Viewing Transformations

CS425: Computer Graphics I

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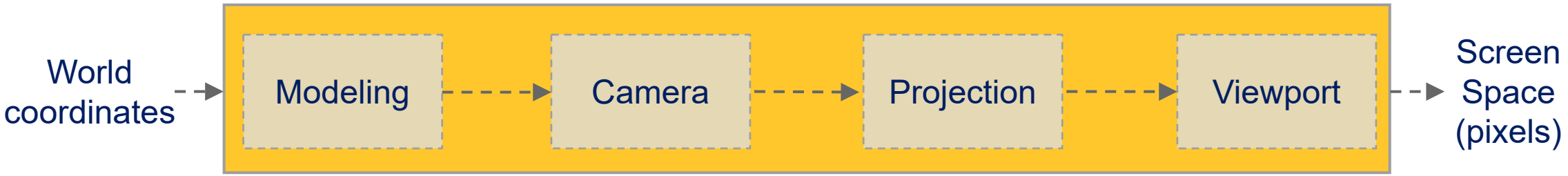
Overview

- Coordinate systems
- Camera
- Viewing transformations
- Orthographic and perspective projections
- Hidden surface removal

Viewing transformations

- Viewing transformation is the mapping of coordinates of points and lines from world coordinates into screen space pixels.



Viewing transformations

- Previously, we saw how transformations can manipulate primitives (points, vectors) in space.
- Today, we will see how transformations can manipulate primitives to 2D screen coordinates (that will be later rasterized).

Perspective projection

Distant objects appear smaller

Parallel lines converge at the horizon

Early paintings

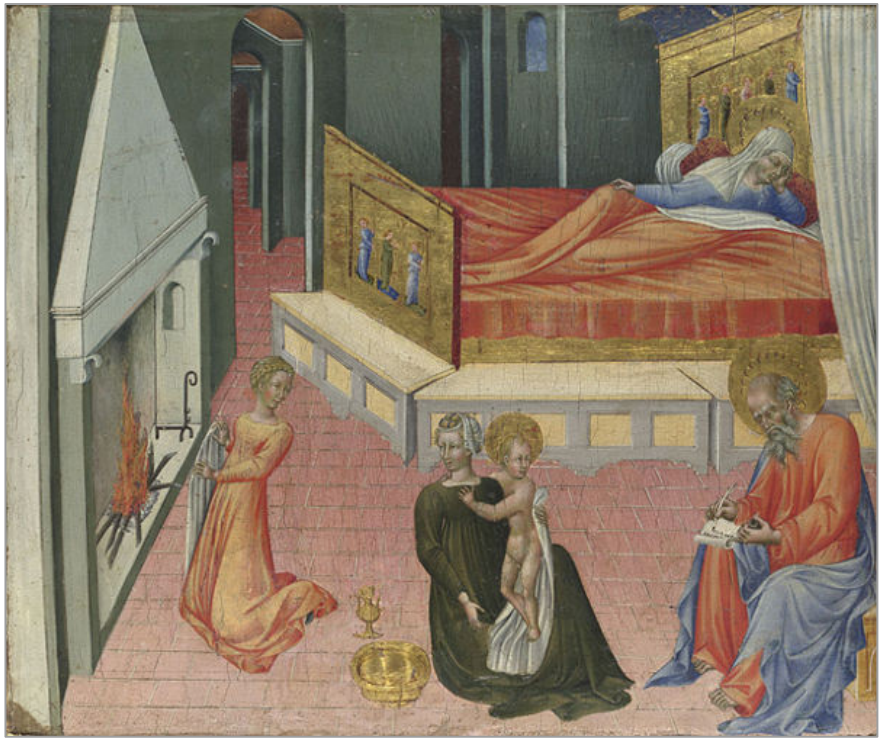
Lorsch Gospels (8th century)

Perspective in art

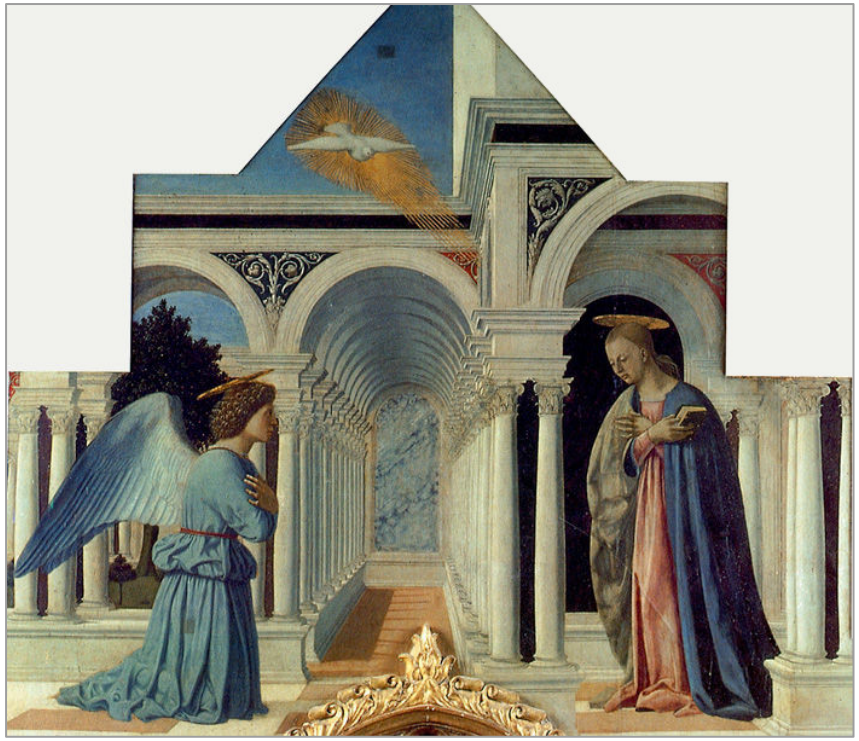
Giotto di Bondone (14th century)

Giotto di Bondone (14th century)

Birth of perspective in art: One-point perspective



Giovanni di Paolo (15th century)



Piero della Francesca (15th century)

Birth of perspective in art: One-point perspective

Perugino (15th century)

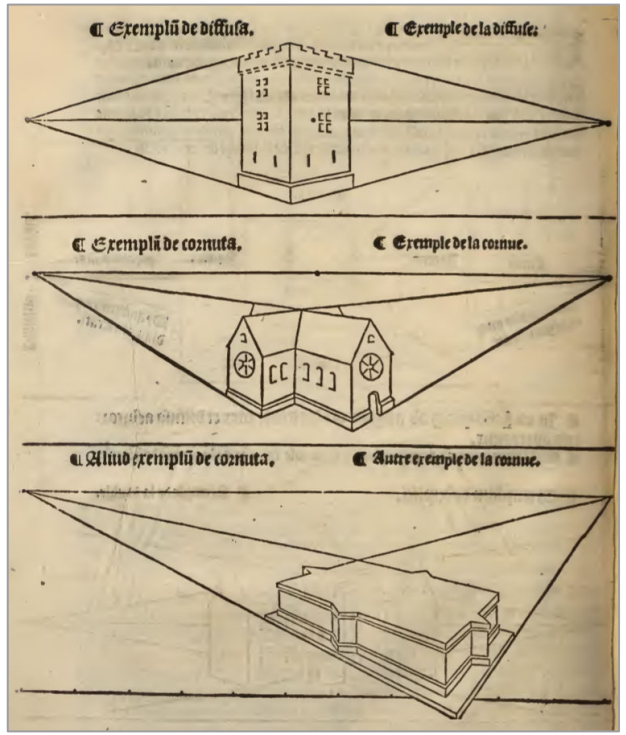
Birth of perspective in art: One-point perspective

The Ideal City (15th century)

Birth of perspective in art: One-point perspective

Rafael (16th century)

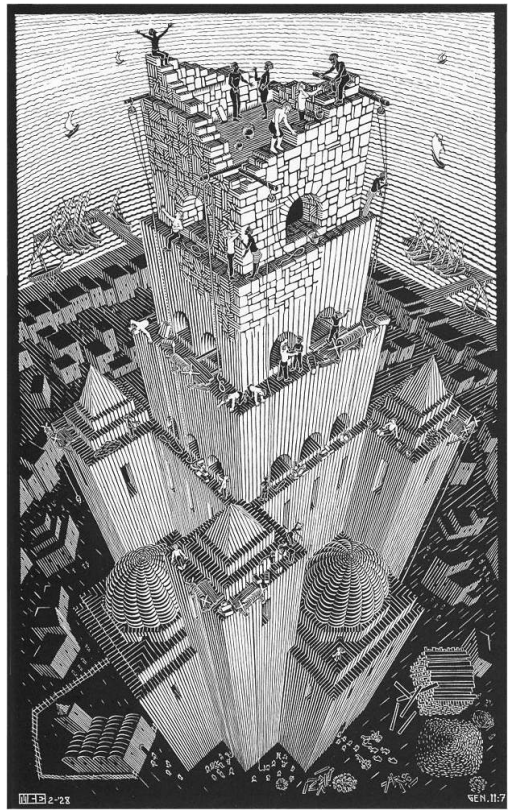
Birth of perspective in art: Two-point perspective



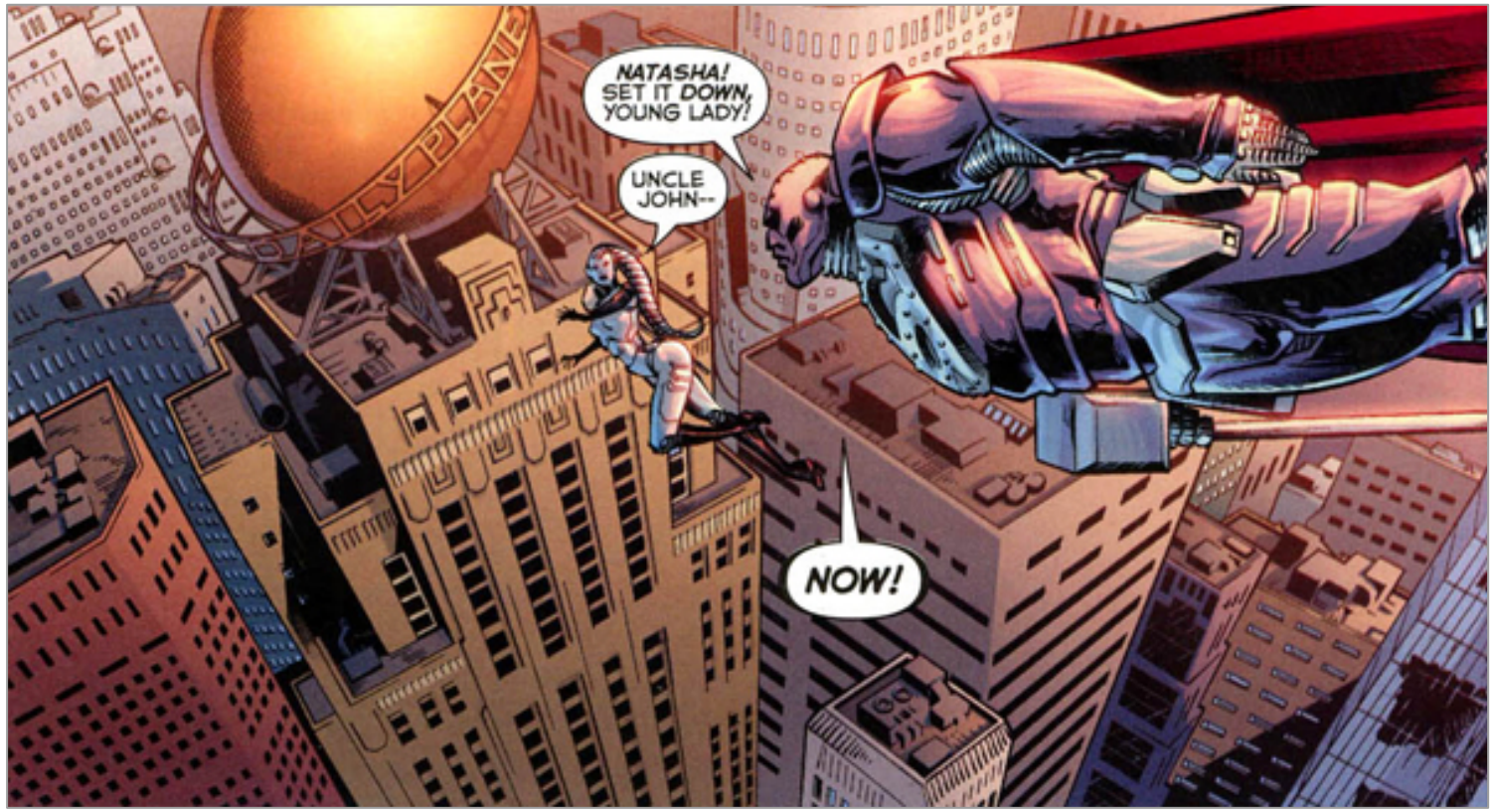
Jean Pélérin (16th century)

Gustave Caillebotte (19th century)

Birth of perspective in art: Three-point perspective



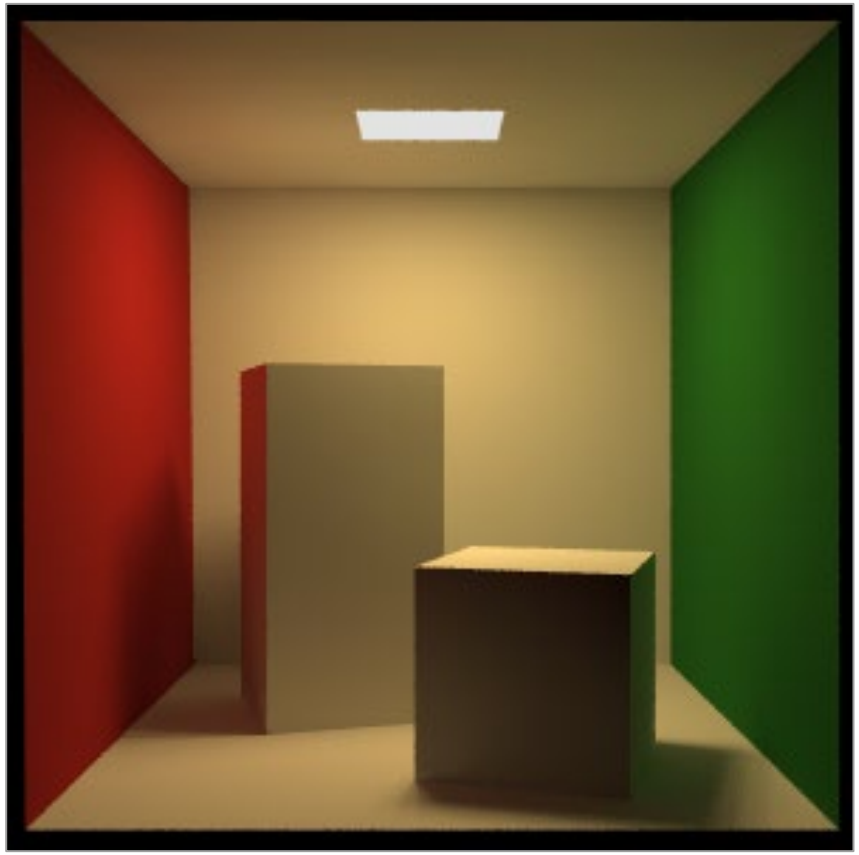
M.C. Escher (1928)

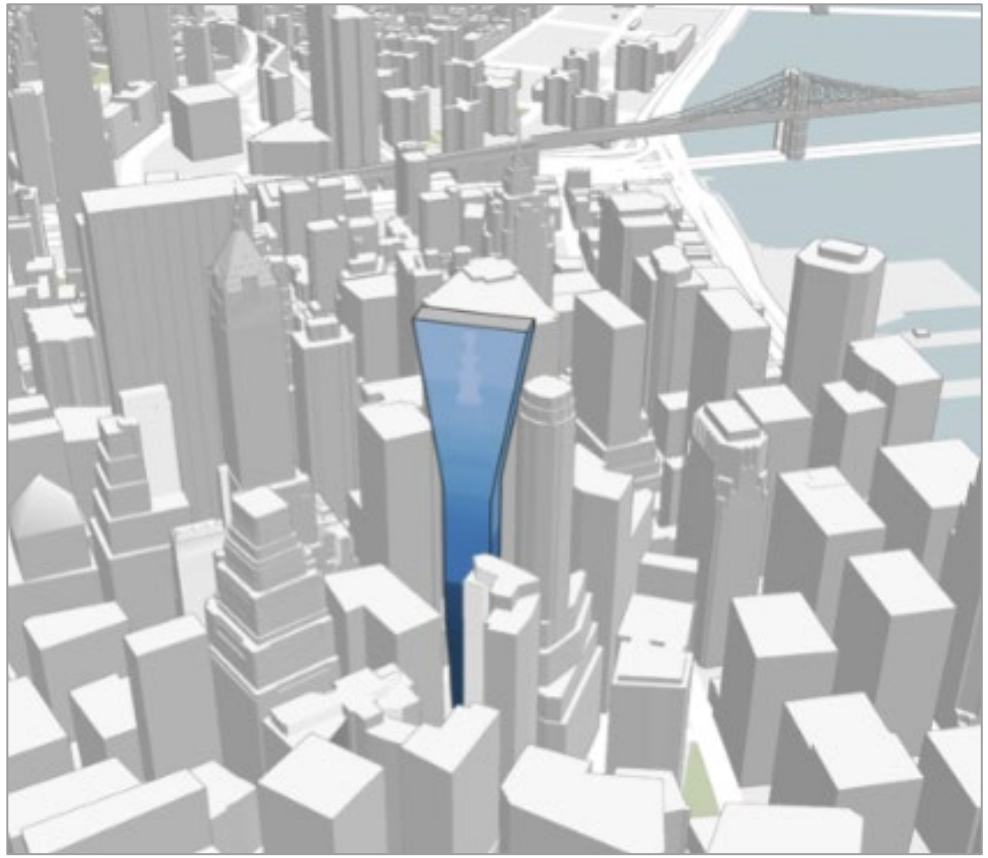


Multi perspective

The Frozen City by Matthias A. K. Zimmermann (2006)

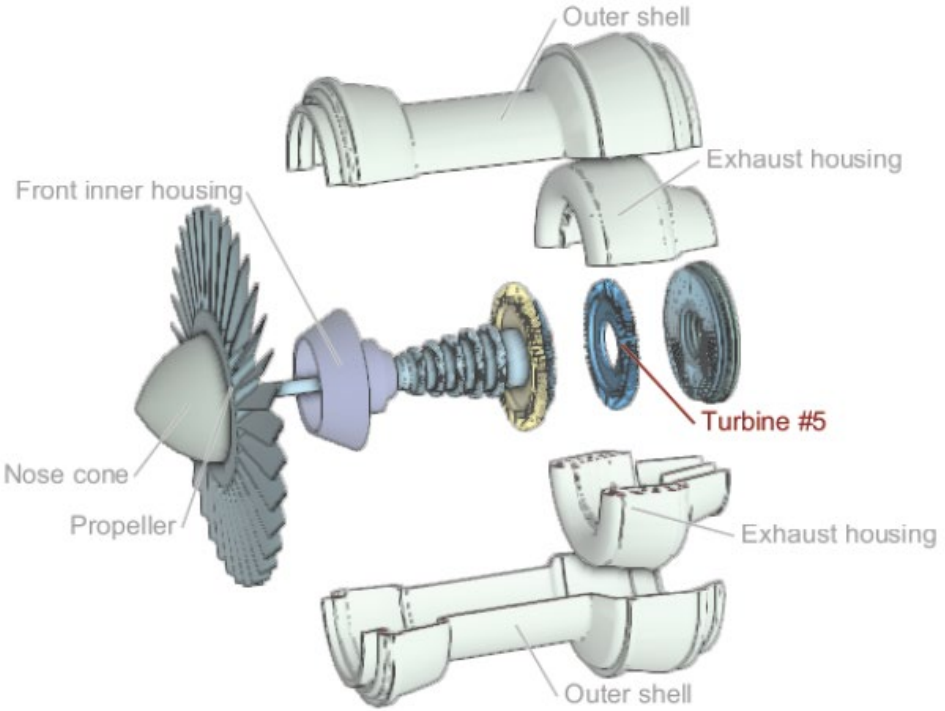
Perspective in computer graphics





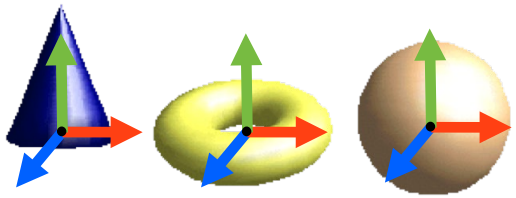
Rejection of perspective in CG

SimCity 2000

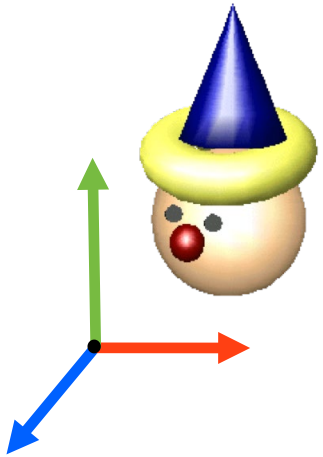


"Automated Generation of Interactive 3D Exploded Views"

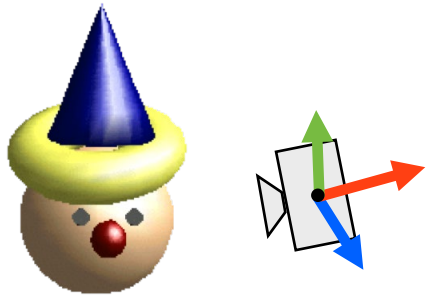
Coordinate spaces



Object coordinates



World coordinates

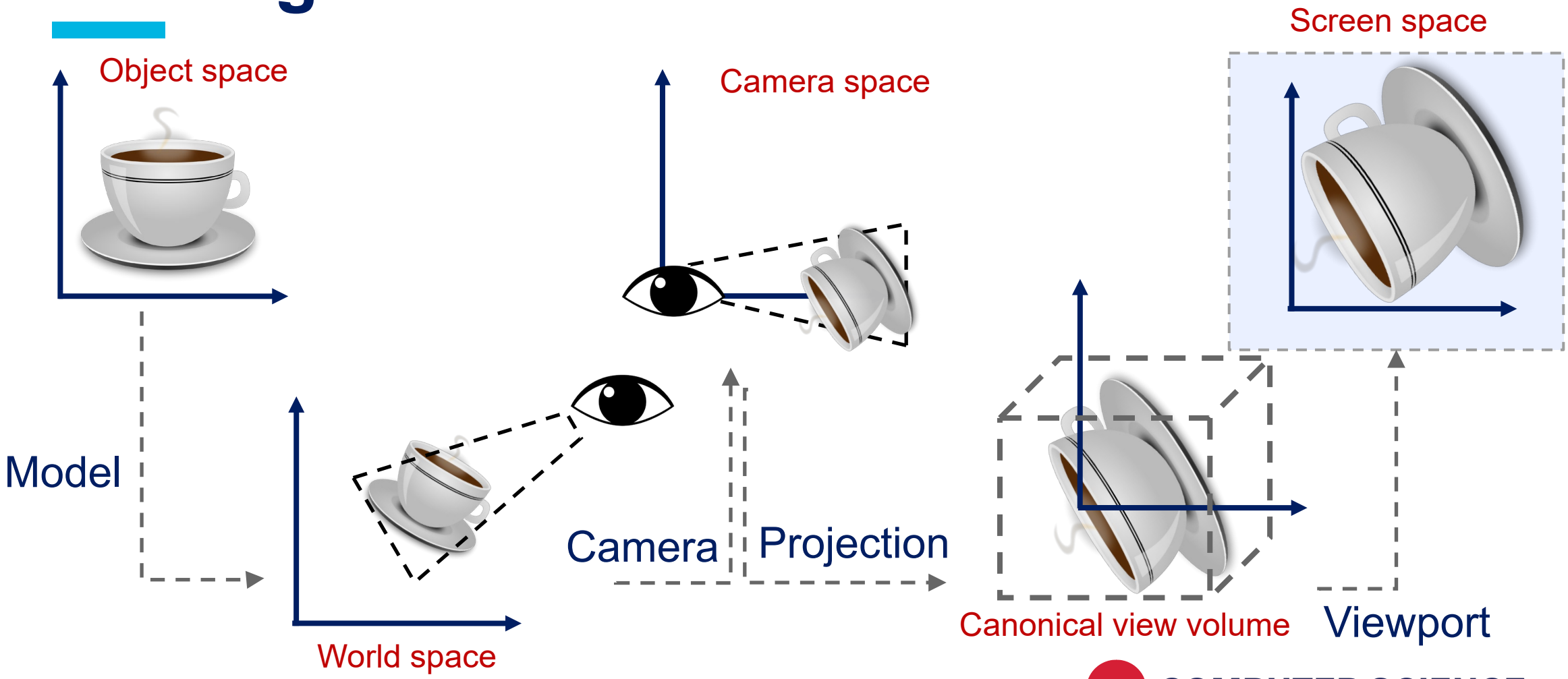


Camera coordinates

Screen coordinates

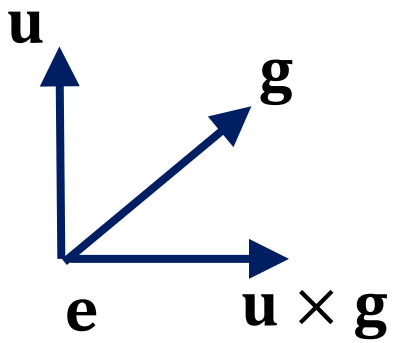
From: Mark Pauly

Viewing transformation



Camera transformation

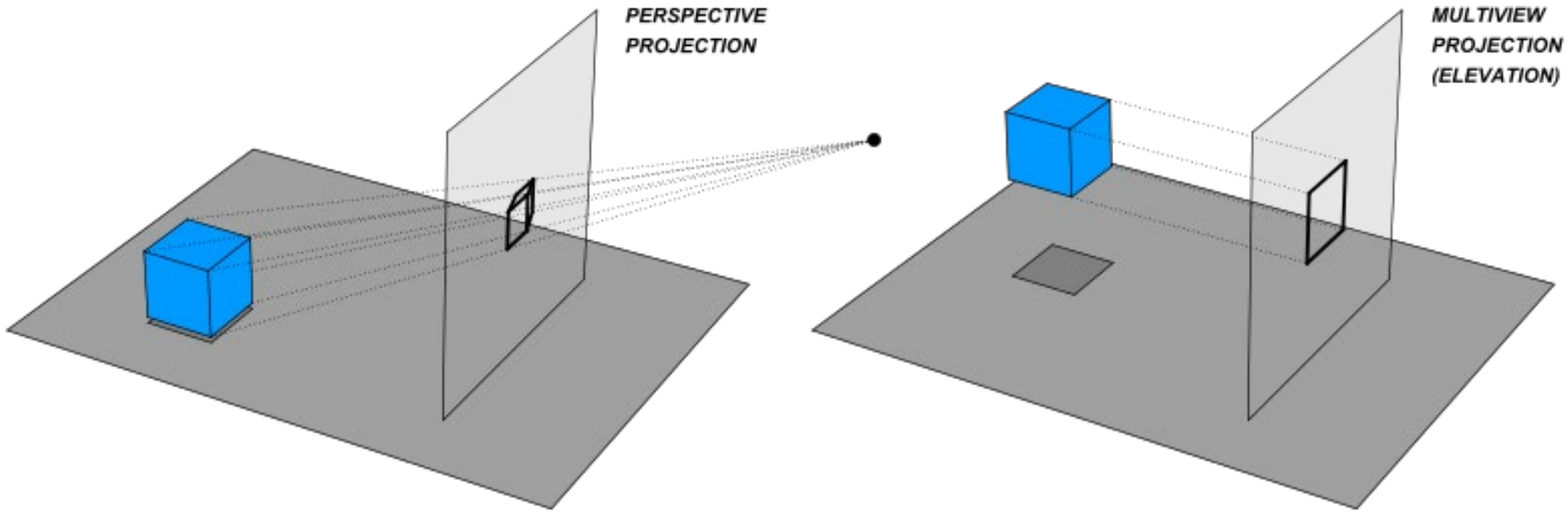
- Construct the camera reference system:
  - The eye position $\mathbf{e}$.
  - The forward direction $\mathbf{d}$.
  - The view-up vector $\mathbf{u}$.
- A view matrix transform all coordinates into view coordinates.



$$\mathbf{M}_{camera2world} = \begin{pmatrix} \mathbf{u} \times \mathbf{g} & \mathbf{u} & \mathbf{d} & \mathbf{e} \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

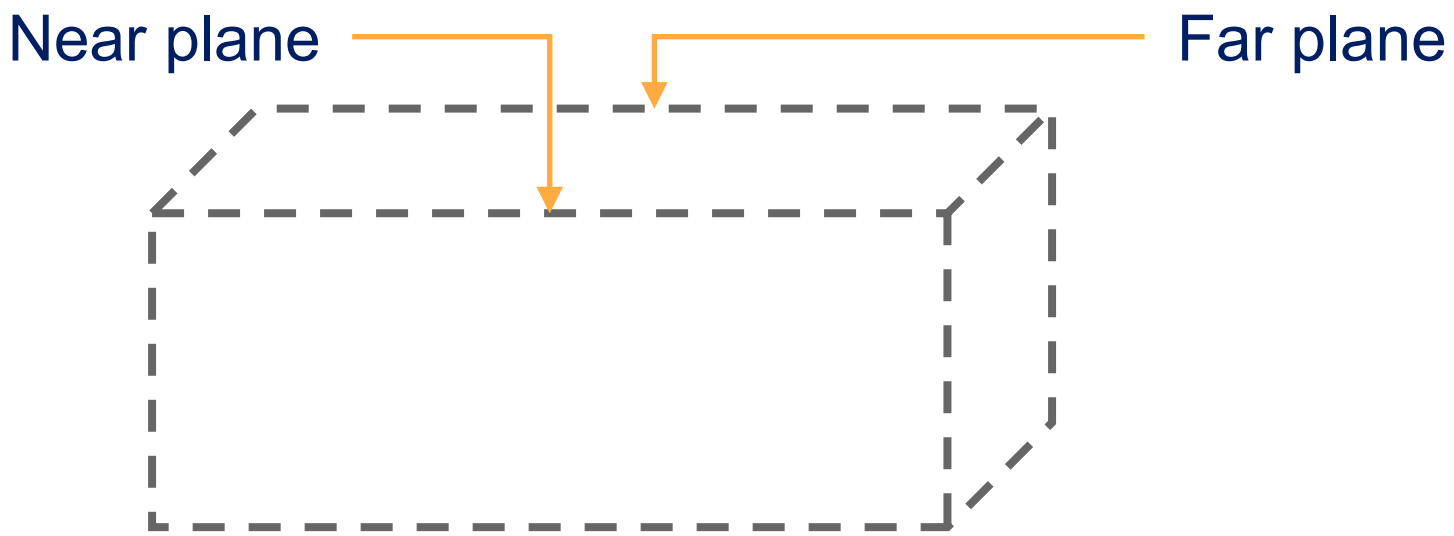
$$\mathbf{M}_{world2camera} = \begin{pmatrix} \mathbf{u} \times \mathbf{g} & \mathbf{u} & \mathbf{d} & \mathbf{e} \\ 0 & 0 & 0 & 1 \end{pmatrix}^{-1}$$

Orthographic and perspective projections

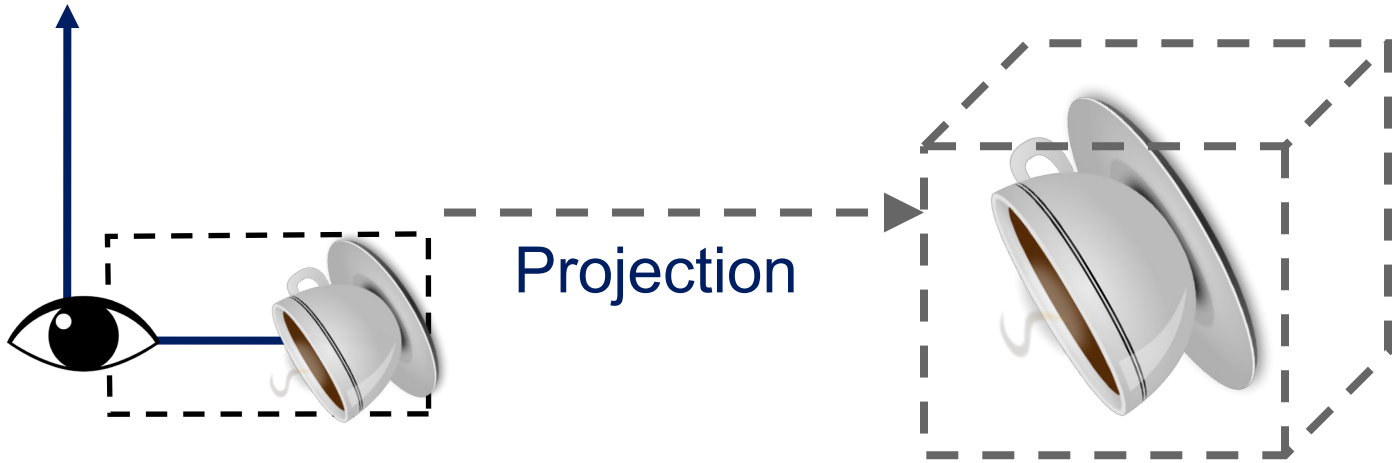


https://en.wikipedia.org/wiki/File:Various_projections_of_cube_above_plane.svg

View frustum

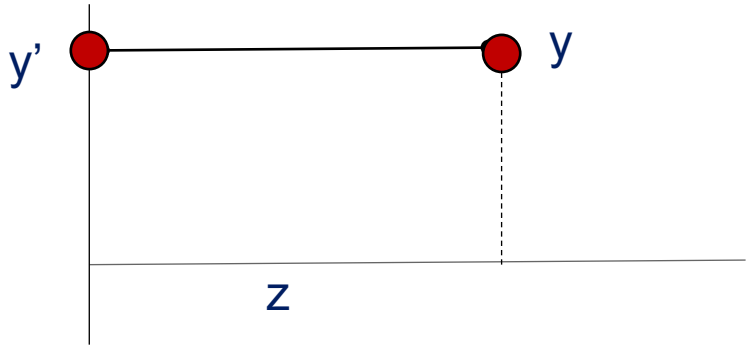


Orthographic transformation



$$\begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} = \begin{pmatrix} \frac{2}{(right-left)} & 0 & 0 & -x_{mid} \\ 0 & \frac{2}{(top-bottom)} & 0 & -y_{mid} \\ 0 & 0 & \frac{-2}{(far-near)} & -z_{mid} \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

Basic orthographic projection

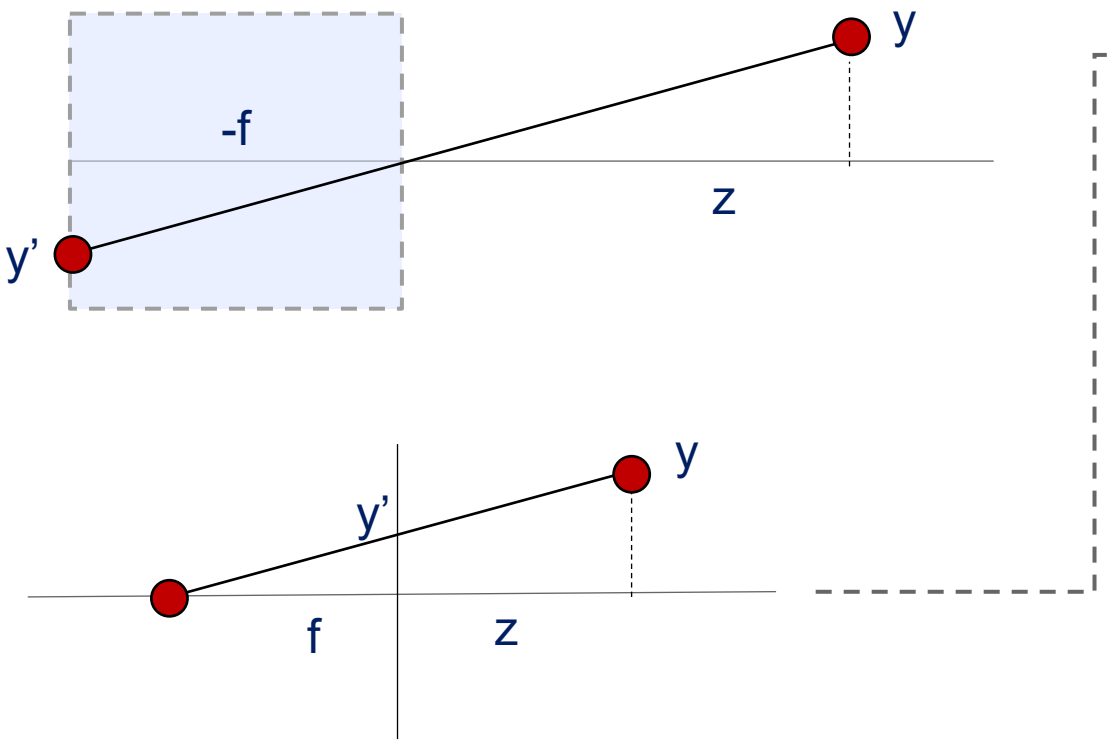


Projection plane
(z = 0)

$$\mathbf{x}' = \mathbf{x}, \mathbf{y}' = \mathbf{y}$$

$$\begin{pmatrix} x' \\ y' \\ z' \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

Basic perspective projection

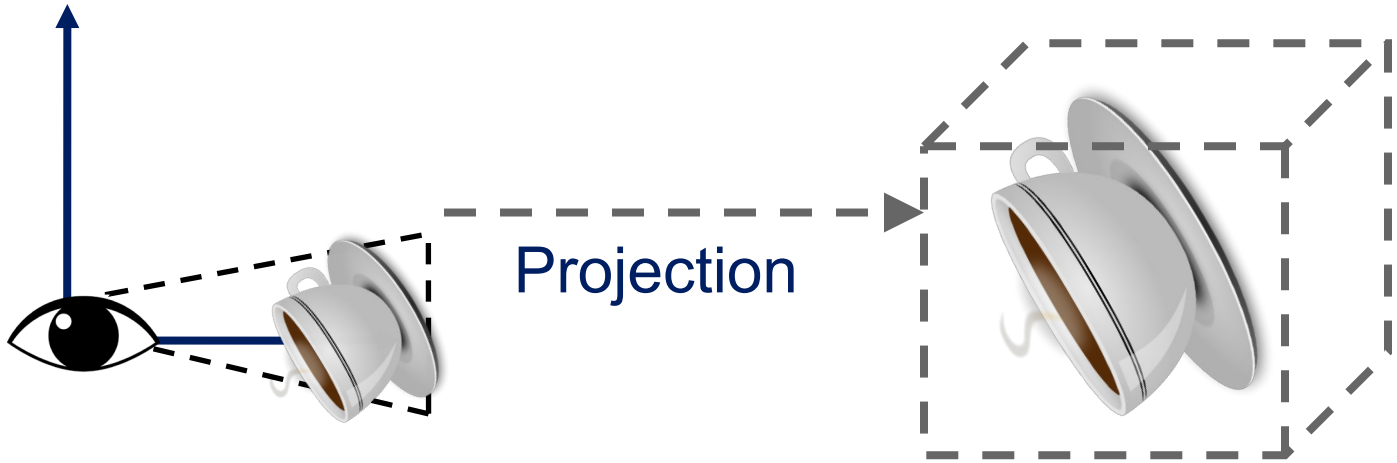


$$x' = f\frac{x}{z}, y' = f\frac{y}{z}$$

$$if\ f = 1:$$
$$x' = \frac{x}{z}, y' = \frac{y}{z}, f = 1$$

$$\begin{pmatrix} x' \\ y' \\ z' \\ 1 \end{pmatrix} \rightarrow \begin{pmatrix} x \\ y \\ z \\ z \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

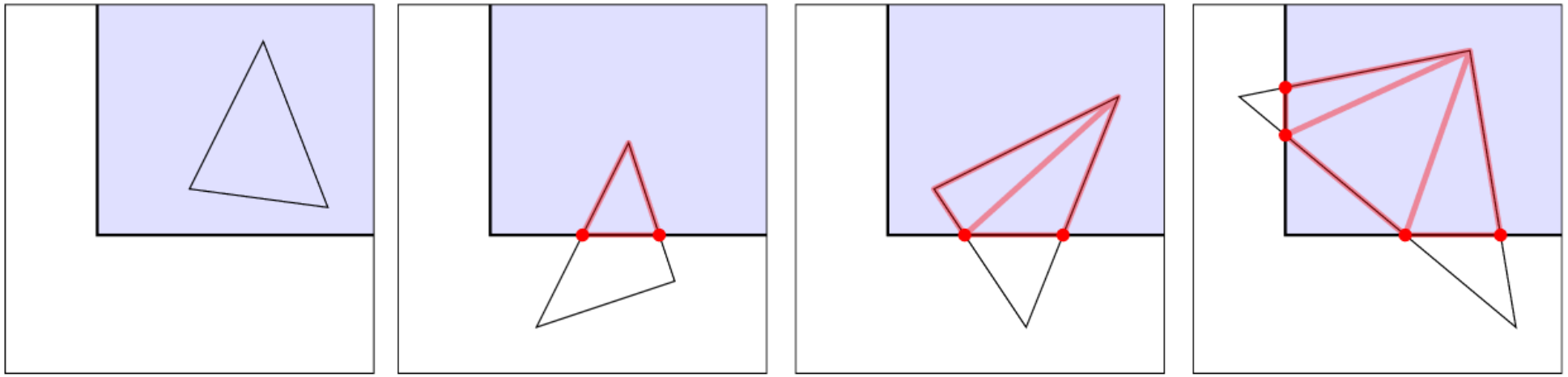
Perspective transformation



$$\begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} = \begin{pmatrix} \frac{2 * near}{(right - left)} & 0 & 0 & \frac{-near * (right + left)}{(right - left)} \\ 0 & \frac{2 * near}{(top - bottom)} & 0 & \frac{-near * (top + bottom)}{(top - bottom)} \\ 0 & 0 & \frac{-2}{(far - near)} & \frac{2 * far * near}{(near - far)} \\ 0 & 0 & -1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

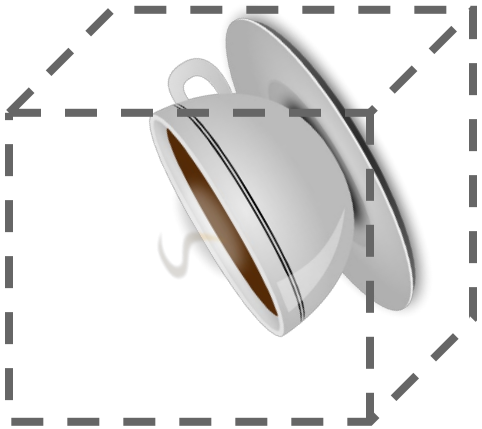
Canonical view volume

- Why?
  - Makes clipping much easier! GL can quickly discard geometry outside -1,1.



From: https://paroj.github.io/gltut/

Viewport transformation



Canonical view volume
$(-1,-1,-1) \times (1,1,1)$

Viewport

Screen space

$$\begin{pmatrix} x_{screen} \\ y_{screen} \\ z_{depth} \\ 1 \end{pmatrix} = \begin{pmatrix} \frac{width}{2} & 0 & 0 & \frac{width}{2} \\ 0 & \frac{height}{2} & 0 & \frac{height}{2} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

```
gl.viewport(0, 0, width, height);
```

Lab

- Draw a cube on screen with size $(-1,-1,-1) \times (1,1,1)$.
  - Apply a model transformation that scales it by 0.5, and tilts it slightly.
  - Apply a projection matrix (orthographic and perspective).