# Sets & Functions

## What every programmer must know about it

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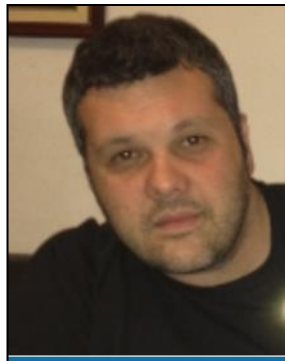


**First year awarded:**
2002

**Number of MVP Awards:**
11

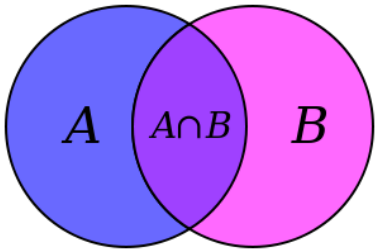
**Technical Expertise:**
Visual C++

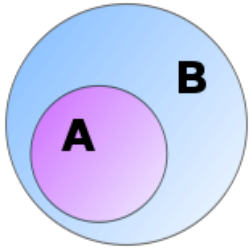
**Technical Interests:**
Visual C#, Visual F#

$$\left\{(x,y) \in \mathbb{R}^2 : \left(\frac{x^2}{3^2} + \frac{y^2}{4^2} - 1 = 0\right) \wedge (x + 2y - 3 = 0)\right\}$$

# Sets

"A collection of well defined and distinct objects" http://en.wikipedia.org/wiki/Set_(mathematics)



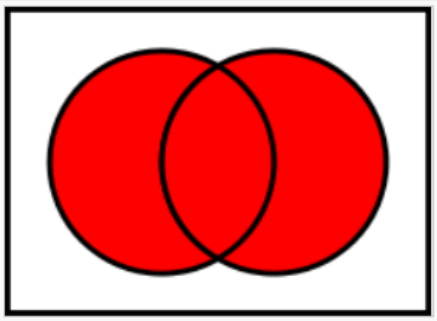

Things like 5, $\frac{3}{7}$, $\sqrt{2}$, $\pi$, $2i+3$ are *elements.*

Things like $\mathbb{N}$, $\mathbb{Z}$, $\mathbb{Q}$, $\mathbb{R}$, and $\mathbb{C}$ are *sets.*

primitive terms in set theory

http://cs.wheaton.edu/~tvandrun/dmfp/slides/a_setelem.pdf

# Set Operations

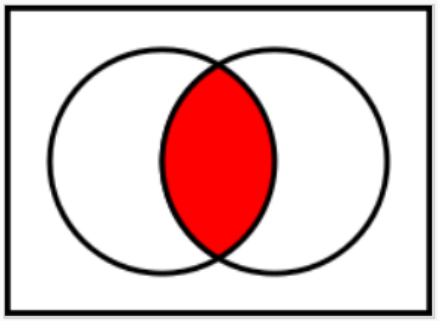


The **union** of $A$ and $B$, denoted $A \cup B$

$\{1, 2\} \cup \{1, 2\} = \{1, 2\}.$

$\{1, 2\} \cup \{2, 3\} = \{1, 2, 3\}.$

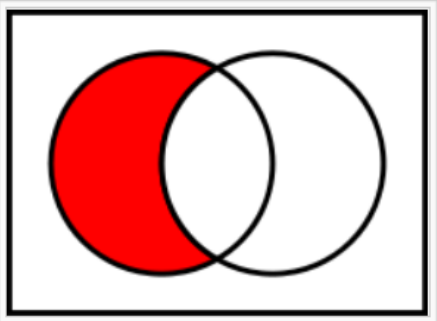
$\{1, 2, 3\} \cup \{3, 4, 5\} = \{1, 2, 3, 4, 5\}$



The **intersection** of $A$ and $B$, denoted $A \cap B$.

$\{1, 2\} \cap \{1, 2\} = \{1, 2\}.$

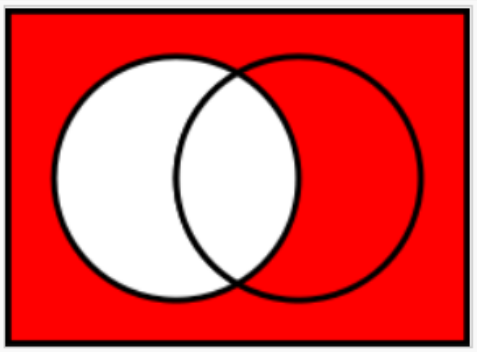
$\{1, 2\} \cap \{2, 3\} = \{2\}.$



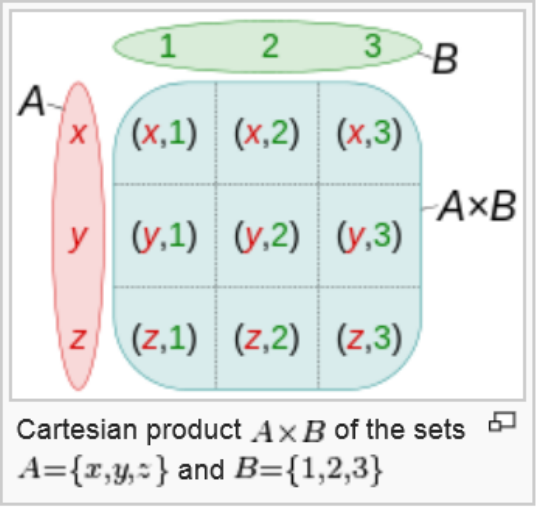
The **relative complement** of $B$ in $A$

$\{1, 2\} \setminus \{1, 2\} = \varnothing.$

$\{1, 2, 3, 4\} \setminus \{1, 3\} = \{2, 4\}.$



The **complement** of $A$ in $U$



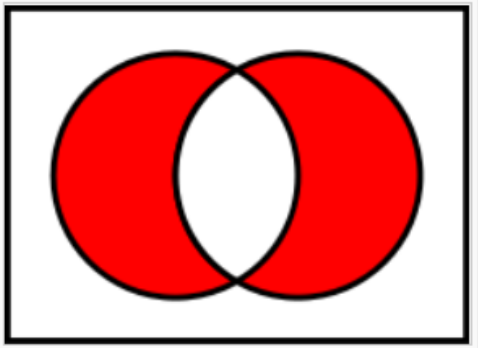

Cartesian product $A \times B$ of the sets $A = \{x, y, z\}$ and $B = \{1, 2, 3\}$

$$A \times B = \{ (a, b) \mid a \in A \text{ and } b \in B \}$$

$A = \{1,2\}$; $B = \{3,4\}$

$A \times B = \{1,2\} \times \{3,4\} = \{(1,3), (1,4), (2,3), (2,4)\}$

$B \times A = \{3,4\} \times \{1,2\} = \{(3,1), (3,2), (4,1), (4,2)\}$



The **symmetric difference** of $A$ and $B$

$$A \Delta B = (A \setminus B) \cup (B \setminus A)$$

symmetric difference of {7,8,9,10} and {9,10,11,12} is the set {7,8,11,12}

http://en.wikipedia.org/wiki/Set_(mathematics) and http://en.wikipedia.org/wiki/Cartesian_product

# Relations

- A Binary Relation from $A$ to $B$ is a subset of $A \times B$

pre-image

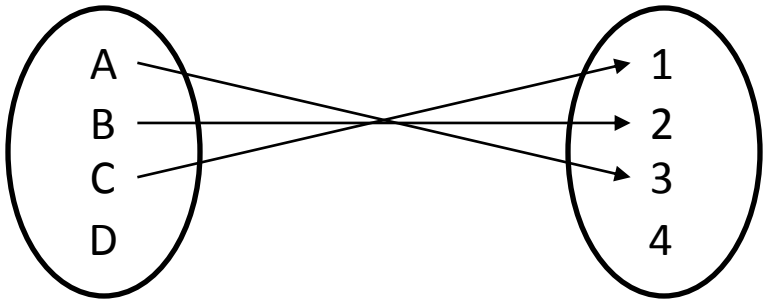
Domain → $A = \{Mark\ Twain, Lewis\ Carroll, Charles\ Dickens, Stephen\ King\}$

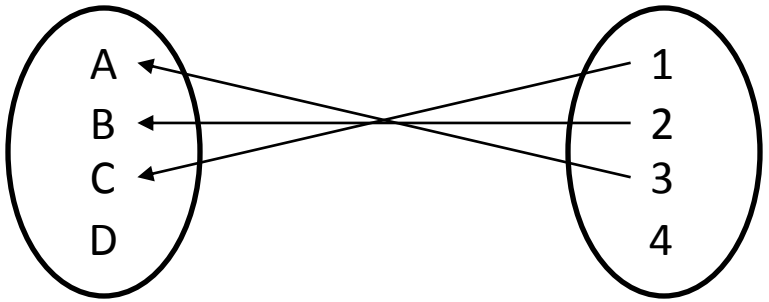
image

Codomain → $B = \{A\ Christmas\ Carol, Alice's\ Adventures\ in\ Wonderland, The\ Adventures\ of\ Tom\ Sawyer, The\ Left\ Hand\ of\ Darkness\}$

$$R = \{(Mark\ Twain, The\ Adventures\ of\ Tom\ Sawyer), (Lewis\ Carroll, Alice's\ Adventures\ in\ Wonderland), (Charles\ Dickens, A\ Christmas\ Carol)\}$$

$$R^{-1} = \{(The\ Adventures\ of\ Tom\ Sawyer, Mark\ Twain), (Alice's\ Adventures\ in\ Wonderland, Lewis\ Carroll), (A\ Christmas\ Carol, Charles\ Dickens)\}$$




The pre-image set of R is {Mark Twain, Lewis Carroll, Charles Dickens}
The image set of R is {The Adventures of Tom Sawyer, Alice's Adventures in Wonderland, A Christmas Carol}

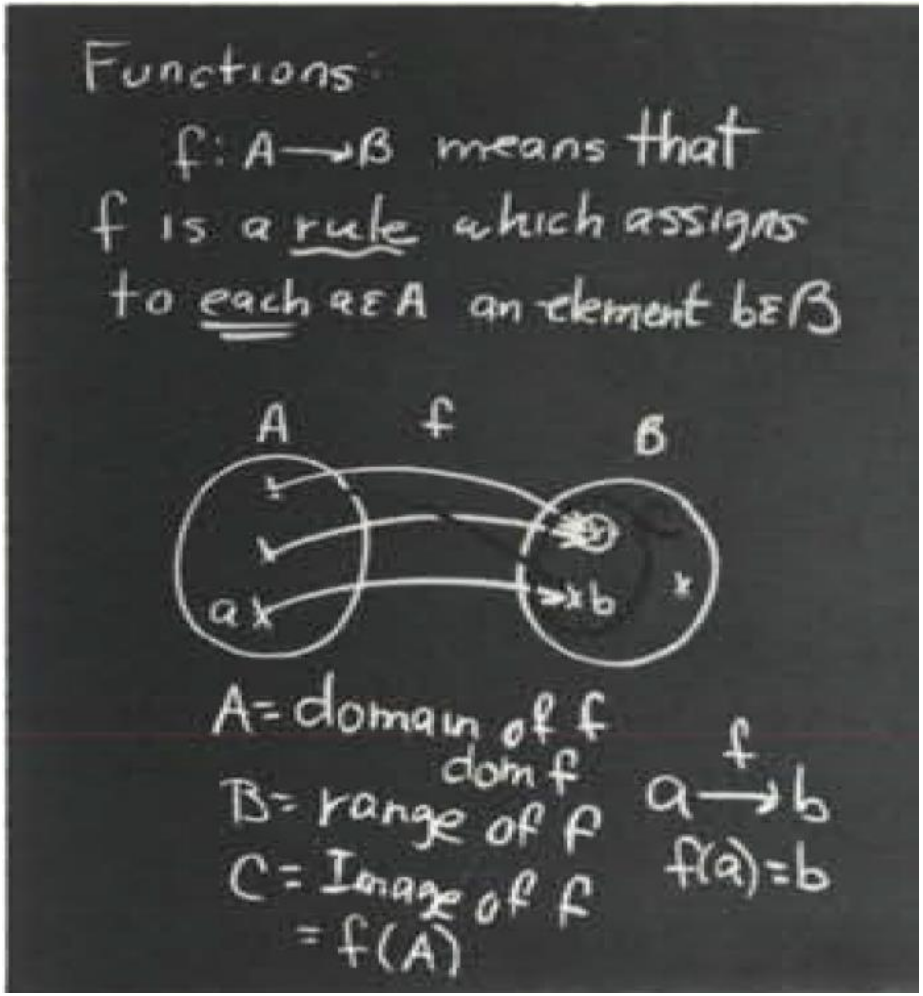
# Functions

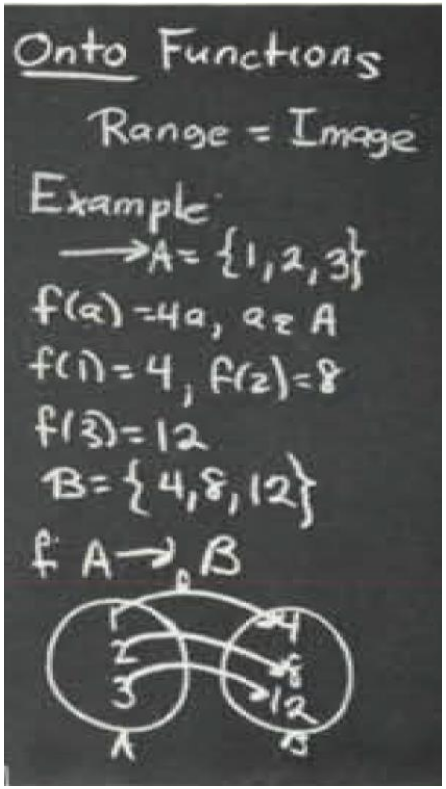
a function is a relation between a set of inputs and a set of permissible outputs - http://en.wikipedia.org/wiki/Function_(mathematics)

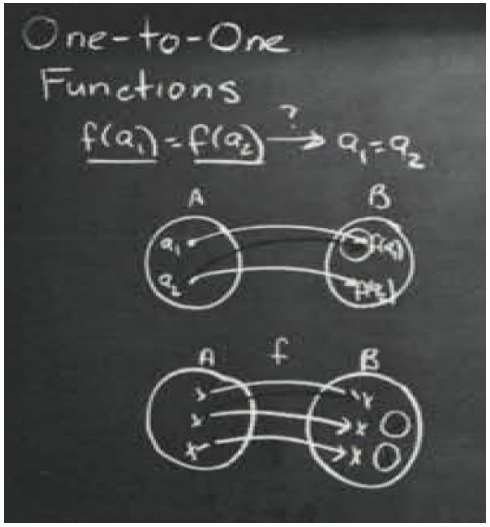
bijective function

surjective function

injective function

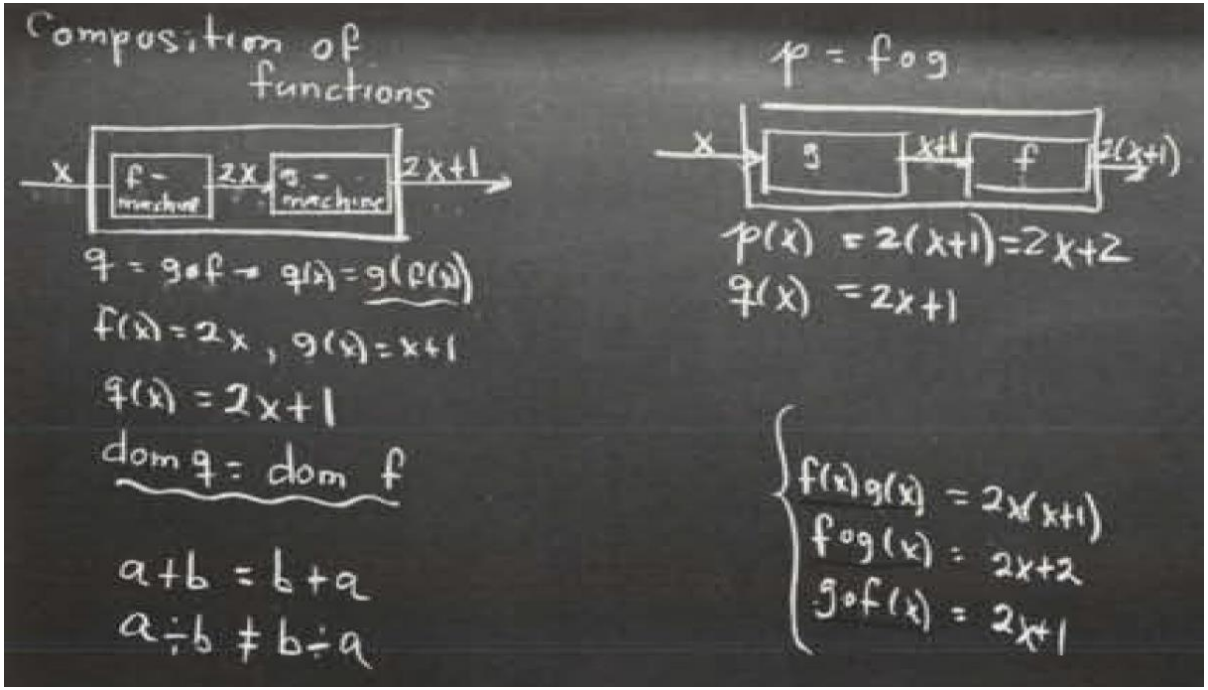


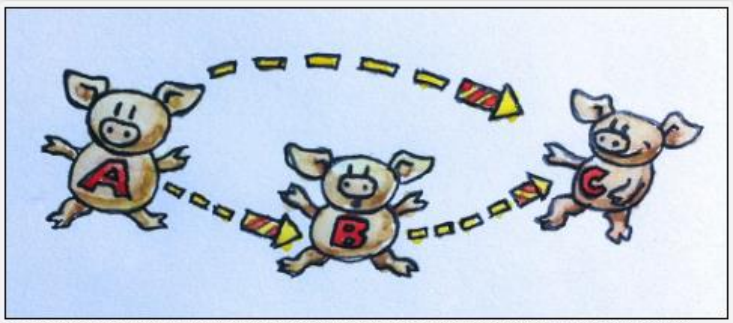

http://ocw.mit.edu/resources/res-18-006-calculus-revisited-single-variable-calculus-fall-2010/part-i-sets-functions-and-limits/lecture-2-functions/

# Composition of Functions

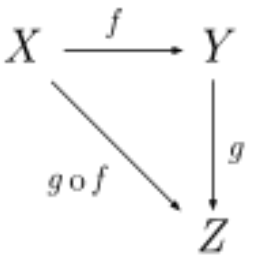



http://ocw.mit.edu/resources/res-18-006-calculus-revisited-single-variable-calculus-fall-2010/part-i-sets-functions-and-limits/lecture-2-functions/

$$\frac{f: a \to b \qquad g: b \to c}{g \circ f: a \to c}$$




In a category, if there is an arrow going from A to B and an arrow going from B to C then there must also be a direct arrow from A to C that is their composition. This diagram is not a full category because it's missing identity morphisms (see later).



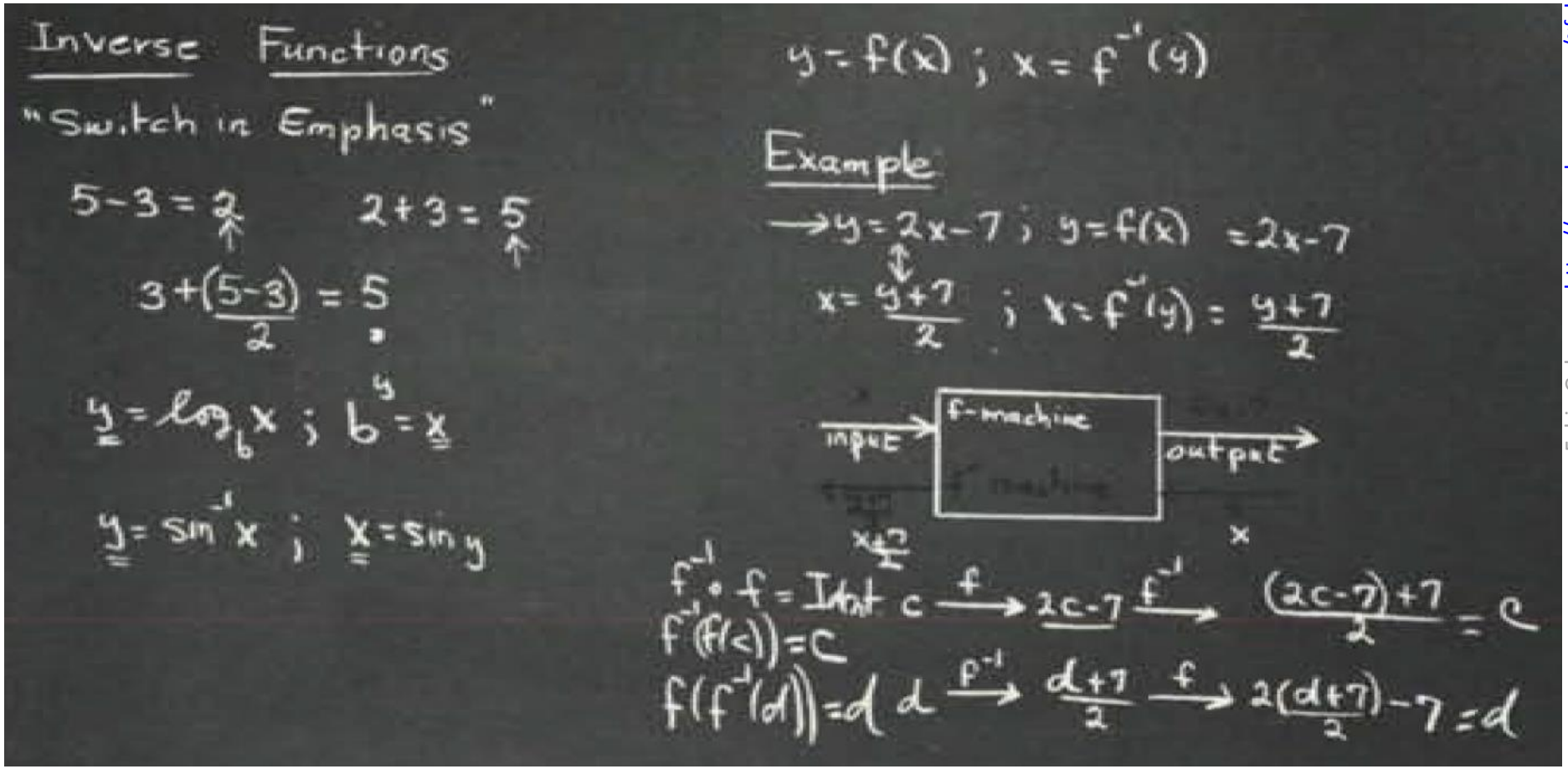

http://bartoszmilewski.com/2014/11/04/category-the-essence-of-composition/

# Inverse Functions

$\log_2 8 = 3 \qquad 2^3 = 8$

$(f \circ f^{-1})(x) = x$

$(f^{-1} \circ f)(y) = y$




http://ocw.mit.edu/resources/res-18-006-calculus-revisited-single-variable-calculus-fall-2010/part-i-sets-functions-and-limits/lecture-3-inverse-functions/

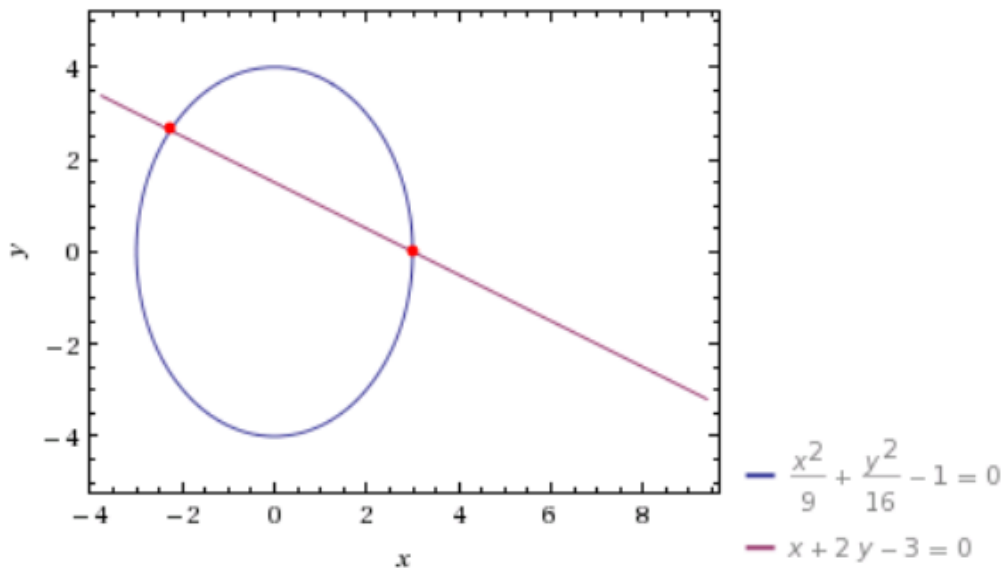
x^2/9 + y^2/16 - 1 = 0 and x + 2y - 3 = 0

Examples Random

Input:

$$\left\{\frac{x^2}{9}+\frac{y^2}{16}-1=0,\ x+2y-3=0\right\}$$

Plot of solution set:




Solutions:

$$x=-\frac{165}{73},\quad y=\frac{192}{73}$$

$$x=3,\quad y=0$$

$$\left\{\left(-\frac{165}{73},\frac{192}{73}\right),(3,0)\right\}$$

http://www.wolframalpha.com/input/?i=x%5E2%2F9+%2B+y%5E2%2F16+-+1+%3D+0+and+x+%2B+2y+-+3+%3D+0