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Interference of probability amplitudes: a simple demonstration within the Hong–Ou–Mandel experiment

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Abstract
The dip in the Hong–Ou–Mandel (HOM) experiment is frequently attributed to the interference of two indistinguishable photons. The width of the dip is inversely proportional to the spectral bandwidth of the light. We confirm that this width is unchanged regardless of where the bandwidth selection happens, i.e. prior to the photons combining in the beam splitter or just before the detectors; the latter case seemingly after the interference of the photons. The equivalence of the results from these two cases is a simple demonstration that the interference in the HOM experiment is not between photons, but rather between probability amplitudes.

Keywords: Hong–Ou–Mandel experiment, quantum interference, spectral filtering
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(Some figures may appear in colour only in the online journal)

1. Introduction

The Hong–Ou–Mandel (HOM) experiment is arguably the simplest and most elegant interference experiment in quantum physics. In the first HOM experiment [1], each of the degenerate signal and idler photons (s and i in figure 1) produced in spontaneous parametric down-conversion was separately coupled to an input of a beam splitter. Detectors at each of the outputs of the beam splitter then recorded the coincidence rate as a function of the relative path delay (Δz in figure 1) between the otherwise indistinguishable signal and idler photons. The coincidence rate obtained shows a minimum (referred to as the 'dip') when the optical paths of the signal and idler- from the crystal to the beam splitter- are matched to within the coherence length, L_c, of the light. The HOM dip has been repeatedly observed in many other similar experiments, particularly with photons from independent sources [2, 3], or even with nondegenerate photons [4, 5]. This latter experiment runs counter to the common notion that indistinguishability between the two photons entering the beam splitter is a requirement in order to observe the HOM dip.

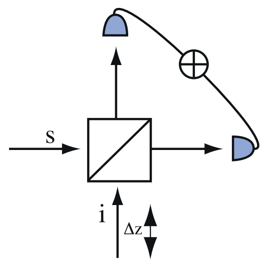

Figure 1. Basic Hong–Ou–Mandel experiment. The signal (s) and idler (i) photons from spontaneous parametric down-conversion combine in a beam splitter. When the optical path difference (Δz) between the photons is zero, they are detected in the same detector, leading to a dip in the coincidence counts between the detectors.

Because the dip disappears as the relative path delay is increased (i.e. when the path length difference makes it possible to distinguish between the signal and idler photons), it is natural to think of the dip as arising from the interference between two individual photons entering the beam splitter. However, as previous works have already pointed out

[1, 6, 7], the interference in the HOM experiment is not the interference between two single photons, but rather the interference between probability amplitudes that describe the various alternatives that can lead to a coincidence event. In the simple arrangement of figure 1, coincidences are registered when the photons impinging on the beam splitter are either both reflected or both transmitted. It is not the indistinguishability between the individual photons that is necessary, but rather, the indistinguishability between alternatives that lead to the detectors clicking after the photons have passed through the interferometer.

The interference of probability amplitudes has been often demonstrated within the Hong–Ou–Mandel experiment. In an experiment by Pittman *et al* [7], they have demonstrated that any path difference incurred between the photons before they come into the beam splitter can actually be compensated after the beam splitter. The HOM dip in their experiment then cannot be interpreted as the interference of the two individual photons, because for the two photons entering the beam splitter, $\Delta z \gg L_c$. The distinguishing information is not limited to the path length. In the 'quantum eraser' experiment by Kwiat *et al* [6], one of the photons was labelled by rotating its polarization (making it orthogonal to the other photon's polarization). Polarizers were then placed right before the detectors, and they show that the visibility of the resulting dip depends on the relative orientation of the polarizers. The polarizers before the detectors can seemingly 'erase' the labelling after the photons have exited the beam splitter. Again, this experiment can only be interpreted as an interference of indistinguishable probability amplitudes rather than indistinguishable photons. The HOM-type interference has also been predicted with photons that have different colours, or spectra [8, 9], and has been recently shown experimentally [5].

Our experiment is simpler than these previous experiments, in that we do not try to label any of the photons combining in the beam splitter. We simply measure the width of the HOM dip for two different narrow-bandwidth spectral filters. In both cases, the filter was placed before the beam splitter (before the individual photons combine) or after the beam splitter (after the photons have combined, just before they reach the detectors). For a given filter, we find that the widths of the HOM dips are the same regardless of where the filter was placed. This is as expected if the dip is indeed a result of the interference of probability amplitudes, but not if the dip is the result of the interference of the individual photons. Despite the simplicity of our demonstration, to the best of our knowledge, no similar result has previously been reported.

2. Hong–Ou–Mandel interference

The dip in the HOM experiment can be elegantly described as a result of the destructive interference of the two indistinguishable events that lead to a coincidence, i.e. when the individual photons are both reflected or both transmitted. If we let r and t be coefficients of reflection and transmission respectively, the probability of a coincidence count for a beam splitter with $r = t = 1/\sqrt{2}$ is given by

$$P_c = |r \times r + t \times t|^2 = \left| \frac{i}{2} \times \frac{i}{2} + \frac{1}{2} \times \frac{1}{2} \right|^2 = 0. \qquad (1)$$

Indistinguishability between the alternatives leading to a coincidence is the necessary condition for complete destructive interference. In many cases, but definitely not always, this means that the photons entering the beam splitter should be indistinguishable themselves. The photons in many HOM-type experiments come from spontaneous parametric down-conversion (SPDC). Here, the absorption of a pump photon (with frequency ω_p) by a nonlinear crystal causes the emission of signal and idler photons (with frequencies ω_s and ω_i, respectively) with a small probability. Being a parametric process, the total energy of the photons is conserved such that $\omega_p = \omega_s + \omega_i$. The phase-matching conditions due to the finite size of the nonlinear crystal, introduce a spread in the frequencies of the signal and idler photons. Hence, to ensure indistinguishability, most experiments of this type use nonlinear crystals cut for degenerate SPDC ($\omega_s = \omega_i$), and identical narrow-band interference filters centred on the degenerate frequency are used to ensure that the signal and idler photons have the same spectra. If we assume that the interference filters have a Gaussian profile, of standard deviation σ (in frequency), the two-photon coincidence count, N_c can be approximated as,

$$N_c \approx 1 - \frac{2RT}{R^2 + T^2} \exp\left[-\frac{(\sigma \Delta z)^2}{2c^2} \right], \qquad (2)$$

where $R = r^2$ and $T = t^2$ (the reflectivity and transmissivity of the beam splitter, respectively), Δz is the optical path difference between the two photons and c is the speed of light [10]. In the general case where the pump spectrum and dispersion effects are considered, the shape of the HOM dip can be asymmetric, detailed analyses can be found in [11, 12].

The interference filters can be placed before the beam splitter (e.g. immediately after the nonlinear crystal) or after the beam splitter (e.g. immediately before the detectors). In the first case, the filters ensure the indistinguishability of the photons *before* they combine in the beam splitter. In the second case, the filters are placed *after* the photons have combined in the beam splitter. This latter case introduces a spectral filtering after the interference of the two photons has taken place, if indeed the two-photon interference in the HOM can be attributed to the interference of the individual photons. However, the dip in the HOM experiment is due to the interference of the probability amplitudes of the outcomes—the coincidence between the detector outputs. The width of the HOM dip with the filters placed just before the detectors or with the filters placed before the beam splitter is then expected to be the same.

3. Experiment and results

We use a 3-mm long β-barium borate crystal (BBO) cut for type-I degenerate, near-collinear SPDC (figure 2) producing signal and idler photons having the same polarization (which, in principle, allows for HOM dips with 100% visibility). The BBO crystal is pumped by a 355-nm laser source with a repetition rate of 100 MHz. A longpass interference filter (IF1, with a cutoff frequency of 500 nm) blocks the pump beam after the BBO crystal. The 710 nm down-converted photons

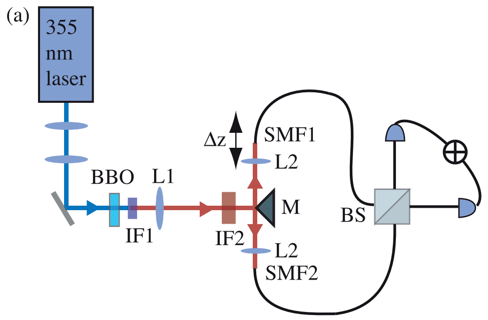

Figure 2. Experiment setup. Type-I SPDC in a BBO crystal results to signal and idler photons with the same polarization. These photons are separately coupled to single-mode fibres (SMF1 and SMF2) which are input to a fibre-optic beam splitter (BS). We consider two configurations. (a) The photons are spectrally filtered by IF2 before they combine in the beam splitter. (b) The spectral filtering is done by the IF2s placed just before the detectors, after the photons have combined in the beam splitter. In both cases, the coincidence count is monitored as the relative path difference (Δz) between the two photons is varied.

are separated by a knife-edge prism mirror (M). The signal and idler photons are focused by a lens ($L1$) onto the facets of separate polarization-maintaining single-mode fibres (SMF1 and SMF2), which are in the far field of the crystal. One of the fibres (SMF1) is mounted on a translation stage that allows us to match the path lengths of the signal and idler photons. For ease of alignment, we employ a balanced fibre-optic beam splitter, which consists of a 50/50 beam splitter connected to two polarization-maintaining input single-mode fibres and two output single-mode fibres. The signal and idler photons enter the beam splitter via the fibres at the input ports which are connected to SMF1 and SMF2. We use interference filters (IF2) centred at 710 nm to select the signal and idler photons.

We consider two cases. For the first case (figure 2(a)), a filter (IF2) is placed just before the mirror, hence the spectrum of the photons are identical before they reach the beam splitter; the fibres at the output ports of the beam splitter are then fed directly to avalanche photodiodes that serve as single-photon detectors. For the second case (figure 2(b)), there is no filter before the beam splitter apart from IF1, instead each of the fibres at the output port of the beam splitter is connected to a short free-space fibre coupler which holds identical interference filters (IF2); the output of the fibre coupler is then fed to the single-photon detectors. The outputs of the detectors in both cases are fed into a coincidence circuit, and we monitor the coincidences as we scan Δz.

Figure 3 show the coincidences (normalized to the constant coincidence count when $\Delta z \gg L_c$, due to the different efficiencies of the two configurations) as a function of the relative path difference (Δz) of the signal and idler photons, for two bandpass filters with different widths. Figure 3(a) shows the coincidences for a 2-nm filter placed before (blue) and after (red) the beam splitter. Figure 3(b) shows the coincidences for a 10-nm filter placed before (blue) and after (red) the beam splitter. As expected, we find a minimum in the coincidences when $\Delta z = 0$. Our results show that regardless of the positioning of the filter with respect to the beam splitter, the shape and width of the HOM dip remains the same. The full width at half maximum (FWHM) for when the 2-nm filter is placed before and after the beam splitter is 141.5 ± 1.2 μm and 142.2 ± 0.8 μm, respectively. As evident in equation (2), the width of the HOM dip is inversely proportional to the width of the interference filter, hence we expect narrower dips for the 10-nm filter. The FWHM for when the 10-nm filter is placed before and after the beam splitter is 41.7 ± 0.5 μm and 46.6 ± 0.6 μm, respectively. These values were calculated from a least-squares Gaussian fit of the data in figure 3. Because the interference from which the dip in the HOM experiment arises is an interference of probability amplitudes, rather than the interference of the individual photons coming together in the beam splitter, we expect that the HOM dip will be the same regardless of where the interference filters are placed, as supported by our experimental results.

We note that the ratio of the widths of the dip for the case of the 2-nm filter and the 10-nm filter, i.e. $\mathrm{FWHM}_{2\ \mathrm{nm}}/\mathrm{FWHM}_{10\ \mathrm{nm}}$ is approximately 3.2, rather than the expected ratio of 5. This is because our filters are not centred exactly at 710 nm (e.g. the 10-nm filters were centred at 712 nm, and the 2-nm filters were centred at 709 nm). The expected ratio, after accounting for this slight nondegeneracy is 3.5, close to what we get from the experiment.

We also consider the case where the filters placed both before and after the beam splitter have the same central wavelength but different bandwidths. We placed a 10-nm filter before the beam splitter (where IF2 is in figure 2(a)) and 2-nm filters after the beam splitter (where the IF2s are in figure 2(b)). The result is shown in figure 4. We compare the resulting HOM dip from this case (red) to our earlier result which employed just the 10-nm filter before the beam splitter (blue). If the interference were due to the individual photons that have passed through the 10-nm bandwidth filters before coming to the beam splitter, we expect the width of the resulting HOM dip (red) to be unaffected by the 2-nm filter. The width of the dip will be the same as that of the blue curve, but with reduced coincidences due to the presence of the 2-nm filters that limit the number of photons reaching the detectors. This is not what we observe in the experiment; the red curve is wider, with a FWHM of 146.0 ± 3.0 μm, similar to the FWHM of the HOM dip when there was only the 2-nm filter (figure 3(a)). The 2-nm filter placed after the beamsplitter seems to 'retroactively' alter the width of the coincidence dip supposedly after the individual photons have interfered. This result highlights how

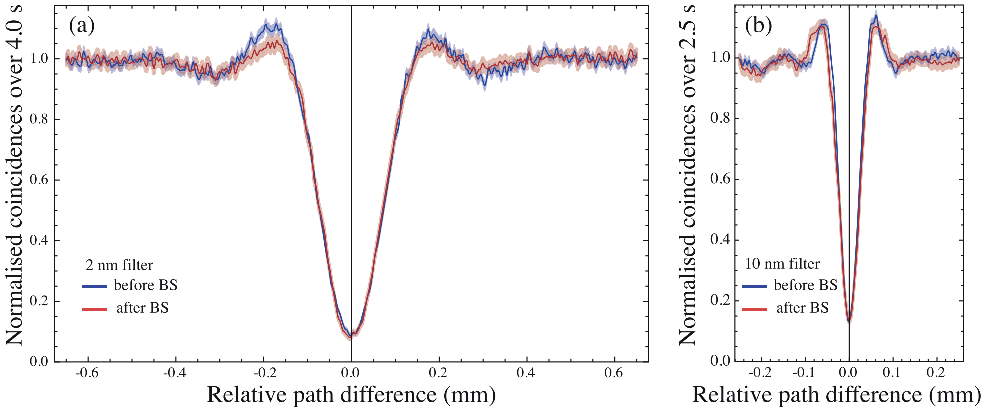

Figure 3. HOM dips. The HOM dips in our experiment is the same regardless of where the interference filter is placed. (a) HOM dip for a 2-nm filter placed before (blue) and after (red) the beam splitter. (b) HOM dip for a 10-nm filter placed before (blue) and after (red) the beam splitter. The width of the dip is inversely proportional to the bandwidth of the filter. The shaded thickness about the trace denote the uncertainty. The uncertainty in the relative path difference is set by the stage, and is 0.055 μm.

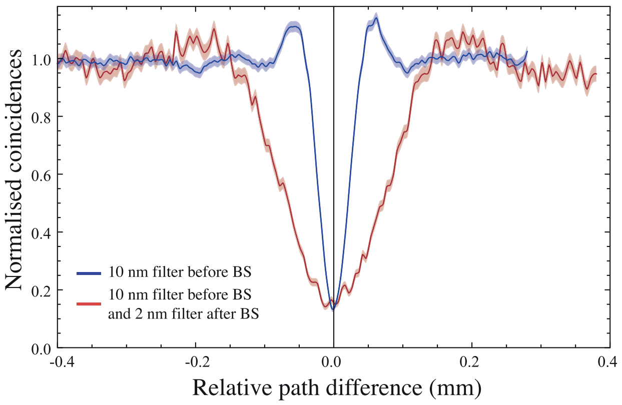

Figure 4. HOM dip for different bandwidth filters before and after the BS. The width of the HOM dip when a 10-nm filter is placed before and a 2-nm filter is place after the BS (red) is wider than the HOM dip which results from having just a 10-nm filter before the BS (blue). The FWHM for the former case is 146.0 μm, closer to the FWHM of an HOM dip resulting from a 2-nm filter (figure 3(a)). The shaded regions denote the uncertainty. The Gaussian curves shown come from a least-squares fit.

the interference resulting to the HOM dip is not simply an interference of the input photons. The width and shape of the HOM dip are determined by the filter with the narrowest bandwidth, regardless of where that filter was placed.

Finally, the effect of spectral filtering on the coincidence counts has also been investigated experimentally in [13], in the context of time–energy entanglement. We do not invoke the concept of entanglement here, but this is generally accepted as intimately related to the spectral and temporal correlations.

4. Conclusion

We have demonstrated a simple experiment illustrating that the dip in the HOM experiment is a result of the interference of the probability amplitudes, rather than the interference of the individual photons entering the beam splitter. We measured the width of the HOM dip for interference filters with different bandwidths. Regardless of whether the same filter is placed before or after the beam splitter, the width of the HOM dip remains unchanged. We also considered the case when the filters are placed both before and after the beam splitter have different bandwidths. We find that the width of the resulting HOM dip can still be affected by the filter placed after the beam splitter. Our results are consistent with what is expected in theory, if the HOM dip is indeed the result of the interference of the probability amplitudes of the events leading to a coincidence count, rather than the interference of individual photons entering the beam splitter.

Acknowledgments

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