Fabio Tonini

Personal Data

Date of Birth	May 16, 1984
Citzenship	Italy

Current Position

Since October 2015	Post Doc at the Freie University of Berlin

Positions Held

April 2013 - September 2015	Post Doc at the Humboldt University of Berlin
January 2012 - January 2013	Scholarship at Scuola Normale Superiore of Pisa
January 2009 - January 2012	Ph.D. student at Scuola Normale Superiore of Pisa under the supervision of Prof. Angelo Vistoli

Education

May 2013	Ph.D. at Scuola Normale Superiore of Pisa
December 2008	Diploma at Scuola Normale Superiore of Pisa
September 2008	Master's Degree in Pure Mathematics, University of Pisa, *with honors*
July 2006	Bachelor's Degree in Pure Mathematics, University of Pisa, *with honors*

PhD's Thesis

Title	*Stacks of ramified Galois covers*
	defended the 2 May 2013
pdf	online link
Advisor	Angelo Vistoli

Master's Thesis

Title	*Rivestimenti di Gorenstein (Gorenstein covers)*
pdf	online link
Advisor	Angelo Vistoli

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Research Interests

- Algebraic Geometry
- Algebraic stacks, Moduli theory
- Action of algebraic groups and Galois covers
- Representation theory
- Algebraic fundamental groups and gerbes

Teaching Experience

2011-2012 Tutor of a group of young students at Scuola Normale Superiore of Pisa

2014 I held a course called *Introduction to algebriac stacks and moduli theory* at Humboldt University

2016 Advisor of Gabriel Lepetit, who wrote a thesis for the first year of Master entitled *Low-degree covers in algebraic geometry*

Languages

English **Good**

Italian **Native**

Invited Talks

June 4-7, 2017 **Representations of the Nori fundamental gerbes**, during Angelo Vistoli's 60 birthday at University of Pisa, Italy.

December 8, 2017 **Representations of the Nori fundamental gerbes**, at Osaka Univesity, Osaka, Japan.

September 4-5, 2017 **Sheafification functors**, during "Stacks in Turin" conference at University of Turin, Italy.

March 15, 2016 **Representations of the Nori fundamental gerbes**, during "Equivariant geometry and algebraic stack" conference at ANU Kioloa Campus, Australia.

December 19, 2014 **Stacks of ramified Galois covers**, at Bordeaux University, Bordeaux, France.

May 15, 2014 **Stacks of ramified Galois covers**, at Hamburg University, Hamburg, Germany.

April 25, 2014 **Stacks of ramified Galois covers**, at Kyoto University, Kyoto, Japan.

April 18, 2014 **Stacks of ramified Galois covers**, at Osaka Univesity, Osaka, Japan.

November 28, 2013 **Stacks of ramified Galois covers**, at Essen University, Essen, Germany.

January 30, 2013 **Stacks of ramified Galois covers**, at KTH, Stockholm, Sweden.

Schools and Conferences Attended

September 11-22, 2017 **Instruments of Algebraic Geometry**, University of Bucharest, Romania.

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September 4-5, 2017	**Stacks in Turin**, University of Turin, Italy.
August 28 - September 1, 2017	**Motives for periods**, Freie University of Berlin, Germany.
March 14-18, 2016	**Equivariant geometry and algebraic stacks**, ANU Kioloa Campus, Australia.
June 1-5, 2015	**Algebraic varieties and their moduli**, Pisa, Italy.
September 8-12, 2014	**Modern Trends in Gromov-Witten Theory**, Hannover, Germany.
August 25-30, 2014	**Recent Developments in Algebraic and Arithmetic Geometry**, Budapest, Hungary.
August 25-30, 2014	**Conference on Algebraic Geometry**, Amsterdam, Netherlands.
May 15-16, 2014	**NoGAGS**, *North German Algebraic Geometry Seminar*, Hamburg, Germany.
December 16-20, 2013	**Fundamental Groups in Arithmetic and Algebraic Geometry**, Pisa, Italy.
May 27-31, 2013	**Syzygies in Berlin**, Berlin, Germany.
May 16-17, 2013	**NoGAGS**, *North German Algebraic Geometry Seminar*, Hannover, Germany.
June 25-29, 2012	**Conference on Galois covers and deformations**, Bordeaux, France.
June 18-22, 2012	**GAel**, *Géométrie Algébrique en Liberté XX*, Grenoble, France.
June 11-15, 2012	**Summer school 'Geometry of moduli'**, Nordfjordeid, Norway.
May 23-26, 2012	**Giornate di Geometria Algebrica e argomenti correlati XI**, Pisa, Italy.
September 18-24, 2011	**Geometric Invariant Theory, Old and New**, Lucekin, Poland.
July 18-29, 2011	**Toric Varieties in Cortona**, Cortona, Italy.
June 20 - July 8, 2011	**Moduli of curves and Gromov-Witten theory**, Institute Fourier, Grenoble, France.
May 23-27, 2011	**Moduli spaces and moduli stacks**, Columbia University, New York.
January 5-14, 2011	**School on moduli spaces**, Isaac Newton Institute for Mathematical Sciences, Cambridge, United Kingdom.
October 13-15, 2010	**Intersection theory on the moduli space of curves**, Humboldt University of Berlin, Germany.

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August 30 - September 3, 2010 **WDT II**, *Workshop in Deformation Theory II*, University 'La Sapienza' of Rome, Italy.

August 9-14, 2010 **Geometry and Algebra of Orbifolds and the McKay Correspondence**, University of Warwick, United Kingdom.

July 4-10, 2010 **Impanga**, *Summer School on Algebraic Geometry*, Institute of Mathematics of Polish Academy of Sciences, Bedlewo, Poland.

May 25-29, 2010 **Giornate di Geometria Algebrica e argomenti correlati X**, Gargnano, Italy.

September 13 - October 3, 2009 **P.R.A.G.MAT.I.C 2009**, *Promotion of Research in Algebraic Geometry for MAThematicians in Isolated Centres*, University of Catania, Italy.

June 8-12, 2009 **GAel**, *Géométrie Algébrique en Liberté XVII*, Lorentz Center, Leiden, Netherlands.

Activities

2009-2011 Organizer and speaker of weekly seminars in algebraic geometry at Scuola Normale Superiore of Pisa

2014 Organizer of weekly student seminars in algebraic geometry at Humboldt University of Berlin

Publications

7. 2017 Tonini, F. & Zhang, L. F-divided sheaves trivialized by dominant maps are essentially finite. *(accepted in the Transactions of the American Mathematical Society).* **online link**, **arxiv link**, 1–21.

6. 2017 Tonini, F. & Zhang, L. Algebraic and Nori fundamental gerbes. *Journal of the Institute of Mathematics of Jussieu.* **online link**, **arxiv link**, 1–43. ISSN: 1474-7480.

5. 2017 Tonini, F. Ramified Galois covers via monoidal functors. *Transformation Groups* **22. online link**, **arxiv link**, 845–868. ISSN: 1083-4362.

4. 2017 Tonini, F. Trace map and regularity of finite extensions of a DVR. *Journal of Number Theory* **172. online link**, **arxiv link**, 133–144. ISSN: 0022314X.

3. 2015 Poma, F., Talpo, M. & Tonini, F. Stacks of uniform cyclic covers of curves and their Picard groups. *Algebraic Geometry* **2. Algebraic Geometry**, **arxiv link**, 26.

2. 2014 Tonini, F. Stacks of ramified covers under diagonalizable group schemes. *International Mathematics Research Notices* **2014. online link**, **arxiv link**, 2165–2244. ISSN: 16870247.

1. 2010 Pons-Llopis, J. & Tonini, F. ACM bundles on del Pezzo surfaces. *Le Matematiche* **64. Le Matematiche**, **arxiv link**, 177–211.

Preprints

5. 2017 Zhang, L., Tonini, F. & Romagny, M. Neukirch-Uchida theorem for purely inseparable field extensions. *arXiv:1711.06898.* **online link**, **arxiv link**, 1–9.

4. 2017 Tonini, F. & Yasuda, T. Moduli of formal torsors. *arXiv:1709.01705.* **arxiv link**, 1–34.

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3. 2017 Tonini, F. & Zhang, L. Essentially Finite Vector Bundles on Normal Pseudo-proper Algebraic Stacks. *arXiv:1702.03751.* **online link**, **arxiv link**, 1–9.

2. 2017 Antei, M., Biswas, I., Emsalem, M., Tonini, F. & Zhang, L. Nori fundamental gerbe of essentially finite covers and Galois closure of towers of torsors. *arXiv:1706.00739.* **arxiv link**, 1–31.

1. 2014 Tonini, F. Sheafification functors and Tannaka's reconstruction. *arXiv:1409.4073.* **online link**, **arxiv link**, 1–35.

Ph.D. thesis

1. 2013 Tonini, F. *Stacks of ramified Galois covers* **online link**, **arxiv link**. PhD thesis (), 1–192.

November 21, 2017

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RESEARCH PROJECT

FABIO TONINI

Past and current research

My general area of research is algebraic geometry and, more precisely, moduli theory. The study of moduli problems led me to form a strong background in commutative algebra, schemes theory and stacks theory, which are the languages used in this context.

Ph.D. thesis and works on ramified G-covers. In my Ph.D. thesis I have introduced the notion of ramified Galois cover under a finite group scheme G (briefly a G-cover) and studied the geometry of the moduli stack G-Cov they form. The study of G-covers essentially splits in two cases, namely the abelian case, where I have provided a toric description of a special irreducible component of G-Cov (see [Ton14b]), and the non-abelian one. This last case is much harder than the abelian one, due to the complexity of the representation theory of a non-abelian group. Torsors under the group G are special cases of G-covers and Tannaka's reconstruction asserts that they correspond to particular strong monoidal functor. Following this point of view I have been able to identify a special class of (non-strong) monoidal functors and extend Tannaka's correspondence between them and G-covers and, more generally, to interpret any cover with an action of a group in terms of a functor. This point of view allowed me to prove results about the geometry of G-Cov and give a divisorial condition assuring that the total space of a G-cover of a normal scheme is normal too (see [Ton17]).

Sheafifying linear functors. Equivariant problems are known to correspond to problems over the stack $\mathrm{B}\,G$ of G-torsors and, using this strategy, I have generalized the correspondence between G-covers and particular monoidal functors replacing $\mathrm{B}\,G$ by an arbitrary algebraic stack (see [Ton14a]): given a collection $\mathcal{C}$ of quasi-coherent sheaves on an algebraic stack $\mathcal{X}$ it is possible to map quasi-coherent sheaves (of algebras) to particular (monoidal) linear functors and conversely. For instance considering $\mathcal{X} = \mathrm{B}\,G$ and by $\mathcal{C}$ the category of locally free sheaves on $\mathrm{B}\,G$ (that is locally free G-representations) one obtains the characterization of G-covers. Quite surprisingly, considering instead by $\mathcal{X}$ a projective scheme and setting $\mathcal{C} = \{\mathcal{O}_{\mathcal{X}}(n)\}_{n\in\mathbb{Z}}$ one recovers the correspondence between quasi-coherent sheaves on $\mathcal{X}$ and graded modules over the coordinate ring of $\mathcal{X}$. More generally this theory works well under the assumption that $\mathcal{C}$ form a generator of the category of quasi-coherent sheaves on the stack $\mathcal{X}$.

Picard group of the moduli of uniform cyclic covers of curves. I have also studied moduli problems of covers of curves and, in particular, described the Picard group of some of those moduli (see the joint work [PTT15] with Mattia Talpo (University of British Columbia, Vancouver) and Flavia Poma).

Nori and algebraic fundamental gerbes and their Tannakian interpretation. During the last year I have been collaborating with Lei Zhang (Freie University of Berlin, Germany) on the study of the Nori fundamental gerbe and algebraic

fundamental gerbes of a stack. Let $\mathcal{X}$ be a geometrically connected and geometrically reduced stack of finite type over a field k. One can associate with $\mathcal{X}$ a profinite gerbe $\Pi_{\mathcal{X}}$ together with a map $\pi\colon \mathcal{X} \longrightarrow \Pi_{\mathcal{X}}$ having the following universal property: all maps from $\mathcal{X}$ to a finite gerbe factor uniquely through π. The gerbe $\Pi_{\mathcal{X}}$ is called the Nori fundamental gerbe of $\mathcal{X}$. Following ideas from [BV14] and [EH10], in [TZ17a] we introduced the notion of algebraic fundamental gerbe of $\mathcal{X}$ and prove that $\Pi_{\mathcal{X}}$ is its profinite quotient, generalizing what is known for smooth varieties. In other words, we introduced a Tannakian category $\mathrm{Strat}_{\infty}(\mathcal{X})$, whose corresponding gerbe is the algebraic fundamental gerbe of $\mathcal{X}$ and showed that the category $\operatorname{Rep}\Pi_{\mathcal{X}}$ of representations of $\Pi_{\mathcal{X}}$ can be identified with the subcategory of $\mathrm{Strat}_{\infty}(\mathcal{X})$ of essentially finite objects. Being essentially finite is an algebraic condition and, in [TZ17b], we find an equivalent but more geometric description by looking at objects of $\mathrm{Strat}_{\infty}(\mathcal{X})$ who are trivialized by some covering. In particular this yields a geometric criterion for the essential finiteness of vector bundles over a pseudo-proper stack of finite type. This criterion has made possible to solve the problem of finding a Galois closure of a tower of torsors, which is the subject of my most recent preprint [ABE+17], joint work with Marco Antei (Université de Nice, France), Indranil Biswas (Tata Institute of Fundamental Research, Bombay, India), Michel Emsalem (Université des Sciences et des Technologies de Lille, France) and Lei Zhang.

Moduli of formal torsors. I am collaborating with Takehiko Yasuda (Osaka University, Japan) about the construction of a moduli stack of Galois covers of a formal power series ring. This moduli is needed in a project about motivic integration and the wild McKey correspondence. Given a finite group G, in [TY17] we consider the fiber category given by $\Delta_G(B) = \mathrm{B}\, G(B[[t]]_t)$, so that the k-rational points of Δ_G are the G-torsors of $k((t))$. More precisely the final moduli stack is a subfibered category of Δ_G, but we will ignore this technicalities here. When G is a semidirect product of a p-group (where $\operatorname{char} k = p > 0$) and a cyclic tame group, we prove that Δ_G is a direct limit of separated Deligne-Mumford stacks whose transition maps are composition of closed embeddings and universal homeomorphisms. In particular Δ_G admits a "ind coarse moduli space" (defined as one can expect).

Neukirch-Uchida theorem for purely inseparable extensions. In the paper [RTZ17] with Lei Zhang and Matthieu Romagny (University of Rennes, France) we work on a variant of the Neukirch-Uchida theorem. The original Neukirch-Uchida theorem describes how morphisms of number fields can be tought of as maps between the corresponding absolute Galois groups. The idea is to consider the same problem for purely inseparable extensions. Instead of the Galois group, we use the local Nori fundamental group $\pi^L(-)$ defined in [TZ17a] and which "parametrizes" torsors under finite and connected group schemes. Given a characteristic p field K we show that the map

$$\begin{aligned} \{\text{p.i. extensions of } K\} &\longrightarrow \{\text{subgroups of } \pi^L(K)\} \\ L/K &\longmapsto \pi^L(L) \subseteq \pi^L(K) \end{aligned}$$

is a order-reverse-embedding. Notice that maps between purely inseparable extensions are unique and indeed, in the category on the right, they correspond to inclusions between subgroups.

Future research

Homotopy exact sequence. I am collaborating with Lei Zhang and Valentina Di Proietto (University of Exeter, United Kingdom) in proving the homotopy exact

sequence for isocrystals. If $f\colon X \longrightarrow S$ is a proper, smooth and geometrically connected map between smooth varieties over a field k, $x \in X(k)$, $s = f(x) \in S(k)$ and X_s is the fiber over s one can ask if the sequence

$$\pi(X_s, x) \longrightarrow \pi(X, x) \longrightarrow \pi(S, s) \longrightarrow 0$$

is exact. Here π denotes a fundamental group attached to a variety over k. The sequence is known to be exact if π is the étale fundamental group or π is the stratified fundamental group. We want to prove that the same is true for the isocrystal fundamental group.

Picard group of compactification of moduli of uniform cyclic covers of curves. In [PTT15] we have computed the Picard group of the stacks of "uniform" cyclic covers of curves. Those spaces admit a natural compactification using admissible covers. We have recently understood that the information on the Picard group of the "open" part should be enough to describe the Picard group of the compactification. We know that the boundary divisors are independent and the last ingredient we need (and we are looking for) is one explicit relation among the given generators. We plan to find this relation by using test curves and, perhaps, Grothendieck-Riemann-Roch. This is a work in progress with Mattia Talpo and Nicola Pagani (University of Liverpool, United Kingdom).

Homogeneous sheafifications for groups of invertible sheaves. As described in the first section, I have shown that one can recover the correspondence between quasi-coherent sheaves on a projective space and graded modules over its coordinate ring from a general theory: a similar association holds when we fix an algebraic stack $\mathcal{X}$ and a collection $\mathcal{C}$ of quasi-coherent sheaves on $\mathcal{X}$. When we consider as $\mathcal{C}$ a subgroup of the Picard group of $\mathcal{X}$ and we set

$$S = \bigoplus_{\mathcal{L} \in \mathcal{C}} \mathrm{H}^0(\mathcal{X}, \mathcal{L})$$

for the associated Cox ring (with an algebra structure opportunely defined) we obtain a way to associate with a quasi-coherent sheaf on $\mathcal{X}$ a graded module on S and conversely. This converse is essentially a homogeneous sheafification of graded modules. The general theory also predict under what conditions the functor $\mathrm{QCoh}\,\mathcal{X} \longrightarrow \text{Graded-Modules}(S)$ is fully faithful. When $\mathcal{X}$ has some more nice geometric property, e.g. $\mathcal{X}$ is a normal variety, the same theory works also if we consider subgroups of the group of reflexive sheaves of rank 1 of $\mathcal{X}$. This seems to generalize some previous results about Cox rings and put them in a common framework and I plan to investigate this subject.

P-moduli spaces and motivic integration. As a continuation of [TY17], our plan is to integrate particular functions (determined via the notion of ramification filtration) over the ind-coarse moduli space of Δ_G. In order to do that and have a space attached to any finite group G we have to introduce the notion of P-moduli spaces: this is a universal map from your preferred moduli problem to an object called a P-scheme, which is a scheme in a category where universally bijective and finitely presented morphisms (e.g. partition in locally closed subsets) are isomorphism. The idea is that integretation just cares about the constructible subsets, not about the opens.

Inverse Galois problem for finite and connected group schemes. I am collaborating with Lei Zhang and Matthieu Romagny on the following problem: given a perfect field k and a finite and connected group scheme G over k does there exists a G-torsor over $k(t)$ which is minimal, that is not induced by a torsor under a subgroup of G? In other words are all finite and connected group schemes over

k a quotient of the local Nori fundamental group $\pi^L(k(t)/k)$? This is analogous to the inverse Galois problem: Galois extensions over a given field with group G are exactly minimal G-torsors over that field. Our idea is that the above question should have a positive answer: we have a moduli space of "candidates" and we have to prove that there is "good" one between them. The same method allowed us to find a minimal torsor for the kernel of any power of the Frobenius of GL_n.

References

[ABE+17] Marco Antei, Indranil Biswas, Michel Emsalem, Fabio Tonini, and Lei Zhang. Nori fundamental gerbe of essentially finite covers and Galois closure of towers of torsors. *arXiv:1706.00739*, jun 2017.

[BV14] Niels Borne and Angelo Vistoli. The Nori fundamental gerbe of a fibered category. *Journal of Algebraic Geometry*, 24(2):311–353, sep 2014.

[EH10] Hélène Esnault and Amit Hogadi. On the algebraic fundamental group of smooth varieties in characteristic p>0. *Transactions of the American Mathematical Society*, 364(5):2429–2442, may 2010.

[PTT15] Flavia Poma, Mattia Talpo, and Fabio Tonini. Stacks of uniform cyclic covers of curves and their Picard groups. *Algebraic Geometry*, 2(1):26, dec 2015.

[RTZ17] Matthieu Romagny, Fabio Tonini, and Lei Zhang. Neukirch-Uchida Theorem for Purely Inseparable Field Extensions. *arXiv:1711.06898*, pages 1–9, nov 2017.

[Ton14a] Fabio Tonini. Sheafification functors and Tannaka's reconstruction. *arXiv:1409.4073*, page 35, sep 2014.

[Ton14b] Fabio Tonini. Stacks of ramified covers under diagonalizable group schemes. *International Mathematics Research Notices*, 2014(8):2165–2244, jul 2014.

[Ton17] Fabio Tonini. Ramified Galois covers via monoidal functors. *Transformation Groups*, 22(3):845–868, sep 2017.

[TY17] Fabio Tonini and Takehiko Yasuda. Moduli of formal torsors. *arXiv:1709.01705*, sep 2017.

[TZ17a] Fabio Tonini and Lei Zhang. Algebraic and Nori fundamental gerbes. *Journal of the Institute of Mathematics of Jussieu*, pages 1–43, jul 2017.

[TZ17b] Fabio Tonini and Lei Zhang. F-divided sheaves trivialized by dominant maps are essentially finite. *(accepted in the Transactions of the American Mathematical Society)*, dec 2017.