Min-Degree Constrained Minimum Spanning Tree Problem with Fixed Centrals and Terminals: complexity, properties and formulations

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Abstract

We present complementary computational results to the paper "Min-Degree Constrained Minimum Spanning Tree Problem with Fixed Centrals and Terminals: complexity, properties and formulations".

Keywords: Min-Degree Constrained Minimum Spanning Tree Problem, Integer Programming, Lagrangian Heuristic.

Tables (1)-(4) compare the linear relaxations for the EUC and NEU instances. For each instance and formulation, we calculate the percentage gap as $\%GAP = 100\frac{BUB-LP}{BLB}$, where LP is the optimal value of the linear relaxation, BUB and BLB are the best upper and lower bound obtained in the time limit, respectively. If the optimal value (OPT) is known, we have $BUB = BLB = OPT$. This is the case of all EUC and NEU instances with $c \leq 300$, 4 EUC instances and 14 NEU instances with $c \geq 400$.

The processing time limit (considering a single processor) for each run is fixed in 18000 seconds (5 hours). If this time limit is exceeded, letter 'T' is used in tables 1 and 2. Letter 'M' indicates that the run stopped by lack of memory.

For $c \geq 400$ (tables 3 and 4), the linear relaxations of $\texttt{FMD}_{\text{sec}}$, $\texttt{FMD}_{\text{int}}$, and $\texttt{FMD}^r_{\text{cut}}$ could not be solved within the time limit and available memory for all EUC and NEU instances (except for EUC400-1 and EUC-400-2 with $\texttt{FMD}_{\text{sec}}$, EUC400-3 and EUC400-4 with $\texttt{FMD}^r_{\text{cut}}$). Istead of using the flags "T" and "M", we do present the time spent when the process stopped. A value greater that $18,000s$ means time limit exceeded; otherwise, it means required memory exceeded. In both cases, %GAP given by the lower bound obtained in the last iteration before stopping the cutting-plane algorithm.

Tables 5 and 6 present the processing times of the integer formulations with the EUC and NEU instances.

Table 7 presents the best upper bound (UB) and best lower bound (LB) obtained by $LA1$ and the corresponding %GAP=100(UB-LB)/LB as well as the number ITER of iterations performed and the processing time T(s) in seconds. This time may slightly exceed the limit of 18,000s because the time checking is only made after a complete iteration of the Lagrangian algorithm. The results corresponding to $LA2$ and $LA3$ are presented in tables 8 and 9, respectively. Note that we calculate the gap using the bounds provided by each algorithm separately (not using the

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best known upper bound). The average results on the Lagrangian algorithms are summarized in Table 10. In Table 11, we can find statistics on the number of times each algorithm obtained the best result (upper bound, lower bound and gap). If a tie occurs, we count one for each.

Table 1: EUC Instances with $c \leq 300$ - Results for the linear relaxations.

Instance		$\texttt{FMD}^r_{\text{cut}}$		$\texttt{FMD}_{\text{sec}}$		$\texttt{FMD}_{\text{int}}$		$\texttt{FMD}^r_{\text{mtz}}$		$\texttt{FMD}^r_{\text{flo}}$	
EUC	OPT	%GAP	T (s)	%GAP	T (s)	%GAP	T (s)	%GAP	T (s)	%GAP	T (s)
60-1	6943	0.000	3.10	0.000	1.45	0.000	448.78	6.517	0.08	8.396	0.64
60-2	6772	0.000	3.29	0.000	0.88	0.000	379.99	5.545	0.08	7.948	0.64
60-3	6709	0.000	2.09	0.000	1.25	0.000	647.07	7.199	0.08	9.296	0.66
60-4	7040	0.000	2.12	0.000	1.33	0.000	515.48	7.230	0.09	7.226	0.66
100-1	8217	0.091	17.13	0.091	5.48	0.091	4740.88	4.016	0.30	9.089	6.50
100-2	8069	0.043	14.11	0.043	5.51	0.043	3412.41	3.204	0.23	8.293	6.55
100-3	8055	0.000	14.10	0.000	10.94	0.000	4841.10	5.767	0.30	7.494	7.36
100-4	8139	0.000	11.67	0.000	22.49	0.000	4872.68	5.185	0.48	7.019	7.99
200-1	11509	0.000	715.47	0.000	117.05		M	4.205	1.17	10.161	40.63
200-2	11305	0.148	816.87	0.148	142.92		M	4.728	1.10	10.328	36.53
200-3	11454	0.183	650.93	0.183	2617.33		M	5.509	4.92	8.012	44.60
200-4	11391	0.250	407.39	0.250	3511.63		M	4.846	5.49	7.093	51.95
300-1	13462	0.037	6876.62	0.037	2515.81		M	4.602	3.88	9.605	127.75
300-2	13004	0.012	7156.78	0.012	1843.10		M	4.568	4.39	9.827	147.55
300-3	12797	0.141	2738.98	0.141	16625.18		M	3.907	62.76	7.902	147.73
300-4	12545	0.207	2518.38		T		M	3.926	68.53	7.477	149.39
Average		0.070	1371.81					5.060	9.62	8.448	48.57

Table 2: NEU Instances with $c \leq 300$ - Results for the linear relaxations.

Instance		$\texttt{FMD}^r_{\text{cut}}$		$\texttt{FMD}_{\text{sec}}$		$\texttt{FMD}_{\text{int}}$		$\texttt{FMD}^r_{\text{mtz}}$		$\texttt{FMD}^r_{\text{flo}}$	
NEU	OPT	%GAP	T (s)	%GAP	T (s)	%GAP	T (s)	%GAP	T (s)	%GAP	T (s)
60-1	3032	0.000	2.71	0.000	1.46	0.000	372.04	0.000	0.07	8.648	0.63
60-2	3872	0.000	1.89	0.000	0.16	0.000	101.18	0.052	0.09	5.147	0.70
60-3	2972	0.050	1.83	0.050	0.53	0.050	562.30	0.084	0.08	8.438	0.63
60-4	4031	0.000	1.05	0.000	0.56	0.000	345.55	0.323	0.09	4.703	0.63
100-1	3079	0.000	16.91	0.000	13.04	0.000	2512.83	0.000	0.28	8.794	6.48
100-2	3858	0.000	20.49	0.000	13.36	0.000	2750.22	0.156	0.35	4.403	7.12
100-3	3084	0.016	14.83	0.016	3.26	0.016	3026.45	0.093	0.50	11.670	6.70
100-4	3655	0.137	7.87	0.137	9.00	0.137	4494.43	0.561	0.57	4.703	7.71
200-1	3326	0.000	993.73	0.000	388.16		M	0.000	1.27	7.428	44.25
200-2	4280	0.000	906.18	0.000	258.49		M	0.327	2.30	6.211	40.66
200-3	3388	0.044	467.03	0.044	153.74		M	0.199	20.80	8.366	46.42
200-4	4223	0.034	661.51	0.034	552.68		M	0.462	21.02	5.572	48.23
300-1	3462	0.000	17709.72		T		M	0.087	27.96	7.674	125.23
300-2	4382	0.000	14044.14		T		M	0.068	34.01	5.730	175.13
300-3	3326	0.090	7569.06	0.090	2678.31		M	0.120	206.50	9.736	128.76
300-4	4001	0.017	7965.08	0.017	13556.42		M	0.264	82.26	6.903	152.94
Average		0.024	3149.00					0.175	24.88	7.133	49.51

Table 3: EUC Instances with $c \geq 400$ - Results for the linear relaxations.

Instance		$\texttt{FMD}^r_{\text{cut}}$		$\texttt{FMD}_{\text{sec}}$		$\texttt{FMD}^r_{\text{mtz}}$		$\texttt{FMD}^r_{\text{flow}}$	
EUC	Best UB	%GAP	T(s)	%GAP	T(s)	%GAP	T(s)	%GAP	T(s)
400-1	15518*	9.256	18197.65	0.026	8875.75	3.989	6.65	9.773	302.03
400-2	15116*	9.548	18026.74	0.060	8661.79	4.303	6.51	10.017	297.34
400-3	15397*	0.013	13882.43	2.780	18002.33	2.535	45.05	5.540	285.84
400-4	15255*	0.100	9412.28	3.407	18002.95	2.324	36.01	5.726	312.12
500-1	17320	11.731	18122.22	0.191	18060.57	5.225	12.08	11.827	627.15
500-2	17161	11.371	18112.33	0.508	18005.01	4.535	11.56	11.454	559.63
500-3	17288	9.195	18014.14	6.323	18097.62	5.184	99.65	9.547	612.40
500-4	17939	8.195	18037.68	9.074	18233.15	4.841	140.97	8.313	598.00
600-1	18983	11.209	12655.51	1.441	18033.17	4.328	22.38	11.245	838.84
600-2	18653	11.234	11346.48	2.750	18140.84	4.469	21.52	11.286	992.17
600-3	19722	10.231	18103.73	12.006	18159.87	6.018	867.67	10.144	1015.51
600-4	19651	8.583	18059.84	12.256	18024.91	4.554	921.46	8.585	1051.18
700-1	20397	10.445	18009.54	2.877	18057.62	4.096	35.16	10.358	1497.71
700-2	19935	11.394	6866.07	3.227	18070.34	4.429	32.3	11.332	1409.28
700-3	21216	8.846	18110.67	13.956	18082.89	5.306	1310.35	8.807	1883.43
700-4	21084	10.061	8581.43	15.817	18011.11	6.034	1388.83	9.924	1791.15
800-1	21617	10.885	8478.99	3.599	18319.63	3.960	41.64	10.787	2325.49
800-2	21326	11.292	18069.36	3.294	18407.64	3.976	42.35	11.137	2521.00
800-3	23964	7.597	16801.28	19.921	18129.02	4.546	740.69	7.394	3005.71
800-4	26246	8.650	13930.06	31.984	16411.62	6.033	1241.81	8.268	3316.81
900-1	22825	11.057	16843.09	3.921	18011.02	4.126	59.66	10.934	2885.74
900-2	22207	11.092	9979.33	3.925	18526.47	4.087	57.34	10.940	2994.80
900-3	27329	8.364	6157.33	31.436	17374.04	5.499	1411.98	8.104	4141.80
900-4	26701	8.712	18068.72	31.382	16380.68	6.058	1496.85	8.506	4231.92
	Average	9.128	14661.12	9.007	17170.00	4.602	418.77	9.581	1645.71
	Std. Deviation	3.052		10.237		0.976		1.749	
Avg. for $g = 1, 2$		10.876	14558.94	2.152	16597.49	4.294	29.10	10.924	1437.60
Avg. for $g = 3, 4$		7.379	14763.30	15.862	17742.52	4.911	808.44	8.238	1853.82

(*) Optimal value.

Table 4: NEU Instances with $c \geq 400$ - Results for the linear relaxations.

Instance		$\mathtt{FMD}^r_{\mathrm{cut}}$		$\mathtt{FMD}_{\mathrm{sec}}$		$\mathtt{FMD}^r_{\mathrm{mtz}}$		$\mathtt{FMD}^r_{\mathrm{flow}}$	
NEU	Best UB	%GAP	T(s)	%GAP	T(s)	%GAP	T(s)	%GAP	T(s)
400-1	3547*	7.417	18004.97	0.470	18029.80	0.056	19.17	7.690	296.01
400-2	4220*	6.105	18004.95	0.926	18517.80	0.095	13.02	6.337	261.45
400-3	3704	15.963	18076.74	6.456	18016.84	6.204	291.98	16.329	333.74
400-4	4322	9.879	18109.64	3.272	18010.15	3.191	408.30	10.142	302.01
500-1	3510*	8.072	18035.86	1.168	18058.10	0.000	17.75	8.135	554.60
500-2	4360*	5.503	18049.12	1.126	18138.17	0.000	25.80	5.551	480.58
500-3	3669	15.880	18040.53	6.935	18068.13	6.548	688.72	15.951	572.69
500-4	4322*	5.750	18049.09	0.641	18445.92	0.042	583.79	5.783	733.49
600-1	3663*	7.105	18291.25	1.252	18013.41	0.000	87.72	7.120	800.22
600-2	4544*	5.313	18036.00	0.911	18143.02	0.022	521.08	5.330	1019.53
600-3	3472*	9.281	18069.37	0.495	18034.68	0.037	1039.87	9.287	942.72
600-4	4620	12.179	18075.02	5.321	18060.56	4.426	1006.60	12.175	1176.85
700-1	3812	7.226	18039.74	1.406	18012.55	0.079	239.67	7.232	1199.38
700-2	4758*	5.616	18002.38	1.582	15027.52	0.000	902.60	5.603	1467.62
700-3	3677*	8.872	18009.09	0.835	15023.31	0.044	2034.53	8.859	1579.08
700-4	4564*	5.390	18000.52	0.940	18320.73	0.029	1581.40	5.364	1597.86
800-1	3943*	7.229	18002.58	1.577	18035.79	0.000	80.97	7.198	2002.50
800-2	4936*	4.477	18006.50	1.241	15473.63	0.000	65.00	4.475	2250.58
800-3	3709*	7.111	6305.36	0.592	18250.55	0.053	2260.13	7.099	2140.82
800-4	4884	9.656	18006.99	4.786	18432.15	3.810	2110.06	9.641	2951.75
900-1	4154	5.974	18111.29	1.814	13910.65	0.096	72.09	5.969	2309.17
900-2	5049	4.860	18004.60	1.752	15284.48	0.258	211.92	4.835	3165.17
900-3	4181	14.611	18001.02	7.119	16342.88	6.218	2728.74	14.553	6369.64
900-4	5078	10.152	18020.64	5.511	17859.98	4.556	3257.42		18100.00
	Average	8.105	17561.04	2.417	17383.20	1.490	843.68	8.064	2191.98
	Std. Deviation	3.441		2.244		2.395		3.528	
Avg. for $g = 1, 2$		5.968	18061.57	1.290	17042.85	0.051	188.07	6.010	1317.23
Avg. for $g = 3, 4$		10.242	17060.52	3.574	17723.55	2.930	1499.29	10.305	3066.72

(*) Optimal value.

Table 5: EUC Instances - Results for integer solution.

Instance		T(s)	
EUC	OPT	$\mathtt{FMD}^r_{\mathrm{cut}}$	$\mathtt{FMD}_{\mathrm{sec}}$
60-1	6943	3.10	1.45
60-2	6772	3.29	0.88
60-3	6709	2.09	1.25
60-4	7040	2.12	1.33
100-1	8217	18.01	13.72
100-2	8069	15.65	7.23
100-3	8055	14.10	10.94
100-4	8139	11.67	22.49
200-1	11509	715.47	117.05
200-2	11305	936.63	248.80
200-3	11454	806.21	3856.10
200-4	11391	2795.47	9031.79
300-1	13463	6928.37	2870.33
300-2	13004	7213.40	2161.36
300-3	12797	5555.20	M
300-4	12545	4208.55	T
400-1	15518	T	9384.88
400-2	15116	T	12121.26
400-3	15397	13982.72	T
400-4	15255	10499.41	T
Instances optimally solved		45%	40%

Table 6: NEU Instances - Results for integer solution.

Instance		T(s)			
NEU	OPT	$\texttt{FMD}^r_{\text{cut}}$	$\texttt{FMD}_{\text{sec}}$	$\texttt{FMD}^r_{\text{mtz}}$ (??)	$\texttt{FMD}^r_{\text{mtz}}$ (??)
60-1	3032	2.71	1.46	0.46	0.55
60-2	3872	1.89	0.16	1.56	0.97
60-3	2972	4.99	1.27	1.47	1.02
60-4	4031	1.05	0.56	5.01	7.99
100-1	3079	16.91	13.04	4.43	1.41
100-2	3858	21.52	20.10	5.70	4.09
100-3	3084	16.45	5.89	12.92	9.22
100-4	3655	13.94	15.06	312.50	384.46
200-1	3326	993.73	388.16	43.79	16.51
200-2	4280	906.18	314.58	1444.25	2058.99
200-3	3388	489.28	222.29	744.91	1326.11
200-4	4223	709.30	854.86	M	M
300-1	3462	17709.72	T	119.36	2580.86
300-2	4382	14111.02	T	402.90	6599.00
300-3	3326	7708.38	3366.65	9281.42	1370.94
300-4	4001	8012.62	M	T	T
400-1	3547	T	T	742.20	15106.18
400-2	4220	T	T	733.38	7220.58
400-3		T	T	T	T
400-4		T	T	T	T
500-1	3510	T	T	419.13	148.13
500-2	4360	T	T	376.33	556.06
500-3		T	T	T	T
500-4	4322	T	T	1644.48	1516.53
600-1	3663	T	T	879.19	1212.99
600-2	4544	T	T	2743.78	M
600-3	3472	T	T	3354.97	4935.26
600-4		T	T	M	M
700-1	3809	T	T	M	1207.72
700-2	4758	T	M	1076.31	M
700-3	3677	T	M	M	6094.65
700-4	4564	T	T	8174.96	M
800-1	3943	T	T	1634.48	M
800-2	4936	T	M	632.51	M
800-3	3709	M	T	10632.29	M
800-4		T	T	M	M
900-1		T	M	M	M
900-2		T	M	M	M
900-3		T	M	M	M
900-4		T	M	M	M
Instances optimally solved		40%	32.5%	67.5%	60%

Table 7: Computational results for $LA1$.

#	EUC					NEU				
	UB	LB	%GAP	ITER	T(s)	UB	LB	%GAP	ITER	T(s)
60-1	6943	6942.12	0.013	38	0.14	3032	3031.11	0.029	43	0.10
60-2	6772	6772.00	0.000	48	0.20	3872	3871.76	0.006	37	0.10
60-3	6709	6708.35	0.010	258	0.73	2972	2970.41	0.053	613	0.96
60-4	7040	7039.20	0.011	238	0.62	4031	4030.22	0.019	229	0.41
100-1	8217	8209.37	0.093	491	3.34	3079	3078.17	0.027	177	2.12
100-2	8069	8065.42	0.044	678	5.22	3858	3857.47	0.014	135	1.14
100-3	8055	8051.94	0.038	943	7.09	3095	3083.04	0.388	698	5.51
100-4	8139	8122.24	0.206	1291	9.04	3715	3647.21	1.859	664	6.30
200-1	11509	11508.53	0.004	376	23.26	3326	3325.25	0.023	293	27.86
200-2	11319	11286.79	0.285	637	74.76	4282	4279.79	0.052	876	120.65
200-3	11536	11339.83	1.730	1186	139.93	3505	3377.59	3.772	619	85.22
200-4	11638	11137.45	4.494	1071	120.41	4335	4212.60	2.906	644	109.60
300-1	13468	13456.49	0.086	695	273.07	3462	3461.52	0.014	428	136.40
300-2	13004	13002.45	0.012	685	184.07	4382	4381.88	0.003	716	344.07
300-3	13127	12735.09	3.077	810	370.22	3725	3305.09	12.705	512	324.30
300-4	12814	12453.70	2.893	740	312.56	4311	3983.66	8.217	571	401.18
400-1	15523	15513.62	0.060	778	811.52	3549	3546.70	0.065	684	873.88
400-2	15119	15106.49	0.083	934	771.66	4222	4219.45	0.060	571	919.60
400-3	15723	15296.38	2.789	769	1405.15	3921	3462.53	13.241	391	993.97
400-4	15655	15178.38	3.140	989	1392.26	4322	4174.77	3.527	707	1738.41
500-1	17320	17291.08	0.167	866	2387.53	3515	3509.47	0.158	680	2455.30
500-2	17161	17153.01	0.047	772	1930.21	4364	4359.12	0.112	466	1718.36
500-3	17406	16936.94	2.769	523	1900.94	3777	3422.53	10.357	544	2480.97
500-4	18116	17626.66	2.776	904	3624.80	4531	4304.51	5.262	679	4172.38
600-1	18983	18964.28	0.099	755	5151.59	3668	3662.39	0.153	363	2444.77
600-2	18658	18646.02	0.064	818	4832.53	4556	4542.89	0.288	692	4752.32
600-3	19972	19263.59	3.677	837	5979.75	3925	3441.52	14.049	720	6685.81
600-4	19990	19159.53	4.335	771	6410.18	4696	4407.32	6.550	687	6945.65
700-1	20397	20375.68	0.105	699	7587.39	3812	3808.33	0.096	531	6065.08
700-2	19935	19894.24	0.205	771	9561.97	4763	4757.44	0.117	841	10499.87
700-3	21529	20694.11	4.034	908	14647.77	4165	3646.97	14.205	431	9943.11
700-4	22051	20027.66	10.103	808	14089.94	4896	4539.37	7.856	509	10045.86
800-1	21617	21592.29	0.114	614	8363.12	3953	3942.04	0.278	478	9816.33
800-2	21332	21302.92	0.137	659	9089.91	4946	4935.14	0.220	582	12931.48
800-3	24769	22645.29	9.378	467	18483.26	4300	3673.64	17.050	563	14238.78
800-4	28985	21155.27	37.011	354	18239.69	4902	4682.36	4.691	591	18377.94
900-1	22831	22779.18	0.228	467	18238.88	4154	4149.37	0.112	453	14272.24
900-2	22241	22077.12	0.742	320	18400.37	5049	5029.88	0.380	393	18356.91
900-3	30010	21222.26	41.408	150	19401.55	4700	3766.77	24.775	305	18295.71
900-4	29271	19952.75	46.702	154	19098.28	5396	4726.73	14.159	309	18648.23

Table 8: Computational results for $LA2$.

#	EUC					NEU				
	UB	LB	%GAP	ITER	T(s)	UB	LB	%GAP	ITER	T(s)
60-1	6943	6942.25	0.011	42	0.09	3032	3031.11	0.029	35	0.09
60-2	6772	6771.18	0.012	43	0.11	3872	3871.33	0.017	55	0.15
60-3	6709	6708.21	0.012	149	0.23	2973	2969.93	0.103	696	1.38
60-4	7040	7039.51	0.007	248	0.35	4031	4030.47	0.013	280	0.59
100-1	8221	8209.26	0.143	395	2.06	3079	3078.83	0.006	431	4.35
100-2	8069	8065.37	0.045	617	3.40	3859	3857.85	0.030	555	3.83
100-3	8055	8053.07	0.024	1828	7.45	3095	3082.76	0.397	697	5.54
100-4	8148	8114.90	0.408	934	5.77	3667	3649.35	0.484	911	6.28
200-1	11509	11508.25	0.006	94	8.88	3327	3325.81	0.036	526	34.09
200-2	11317	11286.82	0.267	634	53.39	4284	4279.48	0.106	629	69.63
200-3	11641	11308.73	2.938	795	87.91	3488	3381.36	3.154	714	62.89
200-4	11567	11188.48	3.383	1284	135.15	4354	4216.48	3.262	854	121.05
300-1	13471	13456.62	0.107	607	237.14	3462	3461.93	0.002	834	302.67
300-2	13004	13002.44	0.012	741	175.34	4383	4381.92	0.025	469	193.41
300-3	13081	12740.05	2.676	866	402.08	3592	3309.59	8.533	619	354.29
300-4	12771	12450.65	2.573	840	279.16	4079	3989.06	2.255	497	328.89
400-1	15523	15513.61	0.061	568	556.35	3547	3546.80	0.006	782	963.45
400-2	15119	15106.33	0.084	692	587.54	4225	4219.44	0.132	516	792.59
400-3	15723	15295.90	2.792	837	1285.63	3911	3466.46	12.824	541	1285.17
400-4	15582	15156.14	2.810	880	1120.80	4368	4176.35	4.589	479	1086.49
500-1	17323	17290.92	0.186	779	2101.44	3516	3508.97	0.200	580	2085.77
500-2	17175	17152.44	0.132	508	1238.81	4364	4358.70	0.122	582	2534.95
500-3	17288	16951.51	1.985	932	2319.18	3799	3428.14	10.818	706	3038.40
500-4	17939	17626.35	1.774	990	3261.69	4559	4305.05	5.899	769	3704.83
600-1	18994	18963.68	0.160	559	3710.44	3673	3662.03	0.300	693	5070.69
600-2	18653	18645.97	0.038	691	4235.74	4560	4542.76	0.380	576	3965.67
600-3	19786	19293.38	2.553	1049	7299.71	3747	3451.56	8.559	787	6757.27
600-4	19651	19192.42	2.389	990	7221.65	4764	4407.95	8.077	403	3715.14
700-1	20418	20374.78	0.212	776	7755.56	3815	3807.99	0.184	533	5685.88
700-2	19947	19894.24	0.265	573	7398.48	4761	4757.14	0.081	778	9396.37
700-3	21216	20697.79	2.504	1052	15505.55	4070	3649.94	11.509	393	6450.09
700-4	21422	20080.31	6.682	972	11710.13	4828	4543.27	6.267	638	12321.80
800-1	21620	21591.43	0.132	883	12240.23	3946	3942.26	0.095	565	14003.49
800-2	21326	21303.68	0.105	757	10063.67	4948	4934.57	0.272	510	18609.50
800-3	24271	22795.97	6.471	670	18243.49	4108	3676.34	11.741	468	18287.22
800-4	27449	22013.18	24.693	408	18271.82	5085	4576.71	11.106	306	18833.85
900-1	22825	22782.61	0.186	712	17794.87	4162	4128.57	0.810	240	18610.77
900-2	22207	22179.60	0.124	703	15113.27	5076	4987.43	1.776	217	19801.86
900-3	28686	22959.66	24.941	254	18400.41	4461	3903.05	14.295	377	18179.89
900-4	28086	21930.94	28.066	266	18383.02	5280	4580.76	15.265	311	18769.49

Table 9: Computational results for $LA3$.

#	EUC					NEU				
	UB	LB	%GAP	ITER	T(s)	UB	LB	%GAP	ITER	T(s)
60-1	7004	6937.30	0.961	643	0.38	3073	3023.20	1.647	578	0.42
60-2	6844	6763.54	1.190	462	0.27	3924	3862.14	1.602	398	0.49
60-3	6714	6708.92	0.076	607	0.25	3081	2964.42	3.933	599	0.51
60-4	7040	7039.25	0.011	195	0.08	4073	4013.46	1.484	397	0.46
100-1	8289	8192.48	1.178	422	1.02	3122	3066.98	1.794	578	2.05
100-2	8148	8048.64	1.235	363	0.80	3955	3846.32	2.826	457	2.97
100-3	8058	8054.17	0.047	1057	1.36	3197	3068.09	4.202	517	1.03
100-4	8153	8130.46	0.277	1061	1.35	3776	3627.21	4.102	656	1.87
200-1	11589	11469.51	1.042	617	9.10	3364	3309.10	1.659	475	17.98
200-2	11484	11243.30	2.141	528	8.69	4348	4241.65	2.507	382	28.73
200-3	11614	11312.75	2.663	942	5.26	3514	3349.18	4.921	557	7.41
200-4	11596	11151.27	3.988	1483	8.24	4404	4176.59	5.445	555	16.61
300-1	13614	13407.58	1.540	413	39.14	3502	3429.18	2.124	525	91.25
300-2	13135	12961.64	1.337	470	35.10	4510	4340.85	3.897	660	168.15
300-3	12958	12734.91	1.752	917	17.71	3566	3280.22	8.712	503	25.01
300-4	12900	12431.68	3.767	609	9.19	4180	3956.52	5.648	609	60.90
400-1	15650	15439.45	1.364	441	119.38	3610	3508.19	2.902	426	215.48
400-2	15240	15040.96	1.323	472	140.36	4285	4172.32	2.701	449	419.97
400-3	15730	15285.97	2.905	969	56.31	3704	3439.05	7.704	487	68.09
400-4	15523	15189.77	2.194	1010	40.02	4360	4139.36	5.330	565	142.60
500-1	17439	17223.71	1.250	390	441.45	3552	3459.23	2.682	441	526.51
500-2	17306	17079.72	1.325	447	417.11	4431	4305.96	2.904	460	976.44
500-3	17402	16940.29	2.726	710	65.32	3669	3388.65	8.273	539	162.60
500-4	18016	17608.86	2.312	816	82.03	4442	4268.75	4.058	554	306.97
600-1	19164	18871.44	1.550	506	838.87	3744	3606.83	3.803	445	877.94
600-2	18832	18549.88	1.521	402	749.14	4676	4486.45	4.225	391	1658.92
600-3	19722	19295.69	2.209	1196	190.16	3802	3410.62	11.475	490	256.99
600-4	19691	19180.56	2.661	977	147.65	4620	4362.55	5.901	479	546.02
700-1	20620	20279.08	1.681	361	1768.96	3842	3754.04	2.343	411	1553.58
700-2	20151	19786.15	1.844	400	1427.52	4846	4701.47	3.074	358	2933.87
700-3	21220	20691.96	2.552	1085	263.34	4029	3615.71	11.430	440	420.33
700-4	21084	20168.66	4.538	1164	259.27	4739	4499.33	5.327	420	1004.26
800-1	21770	21496.79	1.271	414	2205.06	4023	3877.74	3.746	519	2567.69
800-2	21534	21191.92	1.614	583	2247.74	4967	4870.01	1.991	423	5079.25
800-3	23964	23130.54	3.603	1124	463.22	3918	3647.24	7.424	534	679.68
800-4	26246	24508.19	7.091	1427	605.33	4884	4630.79	5.468	362	1618.74
900-1	23057	22615.27	1.953	400	4182.67	4238	4077.64	3.933	336	3702.43
900-2	22465	22051.09	1.877	435	3373.68	5084	4956.41	2.574	306	6078.31
900-3	27329	25916.65	5.450	1287	857.59	4181	3870.28	8.028	683	918.91
900-4	26701	24780.00	7.752	1046	685.73	5078	4781.59	6.199	532	1929.87

Table 10: Summary of computational results for the Lagrangian algorithms.

	EUC Instances			NEU Instances		
Algorithm	%GAP	Std. Dev.	T (s)	%GAP	Std. Dev.	T (s)
			Averages for all instances			
$LA1$	4.579	11.025	5333.12	4.196	6.212	4980.97
$LA2$	3.049	6.812	5180.45	3.594	4.890	5385.99
$LA3$	2.194	1.683	544.15	4.500	2.551	876.78
			Averages for all instances with $g = 1, 2$			
$LA1$	0.129	0.163	4384.54	0.110	0.107	4286.93
$LA2$	0.114	0.082	4163.84	0.230	0.410	5106.46
$LA3$	1.460	0.315	900.32	2.747	0.828	1345.12
			Averages for all instances with $g = 3, 4$			
$LA1$	9.029	14.414	6281.71	8.282	6.637	5675.02
$LA2$	5.984	8.781	6197.06	6.958	5.010	5665.53
$LA3$	2.929	2.139	187.97	6.253	2.491	408.44

Table 11: Statistics of best lower bound, upper bound and percentage gap - Number of instances per Lagrangian algorithm.

Algorithm	EUC Instances			NEU Instances		
	Best UB	Best LB	Best %GAP	Best UB	Best LB	Best %GAP
Counting for all instances						
$LA1$	21	17	18	22	16	20
$LA2$	22	13	14	14	23	11
$LA3$	9	10	8	11	1	9
Counting for all instances with $g = 1, 2$						
$LA1$	15	13	14	17	13	15
$LA2$	12	7	6	8	7	5
$LA3$	0	0	0	0	0	0
Counting for all instances with $g = 3, 4$						
$LA1$	6	4	4	5	3	5
$LA2$	10	6	8	6	16	6
$LA3$	9	10	8	11	1	9