# Viewing Transformations

## CS425: Computer Graphics I

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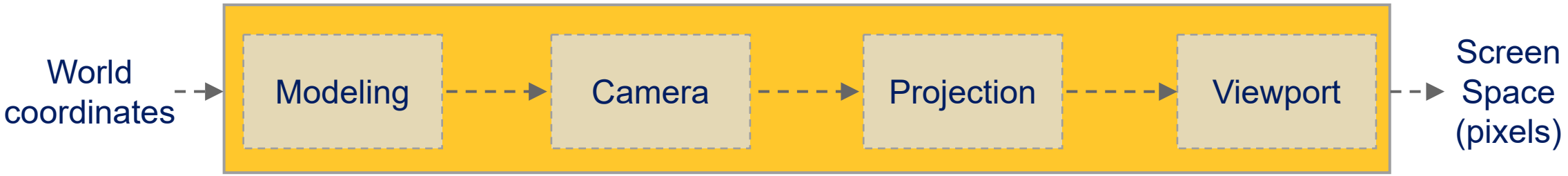
# Overview

- Coordinate systems
- Camera
- Viewing transformations
- Orthographic and perspective projections
- Hidden surface removal

# Viewing transformations

- Viewing transformation is the mapping of coordinates of points and lines from world coordinates into screen space pixels.



# Viewing transformations

- Previously, we saw how transformations can manipulate primitives (points, vectors) in space.
- Today, we will see how transformations can manipulate primitives to 2D screen coordinates (that will be later rasterized).

# Perspective projection

Distant objects appear smaller

Parallel lines converge at the horizon

# Early paintings

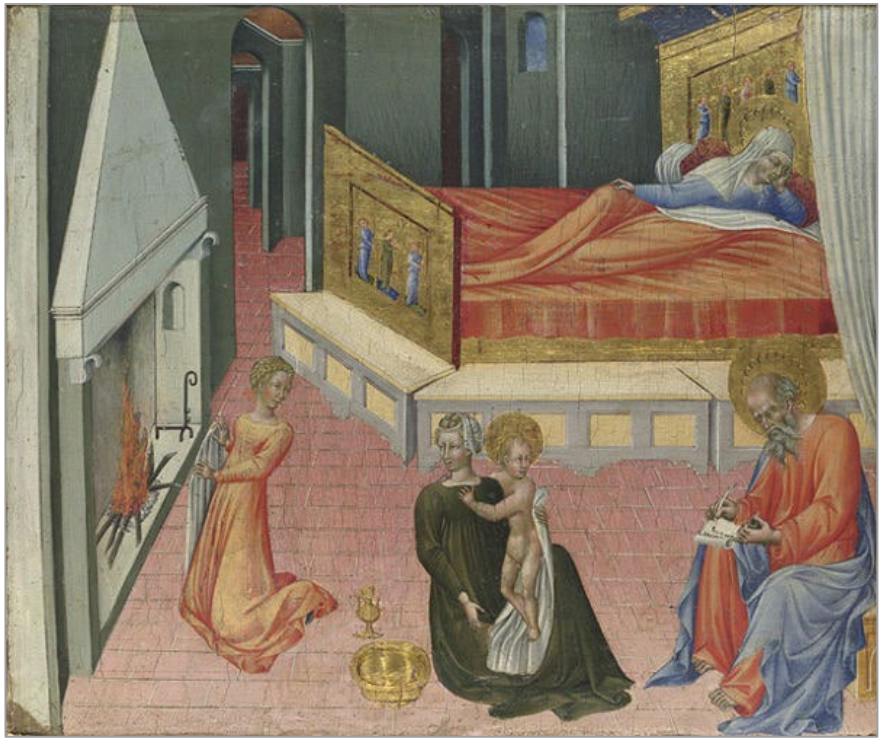
Lorsch Gospels (8th century)

# Perspective in art

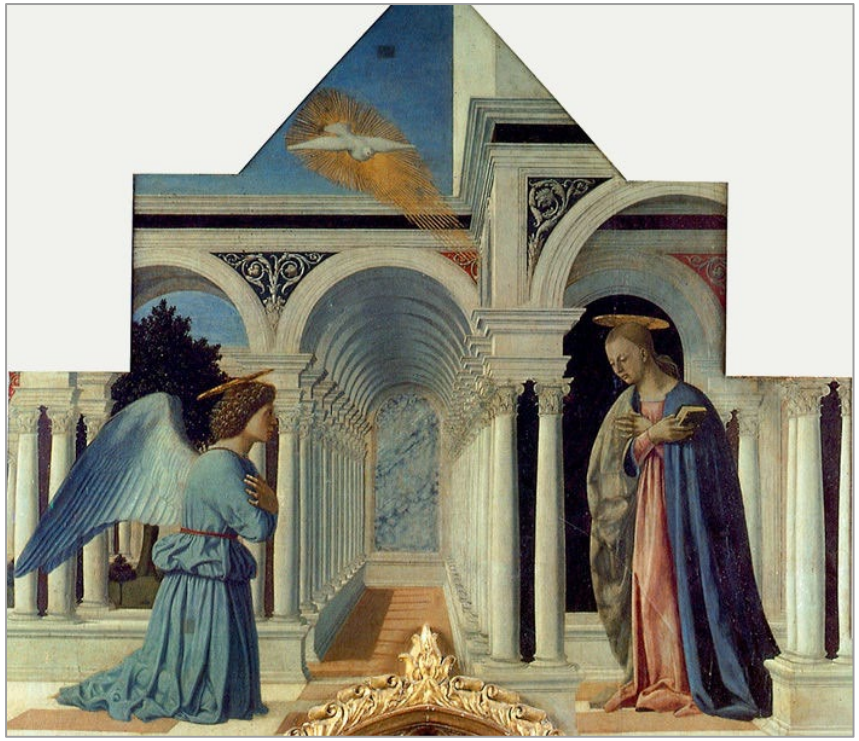
Giotto di Bondone (14th century)

Giotto di Bondone (14th century)

# Birth of perspective in art: One-point perspective



Giovanni di Paolo (15th century)



Piero della Francesca (15th century)

# Birth of perspective in art: One-point perspective

Perugino (15th century)

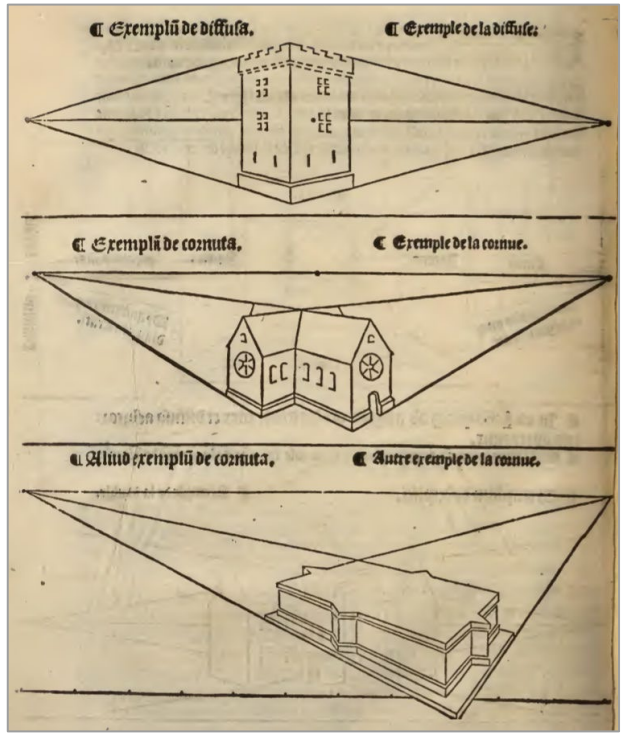
# Birth of perspective in art: One-point perspective

The Ideal City (15th century)

# Birth of perspective in art: One-point perspective

Rafael (16th century)

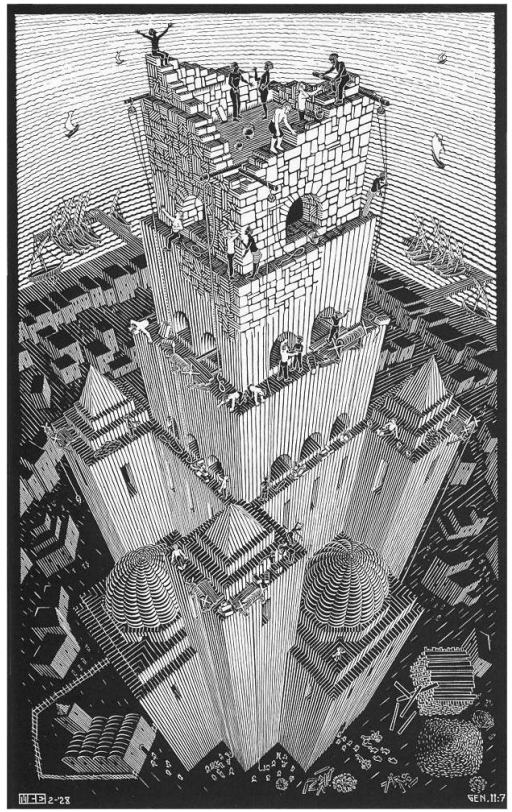
# Birth of perspective in art: Two-point perspective




Jean Pélérin (16th century)

Gustave Caillebotte (19th century)

# Birth of perspective in art: Three-point perspective

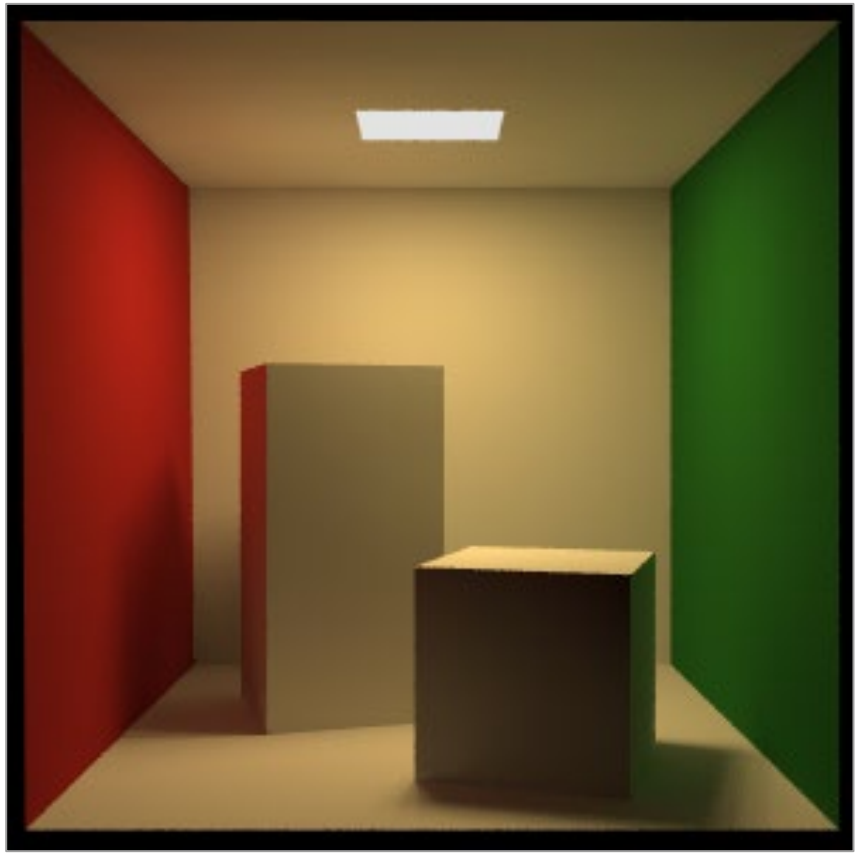


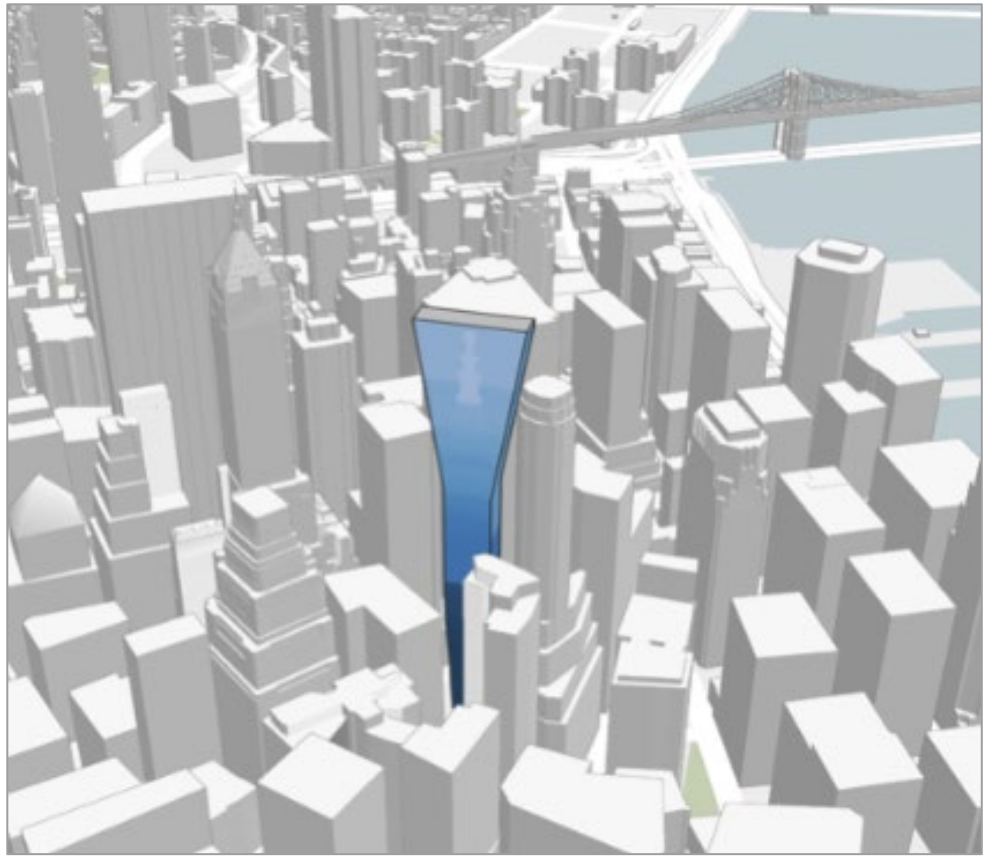
M.C. Escher (1928)

# Multi perspective

The Frozen City by Matthias A. K. Zimmermann (2006)

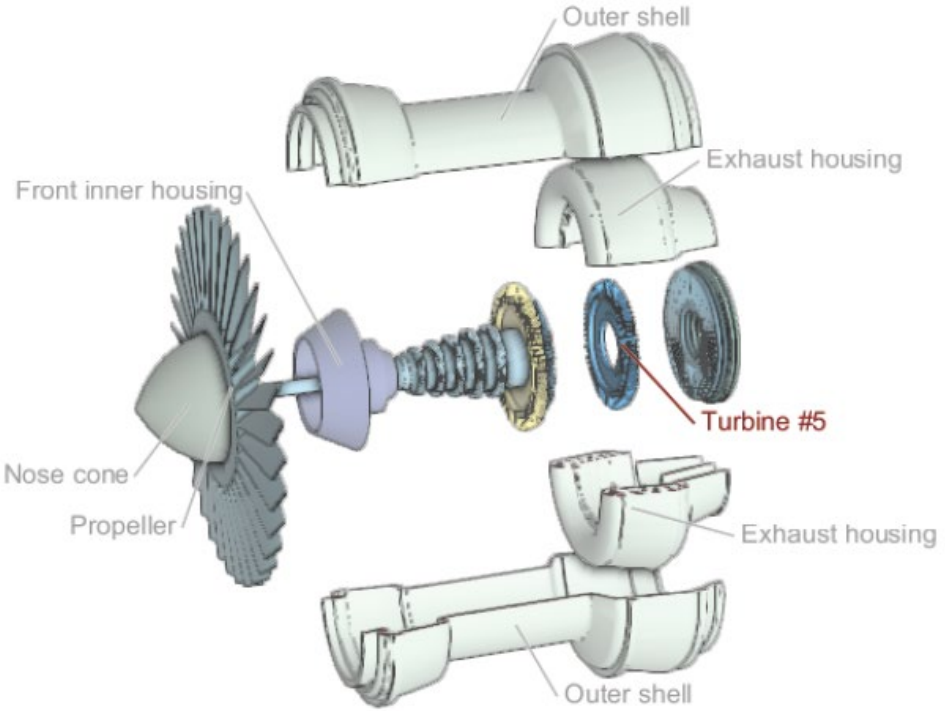
# Perspective in computer graphics





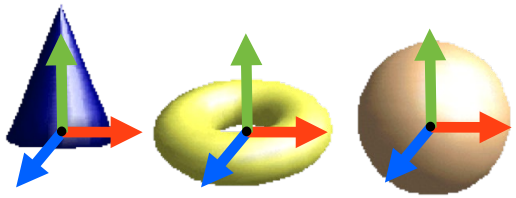
# Rejection of perspective in CG

SimCity 2000

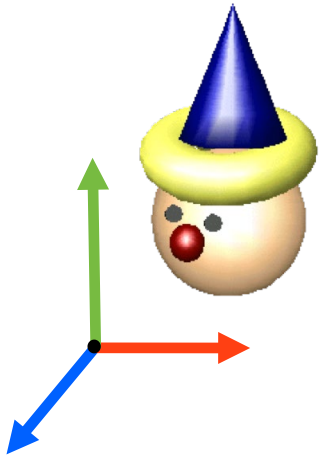



“Automated Generation of Interactive 3D Exploded Views”

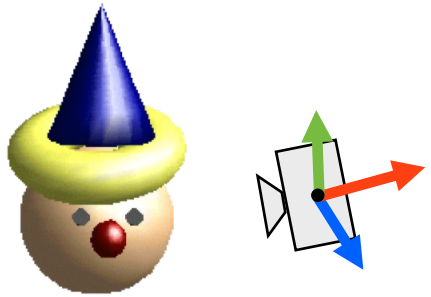
# Coordinate spaces



Object coordinates



World coordinates

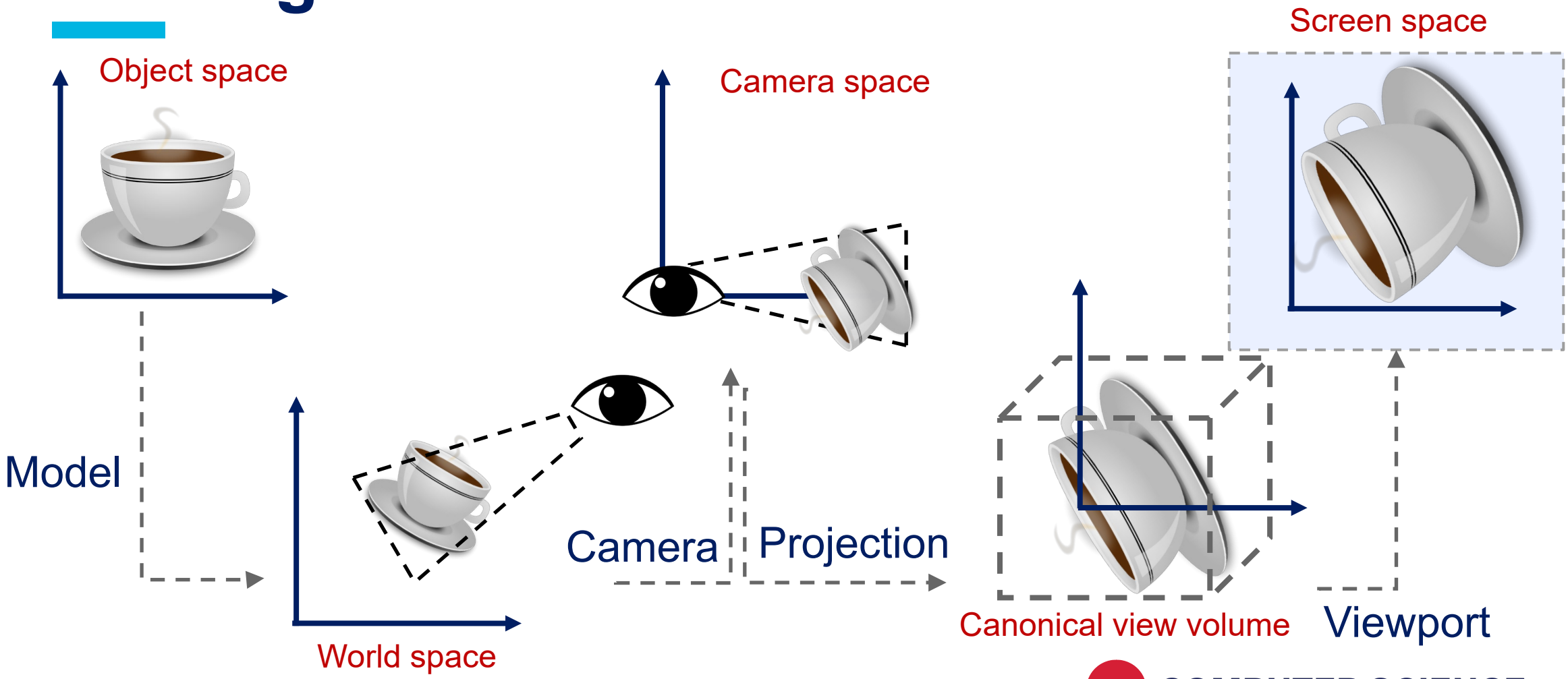


Camera coordinates

Screen coordinates

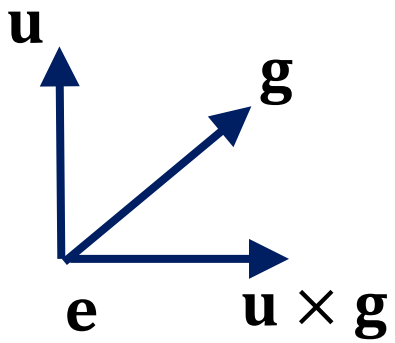
From: Mark Pauly

# Viewing transformation



# Camera transformation

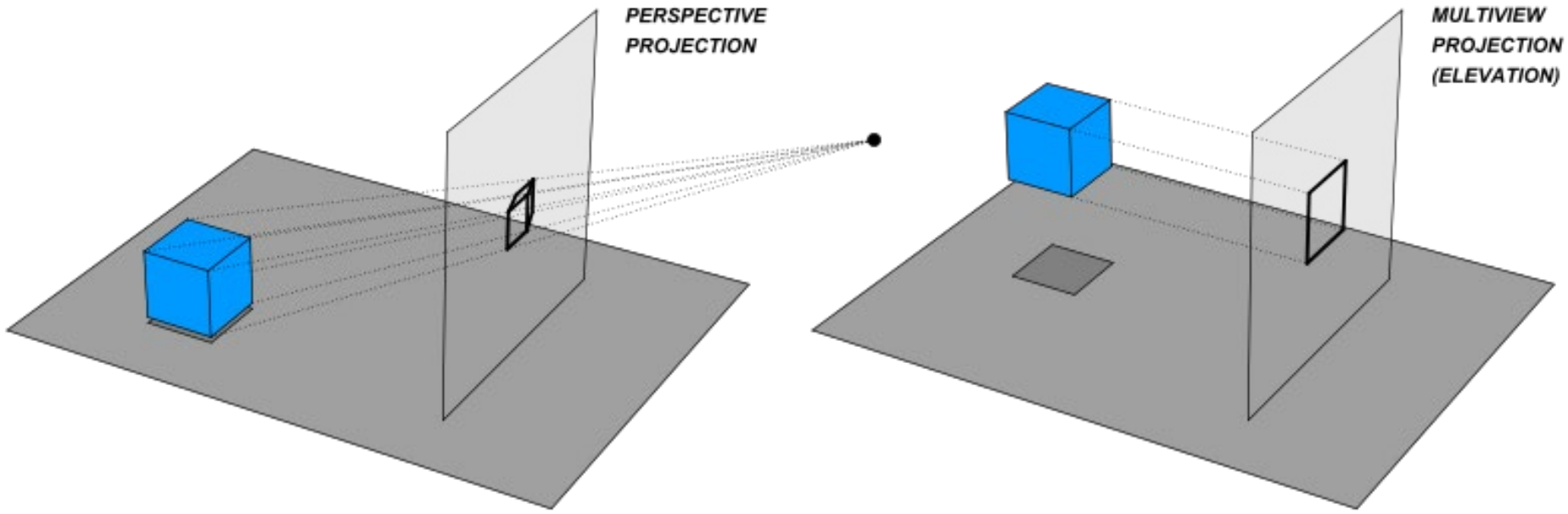
- Construct the camera reference system:
  - The eye position $\mathbf{e}$.
  - The forward direction $\mathbf{d}$.
  - The view-up vector $\mathbf{u}$.
- A view matrix transform all coordinates into view coordinates.




$$\mathbf{M}_{camera2world} = \begin{pmatrix} \mathbf{u} \times \mathbf{g} & \mathbf{u} & \mathbf{d} & \mathbf{e} \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

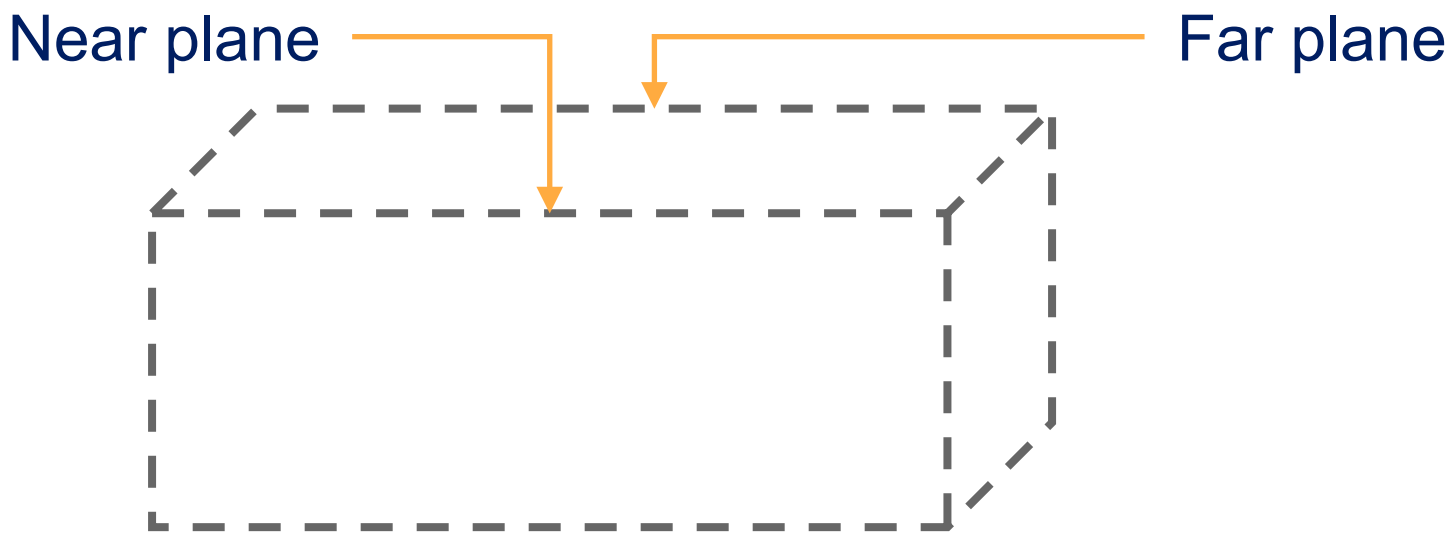
$$\mathbf{M}_{world2camera} = \begin{pmatrix} \mathbf{u} \times \mathbf{g} & \mathbf{u} & \mathbf{d} & \mathbf{e} \\ 0 & 0 & 0 & 1 \end{pmatrix}^{-1}$$

# Orthographic and perspective projections

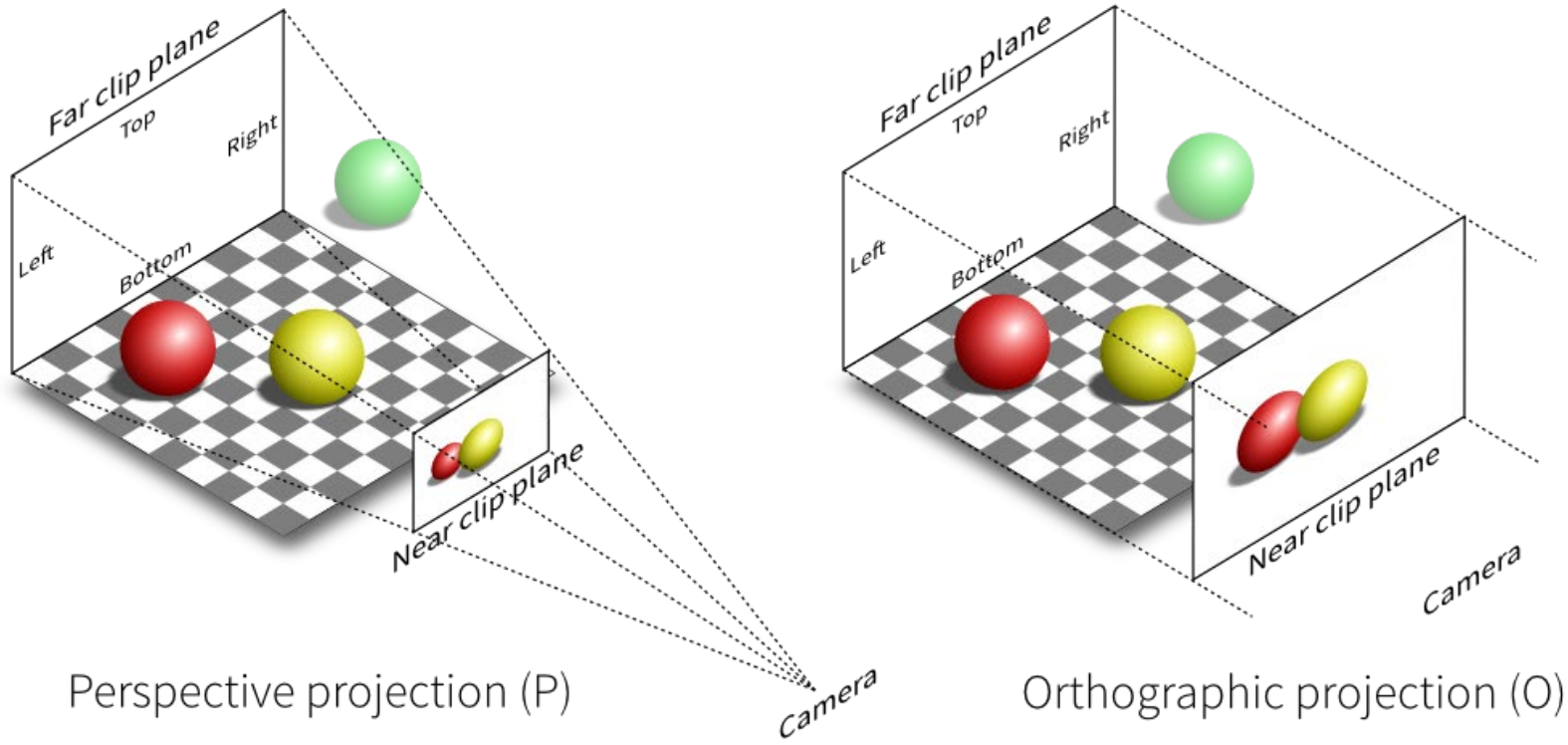



https://en.wikipedia.org/wiki/File:Various_projections_of_cube_above_plane.svg

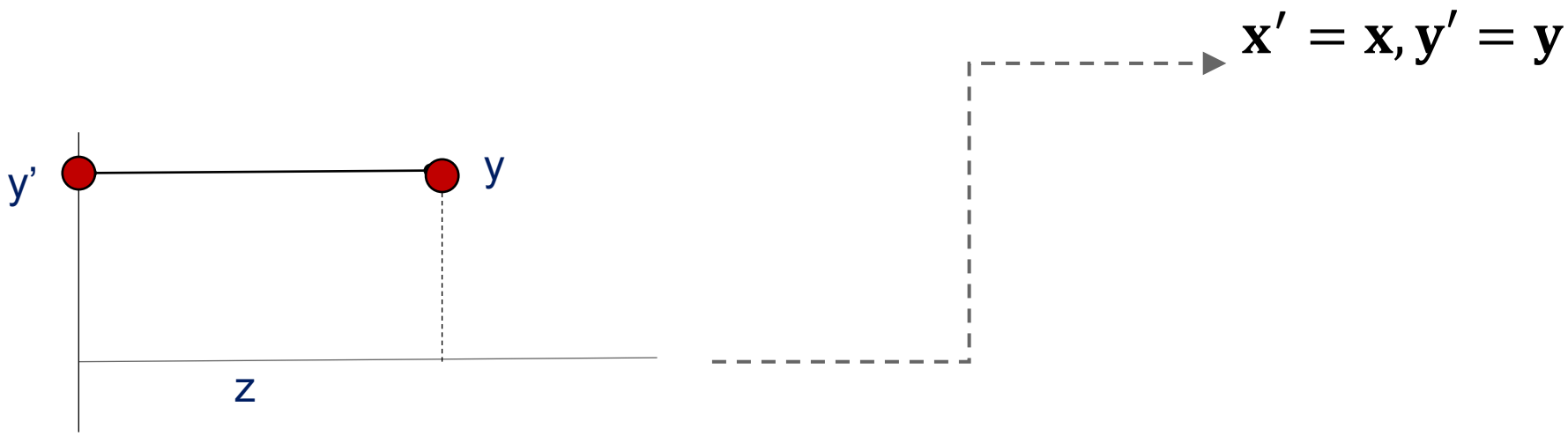
# View frustum



# View frustum

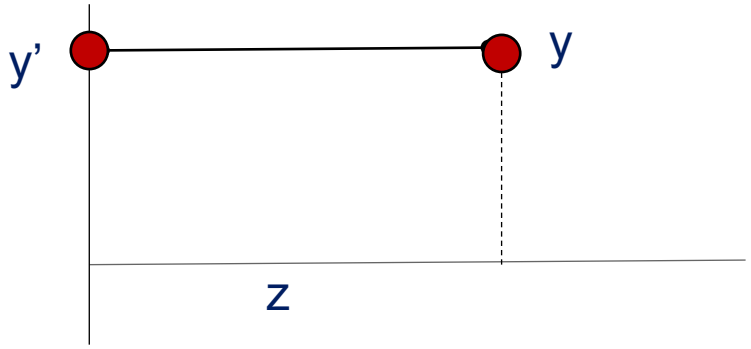


# Basic orthographic projection




Projection plane
(z = 0)

# Basic orthographic projection

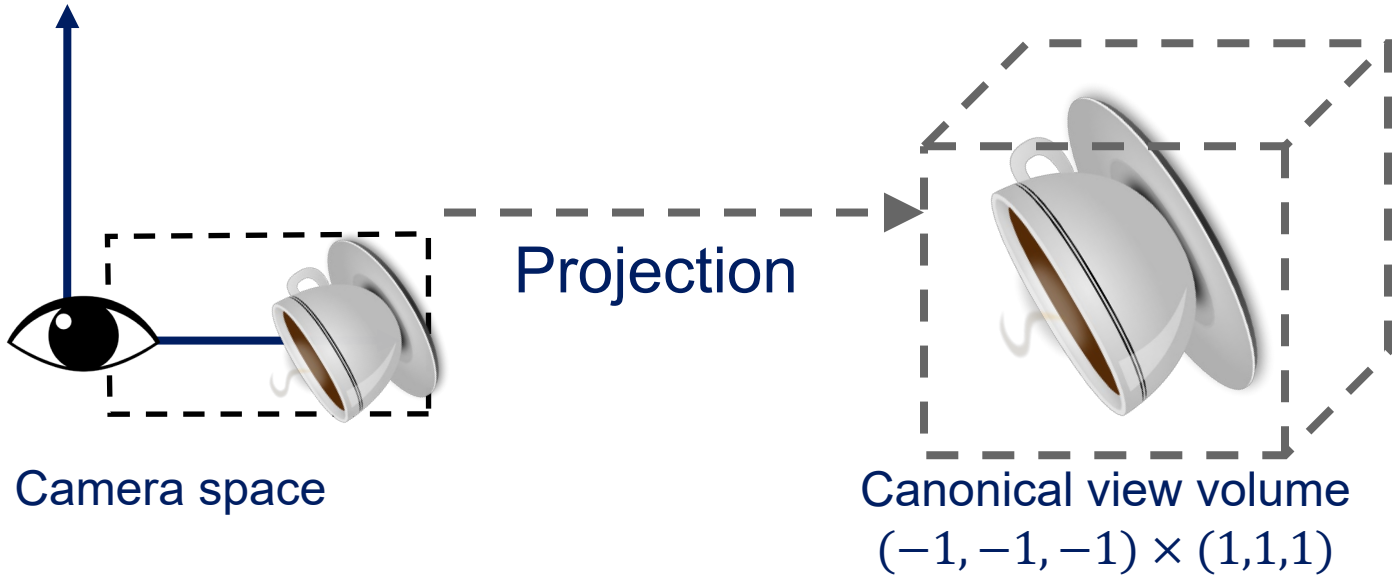



Projection plane
(z = 0)

$$\mathbf{x}' = \mathbf{x}, \mathbf{y}' = \mathbf{y}$$

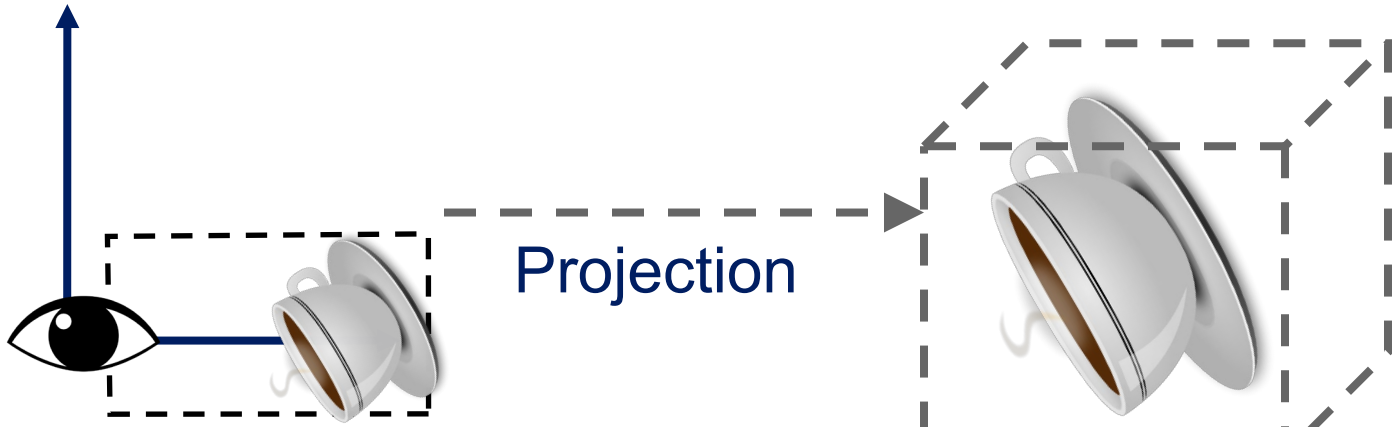
$$\begin{pmatrix} x' \\ y' \\ z' \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

# Orthographic transformation



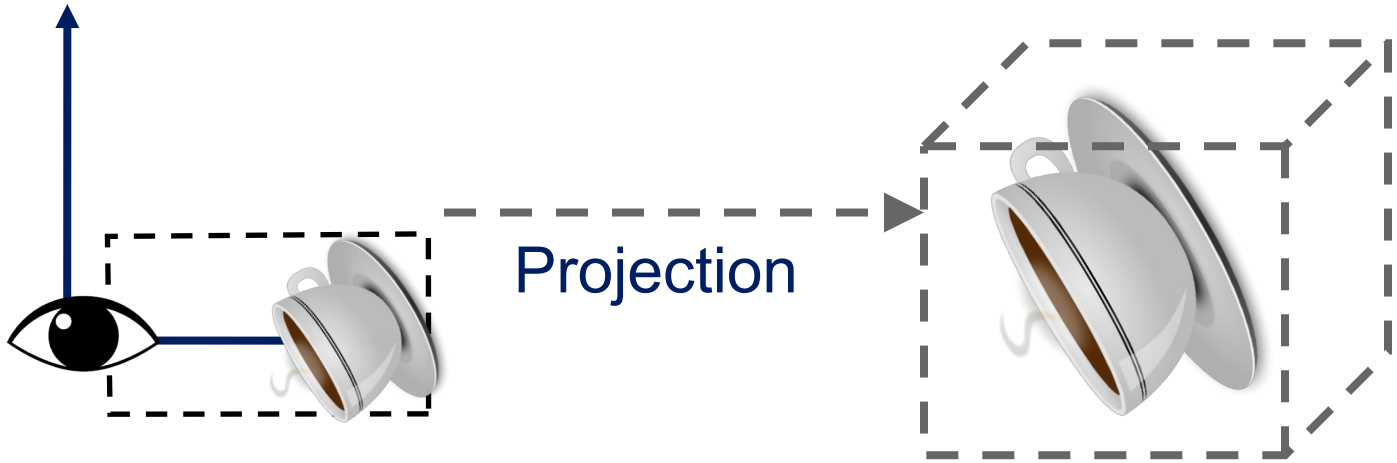

1. Translate to the origin.
2. Scale the volume to a 2-by-2 unit square: (-1,-1) to (+1, +1).
3. Switch coordinate system.

# Orthographic transformation



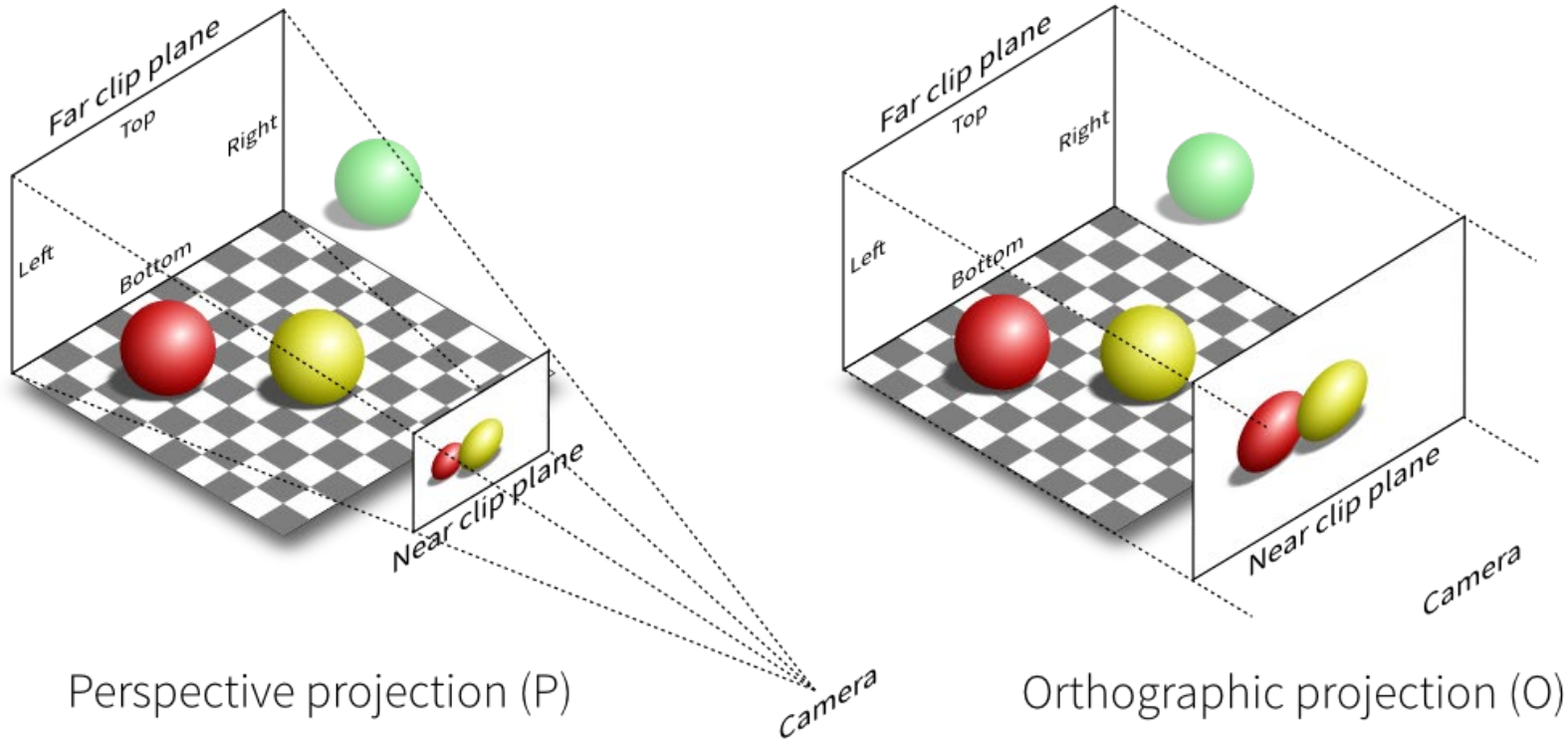

$$\begin{pmatrix} x' \\ y' \\ z' \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \frac{2}{(right-left)} & 0 & 0 & 0 \\ 0 & \frac{2}{(top-bottom)} & 0 & 0 \\ 0 & 0 & \frac{2}{(far-near)} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & -x_{mid} \\ 0 & 1 & 0 & -y_{mid} \\ 0 & 0 & 1 & -z_{mid} \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

# Orthographic transformation

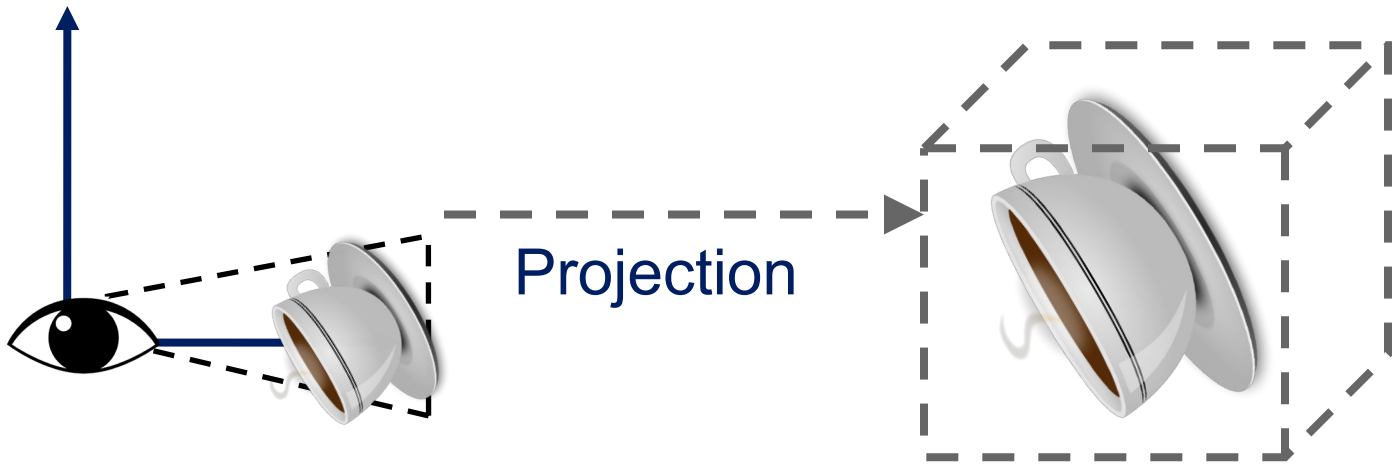



$$\begin{pmatrix} x' \\ y' \\ z' \\ 1 \end{pmatrix} = \begin{pmatrix} \frac{2}{(right-left)} & 0 & 0 & \frac{-(right+left)}{(right-left)} \\ 0 & \frac{2}{(top-bottom)} & 0 & \frac{-(top+bottom)}{(top-bottom)} \\ 0 & 0 & \frac{-2}{(far-near)} & \frac{-(far+near)}{(far-near)} \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

# View frustum



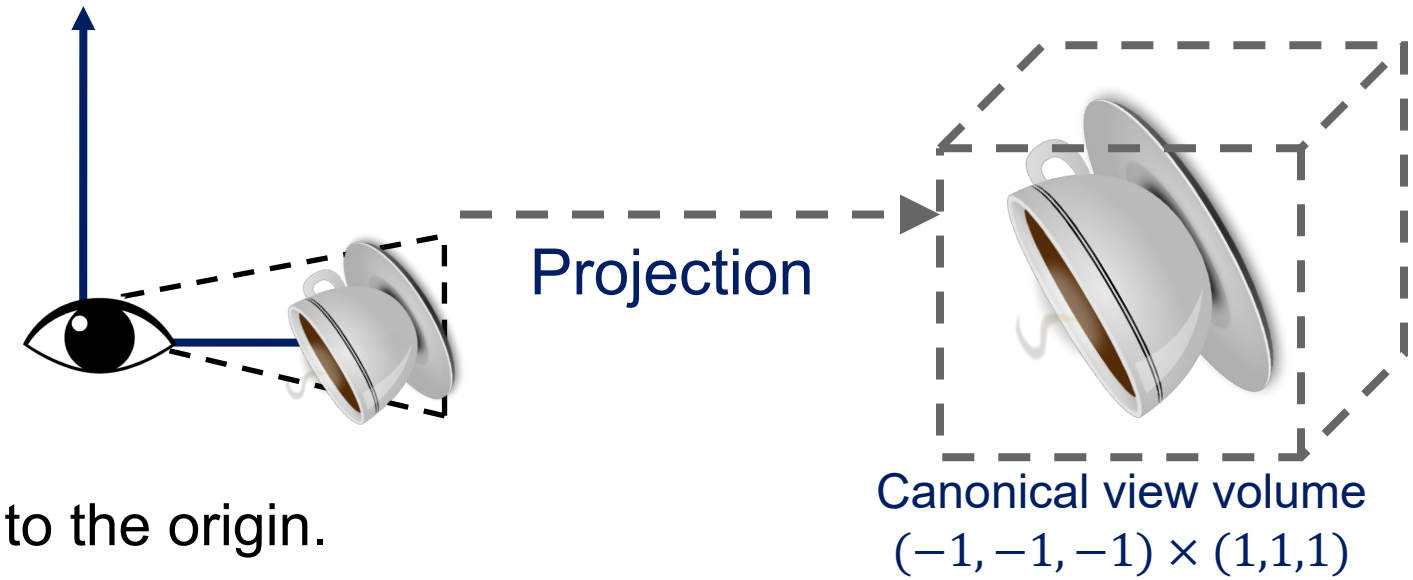
# Perspective projection




1. Translate to the origin.

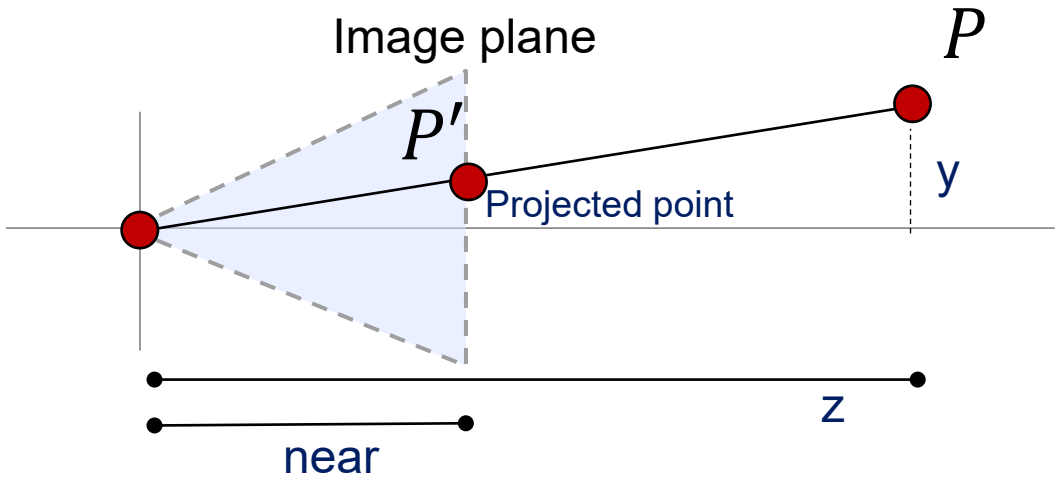
4. Scale the volume to a 2-by-2 unit square: (-1,-1) to (+1, +1).
5. Switch coordinate system.

# Perspective projection

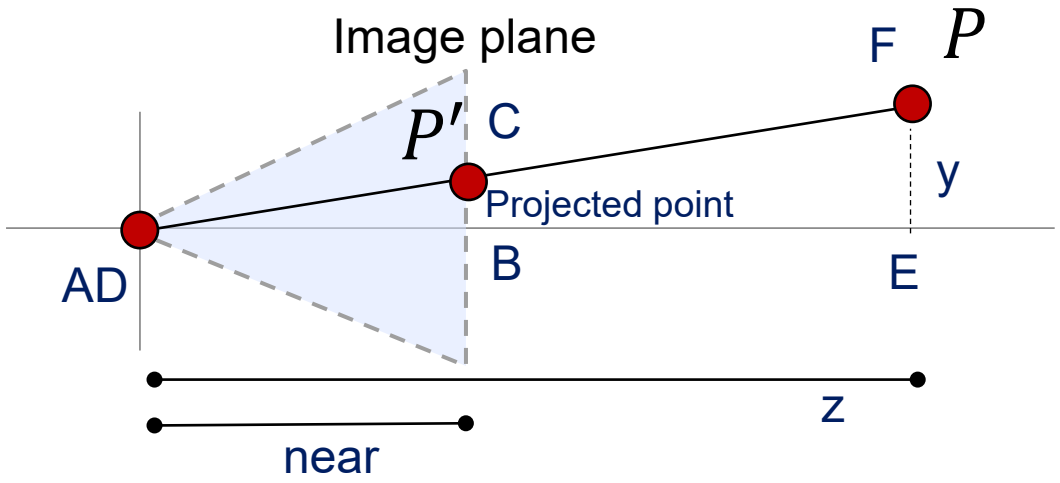



1. Translate to the origin.
2. Scale depth values into normalized range: (-1, +1).
3. Perspective calculation.
4. Scale the volume to a 2-by-2 unit square: (-1,-1) to (+1, +1).
5. Switch coordinate system.

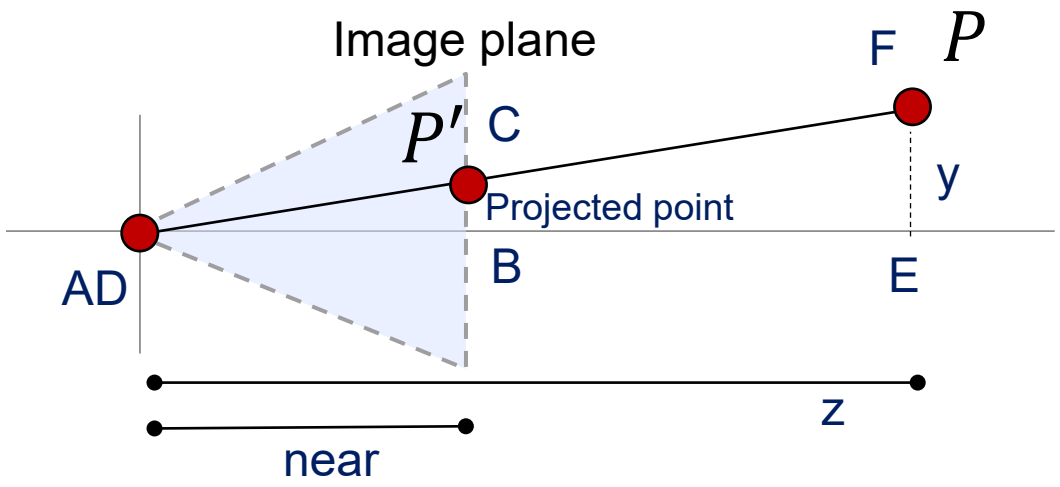
# Perspective calculation



# Perspective calculation



# Perspective calculation




Project point $P'$?

$$\frac{AB}{DE} = \frac{BC}{EF}$$

$$\frac{near}{-P_z} = \frac{P'_y}{P_y}$$

$$P'_y = \frac{near * P_y}{-P_z}$$

$$P'_x = \frac{near * P_x}{-P_z}$$

What is the matrix transformation?

# Perspective calculation

- What is the matrix transformation?
- 4-by-4 transformation matrix is a linear combination of terms:
  - We can calculate $a * x + b * y + c * z + d$
  - But not $\frac{a*x}{z} + \cdots$

# Perspective calculation

- What is the matrix transformation?
- 4-by-4 transformation matrix is a linear combination of terms:
  - We can calculate $a * x + b * y + c * z + d$
  - But not $\frac{a*x}{z} + \cdots$
- Solution: homogeneous coordinates.

$$(x, y, z, w) \rightarrow (\frac{x}{w}, \frac{y}{w}, \frac{z}{w}, 1)$$

# Perspective calculation

- What is the matrix transformation?
- 4-by-4 transformation matrix is a linear combination of terms:
  - We can calculate $a * x + b * y + c * z + d$
  - But not $\frac{a*x}{z} + \cdots$
- Solution: homogeneous coordinates.

$$(x, y, z, w) \rightarrow (\frac{x}{w}, \frac{y}{w}, \frac{z}{w}, 1)$$

$$\begin{pmatrix} x' \\ y' \\ z' \\ 1 \end{pmatrix} = \begin{pmatrix} near & 0 & 0 & 0 \\ 0 & near & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

# Scaling depth values

- Scaling depth values into a normalized range.
- Objective: mapping between (-near,-far) to (-1,+1).
- Naïve solution:

$$-1 = -near$$
$$+1 = -far$$

- However, float point numbers suffer from round-off errors (difference between 0.12345678 and 0.12345677 can have a visual impact.

# Scaling depth values

- Scaling depth values into a normalized range.
- Objective: non-linear mapping between (-near,-far) to (-1,+1).
- More precision for values close to the camera, less precision for values farther from the camera.
- Non-linear mapping with two constants:

$$\frac{c_1}{-z} + c_2$$

# Scaling depth values

- Scaling depth values into a normalized range.
- Non-linear mapping with two constants:

$$\frac{c_1}{-z} + c_2$$

- When $z = -near$, mapping should return $-1$
- When $z = -far$, mapping should return $+1$

$$-1 = \frac{c_1}{-(-near)} + c_2 \text{ and } +1 = \frac{c_1}{-(-far)} + c_2$$

# Scaling depth values

- Scaling depth values into a normalized range.
- Non-linear mapping with two constants:

$$\frac{c_1}{-z} + c_2$$

- When $z = -near$, mapping should return $-1$
- When $z = -far$, mapping should return $+1$

$$-1 = \frac{c_1}{-(-near)} + c_2 \text{ and } +1 = \frac{c_1}{-(-far)} + c_2$$

$$c_1 = 2 * far * \frac{near}{near - far}$$

$$c_2 = \frac{far + near}{far - near}$$

# Scaling depth values

- What is the matrix transformation?
- Same problem as before: 4-by-4 transformation matrix is a linear combination of terms:
  - We can calculate $a * x + b * y + c * z + d$
  - But not $\frac{a*x}{z} + \cdots$
- Solution: homogeneous coordinates (again).

$$\begin{pmatrix} x' \\ y' \\ z' \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -c_2 & c_1 \\ 0 & 0 & -1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

# Perspective projection

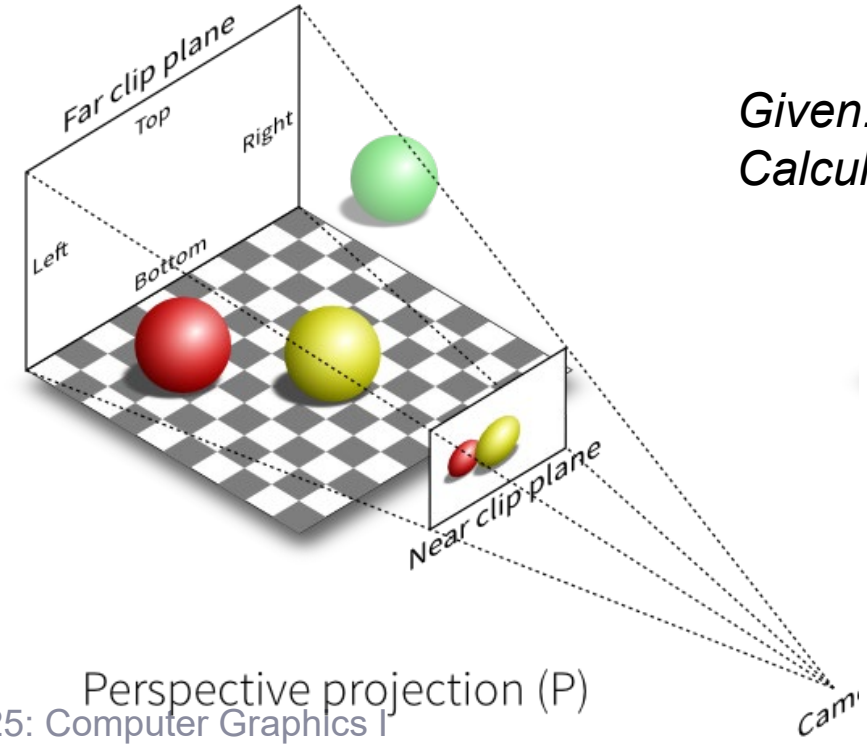
1. Translate to the origin.
2. Scale depth values into normalized range: (-1, +1). (and switch coordinate system)
3. Perspective calculation.
4. Scale the volume to a 2-by-2 unit square: (-1,-1) to (+1, +1).

Note: you only need to perform perspective calculation once.

$$\begin{pmatrix} x' \\ y' \\ z' \\ 1 \end{pmatrix} = \begin{pmatrix} \frac{2}{(right-left)} & 0 & 0 & 0 \\ 0 & \frac{2}{(top-bottom)} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} near & 0 & 0 & 0 \\ 0 & near & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -c_2 & c_1 \\ 0 & 0 & -1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & -x_{mid} \\ 0 & 1 & 0 & -y_{mid} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

# Perspective projection

- How to create a perspective transformation matrix given a view of view (FOV) and aspect ratio width:height?



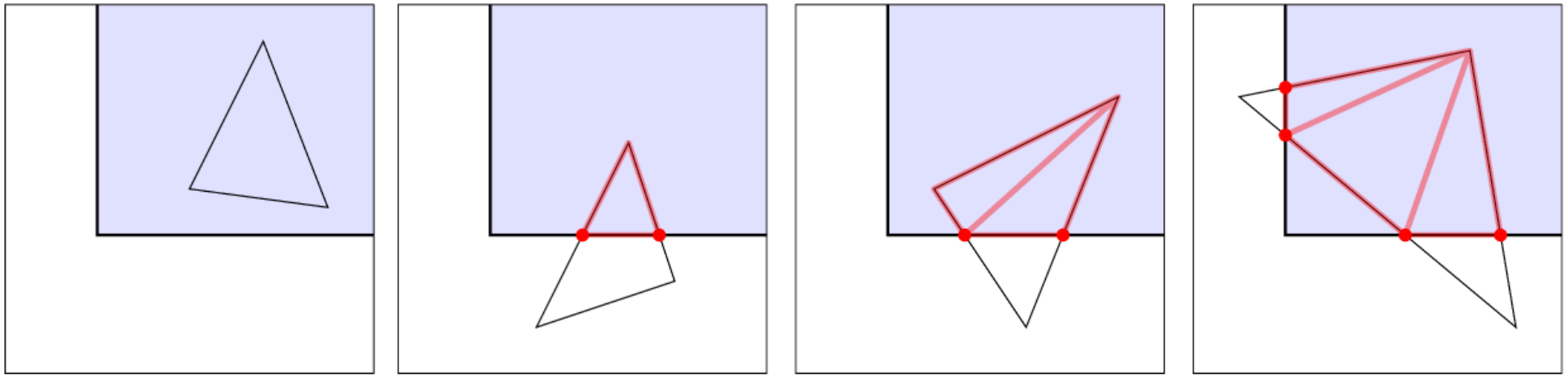

*Given: fov, aspect, near, far*
*Calculate frustum properties (left, right, bottom, top, near, far):*

$$top = near \ * \tan\left(\frac{fov}{2}\right)$$

$$bottom = -top$$
$$right = top * aspect$$
$$left = -right$$

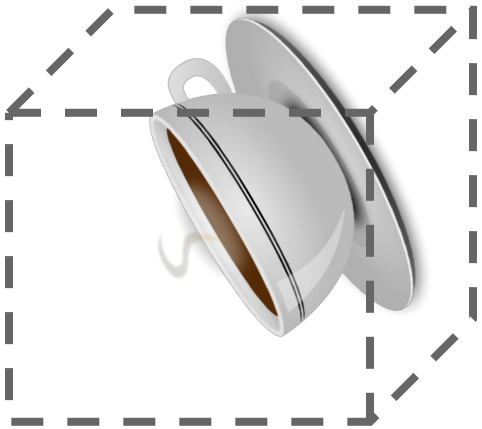
# Canonical view volume

- Why?
  - Makes clipping much easier! GL can quickly discard geometry outside -1,1.



From: https://paroj.github.io/gltut/

# Viewport transformation



Canonical view volume
$(-1,-1,-1) \times (1,1,1)$

Viewport

Screen space

$$\begin{pmatrix} x_{screen} \\ y_{screen} \\ z_{depth} \\ 1 \end{pmatrix} = \begin{pmatrix} \frac{width}{2} & 0 & 0 & \frac{width}{2} \\ 0 & \frac{height}{2} & 0 & \frac{height}{2} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

```
gl.viewport(0, 0, width, height);
```

# Lab

- Draw a cube on screen with size $(-1,-1,-1) \times (1,1,1)$.
  - Apply a model transformation that scales it by 0.5, and tilts it slightly.
  - Apply a projection matrix (orthographic and perspective).