Discrete Mathematica Exercises

4 novembre 2024

Application of Boolean Algebra

Notation

$X, Y \in \mathcal{B} = \{0, 1\}$

$$\begin{aligned} \neg X = \bar{X} = \mathrm{NOT}\, X\,, &\qquad X \vee Y = X + Y = X\, \mathrm{OR}\, Y\,, \\ X \wedge Y = X \cdot Y = X\, \mathrm{AND}\, Y\,, &\qquad X \veebar Y = X \oplus Y = X\, \mathrm{XOR}\, Y\,, \end{aligned}$$

In this notes we use the precedence/priority NOT,AND,OR/XOR. This is a convention, not all the compilers use the same rules `http://www.google.it`.

Thruth tables

X	Y	$X+Y$
0	0	0
0	1	1
1	0	1
1	1	1

X	Y	$X \cdot Y$
0	0	0
0	1	1
1	0	1
1	1	1

X	Y	$X \oplus Y$
0	0	0
0	1	1
1	0	1
1	1	0

Main Formulas

$$\bar{\bar{A}} = A \tag{1}$$

$$A \cdot A = A \tag{2}$$

$$A \cdot 0 = 0 \tag{3}$$

$$A \cdot 1 = A \tag{4}$$

$$A \cdot \bar{A} = 0 \tag{5}$$

$$A + A = A \tag{6}$$

$$A + 0 = A \tag{7}$$

$$A + 1 = 1 \tag{8}$$

$$A + \bar{A} = 1 \tag{9}$$

Exercises

Minimize the following logic expressions

$$Y = \bar{A}\cdot(A+C)+\bar{C}+B\cdot C \qquad [Y = B+\bar{C}] \tag{10}$$
$$Y = \overline{A+A\cdot\bar{B}+C\cdot D} \qquad [Y = \bar{A}\cdot(\bar{C}+\bar{D})] \tag{11}$$
$$Y = \overline{\overline{A+B}\cdot C} \qquad [Y = A+B+\bar{C}] \tag{12}$$
$$Y = \bar{A}\cdot\bar{B}\cdot\bar{C}+A\cdot\bar{B}\cdot\bar{C}+\bar{A}\cdot B\cdot\bar{C}+A\cdot B\cdot\bar{C} \qquad [Y = \bar{C}] \tag{13}$$
$$Y = \overline{\overline{A\cdot B}+C\cdot\bar{D}} \qquad [Y = A\cdot B\cdot\bar{C}+A\cdot B\cdot D] \tag{14}$$
$$Y = A\cdot\bar{B}\cdot C+\bar{A}\cdot\bar{C}\cdot D+A\cdot\bar{C} \qquad [Y = A\cdot\bar{B}+A\cdot\bar{C}+\bar{C}\cdot D] \tag{15}$$
$$Y = B\cdot C\cdot\bar{D}+C\cdot D+A\cdot\bar{B}\cdot C\cdot\bar{D}+\bar{A}\cdot\bar{B}\cdot C \qquad [Y = C] \tag{16}$$
$$Y = \overline{\bar{A}+A\cdot B\cdot C} \qquad [Y = A\cdot(\bar{C}+\bar{C})] \tag{17}$$
$$Y = (A+B)\cdot(A+\bar{B}) \qquad [Y = A] \tag{18}$$
$$Y = A\cdot B\cdot\bar{C}+B\cdot C+\bar{A}\cdot\bar{B}\cdot C \qquad [Y = A\cdot B+\bar{A}\cdot C] \tag{19}$$

Problems

If needed, use sum of products, i.e. minterms, to provide the correct answers

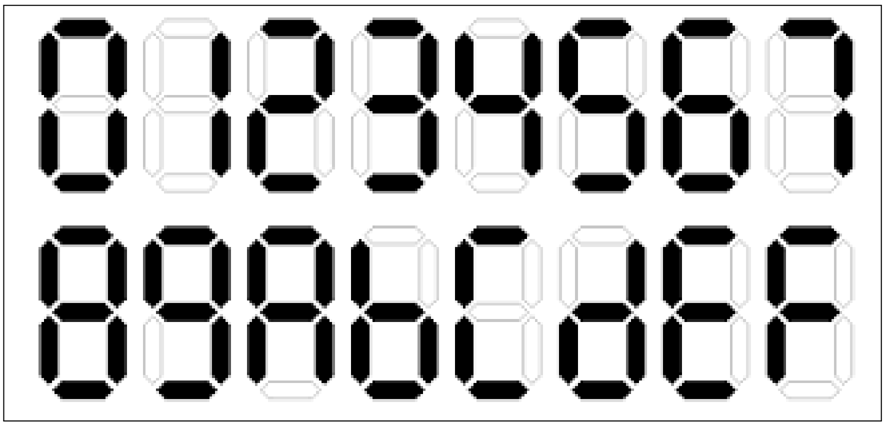
1) Derive the logic expression for $X < Y$ [Answer $Z = \bar{X}\cdot Y$]
2) Derive the logic expression for $X \leq Y$ [Answer $Z = \bar{X}+Y$]
3) Derive the logic expressions for a 4-lines selector [Answer $L_0 = \bar{S}_1\cdot\bar{S}_0$, $L_1 = \bar{S}_1\cdot S_0$, $L_2 = S_1\cdot\bar{S}_0$, $L_3 = S_1\cdot S_0$]

S_1	S_0	L_3	L_2	L_1	L_0
0	0	0	0	0	1
0	1	0	0	1	0
1	0	0	1	0	0
1	1	1	0	0	0

4) Derive the logic expression for an one bit full-adder (answer Sum $S = X\oplus Y$, Carry $C = X\cdot Y$)

X	Y	C	S
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	0

5) Derive the logic expression for a 7 segment display controller capable of displaying numbers from 0 to 9 plus character A, B, C, D, E, and F as reported in the figure below.



Hint: name the i-th segment in the display $S_i = \{0, 1\}$ with $i = 0..6$, you can arrange the segment numbering in the display as you wish. The control logic has 4 lines L_3, L_2, L_1, and L_0 that encode digitally the numbers and the letters.