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Topological Design and Lightpath Routing in WDM Mesh Networks: A Combined Approach*

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Abstract. Multicommodity flow models are commonly used to formulate the logical topology design (LTD) problem and the lightpath routing (LR) problem as mixed integer linear programming (MILP) problems. In general, MILP formulations are intractable even for relatively small networks due to the combinatorial complexity of the problem. In this paper we propose improvements to these models and a method to solve the LTD and the LR problems in a combined manner. The interest is two fold: firstly, by tackling the two problems with separate models, problem instances of realistic size (up to 14 nodes in this paper) can be dealt with. Furthermore, different combinations of optimization models and objective functions can be investigated in a modular manner. Secondly, the mechanisms proposed to combine the problems allow to keep track of the global design problem when solving each individual step.

Keywords: lightpath routing, logical topology design, mixed integer linear programming (MILP), network planning, WDM

1 Introduction

The feasibility of optical add-drop multiplexers (OADM), optical wavelength converters (OWC), optical amplifiers and optical cross-connects (OXC) has been recently demonstrated. The introduction of this technology enables the realization of WDM networks of huge capacity, which find their application in core networks where large volumes of traffic are transported.

The design of networks incorporating this type of equipments requires the development of models that take into account the limitations imposed by these technologies and the particular nature of the transported traffic. Indeed, network planning models developed up today are mainly designed for POTS networks. These models are quite realistic, in part because voice traffic profile is well understood. With the emergence of data applications, design and dimensioning of networks becomes more difficult, in part, because of the unpredictability of data traffic. It is no longer possible to design data networks for specific performance objectives using exactly the same methods. However, some basic principles of design and dimensioning can be extended and adapted to data networks. For example, multicommodity flow models used for topological design and traffic routing in voice networks can be adapted to formulate the problem of topological design and routing of traffic in packet-switched networks. In Ammar [1] a model to design an ATM virtual path network is described. In this paper we adopt a similar approach to address the problem of designing a logical topology made of lightpaths and the routing of packet-switched traffic over this topology. We assume that the traffic demands are known in advance. They may correspond to a long-term traffic forecast.

Clear optical channels, or lightpaths, that do not undergo opto-electronic conversion at intermediate

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nodes can be set between no physically adjacent nodes in the network by assigning a wavelength to the lightpath and switching the wavelength optically. The information traveling on a lightpath is carried optically from end to end.

Because of equipment costs, fiber availability and switching capabilities at the network nodes, it is not possible to set up a full mesh of $N(N-1)$lightpaths between the N nodes of a network. Thus, a particular subset of lightpaths, out of the set of all possible lightpaths, must be selected. This subset is called the logical topology, or virtual topology, seen by the electronic switches at the network nodes. The traffic demands are realized on top of the logical topology by routing them through direct lightpaths, when they exist between the source and destination of the traffic, or through a concatenation of lightpaths otherwise. The logical topology is realized by routing the subset of lightpaths over the physical topology and assigning wavelengths to these lightpaths.

Network design problems are in general very complex because the resulting optimization problems are numerically intractable even for networks with a small number of nodes. This inherent complexity frequently leads to solution approaches based on the decomposition of global design problems into subproblems of tractable complexity; each identified subproblem can be solved by a separate algorithm. Thus, a solution to the global problem is found by sequentially solving the subproblems, i.e., the solution to a first subproblem(output) is the input to the next one. This approach has been adopted, for example, in the design of SONET/SDH networks [5] and has also been used in the framework of WDM network planning [6].

Following a decomposition approach, three subproblems of the network design problem are usually considered in the literature:

- LTD (logical topology design): the definition of the lightpaths to be set as the virtual topology and the routing of traffic demands over the virtual topology.
- LR (lightpath routing): the routing of lightpaths of the virtual topology over the physical topology.
- WA (wavelength assignment): the assignment of wavelengths to lightpaths.

In this paper we only consider the LTD and LR problems under static traffic demands. Fig. 1 illustrates schematically the decomposition of the

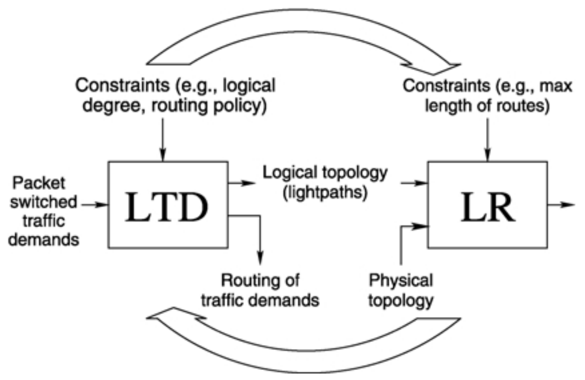


Fig. 1. Decomposition of the optical network design problem into two separate subproblems.

problem: solution methods for each subproblem are considered separately. The solution of the LTD subproblem is the input to the LR subproblem.

A disadvantage of the decomposition approach is the loss of perception of the global problem when handling a particular subproblem. This is because design choices leading to the optimality of one of the subproblems can be detrimental to the subsequent subproblems. It is possible to circumvent this drawback by first solving the subproblems sequentially and then executing a second resolution; this second time, design information of the first phase can be exchanged as feedback between the algorithms that solve the subproblems so that ''bad'' design choices are known *a priori* and, consequently, are avoided. We adopted this approach in the resolution of the problems addressed in this paper. The mentioned subproblems can be formulated as optimization subproblems and solved by means of either exact methods based on MILP models or approximated methods, such as metaheuristics.

In Section 2 we present a multicommodity flow model widely adopted in the literature to solve the LTD problem. The model aims at defining a virtual topology and a routing of traffic demands that minimize the network congestion. Then we propose to add a second optimization step to this model in order to additionally minimize the average hop count. We also propose an evolved version of this model that enforces the atomic routing of the traffic demands, i.e., the whole traffic demand must follow the same route from source to destination. In Section 3 we address the LR problem, for which we present a multicommodity flow problem. In Section 4 we introduce a pruning method that allows to reduce the combinatorial complexity of the LR problem by eliminating from the MILP formulation useless variables. In Section 5 we propose a method to combine the resolution of both the LTD and the LR subproblems. In Section 6 we experimentally evaluate the models presented in the previous sections. Finally, we give our conclusions and perspectives about our ongoing work in Section 7.

2 Logical Topology Design

The design of a logical topology and the routing traffic demands over the topology can be formulated as an optimization problem. Optimality criteria are typically defined in terms of either network implementation cost or performance metrics [1]. Moreover, the constraints of the problem are usually dictated by hardware limitations imposed by transmission and switching equipments. The problem has been extensively investigated in recent years [2–4,6–8]. Most of the proposed models aim at optimizing network performance metrics such as network congestion, throughput and average transit delay, rather than cost metrics. Focus on network performance stems from the fact that the logical topology is the network layer directly seen by the electronic switches, which are typically expensive and, most importantly, offer limited capacity when compared to optical switches.

In this section we present multicommodity flow models of the LTD problem. The models aim at defining the set of lightpaths that constitute the logical topology and the routing of traffic demands over this topology that minimize the network congestion, that is, the amount of traffic passing through the most congested lightpath. The number of lightpaths that can depart from or arrive to any node in the network is constrained to be less or equal than a given parameter δ called the network degree. We begin by introducing the models notations:

Indices:

- i, j — used as subscripts, denote the source and destination nodes of a lightpath.
- s, d — used as superscripts, denote the source and destination nodes of a traffic demand.

Problem parameters:

- N — the number of nodes in the network (the nodes are numbered $1, 2, \ldots, N$).
- (λ^{sd}) — an $(\mathbb{R}_+)^{N \times N}$ matrix describing the amount of traffic flowing from each source node s to each destination node d, expressed in some convenient units (e.g., Mbit/s). The matrix is not necessarily symmetrical.
- δ_{out} — the logical out-degree of the nodes in the network, i.e., the maximum number of lightpaths that can initiate at the electronic switch of the network nodes.
- δ_{in} — the logical in-degree of the nodes in the network, i.e., the maximum number of lightpaths that can be terminated at the electronic switch of the network nodes. It is often assumed that $\delta_{\text{out}} = \delta_{\text{in}} = \delta$, although this is not a strict requirement.

Variables:

- b_{ij} — a binary variable that indicates whether or not there is a lightpath from source node i to destination node j (1 and 0 respectively). Note that lightpaths are directed, therefore, $b_{ij} = 1$ does not imply $b_{ji} = 1$.
- λ_{ij}^{sd} — a real variable indicating the amount of traffic from source s to destination d passing through the lightpath going from i to j.

Computed values:

- λ_{ij} — the total amount of traffic passing through the lightpath going from node i to node j, i.e., $\lambda_{ij} = \sum_s \sum_d \lambda_{ij}^{sd} \, \forall \, i, j$.
- $\lambda_{\max}$ — the amount of traffic passing through the most congested lightpath, i.e., $\lambda_{\max} = \max_{1 \leq i,j \leq N} \lambda_{ij}$. This term is referred to as the maximum congestion level in the network or simply, the congestion.

2.1 Model LTD1

This multicommodity flow model is partially based on the formulation presented in Ramaswami and Sivarajan [2]. The model is defined in terms of b_{ij} (binary) and λ_{ij}^{sd} (real) variables, hence it is a MILP (mixed integer linear program). Once the problem is solved and the values of the variables are known, the variables b_{ij} set to 1 indicate the lightpaths to be established and, for each traffic demand λ^{sd}, the values of the variables λ_{ij}^{sd} indicate the amount of sd traffic flowing on lightpath ij. The model is:

$$\text{Minimize:} \quad \lambda_{\max}. \tag{1}$$

Subject to:

Flow conservation at each node:

$$\sum_i \lambda_{ij}^{sd} - \sum_k \lambda_{jk}^{sd} = \begin{cases} -\lambda^{sd}, & \text{if } j = s \\ \lambda^{sd}, & \text{if } j = d \quad \forall j, s, d. \\ 0, & \text{otherwise,} \end{cases} \tag{2}$$

Total flow on a lightpath:

$$\lambda_{ij} = \sum_s \sum_d \lambda_{ij}^{sd} \le \lambda_{\max}, \quad \forall\, i, j \tag{3}$$

$$\lambda_{ij}^{sd} \le b_{ij} \lambda^{sd}, \quad \forall\, i, j, s, d. \tag{4}$$

Degree constraints:

$$\sum_j b_{ij} \le \delta_{\text{out}}, \quad \forall\, i \tag{5}$$

$$\sum_i b_{ij} \le \delta_{\text{in}}, \quad \forall\, j. \tag{6}$$

Value range constraints:

$$\lambda_{ij}^{sd} \ge 0, \quad \forall\, i, j, s, d \tag{7}$$

$$b_{ij} \in \{0, 1\}, \quad \forall\, i, j. \tag{8}$$

The objective function (1) states that the model aims at minimizing $\lambda_{\max}$, the maximum congestion level in the network. Equation (2) is the flow conservation constraint which states that, for a traffic demand λ^{sd}, at every intermediate node $j (j \neq s, j \neq d)$, the amount of flow entering the node must be equal to the amount leaving it. At the source node $(j = s)$, the traffic only leaves the node, whereas at the destination node $(j = d)$, the traffic only enters the node. Equation (3) ensures that the total amount of traffic passing through any lightpath is at most equal to $\lambda_{\max}$. Equation (4) states that traffic can flow on a lightpath only if the lightpath exists. Moreover, for a traffic demand from s to d, the amount of traffic flowing on the lightpath from i to j cannot be more than the total amount of the traffic demand. Equations (5) and (6) enforce the maximum logical out-degree and in-degree of the nodes, respectively. Note that some nodes with degree smaller that δ can be accepted in the solution. As we shall see later, this allows to find virtual topologies that minimize the congestion while using as less lightpaths as possible. Finally, Equations (7) and (8) constrain the flow indicator variables to be non-negative reals and the lightpath indicator variables b_{ij} to be binary.

2.2 Model LTD1B

It may happen that multiple solutions minimize the congestion for a same problem instance. Thus, once the minimum possible congestion value $\lambda_{\max}^*$ has been found, one can look within the set of solutions for a solution that optimizes a different criterion. For example, a solution that minimizes the average hop distance (the average number of lightpaths that the traffic demands traverse to go from source to destination) for a given $\lambda_{\max}^*$ can be found by modifying the preceding formulation in the following manner:

- first, replace the objective function (1) by

$$\text{Minimize:} \quad \sum_i \sum_j \sum_s \sum_d \lambda_{ij}^{sd}, \tag{9}$$

- then, replace the variable $\lambda_{\max}$ in Equation (3) by the value $\lambda_{\max}^*$.

In order to better understand how this objective function works, we propose the following example. Given a network with $N = 5$ nodes, the traffic matrix T_{ex} shown in Fig. 2, and a logical degree $\delta = 2$, the three solutions shown in Figs 3, 4 and 5, referred to as Solutions 1, 2 and 3 respectively, may be obtained (among others) with model LTD1. In the figures, the arrows represent the computed lightpaths. The three solutions are similar, except for the lightpaths connecting nodes 1, 2 and 5, whose loads are mentioned in the figure, and which deal with the traffic from node 5 to node 2. All the other lightpaths bear a load equal to 10 (their load is not mentioned). The congestion is $\lambda_{\max}^* = 10$ for three solutions, but only Solution 1 minimizes the cost function of model

$$\begin{pmatrix} 0 & 0 & 10 & 0 & 0 \\ 0 & 0 & 10 & 10 & 0 \\ 0 & 0 & 0 & 10 & 10 \\ 10 & 0 & 0 & 0 & 10 \\ 0 & 10 & 0 & 0 & 0 \end{pmatrix}$$

Fig. 2. Matrix T_{ex}.

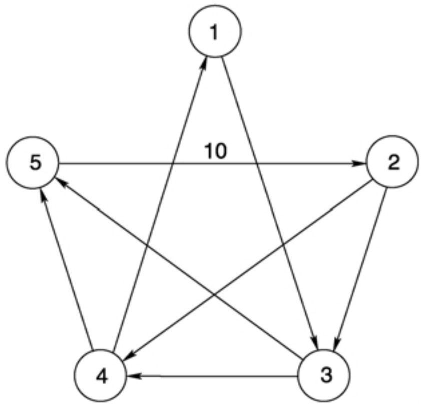


Fig. 3. Solution 1 for the example LTD problem.

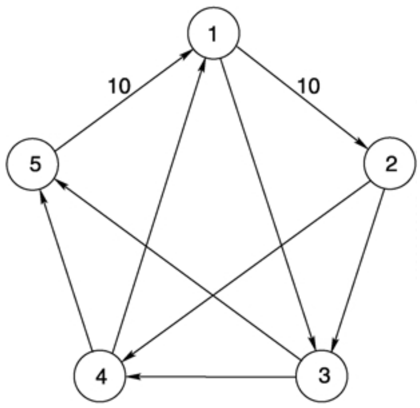


Fig. 4. Solution 2 for the example LTD problem.

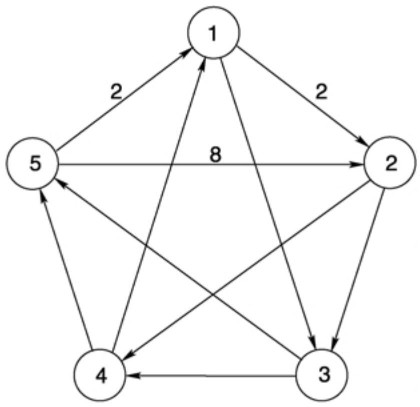


Fig. 5. Solution 3 for the example LTD problem.

LTD1b. This solution reaches the optimal congestion with one lightpath less than the two other solutions, and as such, is better. Clearly, this comes from the fact that, all other lightpaths being the same, the contribution of the considered lightpaths to the cost function is $\lambda_{52}^{52} = 10$ for Solution 1, $\lambda_{51}^{52} + \lambda_{12}^{52} = 10 + 10 = 20$ for Solution 2 and $\lambda_{51}^{52} + \lambda_{12}^{52} + \lambda_{52}^{52} = 2 + 2 + 8 = 12$ for Solution 3.

This leads to a two-steps optimization process. The improvement of this model with respect to the basic one is assessed in Section 6, where a comparison between the results obtained with LTD1 and the ones computed by this model is provided.

2.3 Model LTD2

The two previous models allow a traffic demand λ^{sd} to be arbitrarily split across multiple possible routes. While this property is useful to determine the lower bound of the congestion in the network, the routing of traffic demands obtained in this manner may have little practical relevance. We propose a model that allow the traffic demands to be atomically routed, i.e., the whole traffic from source *s* to destination *d* follows the same route through the network. This idea has been suggested in Banerjee and Mukherjee [4] and called non-bifurcated routing, but the authors mention that they did not experience their model since it led to a significant increase in running time. In our work, since we separately deal with the LTD and LR steps, we have simpler models. Moreover, the formulation in models LTD1 and LTD2 are the same but for the integer constraints. As a result we were able to run them both and to compare their solutions (Section 6).

The model LTD2 is directly inferred from model LTD1 with the introduction of new variables $\varphi_{ij}^{sd} = \lambda_{ij}^{sd}/\lambda^{sd}$ to describe the flow and an additional constraint to avoid traffic splitting: $\varphi_{ij}^{sd} \in \{0, 1\}$, $\forall\, i, j, s, d$. Thus, the model becomes:

$$\text{Minimize:} \quad \lambda_{\max}. \tag{10}$$

Subject to:

No-splitting constraint:

$$\varphi_{ij}^{sd} \in \{0, 1\}, \quad \forall\, i, j, s, d. \tag{11}$$

Flow conservation at each node:

$$\sum_i \varphi_{ij}^{sd} - \sum_k \varphi_{jk}^{sd} = \begin{cases} -1, & \text{if } j = s \\ 1, & \text{if } j = d \\ 0, & \text{otherwise.} \end{cases} \quad \forall j, s, d \tag{12}$$

Total flow on a lightpath:

$$\sum_s \sum_d \varphi_{ij}^{sd} \lambda^{sd} \le \lambda_{\max}, \quad \forall\, i,j \tag{13}$$

$$\varphi_{ij}^{sd} \le b_{ij}, \quad \forall\, i,j,s,d. \tag{14}$$

Degree constraints:

$$\sum_j b_{ij} \le \delta_{\text{out}}, \quad \forall\, i \tag{15}$$

$$\sum_i b_{ij} \le \delta_{\text{in}}, \quad \forall\, j. \tag{16}$$

Value range constraints:

$$b_{ij} \in \{0,1\}, \quad \forall\, i,j. \tag{17}$$

2.4 Model LTD2B

As in model MILP1, once the congestion value $\lambda_{\max}^*$ has been found, the objective function (10) can be replaced by

$$\text{Minimize:} \quad \sum_i \sum_j \sum_s \sum_d \varphi_{ij}^{sd}. \tag{18}$$

This objective function leads to a routing of demands that additionally minimizes the average hop count.

3. Lightpath Routing

The problem of physically routing and allocating wavelengths to a set of lightpaths has been widely studied in recent years [4,7,9–13]. The problem is referred to as routing and wavelength assignment (RWA). In this paper we consider the LR problem alone and not the WA problem.

3.1 Model LR

This section describes a multicommodity flow model introduced in Banerjee and Mukherjee [10] to solve the LR problem. The model aims at determining the physical routes of a set of lightpaths that minimize the lightpath congestion $\phi_{\max}$ (defined below). In the model, a lightpath corresponds to a commodity. An optical switch architecture is assumed wherein a lightpath can be switched from any incoming port to any outcoming port. The notations of the model are:

Indices:

- i,j — used as subscripts, denote the source and destination nodes of a physical link.
- s,d — used as superscripts, denote the source and destination nodes of a lightpath.

Problem parameters:

- N — The number of nodes in the network (the nodes are numbered $1, 2, \ldots, N$).
- (p_{ij}) — A $\{0,1\}^{N\times N}$ matrix that describes the physical topology. Each element p_{ij} indicates whether or not a physical link exists between nodes i and j (1 and 0, respectively). Note that if the links are assumed to be bidirectional, the matrix is symmetrical.
- (λ^{sd}) — A $\{0,1\}^{N\times N}$ matrix that describes the lightpath demands. Each element λ^{sd} indicates whether or not a lightpath must be established from source node s to destination node d (1 and 0, respectively). The matrix is not necessarily symmetrical. Moreover, no more than one lightpath is allowed between any source destination pair of nodes.

Variables:

- ϕ_{ij}^{sd} — A binary variable indicating whether or not the lightpath from node s to node d passes through the physical link going from i to j (1 and 0, respectively).

Calculated values:

- ϕ_{ij} — The total number of lightpaths passing through the link going from i to j, i.e., $\phi_{ij} = \sum_s \sum_d \phi_{ij}^{sd}\ \forall\, i,j.$
- $\phi_{\max}$ — The number of lightpaths passing through the most congested link, i.e., $\phi_{\max} = \max_{1\le i,j\le N} \phi_{ij}$. This term is referred to as the *maximum lightpath congestion* in the network or simply, the *lightpath congestion*.

The LR multicommodity flow models is:

$$\text{Minimize:} \quad \phi_{\max}. \tag{19}$$

Subject to:

Flow conservation at each node:

$$\sum_i \phi_{ij}^{sd} - \sum_k \phi_{jk}^{sd} = \begin{cases} -\lambda^{sd}, & \text{if} \quad j = s \\ \lambda^{sd}, & \text{if} \quad j = d \\ 0, & \text{otherwise.} \end{cases} \quad \forall\, j,s,d. \tag{20}$$

Lightpaths on a physical link:

$$\phi_{ij}^{sd} \leq p_{ij}, \quad \forall\, i,j,s,d \tag{21}$$

$$\phi_{ij} \leq \phi_{\max}, \quad \forall\, i,j. \tag{22}$$

Value range constraint:

$$\phi_{ij}^{sd} \in \{0,1\}, \quad \forall\, i,j,s,d. \tag{23}$$

The objective function and constraints are quite similar to those of the models presented in Section 2 for the LTD problem. The objective function (19) aims at minimizing the maximum lightpath congestion level in the network. Equation (20) enforces flow conservation, whereby a lightpath cannot be terminated at any intermediate node between its source and destination. Equation (21) ensures that lightpaths are routed only on existing physical links and Equation (22) limits the number of lightpaths passing through any physical link to be no greater than $\phi_{\max}$. Finally, Equation (23) ensures that ϕ_{ij}^{sd} variables are binary.

Once the optimal solution to the problem is found, for each lightpath demand λ^{sd} the variables ϕ_{ij}^{sd} indicate the route of the lightpath in the physical topology. The routes are determined so that the lightpath congestion is minimized.

3.2 Model LRb

Similar to the models of Section 2, it may happen that multiple solutions minimize the lightpath congestion for a same problem instance. Thus, once the minimum possible congestion value $\phi_{\max}^{*}$ has been found, one can look within the set of solutions for a solution that optimizes a different criterion. For example, the solution that minimizes the average hop count (the average number of physical links that the lightpaths traverse to go from source to destination) for a given $\phi_{\max}^{*}$ (among others, this prevents lightpaths with loops) can be found by modifying the formulation in the following manner:

- first, replace the objective function (19) by

$$\text{Minimize:} \quad \sum_{i}\sum_{j}\sum_{s}\sum_{d} \phi_{ij}^{sd}. \tag{24}$$

- then, replace the variable $\phi_{\max}$ in Equation (22) by the value $\phi_{\max}^{*}$.

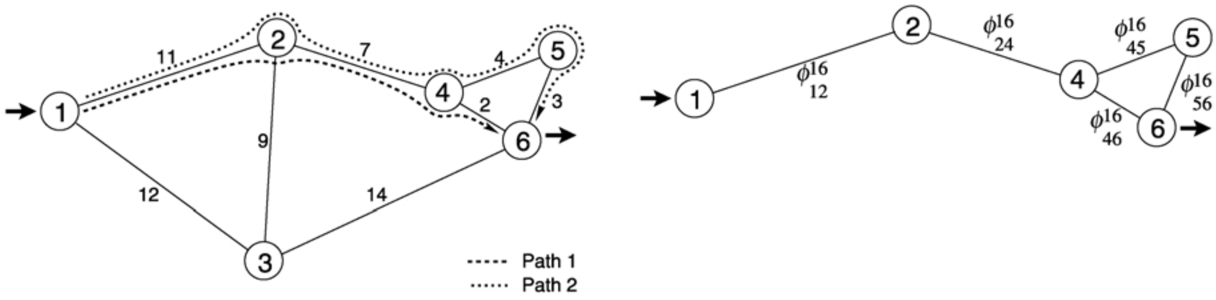
4 Pruning of MILP Formulations

The intractability of multicommodity flow models stems from the fact that the underlying ILP (or MILP) problems are NP-hard [14]. This aspect entails careful formulations and problem size reduction methods in order to enhance the solvability of such models with current ILP solvers. In this section we propose a pruning method aimed at reducing the number of variables used to describe the multicommodity flow model associated to the LR problem. The idea has been suggested by some authors (e.g., Banerjee and Mukherjee [4,10]). Reducing the size of the variable space allows for problem instances of medium size to be solved with current ILP solvers.

The number of variables in the formulation of the LR problem can be reduced by exploiting properties of the problem. It must be noted that the formulation allows a lightpath to be eventually routed on *any* physical link on the network. This is because for each lightpath demand $\lambda^{sd} = 1$, there areas many ϕ_{ij}^{sd} variables as links in the network. Moreover, the routed lightpaths must have certain characteristics imposed, for example, by technological limitations on transmission or switching equipments. These characteristics can be introduced in the model in the form of additional constraints, variables and/or terms on the objective function. However, this solution can rapidly increase the size of the problem. Furthermore, it is possible that the additional constraints cannot be formulated as linear equations.

Alternatively, for each lightpath demand $\lambda^{sd} = 1$ we can prune from the formulation the variables ϕ_{ij}^{sd} corresponding to the physical routes that do not satisfy the characteristics of the lightpath. We refer to these routes as not-valid physical routes. Pruning variables for all the lightpaths in the network reduce the size of the problem not only because of the pruned variables, but because possibly there are equations including exclusively pruned variables. These equations disappear from the formulation.

The selection of variables to be pruned rise two potential problems: first, it may happen that for a lightpath demand $\lambda^{sd} = 1$, a ϕ_{ij}^{sd} variable simultaneously belongs to a *valid* physical route and to a not-valid route. Second, the combination of variables ϕ_{ij}^{sd} that remain in the formulation (i.e., those that are not pruned) may generate not-valid routes. A possible solution to these problems consists in including constraints that explicitly preclude certain routes



(a) Weighted graph $G(V, E, w)$ and $k = 2$ shortest paths for λ^{16}.

(b) The graph induced by the edges belonging to the paths.

Fig. 6. Pruning of variables for a lightpath demand.

from the remaining set of variables. Of course, the number of such constraints must be small enough to avoid increasing significatively the size of the problem.

In what follows, we describe a method to prune the variables' space. We represent the physical topology with an edge-weighted undirected graph $G = (V, E, w)$, where $V = \{v_1, v_2, \ldots, v_N\}$ is the set of vertices that represent the N network nodes, $E = \{e_1, e_2, \ldots, e_M\}$ the set of edges representing the M physical links and $w : E \to \mathbb{R}^+$ is a function that determines the weight of the edges in E. These weights may correspond to the length of the physical links. For each lightpath demand $\lambda^{sd} = 1$, we consider in the formulation the ϕ^{sd}_{ij} variables associated with the k shortest paths in G from vertex s to vertex d.The remaining ϕ^{sd}_{ij} variables are pruned. We use the algorithm presented in Eppstein [15] to calculate the k shortest paths. Alternatively, the same algorithm can be used to calculate the set of paths whose length is below a given threshold.

The pruning method is illustrated in Fig. 6. The subfigure on the left shows the weighted graph $G(V, E, w)$ representing the physical topology and the $k = 2$ shortest paths for the lightpath demand $\lambda^{16} = 1$. The subfigure on the right shows the subgraph induced by the edges belonging to the calculated shortest paths and the associated ϕ^{sd}_{ij} variables. Note that for the lightpath demand $\lambda^{16} = 1$, only five ϕ^{sd}_{ij} variables are considered, instead of 16 (two variables per link, since there is a variable for each direction of the link). The variables $\phi^{16}_{12}, \phi^{16}_{24}, \phi^{16}_{45}, \phi^{16}_{46}, \phi^{16}_{56}$ are used in the ILP formulation of the LR problem.

It must be noted that pruning variables can preclude in some cases to find the minimal lightpath congestion as defined by the LR model. However, this is a valid simplification that helps the solver to find a solution to the ILP problem.

5 Combined Resolution of the LTD and LR Subproblems

As mentioned in Section 1, a disadvantage of the decomposition approach is the loss of perception of the global problem when solving a particular subproblem. We propose a method that combines the resolution of the LTD and LR subproblems. The method aims at circumventing the mentioned drawback by providing a feedback exchange mechanism between the algorithms that solve the two subproblems. In particular, we want to dispose of information about the lightpaths' characteristics (e.g., their length, which is determined by their physical route) when solving the LTD subproblem, so that very long lightpaths are avoided in the final solution.

We use the models presented in Section 2 to solve the LTD subproblem and the models of Section 3 to solve the LR subproblem. The feedback passed from the LR to the LTD solution algorithms allows to use the pruning method (see previous section) in the LTD models.

The idea of exchanging information between the algorithms that solve the subproblems is inspired in the recommendations for optical network planning found in Eurescom [5].

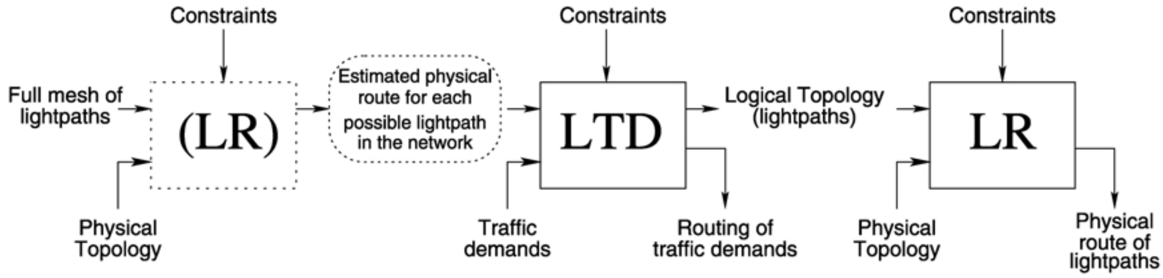


Fig. 7. Combined resolution of the LTD and LR subproblems.

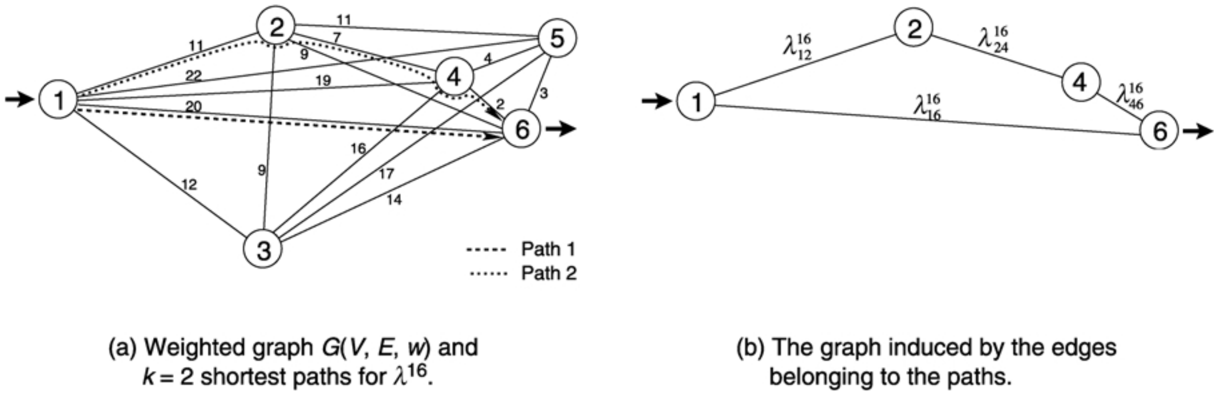


Fig. 8. Pruning of variables for a traffic demand in the LTD multicommodity flow models.

Fig. 7 illustrates the combined resolution method. In a first step, the LR subproblem is solved for a virtual topology consisting of lightpaths between all source–destination pairs in the network (i.e., full mesh). The result of this first step is an estimated physical route for each of the lightpaths that could exist in the network. We stress the term estimated because the definitive route for each lightpath of the final virtual topology may be different. The estimated route of a lightpath however, provides an indication on the lightpath's characteristics, in particular, its estimated length. At this point, a weighted graph $G(V,E,w)$ is used to represent the full mesh of lightpaths with their respective estimated length. The graph $G(V,E,w)$ is then used as a basis to apply the pruning method of Section 4 to the LTD multi-commodity flow models of Section 2. By using the pruning method on this model, we can discard *a priori* certain inappropriate routes for traffic demands.

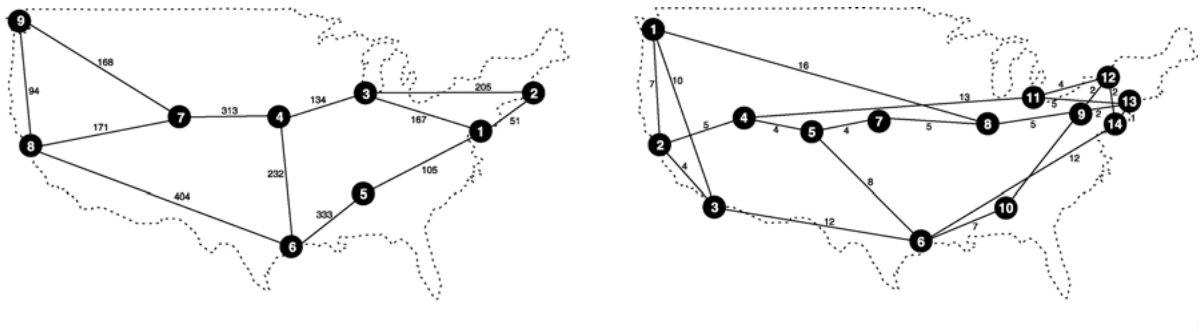
The left part of Fig. 8 illustrates the graph $G(V,E,w)$ associated to the full mesh of lightpaths and their estimated lengths. The right part shows the graph induced by the edges corresponding to the $k=2$ shortest paths[1] for the traffic demand λ^{sd} and the φ_{ij}^{sd} variables used in the formulation.

Once the solution to the LTD problem is found, the set of lightpaths constituting the resulting virtual topology is passed as input to the LR solution algorithm, which finds the definitive physical route for each lightpath of this virtual topology. It may happen that the physical route calculated for a lightpath in this last step is different from the route initially estimated. A deviation criterion could be defined in this casein order to quantify such difference. According to the magnitude of the deviation, the lengths in the full mesh of lightpaths can be updated and the LTD problem solved again.

6 Experimental Results

In this section we experimentally evaluate the LTD and LR models of Sections 2 and 3 and the combined resolution method of Section 5.

An instance of the combined $\mathrm{LTD}+\mathrm{LR}$ problem consists of a physical network, on which the virtual



(a) net 9. (b) net 14.

Fig. 9. Physical networks considered in the experiments.

topology is realized, and of a traffic matrix (λ^{sd}). The elements of the traffic matrix represent a forecast of the packet-switched traffic demand between any two nodes in the network expressed in generic units. The solution to the problem is defined by the set of lightpaths that form the virtual topology, the routing of traffic demands over this topology and the routing of the virtual topology's lightpaths on the physical network. Additionally, the values $\lambda^*_{\max}$ and $\phi^*_{\max}$ of the network congestion and lightpath congestion, respectively, are determined as part of the solution.

Fig. 9 illustrates two physical networks of 9 and 14 nodes considered in the experiments. The networks are denoted **`net9`** and **`net14`**.

For the **`net9`** network we consider two different traffic matrices: the first one, denoted Matrix9a is shown in Fig. 10. The largest element in this matrix is $\lambda^{87} = 847$ traffic units and the average value is 123.63 units. Note that the largest value is much bigger than the average. In the second matrix, denoted Matrix9b, all the elements $\lambda^{sd} = 124 \, \forall \, s \neq d$, so that the traffic is uniformly distributed among all the source destination pairs. For the **`net14`** network we consider the Matrix14a shown in Fig. 11 (used in Ramaswami and Sivarajan [2]) (the largest element is $\lambda^{83} = 21030$ and the average value is 1335.50) and Matrix14b where all the elements $\lambda^{sd} = 1336 \, \forall \, s \neq d$, i.e., the traffic is uniformly distributed. Thus, there are four problem instances that correspond to the combinations **`net9`**-Matrix9a, **`net9`**-Matrix9b, **`net14`**-Matrix14a and **`net14`**-Matrix14b. For sake of simplicity, we refer to the problem instances as Matrix9a, Matrix9b, Matrix14a and Matrix14b.

With some exceptions, the results presented hereafter were obtained using the pruning method of Section 4 and the combined resultion method of Section 5.

Table 1 presents the congestion $\lambda^*_{\max}$ of the logical topologies obtained when models LTD1 and LTD2 are applied to the Matrix9a and Matrix9b instances, using a logical degree δ ranging from 3 to 7. Note that when the LTD2 model is used (atomic routing), the largest element of the considered traffic matrix represents a lower bound on the attainable congestion $\lambda^*_{\max}$. When the LTD2 model is applied to Matrix9a, the lower bound $\lambda^{87} = 847$ is attained with a network degree as small[2] as $\delta = 4$. Since the value of this bound is large (λ^{87} is much bigger than the other elements of the

$$\begin{pmatrix}
0 & 89 & 175 & 1 & 401 & 127 & 143 & 46 & 18 \\
99 & 0 & 26 & 845 & 14 & 8 & 5 & 3 & 1 \\
113 & 175 & 0 & 42 & 19 & 15 & 20 & 2 & 615 \\
83 & 15 & 81 & 0 & 81 & 612 & 115 & 39 & 4 \\
217 & 11 & 95 & 567 & 0 & 60 & 38 & 7 & 5 \\
108 & 8 & 34 & 1 & 45 & 0 & 786 & 12 & 7 \\
84 & 3 & 14 & 184 & 33 & 341 & 0 & 303 & 36 \\
52 & 5 & 8 & 1 & 14 & 19 & 847 & 0 & 4 \\
352 & 30 & 9 & 3 & 155 & 159 & 137 & 76 & 0
\end{pmatrix}$$

Fig. 10. Matrix9a used in the experiments.

$$
\begin{pmatrix}
0 & 109 & 206 & 14 & 45 & 4 & 43 & 145 & 51 & 10 & 7 & 68 & 0 & 33 \\
1171 & 0 & 856 & 62 & 1112 & 777 & 362 & 1579 & 366 & 1661 & 203 & 3781 & 483 & 1319 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
31 & 341 & 1364 & 0 & 190 & 60 & 70 & 288 & 200 & 326 & 307 & 669 & 8 & 401 \\
28 & 6751 & 1902 & 343 & 0 & 403 & 1077 & 6222 & 2402 & 1792 & 45 & 7903 & 997 & 529 \\
0 & 581 & 342 & 552 & 340 & 0 & 261 & 268 & 87 & 387 & 4 & 84 & 6 & 248 \\
175 & 2202 & 10231 & 447 & 2203 & 790 & 0 & 11410 & 1982 & 2195 & 78 & 7140 & 33 & 3284 \\
239 & 6384 & 21030 & 852 & 2821 & 266 & 9708 & 0 & 4395 & 3300 & 1137 & 4863 & 553 & 1385 \\
645 & 1893 & 3735 & 600 & 2499 & 681 & 2506 & 6102 & 0 & 3962 & 1452 & 12750 & 2334 & 76 \\
5 & 3529 & 1026 & 373 & 2234 & 948 & 498 & 5708 & 684 & 0 & 630 & 1764 & 591 & 76 \\
10 & 102 & 313 & 169 & 24 & 6 & 81 & 145 & 58 & 712 & 0 & 84 & 6 & 50 \\
128 & 2615 & 100 & 594 & 2486 & 132 & 549 & 4057 & 2953 & 2237 & 1050 & 0 & 101 & 54 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
73 & 2909 & 1363 & 989 & 3561 & 1207 & 644 & 2879 & 467 & 0 & 399 & 0 & 1075 & 0
\end{pmatrix}
$$

Fig. 11. Matrix14a used in the experiments.

Table 1. Congestion $\lambda^*_{\max}$ obtained with models LTD1 and LTD2 (Matrix9a and Matrix9b).

	Matrix9a		Matrix9b	
δ	LTD1	LTD2	LTD1	LTD2
3	752.16	(923.00)	620.00	620.00
4	524.00	847.00	372.00	496.00
5	492.00	847.00	286.15	372.00
6	492.00	847.00	233.58	248.00
7	492.00	847.00	201.50	248.00

matrix), it is fairly 'easy' to find a solution to the LTD problem that reaches the bound, even for small values of δ. Conversely, when LTD2 is applied to Matrix9b, the lower bound $\lambda^{12} = 124$ is not reached, even for a logical degree $\delta = 7$. It will be seen later that applying the LTD2b model (Section 2.4) to problem instances with a large value of the lower bound leads to network configurations that require less physical ressources than configurations corresponding to problem instances with a smaller value of the lower bound.

On the other hand, the congestion obtained with the LTD1 model is always smaller than the one obtained with LTD2 for both Matrix9a and Matrix9b. This is because the traffic can be arbitrarily split among multiple paths. Furthermore, congestion always decreases with δ. Though traffic splitting (inherent in model LTD1) may have little practical relevance in the context of networks such as current IP networks, it might be interesting to study this issue in networks with traffic engineering functionality, such as MPLS networks. In this context, even if not arbitrarily, different components of a traffic demand can be routed through different explicit routes (e.g., using different LSPs).

Fig. 12 shows the required number of lightpaths computed by the LTD2 and LTD2b models as a function of δ for both Matrix9a and Matrix9b. Less lightpaths are required when using the two-steps optimization model LTD2b than when using the LTD2 model. Indeed additionally minimizing the average hop count has a direct incidence on the number of globally required lightpaths. Furthermore, the difference between the number of lightpaths computed with LTD2 and LTD2b for Matrix9a is greater than the difference computed with the models for Matrix9b. In the LTD2b model, the large value of the lower bound on traffic congestion in the Matrix9a instance ($\lambda^{87} = 847$) implies a solution space greater than the one of the Matrix9b instance (more solutions exist that attain the lower bound). This difference in size of the solution space explains the greater difference between the number of lightpaths computed with LTD2 and LTD2b for Matrix9a.

The congestion is a network performance metric. A metric more related to the network's cost is the number of ports required in the wavelength switches to realize the virtual topology on the physical network. The number of ports depends on the physical route of lightpaths. Indeed, a port is required at each endpoint of a physical link traversed by a lightpath. Thus, the more the lightpaths in the virtual topology and the larger the number of physical links that they span, the more the number of required ports. In the following figures we compare in terms of ports the

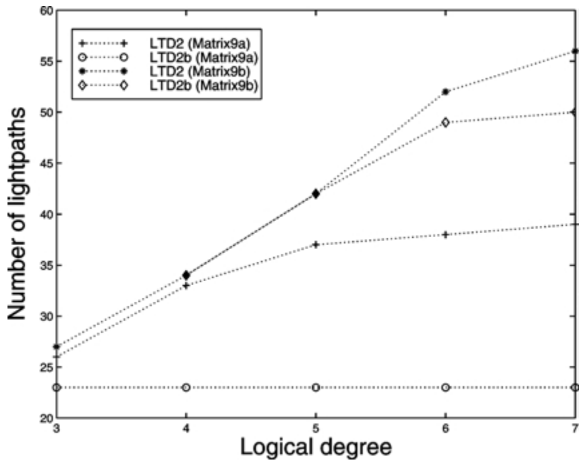


Fig. 12. Required number of lightpaths as a function of δ when using LTD2 and LTD2b.

solutions provided by the combinations of models LTD2-LR, LTD2b-LRb and the solutions of an alternative algorithm [6,17] based on the Tabu Search (TS) meta-heuristic. This algorithm deals with the same problem as the LTD2 model but does not take into account the LR problem, which is solved using the LR model (Section 3.1).

Fig. 13 shows the required number of ports in the wavelength switches as a function of δ when the TS-LR, LTD2-LR and LTD2b-LRb combinations are applied to the Matrix9a and Matrix9b instances. We observe that using the TS algorithm leads in general to a higher requirement of ports. The reason is that the TS algorithm does not take into account any information about the potential cost of lightpaths that it selects to form the virtual topology. Thus, it is unable to discard costly lightpaths as done when the LTD2and LTD2b models are used in combination with the LR and LRb models. We further observe that the LTD2b–LRb combination leads to the lowest requirement of ports. In fact, this combination not only leads to virtual topologies with less lightpaths, but also to physically shorter lightpaths, which has a direct incidence on the number of required ports. Comparing the results on Matrix9a and Matrix9b we note that in the latter case the requirement of ports is larger than in the former. As explained before, the Matrix9a instance has a large value of the lower bound on traffic congestion ($\lambda^{87} = 847$), which renders relatively easy the task of finding configurations that attain this bound with less ressources. It is worth to remind here that, in the case of Matrix9a, a virtual topology with $\delta = 4$ is enough to attain the lower bound. Virtual topologies with larger logical degree only increase the requirement of ports with no benefit. Moreover, for the Matrix9b instance, it is possible to asses the cost in terms of ports of the marginal improvement in congestion in the virtual topology.

Fig. 14 shows the same results as Fig. 13 for the Matrix14a and Matrix14b instances. Note that the relative difference among the different combinations

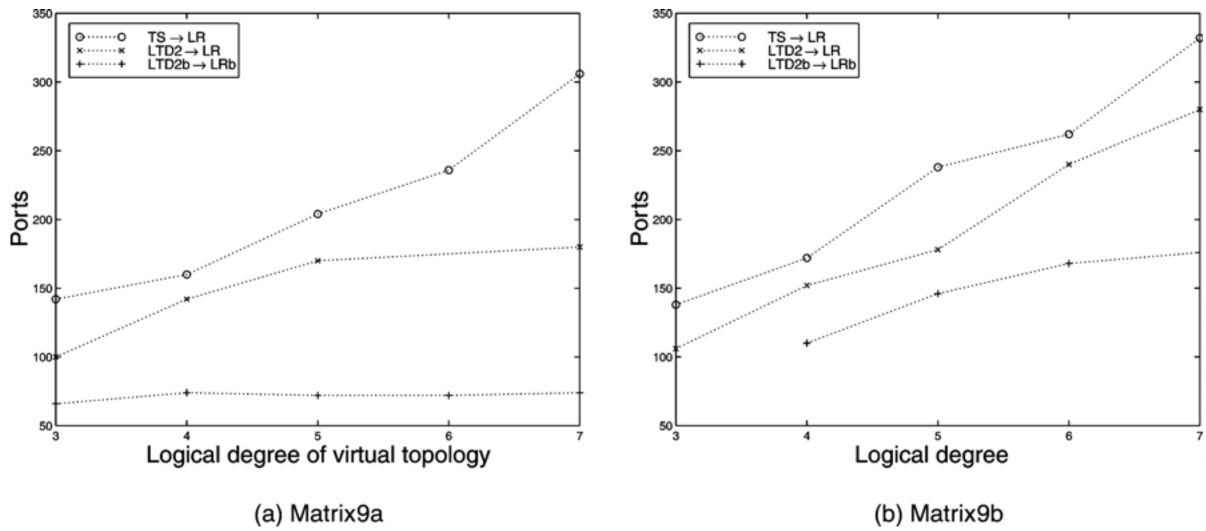
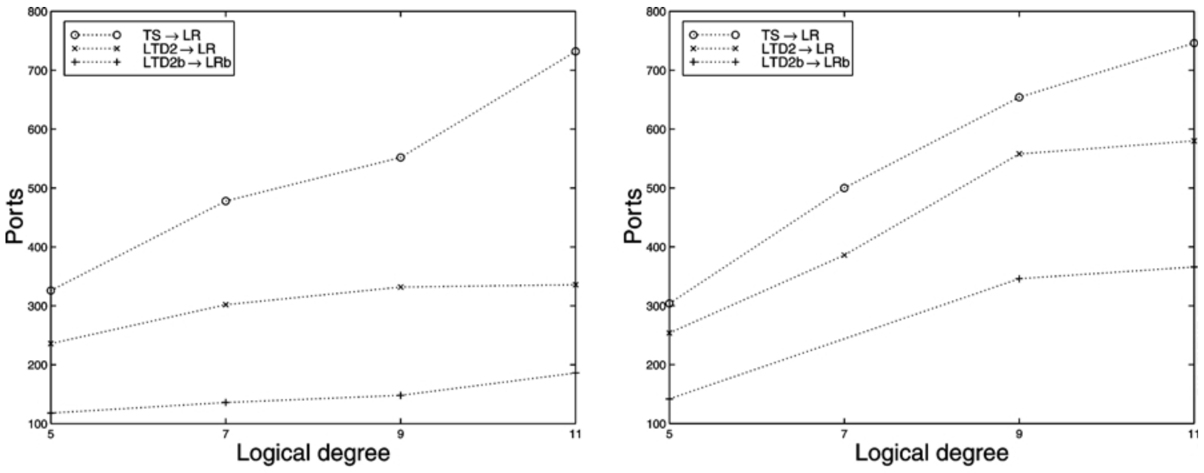


Fig. 13. Comparison of required number of ports in configurations obtained with different design methods (Matrix9a and Matrix9b).



(a) Matrix14a (b) Matrix14b

Fig. 14. Comparison of required number of ports in configurations obtained with different design methods (Matrix14a and Matrix14b).

of models remains in these larger problem instances. We are currently experimenting with other problem instances in order to confirm this pattern of relative difference.

These experiments show the interest of the decomposition approach and a feedback mechanism: not only problem instances of realistic size can be dealt with (the results on atomic routing on large networks are of particular interest), but different combinations of optimization models and objective functions can be investigated in a modular manner and their results compared in terms of the same criterion. Furthermore, the feedback mechanism avoids the loss of perception of the global problem.

7 Conclusions

In this paper we proposed improvements to commonly used multicommodity flow models used to solve the LTD and LR problems. We first added a second optimization step in the LTD model in order to additionally minimize the average hop distance. We then proposed a model that enforces atomic routing of traffic demands. We also developed a pruning method to reduce the number of variables required in both the LR and LTD multicommodity flow formulations. The experiments carried out with these models show that the two steps optimization yields better results than the basic models with respect to the amount of resources required to implement the logical topology. The pruning method proposed in this paper allows to tackle problems of larger size than those typically solved previously by the basic models.

The results obtained by the proposed improvements provided valuable insight on the characteristics of the LTD–LR problem. On the other hand, despite the improvement on the size of addressable problems, we are still limited by the complexity of the underlying ILP problems. We expect to exploit the knowledge of the problem to develop design methods based on meta-heuristics such as TS that take into account the dependency between the LTD and LR subproblems and incorporate a second-step optimization, as done in the LTD2b and LRb models.

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Notes

1. Recall that in the LTD problem, a path corresponds to a concatenation of lightpaths used as a route by a traffic demand.
2. With $\delta = 3$, the obtained congestion is a partial result obtained with the MILP solver, which did not manage to find the optimal solution before the maximum allowed number of iterations.

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