An introduction to PCA

Weekly AI pills

Fabio Brau.

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SSSA, Emerging Digital Technologies, Pisa.

- The aim of Principal Component Analysis
- Derivation
    1. A Geometrical idea
    2. A statistical Derivation
    3. Singolar Value Decomposition
- PCA from Encoder Decoder NN
- Dummy examples

A Geometrical Idea

Let $X \in \mathbb{R}^{N \times n}$ be a dataset of N **observation** within n **variables**.

$$X = \begin{bmatrix} x_1^T \\ \vdots \\ x_N^T \end{bmatrix} = \begin{bmatrix} x^{(1)} & | & \cdots & | & x^{(n)} \end{bmatrix} \tag{1}$$

Notations:

- $x_i \in \mathbb{R}^n$ represents a single **observation**, i.e a **sample** in the feature space.
- $x^{(i)} \in \mathbb{R}^N$ represents the single **variable**, i.e a **column** of the dataset.
- The object $\mathbb{1}_n \in \mathbb{R}^n$ is the unitary columnar vector of length n $\mathbb{1}_n = [1, \cdots, 1]$.