Multiwavelength Optical Networks with Limited Wavelength Conversion

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***Abstract*— This paper proposes optical wavelength division multiplexed (WDM) networks with limited wavelength conversion that can efficiently support *lightpaths* (connections) between nodes. Each lightpath follows a route in a network and must be assigned a channel on each link along the route. The *load* $\lambda_{\max}$ of a set of lightpaths is the maximum over all links of the number of lightpaths that use the link. At least $\lambda_{\max}$ wavelengths will be needed to assign channels to the lightpaths. If the network has full wavelength conversion capabilities, then $\lambda_{\max}$ wavelengths are sufficient to perform the channel assignment.**

Ring networks with fixed wavelength conversion capability within the nodes are proposed that can support all lightpath sets with load $\lambda_{\max}$ at most $W - 1$, where W is the number of wavelengths in each link. Ring networks with a small additional amount of wavelength conversion capability within the nodes are also proposed that allow the support of any set of lightpaths with load $\lambda_{\max}$ at most W. A star network is also proposed with fixed wavelength conversion capability at its hub node that can support all lightpath sets with load $\lambda_{\max}$ at most W. These results are extended to tree networks and networks with arbitrary topologies. This provides evidence that significant improvements in traffic-carrying capacity can be obtained in WDM networks by providing very limited wavelength conversion capability within the network.

***Index Terms*— Lightpaths, optical networks, routing, wavelength conversion, wavelength division multiplexing.**

I. Introduction

WAVELENGTH division multiplexing (WDM) is an important approach to utilize the large available bandwidth in a single-mode optical fiber. WDM is basically frequency division multiplexing in the optical frequency domain, where on a single optical fiber there are multiple communication channels at different *wavelengths* (corresponding to carrier frequencies). There has been a great deal of interest in WDM networks that employ wavelength routing. These networks support *lightpaths*, which are end-to-end circuit-switched communication connections that traverse one or more links and use one WDM channel per link. Lightpaths could serve as the physical communication links for a variety of high-speed networks such as asynchronous transfer mode (ATM) networks.

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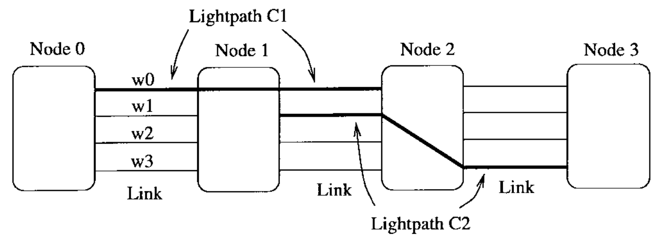


Fig. 1. A network of four nodes $\{0, 1, 2, 3\}$, and links between the following pairs of nodes: (0,1), (1,2), and (2,3). There are four wavelengths $\{\omega_0, \omega_1, \omega_2, \omega_3\}$. The channels are shown as lines between nodes.

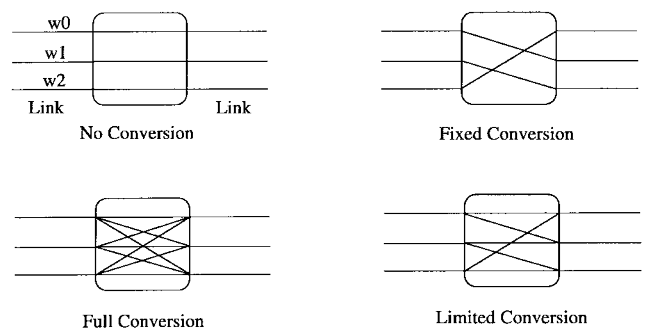


Fig. 2. Different types of wavelength conversion.

An example of a WDM wavelength routing network is shown in Fig. 1. It is composed of four nodes with three links, each having four WDM channels at wavelengths $\{\omega_0, \omega_1, \omega_2, \omega_3\}$. Switching is done at each node so that channels may be connected to form lightpaths. Note that if channels at different wavelengths are to be connected then *wavelength conversion* devices are needed that can shift the wavelength of an optical signal. For example, lightpath $C2$ needs a converter at node 2 because it is composed of two WDM channels at different wavelengths (ω_1 and ω_3). However, the lightpath $C1$ is composed of two WDM channels at the same wavelength ω_0 on links 1 and 2. Hence, it does not need a wavelength converter. The advantage of wavelength conversion is that WDM channels will be used more efficiently, but the disadvantage is increased cost and complexity.

The different types of wavelength conversion possible within the node are illustrated in Fig. 2. *No conversion* means no wavelength shifting, and thus only channels with the same wavelengths may be connected. *Full conversion* means that any wavelength shifting is possible, and so channels may be connected regardless of their wavelengths. This provides the most efficient use of the channels, but it is the most expensive to implement. *Limited conversion* means that there is wavelength shifting but it may be restricted so that not all combinations of channels may be connected. In this paper, we will consider networks with limited wavelength conversion that perform as well as networks with full conversion but with only a "small" amount of conversion capability. Presumably, a small amount of conversion capability will be easier or cheaper to implement than full conversion capability and will provide better channel efficiency than no conversion. *Fixed conversion* is a very restricted form of limited conversion, where at each node each channel may be connected to exactly one predetermined channel on all other links. Note that no conversion is a degenerate case of fixed conversion.

Within each node, the wavelength conversion can be done all optically or by receiving the signal, switching it electronically, and retransmitting it on another wavelength, i.e., *optical–electrical–optical* (O-E-O) conversion. The all-optical approach uses optical wavelength converter devices. In some of these devices, such as those based on *four-wave mixing* [26], the conversion efficiency is a strong function of the input and output wavelengths, naturally leading to limited conversion capability. In particular, it may lead to a limit on the range of wavelength shifting. Otherwise we can save on the number of such devices required in a node. In the O-E-O approach, we can implement limited conversion using fewer electronic switches than would be needed for full conversion.

A. Network Model

We assume that the links, WDM channels, and lightpaths are *bidirectional* (i.e., full duplex). The number of wavelengths (and channels) in each link is denoted by W, and the wavelengths are denoted by $\{\omega_0, \omega_1, \cdots, \omega_{W-1}\}$, where $\omega_0 < \omega_1 < \cdots < \omega_{W-1}$. Such links may be implemented with two fibers, each carrying W wavelengths in opposite directions, and each bidirectional channel will be composed of two unidirectional channels. Our assumption that a bidirectional channel is at a particular wavelength implies that its unidirectional channels have the same wavelength. However, in practical systems, this may not be a restriction, i.e., the two unidirectional channels can be at different wavelengths. In Section IV, we will briefly discuss this less restricted case.

For simplicity, we will assume that that all pairs of nodes have at most one link between them. Each node has switching and conversion capability to connect channels to form lightpaths. We will refer to two channels that may be connected to one another at a node as being *attached* at the node. For example, in Fig. 2, the channels that are attached are indicated by the lines within the nodes.

We characterize the amount of switching and conversion capability at a node by its *wavelength degree*, which is defined next. A node has *wavelength degree* k (for some integer k) if for each pair of incident links a channel in a link is attached to at most k other channels in the other link. For example, the limited conversion node shown in Fig. 2 has wavelength degree two. Also note that a node with no or fixed conversion has wavelength degree one, while a node with full conversion has wavelength degree W.

Now a network supports sets of lightpaths. A lightpath is specified by a path in the network that is referred to as its *route*. A lightpath is realized by a set of channels, one on each link along its route, so that consecutive channels are attached through their common node. Such a set of channels is referred to as a *channel assignment* for the route. Obviously, a channel assignment provides a contiguous full duplex pathway for the communication signals of its lightpath.

A set of lightpaths is specified by a set of routes, one route per lightpath. A set of routes will be referred to as a *request*. A *channel assignment* for a request is a collection of channel assignments, one per route of the request, such that no two routes share a channel, i.e., the channel assignments are channel disjoint. Note that a channel assignment for a request realizes the corresponding lightpaths. An important parameter of a request is its *load*, which is the value $\lambda_{\max} = \max_{e \in E} \lambda_e$, where λ_e denotes the number of routes using link e and E denotes the set of links in the network. Clearly, for a request, $\lambda_{\max} \le W$ is a necessary condition for the existence of a channel assignment.

B. Organization

In this paper, we propose ring and star networks with limited wavelength conversion to support sets of lightpaths efficiently. In Section II, we discuss our results for ring networks. We give a ring network with one node having fixed wavelength conversion and the rest of the nodes with no wavelength conversion such that all requests with load $\lambda_{\max} \le W - 1$ have channel assignments. We also give a ring network with two nodes with wavelength degree two and the rest of the nodes with no wavelength conversion such that all requests with load $\lambda_{\max} \le W$ have channel assignments. Note that the networks that have channel assignments for all requests with load at most W utilize the channels as efficiently as networks with full wavelength conversion at all nodes. Also note that the first deployed WDM networks are likely to be rings, as seen from several recent testbeds (see, for example [5], [22]).

In Section III, we discuss our results for star networks as well as extensions to tree networks, and networks with arbitrary topologies, where route lengths are at most two. We present a star network that has fixed wavelength conversion and has channel assignments for all requests with load $\lambda_{\max} \le W$, when W is an even number. In Section IV, we provide conclusions and discuss how our results can be extended when channels and lightpaths are *unidirectional*.

Note that we consider channel assignments for sets of lightpaths, to be determined all at one time. Thus, if a new lightpath is included into an existing set of lightpaths, then the channel assignments for the existing set may have to be recomputed. In this sense, the channel assignment is done *off-line*. There is also the more practical consideration of *on-line* channel assignment, i.e., setting up new lightpaths without changing the assignment for existing lightpaths. Although we only consider the off-line case, we believe that its understanding can lead to fundamental insights into the on-line case, just as understanding *rearrangeable nonblocking networks* can help to understand efficient *wide-sense* and *strict-sense nonblocking networks* [12]. Also, off-line channel assignment will be more efficient in utilizing channels than on-line channel assignment.

C. Related Work

Previous work focuses primarily on networks with either no wavelength conversion or networks with full wavelength conversion. For networks with no wavelength conversion, there are a number of channel assignment results. The channel assignment problem is known to be NP-complete [6] and remains NP-complete even if the network is a ring [8]. Several heuristic channel assignment schemes have been proposed [2], [20], [6], [14], [4], [24]. There are also channel assignment schemes with provable worst-case performance for special topologies. In particular, for a ring topology, a channel assignment can be found for a request if $2\lambda_{\max} - 1 \le W$ [23], [8]. Also, sample requests can easily be constructed that require $W = 2\lambda_{\max} - 1$ wavelengths. For a star topology, a channel assignment can be found for a request if $(3/2)\lambda_{\max} \le W$ [18]. Again, sample requests can also be easily constructed that require $W = (3/2)\lambda_{\max}$ wavelengths. For networks with directed WDM channels that support directed lightpaths, there are channel assignment algorithms for tree and ring networks [17]. For trees, $15\lambda_{\max}/8 \le W$ is sufficient for a channel assignment, and there are sample requests that require $W = 3\lambda_{\max}/2$ wavelengths [17]. For rings, $2\lambda_{\max} - 1 \le W$ is sufficient for a channel assignment, and there are sample requests that require $W = 2\lambda_{\max} - 1$ wavelengths which follows from the results for undirected rings and lightpaths as stated above.

Variants of the limited conversion model are considered in [15], [16], [25], [21]. In [15], [16], it is assumed that each node has a limited number of wavelength converters and that each converter has no restrictions on the wavelengths of the channels it can connect. Here, the restriction is on the *number* of wavelength conversions at a node. In [25], a network with limited wavelength conversion is used to study the performance due to limited wavelength shifting capability of devices based on four wave mixing. The converters allow wavelengths to be shifted within a given range. Also, the work in [21] studies *sparse wavelength conversion*, where networks are comprised of a mix of nodes having full and no wavelength conversion. The channel assignment in these papers [15], [16], [25], [21] are simple heuristics, and their performance analyses are based upon probabilistic models and techniques (i.e., compute blocking probabilities of setting up lightpaths). This may not be an appropriate traffic model for networks where lightpaths are set up and taken down primarily for provisioning high-speed connections. Rather than block a lightpath request, the network operator is likely to add capacity to the network to support that request.

There are some recent results on the *on-line* channel assignment problem for ring networks [9], [11]. The problem of recovering from link and node faults in ring networks using limited wavelength conversion is addressed in [10].

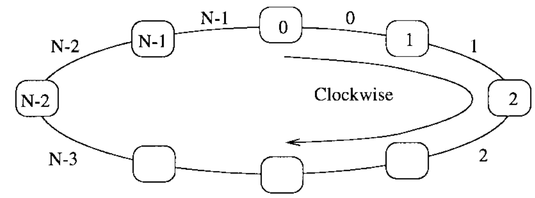


Fig. 3. A ring network.

II. RINGS

In this section, we will consider ring networks. To simplify the discussion, we will assume in this section that the ring networks have a *clockwise direction*, as shown in Fig. 3, and that routes of lightpaths have a *direction* that goes clockwise. The directions only provide a logical orientation for the rings and routes, and the communication transmissions on the links, WDM channels, and lightpaths are still bidirectional (i.e., full duplex).

Let N denote the number of nodes in the network, and let the nodes be numbered $0, 1, \cdots, N-1$ consecutively in the clockwise direction, as shown in Fig. 3. The links are also numbered $0, 1, \cdots, N-1$ in the clockwise direction such that for each $i = 0, 1, \cdots, N-1,$ the link between node i and node $(i+1)$ mod N is numbered i.

Most of the results of this section assume that there is a collection of lightpaths to be set up, and their set of routes (i.e., a *request*) is already given. However, we should note that if their routes are unspecified then there is an algorithm that can compute routes with minimum load given only the terminating nodes of the lightpaths [7]. Another routing algorithm that is popular, practical, and performs well in many cases is simple shortest path routing. As shown below, even in the worse case it finds routes with load at most twice the minimum possible.

Theorem 1: Suppose we are given a request of source–destination pairs and the minimum possible load for satisfying this request is $\lambda_{\max}$. Then shortest-path routing yields a load of at most $2\lambda_{\max}$.

Proof: Suppose shortest-path routing yields a load λ_{sp}. Consider a link i with load λ_{sp}. Rerouting k lightpaths using link i using their longer routes on the ring can reduce the load on link i to $\lambda_{sp} - k$. Note that since all these lightpaths are routed on paths of length at most $\lfloor N/2 \rfloor$ initially, their longer routes on the ring will all use the link $(\lfloor N/2 \rfloor + i) \bmod N$, increasing its load by k. Therefore, an optimal routing algorithm would have a load given by $\lambda_{\max} \ge \min_k \max\{\lambda_{sp} - k, k\}$, or $\lambda_{\max} \ge \lceil \lambda_{sp}/2 \rceil$. □

The next three ring networks have channel assignments for all requests with load at most $W - 1$. They have fixed conversion at one or two nodes and no conversion at the other nodes. Loosely speaking, the networks have their channels forming a single sequence of channels going clockwise around the ring exactly W times such that consecutive channels are attached, and the first and last channels are attached. To be more specific about this property, we will use the following definitions. For a ring network, a *multicycle of channels* (MCC) is a sequence of distinct channels $(c_0, c_1, \cdots, c_{k-1})$, where k is a multiple of N, that starts at some node u, goes clockwise around the ring one or more times, and ends at node u.

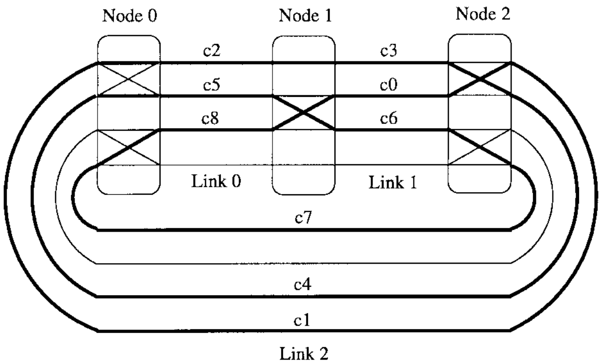


Fig. 4. An MCC $(c_0, c_1, \cdots, c_8)$ in a ring network that starts and ends at node 0 and has multiplicity 3.

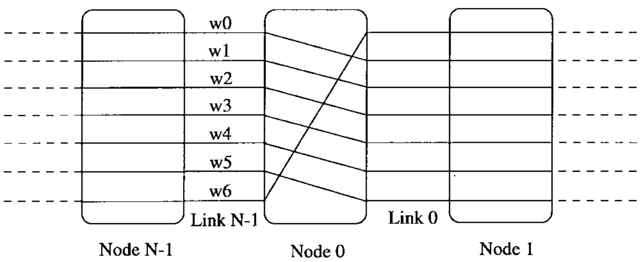


Fig. 5. Ring network 1 when $W = 7$.

In addition, for $i = 0, 1, \cdots, k-1$, the channels c_i and $c_{(i+1) \bmod k}$ are attached. The *multiplicity* of an MCC is the number of times it goes around the ring, i.e., it is equal to k/N. Fig. 4 shows an MCC with multiplicity 3. A ring network that has wavelength degree one at every node and has a single MCC with multiplicity W (i.e., it traverses all the channels in the network) will be referred to as a *single-MCC* ring network. The next three networks are single-MCC ring networks.

Ring Network 1: There is one node, say node 0, with fixed conversion, and the rest of the nodes have no conversion. At node 0, for $i = 0, 1, \cdots, W-1$, the channel at wavelength ω_i in link $N-1$ is attached to the channel at wavelength $\omega_{i+1 \bmod W}$ in link 0. Fig. 5 shows node 0 when $W = 7$.

Ring network 1 requires large wavelength shifting to connect channels at wavelengths ω_0 and ω_{W-1}. The next two ring networks are single-MCC ring networks with much less wavelength shifting.

Ring Network 2: There is one node, say node 0, with fixed conversion and the rest of the nodes have no conversion. For brevity, we will only describe the network when W is odd (the case when W is even is similar). Basically, the channels are attached so that there is a single MCC that visits channels one wavelength at a time, by first going through the even indexed wavelengths in the order $\omega_0, \omega_2, \cdots, \omega_{W-1}$, and then going through the odd indexed wavelengths in the order $\omega_{W-2}, \omega_{W-4}, \cdots, \omega_3, \omega_1$. To accomplish this, the channels are attached at node 0 as follows. For $i = 0, 2, 4, \cdots, W-3$, the channel at ω_i in link $N-1$ is attached to the channel at ω_{i+2} in link 0. The channel at ω_{W-1} in link $N-1$ is attached to channel at ω_{W-2} in link 0. For $i = W-2, W-4, \cdots, 5, 3$, the channel at ω_i in link $N-1$ is attached to the channel at ω_{i-2}. The channel at ω_1 in link $N-1$ is attached to channel at ω_0 in link 0. Fig. 6 shows node 0 when $W = 7$.

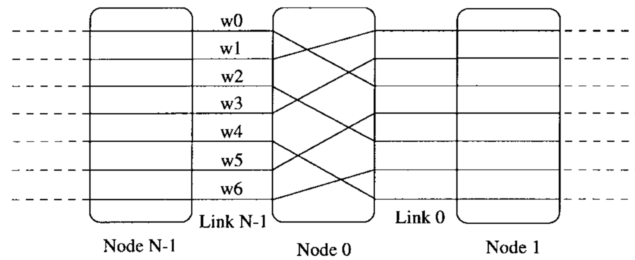


Fig. 6. Ring network 2 when $W = 7$.

Ring Network 3: There are two nodes with fixed conversion, referred to as the "conversion even" (CE) and "conversion odd" (CO) nodes, and the rest of the nodes have no conversion. Fig. 7 shows the CE and CO nodes when $W = 7$. At the CE node, for even values of $i \leq W-2$, channels at ω_i are attached to channels at ω_{i+1}. If W is odd, then channels at ω_{W-1} are attached to each other. At the CO node, for odd values of $i \leq W-2$, channels ω_i are attached to channels at ω_{i+1}. Channels at ω_0 are attached to each other, and if W is even then channels at ω_{W-1} are attached to each other.

It is a little less obvious than for ring networks 1 and 2 that this is a single MCC ring network, so we will show an MCC of multiplicity W. We will do this recursively by defining, for $i = 0, 1, \cdots, k-1$, a (clockwise) sequence of attached channels G_i at wavelengths $\{\omega_0, \omega_1, \cdots, \omega_i\}$. G_0 is the sequence at ω_0 that starts and ends at node CE. For even valued i, G_i starts at node CE, goes through the channels at ω_i until it reaches CO, goes through G_{i-1}, and then finally goes through channels at ω_i until it returns to CE. Similarly, for odd valued i, G_i starts at CO, goes through the channels at ω_i until it reaches CE, goes through G_{i-1}, and then finally goes through channels at ω_i until it returns to CO. G_{W-1} is an MCC with multiplicity W.

Theorem 2: For a single-MCC ring network (e.g., ring networks 1–3) with W wavelengths, any request with load at most $W-1$ has a channel assignment.

Before proving Theorem 2, we will define additional structure for a request and its routes that will help determine a channel assignment. A sequence of routes $(p_0, p_1, \cdots, p_{k-1})$, for some k, is referred to as a *multicycle of routes* (MCR) if, for $i = 0, 1, \cdots, k-1$, the first node of p_i is the last node of $p_{(i+1) \bmod k}$. Hence, an MCR is just a sequence of consecutive routes that circumvents the ring clockwise one or more times. The number of times it goes around the ring is called its *multiplicity*. (Fig. 8 shows an MCR with multiplicity three.) Note that if an MCR $(p_0, p_1, \cdots, p_{k-1})$ has the same multiplicity as some MCC then the channels in the MCC provide a channel assignment for the routes of the MCR. To show one such assignment, let us assume without loss of generality that the MCR and MCC start at the same node, and for $i = 0, 1, \cdots, k-1$, let n_i denote the length (in hops) of route p_i. Then a channel assigment is to assign the first n_0 channels of the MCC to p_0, assign the second n_1 channels of the MCC to p_1, and so forth. (Fig. 9 shows a channel assignment for the MCR $(p_0, p_1, \cdots, p_7)$ in Fig. 8

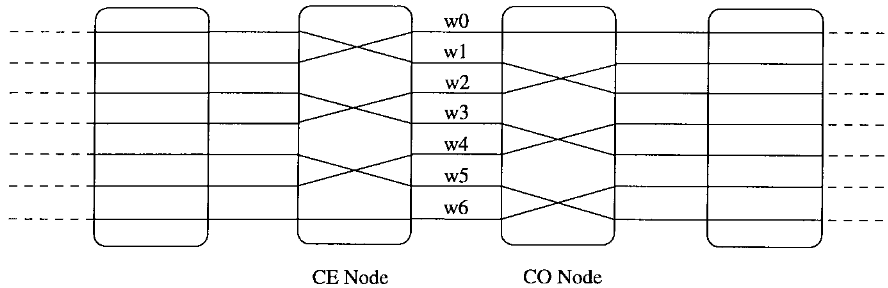


Fig. 7. Ring network 3 when $W = 7$. In this case, the CE and CO nodes are adjacent.

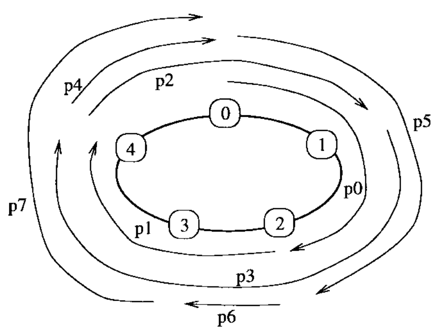


Fig. 8. An MCR $(p_0, p_1, \cdots, p_7)$ is shown that starts and ends at node 0 and has multiplicity 3. The clockwise direction of the routes are just a logical orientation to simplify discussion, and the corresponding lightpaths have bidirectional communication.

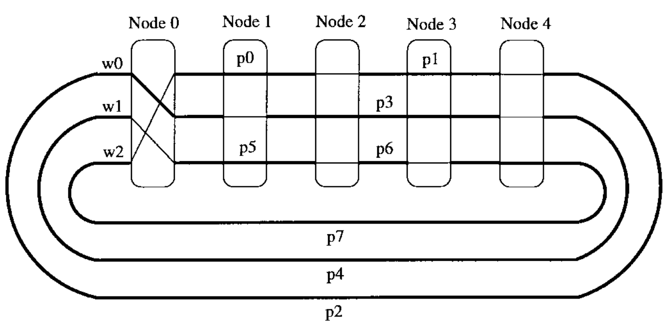


Fig. 9. A channel assignment for the MCR $(p_0, p_1, \cdots, p_7)$ from Fig. 6 using the channels from an MCC that starts from node 0 and channel ω_0.

using the channels from an MCC that starts from node 0 and channel ω_0.)

We will consider requests such that the number of routes traversing any link is the same. Such requests will be referred to as *uniform*. Given a uniform request R, we can define a collection of MCR's $\{M_0, M_1, \cdots, M_{k-1}\}$ (for some k) with the property that each route of R is in exactly one MCR, and the MCR's are composed only of the routes of R. The collection is called an *MCR partition* for R.

An MCR partition can be found from a uniform request R as follows. The MCR's of the MCR partition are found one at a time, forming a sequence of MCR's $M_0, M_1, \cdots, M_{k-1}$, where the value of k will be apparent shortly. For $i = 0, 1, \cdots, k-1, M_i$ is computed from a set of routes R_i, which is defined to be the set of routes in R that are not in any of the MCR's $\{M_0, M_1, \cdots, M_{i-1}\}$ (here, $R_0 = R$). Thus, R_i is the set of routes that have not already been assigned to an MCR. Note that each R_i is a uniform request because it is the uniform request R excluding routes from MCR's. Obviously, k is the value when $\{M_0, \cdots, M_{k-1}\}$ is an MCR partition.

Now each M_i is constructed from routes in R_i as follows. Start from a node u that has an outgoing route from R_i, and traverse a sequence of distinct routes $p_0, p_1, \cdots$ in R_i, where p_{i+1} starts where p_i ends for $i = 0, 1, \cdots$. Stop the traversing when node u is returned to. Note that the traversing will eventually stop at node u because R_i is a uniform request and thus has the property that the number of routes that begin at a node equals the number that end at the node. (Therefore, while traversing the routes, if we reach a node $v \neq u$ at the end of a route, then v has one more untraversed outgoing route than untraversed incoming route.) The sequence of routes forms an MCR, which is M_i. This completes our description of how to construct M_i and also completes our description of how to find an MCR partition.

Example 1: The following is a construction of an MCR partition from the request $R = \{p_0, p_1, \cdots, p_7\}$ shown in Fig. 8. First, $R_0 = R = \{p_0, p_1, \cdots, p_7\}$. Note that nodes $\{0, 2, 4\}$ each have two routes that start from it and two routes that end at it. For example, node 0 has routes $\{p_0, p_5\}$ that start from it, and routes $\{p_4, p_7\}$ that end at it. Nodes $\{1, 3\}$ each have a route that starts from it and a route that ends at it. Starting from node 0, $(p_0, p_1, \cdots, p_4)$ forms a sequence of consecutive and distinct routes that returns to 0. This sequence becomes M_0.

Next, $R_1 = \{p_5, p_6, p_7\}$, which is R excluding the routes in M_1. Nodes $\{0, 2, 3\}$ each have, in R_1, one route that starts from it and one route that ends at it. Starting from node 2, (p_6, p_7, p_5) forms a sequence of consecutive and distinct routes that returns to 2. This sequence becomes M_1. Since all routes have been assigned to an MCR, $\{M_0, M_1\}$ becomes the MCR partition.

Proof of Theorem 2: Without loss of generality, assume that R is uniform with load $W - 1$ (otherwise, we can add one-hop *dummy routes*). There is an MCR partition $\{M_0, M_1, \cdots, M_{k-1}\}$ for R, where k is the number of MCR's in the partition. We will transform the MCR partition into a single MCR with multiplicity W by adding dummy routes as follows.

For $i = 0, 1, \cdots, k-1$, let u_i denote the node where MCR M_i starts and ends. Without loss of generality, let $u_0 \leq u_1 \leq \cdots \leq u_{k-1}$. Let $D = \{d_0, d_1, \cdots, d_{k-1}\}$ be a collection of dummy routes such that for $i = 0, 1, \cdots, k-1$, d_i starts at node u_i, ends at node $u_{(i+1)} \bmod k$, and goes clockwise around the ring. However, if $u_i = u_{(i+1) \bmod k}$ then the route d_i has zero length, i.e., it is a path that starts and ends at node $u_i = u_{(i+1) \bmod k}$ but does not traverse any links. Note that each link of the ring network has at most one dummy route of D traversing it because $u_0 \le u_1 \le \cdots \le u_{k-1}$, i.e., the routes in D have load at most one. Let M be the sequence of routes $M = (M_0, d_0, M_1, d_1, \cdots, M_{k-1}, d_{k-1})$, and note that it is an MCR because for $i = 0, 1, \cdots, k-1$, the dummy route d_i starts at the end of M_i and ends at the beginning of $M_{(i+1) \bmod k}$.

The routes of M have load that is either $W-1$ or W because R has load $W-1$ and the routes in D have load at most one. If the routes of M have load $W-1$, then another dummy route d_k can be appended to M that starts and ends at node u_0 (the starting and ending point for M) and circumvents the ring exactly once. Then M will be an MCR with multiplicity W. Since M is an MCR with multiplicity W and the single-MCC ring network has an MCC with multiplicity W, there is a channel assignment for the routes of M. Thus, there is a channel assignment for R. □

Example 2: Suppose the request $\{p_0, p_1, \cdots, p_7\}$ in Fig. 8 is for a five node ring network with $W = 4$. Since the request has load $W - 1 = 3$, we can find a single MCR with multiplicity W which may include some dummy routes. In particular, consider the MCR partition $\{M_0, M_1\}$, where $M_0 = (p_0, p_1, \cdots, p_4)$ and $M_1 = (p_6, p_7, p_5)$. Note that M_0 starts and ends at node $u_0 = 0$, while M_1 starts and ends at node $u_1 = 2$. Let d_0 be a dummy route from u_0 to u_1, and let d_1 be a dummy route from u_1 to u_0. The new MCR $M = (p_0, p_1, \cdots, p_4, d_0, p_6, p_7, p_5, d_1)$ has multiplicity W. If the network is a single MCC ring network, then the routes of M have a channel assignment, and so the request has a channel assignment.

The following theorem states that with fixed conversion, some requests with load W do not have channel assignments, proving that Theorem 2 provides the best possible design for a fixed-conversion ring network. The proof of this theorem is given in the Appendix.

Theorem 3: For any ring network with W wavelengths, fixed wavelength conversion at every node, and a sufficiently large number of nodes N, there is a request with load W that does not have a channel assignment.

By allowing a bit more wavelength conversion, a ring network may be designed to have channel assignments for all requests with load at most W. The next three limited conversion ring networks have this property, and so they can support all requests that can be supported by a ring network with full conversion at every node. These networks are *multi-MCC ring networks*, which we define next. A vector is called a *W-sum vector* if it is integer, nonnegative, has at most W elements, and the elements sum to W. The ring network is referred to as a *multi-MCC ring network* if for each W-sum vector $(m_0, \cdots, m_{k-1})$ (for some k), it has a collection of channel-disjoint MCC's $\{H_0, H_1, \cdots, H_{k-1}\}$ such that, for $i = 0, 1, \cdots, k-1$, H_i has multiplicity m_i.

Ring Network 4: One node, say node 0, has full conversion and the rest of the nodes have no conversion.

Ring Network 5: One node, say node 0, has limited conversion and the rest of the nodes have no conversion. At the node 0, for $0 \le i, j \le W-1$, channels at wavelengths ω_i and ω_j are attached if and only if $|i - j| \le 2$. Thus, channels are attached if they are within two wavelengths of each other. Fig. 10 shows node 0 when $W = 7$.

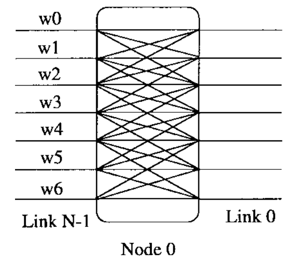


Fig. 10. Node 0 in ring network 5 for $W = 7$.

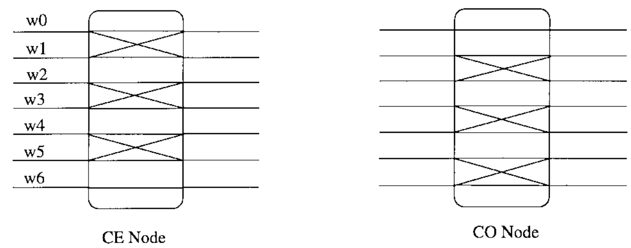


Fig. 11. The CE and CO nodes in ring network 6 for $W = 7$.

Ring Network 6: Two nodes have wavelength degree two, which will be referred to as the CE and CO nodes, and the rest of the nodes have no conversion. (Note that the nodes with wavelength conversion are named this way because they have a similar function as the CE and CO nodes in ring network 3.) Fig. 11 shows the CE and CO nodes when $W = 7$. At the CE node, for even $i \in \{0, 1, \cdots, W-1\}$, channels at wavelengths $\{\omega_i, \omega_{i+1}\}$ are attached. At the CO node, for odd $i \in \{0, 1, \cdots, W-1\}$, channels at wavelengths $\{\omega_i, \omega_{i+1}\}$ are attached. Also, at both the CE and CO nodes, channels at the same wavelength are attached.

Theorem 4: Ring networks 4–6 are multi-MCC ring networks.

Proof: Consider an arbitrary W-sum $(m_0, m_1, \cdots, m_{k-1})$ for some k. We will show that in each network there is a collection of channel-disjoint MCC's $\{H_0, H_1, \cdots, H_{k-1}\}$, where for $i = 0, 1, \cdots, k-1, H_i$ has multiplicity m_i. First note that the W wavelengths can be partitioned into k disjoint sets of contiguous wavelengths such that the ith set $(i = 0, 1, \cdots, k-1)$ has m_i wavelengths.

To complete the proof, we will show that each H_i can be formed from the ith set of wavelengths, which we will denote by $\{\omega_n, \omega_{n+1}, \cdots, \omega_{n+m_i-1}\}$ for some n. (Loosely speaking, the MCC H_i can be formed in ring networks 4–6 because their channels are attached so that they are generalizations of ring networks 1, 2, and 3, respectively.) For ring network 4, H_i starts and ends at node 0 (the full conversion node), and visits channels one wavelength at a time in the order $\omega_n, \omega_{n+1}, \cdots, \omega_{n+m_i-1}$. For ring network 5, H_i starts and ends at node 0, and visits channels one wavelength at a time starting from ω_n, visiting the even indexed wavelengths to reach ω_{n+m_i-1}, and then visiting the odd indexed wavelengths to return to ω_n. For ring network 6, H_i can be built up recursively similar to the way the MCC in ring network 3 is defined. First, if n is even (odd), let G_n be the clockwise sequence of channels at wavelength ω_n, starting and ending at node CE (CO). For $j = n+1, n+2, \cdots, n+m_i-1$, if j is even (odd), let G_j be the clockwise sequence of channels starting at node CE (CO) that goes through channels at ω_j until it reaches CO (CE), goes through G_{j-1}, and then finally goes through channels at ω_j until it returns to CE (CO). H_i is G_{n+m_i-1}. □

Theorem 5: For a multi-MCC ring network, any request with load at most W has a channel assignment.

Proof: Consider a request with load at most W. Without loss of generality, we may assume that the request is uniform and has load W. (Otherwise, we can make it so by adding *dummy* one-hop routes.) There is an MCR partition $\{M_0, M_1, \cdots, M_{k-1}\}$ for the request (for some k), and for $i = 0, 1, \cdots, k-1$, let m_i be the multiplicity of M_i. The vector $(m_0, m_1, \cdots, m_{k-1})$ must be a W-sum vector because the request has load W. Note that the network has a collection of channel-disjoint MCC's $\{H_0, H_1, \cdots, H_{k-1}\}$ such that for $i = 0, 1, \cdots, k-1, H_i$ has multiplicity m_i. A channel assignment for the request is, for $i = 0, 1, \cdots, k-1$, to assign the channels of H_i to the routes of M_i.

Corollary 1: For ring networks 4–6, any request with load at most W has a channel assignment.

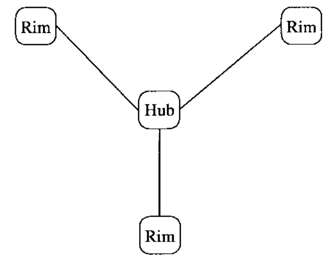
Another kind of multi-MCC ring network is described in [19]. It has its channels attached to form a *permutation interconnection network* such as the Benes network [1]. Although the ring network requires more switches/wavelength converters than ring network 6, it is useful in handling network failures. This topic is explored in more detail in [10].

III. Stars, Trees, and Meshes

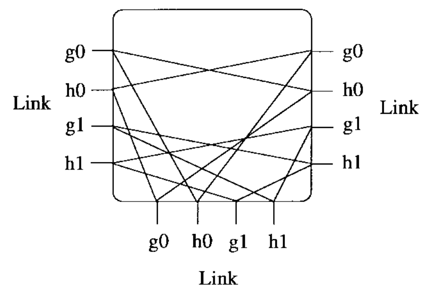
In this section, we will first consider star networks and then more general networks. Throughout this section, we will assume that W is even, and the wavelengths are partitioned into two disjoint sets of equal size, denoted by $\{g_0, g_1, \cdots, g_{(W/2)-1}\}$ and $\{h_0, h_1, \cdots, h_{(W/2)-1}\}$. For example, we could have for $i = 0, 1, \cdots, (W/2)-1, g_i = \omega_{2i}$ and $h_i = \omega_{2i+1}$.

A star network consists of a *hub* node, one or more *rim* nodes, and all links are incident to the hub (see Fig. 12). We consider hub nodes with fixed wavelength conversion such that, for $i = 0, 1, \cdots, (W/2)-1$, channels at g_i are only attached to channels at h_i, as shown in Fig. 12. Such a node will be referred to as having *fixed conversion wavelength pairs* (FCWP's). Notice that channels at the same wavelength are not attached.

As in the previous section, requests will be given structure that will lead to efficient channel assignments. In particular, the routes of a request will be given *directions* to provide for a logical orientation. However, it should be noted that the communication transmissions on the links, WDM channels, and lightpaths are still bidirectional. If the directions are such that at each link exactly half the traversing routes are directed one way and the other half directed the other way, then the request is said to have a *balanced set of directions*. For example, Fig. 13(a) shows a request for a star network, and Fig. 13(b) shows a balanced set of directions for the request. The following lemma considers tree networks, of which star networks are a special case.

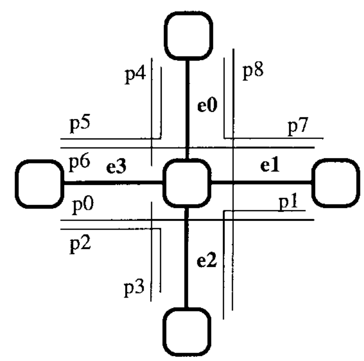


(a)

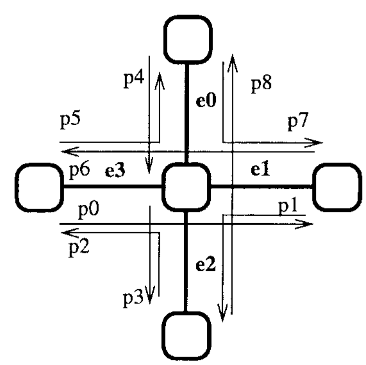


(b)

Fig. 12. A star network with three rim nodes is shown as well as how channels are attached at its hub when $W = 4$. (a) Star network. (b) Hub node.



(a)



(b)

Fig. 13. (a) Request $\{p_0, p_1, \cdots, p_8\}$ for a star network of four links $\{e_0, e_1, e_2, e_3\}$. (b) A balanced set of directions for the request.

Lemma 1: Consider a tree network T with W even. Any uniform request $R = \{p_0, p_1, \cdots, p_{m-1}\}$ with load W may have its routes directed such that it has a balanced set of directions.

Proof: First, we will show that each node in T is an end node for an even number of routes in R. For each node u, let E_u denote the set of links that are incident to it, and for each link e, let α_e be the number of routes that traverse it. Since each link has exactly W routes traversing it, $\Sigma_{e \in E_u}\ \alpha_e = |E_u| \cdot W$. Now let $n_{u,1}$ denote the number of routes that have u as an end node, and let $n_{u,2}$ denote the number of routes that have u as an intermediate node. Then $n_{u,1}+2n_{u,2} = \Sigma_{e \in E_u}\ \alpha_e$. Thus, $n_{u,1}+2n_{u,2} = |E_u| \cdot W$. Since both W and $2n_{u,2}$ are even, $n_{u,1}$ must be even for each node u.

Next, we will find a balanced set of directions for the routes in R. Initially, the routes are undirected. Starting from an end node u of some undirected route, we traverse a sequence of distinct undirected routes $p_0, p_1, \cdots$, such that route p_{i+1} begins at the node where p_i ends (for $i = 0, 1, \cdots$), until we return to u. Note that we eventually return to u because each node is an end node for an even number of undirected routes. (Therefore, while traversing the routes, if we reach a node $v \neq u$ at the end of a route, then v is an end node of an odd (nonzero) number of undirected, untraversed routes.) For routes in the sequence, we assign directions according to the way they were traversed, forming a directed *tour* of routes. Since each node is still an end node for an even number of the remaining undirected routes, we can find another directed tour. Continuing in this way in finding directed tours, all routes will be included in some tour and be given directions.

To check that the set of directions is balanced for R, it is sufficient to check that the directions of the routes in a tour are balanced. Notice that the tour must follow a tree and that it ends where it begins. Thus, the number of times it crosses a link in one direction equals the number of times it crosses the same link in the opposite direction. Hence, the directions of the routes in the tour are balanced. □

Theorem 6: Consider a star network with N nodes, W even, and where the hub node has FCWP. Then any request with load at most W has a channel assignment.

Proof: First, let $\{e_0, e_1, \cdots, e_{N-2}\}$ denote the links of the star network. We will find a channel assignment for a request R, and we will assume without loss of generality that it is uniform with load W (otherwise, we can add dummy one-hop routes). Due to Lemma 1, we also assume without loss of generality that R has a balanced set of directions.

Let $R_2 = \{p_0, p_1, \cdots, p_{m-1}\}$ denote the two-hop routes of R, where m is the number of such routes. The other routes of R are one-hop routes, and denote these by R_1. We will first find a channel assignment for the routes in R_2. The assignment gives each route p_i $(i = 0, 1, \cdots, m-1)$ in R_2 a number $n(i) \in \{0, 1, \cdots, (W/2)-1\}$ referred to as its *label*. Given the label, the channels assigned to p_i are at wavelength $g_{n(i)}$ on its first link and $h_{n(i)}$ on its second link. This assignment insures that the channels for a route are attached because the hub has FCWP. To insure that routes do not share channels, routes that have a common first or second link should have different labels.

To find such labels, we will define a (fictitious) bipartite multigraph B which has two sets of vertices

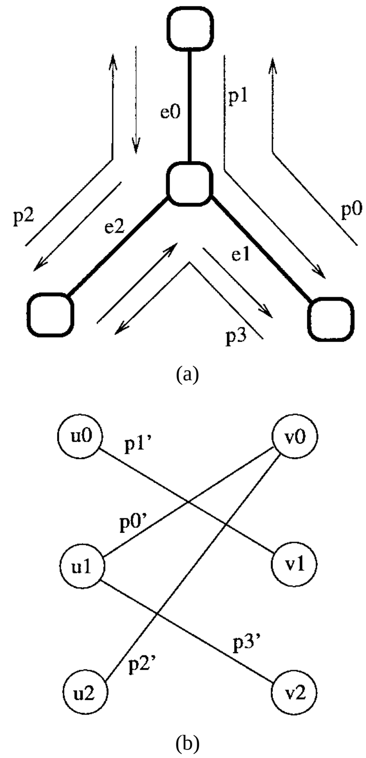


Fig. 14. (a) A star network with links $\{e_0, e_1, e_2\}$ and a request, and (b) the corresponding bipartite multigraph B for the two-hop routes $\{p_0, \cdots, p_3\}$.

$\{u_0, u_1, \cdots, u_{N-2}\}$ and $\{v_0, v_1, \cdots, v_{N-2}\}$, and a set of edges $\{p'_0, p'_1, \cdots, p'_{m-1}\}$. (Recall that a multigraph is a graph that may have more than one edge between a pair of vertices.) Each pair $\{u_i, v_i\}$ $(i = 0, 1, \cdots, N-2)$ of vertices corresponds to link e_i (in the star), and each edge p'_j $(j = 0, 1, \cdots, m-1)$ corresponds to a route p_j. Each edge p'_i $(i = 0, 1, \cdots, m-1)$ goes between vertices $u_{f(i)}$ and $v_{s(i)}$, where $s(i)$ and $f(i)$ are integers such that $e_{f(i)}$ and $e_{s(i)}$ are the first and second links, respectively, of route p_i. (Fig. 14 shows a request and the corresponding B.) Notice that for $i = 0, 1, \cdots, N-2$, the edges incident to u_i (v_i) correspond to routes in R_2 that use e_i as their first (second) link. This implies two properties. First, since R has a balanced set of directions, each vertex in B has at most $W/2$ incident edges. Second, if we can find labels from the set $\{0, 1, \cdots, (W/2)-1\}$ for edges in B such that edges incident to a common node have distinct labels, then these labels can be used for the routes in R_2 (i.e., the label for p'_i is used for $n(i)$ for $i = 0, 1, \cdots, m-1$).

Such labels can be found for the edges of B by using a scheduling algorithm for satellite switched/time division multiple access (SS/TDMA) systems [13]. The algorithm finds labels for the edges so that edges incident to a common vertex have distinct labels. In addition, the total number of distinct labels used by the algorithm equals the maximum number of incident edges at any vertex in B. Since there are at most $W/2$ edges incident to any vertex in B, the labels can be restricted to the set $\{0, 1, \cdots, (W/2)-1\}$. This completes the description of the channel assignment for the routes in R_2.

To complete the channel assignment for R, we have to find channel assignments for the routes in R_1. This is trivial since the routes in R_1 are one-hop. □

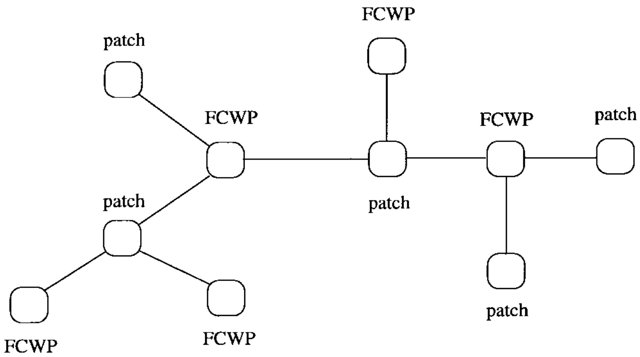


Fig. 15. A tree network with leaf nodes, patch nodes, and nodes with FCWP.

The following is a consequence of the theorem.

Theorem 7: Consider an arbitrary topology network G with W even and where all nodes with two or more incident links have FCWP. There is a channel assignment for each request that has load at most W and routes with at most two hops.

Proof: We will find a channel assignment for a request $R = \{p_0, p_1, \cdots, p_{m-1}\}$ (for some m) with load at most W. We will use a (fictitious) star network $G_{\rm star}$ to represent G. It has the same number of links and the same wavelengths $\{\omega_0, \omega_1, \cdots, \omega_{W-1}\}$. Distinct labels are given to the links in G, and corresponding labels are given to the links in $G_{\rm star}$ such that if there is a link labeled e in G then there is a link labeled e in $G_{\rm star}$. (Thus, the link e in $G_{\rm star}$ will represent the link e in G.)

Let $R' = \{p'_0, p'_1, \cdots, p'_{m-1}\}$ be a set of routes for $G_{\rm star}$ such that for $i = 0, 1, \cdots, m-1$, p'_i and p_i traverse links with the same labels. Since R' has the same load as R, we can apply Theorem 6 to find a valid channel assignment for R'. We can then find a channel assignment for R by assigning to each p_j $(j = 0, 1, \cdots, m-1)$ the channels corresponding to the ones assigned to p'_j. Thus, if a channel at wavelength ω_k (for some k) in link e in $G_{\rm star}$ is assigned to p'_j, then the channel at ω_k in link e in G is assigned to p_j. Notice that channels assigned to a two-hop route in R are still attached since their common node in G has FCWP. Thus, the channel assignment for R is valid because the channel assignment for R' is valid.

In the next theorem, we consider a tree network. Some of its nodes, referred to as *patch* nodes, will have channels at wavelengths $\{g_0, g_1, \cdots, g_{(W/2)-1}\}$ attached to channels at wavelengths $\{h_0, h_1, \cdots, h_{(W/2)-1}\}$. Note that patch nodes have wavelength degree $W/2$.

Theorem 8: Consider a tree network with W even and its nodes are either patch nodes or nodes having FCWP's. Suppose patch nodes only have FCWP nodes as neighbor nodes, and FCWP nodes only have patch nodes as neighbor nodes. Then any request with load at most W has a channel assignment.

Proof: First, note that each link is incident to some FCWP node and some patch node (see Fig. 15.) Thus, the links can be partitioned into subsets such that links incident to a common FCWP are in a subset. Note that a subset of links forms a star subnetwork with its corresponding FCWP node as the hub.

For each request R, a channel assignment can be determined as follows. First, assume without loss of generality that R is uniform with load W (otherwise, we can add dummy one-hop routes). Second, find a balanced set of directions for R, which is possible by Lemma 1. Next, "cut" routes into smaller "residual routes" at the patch nodes, while having the residual routes keep the same directions as the routes they were cut from. Note that routes that do not go through patch nodes will not be "cut," but refer to these as "residual routes" too. Note that each residual route is contained entirely in some star subnetwork. Thus, we are left with a collection of star subnetworks, each with its own set of routes with a balanced set of directions. For each star subnetwork, we can find a channel assignment for its residual routes by using the directions of the routes and the channel assignment method in the proof of Theorem 6. However, the assignment should have the additional restriction that each one-hop residual route is assigned to a wavelength in $\{g_0, \cdots, g_{(W/2)-1}\}$ ($\{h_0, \cdots, h_{(W/2)-1}\}$) if it is directed into (out of) a hub node. Such an assignment is possible because, at each link, there are $W/2$ channels dedicated to routes that traverse the link in any particular direction (more specifically, the channels at $\{g_0, \cdots, g_{(W/2)-1}\}$ are for one direction and the channels at $\{h_0, \cdots, h_{(W/2)-1}\}$ are for the other direction) and there are $W/2$ routes that traverse the link in any particular direction. The channel assignment for R is to assign to each route $p \in R$ the channels of p's residual routes.

We now turn to check the validity of the channel assignment for R. Note that the routes of R do not share channels since they were pieced together from channel assignments of their residual routes. Now we need to check if the channels of routes are attached. Note that consecutive channels of a route meet either at patch nodes or at nodes with FCWP. Consecutive channels are attached at nodes with FCWP because they correspond to the channels assigned to a residual route. They are also attached at patch nodes because one of them corresponds to a residual route directed into the patch node (thus, it has a wavelength in $\{h_0, \cdots, h_{(W/2)-1}\}$), and the other corresponds to a residual route directed out of the patch node (thus, it has a wavelength in $\{g_0, \cdots, g_{(W/2)-1}\}$). □

It should be noted that *leaf* nodes (i.e., ones that have only one incident link) in the tree network will not require any wavelength conversion.

IV. Conclusion

We considered a network model with bidirectional links, WDM channels, and lightpaths for ring, star, and tree networks, as well as arbitrary topology networks with restrictions on lightpath route lengths. We showed that there are ring and star networks with minimal wavelength conversion capabilities that can perform off-line channel assignment as well as networks with full wavelength conversion. In fact, it was shown that there are ring and star networks that only require wavelengths to be shifted to their nearest wavelengths, which is very minimal wavelength shifting.

Note that each bidirectional WDM channel was assumed to be implemented using two unidirectional channels, in opposite directions, but at the same wavelength. If the unidirectional channels could be at different wavelengths, then (perhaps) there may be networks with even less wavelength conversion but with the same performance. This is the case for star networks. In particular, consider one with no wavelength conversion and an even number of wavelengths W split into two equal-sized sets $\{g_0, g_1, \cdots, g_{(W/2)-1}\}$ and $\{h_0, h_1, \cdots, h_{(W/2)-1}\}$. For this network, a bidirectional channel is composed of two unidirectional channels, one at g_i and the other at h_i for some i. The bidirectional channel is referred to as a "g_i" ("h_i") channel if its unidirectional channel going into the hub node has wavelength g_i (h_i). Thus, bidirectional channels are attached if they are labeled g_i and h_i for some i. Hence, the network is the same as the star network considered in Theorem 6 except it is implemented without wavelength conversion.

It should also be noted that the results of this paper can be extended to the case when links, WDM channels, and lightpaths are unidirectional. The results for ring networks in Section II can be extended in a straightforward way. If the channels, links, and lightpaths are unidirectional in the clockwise direction, then the results for rings are easily extended because in Section II the routes have clockwise directions.

The results for stars in Section III can also be extended for the following modified star network. Each bidirectional link is composed of two unidirectional ones going in opposite directions. Each unidirectional link has channels at $\{\omega_0, \omega_1, \cdots, \omega_{W-1}\}$. The hub node has no conversion. Lightpaths are unidirectional and have routes directed accordingly. We will refer to this star as the *directed star*, and the stars in Section III as the *undirected stars*.

Note that the channels directed into (out of) the hub in the directed star are analagous to the channels at $\{g_0, g_1, \cdots\}$ ($\{h_0, h_1, \cdots,\}$) in the undirected star. In addition, if $g_i = h_i$ for all i, then the no-conversion hub in the directed star is analagous to the FCWP hub in the directed star. Then the arguments used in the proof of Theorem 6 can be used to show that a channel assignment exists for any set of directed routes on the directed star such that the number of routes traversing a link in any direction is at most W. Note that the more general case of trees with unidirectional channels, unidirectional lightpaths, and no conversion is treated in [17]. Also note that we can extend our results for tress in Theorem 8 for unidirectional channels and lightpaths just as we did for stars. Then patch nodes have full conversion, and FCWP nodes have no conversion.

Extending the results to arbitrary topologies is an interesting open problem. Considering dynamic lightpaths, where they are set up and terminated at arbitrary times, is also an important area of investigation. As mentioned earlier, this has been considered in [9], [11]. To guarantee channel assignments of arriving lightpaths while not disturbing existing ones, lightpath load was required to be low. Our off-line results may imply that if high channel utilization is required and conversion device costs are high then limited conversion architectures with channel rearrangement should be considered.

APPENDIX
PROOF OF THEOREM 3

Proof: Consider a ring network with W wavelengths, fixed wavelength conversion at nodes, and $N \geq 2W$ (i.e., N is sufficiently large). Notice that since the network has fixed wavelength conversion, attached channels form MCC's that are channel-disjoint. Let $\{H_0, H_1, \cdots, H_{k-1}\}$ denote the MCC's where k is the number of MCC's, and for $i = 0, 1, \cdots, k-1$, let m_i denote the multiplicity of H_i.

We will consider requests R that are uniform with load W. For such a request, suppose that there is a channel assignment. Consider the routes with channels from H_i (for some $i = 0, 1, \cdots, k-1$). Note that the routes only have channels from H_i because the channels for the routes must be attached. In addition, the routes use up all the channels from H_i because R is uniform with load W. Since the routes correspond to channels in the MCC H_i of multiplicity m_i, they can form an MCR of multiplicity m_i. Since this is true for $i = 0, 1, \cdots, k-1$, there must be an MCR partition $\{M_0, M_1, \cdots, M_{k-1}\}$ for R, where M_i has multiplicity m_i for $i = 0, 1, \cdots, k-1$. For the rest of the proof, we will define requests R that are uniform with load W and do not have channel assignments, or equivalently, do not have an MCR partition $\{M_0, M_1, \cdots, M_{k-1}\}$ (for some k), where M_i has multiplicity m_i for $i = 0, 1, \cdots, k-1$.

We consider two cases. In the first case, suppose for some $j, m_j > 1$. Then we will define a request R that only has MCR's with multiplicity one. In particular, let $R = \{p_0, p_1, \cdots, p_{W-1}, p'_0, p'_1, \cdots, p'_{W-1}\}$ such that, for $i = 0, 1, \cdots, W-1, p_i$ starts at node i, goes clockwise around the ring, and ends at node $W+i$, and p'_i starts at node $W+i$, goes clockwise around the ring, and ends at node i. It is straightforward to verify that R is uniform with load W. Now note that by having N sufficiently large, for $i = 0, 1, \cdots, W-1$, p_i and p'_i are the only routes that have nodes i and $W+i$ as terminating nodes. Thus, the only MCR partition for R is $\{M_0, M_1, \cdots, M_{W-1}\}$, where M_i equals (p_i, p'_i) and has multiplicity one for $i = 0, 1, \cdots, W-1$. Since $m_j \neq 1, R$ has no channel assignment.

In the second case, suppose $k = W$ and, for $i = 0, 1, \cdots, k-1$, $m_i = 1$. We will define a request R that can only form an MCR with multiplicity W. In particular, let $R = \{p_0, p_1, \cdots, p_W\}$, where p_0 is a route that goes clockwise around the ring from node 0 to node W, and, for $i = 1, 2, \cdots, W, p_i$ goes clockwise around the ring from node i to node $i-1$. It is straightforward to verify that R is uniform with load W. In addition, by having N sufficiently large, for $i = 0, 1, \cdots, W$, routes p_i and $p_{(i+1) \bmod (W+1)}$ are the only ones in R that have the terminating node i. Thus, the only MCR in R is $(p_0, p_1, \cdots, p_W)$, which has multiplicity W. Since the multiplicities of all the MCC's are one, R has no channel assignment.

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