Theorem 7

Problem LP

Let S be a set of m linear inequalities of the form

$$x_j - x_i \leq a_{ij}$$

on the unknowns $x_1, x_2, \ldots, x_n$, where a_{ij} are given real constants.

Any such system can be satisfied — or determined to be inconsistent — in $\mathcal{O}(mn)$ time by the Bellman-Ford algorithm.

Theorem 7

Solving Problem LP

Construct a graph G such that

- For each inequality $x_j - x_i \leq a_{ij}$ in S, there is an edge that goes from x_i to x_j in G, weighted with a_{ij}.

- Add an extra node x_s and connect it with all the other nodes, setting the weight on the newly-created edges to 0.

Run the Bellman-Ford algorithm on such graph with source x_s. If the algorithm terminates, the shortest-paths distances found are a possible solution of S. Otherwise, if it detects a negative cycle, the system is inconsistent.