Notes

Fabio Brau

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Contents

Chapter 1

Markov Chains

In the section we will introduce the model of a classical markov chain with discrete time for which each state belongs to some finite or contouble set of possible state. In the next section we extend the definition to states with a continuous states or in general a continuous density function.

1.1 Background

1.2 Countable States Markov Chains

Let $(X_t)_{t\in\mathbb{N}}$ a sequence of random variables. Let us assume that each instant $t \in \mathbb{N}$ the variable X_t takes values in a countable state space S.

Definition 1 (Markov Property)**.** The sequence $(X_t)_{t\in\mathbb{N}}$ satisfies the Markov property if for each time t and for each states $s, s_0, \dots, s_n \in S$

$$\mathbb{P}(X_{n+1} = s \mid X_0 = s_0, \dots, X_n = s_n) = \mathbb{P}(X_{n+1} = s \mid X_n = s_n). \tag{1.1}$$

That is, the state assumed at a certain instant t only depends on the previous state and not on the all history.

Observe that, since we are assuming that S is finite, then we are assuming that there exists an enumeration $S = \{s_1, \dots, s_n, \dots\}$. Hence, for the sake of simplicity, and without loss of generality, we can assume that $S = \mathbb{N}$, from which $X_t \in \mathbb{N}$ for each t.