①

$$Y = \bar{A}(B+C) + \bar{C} + BC$$

$$Y = \bar{A}B + \bar{A}C + \bar{C} + BC$$

} distributive property ?

$$Y = \overline{A + A\bar{B} + CD} = \bar{A}\;\overline{A\bar{B}}\;\overline{CD} = \bar{A}\cdot(\bar{A} + \overset{\to B}{\bar{\bar{B}}})\cdot$$

$$\cdot(\bar{C} + \bar{D}) = (\bar{A}\bar{A} + \bar{A}B)(\bar{C} + \bar{D}) = (\bar{A} + \bar{A}B)(\bar{C} + \bar{D})$$

$$= \bar{A}(1 + \bar{B})(\bar{C} + \bar{D}) = \bar{A}(\bar{C} + \bar{D})$$

deM for $\overline{A+B}$ and $C$

$$Y = \overline{\underbrace{\boxed{\overline{A+B}}}_{X}\,\underbrace{\boxed{C}}_{Y}} = \underbrace{\overline{\overline{A+B}}}_{\bar{\bar{X}}} + \underbrace{\bar{C}}_{\bar{Y}} = \underbrace{A+B}_{X} + \underbrace{\bar{C}}_{\bar{Y}} \leftarrow$$

$$\overline{X+Y} = \bar{X}\,\bar{Y} \qquad \overline{\sum_i x_i} = \prod_i \bar{x}_i \qquad \overline{\bigvee_i x_i} = \bigwedge_i \bar{x}_i$$

$$\overline{XY} = \bar{X} + \bar{Y} \qquad \overline{\prod_i x_i} = \sum_i \bar{x}_i$$

$+ \longrightarrow \vee \longrightarrow$ OR

$\cdot \longrightarrow \wedge \longrightarrow$ AND

2

$$Y = \bar A \bar B \bar C + A \bar B \bar C + \bar A B \bar C + A B \bar C$$

$$Y = \bar C \left( \bar A \bar B + A \bar B + \bar A B + A B \right)$$

$$Y = \bar C \left[ \bar A (\bar B + B) + A (\bar B + B) \right]$$

$$Y = \bar C (\bar A + A)(\bar B + B)$$

$$Y = \bar C$$

$XY = YX$

commutative

| X | Y | R |
|---|---|---|
| 0 | 1 | 1 |
| 1 | 0 | 1 |

$$Y = \overline{\overline{AB} + C\bar D} = \overline{\overline{AB}}\ \overline{C \bar D} = AB(\bar C + D) = AB\bar C + ABD$$

$$Y = A\bar B C + \bar A \bar C D + A \bar C = A(\bar B C + \bar C) + \bar A \bar C D =$$

$$= A(\bar C + \bar B) + \bar A \bar C D = A\bar C + A\bar B + \bar A \bar C D =$$

$$= \bar C (A + \bar A D) + A \bar B = \bar C (A + D) + A\bar B =$$

$$= A\bar C + \bar C D + A \bar B$$

$$\left[(\bar{P}+R)(\bar{Q}+R)\right]+(P+Q)$$

$\bar{P}\bar{Q}+\bar{P}R+R\bar{Q}+RR+P+Q$

dM $\downarrow$ $\quad\downarrow$ ✱

$P+Q+\bar{P}R+R\bar{Q}+R+P+Q$

2

$P+Q+\bar{P}R+R(1+\bar{Q})$

$P+Q+\bar{P}R+R$

$P+Q+R(1+\bar{P})$

$\boxed{P+Q+R}$ ?

&

| X | Y ✱ | Z |
|---|---|---|
| 0 | 0 | 0 |
| 1 | 1 | 1 |

$X+1=$

| X | Y | Z |
|---|---|---|
| 0 | 1 | 1 |
| 1 | 1 | 1 |

or

| X | Y | Z |
|---|---|---|
| 0 | 0 | 0 |
| 1 | 1 | 1 |

<u>Relation(s)</u> Two sets A, B $\quad A\times B$

$p(a,b) \to T,\ \to F$ $\quad a\in A,\ b\in B$

$R = aRb$ if $p(a,b)$ is true

$\hookrightarrow (a,b)\in R$ $\quad R\subseteq A\times B$ $\hookrightarrow$ cartesian product

$A=\{a,b\}$
$B=\{1,2\}$
$R=\{(a,1),(b,1),(a,2),(b,2)\}$

<u>example</u>

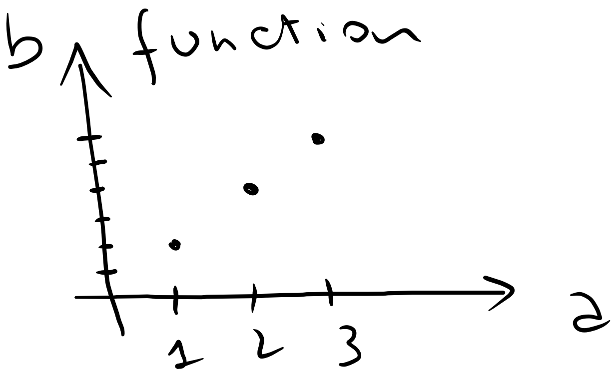
$\boxed{p(a,b)}$

a is half b $\quad R = aRb$

$A=\{1,2,3\}$
$B=\{2,4,6\}$

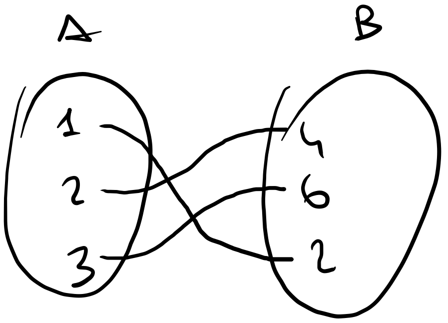
$R=\{(1,2),(2,4),(3,6)\}$ ✱

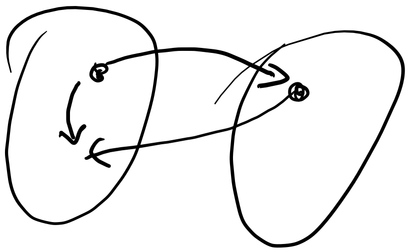
function




| B \ A | 1 | 2 | 3 |
|---|---|---|---|
| 6 | 0 | 0 | 1 |
| 4 | 0 | 1 | 0 |
| 2 | 1 | 0 | 0 |

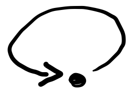
double entry table

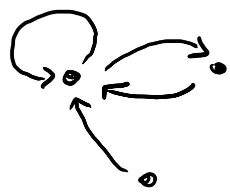


## (Directed) Graphs

reflexive $aRa$

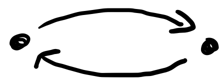




antireflexive $a \not R a$

symmetric $aRb \quad bRa$

$(a,b) \in R$



transitive $aRb \quad bRc$

$aRc$