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Editors  
Physical Review X Quantum

Dear Editors,

We would appreciate your considering the manuscript “Beyond Density Matrices: Geometric Quantum States” for publication in *Physical Review X Quantum*.

*PRX Quantum* is the ideal venue for this timely work since the study of dynamical and equilibrium properties of quantum systems has come to a turning point. In only the last decade or so the early 20th century’s extensive and well-known body of results that established quantum mechanics has found a powerful conceptual ally in information theory. The resulting synergy is gradually realizing Feynman’s original vision of a quantum computer.

While there has been much progress since the early days, recent insights into the nonequilibrium behavior of small systems has highlighted a lack of quantum information tools that appropriately address and model how interactions with a structured environment affect a quantum system’s dynamics. In this, we refer to the environment in a general sense: *Anything with which the quantum system under study interacts*. Despite the many analysis tools available, they each have concrete drawbacks when inferring dynamical behavior from available data. A good example, of immediate interest to quantum computing, arises when diagnosing crosstalk between qubits in the variously newly-proposed quantum platforms.

We believe part, if not all, of these challenges arise from misconceptions in what is considered the *state of a quantum system*. Addressing these challenges is our manuscript’s main focus. We accomplish this by leveraging earlier attempts at *geometric quantum mechanics*—a differential-geometric formulation of quantum mechanics that removes distracting redundancies embedded in traditional Hilbert-space formulations and properly identifies a quantum system’s *state space* as a projective manifold whose geometry and topology encode quantum physics’ distinctive character. (Bengtsson and Życzkowski’s *Geometry of Quantum States* discusses the fundamentals.)

Building on this formalism, this manuscript develops an appropriate measure-theoretic formalism on the projective manifold of pure states and advocates a new notion of state, dubbed *geometric quantum state*. At its core, it is a measure (probability distribution) on the manifold of pure quantum states. We show that the standard density-matrix formalism is a special case and so the new notion of quantum state is more fundamental. Specifically, we show how to compute density matrices from geometric quantum states and then turn to explore the latter’s relevance in two key situations. The first considers a discrete system interacting

with a continuous one. The second is a thermodynamic setting in which a small system interacts with a large, possibly structured, environment.

The net result is a more accurate picture of the state of a quantum system—a richer description of its behavior, both at the static and at the dynamic levels. As a concrete example, we should note that this manuscript has a companion “Geometric Quantum Thermodynamics”, also submitted to *PRX Quantum*. There, the geometric approach is exploited to introduce a new foundation for quantum thermodynamics of small systems, based on geometric quantum states and not density matrices. Moreover, there we also clearly show that our work provides concrete, actionable results—results that can be experimentally observed.

Given this, the findings should be of broad interest to research communities working with quantum physics at large and, in particular, to those focused on quantum information science and computing. For these reasons *PRX Quantum* strikes us as the proper venue.

Due to the work’s interdisciplinary nature—combining as it does stochastic thermodynamics, information and computation theories, and stochastic processes—we recommend several researchers with the appropriate background:

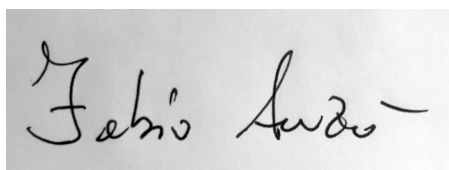
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Thanks in advance for your consideration.

Best regards,



James P. Crutchfield, Director  
Distinguished Professor of Physics



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