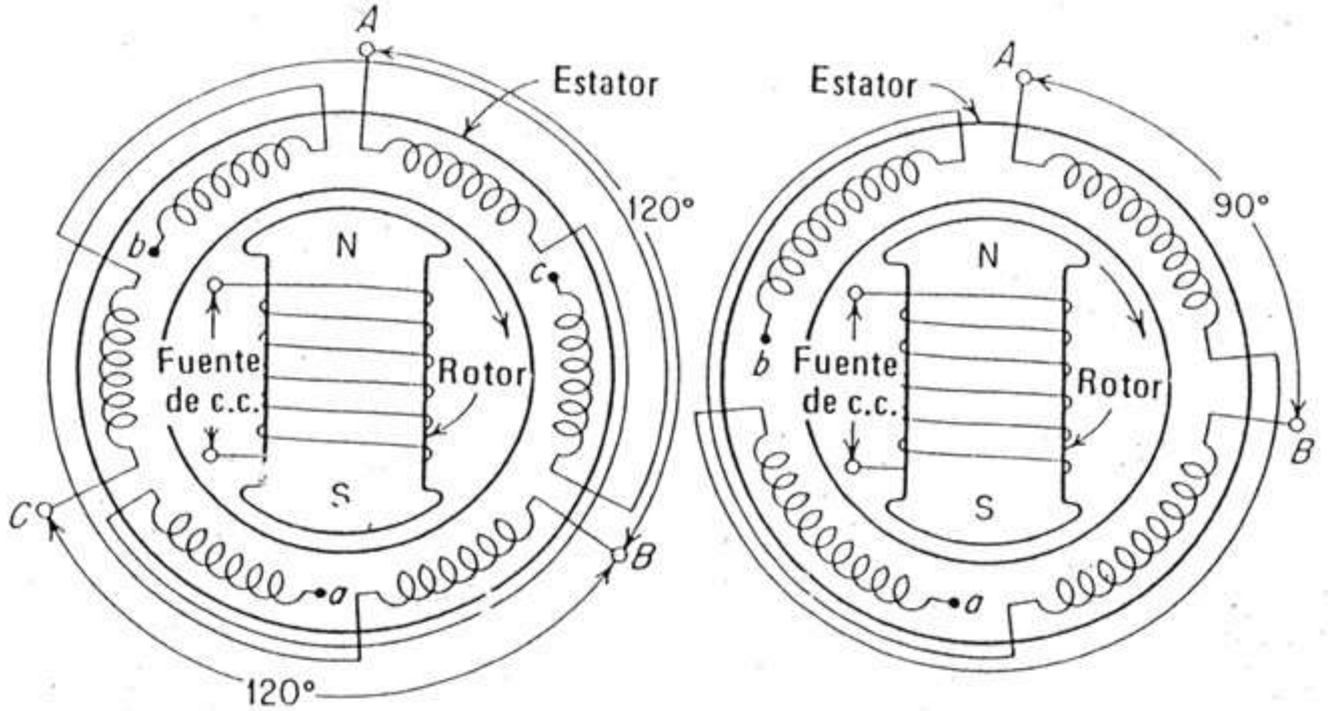
SISTEMAS POLIFÁSICOS



(*a*) Tres fases

(*b*) Dos fases

FIG. 18.1.—Esquema de devanados polifásicos.

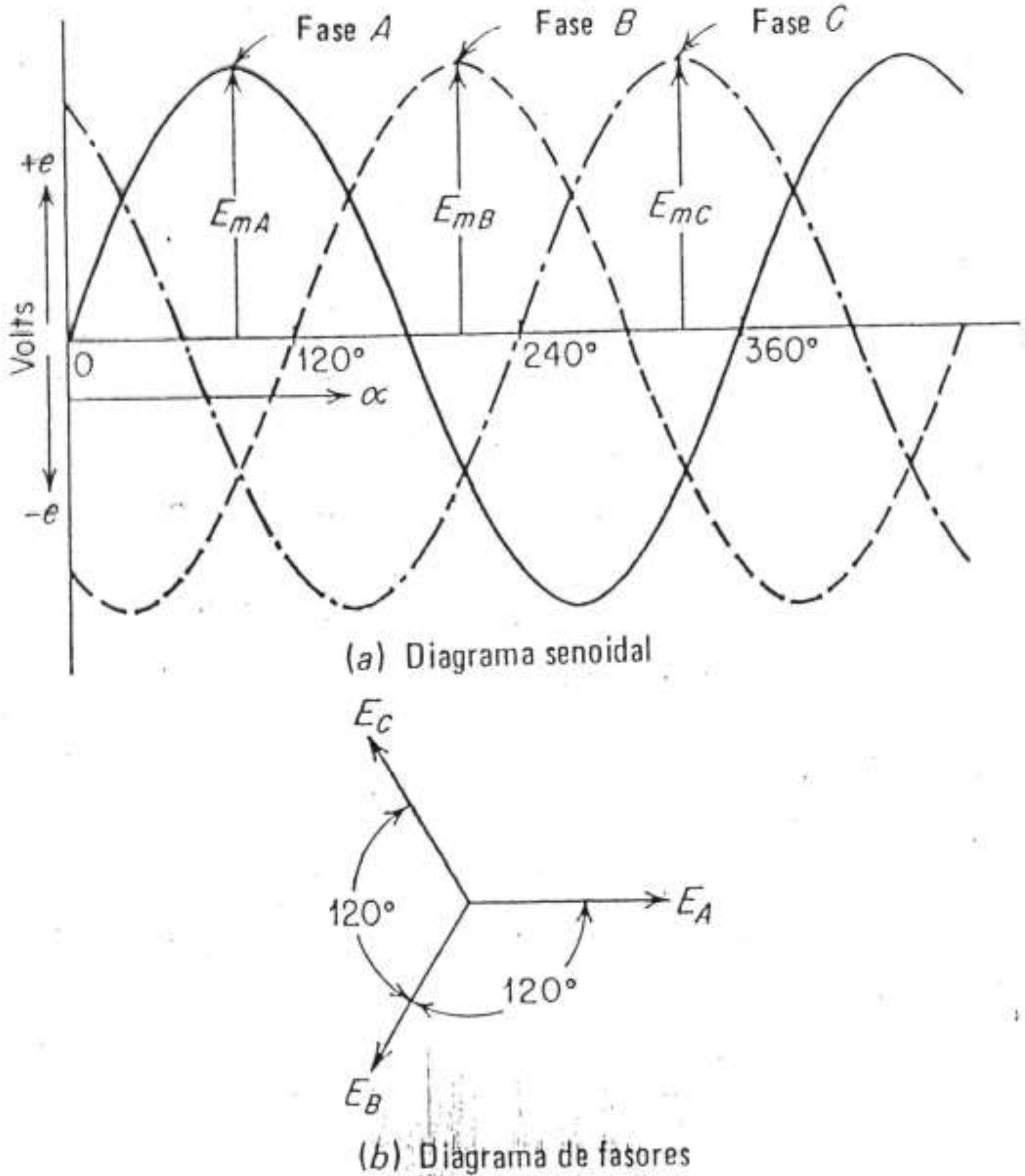
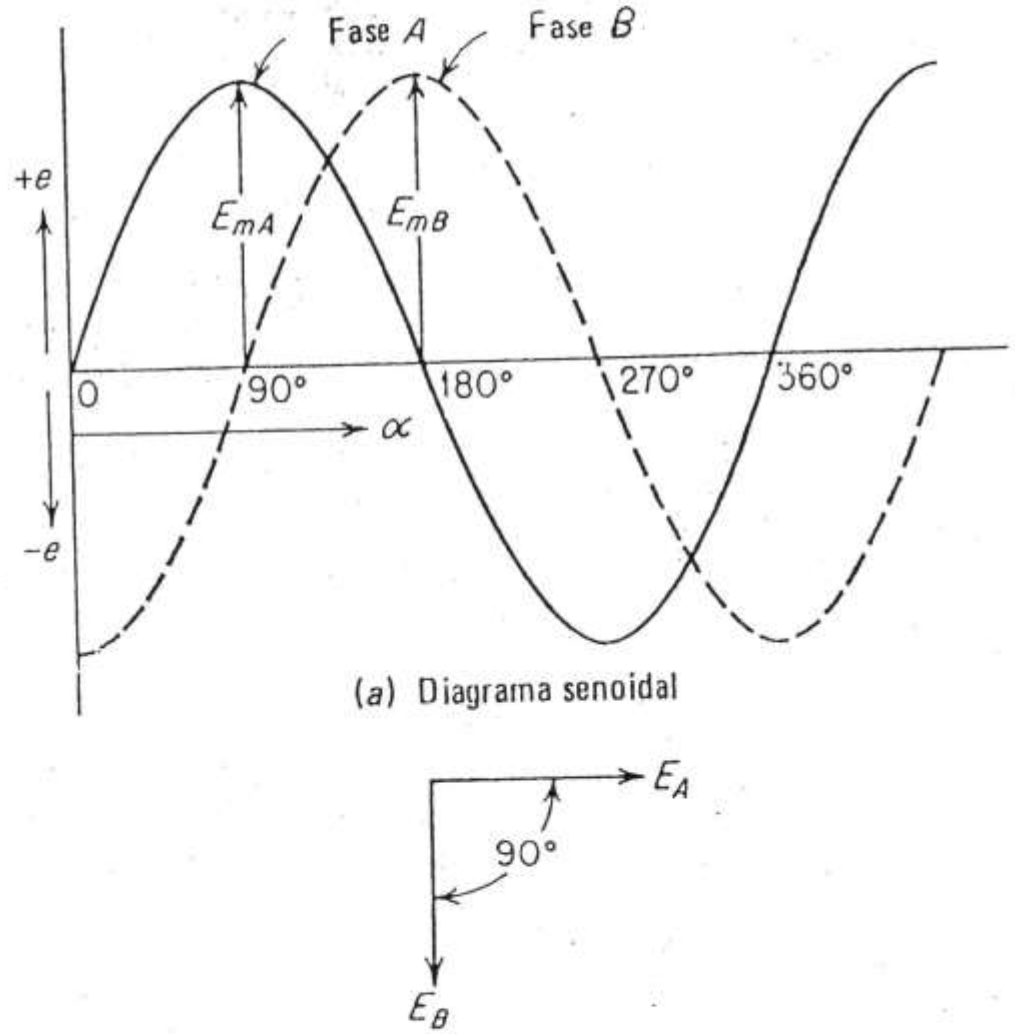


FIG. 18.2.—Diagrama de tensión para un sistema trifásico con una secuencia de fases $A-B-C$.



(a) Diagrama senoidal

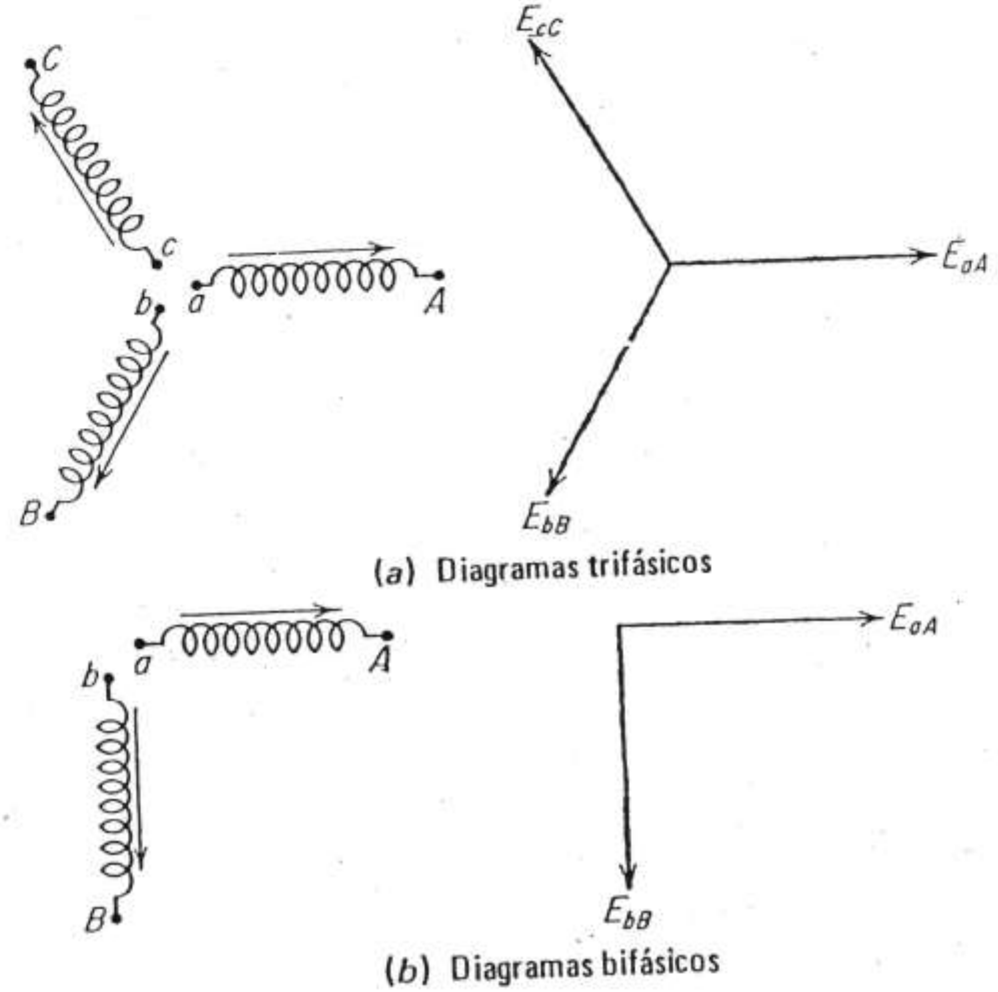
(b) Diagrama de fasores

FIG. 18.3.—Diagramas de tensión para un sistema bifásico con una secuencia $A-B$.

$\hat{E}$

$$
\begin{aligned}
e_A &= E_m \text{ sen } \alpha \\
e_B &= E_m \text{ sen } (\alpha - 120°) \\
e_C &= E_m \text{ sen } (\alpha - 240°)
\end{aligned}
$$

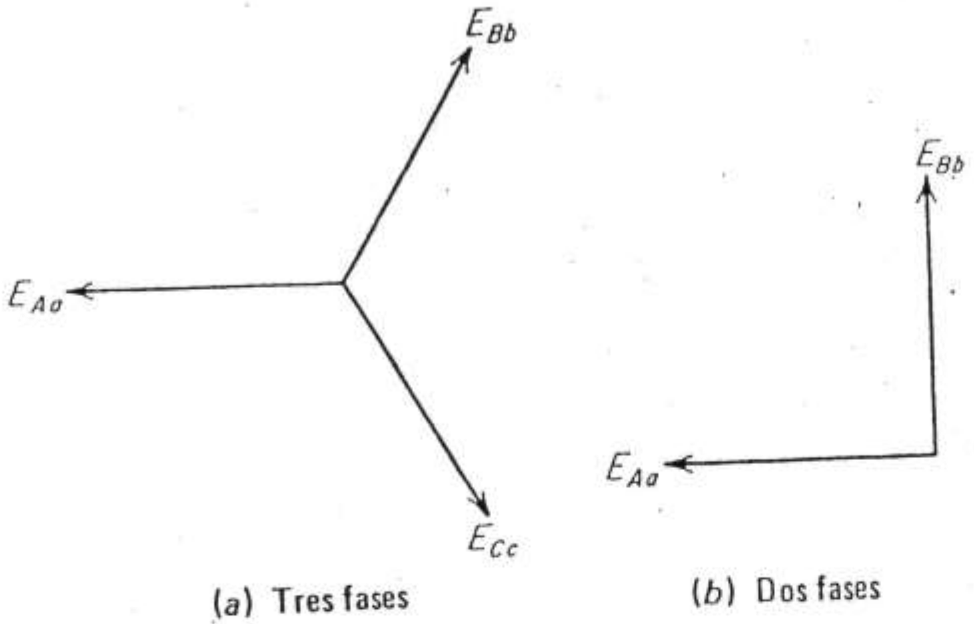
$$
\begin{aligned}
e_A &= E_m \text{ sen } \alpha \\
e_B &= E_m \text{ sen } (\alpha - 90°)
\end{aligned}
$$



(*a*) Diagramas trifásicos

(*b*) Diagramas bifásicos

FIG. 18.4.—Diagramas con notación de doble subíndice.



(*a*) Tres fases (*b*) Dos fases

FIG. 18.5.—Diagramas con notación de doble subíndice, opuesta a la de la fig. 18.4.

La conexión en estrella o Y trifásica

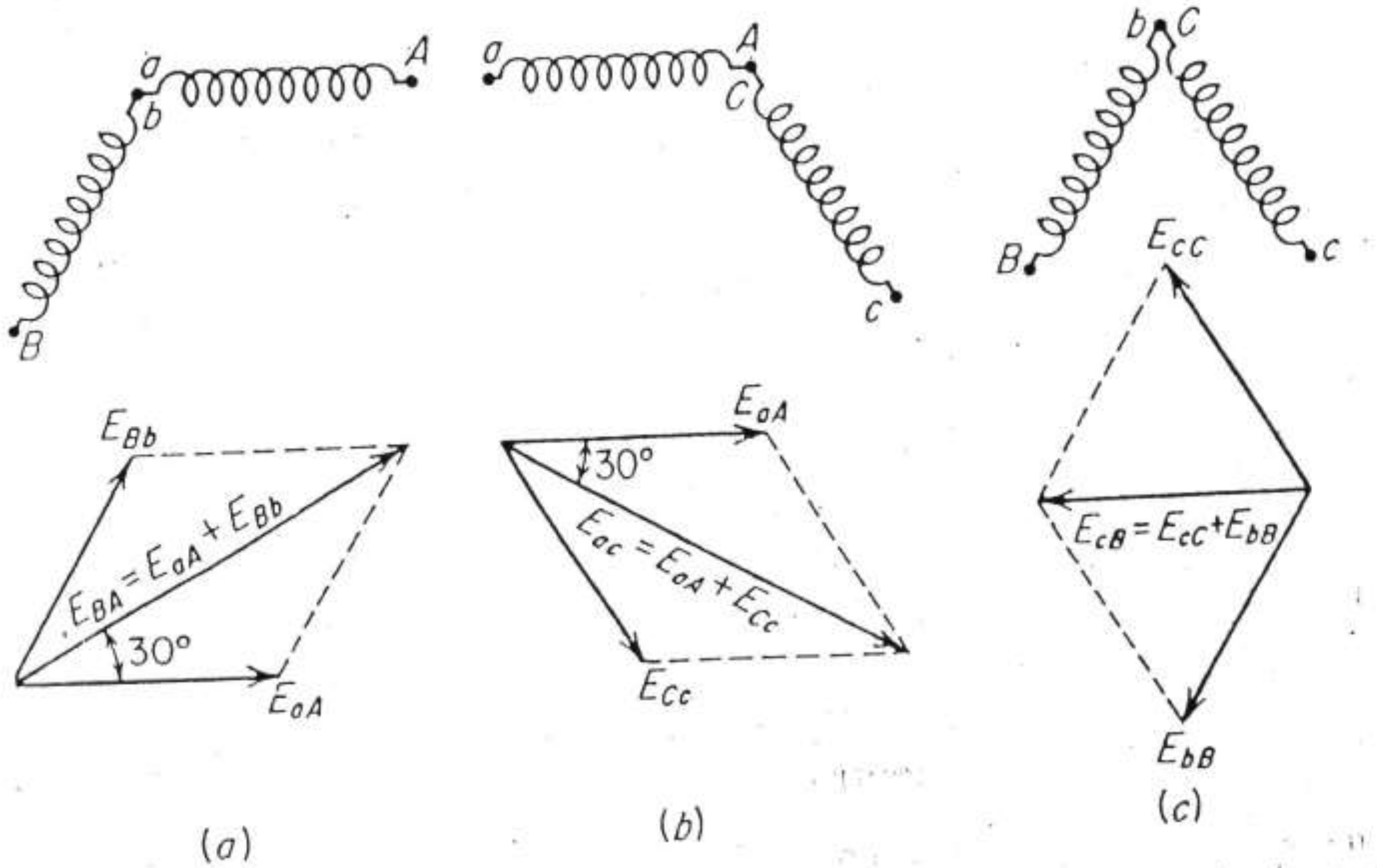
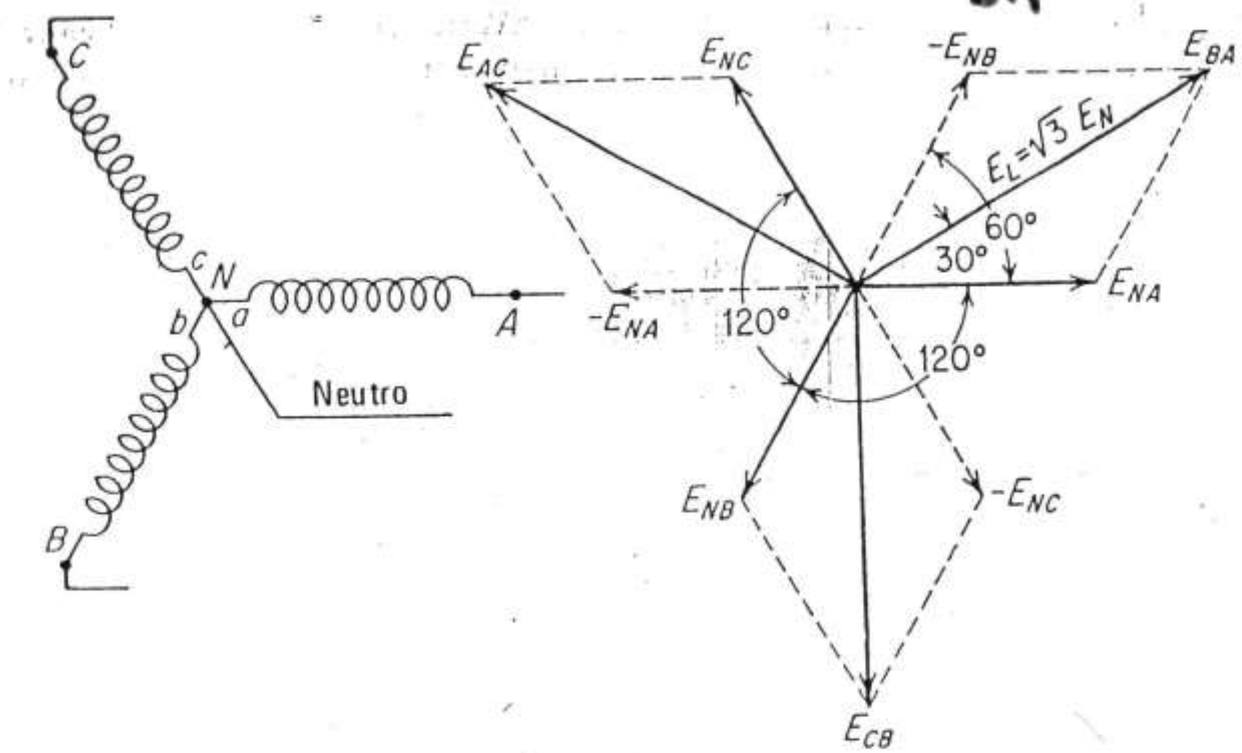


FIG. 18.6.—Devánados de alternador interconectados, y determinación de las tensiones en los terminales.

E_{BN}



(*a*) Conexiones (*b*) Diagrama de fasores

FIG. 18.7.—Sistema trifásico en estrella.

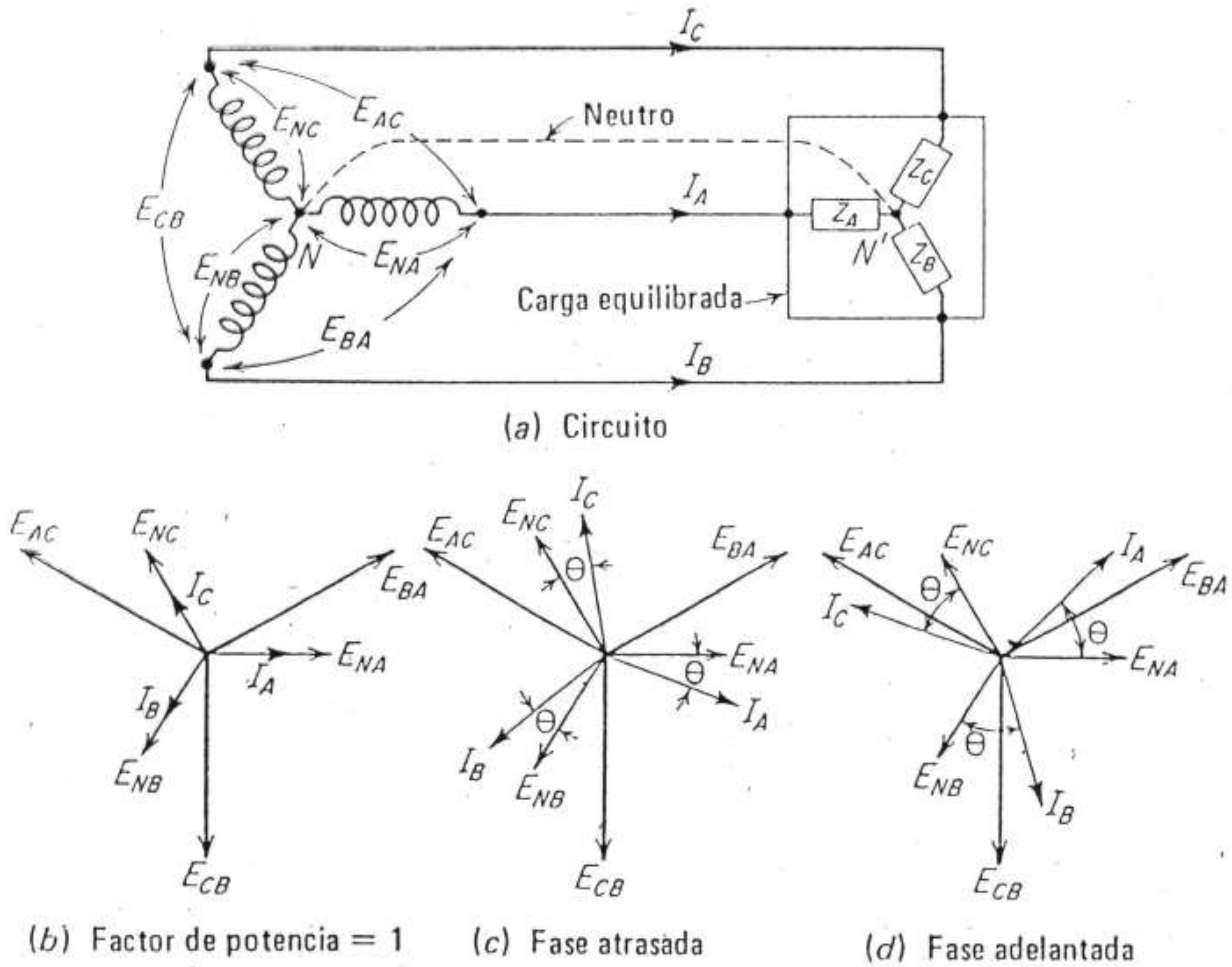


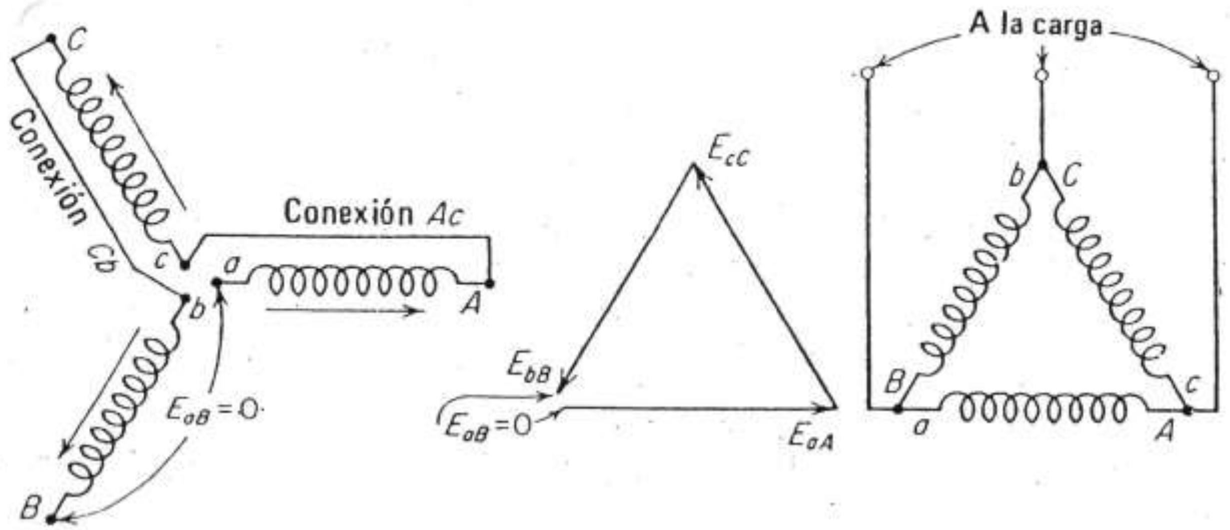
FIG. 18.8.—Diagramas de una fuente trifásica en estrella que alimenta cargas equilibradas con factores de potencia unidad, en atraso y en adelanto.

Carga Equilibrada

$$I_N = 0$$

$$I_N = -(I_A + I_B + I_C)$$

La conexión trifásica en delta o triángulo



(*a*) Combinación original con dos interconexiones (*b*) Diagrama demostrativo de $E_{aB} = 0$ (*c*) Triángulo completo

FIG. 18.9.—Desarrollo de la conexión en triángulo o delta.

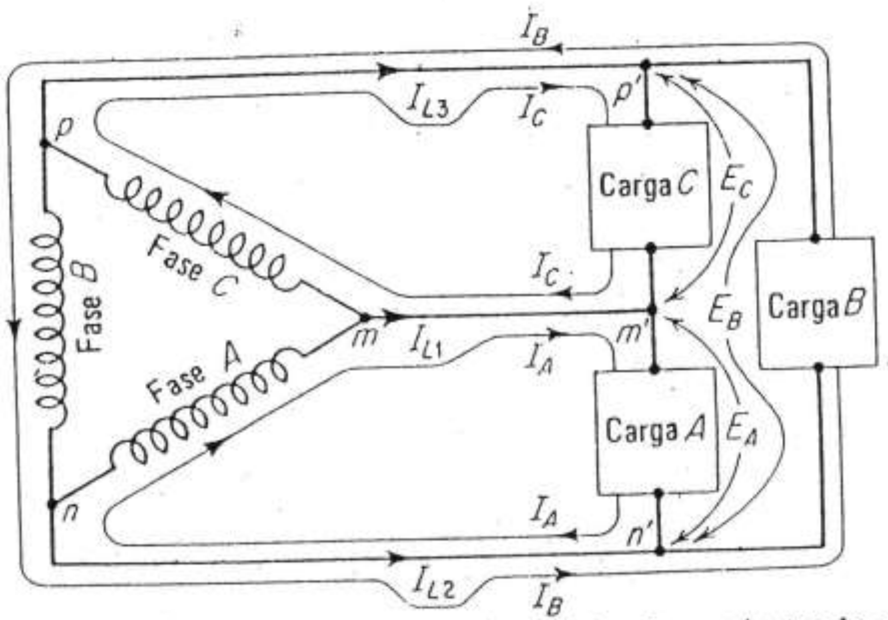
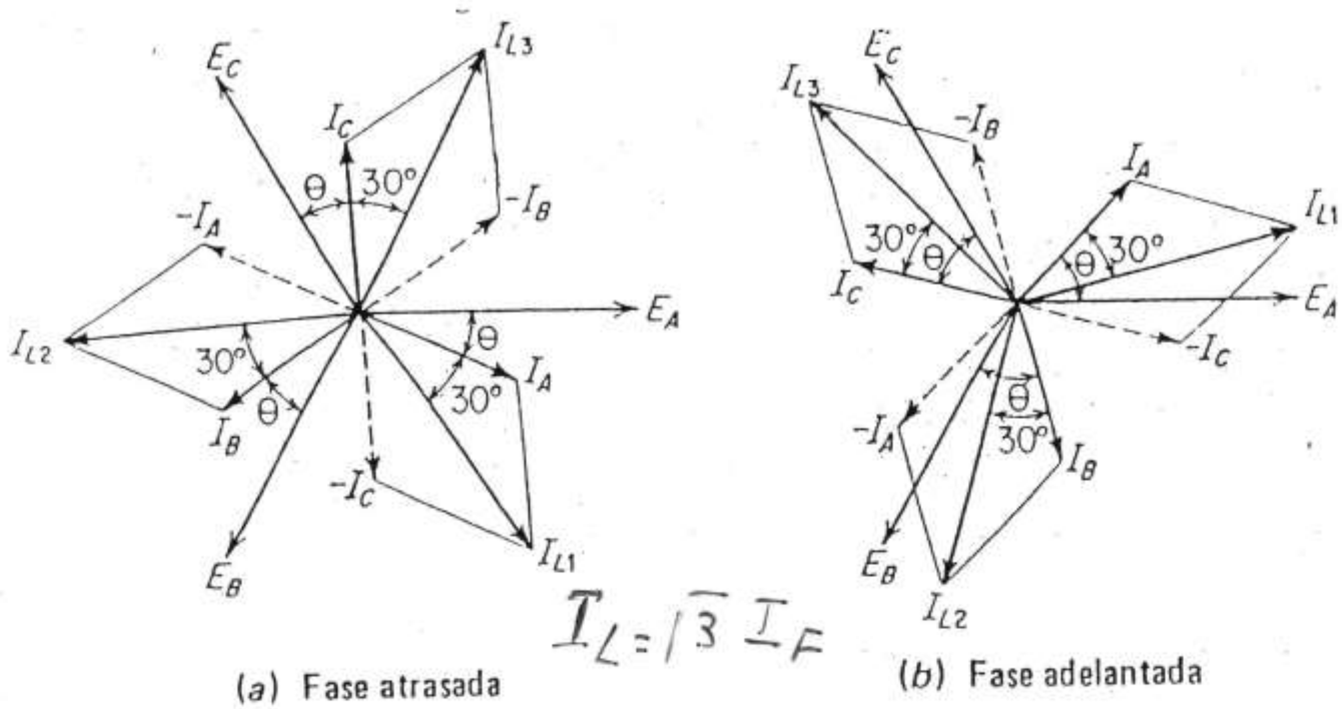


FIG. 18.10.—Una fuente conectada en triángulo suministrando corrientes de carga indicadas con I_A, I_B e I_C. Las tres corrientes de línea I_{L_1}, I_{L_2} y I_{L_3}, son las diferencias geométricas de las corrientes en las cargas.

$$\text{Corriente en } mm' = \mathbf{I}_{L1} = \mathbf{I}_A - \mathbf{I}_C$$
$$\text{Corriente en } nn' = \mathbf{I}_{L2} = \mathbf{I}_B - \mathbf{I}_A$$
$$\text{Corriente en } pp' = \mathbf{I}_{L3} = \mathbf{I}_C - \mathbf{I}_B$$

$$\mathbf{I}_{L1} + \mathbf{I}_{L2} + \mathbf{I}_{L3} = 0$$



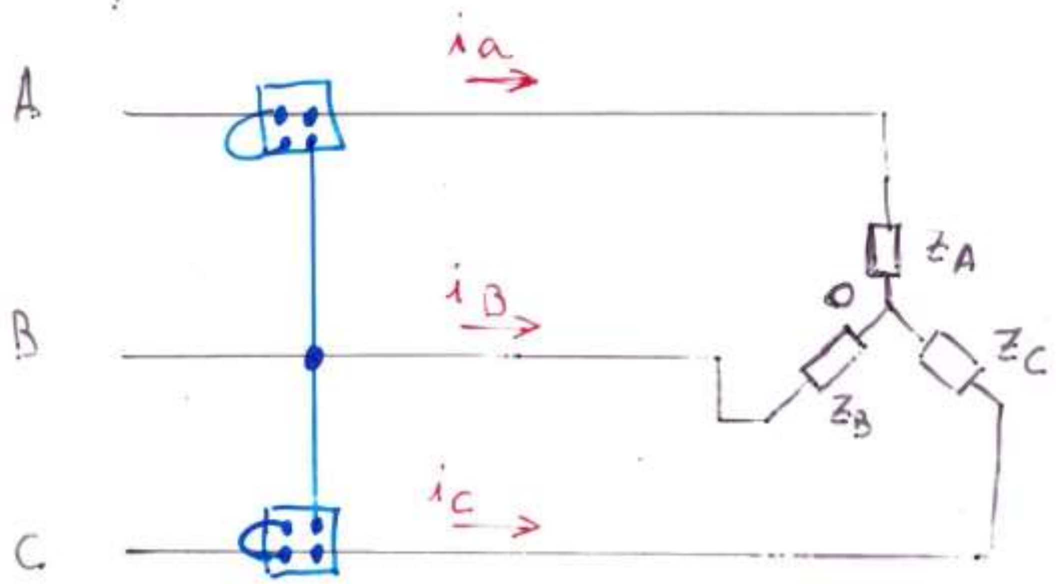
$I_L = \sqrt{3}\, I_F$

FIG. 18.11.—Diagrama de fasores que representan una fuente conectada en triángulo alimentando cargas equilibradas con fases en atraso y en adelanto.

Potencia en sistema Trifasico

Metodo de los dos Watimetros

1)



La potencia instantánea total:

$$p = v_{AO}\, i_A + v_{BO}\, i_B + v_{CO}\, i_C$$

$$i_A + i_B + i_C = 0$$

$$i_B = -i_A - i_C$$

$$p = v_{AO}\, i_A - v_{BO}\, i_A - v_{BO}\, i_C + v_{CO}\, i_C$$

$$p = (v_{AO} - v_{BO})\, i_A + (v_{CO} - v_{BO})\, i_C$$

P_{tvp} :

$$v_{AO} - v_{BO} = v_{AB}$$

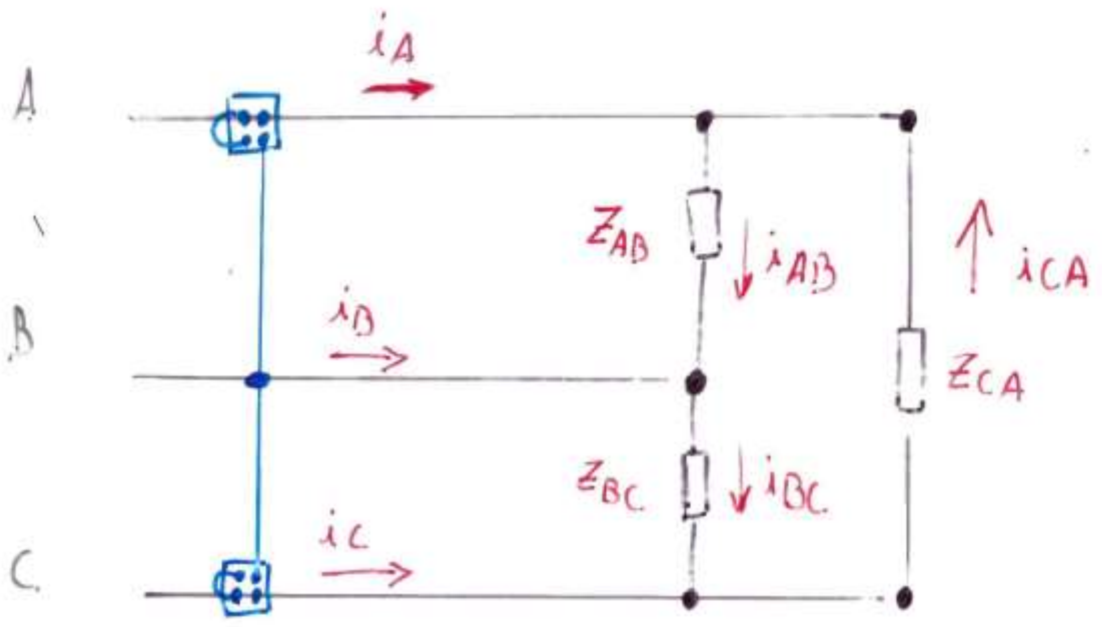
$$v_{CO} - v_{BO} = v_{CB}$$

$$p = v_{AB}\, i_A + v_{CB}\, i_C$$

$$P = \frac{1}{T}\int_0^T p\,dt = \frac{1}{T}\int_0^T v_{AB}\, i_A\, dt + \frac{1}{T}\int_0^T v_{CB}\, i_C\, dt$$

$$P = P_A + P_C$$

2)



La potencia instantánea total:

$$p = v_{AB}\, i_{AB} + v_{BC}\, i_{BC} + v_{CA}\, i_{CA}$$

$$v_{AB} + v_{BC} + v_{CA} = 0 \quad ; \quad v_{CA} = -v_{AB} - v_{BC}$$

$$p = v_{AB}\, i_{AB} + v_{BC}\, i_{BC} - v_{AB}\, i_{CA} - v_{BC}\, i_{CA}$$

$$p = v_{AB}\,(i_{AB} - i_{CA}) + v_{BC}\,(i_{BC} - i_{CA})$$

$$p = v_{AB}\,(i_{AB} - i_{CA}) - v_{BC}\,(i_{CA} - i_{BC})$$

$$p = v_{AB}\,(i_{AB} - i_{CA}) + v_{CB}\,(i_{CA} - i_{BC})$$

$$i_{AB} - i_{CA} = i_A$$

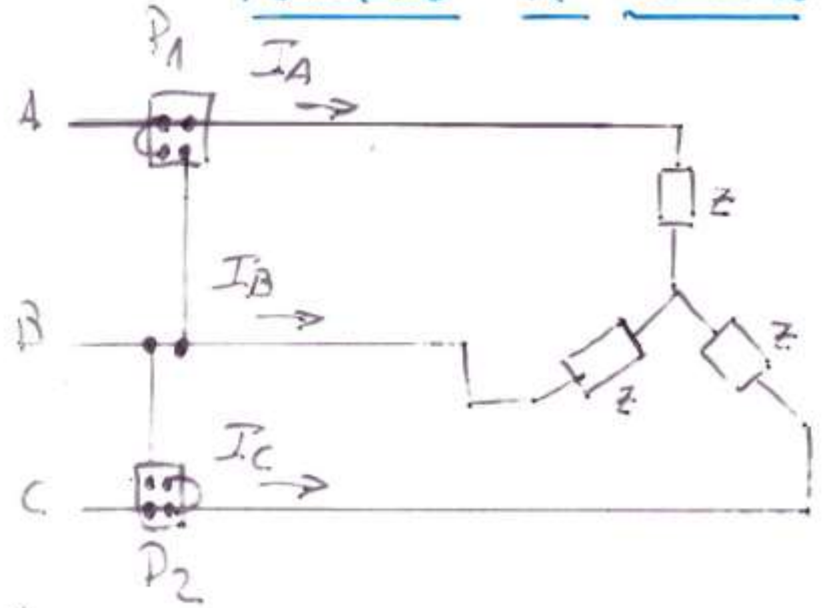
$$i_{CA} - i_{BC} = i_C$$

$$p = v_{AB}\, i_A + v_{CB}\, i_C$$

$$P = \frac{1}{T}\int_0^T p\, dt = \frac{1}{T}\int_0^T v_{AB}\, i_A\, dt + \frac{1}{T}\int_0^T v_{CB}\, i_C\, dt$$

$$\underline{\underline{P = P_A + P_C}}$$

Potencia en Sistema Trifasico



sec. Directa ; $f_p = \cos\theta$
en atraso.

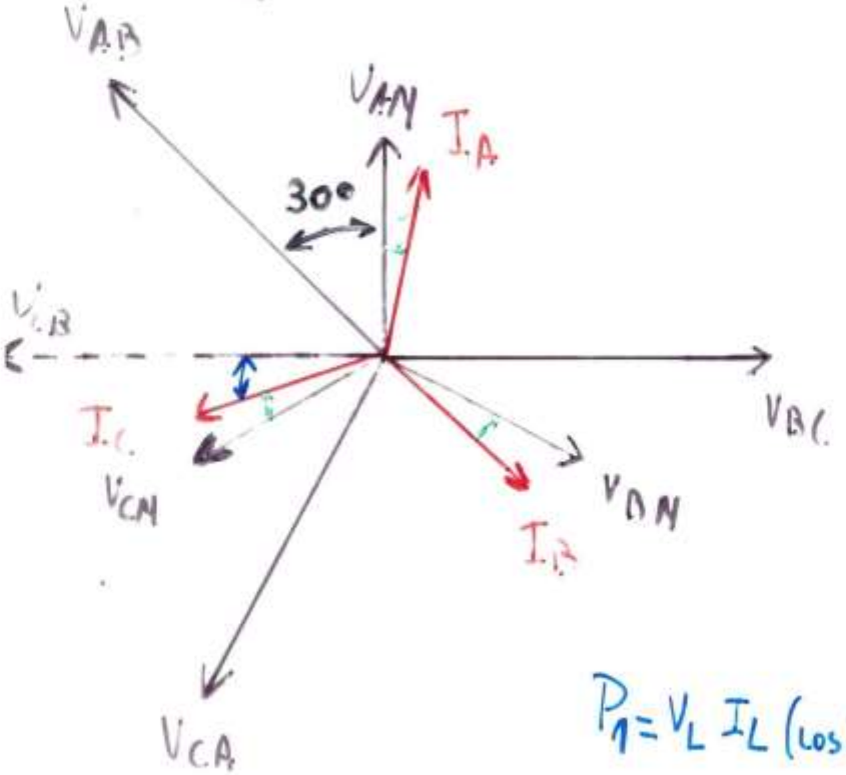
$$P_1 = V_{AB}\, I_A \cos(30^\circ + \theta)$$

$$P_2 = V_{CB}\, I_C \cos(30^\circ - \theta)$$

Sist. Equilibrado

$$V_{AB} = V_{CB} = V_L$$

$$I_A = I_C = I_L$$



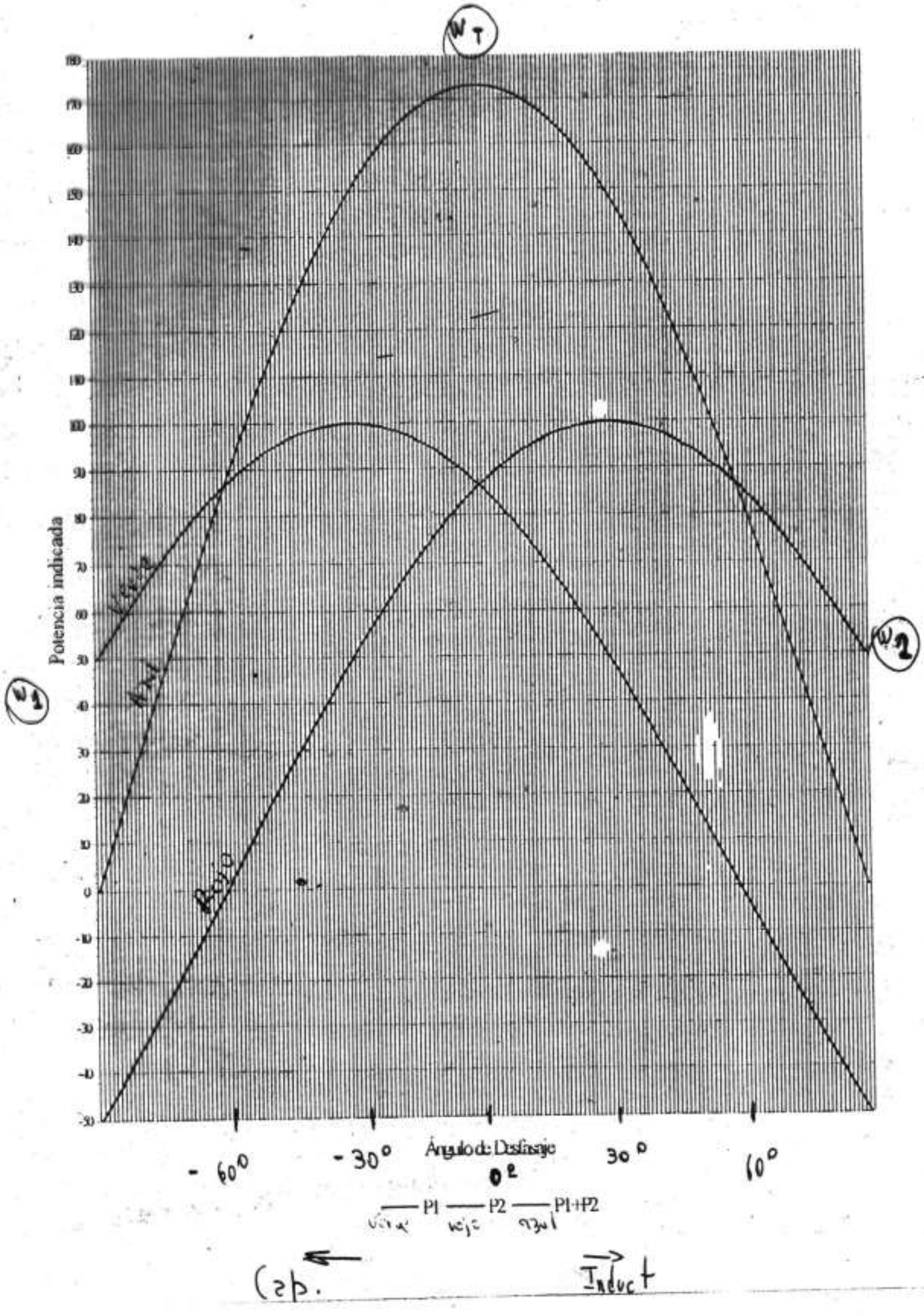
$$P_1 = V_L I_L (\cos 30^\circ \cos\theta - \operatorname{sen} 30^\circ \operatorname{sen}\theta) = V_L I_L \left(\frac{\sqrt{3}}{2}\cos\theta - \frac{1}{2}\operatorname{sen}\theta\right)$$

$$P_2 = V_L I_L (\cos 30^\circ \cos\theta + \operatorname{sen} 30^\circ \operatorname{sen}\theta) = V_L I_L \left(\frac{\sqrt{3}}{2}\cos\theta + \frac{1}{2}\operatorname{sen}\theta\right)$$

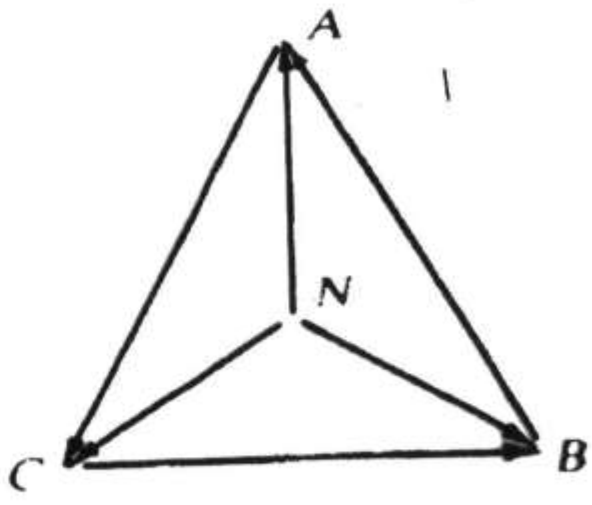
$$P = P_1 + P_2 = \sqrt{3}\, E_L I_L \cos\theta$$

$$\operatorname{tg}\theta = \sqrt{3}\left(\frac{P_2 - P_1}{P_2 + P_1}\right)$$

Valores que pueden adoptar los vatímetros, en conexión Aron, en un sistema trifásico equilibrado en función de la naturaleza de la carga



FASORES DE LAS TENSIONES

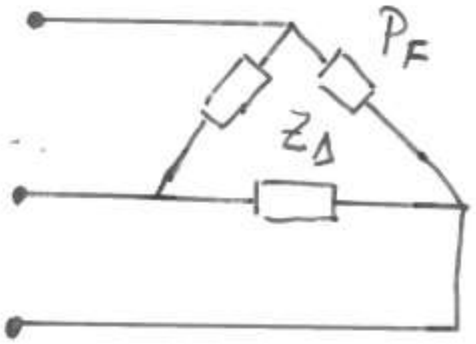


$$\mathbf{V}_{AB} = V_L\angle 120^\circ$$
$$\mathbf{V}_{BC} = V_L\angle 0^\circ$$
$$\mathbf{V}_{CA} = V_L\angle 240^\circ$$
$$\mathbf{V}_{AN} = (V_L/\sqrt{3})\angle 90^\circ$$
$$\mathbf{V}_{BN} = (V_L/\sqrt{3})\angle -30^\circ$$
$$\mathbf{V}_{CN} = (V_L/\sqrt{3})\angle -150^\circ$$

(*a*) Secuencia *ABC*

Potencia en cargas Trifasicas Equilibradas

$\underline{\Delta}$:



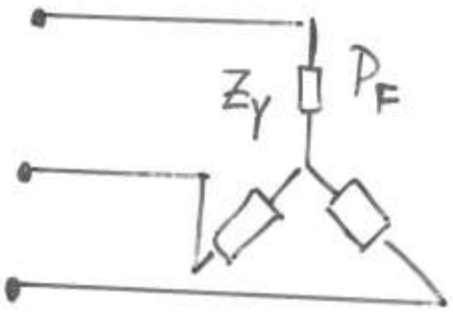
$P_F = V_L I_F \cos\theta$

$P_T = 3 V_L I_F \cos\theta$

$\rightarrow I_L = \sqrt{3}\, I_F$

$P_T = \sqrt{3}\, V_L I_L \cos\theta$ (1)

$\underline{Y}$:



$P_F = V_F \cdot I_L \cos\theta$

$P_T = 3 V_F I_L \cos\theta$

$\rightarrow V_L = \sqrt{3}\, V_F$

$P_T = \sqrt{3}\, V_L I_L \cos\theta$ (2)

(1) = (2)

- Potencia Media o Activa

$$P = V_{ef} . I_{ef} \cos\theta$$

$$Z = R + JX = |Z| \underline{\angle\theta} \quad ; \quad \cos\theta = \frac{R}{|Z|}$$

$$P = V_{ef} . I_{ef} \frac{R}{|Z|}$$

$$P = \frac{V_{ef}^2}{|Z|^2} R = R \, I_{ef}^2$$

$$f_p = \frac{P}{V_{ef} \, I_{ef}} \qquad 0 \leq f_p \leq 1$$

- Potencia Reactiva

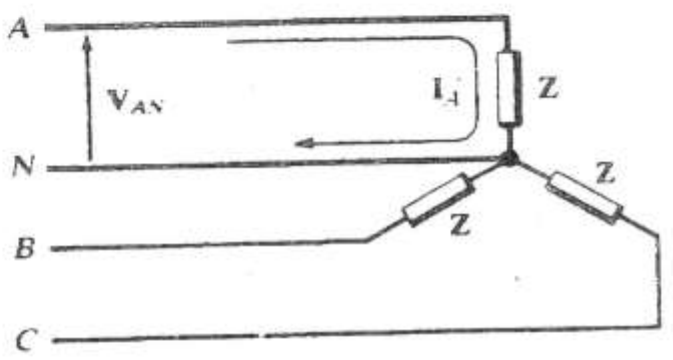
$$Q = V_{ef} \, I_{ef} \operatorname{sen}\theta$$

$$Z = R + JX = |Z| \underline{\angle\theta} . \qquad \operatorname{sen}\theta = \frac{X}{|Z|}$$

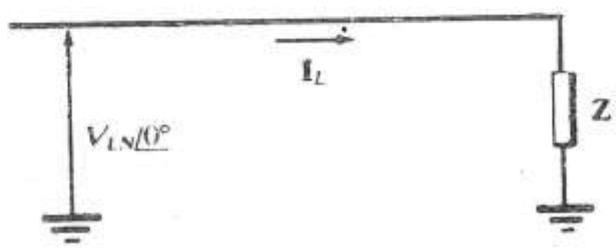
$$Q = V_{ef} \, I_{ef} \frac{X}{|Z|}$$

$$Q = \frac{V_{ef}^2}{|Z|^2} X = X . I_{ef}^2$$

CIRCUITO MONOFÁSICO EQUIVALENTE DE CARGAS TRIFÁSICAS EQUILIBRADAS



(a)



(b)

EQUIVALENCIA ESTRELLA-TRIÁNGULO

Transformación Y a Δ

$$\mathbf{Z}_A = \frac{\mathbf{Z}_1\mathbf{Z}_2 + \mathbf{Z}_1\mathbf{Z}_3 + \mathbf{Z}_2\mathbf{Z}_3}{\mathbf{Z}_3}$$

$$\mathbf{Z}_B = \frac{\mathbf{Z}_1\mathbf{Z}_2 + \mathbf{Z}_1\mathbf{Z}_3 + \mathbf{Z}_2\mathbf{Z}_3}{\mathbf{Z}_2}$$

$$\mathbf{Z}_C = \frac{\mathbf{Z}_1\mathbf{Z}_2 + \mathbf{Z}_1\mathbf{Z}_3 + \mathbf{Z}_2\mathbf{Z}_3}{\mathbf{Z}_1}$$

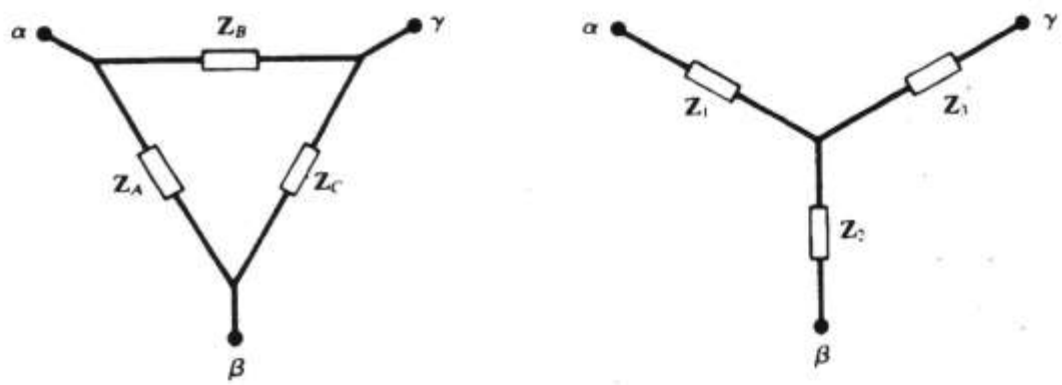
Transformación Δ a Y

$$\mathbf{Z}_1 = \frac{\mathbf{Z}_A\mathbf{Z}_B}{\mathbf{Z}_A + \mathbf{Z}_B + \mathbf{Z}_C}$$

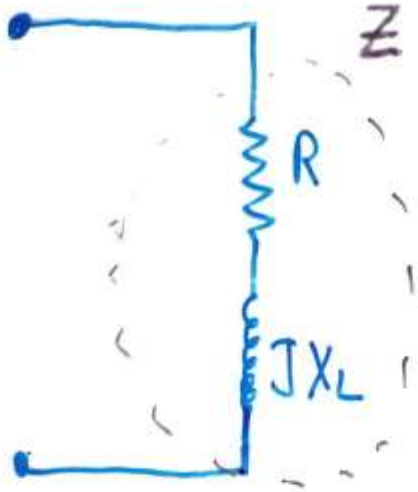
$$\mathbf{Z}_2 = \frac{\mathbf{Z}_A\mathbf{Z}_C}{\mathbf{Z}_A + \mathbf{Z}_B + \mathbf{Z}_C}$$

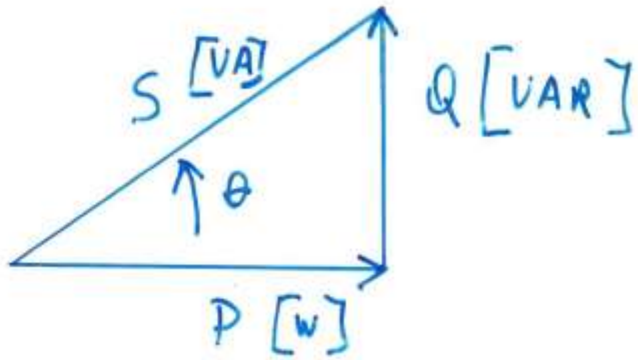
$$\mathbf{Z}_3 = \frac{\mathbf{Z}_B\mathbf{Z}_C}{\mathbf{Z}_A + \mathbf{Z}_B + \mathbf{Z}_C}$$

Obsérvese que si las tres impedancias de una conexión son iguales, entonces lo son las de la conexión equivalente, siendo $\mathbf{Z}_\Delta/\mathbf{Z}_Y = 3$.



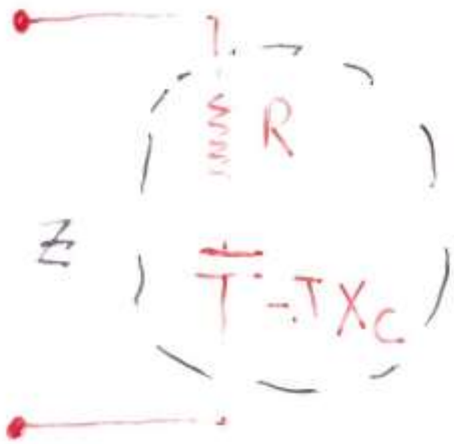
(*a*) Conexión en triángulo (Δ) (*b*) Conexión en estrella (Y)

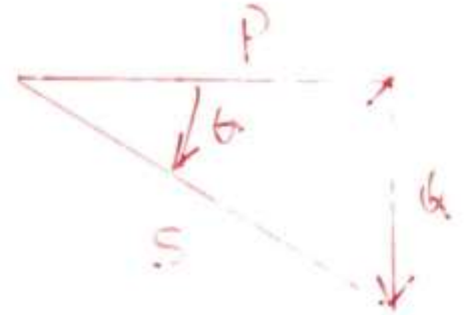




$$S = P + JQ$$

$$|S| = \sqrt{P^2 + Q^2} = V_{ef}\, I_{ef}$$





$$P = \mathrm{Re}[S] = V_{ef} . I_{ef} \cos\theta$$

$$Q = \mathrm{Im}[S] = V_{ef}\, I_{ef} \operatorname{sen}\theta$$

$$S = V_{ef} . I_{ef}$$

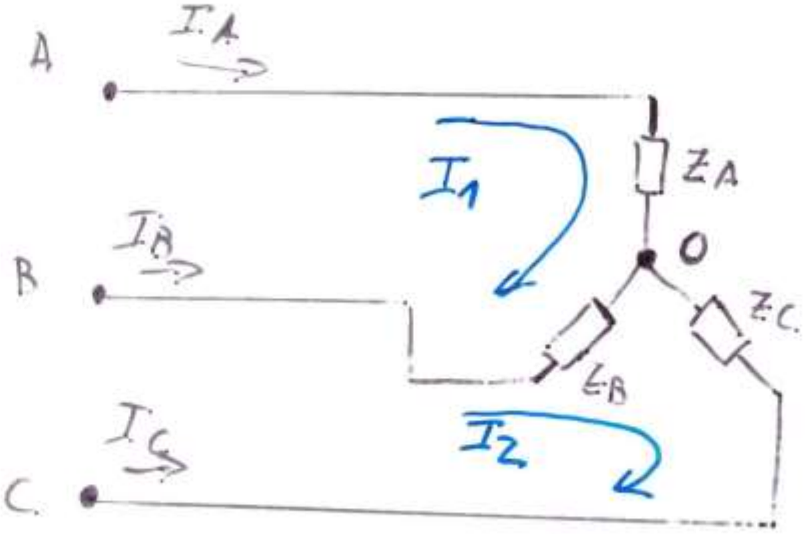
Sist. Trifasico

$$P = \sqrt{3}\, V_L I_L \cos\theta \; ; \; Q = \sqrt{3}\, V_L I_L \operatorname{sen}\theta \; ; \; S = \sqrt{3}\, V_L I_L$$

$$fp = \frac{P}{S}$$

Carga Desequilibrada

Ejemplo Conexión Estrella con tres conductores



Sec. Inversa CBA

$V_L = 208$ Volt ; $Z_A = 6\angle 0^\circ$;

$Z_B = 6\angle 30^\circ$; $Z_C = 5\angle 45^\circ$

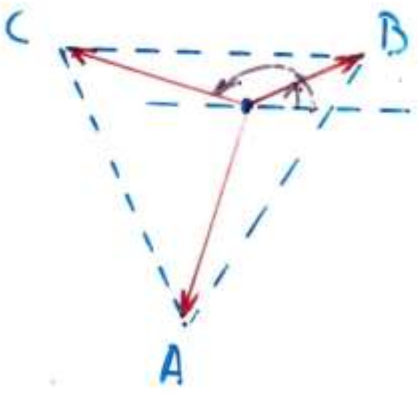
- I_L ? ; V_f ?
- Construir Triangulo de Tensiones
- V_{OM} ?

$$\begin{bmatrix} Z_A + Z_B & -Z_B \\ -Z_B & Z_B + Z_C \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} V_{AB} \\ V_{BC} \end{bmatrix}$$

$$\begin{bmatrix} 6\angle 0^\circ + 6\angle 30^\circ & -(6\angle 30^\circ) \\ -(6\angle 30^\circ) & 6\angle 30^\circ + 5\angle 45^\circ \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 208\angle 240^\circ \\ 208\angle 0^\circ \end{bmatrix}$$

$I_1 = 23{,}3\angle 261{,}1^\circ$

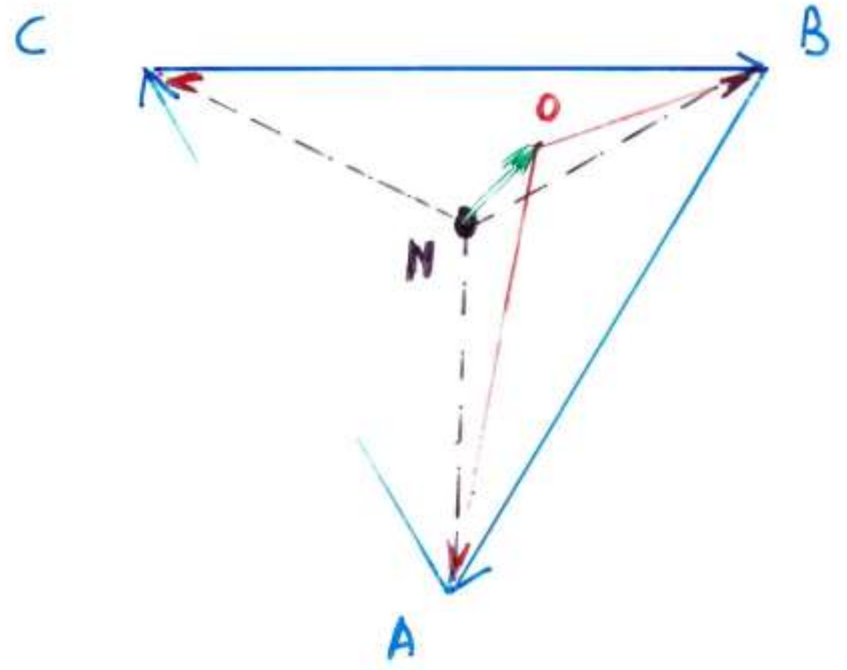
$I_2 = 26{,}5\angle -63{,}4^\circ$



$I_A = I_1 = 23{,}3\angle 261{,}1^\circ$

$I_B = I_2 - I_1 = 26{,}5\angle -63{,}4^\circ - (23{,}3\angle 261{,}1^\circ) = 15{,}4\angle -3{,}5^\circ$

$I_C = -I_2 = 26{,}5\angle 116{,}6^\circ$



$V_{AN} = 120 \angle -90°$

$V_{BN} = 120 \angle 30°$

$V_{CN} = 120 \angle 150°$

$V_{AO} = I_A \cdot Z_A = (23{,}3 \angle 261{,}1°)(6 \angle 0°) = 139{,}8 \angle 261{,}1°$

$V_{BO} = I_B \cdot Z_B = (15{,}4 \angle -2{,}5°)(6 \angle 30°) = 92{,}7 \angle 27{,}5°$

$V_{CO} = I_C \cdot Z_C = (26{,}5 \angle 116{,}6°)(5 \angle 45°) = 132{,}5 \angle 161{,}6°$

$V_{ON} = V_{AN} + V_{OA} = 120 \angle -90° - (139{,}8 \angle 261{,}1°)$

$V_{ON} = 28{,}2 \angle 39{,}1°$

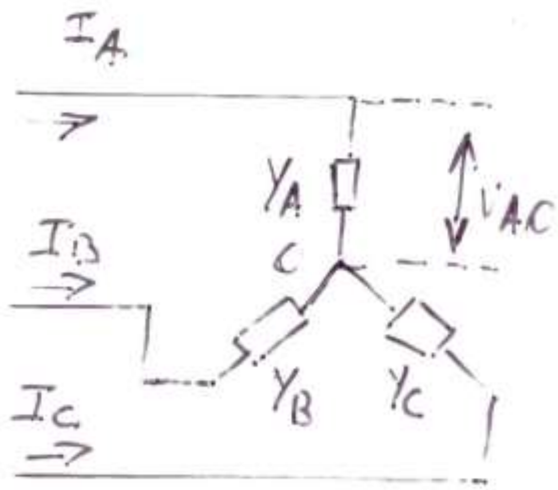
$V_{ON} = V_{BN} + V_{OB}$ ✓

$V_{ON} = V_{CN} + V_{OC}$ ✓

Otro metodo para encontrar V_{ON} :

$I_A = V_{AO}\, Y_A \;;\; I_B = V_{BO}\, Y_B$

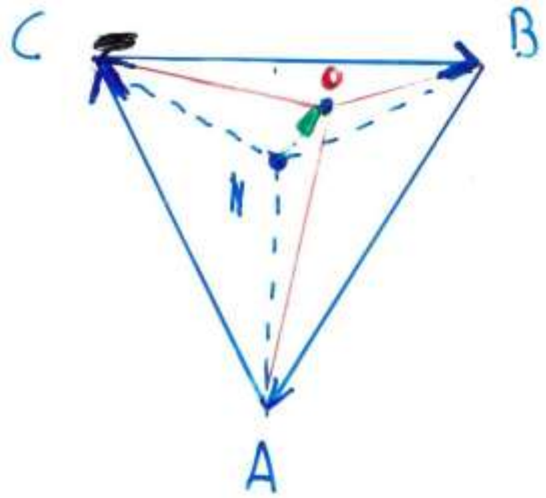
$I_C = V_{CO}\, Y_C$



$\rightarrow \; I_A + I_B + I_C = 0$

$V_{AO} Y_A + V_{BO} Y_B + V_{CO} Y_C = 0$

Del ∇ de Tensiones :



$V_{AO} = V_{AN} + V_{NO} \;;\; V_{BO} = V_{BN} + V_{NO}$

$V_{CO} = V_{CN} + V_{NO}$

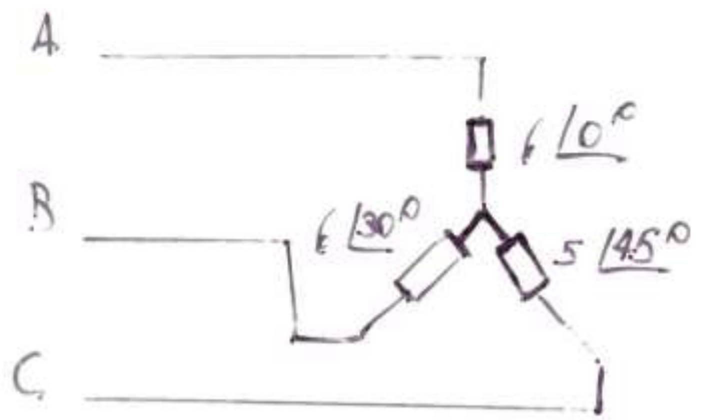
Reemplazando :

$$(V_{AN} + V_{NO}) Y_A + (V_{BN} + V_{NO}) Y_B + (V_{CN} + V_{NO}) Y_C = 0$$

$$V_{AN} Y_A + V_{BN} Y_B + V_{CN} Y_C = - V_{NO} (Y_A + Y_B + Y_C)$$

$$V_{ON} = \frac{V_{AN} Y_A + V_{BN} Y_B + V_{CN} Y_C}{Y_A + Y_B + Y_C}$$

De acuerdo al ejemplo dado:



Sec. Inversa ; $V_L = 208$ Volt.

- I_L ? ; V_f ?

- V_{ON} ?

$$V_{ON} = \frac{V_{AN} Y_A + V_{BN} Y_B + V_{CN} Y_C}{Y_A + Y_B + Y_C}$$

$Y_A = 1/6\angle 0^\circ = 0{,}1667\angle 0^\circ = 0{,}1667$

$Y_B = 1/6\angle 30^\circ = 0{,}1667\angle -30^\circ = 0{,}1443 - J0{,}0833$

$Y_C = 1/5\angle 45^\circ = 0{,}20\angle -45^\circ = 0{,}1414 - J0{,}1414$

$Y_A + Y_B + Y_C = 0{,}4524 - J0{,}2247 = 0{,}504\angle -26{,}5^\circ$

$V_{AN} Y_A = 120\angle -90^\circ (0{,}1667\angle 0^\circ) = 20\angle -90^\circ = -J20$

$V_{BN} Y_B = 120\angle 30^\circ (0{,}1667\angle -30^\circ) = 20\angle 0^\circ = 20$

$V_{CN} Y_C = 120\angle 150^\circ (0{,}20\angle -45^\circ) = 24\angle 105^\circ = -6{,}2 + J23{,}2$

$V_{AN} Y_A + V_{BN} Y_B + V_{CN} Y_C = 13{,}8 + J3{,}2 = 14{,}1\angle 13{,}1^\circ$

$$V_{ON} = \frac{14{,}1\angle 13{,}1^\circ}{0{,}504\angle -26{,}5^\circ} = 28{,}0\angle 39{,}6^\circ$$

Los V_f se obtienen a partir V_{NO}

$$V_{AO} = V_{AN} + V_{NO} = 120 \angle{-90^\circ} - (28{,}0 \angle{39{,}6^\circ}) = 139{,}5 \angle{261^\circ}$$

$$V_{BO} = V_{BN} + V_{NO} = 120 \angle{30^\circ} - (28{,}0 \angle{39{,}6^\circ}) = 92{,}5 \angle{27{,}1^\circ}$$

$$V_{CO} = V_{CN} + V_{NO} = 120 \angle{150^\circ} - (28{,}0 \angle{39{,}6^\circ}) = 132{,}5 \angle{161{,}45^\circ}$$

Los I_L serán:

$$I_A = V_{AO} Y_A = 139{,}5 \angle{261^\circ} \cdot (0{,}1667 \angle{0^\circ}) = 23{,}2 \angle{261{,}1^\circ}$$

$$I_B = V_{BO} \cdot Y_B = 92{,}5 \angle{27{,}1^\circ} \cdot (0{,}1667 \angle{-30^\circ}) = 15{,}4 \angle{-2{,}9^\circ}$$

$$I_C = V_{CO} Y_C = 132{,}5 \angle{161{,}45^\circ} \cdot (0{,}20 \angle{-45^\circ}) = 26{,}5 \angle{116{,}45^\circ}$$