

Homework #1

Getting the Bayesian feet wet – enjoying the posterior

1. Simulate data from a linear regression model with the following design matrix and coefficients, respectively: $X = \text{cbind}(\text{rep}(1, \text{nobs}), \text{runif}(\text{nobs}), \text{runif}(\text{nobs}))$, $\beta = c(-1, 3, 6)$, setting the observation error variance equal to 1. (In preparation for assignment (3), also produce the probit version of each of the simulated data sets.) Use `runireg` from the `bayesm` package and convince yourself of the asymptotic consistency of the Bayes estimator, i.e., the posterior mean of the regression coefficients. Start with `nobs=5` or something similarly small and crank `nobs` up to demonstrate asymptotic consistency (no data replications required). For each differently sized data set, compare what you get from `runireg` to what you get from OLS. You can use the default prior for β in `runireg`. However, I recommend specifying an uninformative prior for the observation error variance, i.e., `Prior=list(nu=0, ssq=0)`.
2. Now simulate a medium-sized data set using the same parameter settings, but with the following design matrix: `h1f= runif(nobs); X=cbind(rep(1,nobs), h1f, h1f)`. This design has two perfectly correlated covariates. Investigate the joint posterior of the regression coefficients in some detail and document what you find. Finally, compare to what you get from OLS.
3. Repeat the demonstration from assignment (1) using a probit model and using default priors. Compare the posteriors you got from the linear data in assignment (1) to those from the probit data for different sample sizes. How would you characterize the trade-off between sample size and having access to the linear observations or only to the probit observations?