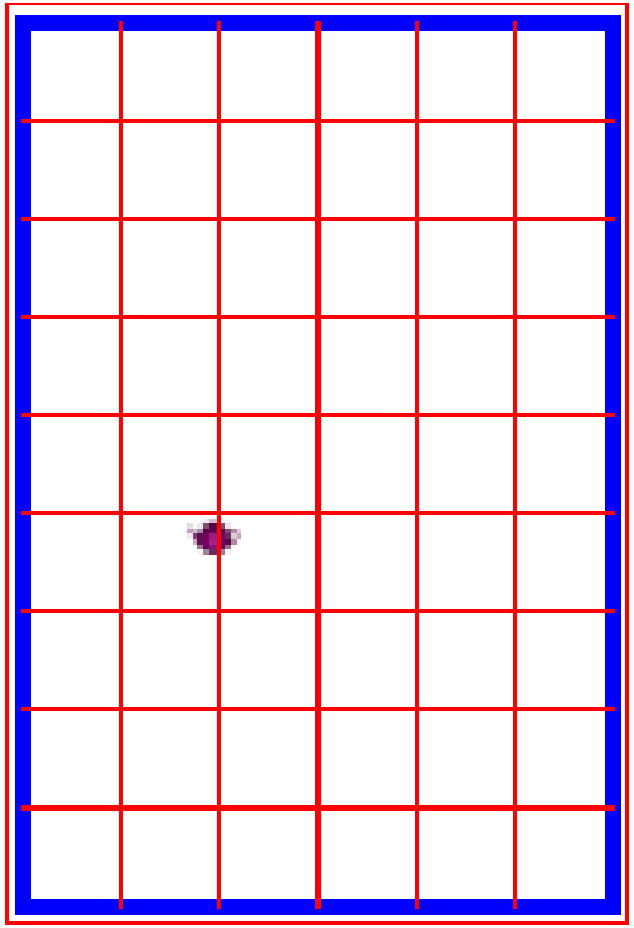


Mitsuba renderer

peterguthrie.net

Uniform grid: drawbacks

- When uniform grids do not perform well? Non-uniform distribution of objects.
- "Teapot in a stadium" problem: uniform grids cannot adapt to local density of objects.

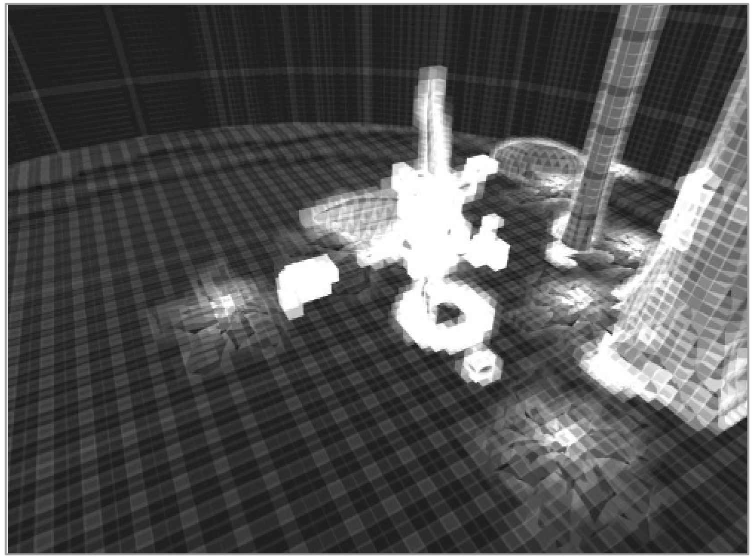




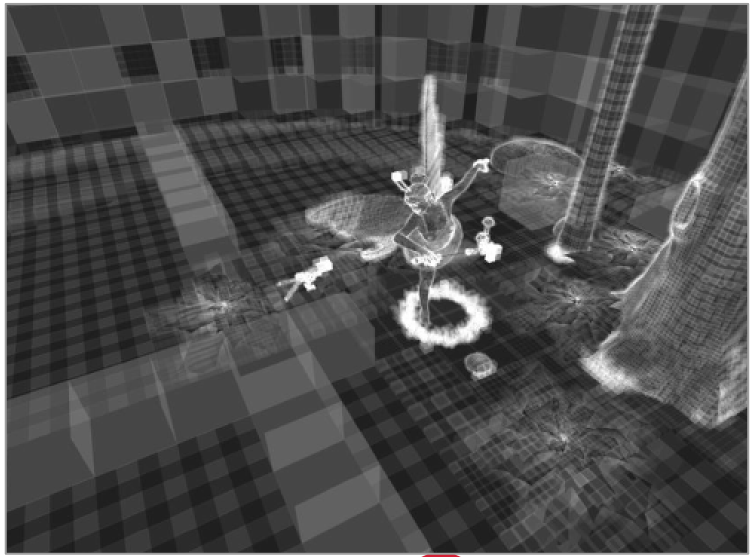
Uniform grid: drawbacks

- Assumes objects uniformly distributed in space.
- What happens when assumption does not hold?
  - Many empty cells.
  - Few cells with too many points.
- Change cell size?
  - Too small: memory occupancy too large.
  - Too big: too many objects in one cell.

Nested grids

- Possible solution to “teapot in a stadium” problem.
- Hierarchy of uniform grids: each cell is itself a grid.
- Fast building & traversal.

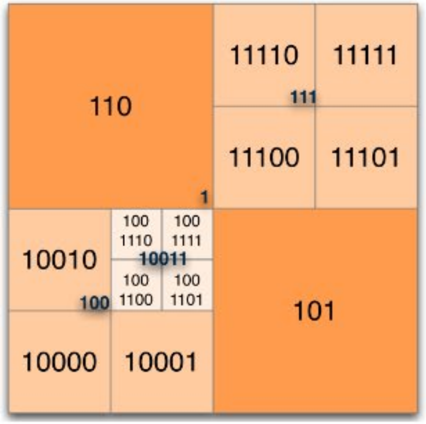


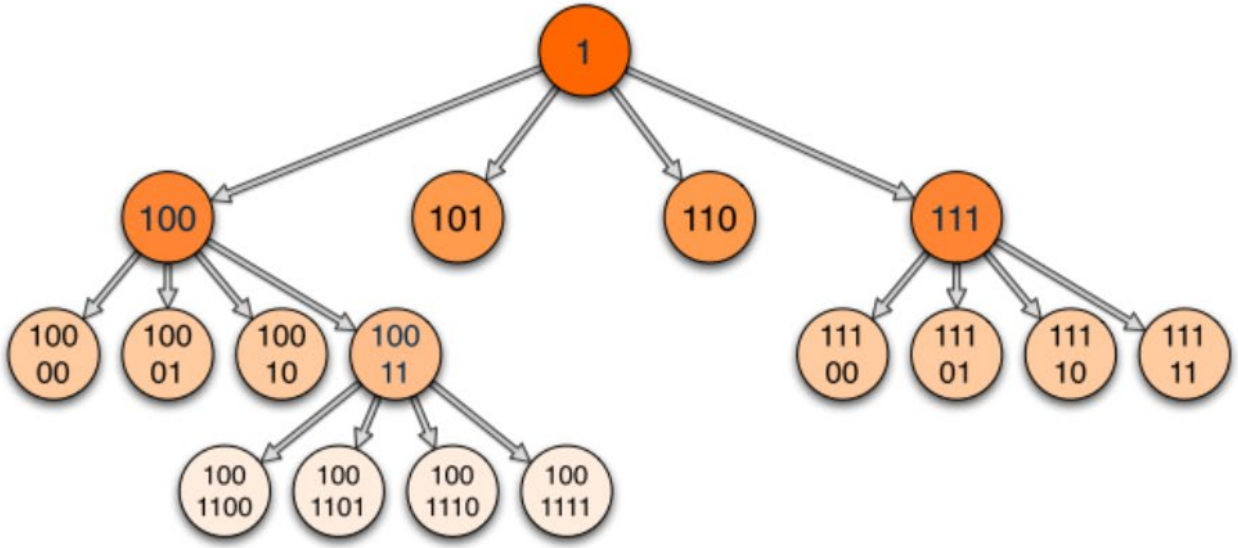


Philipp Slusallek

Quadtree

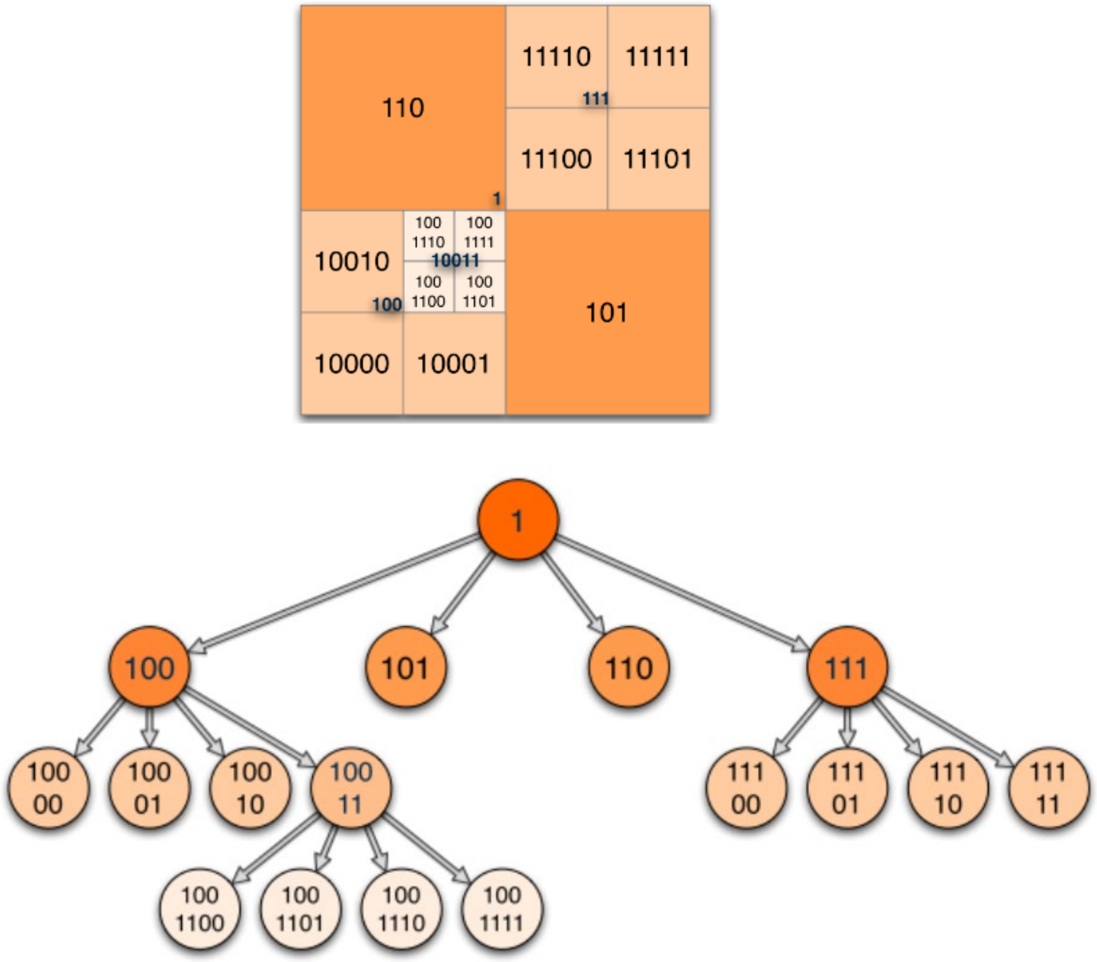
- Hierarchical structure that stores regular grids at each level.
- Adaptive subdivision: adjust depth to local scene complexity.





Quadtree

- Rooted tree in which every internal node has four children.
- Every node corresponds to a square.
- Tree: branching factor 4 or 8.
- Each node: splits into all dimensions at once (in the middle).
- Construction: continue splitting until end nodes have few objects (or limit level reached).



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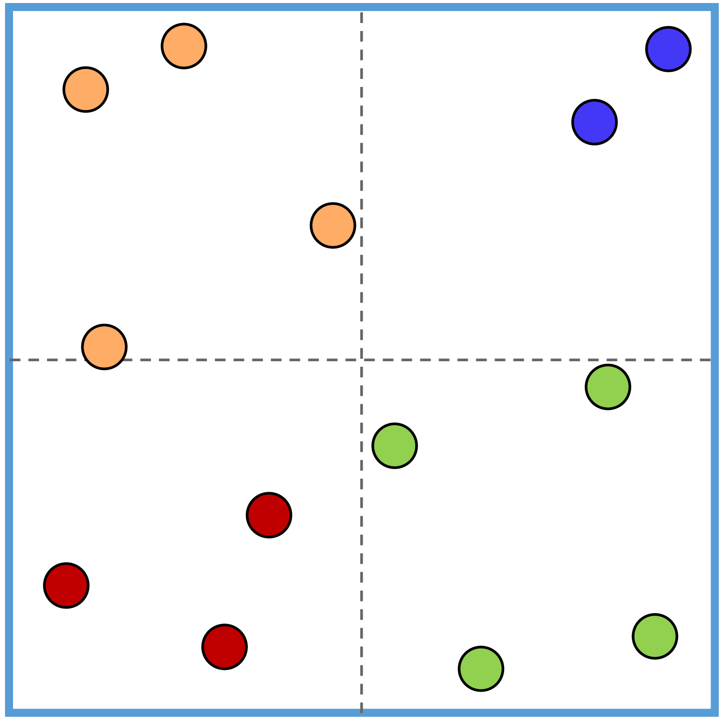
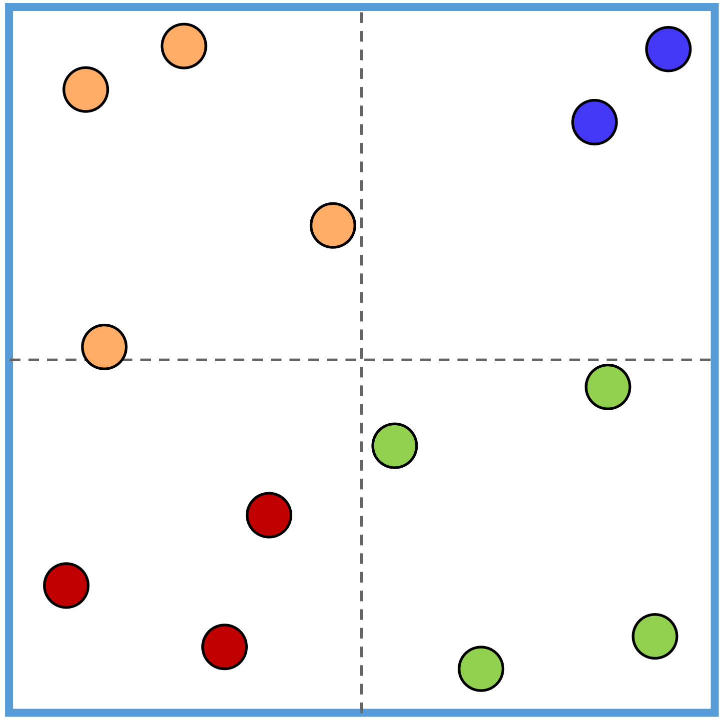
Quadtree: construction

- Split the top level.



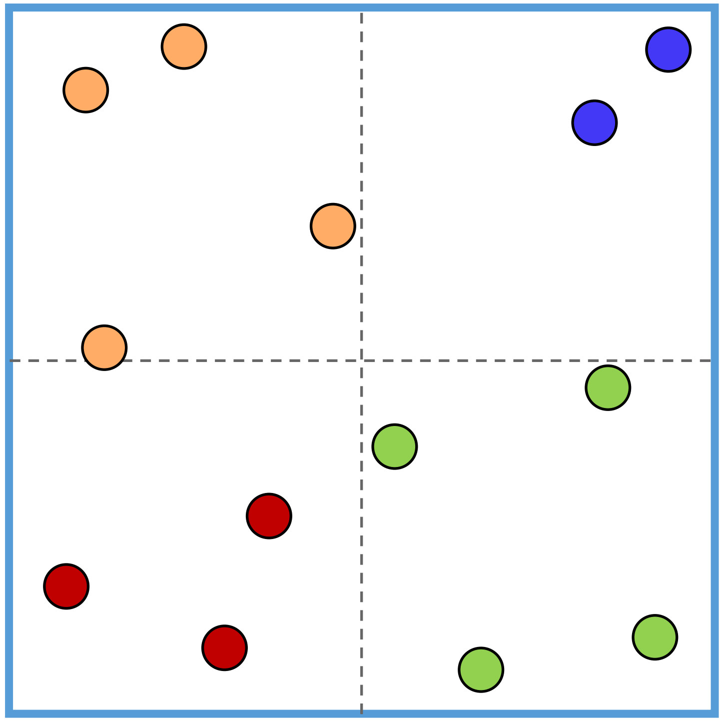
Quadtree: construction

- Split the top level.
- Can we stop?



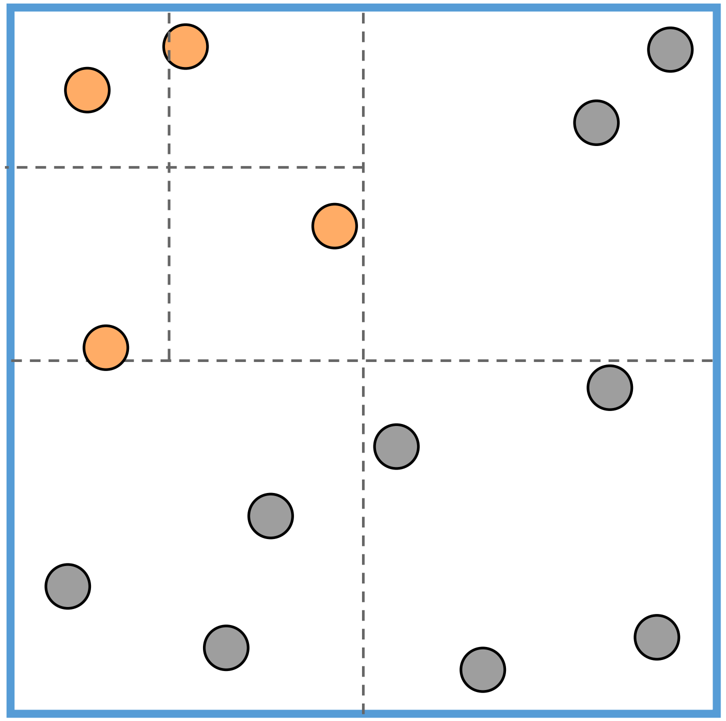
Quadtree: construction

- Split the top level.
- Can we stop? No, split the next level.



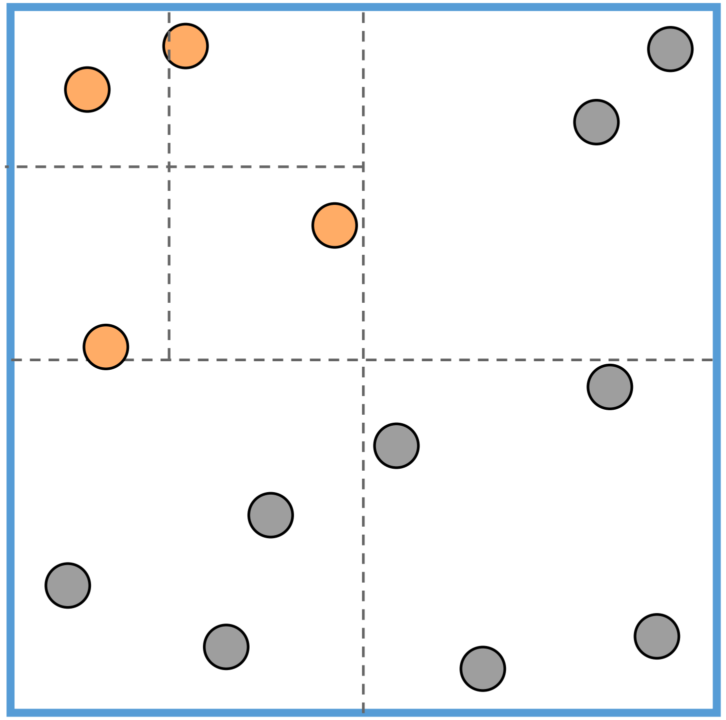
Quadtree: construction

- Split the top level.
- Can we stop? No, split the next level.
- Split top-left.



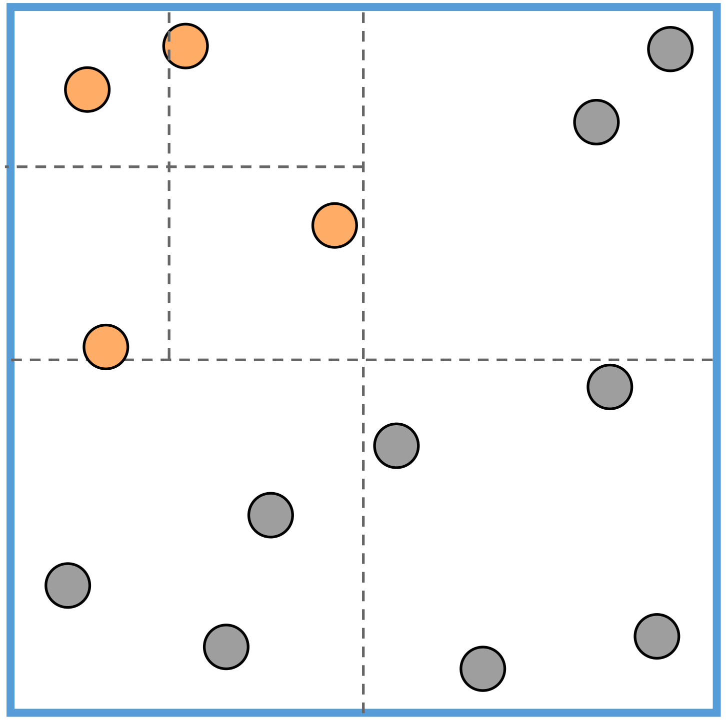
Quadtree: construction

- Split the top level.
- Can we stop? No, split the next level.
- Split top-left.
- Can we stop top-left?



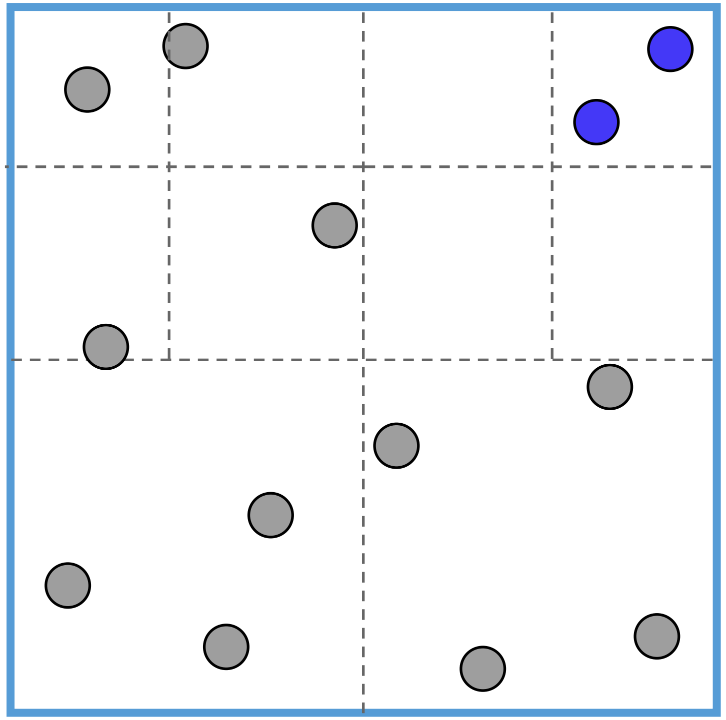
Quadtree: construction

- Split the top level.
- Can we stop? No, split the next level.
- Split top-left.
- Can we stop top-left? Yes.



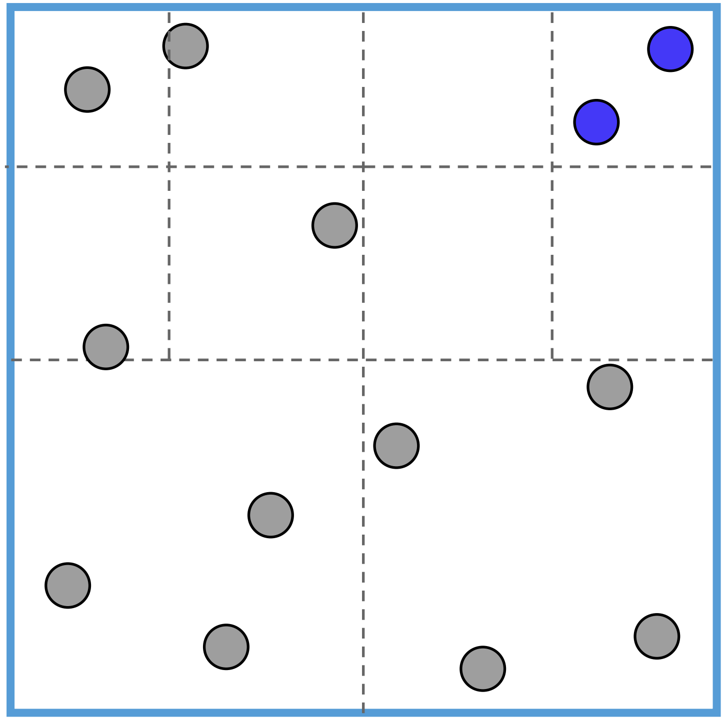
Quadtree: construction

- Split the top level.
- Can we stop? No, split the next level.
- Split top-left.
- Can we stop top-left? Yes.
- Split top-right.



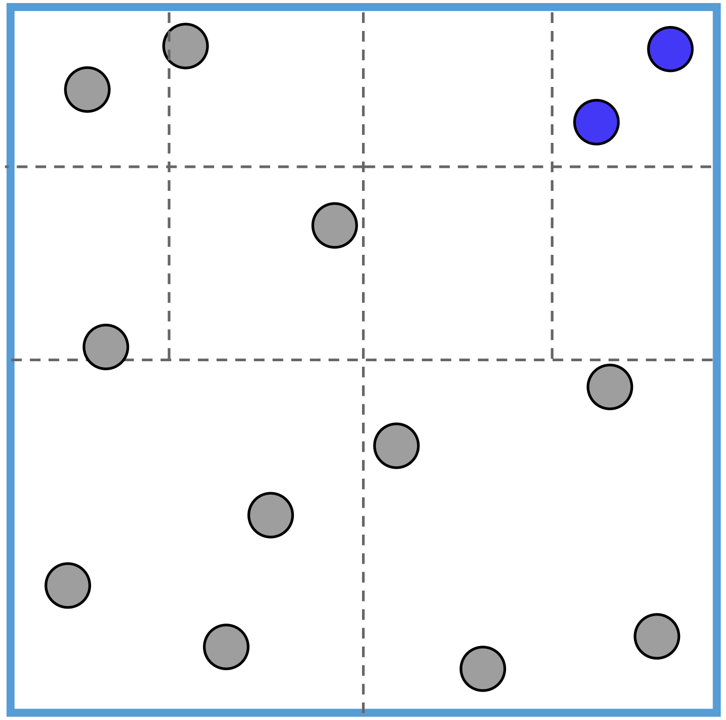
Quadtree: construction

- Split the top level.
- Can we stop? No, split the next level.
- Split top-left.
- Can we stop top-left? Yes.
- Split top-right.
- Can we stop top-right?



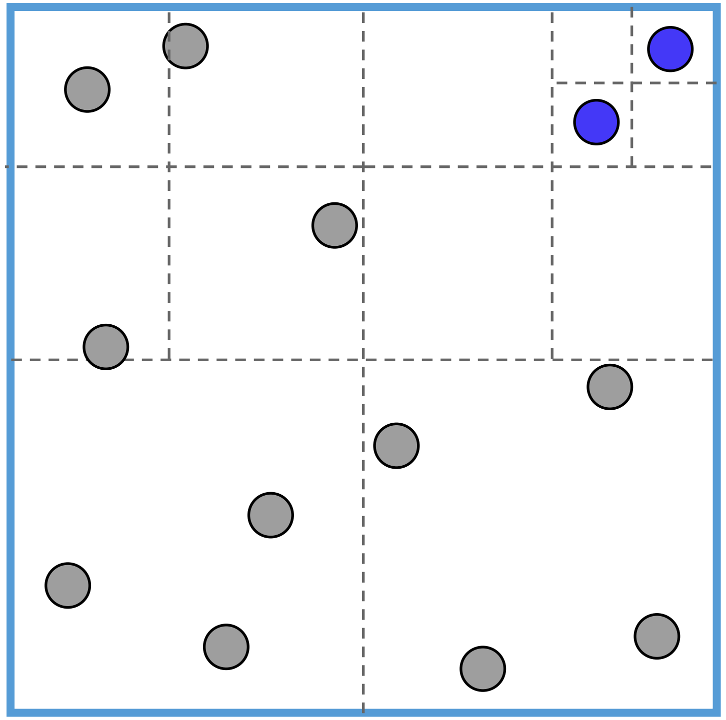
Quadtree: construction

- Split the top level.
- Can we stop? No, split the next level.
- Split top-left.
- Can we stop top-left? Yes.
- Split top-right.
- Can we stop top-right? No.



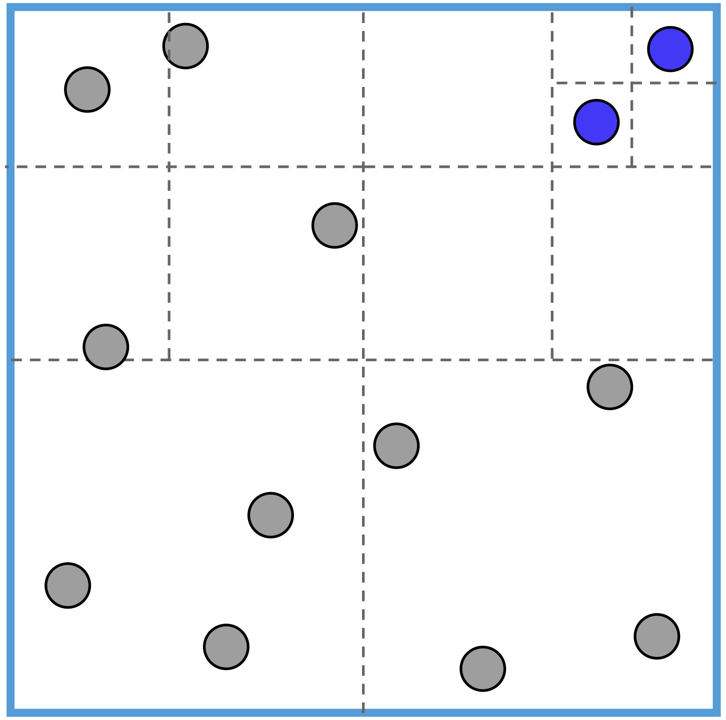
Quadtree: construction

- Split the top level.
- Can we stop? No, split the next level.
- Split top-left.
- Can we stop top-left? Yes.
- Split top-right.
- Can we stop top-right? No.
- Split top-right.



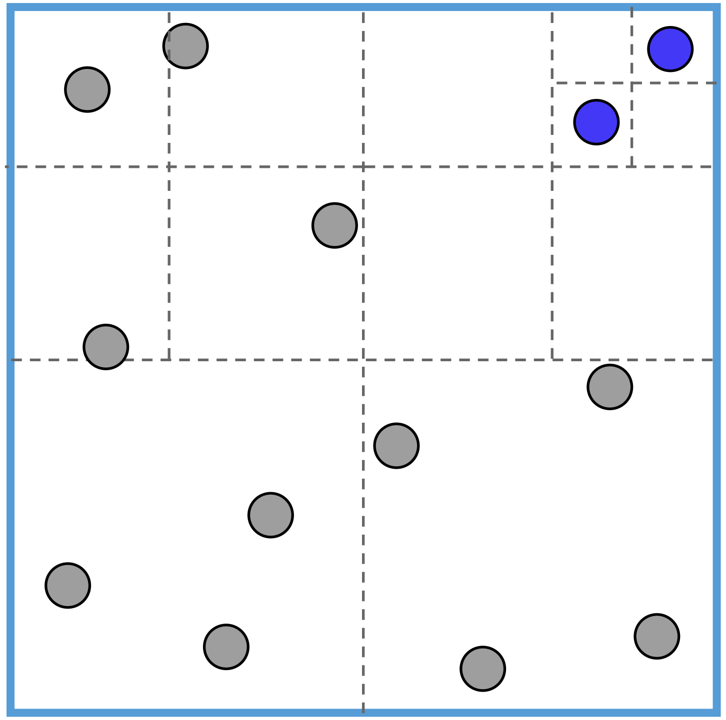
Quadtree: construction

- Split the top level.
- Can we stop? No, split the next level.
- Split top-left.
- Can we stop top-left? Yes.
- Split top-right.
- Can we stop top-right? No.
- Split top-right.
- Can we stop top-right?

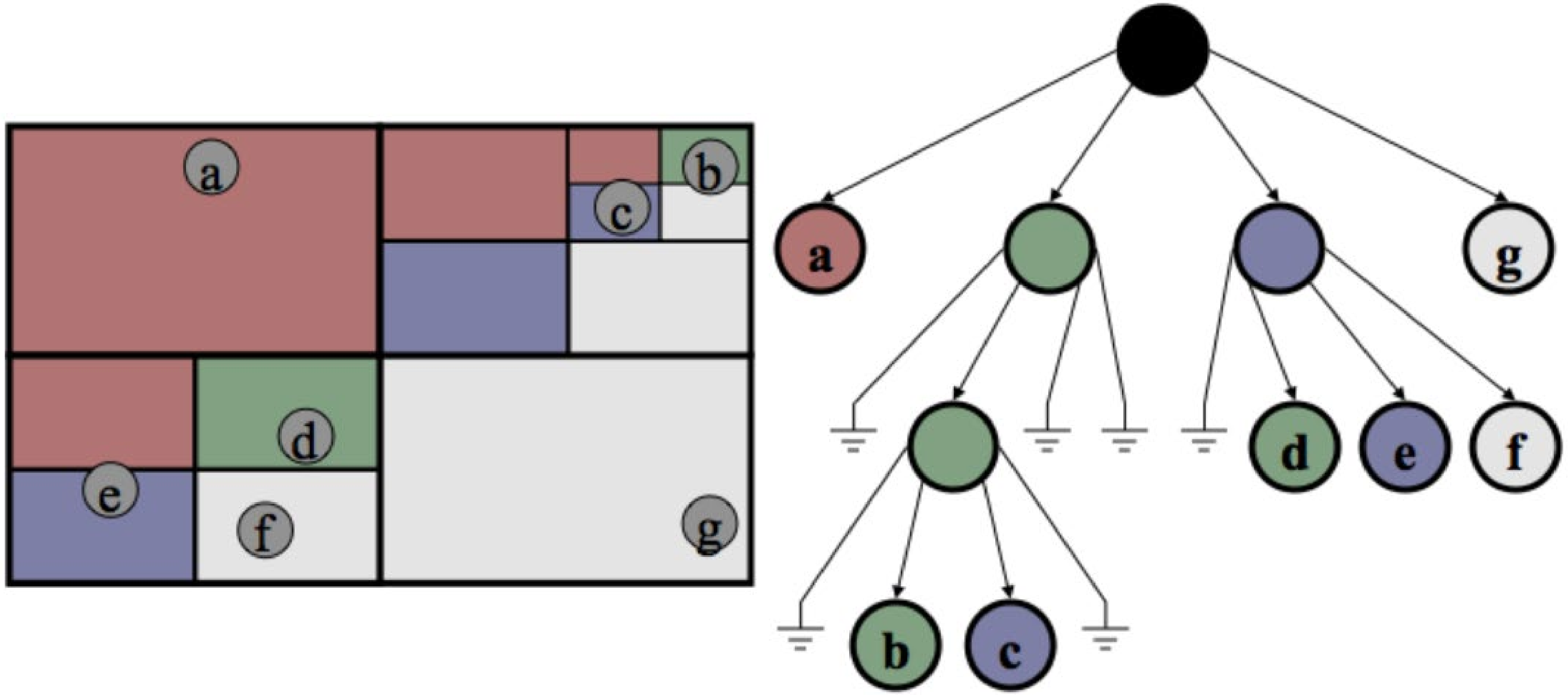


Quadtree: construction

- Split the top level.
- Can we stop? No, split the next level.
- Split top-left.
- Can we stop top-left? Yes.
- Split top-right.
- Can we stop top-right? No.
- Split top-right.
- Can we stop top-right? Yes.



Quadtree: construction



Quadtree: construction

- Construction:
  - Input: set of objects P inside a square S $(x_1, y_1) \times (x_2, y_2)$, tree node v
  - If $|P| \leq 1$:
    - Quadtree consists of a single leaf with P.
  - Else:
    - P_{00}: set of points that fall in the bottom-left corner of S.
    - P_{01}: set of points that fall in the bottom-right corner of S.
    - …
    - v_{00}: node with points of P_{00}.
    - v_{01}: node with points of P_{01}.
    - …
    - Append v_{00}, v_{01}, v_{10}, v_{11} to v.

Quadtree: query

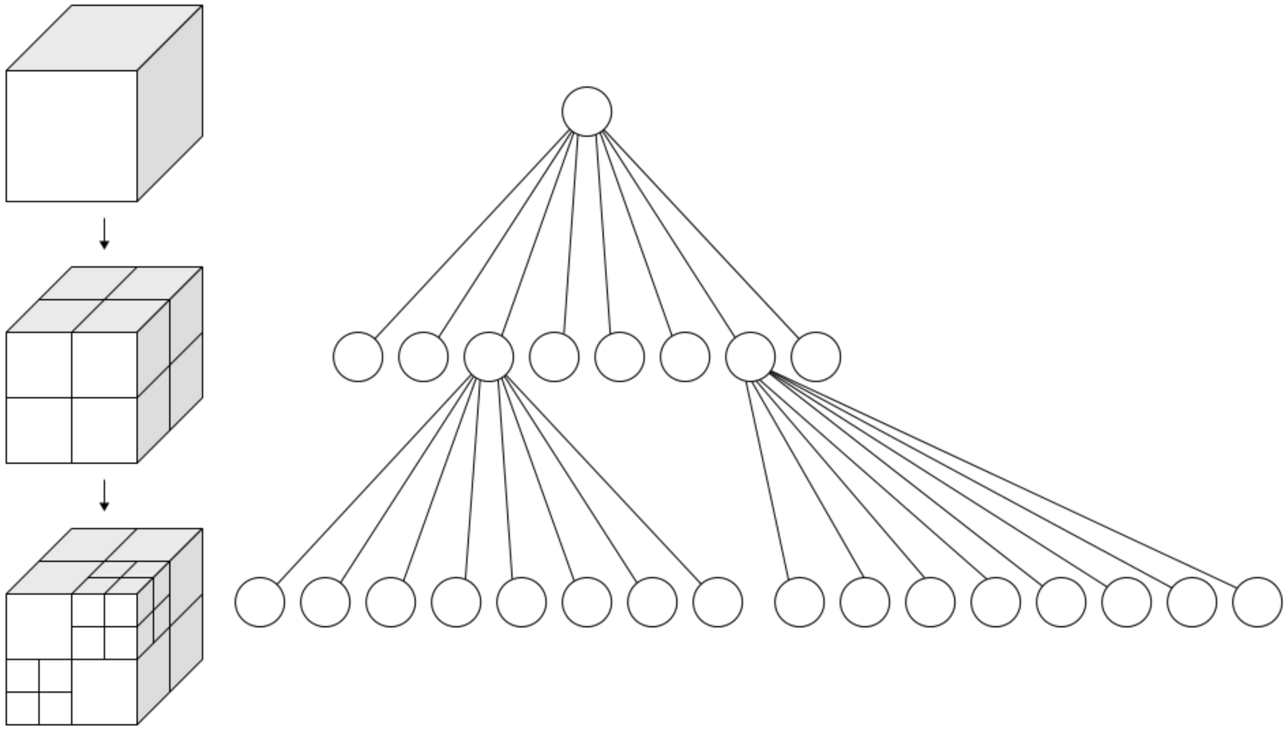
- Query:
  - Input: range query r $(x_1, y_1) \times (x_2, y_2)$, tree node v.
  - If v is a leaf:
    - Search points of v inside range r.
  - If v_{00} inside range r :
    - Query(v_{00}, r)
  - If v_{01} inside range r :
    - Query(v_{01}, r)
  - If v_{10} inside range r :
    - Query(v_{10}, r)
  - If v_{11} inside range r :
    - Query(v_{11}, r)

Quadtree: complexity

- Build time: $O(n)$
- Space: $O(n)$
- Query (range): $O(\sqrt{n} + k)$

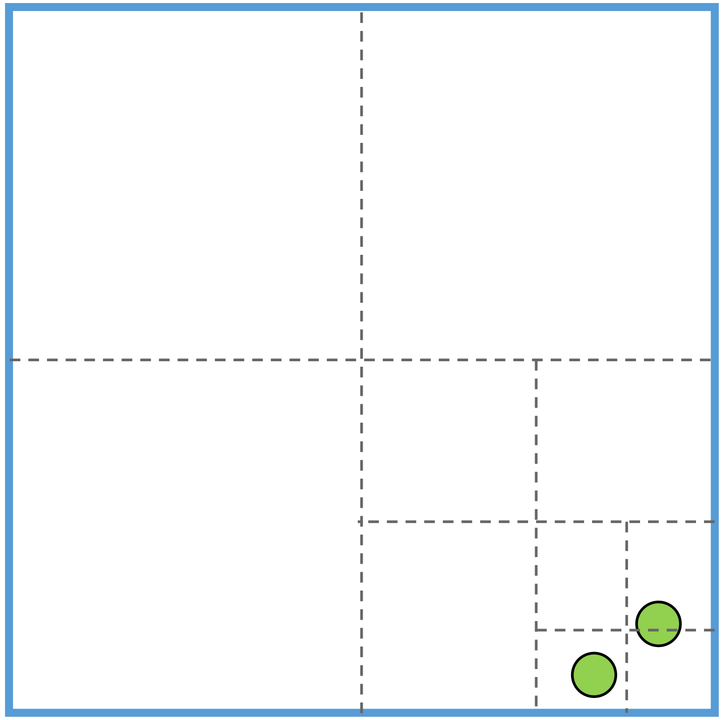
Octree

- Each inner node contains 8 equally sized voxels.
- A 3D quadtree.



Quadtree and octree: drawbacks

- Grater ability to adapt to location of scene geometry than uniform grid.
- But very long tree to store points that are concentrated in a small region.
- Many nodes will contain zero objects.

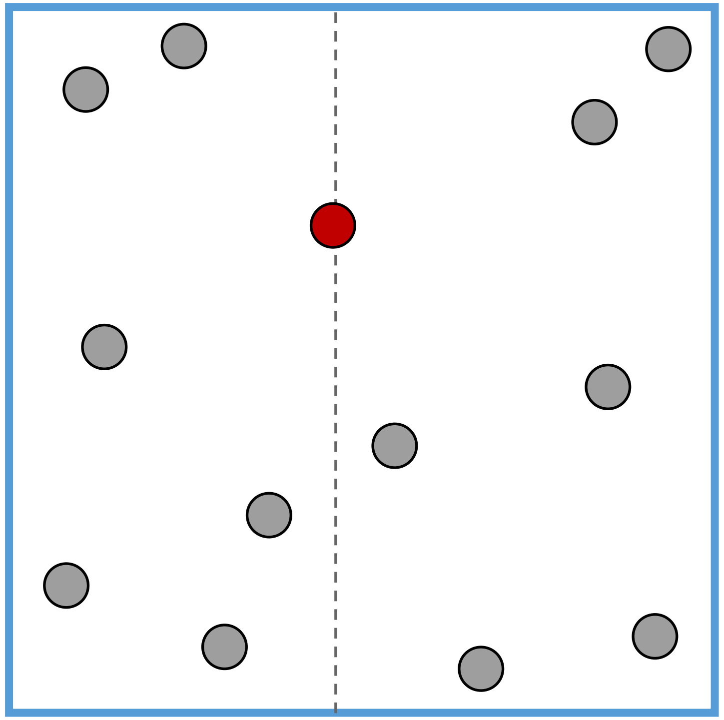


K-d tree

- Differently from quadtrees and octrees, k-d trees only split **one** dimension at each level.
- Where to split? Middle? Median? Proportional to surface area?
- At each level:
  - Quadtree creates 4 equal sized cells.
  - Octree creates 8 equal sized cells.
  - K-d tree creates 2 non-equal sized cells (2D case).

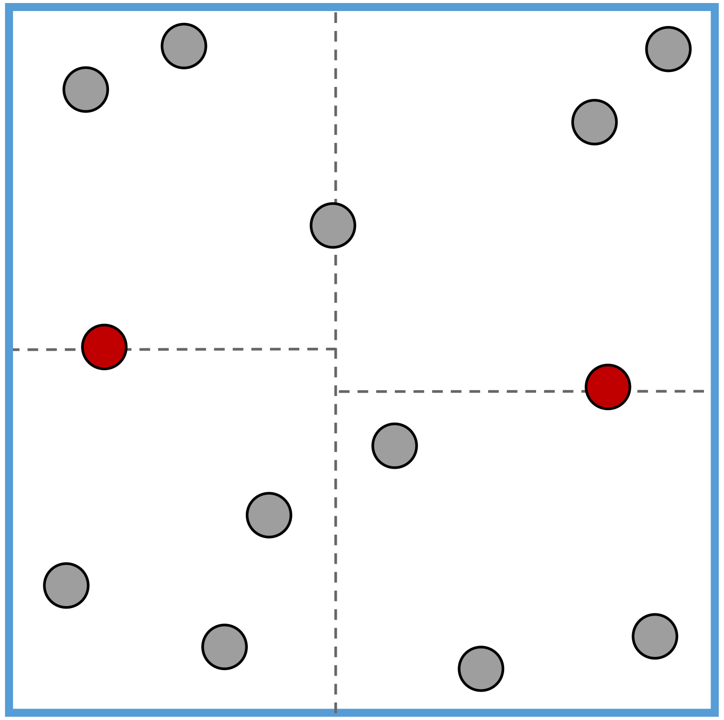
K-d tree: construction

- First split: x dimension (median point).



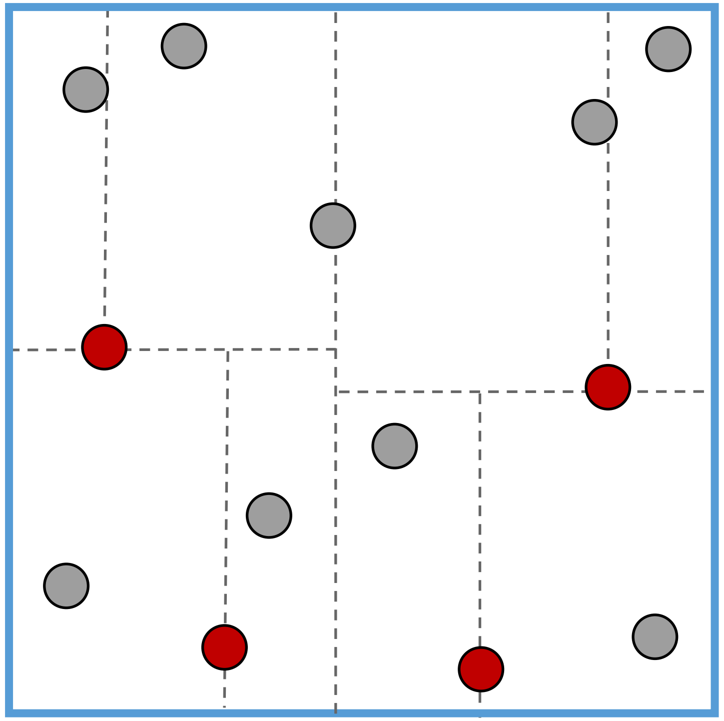
K-d tree: construction

- First split: x dimension (median point).
- Second split: y dimension.



K-d tree: construction

- First split: x dimension (median point).
- Second split: y dimension.
- Repeat, alternating split dimensions



K-d tree: construction

- Construction:
  - Input: set of objects P inside a square S $(x_1, y_1) \times (x_2, y_2)$, tree node v
  - If $|P| \leq 1$:
    - K-d tree consists of a single leaf with P.
  - Else:
    - If depth is even:
      - Split P into P_0 and P_1, along a vertical line through the y axis.
    - Else:
      - Split P into P_0 and P_1, along a vertical line through the x axis.
    - v_0: $build(v, P_0, depth + 1)$.
    - v_1: $build(v, P_1, depth + 1)$.
    - …
    - Append v_0, v_1 to v.

K-d tree: query

- Query:
  - Input: range query r $(x_1, y_1) \times (x_2, y_2)$, tree node v.
  - If v is a leaf:
    - Search points of v inside range r.
  - If v_0 inside range r :
    - Query(v_0, r)
  - If v_1 inside range r :
    - Query(v_1, r)

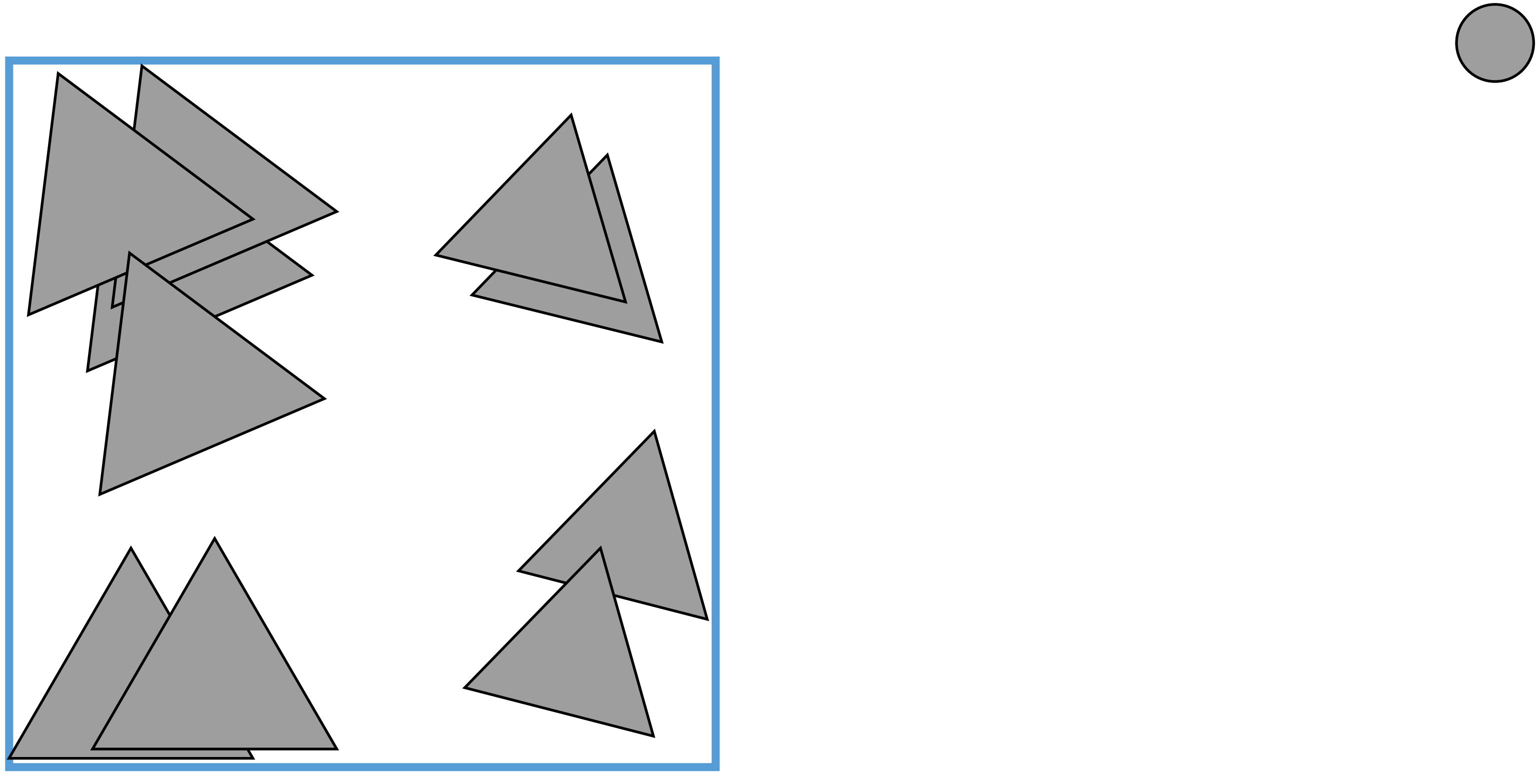
K-d tree: complexity

- Build time: $O(nlogn)$
- Space: $O(n)$
- Query: $O(\sqrt{n} + k)$

Bounding volume hierarchies (BVH)

- Goal: use the scene hierarchy given by the scene graph (instead of a spatial derived one).
- Associate a bounding volume to each node.
  - A bounding volume of a node bounds **<u>all</u>** objects in the subtree.
  - Nodes are aggregated objects.
- Construction and update is fast.
  - Bottom-up: recursive.
- Querying it:
  - Top-down: visit.

Bounding volume hierarchies (BVH)



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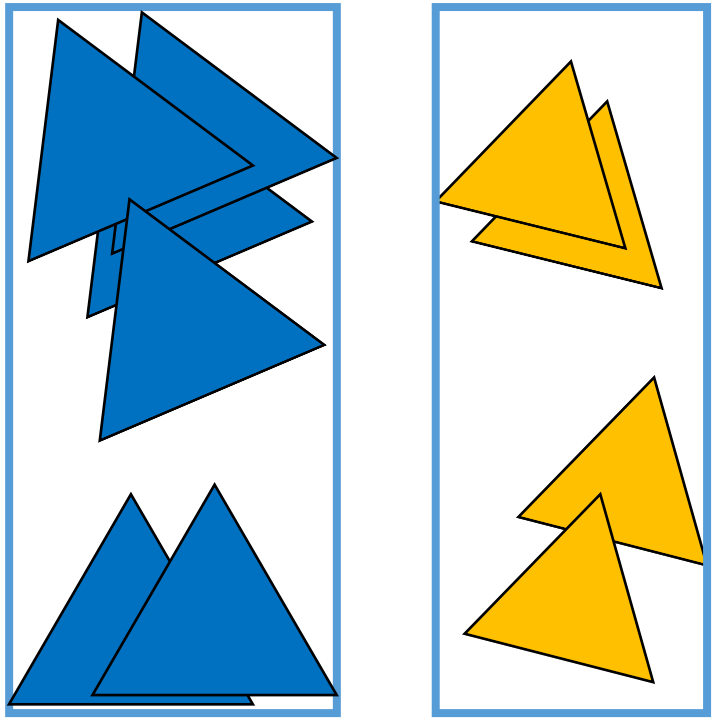
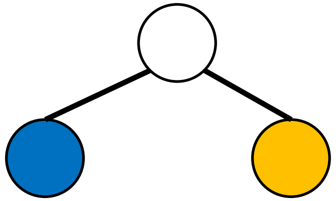
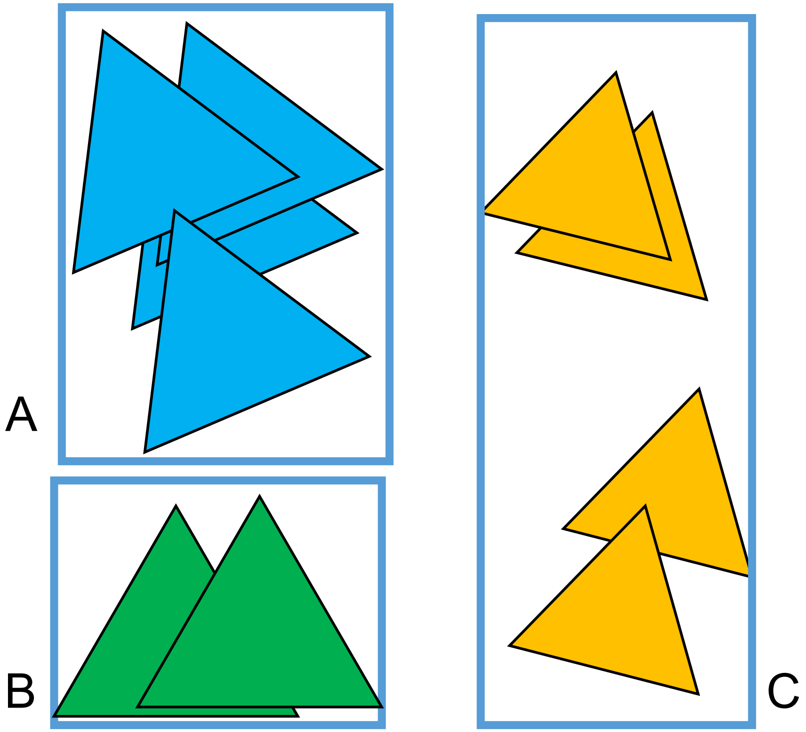
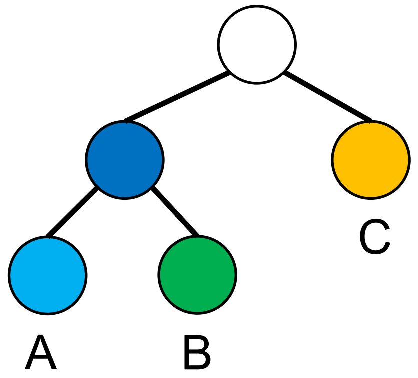
Bounding volume hierarchies (BVH)





Bounding volume hierarchies (BVH)





- Leaf nodes: contain *small* list of primitives.
- Interior nodes:
    - Proxy for a large subset of primitives.
    - Stores bounding box for all primitives in subtree.

Bounding volume hierarchies (BVH)

- How to split the scene?
- Primitive list of current node P is sorted based on the centroids of primitive bounding boxes. Ordered list is split into P_0 and P_1.
- Different heuristics:
  - Edge volume heuristic (EVH).
  - Split bounding volume hierarchy (SBVH).
  - Surface area heuristic (SAH):

$$C = C_t + \frac{SA(B_1)}{SA(B)}|P_1|C_i + \frac{SA(B_2)}{SA(B)}|P_2|C_i$$

Bounding volume hierarchies (BVH)

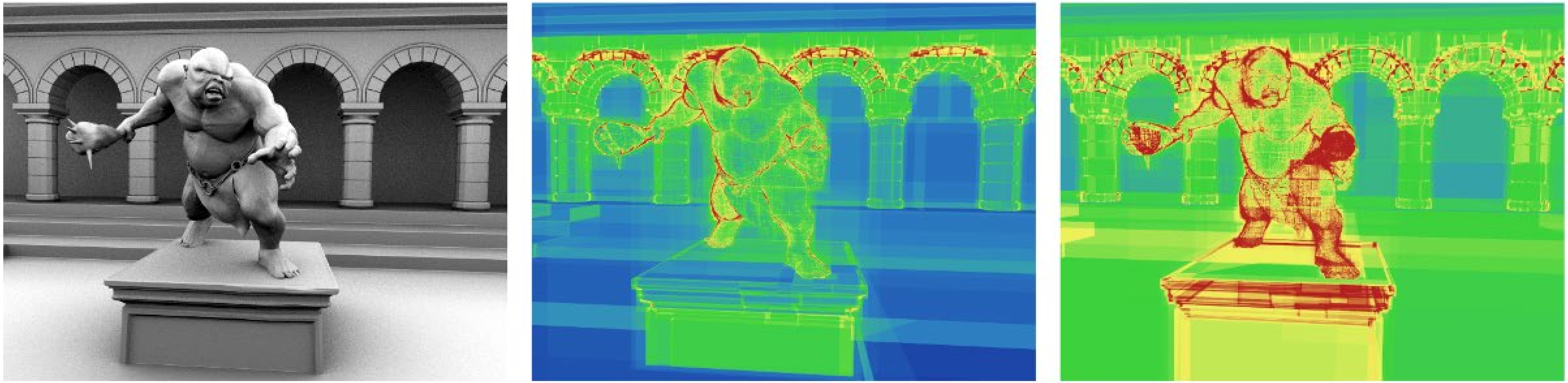


Figure 1: *Sample scene consisting of roughly 1.9 million triangles (left). Our method (middle) results in a significant reduction of ray shooting costs compared to a regular bounding volume hierarchy (right). The heat views visualize the summed number of traversal steps and primitive intersections for primary rays.*

[Stich et al., 2009]

Object vs. space partitioning

- Object partitioning:
  - Hierarchically partition **objects** into groups.
  - Spatial index is created by spatially bounding each subgroup.
- Space partitioning:
  - Hierarchically partition **space** into subspaces.
  - Subspaces are non-overlapping and completely fill parent space.
  - Tree or table structure.

Summary

- Choose the right structure considering the operations and data.
- Uniform grid:
    - The most parallelizable (to update, construct, use).
    - Constant time access (best!).
    - Quadratic / cubic space (2D, 3D).
    - Good performance under uniform distribution of objects.
- Quadtree, octree, k-d tree:
    - Compact.
    - Simple.
    - Non-constant accessing time.
    - Good performance under non-uniform distribution of objects.
- BVH:
    - Simple construction.
    - Ideal for dynamic parts of the scene.
    - Not necessarily very efficient to access.
        - May need to traverse multiple children.
        - If you do not have a scene-graph you need to create one.