FET Bayesian Methods - HW3 - 7751512

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Question 1

```
## Starting Gibbs Sampler for Binary Probit Model
     with 100 observations
## Table of y Values
## y
## 0 1
## 51 49
##
## Prior Parms:
## betabar
## [1] 0 0 0
## A
        [,1] [,2] [,3]
## [1,] 0.01 0.00 0.00
## [2,] 0.00 0.01 0.00
## [3,] 0.00 0.00 0.01
##
## MCMC parms:
## R= 50000 keep= 1 nprint= 100
##
##
  MCMC Iteration (est time to end - min)
##
   100 (0.0)
##
    200 (0.0)
    300 (0.0)
##
##
   400 (0.0)
##
   500 (0.0)
##
   600 (0.0)
##
   700 (0.0)
##
   800 (0.0)
   900 (0.0)
   1000 (0.0)
##
##
   1100 (0.0)
##
   1200 (0.0)
   1300 (0.0)
##
   1400 (0.0)
   1500 (0.0)
##
  1600 (0.0)
  1700 (0.0)
## 1800 (0.0)
## 1900 (0.0)
## 2000 (0.0)
```

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2100 (0.0)
##
##
    2200 (0.0)
    2300 (0.0)
##
    2400 (0.0)
##
##
    2500 (0.0)
##
    2600 (0.0)
##
    2700 (0.0)
##
    2800 (0.0)
##
    2900 (0.0)
##
    3000 (0.0)
##
    3100 (0.0)
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    3200 (0.0)
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    3300 (0.0)
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    3400 (0.0)
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    3500 (0.0)
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    3600 (0.0)
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    3700 (0.0)
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    3800 (0.0)
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    3900 (0.0)
##
    4000 (0.0)
##
    4100 (0.0)
##
    4200 (0.0)
##
    4300 (0.0)
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    4400 (0.0)
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    4500 (0.0)
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    4600 (0.0)
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    4700 (0.0)
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    4800 (0.0)
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    4900 (0.0)
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    5000 (0.0)
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    5100 (0.0)
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    5200 (0.0)
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    5300 (0.0)
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    5400 (0.0)
    5500 (0.0)
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    5600 (0.0)
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    5700 (0.0)
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    5800 (0.0)
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    5900 (0.0)
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    6000 (0.0)
##
    6100 (0.0)
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    6200 (0.0)
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    6300 (0.0)
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    6400 (0.0)
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    6500 (0.0)
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    6600 (0.0)
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    6700 (0.0)
##
    6800 (0.0)
    6900 (0.0)
##
##
    7000 (0.0)
```

7100 (0.0)

7200 (0.0) 7300 (0.0)

7400 (0.0)

##

##

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7500 (0.0)
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##
    7600 (0.0)
    7700 (0.0)
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    7800 (0.0)
##
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    7900 (0.0)
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    8000 (0.0)
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    8100 (0.0)
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    8200 (0.0)
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    8300 (0.0)
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    8400 (0.0)
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    8500 (0.0)
    8600 (0.0)
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    8700 (0.0)
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    8800 (0.0)
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    8900 (0.0)
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    9100 (0.0)
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    9300 (0.0)
    9400 (0.0)
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    9500 (0.0)
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    9600 (0.0)
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    9700 (0.0)
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    9800 (0.0)
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    10000 (0.0)
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    10100 (0.0)
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    10200 (0.0)
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    10300 (0.0)
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    10400 (0.0)
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    10500 (0.0)
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    10600 (0.0)
##
    10700 (0.0)
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    10800 (0.0)
    10900 (0.0)
##
    11000 (0.0)
##
##
    11100 (0.0)
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    11200 (0.0)
##
    11300 (0.0)
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    11400 (0.0)
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    11500 (0.0)
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    11600 (0.0)
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    11700 (0.0)
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    11800 (0.0)
##
    11900 (0.0)
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    12000 (0.0)
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    12100 (0.0)
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    12200 (0.0)
    12300 (0.0)
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    12400 (0.0)
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12500 (0.0)

12600 (0.0) 12700 (0.0)

12800 (0.0)

##

##

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12900 (0.0)
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    13000 (0.0)
    13100 (0.0)
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    13500 (0.0)
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    13600 (0.0)
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    13700 (0.0)
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    13800 (0.0)
    13900 (0.0)
    14000 (0.0)
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    14400 (0.0)
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    14700 (0.0)
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    14900 (0.0)
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    15000 (0.0)
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    16400 (0.0)
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    16500 (0.0)
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    16600 (0.0)
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    16700 (0.0)
##
    16800 (0.0)
    16900 (0.0)
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    17000 (0.0)
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    17100 (0.0)
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    17200 (0.0)
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    17300 (0.0)
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    17500 (0.0)
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    17600 (0.0)
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- ## 28800 (0.0) ## 28900 (0.0)
- ## 29000 (0.0)

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31800 (0.0) ## 31900 (0.0)

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34100 (0.0)

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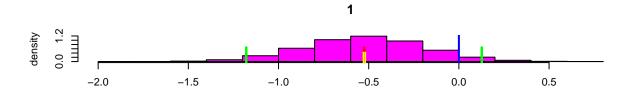
34400 (0.0)

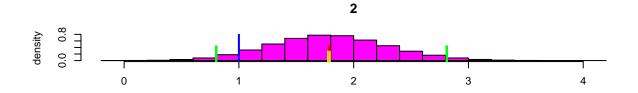
- 34500 (0.0) ##
- ## 34600 (0.0)
- 34700 (0.0) ##
- 34800 (0.0) ##
- ## 34900 (0.0)
- ## 35000 (0.0)
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- ## 35400 (0.0)
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- ## 35600 (0.0)
- ## 35700 (0.0)
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- ## 35900 (0.0) ##
- 36000 (0.0) ## 36100 (0.0)
- ## 36200 (0.0)
- ##
- 36300 (0.0)
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- ## 36500 (0.0)
- ## 36600 (0.0)
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- ## 37000 (0.0)
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- ## 38800 (0.0)
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- ## 39200 (0.0)
- 39300 (0.0) ##
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- ## 39500 (0.0)
- ## 39600 (0.0) 39700 (0.0) ##
- ## 39800 (0.0)

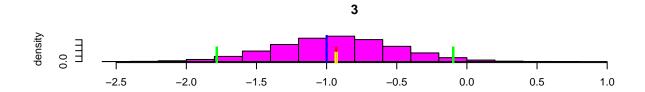
- ## 39900 (0.0)
- ## 40000 (0.0)
- ## 40100 (0.0)
- ## 40200 (0.0)
- ## 40300 (0.0)
- ## 40400 (0.0)
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- ## 41000 (0.0)
- ## 41100 (0.0)
- ## 41200 (0.0)
- ## 41300 (0.0)
- ## 41400 (0.0)
- ## 41500 (0.0)
- ## 41600 (0.0)
- ## 41700 (0.0)
- ## 41800 (0.0)
- ## 41900 (0.0)
- ## 42000 (0.0)
- ## 42100 (0.0)
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- ## 42900 (0.0)
- ## 43000 (0.0)
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- ## 44000 (0.0)
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- ## 44300 (0.0) ## 44400 (0.0)
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- ## 44700 (0.0)
- ## 44800 (0.0)
- ## 44900 (0.0)
- ## 45000 (0.0)
- ## 45100 (0.0) ## 45200 (0.0)

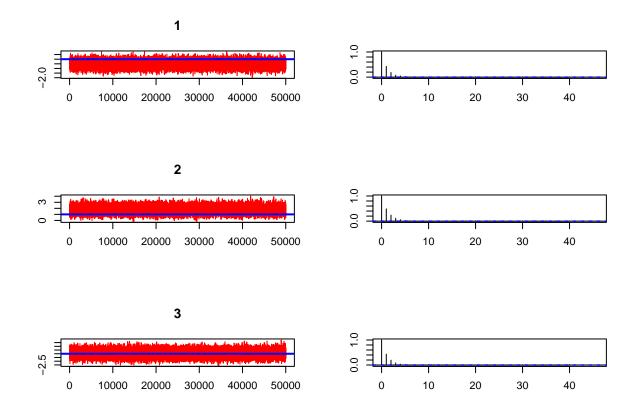
```
## 45300 (0.0)
##
    45400 (0.0)
    45500 (0.0)
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##
    45600 (0.0)
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    45700 (0.0)
##
    45800 (0.0)
    45900 (0.0)
    46000 (0.0)
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    46100 (0.0)
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    46200 (0.0)
    46300 (0.0)
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    46400 (0.0)
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    46500 (0.0)
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    46600 (0.0)
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    46700 (0.0)
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    46800 (0.0)
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    46900 (0.0)
   47000 (0.0)
##
##
   47100 (0.0)
   47200 (0.0)
##
##
   47300 (0.0)
##
   47400 (0.0)
##
   47500 (0.0)
##
    47600 (0.0)
##
   47700 (0.0)
   47800 (0.0)
##
   47900 (0.0)
##
    48000 (0.0)
##
   48100 (0.0)
##
    48200 (0.0)
    48300 (0.0)
##
##
    48400 (0.0)
##
    48500 (0.0)
##
   48600 (0.0)
    48700 (0.0)
##
    48800 (0.0)
##
##
   48900 (0.0)
##
    49000 (0.0)
    49100 (0.0)
##
##
    49200 (0.0)
##
    49300 (0.0)
   49400 (0.0)
##
##
    49500 (0.0)
##
   49600 (0.0)
   49700 (0.0)
## 49800 (0.0)
## 49900 (0.0)
## 50000 (0.0)
    Total Time Elapsed: 0.03
## Summary of Posterior Marginal Distributions
## Moments
     tvalues mean std dev num se rel eff sam size
           0 -0.52
                      0.33 0.0026
## 1
                                       2.7
                                               15000
```

```
## 2    1 1.79    0.51 0.0043    3.2    11250
## 3    -1 -0.93    0.43 0.0033    2.6    15000
##
## Quantiles
## tvalues 2.5%    5%    50%    95%    97.5%
## 1    0 -1.2 -1.07 -0.52 0.018 0.126
## 2    1 0.8 0.96 1.78 2.644 2.811
## 3    -1 -1.8 -1.64 -0.93 -0.228 -0.097
## based on 45000 valid draws (burn-in=5000)
```









Given the model structure, I expect that we cannot tell apart the effects of θ_2 and θ_3 on their joint distribution; hence, we cannot estimate the parameters θ_2, θ_3 consistently. The chunk above produces a long pdf because iterations get printed; I don't know how to fix that.

Question 2

The model is a Multinomial, where we use the following distributional objects to obtain y_i:

$$p(X_i|\beta, \sigma_{12}, z_i), p(\beta|X_i, \sigma_{12}, z_i), p(\sigma_{12}|X_i, \beta, z_i), p(z_j|X_i, \beta, \sigma_{12}, y_i, z_{-j}), p(y_i|z_i)$$
(1)