Discrete Fourier Transform

Digital Signal Processing with a focus on audio signals

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Review: road map to Fourier Kingdom

	APERIODIC in time *CONTINUOUS in frequency*	*PERIODIC in time* *DISCRETE in frequency*
CONTINUOUS in time *APERIODIC in frequency*	Fourier Transform $$X(j\Omega) = \int_{-\infty}^{\infty} x(t) e^{-j\Omega t} dt$$ $$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\Omega) e^{j\Omega t} d\Omega$$	Fourier Series $$a_k = \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} x(t) e^{-j\frac{2\pi k t}{T}} dt$$ $$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{j\frac{2\pi k t}{T}}$$
DISCRETE in time *PERIODIC in frequency*	Discrete-Time Fourier Transform (DTFT) $$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n}$$ $$x(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{j\omega}) e^{j\omega n} d\omega$$	?

Motivations behind the DFT

- In the previous lesson we introduced the DTFT, which operates on discrete signals of indefinite length
- In practice, we usually want to obtain the Fourier transform using digital computation, thus needing:
  - finite signal length
  - finite set of frequencies
- The DFT provides a mean for achieving this!

From the DTFT to the DFT

- We start from the DTFT definition

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n} \, ,$$

  which is a 2π periodic continuous function of ω

- We sample it at N uniformly spaced samples between 0 and 2π (sample frequency equal to an integer multiple of its period), i.e.

$$\omega_k = \frac{2\pi}{N} k \, , \quad k \in \mathbb{Z}$$

- The sampling process leads to

$$X'(e^{j\omega}) = X(e^{j\omega}) \sum_{k=-\infty}^{\infty} \delta\left(\omega - \frac{2\pi}{N} k\right)$$

From the DTFT to the DFT (cont')

- We apply the convolution theorem for the DTFT, obtaining

$$x'(n) = \mathrm{DTFT}^{-1}\left\{X'(e^{j\omega})\right\} = x(n) * \mathrm{DTFT}^{-1}\left\{\sum_{k=-\infty}^{\infty} \delta\left(\omega - \frac{2\pi}{N}k\right)\right\}$$

- It can be proved that

$$\mathrm{DTFT}^{-1}\left\{\sum_{k=-\infty}^{\infty} \delta\left(\omega - \frac{2\pi}{N}k\right)\right\} = \frac{N}{2\pi}\sum_{p=-\infty}^{\infty} \delta(n - Np)$$

- Finally we have:

$$x'(n) = x(n) * \frac{N}{2\pi}\sum_{p=-\infty}^{\infty} \delta(n - Np) = \frac{N}{2\pi}\sum_{p=-\infty}^{\infty} x(n - Np)$$

From the DTFT to the DFT (cont')

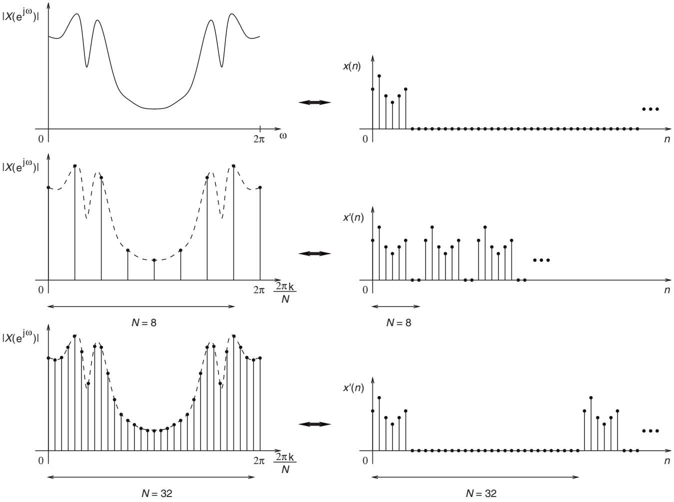
- The last equation indicates that, from a sampled DTFT, we can recover a signal $x'(n)$ consisting of a sum of periodic replicas of the original discrete signal $x(n)$
- This means that, in order to exactly recover $x(n)$ (by isolating a single repetition), its length L must be not greater than N
  - in other words, we require the signal $x(n)$ to be *time-limited*
  - this is the equivalent of the sampling theorem, applied to sampling in the frequency domain
- Therefore, if $L \leq N$, we can recover the signal as

$$x(n) = \frac{2\pi}{N} x'(n) , \quad \text{for } \ 0 \leq n \leq N-1$$

- If $L > N$ the replicas overlap, thus introducing *time-domain aliasing*

From the DTFT to the DFT (cont')

Example: sampling the DTFT with a different number of samples N and its effect in the time-domain:



From the DTFT to the DFT (cont')

From the previous picture, we observe that:

- sampling the DTFT corresponds to generating replicas of the original signal in the time domain
- increasing the number of samples N corresponds to increase the distance between replicas in the time domain
- clearly, in order to avoid that the replicas overlap, we must satisfy $N \geq L$ (sampling theorem)

The Discrete Fourier Transform (DFT) and its inverse (IDFT)

- Considering a signal $x(n)$ of length N, its DFT is defined as

$$X(k) \triangleq \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi kn}{N}} , \quad \text{for } 0 \leq k \leq N-1$$

- The IDTFT is computed as

$$x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k) e^{j\frac{2\pi kn}{N}} , \quad \text{for } 0 \leq n \leq N-1$$

DFT periodicity

The DFT can be seen as a periodic transform in both frequency and time domain

- **Frequency domain**:

$$X(k) = \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi kn}{N}} = X(k+mN) \,, \quad \text{for } m \in \mathbb{Z} \,,$$

  corresponding to the fact that we are sampling the DTFT, which is itself 2π-periodic

- **Time domain**:

$$x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k) e^{j\frac{2\pi kn}{N}} = x(n+mN) \,, \quad \text{for } m \in \mathbb{Z} \,,$$

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DFT periodicity: modulo indexing

- A convenient way to account for periodicity is to consider the periodic extensions using modulo indexing

- We will use the following notation:
  - $X(k)_N \triangleq X(k \mod N)$ for the frequency domain
  - $x(n)_N \triangleq x(n \mod N)$ for the time domain

- Remember that all the operations involving the DFT/IDFT are implicitly circular (modulo N)!

Properties of the DFT

- Most of the properties of the DFT are analogous to those of the DTFT

- However, we must pay attention to the fact that, **for all the operations involving shifts, modulo N indexing must be considered**

Properties of the DFT (cont')

- **Linearity**

$$k_1 x_1(n) + k_2 x_2(n) \quad \longleftrightarrow \quad k_1 X_1(k) + k_2 X_2(k)$$

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- **Frequency-shift theorem**

$$e^{j\omega_k l} x(n) \quad \longleftrightarrow \quad X(k+l)_N$$

Properties of the DFT (cont')

- **Circular convolution in time**

$$\sum_{l=0}^{N-1} x(l)h(n-l)_N = \sum_{l=0}^{N-1} x(n-l)_N h(l) \quad \longleftrightarrow \quad X(k)H(k)$$

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- **Parseval's theorem**

$$\sum_{n=0}^{N-1} |x(n)|^2 = \frac{1}{N}\sum_{n=0}^{N-1} |X(k)|^2$$

Properties of the DFT (cont')

- When $x(n)$ is a **real sequence**:

$$\mathrm{Re}\{X(k)\} = \mathrm{Re}\{X(-k)_N\}$$
$$\mathrm{Im}\{X(k)\} = -\mathrm{Im}\{X(-k)_N\}$$

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- When $x(n)$ is an **imaginary sequence**:

$$\operatorname{Re}\{X(k)\} = -\operatorname{Re}\{X(-k)_N\}$$
$$\operatorname{Im}\{X(k)\} = \operatorname{Im}\{X(-k)_N\}$$

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Symmetry properties (cont')

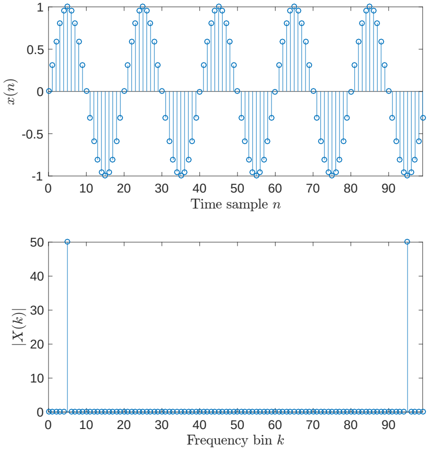
- Using the above definitions, the following symmetry properties hold:

$x(n)$		$X(k)$
real and even	$\longleftrightarrow$	real and even
imaginary and even	$\longleftrightarrow$	imaginary and even
real and odd	$\longleftrightarrow$	imaginary and odd
imaginary and odd	$\longleftrightarrow$	real and odd
conjugate symmetric	$\longleftrightarrow$	real

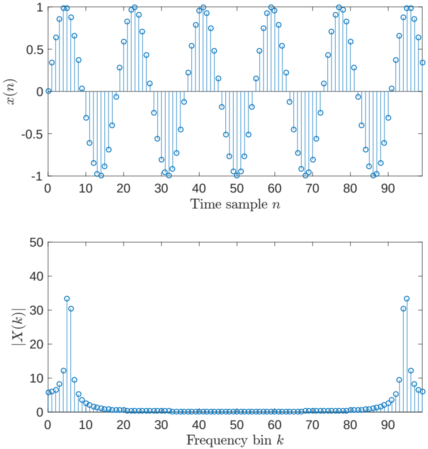
DFT: spectral leakage

Example: DFT of sinusoid segment of length $N = 100$, $x(n) = \sin(2\pi a n/N)$

Integer a ($a = 5$)



Non integer a ($a = 5.5$)



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- Only the DFT sinusoids are not cut off at the window boundaries
- To understand spectral leakage, remember that taking the DFT means computing samples of the DTFT of infinite periodic extension of the considered N-samples time window:
  - discontinuities at the end of repeated blocks
  - these "glitches" every N samples can be considered as a source of new energy over the entire spectrum

Reducing the spectral leakage: windowing

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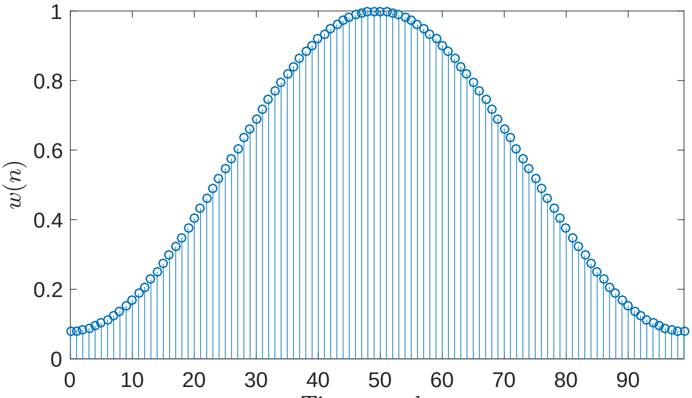
- Spectral leakage can be reduced using a non-rectangular window function
- A typical window is the "raised cosine" one (aka *Hamming window*), which tapers data gracefully to zero at both endpoints:

$$w_H(n) = 0.54 + 0.46 \cos\left[\frac{2\pi(n - N/2)}{N}\right] , \quad n = 0, 1, \ldots N - 1$$



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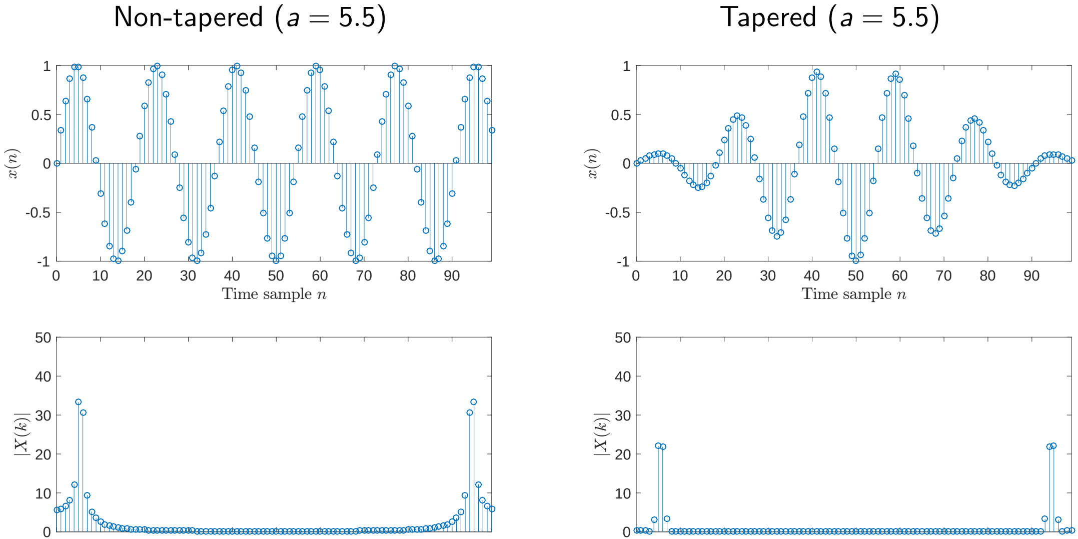
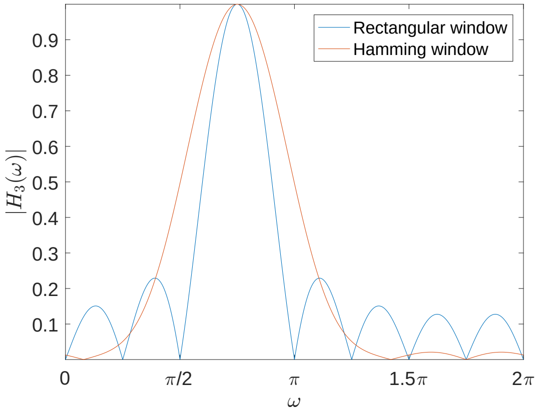


Reducing the spectral leakage: windowing (cont')

- Using a tapered window, the mainlobe widens and the sidelobes decrease in the DFT response
- Using no windows is the same of using a rectangular window
- Example for $N = 8$, $k = 3$:



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- In other words, the **spectral resolution depends on the width of the main lobe** of the sinc function
  - what happens if more than one sinusoidal component are present in the signal?

DFT spectral resolution (cont')

- Consider a signal containing two sinusoidal components at frequencies ω_a and ω_b, truncated with a N-length rectangular window:

$$x(n) = \left[e^{j\omega_a n} + e^{j\omega_b n}\right] \cdot w_R(n)$$

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- The DFT coincides with sampling the DTFT of $x(n)$, which now consists in two impulses convolved with the sinc function:
$$\begin{aligned} X(k) &\triangleq X(e^{j\omega})\Big|_{\omega=\omega_k} \\ &= \left([\delta(\omega - \omega_a) + \delta(\omega - \omega_b)] * \frac{1 - e^{-j\omega N}}{1 - e^{-j\omega}} \right)\Bigg|_{\omega=\omega_k} \end{aligned}$$

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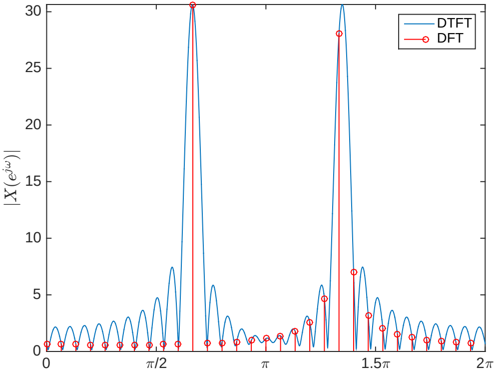
- The two impulses at frequencies ω_a and ω_b turn to be blurred due to the convolution:
  - it may happen that ω_a and ω_b are too close and the frequency resolution provided by the DFT is not enough
  - in this case we are not able to resolve the two frequency components! (see example on the next slide)

DFT spectral resolution (cont')

Example: two complex sinusoids with freq. distance $\Delta_\omega = |\omega_a - \omega_b|$, $N = 30$

Case 1: $\Delta_\omega = 10.2 \times 2\pi/N$

- $\omega_a = 10 \times 2\pi/N$
- $\omega_b = 20.2 \times 2\pi/N$



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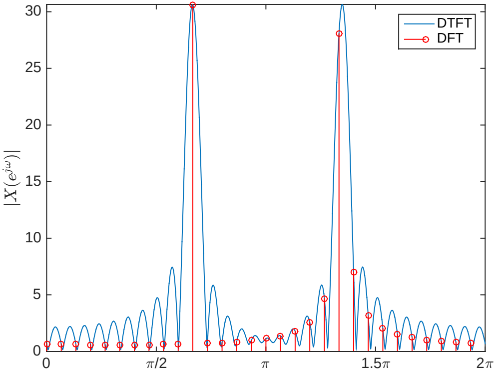
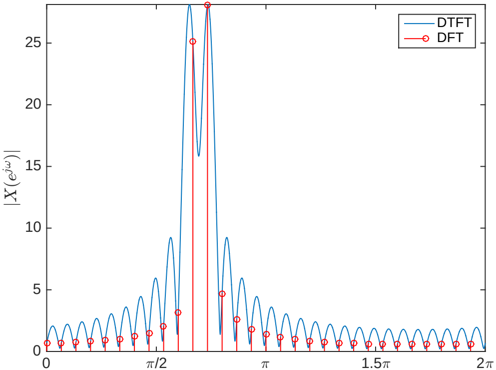


Case 2: $\Delta_\omega = 0.8 \times 2\pi/N$

- $\omega_a = 10 \times 2\pi/N$
- $\omega_b = 10.8 \times 2\pi/N$



DFT spectral resolution (cont')

- Clearly, the spectral resolution is governed by the choice of the segment length N, which determines the distance between two adjacent frequency bins:

$$\omega_k - \omega_{k-1} = \frac{2\pi}{N}\ , \quad k = 1, \ldots N-1$$

- If the two ideal impulses at frequencies ω_a and ω_b fall inside the mainlobe of the DFT response, they will be "absorbed" in a single, wider "blurred" peak

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- If the two ideal impulses at frequencies ω_a and ω_b fall inside the mainlobe of the DFT response, they will be "absorbed" in a single, wider "blurred" peak
- In order to guarantee an effective separation of the two frequency components ω_a and ω_b, we must ensure that:

$$\Delta\omega = |\omega_a - \omega_b| > 2 \times \frac{2\pi}{N}$$

DFT spectral resolution: the general case

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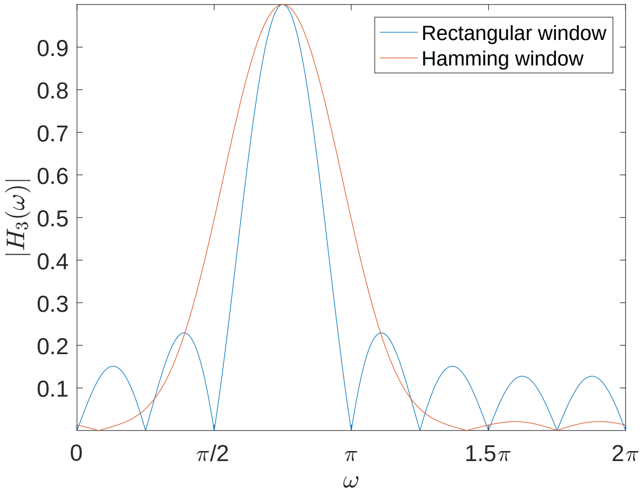
- In general, we can say that the "blurring" of the impulses is determined by the shape of the DTFT of the chosen window
- To be more precise, the **spectral resolution depends on the width B_ω of the mainlobe of the observation window**:
  - $B_\omega = \frac{4\pi}{N}$ for rectangular window (i.e., no windowing)
  - $B_\omega = \frac{8\pi}{N}$ for Hamming's window
  - ... (there exists a number of different windows!)

DFT spectral resolution: the general case (cont')

Recall the previous example (DFT response for $N=8$, $k=3$):



DFT spectral resolution: the general case (cont')

- The general formula for guaranteeing spectral separation is

$$\Delta_\omega > B_\omega = Q \times \frac{2\pi}{N} \ ,$$

  where Q is a constant depending on the type of window (e.g., $Q = 2$ for rectangular window, $Q = 4$ for Hamming window)

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To conclude:

The **spectral resolution** of the DFT is **governed by the width of the mainlobe** of the used window, and **increases by increasing the window length** N.

DFT interpolation via zero-padding

- Let us go back for a while to the DFT definition:

$$X(k) = \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi kn}{N}} \ , \quad 0 \le k \le N-1$$

- Suppose that we want to compute an N-points DFT but the signal length is $L < N$

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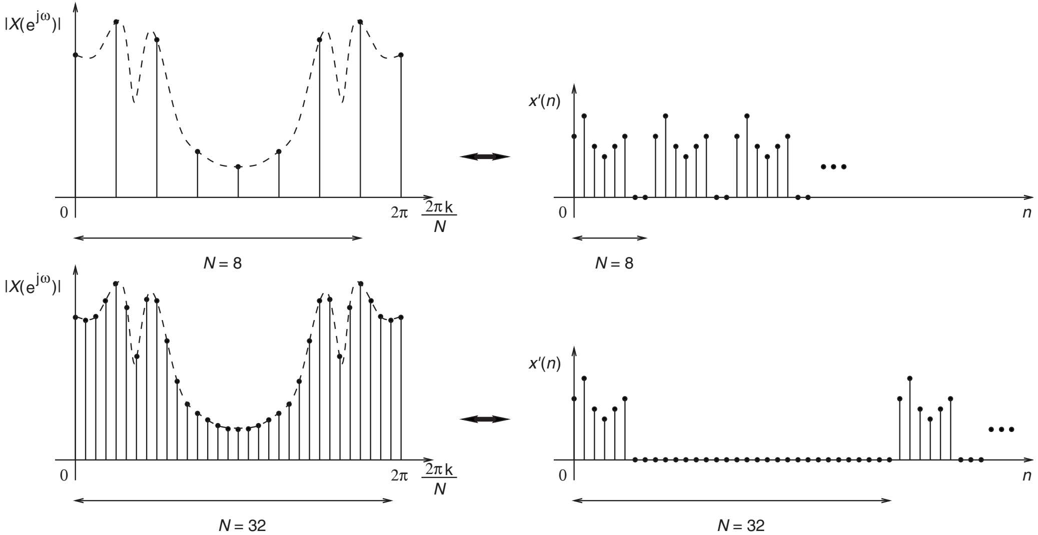
- Note that:
  - adding an infinite number of zeros than $N \to \infty$: this corresponds to computing the DTFT
  - roughly speaking, by increasing N (i.e., by appending more zeros), we provide a better approximation of the DTFT

DFT interpolation via zero-padding (cont')

Example:



DFT interpolation via zero-padding (cont')

In the picture, the signal has a length $L = 6$. The DFT is computed with $N = 8$ and with $N = 32$, and we observe that:

- the larger the number of zeros padded on $x(n)$ for the calculation of the DFT, the more it resembles its DTFT. This happens because of the larger number of samples taken within $[0, 2\pi]$
- the larger the amount of zero-padding the greater the computational and storage requirements involved in the DFT computation

DFT interpolation via zero-padding (cont')

In the picture, the signal has a length $L = 6$. The DFT is computed with $N = 8$ and with $N = 32$, and we observe that:

- the larger the number of zeros padded on $x(n)$ for the calculation of the DFT, the more it resembles its DTFT. This happens because of the larger number of samples taken within $[0, 2\pi]$
- the larger the amount of zero-padding the greater the computational and storage requirements involved in the DFT computation

To conclude:

Zero-padding acts as an interpolator in the frequency domain

Zero-padding DOES NOT INCREASE spectral resolution!

- Zero-padding is effective to interpolate between frequency bins of the DFT

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- Zero-padding is effective to interpolate between frequency bins of the DFT
- **What is the effect of zero-padding on spectral resolution**?
  - Answer: **NOTHING**!
  - Indeed, adding zeros we are not really increasing the segment length!
  - i.e., no further information is added to the analyzed segment!

Summary on interpolation, windowing and spectral resolution

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  - resolution can be always increased by increasing the window length N

Summary on interpolation, windowing and spectral resolution

- Windowing and interpolation have a different impact on the DFT, it is important to not confuse them!
- **Windowing**:
  - a tapered window reduces the spectral leakage (peaks turn to be more defined)
  - however, a tapered window also reduces the spectral resolution, due to the wider mainlobe
  - resolution can be always increased by increasing the window length N
- **Interpolation**:
  - zero-padding corresponds to interpolating in the DFT domain
  - it **increases** the available **frequency bins**, but the **spectral resolution remains unaltered**: it does not increase the capability of resolving close sinusoidal peaks

Review: road map to Fourier Kingdom

	***APERIODIC** in time* ***CONTINUOUS** in frequency*	***PERIODIC** in time* ***DISCRETE** in frequency*
***CONTINUOUS** in time* ***APERIODIC** in frequency*	Fourier Transform $X(j\Omega) = \int_{-\infty}^{\infty} x(t)e^{-j\Omega t}dt$ $x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\Omega)e^{j\Omega t}d\Omega$	Fourier Series $a_k = \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} x(t)e^{-j\frac{2\pi kt}{T}}dt$ $x(t) = \sum_{k=-\infty}^{\infty} a_k e^{j\frac{2\pi kt}{T}}$
***DISCRETE** in time* ***PERIODIC** in frequency*	Discrete-Time Fourier Transform (DTFT) $X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x(n)e^{-j\omega n}$ $x(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})e^{j\omega n}d\omega$	Discrete Fourier Transform (DFT) $X(k) = \sum_{n=0}^{N-1} x(n)e^{-j\frac{2\pi kn}{N}}$ $x(n) = \frac{1}{N}\sum_{k=0}^{N-1} X(k)e^{j\frac{2\pi kn}{N}}$