WDM network design by ILP models based on flow aggregation

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Abstract— **Planning and optimization of WDM networks has raised much interest among the research community in the last years. Integer Linear Programming (ILP) is the most used exact method to perform this task and many studies have been published concerning this issue. Unfortunately many works have shown that, even for small networks, the ILP formulations can easily overwhelm the capabilities of today state-of-the-art computing facilities. So in this article we focus our attention on ILP model computational efficiency in order to provide a more effective tool in view of direct planning or other benchmarking applications. Our formulation exploits flow aggregation and consists in a new ILP formulation that allows us to reach optimal solutions with less computational effort compared to other ILP approaches. This formulation applies to multifiber mesh networks with or without wavelength conversion. After presenting the formulation we discuss the results obtained in the optimization of case-study networks.**

I. Introduction

In recent optical networks the introduction of Wavelength Division Multiplexing (WDM) technique has opened the road to a new paradigm of transport infrastructure evolution characterized by high capacity and high reliability. On the switching equipment side, Optical Cross Connects (OXC) systems have become available, beside the more mature Optical Add-Drop Multiplexers. This opened up the road to the possibility of deploying complex WDM networks based on mesh topologies, while in the past single ring or overlaid multi-ring were the most used architectures for WDM. In order to transfer data between two nodes, an optical connection needs to be set up and routed at the optical layer as in a circuit-switched network.

The increase in WDM complexity brought the need for suitable network planning strategies into the foreground. Problems such as optimal dimensioning, routing and resource allocation for optical connections must be continuously solved by new and old operators, to plan new installations or to update and expand the existing ones. These problems can no longer be manually solved in complex network architectures, as it usually happened in the earlier experimental WDM installations. Computer-aided planning tools and procedures are needed for the future which can determine how to utilize efficiently the network resources in a reasonable computational time.

Since some years ago research on optical networks has been investigating design and optimization techniques. The various proposed solutions can be classified into two main groups: heuristic methods and exact methods. The former return sub-optimal solutions that in many cases are acceptable and have the advantage of requiring a limited computational effort. The latter are much more computationally intensive and do not scale well with the network size, being even not applicable in some cases. However since the exact methods are able to identify the absolute optimal solution, they play a fundamental role either as direct planning tools or as benchmarks to validate and test heuristic methods.

The work we are presenting concerns exact methods to plan and optimize multifiber WDM networks. In particular we focus on Integer Linear Programming (ILP), a widespread technique to solve exact optimization: we propose a new formulation of the optimization problem that we call *source formulation*, in that it exploits the aggregation of all the flows generated in a single source node [1]. Our source formulation is equivalent to the well known flow formulation, but it allows a relevant reduction of the number of variables and of constraints, thus sensibly diminishing computation time and memory occupancy during optimization runs.

The paper summary is as follows. In section II we introduce our solution by presenting a short review of the literature regarding ILP applications to WDM optimization. In section III the source formulation is presented and explained into details in the two versions for network with or without wavelength conversion. Finally, in section IV results obtained by applying the source formulation to case-study networks are shown and new the formulation is compared to the traditional flow and route formulations to point out the advantages of the method we are proposing. An appendix is finally dedicated to show the equivalence of flow and source formulation.

II. WDM network optimization by integer linear programming

Network design and planning is carried out with different techniques according to the type of traffic the network has to support. We investigate the static traffic case in which a known set of permanent connection requests is assigned *a priori* to the network. The connections requested by the nodes at a given time to a WDM network all together form the offered traffic matrix virtual topology (alias virtual topology). Each request is for one or more point-to-point optical circuits (lightpaths) able to carry a given capacity from the source termination to the destination termination. We assume that all the WDM channels carry the same capacity. Lightpaths are routed and switched by the OXCs of the network and the two lightpath terminations are located in the source and destination OXCs.

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We assume that the channels composing the lightpath (one for each fiber it crosses) may have different wavelengths or may be all at the same wavelength, according to the availability of the wavelength conversion function in the transit OXCs. To simplify, we have considered two extreme cases referring to definitions introduced in [2]: the Virtual Wavelength Path (VWP) network case, in which all the OXC's are able to perform full wavelength conversion, and the Wavelength Path (WP) network case, in which no wavelength conversion is allowed in the whole network and lightpaths are subject to the "wavelength continuity" constraint, that is absent in the VWP case. It's important noting that wavelength assignment to lightpaths in WP case is an NP-complete problem (it is equivalent to the well-known graph-coloring problem) [3].

Today WDM networks are often designed in order to be resilient to failures that may occur to switching or transmission equipment. Though automatic lightpath protection is very important today (given the high bit-rates that a WDM channel usually carries, e.g. 2.5 to 40 Gbit/s), this feature will not be covered in this work, for the reasons that will be explained later on.

Static optimization of a WDM network can be summarized as follows: given a static traffic matrix, find the optimum values of a set of network variables that minimizes a given cost (or objective) function, under a set of constraints. The choice of variables, cost function and constraints greatly varies from case to case. In the past most of studies regarding WDM network planning were aimed at virtual topology optimization with single-fiber WDM links [4], [5]. The cost function to be optimized was either the number of wavelengths necessary to route the static traffic or the network load (the number of channels routed on the most loaded link of the network) [6]. In Ref. [6] the authors introduce an ILP model based on aggregated flows applied to virtual topology optimization. In the work we are proposing the virtual topology optimization is accompanied by cost minimization of a multi-fiber physical network: the number of fibers per link needed to support a preassigned traffic matrix is a variable of the problem to be minimized, while the amount of wavelengths per fiber is preset [7].

WDM network optimization by ILP has been widely studied in literature. We can subdivide research contributions in two groups according to the type of networks they are applied to:

- WDM networks with single-fiber links;
- multifiber WDM networks.

In the first group the problem consists in optimal routing and wavelength assignment (RWA) of the lightpaths. This is a NP-complete problem, as it was demonstrated in Refs. [8], [3]. Two basic methods have been defined to model the RWA problem: *flow formulation*(FF) and *route formulation* (RF) [9]. In the former the basic variables are the flows on each link relative to each source-destination OXC pair; in the latter the basic variables are the paths connecting each source-destination termination pair. Both these formulations have been employed to solve various sorts of problems and to investigate different aspects of WDM networks. For example, in Ref. [9] the optimization is carried out in order to emphasize the difference between WP and VWP scenarios. Ref. [6] studies the effects of imposing a constraint on the average delay seen by a source-destination pair and the amount of processing required at the nodes, while in Ref. [10] possible utilization of bounds derived from the two formulations by relaxation of the integer constraints are studied and compared. In other works, the authors have selected as cost functions the number of wavelengths [11], [9] or the total number of WDM channels in the network [12], [13]. In Ref. [14] authors propose new ILP formulations, which tend to have integer optimal solutions even when the integrality constraints are relaxed, thereby allowing the problem to be solved optimally by fast and highly efficient linear (not integer) programming methods. In Ref. [15] an exact linear formulation was presented for the logical topology design problem with no wavelength converters. In Ref. [16] the authors have investigated the so called RWA-P, i.e the RWA problem while allowing for degradation of routed signals by optical components.

In optimization of multifiber WDM networks optimal allocation of fibers has also to be solved, thus complicating the problem of lightpath set up into routing, fiber and wavelength assignment (RFWA). Solving RFWA becomes really challenging even with relatively small networks, especially because routing and wavelength assignment is coupled to dimensioning. In this case a new set of variables representing the number of fibers of each physical link must be considered in addition to the flow or the route variables defined above for the two formulations. This implies that RFWA has also to include the highly complex localization problem. The choice of complex cost functions such as those comprising node or duct cost makes the achievement of ILP optimal solution very challenging even for very small networks [17] (this is even worse in the case of non-linear objective function that require integer non-linear programming [18]).

When the problem becomes computationally impractical, route formulation becomes more useful than flow formulation. If it is acceptable that RFWA is performed in a constrained way, then the solution complexity of the route formulation can be controlled. For example, all the lightpaths can be constrained to be routed along the first k shortest paths connecting the source to the destination. Differently from the flow formulation, the complexity of which is strictly dependent on physical and virtual topologies, the complexity of the route formulation decreases with the number of paths that can be employed to route the lightpaths. Multifiber network optimization with route formulation and constrained routing has been studied in Refs. [19], [20], [21], [9], [22], [23].

Beside route formulation with constrained routing, other methods to control complexity have been proposed. A possibility is to stop the branch-and-bound algorithm (typically used to solve ILP problems) after finding the first or a pre-definite number of integer solutions. Ref. [17] shows that acceptable results (though quite far from the optimal solution) can be obtained when the branch-and-bound duration is fixed to 10 minutes. Ref. [24] proposed that the whole RFWA problem can be solved as a sequence of simpler problems (e.g. first routing, then fiber assignment, and so on). Other possible approaches are: exploitation of lagrangean relaxation [25], [23], relaxation of integer constraints [19] and randomized routing [12].

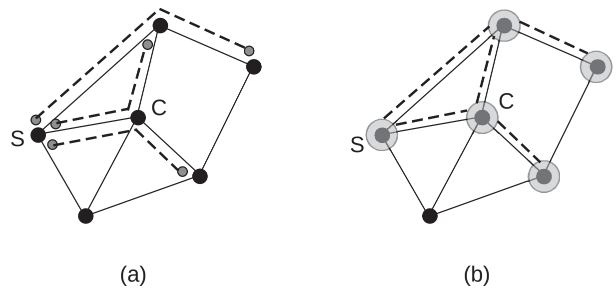


Fig. 1. Example of three distinct source-destination commodities (a) and the corresponding single source commodity (b), which will be exploited in source formulation

Undoubtedly the massive need for computational resources (i.e. processing time and memory occupation) represents the main obstacle to an efficient application of ILP in optical network design. Constrained routing and the other simplification techniques are able to overcome this limitation, but the solution they produce is only an approximation of the actual optimal network design. The great advantage of ILP over heuristic methods is the ability to guarantee that the obtained solution is the absolute optimum value. Any of the above techniques aimed at reducing the computational burden implies that the ILP approach loses its *added value*, even if the approximated solutions may be close to the exact one. Our work develops and applies a new formulation of RFWA problem which is able to prune variable multiplicity without introducing any approximation, thus preserving the *added value* of mathematical programming.

III. SOURCE FORMULATION OF THE RFWA PROBLEM

Let us consider a multifiber WDM network environment under static traffic, in which the number of wavelengths per fiber W is given *a priori*, while the fiber numbers of each physical link are variables of the problem.

Traditional ILP formulations based on flow or route paradigm[1] solve the RFWA problem managing source-destination commodity, that is to say that these formulations route static connection requests identified by a source and a destination node on the graph representing the WDM network (see fig. 1(a)).

In our proposal the ILP formulation will consider all the connections originating from a single source OXC as a single commodity (see fig.1(b)). Let us observe that single source commodity on link $S-C$ assumes value equal to 2, because on that link there are two source -destination commodities having origin in node S. Thanks to this new model (that from now on we will call *source formulation* or SF), we are able to prune the number of variables associated to traffic flows, thus reducing computational time and memory occupation compared to the flow formulation.

In order to minimize the number of fibers needed to support a certain amount of traffic, source and flow formulations are equivalent. In appendix we show the equivalence of these two formulations by describing how to obtain an equivalent FF (SF) solution, given a SF (FF) solution. In other words, if the objective is to evaluate the number and the distribution of the fibers in the network, we can simply apply SF in order to achieve the solution. Then, if we are interested also in the details of the RWA (i.e the routing and wavelength assignment of each connection request), we have to transform the SF solution in a FF (or equivalent solution). This second step is absolutely negligible from a complexity point of view: if the first SF step is a localization problem, the second step (needed to transform the SF solution in detailed RFWA description) has the same complexity of a mere max-flow algorithm (for further details refer to the appendix). So all the computational times reported in the following are related to the SF step, disregarding the possible following transformation.

We explain now the details of the source formulation, for which two different versions are reported related to networks with or without wavelength conversion capability.

A. Source formulation for VWP networks

First we consider a VWP network, provided with full wavelength conversion as defined in II. The physical topology is modeled by the graph $\mathcal{G} = \mathcal{G}(\mathcal{N}, \mathcal{A})$. Physical links are represented by the undirected edges $l \in \mathcal{A}$ with $|\mathcal{A}| = L$, while the nodes $i \in \mathcal{N} = \{1, 2, \cdots N\}$, with $|\mathcal{N}| = N$, represent the OXCs. Each link is equipped with a certain amount of unidirectional fibers in each of the two directions; fiber direction is identified by the binary variable k. Finally, the virtual topology is represented by the set of known terms $C_{i,j}$, each one expressing the number of connections that must be established from the source node i to the destination node j. Unidirectional point-to-point connections are considered (thus, in the general case, $C_{i,j} \neq C_{j,i}$).

The variables in the source formulation are the following:

- $x^i_{l,k}$ is the number of WDM channels on link l on fibers having direction k which have been allocated to lightpaths generated at node i;
- $F_{l,k}$ is the number of fibers on link l with direction k.

It should be noted that the flow variables $x^i_{l,k}$ are defined in such a way that all the traffic originating from the same node and traveling on the same link in the same direction is represented in an aggregated form, regardless of the destination. This is the main aspect that differentiates source from flow formulation.

The following additional symbols are defined:

- (l, k) identifies the set of fibers of link l that are directed as indicated by k; for sake of clarity, in the following we name (l, k) a "unidirectional link";
- I^+_i is the set of "unidirectional links" having the node i as one extreme and leaving the node; analogously, I^-_i is the set of "unidirectional links" having the node i as a one extreme and pointing towards the node;
- $S_i = \sum_j C_{i,j}$ is the total number of requested connections having node i as source.

Now we can detail the source formulation. The cost function to be minimized is the total fiber number

$$\min \sum_{(l,k)} F_{l,k}$$

[1]From now on the flow formulation case will be considered the main term of comparison.

Actually the source formulation can be very easily extended to solve optimization problems based on the length metric. The only change that must be made regards the cost function, which becomes

$$\min \sum_{(l,k)} F_{l,k} \cdot p_l$$

where p_l is the geographical length of link l.

The set of constraints is the following

$$\sum_{(l,k)\in I_i^+} x_{l,k}^i = S_i \qquad \forall\ i; \qquad (1)$$

$$\sum_{(l,k)\in I_j^+} x_{l,k}^i = \sum_{(l,k)\in I_j^-} x_{l,k}^i - C_{i,j} \qquad \forall\ (i,j), j \neq i; \qquad (2)$$

$$\sum_i x_{l,k}^i \leq W \cdot F_{l,k} \qquad \forall\ (l,k); \qquad (3)$$

$$x_{l,k}^i \text{ integer} \qquad \forall\ i, (l,k); \qquad (4)$$

$$F_{l,k} \text{ integer} \qquad \forall (l,k); \qquad (5)$$

Constraint (1) is a solenoidality constraint which imposes that the total flow (number of lightpaths) generated by node i and exiting from it must be equal to the total number of connection requests having node i as source. Note that the solenoidality constraint is not applied on each node-pair (by which a connection is requested) but on the aggregated traffic relative to a source node: therefore it is not dependent on destinations.

Constraint (2) is again a solenoidality constraint. It corresponds to the following sequence. Let us take a node i. We express the flow conservation condition for each other node of the network $j \neq i$, considering only traffic having i as source node. This condition states that the total flow generated by i and leaving j is given by the total flow generated by i and incident on j minus the number of requested connections having i as source and j as destination ($C_{i,j}$).

In Fig. 2 we show the different application of the solenoidality constraint in the flow and source formulation cases using two simple examples. The first example shown in Fig. 2a refers to solenoidality constraint in the classical flow formulation. A single connection request has been routed between source node s and destination node d through nodes a and c (dotted line). The flows associated to this connection are represented by a solid arrow in the round windows that magnify the situation in nodes s, c and d: in the source (destination) the node leaving (entering) flow is equal to the offered traffic (i.e. a traffic unit), while in the intermediate nodes the leaving flows equal the entering flows (e.g., in node c the leaving arrow has a correspondent entering arrow).

Fig. 2b refers to solenoidality constraint working in source formulation case: a simple network case with two connection requests (between s and c and s and d) is shown. At the source node s, the sum of the leaving flows is enforced to be equal to the sum of the traffics to be routed towards all the destinations (in this example two traffic units, one destined to node c, the other destined to node d). In the other nodes the sum of entering flows equals the sum of leaving flows plus the traffic that is dropped at that node (e.g node c in Fig. 2b

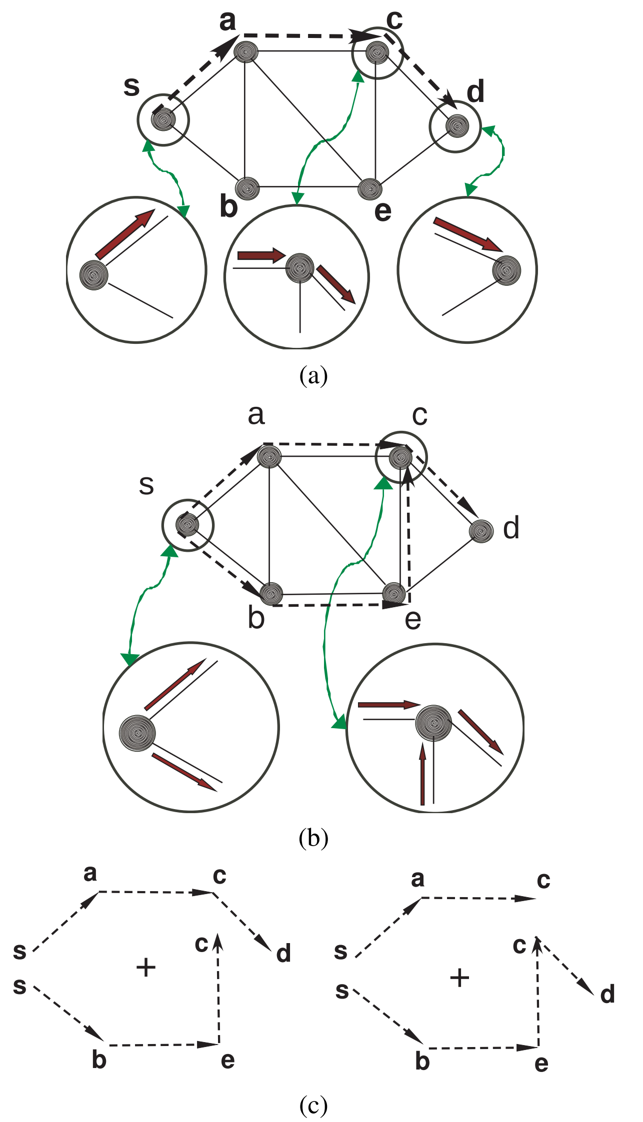


Fig. 2. The solenoidality constraint in flow formulation (a) and source formulation (b). In (c), two admissible solutions derivable from the previous source formulation outcome

we have two entering flows and just one leaving due to the flow which is dropped at that node).

It is worth noting that the source formulation does not return a detailed mapping of routing (i.e. a path for each single connection request), even if it optimally assigns the number of fibers needed to support the traffic; a second step[2] must be used to identify the routing of the connections. In other words the source formulation loses the information of the routing of each single connection due to the aggregation of flows on the basic variable. Let us refer to Fig. 2c: over the source formulation outcome shown in Fig. 2b, we can map two distinct (yet admissible) routing assignments (RA): in a first RA the two connections are routed on the two paths $s-a-c-d$ and $s-b-e-c$, while a second admissible RA could be $s-b-e-c-d$ and $s-a-c$.

The capacity constraint (3) allows us to dimension the physical network capacity. In order to ensure a feasible resource

[2]for further details see the appendix

allocation it imposes that on each link the sum of flows generated by all the nodes is smaller than the product of the number of fibers by the number of wavelengths per fiber. The remaining constraints (4 and 5) enforce variable integrity.

Let us now discuss the source formulation complexity for a VWP network. Table I shows the relations expressing the total number of variables and constraints as functions of the physical topology size and the number of node pairs requiring connections. The corresponding relations for the flow and route formulation are reported for comparison (symbols reported in Table I have been previously described, except for R that represents the mean number of possible alternative routes between two nodes in the network). In the table, C is the number of source-destination node-pairs requiring connections, that is upper-bounded by the number of node pairs of the virtual topology $C = N(N-1)$.

The number of variables of the source formulation grows with the product of the number of links by the number of nodes. In the flow formulation it grows instead with the product of the number of links by the number of node pairs requesting connections. So, from the variable number point of view, source formulation should be more efficient than flow formulation under the condition $C > N$, that is presumably a common situation in real networks. If there is at least one lightpath requested by each node pair, then we could set $C = N(N-1)$ (the maximal value that could be achieved by C). Source formulation in this case allows a reduction of the number of variables by a factor N compared to flow formulation. The same order of reduction is obtained on the number of constraints, whose complexity decreases from $NC(\simeq N^3)$ to N^2. The previous comparison is focused on the difference between flow and source formulations. As far the route formulation without constrained routing is concerned, we can easily notice that the variables number is dependent on the term $C \cdot R$, i.e. the total number of possible alternative paths for each node pair requiring connections in a network. This means that the number of variables tends to grow very quickly with network connectivity and dimension, so that, for example in our case study-networks, flow formulation is modelized by a lower number of variables than route formulation.

B. Source formulation for WP networks

The source formulation can be extended to networks without wavelength conversion capability. ILP complexity in the WP case grows with the number of wavelengths per fiber W and constraints become more complicated because wavelength continuity has to be imposed on the lightpaths. Nevertheless the advantages of the source over flow formulation are still relevant.

The cost function is the same as in the VWP case III-A. A new index $\lambda \in \{1, 2, \dots W\}$ must be added to identify the wavelength of the WDM channels, in order to impose the wavelength continuity constraint along a lightpath. Flow variables defined in the VWP case are transformed: $x^i_{l,k,\lambda}$ now indicates the number of WDM channels having wavelength λ which on the "unidirectional link" (l,k) carry lightpaths generated at node i. The known terms S_i and $C_{i,j}$ have to be split, originating the new variables $s_{i,\lambda}$ and $c_{i,j,\lambda}$.

The set of constraints is modified as shown below:

$$\sum_{(l,k)\in I_i^+} x^i_{l,k,\lambda} = s_{i,\lambda} \qquad \forall\ (i,\lambda); \tag{6}$$

$$\sum_{\lambda} s_{i,\lambda} = S_i \qquad \forall\ i; \tag{7}$$

$$\sum_{(l,k)\in I_j^-} x^i_{l,k,\lambda} = \sum_{(l,k)\in I_j^+} x^i_{l,k,\lambda} - c_{i,j,\lambda} \qquad \forall\ (i,j,\lambda), j \neq i; \tag{8}$$

$$\sum_{\lambda} c_{i,j,\lambda} = C_{i,j} \qquad \forall\ (i,j) \quad i \neq j; \tag{9}$$

$$\sum_{i} x^i_{l,k,\lambda} \leq F_{l,k} \qquad \forall\ (l,k,\lambda); \tag{10}$$

$$\begin{aligned} x^i_{l,k,\lambda} &\ \text{integer} && \forall\ (i,l,k,\lambda); \\ F_{l,k} &\ \text{integer} && \forall\ (l,k); \\ s_{i,\lambda} &\ \text{integer} && \forall\ (i,\lambda); \\ c_{i,j,\lambda} &\ \text{integer} && \forall\ (i,j,\lambda),\ i \neq j; \end{aligned}$$

The solenoidality constraints are split into the sets (6) and (8) in order to impose flow conservation independently for each wavelength. Also the capacity constraint (10) is modified. The new constraints (7) and (9) express the distribution of the total number of connections among the different wavelengths for a source node and for a source-destination pair respectively.

Table I compares the complexity of source and flow formulations also in the WP case. In both formulations the number of constraints and variables increases linearly with W. It is important to notice that the increase of W in the WP scenario is accompanied not only by a growth of variable and constraint numbers, but also by the extension of range of possible values that the variable can take. Although not directly arguable from the table, this has a great impact on computational time and memory requirement.

The advantage of the source formulation can be evaluated in a simple way by considering a fully-connected virtual topology in which $C = N(N-1)$. Under such assumption the dominant term of the number of variables is $2W \cdot L \cdot N$ and $2WL \cdot N^2$ for source and flow formulation, respectively. As for the number of constraints, the two dominant terms are $W \cdot N^2$ and $W \cdot N^3$, respectively.

Finally, we shall mention a limitation of the source formulation. Unfortunately, this formulation can not be extended to optimize path-protected WDM networks. In fact path protection requires to route lightpaths under the link-disjoint constraint, so that a working lightpath can not share any physical link with its protection lightpath. The basic variables $x^i_{l,k}$ contains information concerning all the connections having the same source node aggregated together. No explicit reference can be inferred regarding lightpaths having the same source and the same destination, so that the link-disjoint constraint can not be enforced. Anyway other protection techniques, such as link protection, could be planned using source formulation. In fact an approach to link protection consists in providing for each link (i.e for all its fibers) an alternative route in order to face link failure: such a feature doesn't need information related to

TABLE I

COMPARISON ON CONSTRAINT AND VARIABLE NUMBERS BETWEEN SOURCE AND FLOW FORMULATIONS.

formulation	constraints	variables
VWP source	$2L + N^2$	$2L(1+N)$
VWP flow	$2L + N \cdot C$	$2L(1+C)$
VWP route	$2L + C$	$C \cdot R + 2L$
WP source	$W(2L + N^2) + C + N$	$W(N + 2L \cdot N + C) + 2L$
WP flow	$W(2L + (N-1) \cdot C) + C$	$WC(1+2L) + 2L$
WP route	$C \cdot (W+1) + 2L \cdot W$	$C \cdot R \cdot W + C \cdot W + 2L$

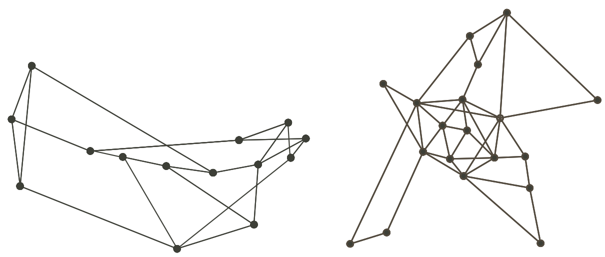


Fig. 3. Physical topologies of two case-study networks: (a) NSFNET and (b) EON.

TABLE II

ILP VARIABLES AND CONSTRAINTS FOR NSFNET AND EON IN THE VWP CASE.

network/formul.	constraints	variables
NSFNET/source	240	660
NSFNET/flow	1556	4796
NSFNET/route	152	14604
EON/source	439	1560
EON/flow	6576	26754
EON/route	420	$\cong 3 \cdot 10^8$

traffic destination node. A source formulation based model for link protection in both dedicated and shared cases is currently under study.

IV. CASE STUDIES AND RESULT COMPARISON

In this section we present and discuss the results obtained by ILP optimization exploiting source formulation on two case-study network types in comparison with results obtained using traditional flow or route formulations. Well-known mesh networks are considered first, that is the National Science Foundation Network (NSFNET) and the European Optical Network (EON). Then a class of networks called "wheel networks" are considered, in which the variation of the connectivity index [2], [11] defines a set of topologies ranging from the ring to the full-mesh network.

A. NSFNET and EON

Data regarding the physical topology of NSFNET and EON, represented in Fig. 3a and 3b, have been taken from Ref. [21][3] and Ref. [26], respectively. NSFNET has 14 nodes and 22 links, while EON has 19 nodes and 39 links. The virtual topologies are based on the static (symmetric) traffic matrices derived from real traffic measurements which are reported in the same references. The two traffic matrices comprise 360 and 1380 unidirectional connection requests for NSFNET and EON, respectively, while the distinct node pairs requiring connections [4] are respectively 108 and 342. Both VWP and WP cases have been analyzed.

Table II shows the number of variables and constraints that are involved in the ILP problem applied to the two networks in the VWP case. They clearly show the advantage achieved using the aggregation of flows. Data are computed using the relations reported in Table I except the number of variables in route formulation, which needs as input variable all admissible paths in the network between nodes requiring connections. So, in order to run an optimization based on non-constrained route formulation we have precomputed all the possible alternative paths using a greedy routine: our algorithm takes about five hours to compute the 14604 paths in the NSFNET network, while in the EON, due to its greater dimension, our algorithm takes about ten hours to compute about $1.57 \cdot 10^5$ paths between a single node and all the other nodes taken as destinations. The huge number of variables in this last case induced us not to proceed on EON optimization based on unconstrained route formulation.

For the WP case, a comparison can be done taking into account route formulation complexity. In this latter case, as we can argue from Fig. 4, the number of constraints is smaller than in source formulation, but the number of variables grows rapidly. In fact it is associated to the number of all the possible routes connecting each node couple, that increases exponentially with network dimension and in particular it is influenced by the connectivity index of the network. In the following we will see how the increase in network dimension and connectivity will affect ILP model performance.

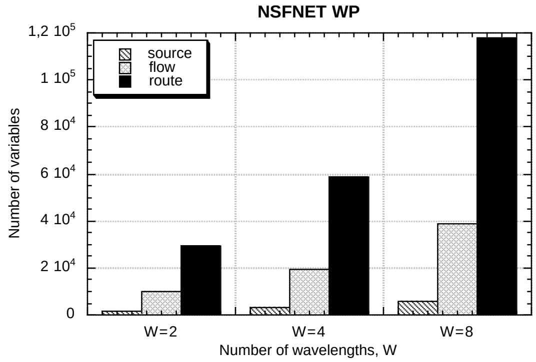
To solve the ILP problems we used the software tool CPLEX 6.5 based on the branch-and-bound method [27]. As hardware platform a workstation equipped with a 1 GHz processor was used. The available memory (physical RAM + swap) amounted to 900 MByte.

Before the presentation of numerical results, it is crucial to remember that the the source and the flow formulation are equivalent (see appendix). This equivalence is confirmed in all the network cases in which both formulations succeed in finding the optimum value: this value in fact results to be the same in the two formulations.

We have already shown the advantage of source formulation versus flow and route formulation in terms of variable

[3] The reported NSFNET topology is actually the NSFNET T1 backbone [6] with the addition of one extra link.

[4] Each node pair can require more than one connection.



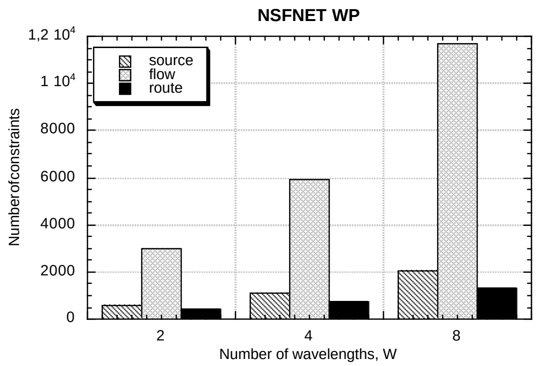


Fig. 4. ILP variables and constraints for NSFNET in the WP case.

TABLE III
VWP NSFNET OPTIMIZATION: COMPUTATIONAL TIME.

W	source form.	flow form.	route form.
2	**27 m**	**1.5 h**	**1.6 h**
4	55 m	3.7 h	1.8 h
8	**36 s**	**26 m**	**1.5 h**
16	**3 m**	9 h	**3 h**
32	**19 m**	7,9 h	**6 h**

and constraint numbers. It is important to see how much this advantage affects the actual computational performance of ILP. Tables III and IV display computational time and memory occupation measurements of NSFNET optimization in the VWP case (s, m, h and d stand for seconds, minutes, hours and days respectively, while MB stands for mega-byte). Computational times in bold are associated to runs succeeding in finding optimal values.

To clearly understand the reported data, a particular aspect of ILP must be clarified. The branch-and-bound algorithm progressively occupies memory with its data structure while it is running. When the optimal solution is found, the algorithm stops and the computational time and the final memory occupation can be measured. In some cases, however, all the available memory is filled up before the optimal solution can be found. In this cases CPLEX returns the best but non-optimal solution that branch-and-bound has been able to find and forces the execution to quit. These cases are identified by the *out-of-memory* tag (O.O.M.) and the computational time measures how long it has taken to fill up memory. This integer solution, forced to be returned because of the limited amount of memory, is associated to the so-called *gap* parameter that expresses the percentage difference between the integer solution found and the minimal possible value the solution could reach (i.e. a lower bound returned by branch and bound algorithm). This parameter returns an estimation of the quality of the non-optimal integer solution found in terms of maximal possible distance from the optimum. From tables III and IV we can see that the out-of-memory event is less frequent with the source formulation than with flow formulation. Moreover source formulation always requires a smaller memory amount and a shorter run duration than the other two formulations. The gap in run duration between source and the other two tends to increase with the W parameter. This is probably due to the extension of the range of the possible values that the variable can take.

TABLE IV
VWP NSFNET OPTIMIZATION: MEMORY OCCUPATION.

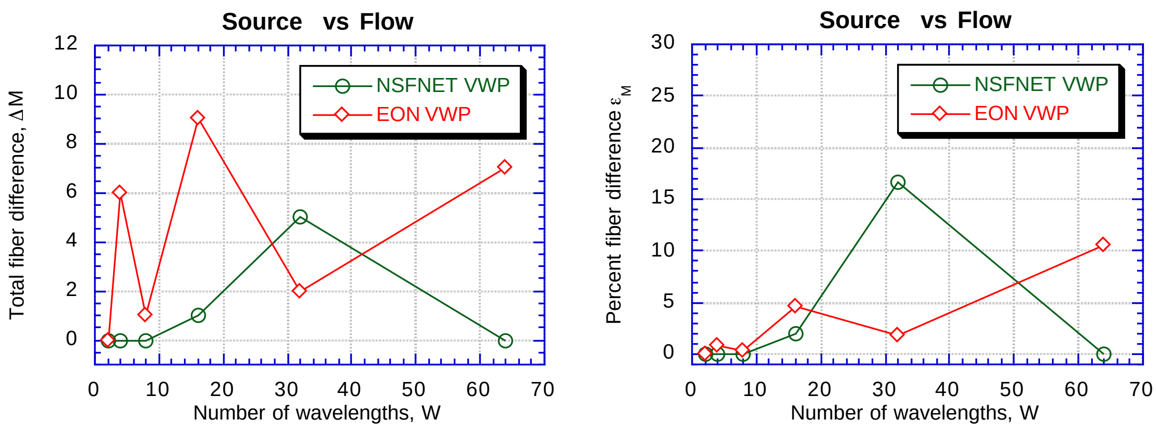
W	source form.	flow form	route form.
2	0,39 MB	1,3 MB	33 MB
4	O.O.M	O.O.M.	O.O.M.
8	5 MB	42 MB	432 MB
16	47 MB	O.O.M.	852 MB
32	180 MB	O.O.M.	750 MB

TABLE V
VWP EON OPTIMIZATION: COMPUTATIONAL TIME AND *gap* BETWEEN INTEGER SOLUTION FOUND AND LOWER BOUND RETURNED BY BRANCH AND BOUND ALGORITHM

EON	source form.		flow form.	
W	time	gap	time	gap
2	**1 m**	0%	**3.5 h**	0%
4	2.2 h	0.36 %	15.6 h	1.2 %
8	2.2 h	0.87 %	20.3 h	2.5 %
16	2.3 h	1.74 %	19h	6.2 %
32	1.7 h	9 %	46h	15 %

Table V shows resource occupation comparison between source and flow formulation in EON network (the route formulation is not feasible due to the huge number of variables); again computational times in bold are associated to runs succeeding in finding optimal values. We will show here the *gap* parameter to compare the quality of integer solution found: neither source formulation nor flow formulation succeeds in demonstrating the optimality of returned integer solution except for $W = 2$, but quality of source formulation solutions is evidently better than quality of flow solutions. So O.O.M event happens in all the optimizations, while for $W = 2$ the amount of occupied memory is about 10 MB in both cases.

NSFNET optimization in the WP scenario takes a very long time with the hardware we employed. In some cases it was too long to wait either for the optimal result or for an out-of-memory event. Thus in table VI we have reported the time necessary to fill up of the first 100 MB of memory. The case with $W \geq 16$ has proved to be too complex to be solved in a reasonable time and therefore it has been omitted (except for $W = 16$ in the source formulation). The speed of the branch-and-bound algorithm applied to the flow formulation decreases dramatically for high values of W. Although in the source formulation the speed does not decrease so much, the model simplification allows a significant computational time



(a) (b)

Fig. 5. Source-flow comparison on the final number of fibers in the VWP case, as absolute (a) and percent relative difference (b).

TABLE VI
WP NSFNET OPTIMIZATION: TIME REQUIRED TO FILL UP 100 MB OF MEMORY.

W	source form.	flow form.	route form.
2	**1.7 h**	1 h	**10.3 h**
4	8 h	60 h	9 h
8	57h	12d	39 h
16	6.2d	–	–

decrement compared to flow formulation. The route formulation, despite the great difference on the number of variables, has comparable results regarding computational performance; this is probably due to the structure of route variables that make simpler to set the wavelength continuity constraints.

Now we are going to compare the source and the flow formulations on the basis of the final value of the cost function. In all the cases in which, for both formulations, the branch-and-bound ends up before an out-of-memory event, the final values obtained are coincident, thus proving the equivalence of source and flow formulation. In all the other cases, the best integer solution returned by the SF is smaller than or equal to the FF best integer solution. Furthermore, the *gap* parameter associated integer solutions is always smaller for SF than for FF, showing that SF optimization runs are able to get closer to lower bound of the problem.

We focus our analysis on the performance comparison between flow and source formulation. The following parameters are introduced:

- $M_{source}(M_{flow})$: total fiber number returned by ILP based on the source (flow) formulation;
- ΔM: difference $M_{flow} - M_{source}$;
- ϵ_M: percent relative difference $100 \cdot (M_{flow} - M_{source})/M_{source}$;

We will show results concerning the two case-study networks.

The difference between the two formulations obtained in the VWP scenario are represented in Fig. 5a (absolute values) and in Fig. 5b (percent) as functions of the parameter W. The absolute difference is on average greater for the EON which has a larger number of nodes and links. Convergence between the two formulations occurs, for example, in the NSFNET

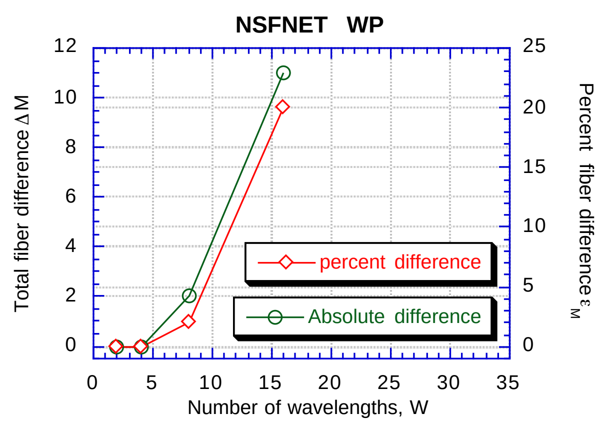


Fig. 6. Source-flow comparison on the final number of fibers in the WP case for NSFNET.

case for $W = 2$ and $W = 8$, in accordance with table IV[5]. It should be noted that sub-optimal solutions with the flow formulation can be up to 18% worse than the corresponding solutions produced by the source formulation.

In Fig. 6 ΔM and ϵ_M are displayed for NSFNET in the WP case. In this case the strong increase of variable and constraint number with W causes a relevant increase of the differences between the two formulations. It is worth noting that both SF or FF are unable to reach the end of optimization runs (except for the $W = 2$ case). We evaluate the quality of the integer solution not only comparing its numerical value (as shown in Fig. 6), but also comparing the *gap* parameter associated to the integer solution: for W=2,4,8,16 the value of the *gap* is equal to 0%, 1,9%, 5.5% and 12% for SF and 0,3%, 2,4%, 8%, and 30% for FF. In conclusion, integer solutions provided by SF outperform solutions provided by FF as far as the numerical values, computational times and proximity to the lower bound are concerned.

Up to this point of the paper we have described multifiber WDM network optimization having the total number of fibers

[5]in the case $W = 4$, SF and FF return the same integer solution, which is very likely to be the optimal, but the both of them fail in proving the optimality of integer solution found

as cost function. This interpretation of network cost is called *hop metric* and it models a situation in which all the fibers of the network have the same cost. However in real networks the cost of a link also depends on its geographical length, which for example determines the number of optical line amplifiers that must be installed. Measuring the cost of a fiber in this situation becomes much more complicated and the hop metric is not appropriate any more. Another simple alternative is the *length metric*, which assigns a cost to each fiber proportional to the geographical length of the link it belongs to. Although still not completely realistic (e.g. it does not take into account that the cost of the duct should be shared by all the fibers of a link), it could be useful in many situations (e.g. when the cost of optical line amplifiers is an important issue). Clearly, the hop metric can be regarded as a particular case of length metric in which all the links have unity length[6].

We have tested source formulation based on length metric on NSFNET and EON in the VWP case. In a fashion similar to hop metric, the following parameters have been defined

- $P_{source}(P_{flow})$: total fiber length returned by ILP based on the source (flow) formulation;
- ΔP: difference $P_{flow} - P_{source}$;
- ϵ_P: percent relative difference $100 \cdot (P_{flow} - P_{source})/P_{source}$;

Link lengths p_l were assigned for the two networks according to Refs. [21], [26].

Fig. 7 displays the percent relative difference between the total fiber lengths obtained applying the source and the flow formulation. The same conclusions drawn for the hop metric can be extended to these new optimization experiments. Source formulation performs better in all the cases in which an out-of-memory event occurs; otherwise, the results are coincident, but source formulation converges more rapidly (computational times are omitted for brevity). It is worth noting that the length metric results in more solutions found for the considered set of W with respect to hop metric case: this is because there are less tie-breaks in B&B algorithm than in the hop metric case, where the weight assigned to each link is the same. In confirmation of this observation, the values of the *gap* parameter for the length metric are always smaller than those reported for hop metric in Table V.

Finally we show a comparison between the ILP optimization carried out by source formulation and the optimization by the heuristic approach described in Ref. [28]. Let us consider NSFNET and the hop metric. In Fig. 8 ILP and heuristic final results are displayed in the VWP (Fig. 8a) and WP case (Fig. 8b).

The comparison shows that the results of the two techniques are quite close: the heuristic approach is able to provide good sub-optimal results, but only the exact approach allows to reach the absolute optimum (or to come closer to it when limitations on memory or computational time prevents branch-and-bound to converge). As far as the computational time is concerned, we have noticed that heuristic and source-formulation ILP behave similarly for VWP networks. In the WP case, however, heuristic methods are much faster than ILP, even when source formulation is adopted. It can be noticed in Fig. 8b that heuristic has been the only possible approach to obtain a result with $W = 32$, given the hardware limitations of our workstation. Concluding, SF has provided a useful benchmark to evaluate the performance of a heuristic strategy on NSFNET and EON, that can be considered two significant test-case networks.

[6]Neither hop nor length metric take the node cost into account. Node-cost optimization issues are not covered by this paper.

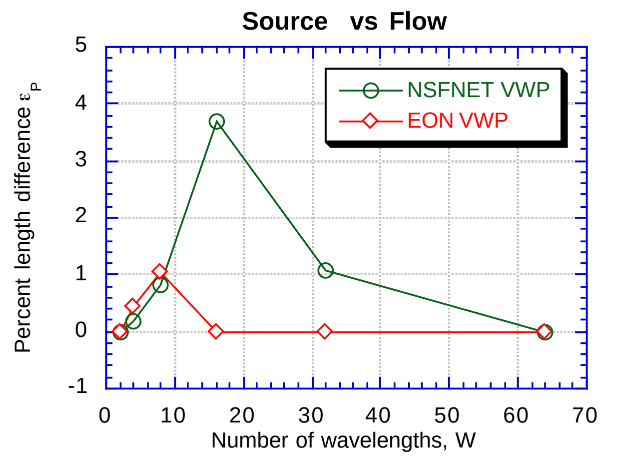


Fig. 7. Source-flow comparison on the final total fiber length in the VWP case, as percent relative difference.

TABLE VII

WHEEL NETWORKS OPTIMIZATION: RESULTS ON FIBER NUMBER

W	$\alpha = 0.29$	$\alpha = 0.43$	$\alpha = 0.57$	$\alpha = 0.71$	$\alpha = 1$
2	**64**	**44**	**40**	**38**	38
4	**32**	**24**	**23**	*23*	23
8	**16**	**15**	**15**	*15*	15
16	**14**	**10**	**10**	**10**	*10*
32	**8**	**8**	**8**	**8**	**8**

B. Wheel networks

In order to show the effectiveness of our source formulation, we have performed optimization experiments also on the set of 8-node "wheel networks" shown in Fig. 9. This network class defines topologies with increasing connectivity degree, starting from the ring network and ending with a full mesh network. This is obtained by increasing the number of edges with respect to initial ring topology, so that the connectivity index α (i.e. the ratio between the number of links in the considered network and the number of link in the full-mesh network case) assumes the values 0.29 (ring), 0.43, 0.57, 0.71, 1 (full-mesh). Again, we have assumed different values of W, that is $W = \{2, 4, 8, 16, 32\}$. This new class of network topologies will allow us to better appreciate the behavior of SF with respect to FF and RF, while varying one crucial network parameter, the connectivity index. We assume that offered traffic is uniform and equal to one connection request for each node couple.

The numerical results obtained by SF for these networks are summarized in Table VII. Let us now analyze in detail these results. SF, FF and RF all lead to the optimum solution for $\alpha = 0.29$, 0.43, 0.57 and all values of W, for α=0.71

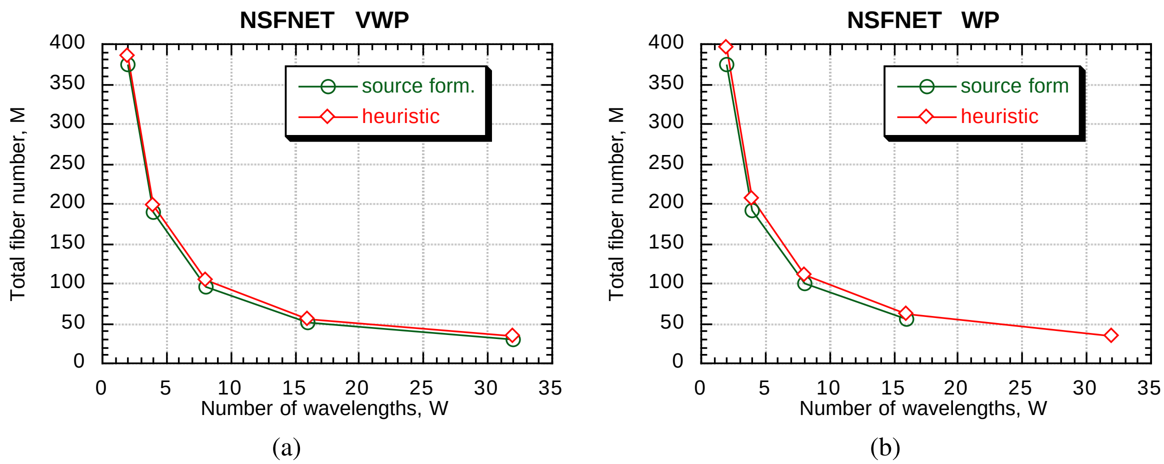


Fig. 8. NSFNET total fiber number optimized by ILP source formulation and by a deterministic heuristic, in the VWP (a) and WP (b) cases.

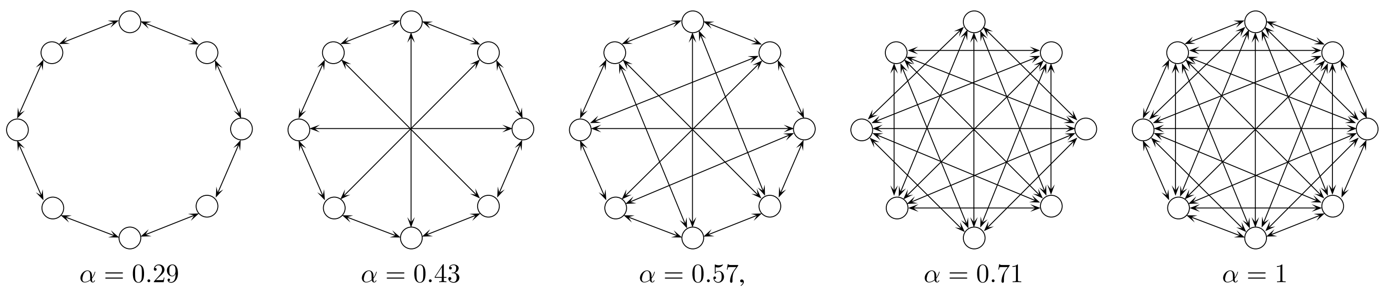


Fig. 9. Wheel networks with different connectivity degrees.

with $W = 2, 16, 32$ and for α=1 with $W = 16, 32$ (the corresponding numerical values are in bold in Table VII. When all the three formulations succeed in finding optimal values, SF takes sensibly lower computational times than the other two formulations (see the corresponding values in Fig. 10, where we have drawn the computational times for each value of connectivity index).

Moreover SF allows us to obtain optimal values also in some network cases in which FF and RF fail; in fact for α= 0.71 with $W = 4, 8$ and α=1 with $W = 16$ SF succeeds in finding optimal values that are not reached by FF and RF (these values are reported in italic style). For these three cases, Fig. 10 campares the computational times SF takes to reach the optimal value with the computational times required by FF and RF to reach their best integer values under O.O.M. condition.

Finally in the cases α=1 with $W = 2, 4, 8$ the three formulations fail in proving the optimality of the best integer solution returned, so the computational times refer all to O.O.M cases (time needed to occupy the whole memory). So, to better appreciate the performance of SF in full-mesh network, in Table VIII we have reported the best integer solutions reached by the three formulations before O.O.M event[7]. The results obtained by SF are strong candidates to be the effective optimal values, because they are equal to the optimal values for α=0.71. FF shows worse results than SF on fiber number, but in particular RF returns results far from SF, due to the exponential increase of admissible paths in the full-meshed case.

[7]Except for $W = 32$ for which all the three approaches find the optimum solution

TABLE VIII

"WHEEL" NETWORK WITH α=1: RESULTS ON FIBER NUMBER

W	SF	FF	RF
2	38	39	39
4	23	25	28
8	15	15	17
16	10	10	13

Concluding, Fig. 10 clearly shows the difference in computational time among the three approaches: SF outperforms FF and RF for all the values of W[8]. In particular, for increasing values of α, a larger number of variables and constraints results in larger computational times, but SF keeps returning the better results.

V. Conclusions

We have presented and discussed a novel formulation, called source formulation, to solve static-traffic WDM network optimization by ILP. This formulation has been defined for multifiber networks with or without wavelength conversion capability supporting unidirectional unprotected optical connections. Thanks to the source formulation, we are able to substantially prune the multiplicity of both variables and

[8]We have omitted to report the ring network case, which is rapidly solved by each of the three approaches. Anyway a deeper analysis of this simple case shows that B&B algorithm explores a lower number of nodes in the case of SF compared to FF and RF.

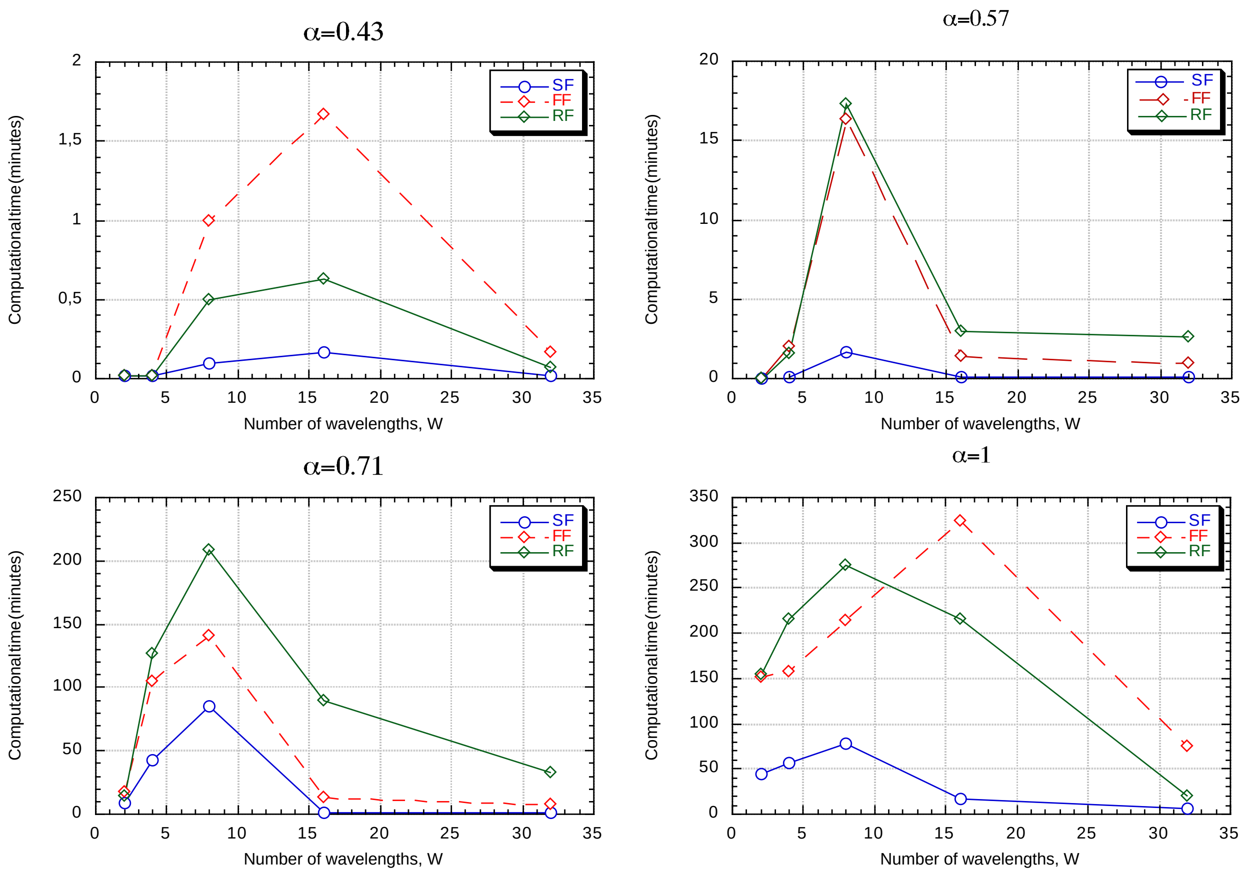


Fig. 10. Computational time fo SF, FF, RF for different α and W values

constraints compared to the well-known flow formulation. Exploiting source formulation we thus obtain a competitive optimization tool capable to solve ILP problems with relatively low computational time and memory occupation. The case-studies discussed in the paper prove the advantages of source formulation in several network-planning experiments.

Appendix

As we have seen in section III SF manages single sources commodities, while FF manages source-destination commodities. From the ILP solutions point of view (i.e. if we consider the integrality constraint on capacity and flow variables), the two formulation are equivalent. To prove this last statement, in the following we will show how to obtain a SF solution from a FF solution and vice versa. It is worth noting that the value of the objective function does not change passing from a model to the other. So, if we can always obtain a FF (SF) solution from a SF (FF) solution, then the solution that minimizes SF (FF) will minimize also a corresponding solution in FF and the optimal RFWA can be obtained equivalently by means of SF or FF.

Let us consider the VWP case. The variable aggregation that has been exploited to transform FF in SF formulation is:

$$\sum_{j} x_{l,k}^{i,j} = x_{l,k}^{i}, \tag{11}$$

The solenoidality and capacity constraint in flow formulation can be written as follows:

$$\sum_{(l,k)\in I_m^+} x_{l,k}^{i,j} - \sum_{(l,k)\in I_m^-} x_{l,k}^{i,j} = \begin{cases} v_{i,j} & \text{if m = } j \\ -v_{i,j} & \text{if m = } i \\ 0 & \text{otherwise} \end{cases}$$
$$\forall\ m, i, j (i \neq j); \tag{12}$$

$$\sum_{i,j\ \ i\neq j} x_{l,k}^{i,j} \leq W_l \cdot F_{l,k} \qquad \forall\ (l,k); \tag{13}$$

A. From FF to SF

Let us consider a generic admissible solution for FF $(\overline{F_{l,k}}, \overline{x_{l,k}^{i,j}})$. To obtain a SF solution, just sum the flows with fixed source on all the destination to obtain a SF routing. The capacity constraint 3 is automatically enforced combining 13 and 11.

B. From SF to FF

Let us consider a generic admissible solution for SF $(\overline{F_{l,k}}, \overline{x_{l,k}^{i}})$. Let us isolate a single source commodity with fixed source $\bar{i}$ [9]. Let us try to route the FF flows $x_{l,k}^{\bar{i},j}$ (having $\bar{i}$ as source node and any possible destination d) on the commodity identified by $x_{l,k}\bar{i}$, considering its value as a term

[9]Then the following FF on SF mapping can be extended to all the single source commodity returned by SF.

of capacity for each link of the graph. To route the traffic we can use the usual techniques of max-flow theory loading flows along paths with positive remaining capacity, until all FF commodities are dealt with. The capacity of the single source commodity will be enough to support all the traffic, because from 11:

$$\sum_{j} x_{l,k}^{\bar{i},j} \leq x_{l,k}^{\bar{i}} \tag{14}$$

Alternatively for each single source commodity from SF we could solve this simple ILP problem (as stated before, i is fixed):

$$\min \sum_{j} \sum_{l,k} x_{l,k}^{\bar{i},j} \tag{15}$$

$$\sum_{(l,k)\in I_m^+} x_{l,k}^{\bar{i},j} - \sum_{(l,k)\in I_m^-} x_{l,k}^{\bar{i},j} = \tag{16}$$

$$\begin{cases} C_{\bar{i},m} & \text{if m} = j \\ -C_{\bar{i},m} & \text{if m} = \bar{i} \\ 0 & otherwise \end{cases} \quad \forall\, j, m; \tag{17}$$

$$\tag{18}$$

and, as capacity constraint, we can use 14.

References

[1] D. Bienstock and O. Gonluk, “Computational experience with a difficult mixed-integer multi-commodity flow problem,” *Mathematical Programming*, vol. 68, no. 32, pp. 213–237, 1995.

[2] K.-I. Sato, *Advances in Transport Network Technologies - Photonic Networks, ATM, and SDH*, Artech House, Norwood, MA, first edition, 1996.

[3] I. Chamtlac, A. Ganz, and G. Karmi, “Lightpath communications: an approach to high-bandwidth optical WAN’s,” *IEEE/ACM Transactions on Networking*, vol. 40, no. 7, pp. 1172–1182, July 1992.

[4] B. Mukherjee, *Optical communication networks*, McGraw-Hill, 1997.

[5] R.Ramaswami and K.N.Sivarajan, *Optical Networks: A Pratical Perspective*, Morgan Kaufmann, 1998.

[6] R. Ramaswami and K. N. Sivarajan, “Design of logical topologies for wavelength-routed optical networks,” *IEEE Journal on Selected Areas in Communications*, vol. 14, pp. 840–851, June 1996.

[7] S. Baroni, P. Bayvel, R. J. Gibbens, and S. K. Korotky, “Analysis and design of resilient multifiber wavelength-routed optical transport networks,” *Journal of Lightwave Technology*, vol. 17, pp. 743–758, May 1999.

[8] S. Even, A. Itai, and A. Shamir, “On the complexity of timetable and multicommodity flows problems,” *SIAM Journal of computing*, vol. 5, pp. 691–703, 1976.

[9] N. Wauters and P. M. Deemester, “Design of the optical path layer in multiwavelength cross-connected networks,” *IEEE Journal on Selected Areas in Communications*, vol. 14, pp. 881–891, june 1996.

[10] R. Ramaswami and K. N. Sivarajan, “Routing and wavelength assignment in all- optical networks,” *IEEE/ACM Transactions on Networking*, vol. 3, pp. 489–500, Oct 1995.

[11] S. Baroni and P. Bayvel, “Wavelength requirements in arbitrarily connected wavelength-routed optical networks,” *Journal of Lightwave Technology*, vol. 15, no. 2, pp. 242–251, Feb 1997.

[12] D. Banerjee and B. Mukherjee, “A practical approach for routing and wavelength assignment in large wavelength-routed optical networks,” *IEEE Journal on Selected Areas in Communications*, pp. 903–908, June 1996.

[13] M. Alanialy and E. Ayanoglu, “Provisioning algorithms for WDM optical networks,” *IEEE/ACM Transactions on Networking*, vol. 3, pp. 767–778, Oct 1999.

[14] Asuman E. Ozdaglar and Dimitri P. Bertsekas, “Routing and wavelength assignment in optical networks,” *IEEE/ACM Transactions on Networking*, vol. 11, no. 2, pp. 259–272, Apr 2003.

[15] Rajesh M. Krishnaswamy and Kumar N. Sivarajan, “Design of logical topologies: A linear formulation for wavelength-routed optical networks with no wavelength changers,” *IEEE/ACM Transactions on Networking*, vol. 9, no. 2, pp. 186–198, Apr 2001.

[16] M. Ali, B. Ramamurthy, and Deogun, “Routing and wavelength assignment with power considerations in optical networks,” *Computer Communications*, , no. 32, pp. 539–555, 2000.

[17] B. Van Caenegem, W. Van Parys, F. De Turck, and P. M. Deemester, “Dimensioning of survivable WDM networks,” *IEEE Journal on Selected Areas in Communications*, pp. 1146–1157, Sept 1998.

[18] C. M. Lee, C. C. Hui, F. Tong, and P. Yum, “Network dimensioning in WDM based all optical networks,” *Proceedings,IEEE GLOBECOM ’98*, vol. 1, pp. 328–333, 1998.

[19] D. Banerjee and B. Mukherjee, “Wavelength-routed optical networks: linear formulation, resource budgeting tradeoffs and a reconfiguration study,” *IEEE/ACM Transactions on Networking*, pp. 598–607, Oct 2000.

[20] F. Glenstrup, “Full design of robust optical networks,” *15th Nordic Teletraffic Seminar*, 2000.

[21] Y. Miyao and H. Saito, “Optimal design and evaluation of survivable WDM transport networks,” *IEEE Journal on Selected Areas in Communications*, vol. 16, pp. 1190–1198, Sept 1999.

[22] M. Shiva Kumar and P. Sreenivasa Kumar, “Static lightpath establishment in WDM networks- new ilp formulations and heuristic algorithms,” *Computer Communications*, , no. 25, pp. 109–114, 2002.

[23] M. Saad and Z.Luo, “A Lagrangean Decomposition Approach for the Routing and Wavelength Assignment in Multifiber WDM Networks,” in *Proceedings, GLOBECOMM 2002*, Nov 2002.

[24] F. Poppe and P. Demeester, “Wavelength requirement of mesh-restorable multi-wavelength optical networks,” *IEEE/ACM Transactions on Networking*, vol. 3, pp. 767–778, Oct 1999.

[25] H. Yen and F. Y. Lin, “Near optimal design of lightpath routing and wavelength assignment in purely optical WDM networks,” *Proceedings, ONDM ’01*, 2001.

[26] A. Fumagalli, I. Cerutti, M. Tacca, F. Masetti, R. Jagannathan, and S. Alagar, “Survivable networks based on optimal routing and WDM self-heling rings,” *Proceedings, IEEE INFOCOM ’99*, vol. 2, pp. 726–733, 1999.

[27] ILOG, “Ilog cplex 6.5, user’s manual,” March 1999.

[28] G. Maier, A. Pattavina, L. Roberti, and T. Chich, “Static-lightpath design by heuristic methods in multifiber WDM networks,” *SPIE Optical Network Magazine*, vol. 3, no. 5, pp. 52–66, Sept.-Oct. 2002.

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