Beyond Density Matrices: Geometric Quantum States

Dear Editor,

We are pleased to submit our manuscript “Beyond Density Matrices: Geometric Quantum States” for consideration in Physical Review X Quantum.

The study of both dynamical and equilibrium properties of quantum systems is at a turning point. The large body of work produced at the beginning of the 20th century to establish quantum mechanics has, in the past 40 years, found a powerful ally into Information Theory. As a result of this synergy, Quantum Information Science is slowly turning Feynman’s original vision of a quantum computer into reality.

While much progress has been realized in the fundamental areas, recent insights into the non-equilibrium behavior of small classical systems have highlighted a lack of Quantum Information Theoretic tools to appropriately address and model how interactions with a structured environment can affect the dynamics of the quantum system under study. Here we use environment in a general sense: As “anything our quantum system under study is interacting with”. Despite many available tools of analysis, this has concrete drawbacks when trying to infer the dynamical behavior from available data. For example, when trying to address cross-talks between qubits in new quantum platforms.

We believe part, if not all, of the issue to be due to a fundamental misconception in what the “state of a quantum system” is considered to be. This is the main focus of the paper.

To appropriately address this issue, we leverage a geometric formulation of quantum mechanics in which one gets rid of the redundancy of the Hilbert space formulation, and correctly identifies the “space of the states” of a quantum system. This has the characteristics of a projective manifold. See the book “Geometry of Quantum States”, by Bengtsson and Zyczkowski for its fundamentals.

Building on *Geometric Quantum Mechanics*, in this manuscript we put forth various arguments in favor of a new notion of state, dubbed *Geometric Quantum State.* At its core, this is a probability distribution (a measure) on the manifold of quantum states. We correctly identify its relation with the standard density matrix formalism, which suggests that such notion of quantum state is more fundamental. Indeed, we explicitly show how to compute density matrices from geometric quantum states. We also explore the relevance of such notion of state in two key situations. First, when a discrete system interacts with a continuous one. Second, in a thermodynamic setting in which a small system interacts with a large, possibly structured, environment.

These results clearly introduce a more accurate description of the state of a quantum system, thus producing a richer description of its behavior, both at the static and at the dynamic level. We would also like to mention that this manuscript has a companion paper, also submitted to Physical Review X Quantum, called “Geometric Quantum Thermodynamics”. There, the same approach is exploited to propose a new foundation for the Quantum Thermodynamics of small systems, based on Geometric Quantum States and not density matrices. Moreover, there we also clearly show that our work provides concrete results, which can be measured in a quantum thermodynamic setting.

We believe the findings of our work to be of interest for the community of people working with quantum systems at large and, in particular, to the community focused on Quantum Information Science.

Your truly,

Fabio Anza and James P. Crutchfield