Diffusion Models: DALL-E

Deep Learning and Neural Networks: Advanced Topics

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March 1, 2023

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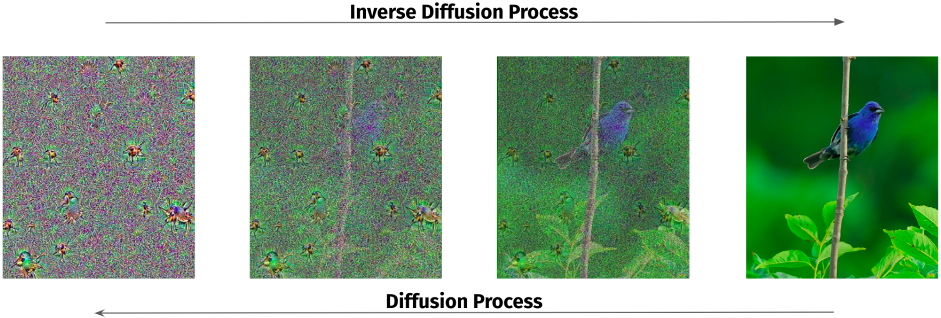
Introduction

Diffusion Models

Overview

Diffusion models are generative models that aim at denoising data



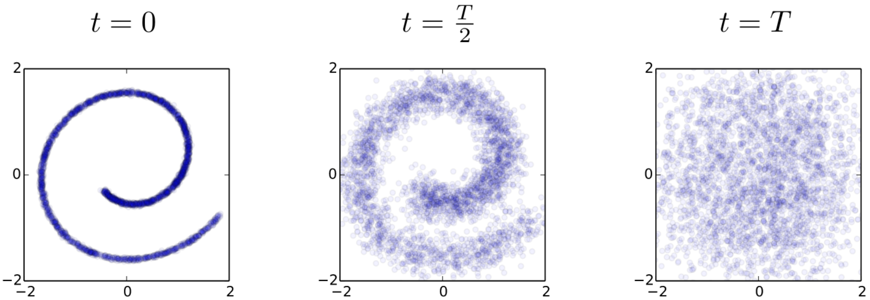
Timeline

2015) *...Non-equilibrium Thermodynamics*. Sohl-Dickstein et al. ICML

2020) *Denoising Diffusion Probabilistic Models*. Ho et al. NeurIPS.

2021) *Score-Based Generative Modeling Through SDE*. Song et al. ICLR.

Deep Unsupervised Learning using Non-Equilibrium Thermodynamics



Diffusion process as a **Markov Chain** with **Continuous State Space** and **Discrete Time**.[1]

[1] Sohl-Dickstein et al., “Deep Unsupervised Learning using Nonequilibrium Thermodynamics”.

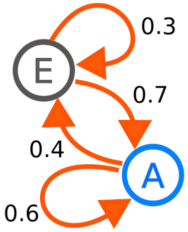
Reminder: Markov Chains with Discrete Time

Informal Definition

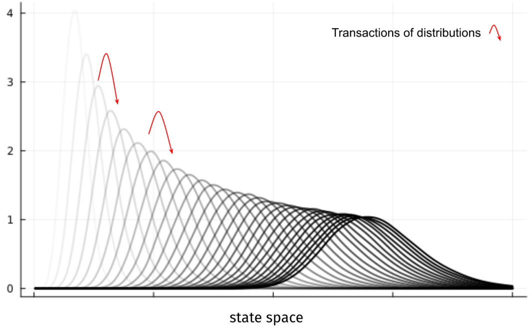
A sequence of random variables $\mathbf{x}^{(0)}, \mathbf{x}^{(1)}, \cdots, \mathbf{x}^{(t)}, \cdots$, such that:

- $\mathbf{x}^{(t)} \in S$, where S **State Space**
- The future $\mathbf{x}^{(t+1)}$ depends on the present $\mathbf{x}^{(t)}$ but not on the past $\mathbf{x}^{(t-1)}$

Discrete State Space S



Continuous State Space S



Reminder: MCDT with Discrete State Space

Definition

A sequence $\left\{\mathbf{x}^{(t)}\right\}_{t\in\mathbb{N}} \subseteq S$, a matrix $P = \left(p_{ij}\right)$.

- **Discrete state space**: $S = \{s_0, \cdots, s_n, \cdots\}$
- **Markov Property:** $\mathbf{x}^{(t+1)}$ not dep. $\mathbf{x}^{(0)}, \cdots, \mathbf{x}^{(t-1)}$.
- **Transaction Matrix**: $\mathbb{P}\left(\mathbf{x}^{(t+1)} = s_j | \mathbf{x}^{(t)} = s_i\right) = p_{ij}$

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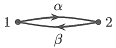
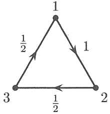
P is a stochastic matrix!

$$\forall i, \quad \sum_{j\in\mathbb{N}} p_{ij} = 1$$

$$P = \begin{pmatrix} 1-\alpha & \alpha \\ \beta & 1-\beta \end{pmatrix}$$



$$P = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 1/2 & 1/2 \\ 1/2 & 0 & 1/2 \end{pmatrix}$$



Reminder: DTMC with Continuous State Space

Let assume $\mathbf{x}, \mathbf{y} \in S$ where S continuous state space (e.g. $S = \mathbb{R}^d$).

Joint Distribution $p(\mathbf{x}, \mathbf{y})$

$$\mathbb{P}\left(\mathbf{x} \in A \,|\, \mathbf{y} \in B\right) = \int_A \int_B p\left(\mathbf{x}, \mathbf{y}\right) \, d\mathbf{x} \, d\mathbf{y}$$

Transactional Kernel $p(\mathbf{x} \,|\, \mathbf{y})$

$$p\left(\mathbf{x}, \mathbf{y}\right) = p(\mathbf{x} \,|\, \mathbf{y}) \, p\left(\mathbf{y}\right)$$

Marginal Distribution $p(\mathbf{x})$

$$p\left(\mathbf{x}\right) = \int_S p\left(\mathbf{x}, \mathbf{y}\right) \, d\mathbf{y} = \int_S p(\mathbf{x} \,|\, \mathbf{y}) \, p\left(\mathbf{y}\right) \, d\mathbf{y}$$

Markov Chains with Discrete Time

Definition
A sequence of random variables $\{\mathbf{x}^{(t)}\}_{t\in\mathcal{T}} \subseteq S$, such that the future $\mathbf{x}^{(t+1)}$ depends on the present $\mathbf{x}^{(t)}$ but not on the past $\mathbf{x}^{(t-1)}$.

- **Discrete Time Property**
  $\mathbf{x}^{(0)}, \mathbf{x}^{(1)}, \cdots, \mathbf{x}^{(t)}, \cdots$
- **Markov Property**
  $\mathbb{P}\left(\mathbf{x}^{(t+1)} \in A \mid \mathbf{x}^{(0)}, \ldots, \mathbf{x}^{(t)}\right) = \mathbb{P}\left(\mathbf{x}^{(t+1)} \in A \mid \mathbf{x}^{(t)}\right)$

Discrete State Space S

Continuous State Space S

Broader Impacts

CLIP Model

“We also found discrepancies across gender and race for people categorized into the ‘crime’ and ‘non-human’ categories...”[2]

[2] Radford et al., “Learning Transferable Visual Models From Natural Language Supervision”.

Thanks for the attention

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