Problem Set 1 - Monetary and Fiscal Policy: Theory and Practice

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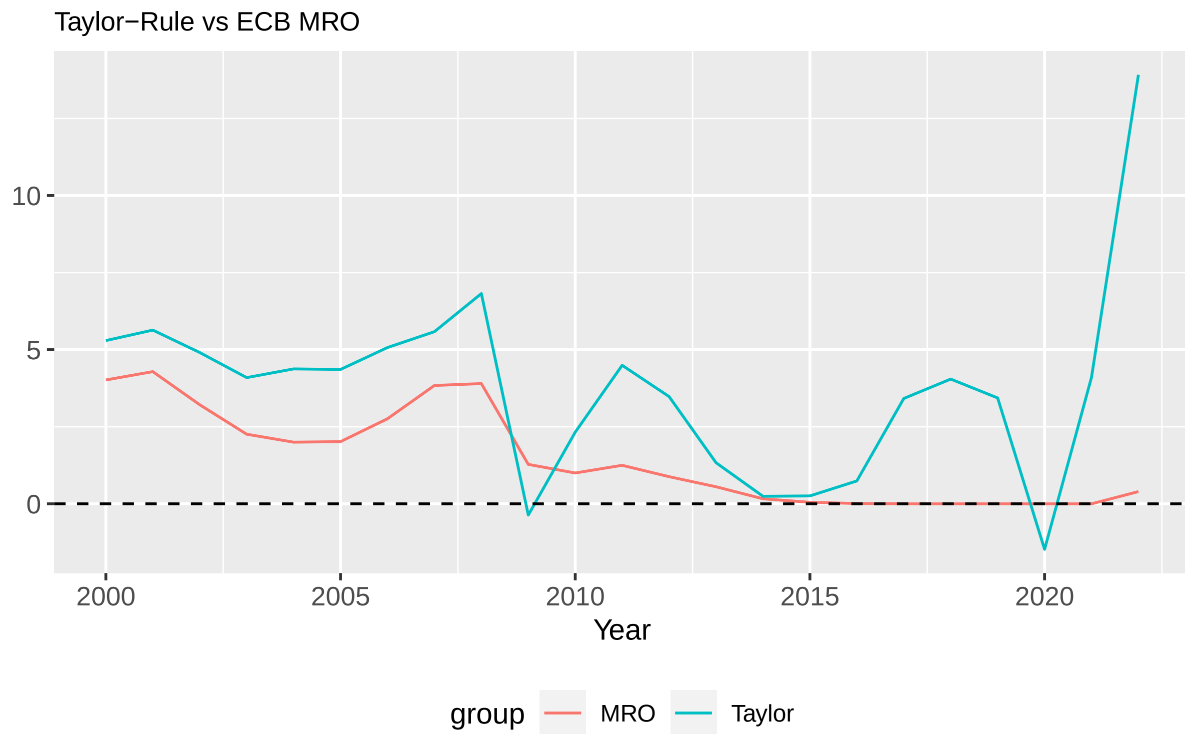
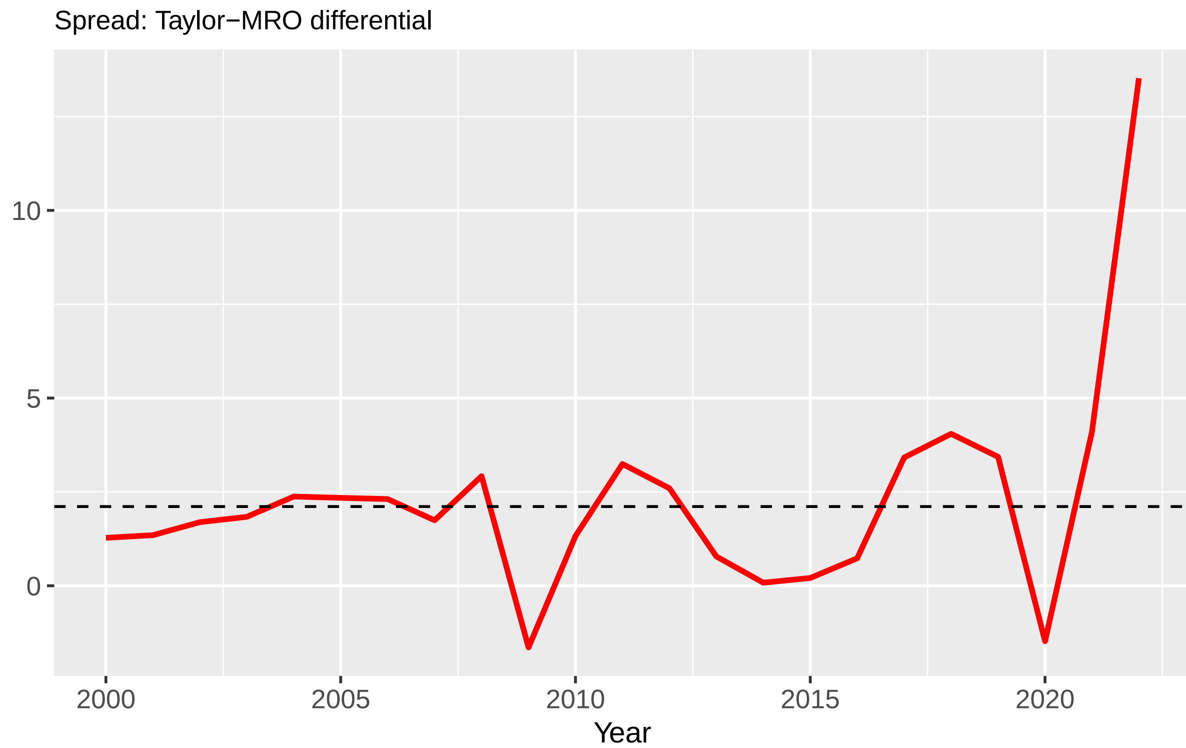
Taylor Rules

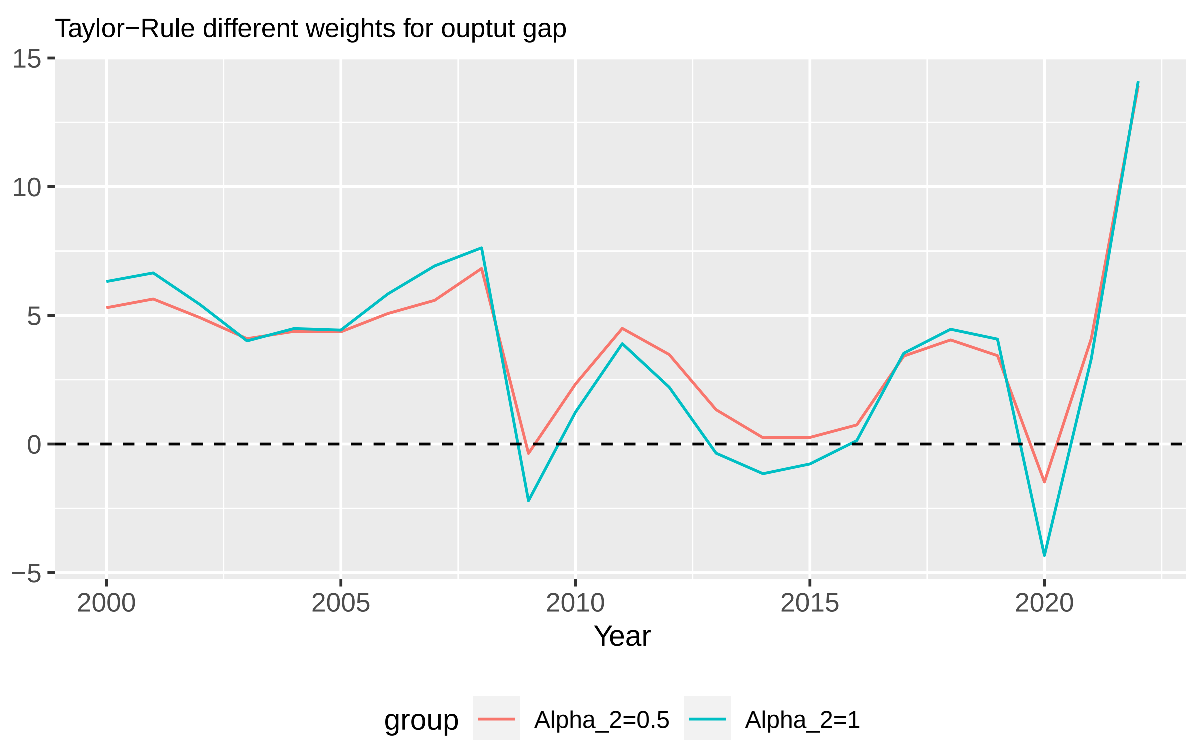
To perform the task, I have used RStudio, which includes the packages *rdbnomics* and *ecb*, allowing to download directly nicely formatted time series on macroeconomic data. I downloaded daily data on MRO rates and averaged them by year, in order to make these data conformable with the yearly format of the AMECO time series.

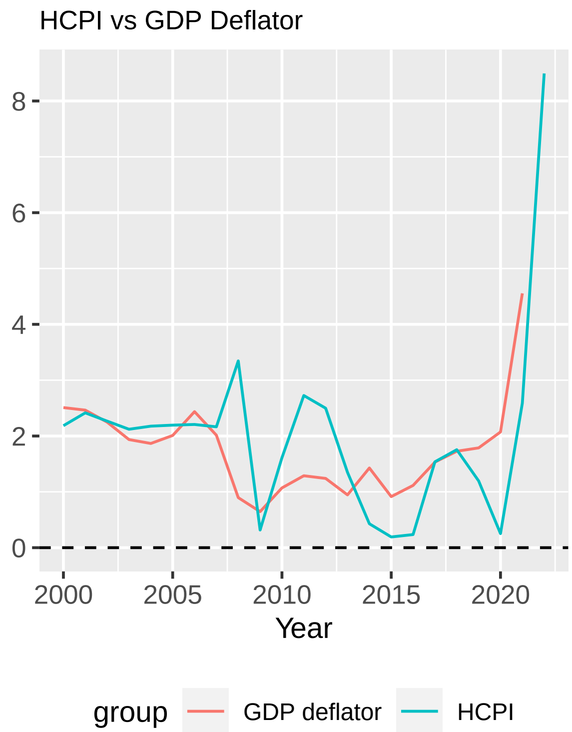


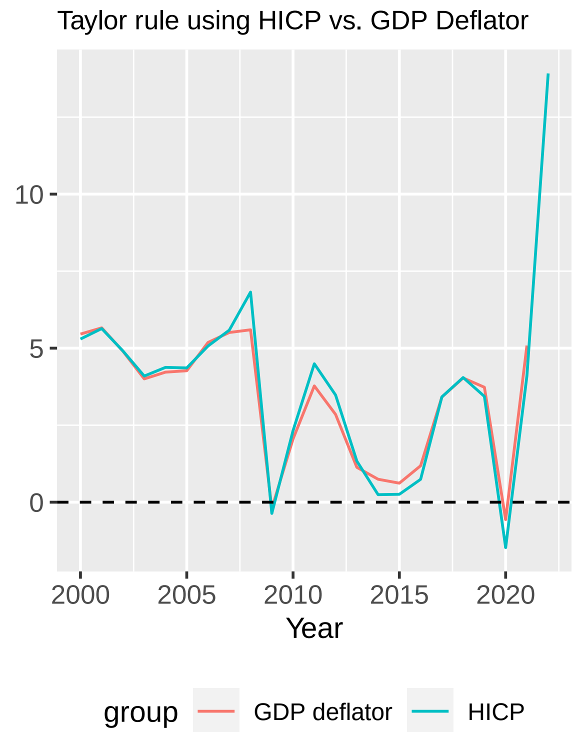
The spread seems to suggest that the Taylor Rule, outside of the financial crisis, has prescribed a tighter monetary policy than the one implemented by the ECB. Therefore, the ECB can be, on the basis of the Taylor Rule, be blamed of having been too loose in its monetary policy and, during the financial crisis, can be blamed of having reversed its stance too strongly. A clear shortcoming of the Taylor Rule is that it recommends a policy rate at time t based on the information on macroeconomic variables observed at time t. Such shortcoming would be overcome by a Taylor rule that recommends a policy rate at time t based on observations at time $t - n$, in order to allow policy makers to actually implement the rule's prescriptions.



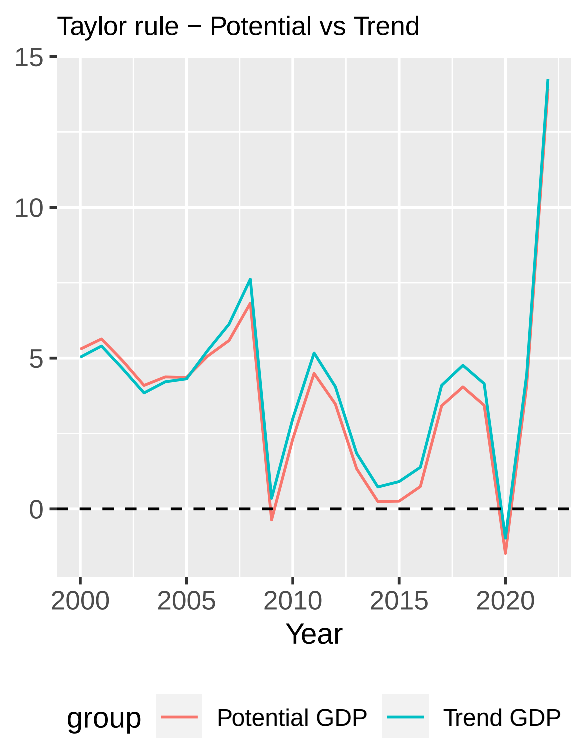
Assigning a larger coefficient $\alpha_2 = 1$ to the output gap yields a more volatile, or rather extreme, Taylor Rule. This is particularly true when the output gap is particularly large, e.g. during recessions.

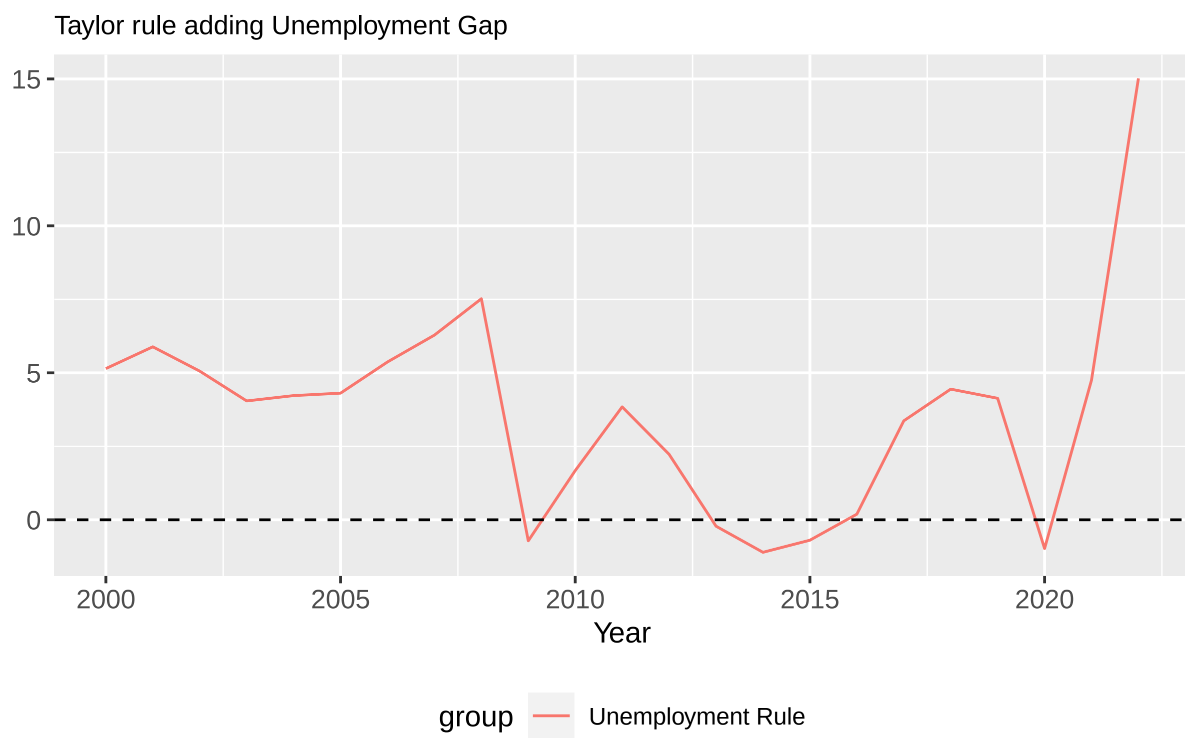






Despite trying different measures of inflation, the prescriptions of the Taylor Rule do not seem to change significantly.





I chose to set $U^n = 3\%$ and $\alpha_3 = 0.5$. Adding the unemployment gap does not change significantly the predictions of the Taylor Rule.

Optimal monetary policy at the effective lower bound (LB) in a simple model of inflation

Consider the simple linear process governing inflation:

$$\pi_t = -a(i_t - i^*) + bq_t + pi_{t-1} + \epsilon_t \tag{1}$$

Assume that the central bank minimizes the following loss function:

$$L(\pi_t) = E_t(\pi_t - \pi^*)^2 \tag{2}$$

1. Find the optimal interest rate when the LB is not binding.

$\min\limits_{i_t, q_t} E_t(\pi_t - \pi^*)^2$ s.t. $\pi_t = -a(i_t - i^*) + bq_t + \pi_{t-1} + \epsilon_t \rightarrow \min\limits_{i_t, q_t} E_t(-a(i_t - i^*) + bq_t + \pi_{t-1} + \epsilon_t - \pi^*)^2 \rightarrow \min\limits_{i_t, q_t}\{E_t(\pi_t - \pi^*)^2 + Var(\pi_t)\} = \min\limits_{i_t, q_t}\{E_t(-a(i_t - i^*) + bq_t + \pi_{t-1} + \epsilon_t)^2 + \sigma_\epsilon^2 + \sigma_b^2 q_t^2\}$

We take the F.O.C.:

$\frac{\partial}{\partial i_t} : -2aE_t\{-a(i_t - i^*) + bq_t + \pi_{t-1} + \epsilon_t - \pi^*\} \overset{!}{=} 0$, since $E[e_t] = 0$ we have that $i_t = i^* + \frac{1}{a}(bq_t + \pi_{t-1} - \pi^*)$ and that $q_t = \frac{\bar{b}^2}{\bar{b}^2 + \sigma_b^2} q_t \rightarrow q_t = 0 \rightarrow i_t = i^* + \frac{1}{a}(\pi_{t-1} - \pi^*)$.

When the LB is not binding, the interest rate instrument is superior to quantitative easing because it is not uncertain. Hence, when the interest rate tool is available, it will be optimal to use only this and leave $q_t = 0$. Finally, the parameter $\frac{1}{a}$ represents the elasticity of inflation with respect to the interest rate. To obtain the expected inflation, we plug the solution we obtained into the inflation process:

$\pi_t = -a(i_t - i^*) + bq_t + pi_{t-1} + \epsilon_t \rightarrow \pi_t = \pi^* - \pi_{t-1} + \pi_{t-1} + \epsilon_t \rightarrow E_{t-1}\pi_t = E[\pi^* + \epsilon_t] = \pi^*$.

With unconstrained traditional monetary policy, the central bank is able to target inflation in expectation.

2. What about the case when $i_t = i_{LB}$?.

We take a new F.O.C. with respect to q_t only, since the $i_t = i_{LB}$ constraint is binding and the F.O.C. does not hold.

$\frac{\partial}{\partial q_t} : -2aE_t\{-a(i_t - i^*) + bq_t + \pi_{t-1} + \epsilon_t - \pi^*\}\bar{b} + 2\sigma_b^2 q_t \overset{!}{=} 0 \rightarrow (b^2 + \sigma_b^2) * q_t = -\bar{b}(-a(i_t - i*) + \pi_{t-1} - \pi^*$ which recalling that $i_t = i_{LB}$: gives us the expression $q_t^* = -\frac{\bar{b}}{\bar{b}^2+\sigma_b^2}(-a(i_{LB} - i^*) + \pi_{t-1} - \pi^*)$.

Since the traditional monetary tool is not available, the central bank needs to resort to QE. How much QE is used is a function of how far is i_{LB} from i^*.

It follows that the behaviour of inflation expectations is: $\pi_t = -a(i_t - i^*) + bq_t^* + pi_{t-1} + \epsilon_t \rightarrow \pi_t = -a(i_{LB} - i^*) + \pi_{t-1} - \frac{\bar{b}}{\bar{b}^2+\sigma_b^2}(-a(i_{LB} - i^*) + \pi_{t-1} - \pi^*) + \epsilon_t \rightarrow$

$$E_{t-1}\pi_t = -a(i_{LB} - i^*)(1 - \frac{\bar{b}}{\bar{b}^2 + \sigma_b^2}) + \pi_{t-1}(1 - \frac{\bar{b}}{\bar{b}^2 + \sigma_b^2}) + \frac{\bar{b}}{\bar{b}^2 + \sigma_b^2}\pi^* = \frac{\bar{b}}{\bar{b}^2 + \sigma_b^2}(\pi_{t-1} - a(i^{LB} - i^*)).$$

By observing that the lower bound constraint is binding, we deduce that $i^{LB} > i^*$, thus implying that $\pi_{t-1} - a(i^{LB} - i^*) < 0$.

This eventually implies $E_{t-1}\pi_t = \frac{\sigma_b^2}{\sigma_b^2+\bar{b}^2}(\pi_{t-1} - a(i_{LB} - i^*)) + \frac{\bar{b}^2}{\bar{b}^2+\sigma_b^2}\pi^* < \pi^*$, i.e. that the inflation rate today is below target.

3.Adding the output gap to the model

We can rewrite the model as:

$\pi_t = -a(i_t - i^*) + bq_t + c(Y_t - Y^*) + \pi_{t-1} + \epsilon_t$, with $b \overset{N}{\sim} (0, \sigma_b^2), \epsilon_t \overset{N}{\sim} (0, \sigma_\epsilon^2$ and $Y_t \overset{N}{\sim} N(Y^*, \sigma_Y^2)$.

My intitution suggests that i_t and $Y_t - Y^*$ should be positively correlated: a positive (negative) output shock should increase (decrease) demand, which in turn increases (decreases) inflation expectations and therefore induces a tighter (softer) monetary stance, as experienced in the post-pandemic context when central banks were in 2021 considering raising interest rates.

On the other hand, the relationship between q_t and $Y_t - Y^*$ should be negative, because a positive (negative) output gap will raise the inflationary pressure and require a lower amount of q_t to reach the target π^*.

Optimal Monetary Policy in the Svensson (1997) Model

1. Assume that $\pi_{t-1}^e = \pi_t$. The F.O.C. is obtained as follows:

$\min_{y_t}\{E_t[(\pi_{t+1} - \pi^*)^2 + \lambda y_t^2\}$ s.t. $\pi_{t+1} = \beta(\gamma\pi_{t+1}^e + (1-\gamma)\pi_t) + \alpha y_t + \epsilon_{t+1}$.

Recalling the decomposition of the variance of a variable and using the fact that y_t is not random at t, we obtain that

$\min_{y_t}\{E_t[(\pi_{t+1} - \pi^*)^2 + \lambda y_t^2\} = \min_{y_t}\{(E_t[(\pi_{t+1} - \pi^*))^2 + Var(\pi_{t+1}) + \lambda y_t^2\} = \min_{y_t}\{(E_t^2[(\beta\pi_t + \alpha y_t - \pi^*) + Var(\pi_{t-1}) + \lambda y_t^2\}$

$\frac{\partial}{\partial y_t} : 2\alpha E_t[\beta * \pi_t + \alpha y_t - \pi^*] + 2\lambda y_t \overset{!}{=} 0 \rightarrow y_t = \alpha\frac{\pi^* - \beta\pi_t}{\lambda + \alpha^2}$.

We now look for a law of y_t as a function of current inflation and model parameters. Given $y_t = \alpha\frac{\pi^*-\beta\pi_t}{\lambda+\alpha^2}$, we can plug it into the inflation formula and take expectations to yield $E_t\pi_{t+1} = \frac{\beta\lambda\pi_t+\alpha^2\pi^*}{\lambda+\alpha^2}$. Note that $\lambda = 0$ implies that the central bank manages to hit the target in expectations.

2. Assume that $\beta = 1$ and $\pi_{t-1}^e = E_t\pi_{t-1}$. We derive again the F.O.C.:

$\underset{y_t}{min}\{E[(\pi_{t+1} - \pi^*)^2 + \lambda y_t^2]\}$ s.t. $\pi_{t+1} = \beta(\gamma E_t\pi_{t+1} + (1-\gamma)\pi_t) + \alpha y_t + \epsilon_{t+1} \rightarrow \underset{y_t}{min}\{(\beta(\gamma E_t\pi_{t+1} + (1-\gamma)\pi_t) + \alpha y_t - \pi^*)^2 + \lambda y_t^2\}$.

$2\alpha[\beta(\gamma E_t\pi_{t+1} + (1-\gamma)\pi_t) + \alpha y_t - \pi^*] + 2\lambda y_t = 0 \rightarrow y_t = \alpha\frac{\pi^* - (\gamma E_t \pi_{t+1} + (1-\gamma)\pi_t)}{\lambda + \alpha^2}$.

Note that in this case $E_t\pi_{t+1}$ is multiplied by γ.

We now derive $E_t\pi_{t+1}$ in terms of π_t and y_t. We plug the solution for y_t into the Phillips Curve:

$\pi_{t+1} = \gamma E_t\pi_{t+1} + (1-\gamma)\pi_t + \alpha y_t + \epsilon_{t+1} \rightarrow E_t\pi_{t+1} = \frac{(1-\gamma)\lambda}{\alpha^2+\lambda(1-\gamma)}\pi_t + \frac{\alpha^2}{\alpha^2+\lambda(1-\gamma)}$.

When plugging this into the y_t focs, we obtain that $y_t = \alpha(1-\gamma)\frac{\pi^* - \pi_t}{\alpha^2+\lambda(1-\gamma)}$.

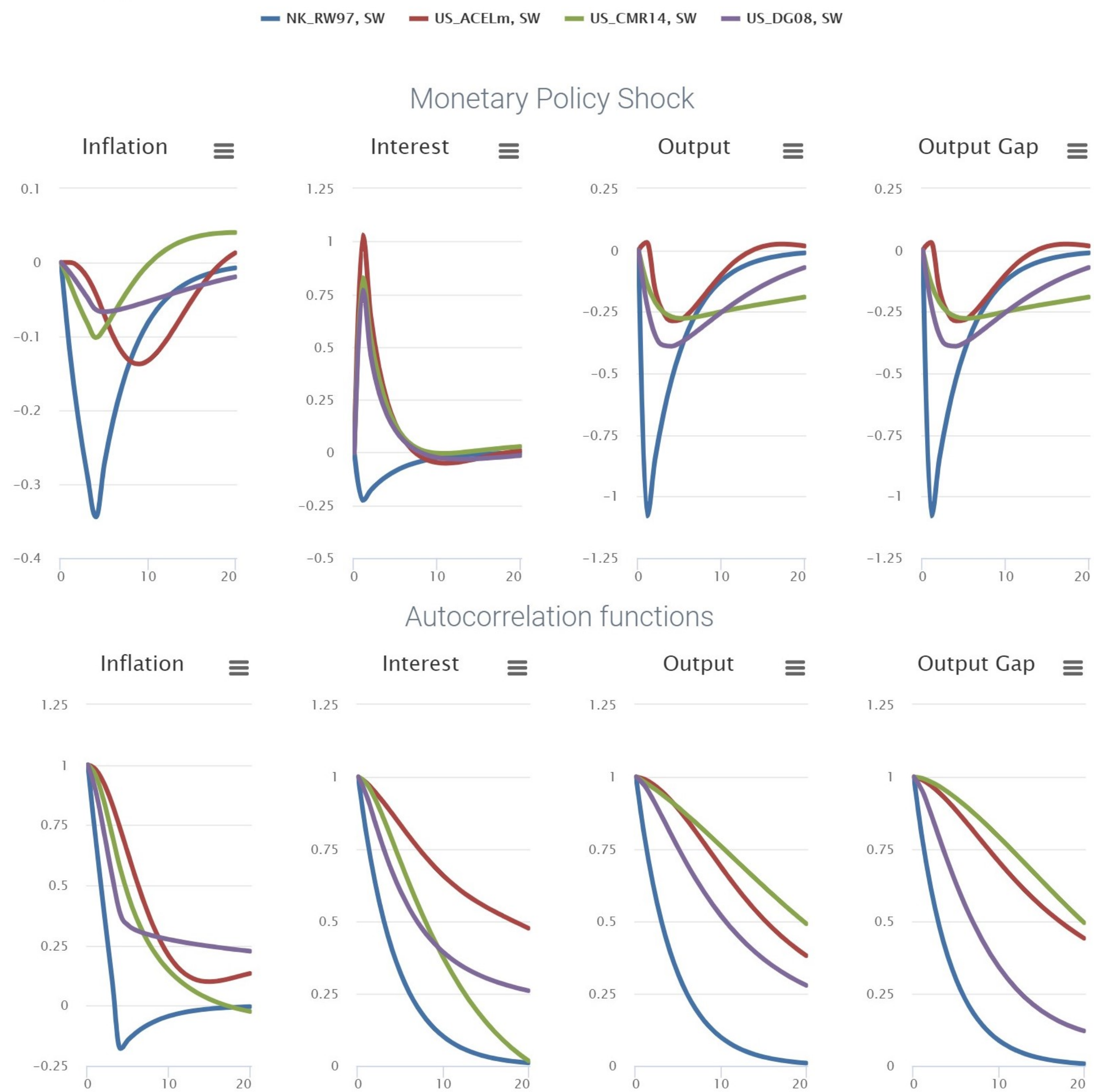
We then turn to derive the reduced form PC:

$\pi_{t+1} = (\gamma E_t\pi_{t+1} + (1-\gamma)\pi_t) + \alpha y_t + \epsilon_{t+1} \rightarrow E_t\pi_{t+1} = \frac{(1-\gamma)\lambda}{\alpha^2+\lambda(1-\gamma)}\pi_t + \frac{\alpha^2}{\alpha^2+\lambda(1-\gamma)}\pi^*$ and since $\pi^* = 0$, $\pi_{t+1} = (1-\gamma)\frac{\lambda+\alpha^2}{\lambda(1-\gamma)+\alpha^2}$.

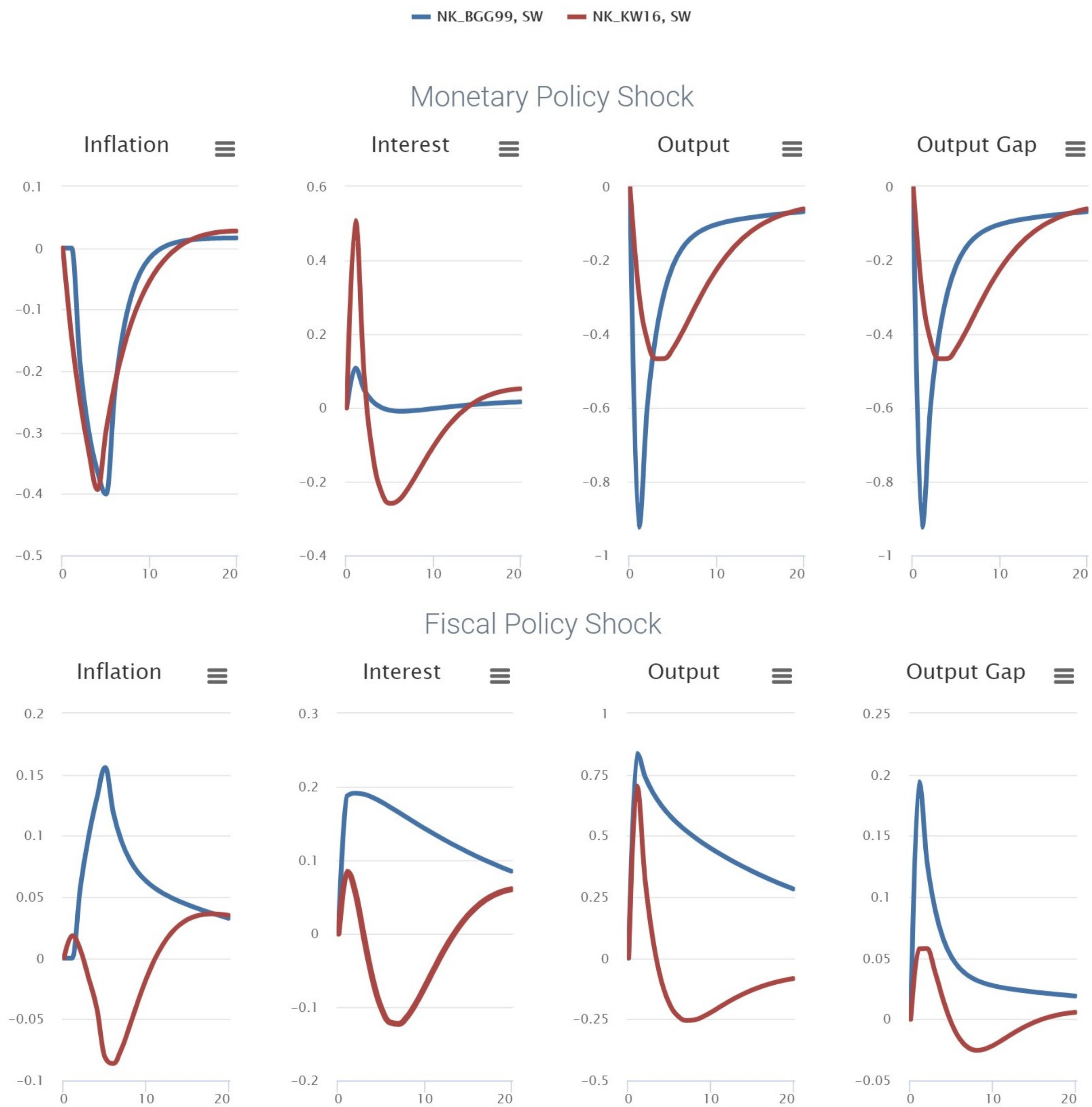
Let's suppose now that $\lambda = 0$. What would be the behaviour of the PC?

The PC becomes $\pi_{t+1} = (1-\gamma)\pi_t + \alpha y_t + \epsilon_{t+1}$. Note that now $c_1(\lambda = 0) < c_1(\lambda > 0)$, which implies that λ is a sensitivity parameter for future inflation with respect to the current level of inflation.

MMB Model Comparison Exercise



Using a SW07 rule, we simulated the IRFs for a Monetary Policy Shock. The 2nd Generation models I chose are Rotemberg & Woodford (1997) and Altig et al. (2005), as suggested by the class material. The 3rd Generation models I chose are Christiano et al. (2014) and De Graeve (2008). In light of the findings, we can say that 3rd generation models are superior, especially when compared to RW97. This fact can be explained in terms of Model Uncertainty, i.e. 3rd Generation models present richer dynamics and present more persistent shocks as can be observed from inspection of the autocorrelation functions.



Different implementation of financial frictions does yield different IRFs, especially in the presence of a Fiscal Shock, thus suggesting different monetary policy stances. This difference is particularly strong regarding the interest rate and inflation.