

Variable Aggregation in the ILP Design of WDM Networks with Dedicated Protection

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Abstract: In wavelength-division-multiplexing (WDM) networks a link failure may cause the failure of several high-bit-rate optical channels, thereby leading to large data loss. Recently, various protection and restoration mechanisms have been proposed to efficiently deal with this problem in mesh networks. Among them, dedicated path protection (DPP) is a promising candidate because of its ultra-fast restoration time and robustness. In this work we investigate the issue of planning and optimization of WDM networks with DPP. Integer linear programming (ILP), in particular, is one of the most common exact method to solve the design optimization problem for protected WDM networks. Traditional ILP formalizations to solve this problem rely on the classical flow or route formulation approaches, but both these approaches suffer from an excessively high computational burden. In this paper, we present a variable-aggregation method that has the ability of significantly reducing the complexity of the traditional flow formulation. We compare also the computational burden of flow formulation with variable aggregation both with the classical flow and route formulations. The comparison is carried out by applying the three alternative methods to the optimization of two case-study networks.

Index Terms: Dedicated-path protection (DPP), design methodology, integer-linear programming (ILP), link failure, wavelength-division-multiplexing (WDM) networks.

I. INTRODUCTION

Optical networks provide a transport infrastructure with very high capacity, thanks to wavelength-division-multiplexing (WDM) technology. These networks are based on switching and routing of optical circuits in space and wavelength switching domains. Recently, on the switching equipment side, optical cross connects (OXC) systems have become available, beside the more mature optical add-drop multiplexers. This opened up the road to the deployment of complex WDM networks based on mesh topologies, while in the past ring-based structures were the most used architectures for WDM.

The increase in WDM-networks complexity brought the need for suitable network planning strategies into foreground. Problems such as optimal routing and resource allocation for optical connections must be continuously solved by new and old operators, to plan new installations or to update and expand the existing ones. These problems can no longer be manually solved in complex network architectures: Computer-aided planning tools and procedures are needed for the future which can achieve an efficient utilization of network resources in a reasonable computational time.

On the other hand, the huge bandwidth of WDM also requires efficient survivability mechanisms, because the failure of a network element (usually a node or a link) can cause a large amount of data loss [1]; a highly available WDM layer is crucial to enable quality-of-service (QoS) sensitive applications over it. In this paper, the WDM network design is developed in order to guarantee network survivability against a link failure; undoubtedly protection technique adoption will be paid off by a more complex network design: This has to include in the optimization an additional capacity term needed to reroute optical connections after a link failure.

Various design and optimization techniques for protected WDM networks has been investigating and the proposed solutions can be classified into two main groups: Heuristic methods and exact methods. The former returns sub-optimal solutions that in many cases are acceptable and have the advantage of requiring a limited computational effort. The latter are much more computationally intensive and do not scale well with the network size, being even not applicable in some cases; however since they are able to identify the absolute optimal solution, they play a fundamental role either as direct planning tools or as benchmarks to validate and test the heuristic methods.

The work we are presenting concerns exact methods to plan and optimize resilient multifiber WDM networks. In particular, we rely on integer linear programming (ILP), a widespread technique to solve exact optimization. In [2], we have proposed a new formulation of the optimization problem, called *source formulation*, which allows us a relevant reduction of the computational weight burden. Unfortunately source formulation cannot be extended to protected WDM networks; in this paper, we focus our attention on the dedicated-path-protection (DPP) strategy: First we present two traditional ILP approaches to solve the problem, then we propose an alternative and more scalable model to obtain optimal results with less computational effort.

The rest of the paper is organized as follows. In Section II, we introduce our network model and present a short review of the literature regarding ILP application to WDM path protected network optimization. In Section III, classical formulations for dedicated-path protection are presented and explained into details. In Section IV, we discuss and propose a variable aggregation which allows us to significantly simplify the traditional flow formulation. Finally, in Section V, results obtained by applying the three formulations to case-study networks are shown, showing the advantages of the new model we are proposing.

II. RESILIENT WDM NETWORK OPTIMIZATION BY INTEGER LINEAR PROGRAMMING

Network design and planning is carried out with different techniques according to the type of traffic the network has to

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support. We investigate the static traffic case in which a known set of permanent connection requests is assigned *a priori* to the network. Each request is for a point-to-point optical circuit (lightpath) able to carry a given capacity from the source optical termination to the destination termination and each node pair may request more than one connection. Though dynamic lightpath provisioning is becoming more and more important, in this work we wish to consider only the static situation, leaving for future development the extension to dynamic traffic conditions.

Lightpaths are routed and switched by the OXCs: The channels composing the lightpath may have different wavelengths or may be all at the same wavelength, according to the availability of the wavelength conversion function in the transit OXCs. To simplify, we consider the two extrem cases: In virtual wavelength path (VWP) network cases, all the OXCs are able to perform full wavelength conversion; on the contrary, in wavelength path (WP) network case, no wavelength conversion is allowed in the whole network and lightpaths are subject to the “wavelength continuity” constraint.

WDM networks today are often designed in order to be resilient to failures that may occur to switching or transmission equipment. For this study, we focus only on link failures since fiber cuts has been shown to be the predominant form of failures in telecom networks [3]. Path protection is a well-known approach to survive single link failures in an optical network: For each connection a backup path is statically reserved along with a working path between the source and destination nodes during call setup. In DPP (also called 1 + 1 protection), each primary path has a dedicated link-disjoint backup path. In shared-path protection (SPP), a link-disjoint backup path of a given connection can share WDM channels reserved for spare lightpaths associated to other connection requests. We investigate the DPP case, while ILP formulations for SPP can be found in [4] and [5].

Since optical WDM network design is essentially an optimization problem, its exact solution can be found by applying ILP. WDM network optimization by ILP has been widely studied in literature and in the following we focus our attention on ILP formulations for path-protected scenarios. We can subdivide research contributions in two groups according to which type of networks they are applied to:

- WDM networks with single-fiber links;
- multifiber WDM networks.

As for the first group, the problem consists in the optimal routing and wavelength assignment (RWA) of the lightpaths. This is an NP-complete problem, as it was demonstrated in [6] and [7]. Two basic methods has been defined to model the RWA problem: *Flow formulation* and *route formulation* [8]. In the former the basic variables are the flows on each link relative to each source-destination OXC pair (or connection requests); in the latter the basic variables are the paths connecting each source-destination pair. Both these two formulations have been employed to solve various sorts of problems and to investigate different aspects of WDM networks. Solving RWA problem has been often associated to survivability matters: A complete description of the different protection strategies is shown in [9] using ILP models.

In optimization of multifiber WDM networks optimal allocation

of fibers has also to be solved, thus complicating the problem of lightpath set up into routing, fiber, and wavelength assignment (RFWA). Solving RFWA becomes really challenging even with relatively small networks, especially because RWA is coupled to dimensioning (fiber assignment). This implies that RFWA scales from an NP-complete multicommodity flow problem to an NP-hard localization problem. The protection issue is taken into account also in works facing RFWA problem: In [10] an ILP model for path protection is presented; in [11] path protection is studied under the different hypothesis of dedicated and shared backup paths; an exhaustive analysis of protection strategies based on ILP models can be found in [12], where link and path protection are described.

When the problem becomes computationally impractical, a typical simplification strategy is to set routing constraints. For example, all the lightpaths can be constrained to be routed along the first k -shortest paths connecting the source to the destination. In this cases route formulation becomes more useful than flow formulation since the number of its variables can be constrained. In other words, the flow formulation has a complexity which is strictly dependent on physical topology and offered traffic, while the size of the route formulation decreases with the number of paths that can be employed to route the lightpaths. Multifiber network optimization with route formulation and constrained routing has been studied in [8], [10], [11], and [13].

Beside route formulation with constrained routing, other methods to control complexity have been proposed. A possibility is to stop the branch-and-bound algorithm (typically used to solve ILP problems) after having found the first or a pre-definite number of integer solutions. [12] shows that acceptable results (though quite far from the optimal solution) can be obtained when the branch-and-bound duration is fixed to 10 minutes. [14] proposed that the whole RFWA problem can be solved as a sequence of simpler problems (e.g., first routing, then fiber assignment). Other possible approaches are: Exploitation of lagrangean relaxation [15], relaxation of integer constraint [13], and randomized routing [16].

Finally, to complete the overview on the possible approaches to soothe the ILP computational burden in complex design problems, it is worth to remember that a large body of research is available in the operations research literature regarding ILP resolution algorithms that exploit efficient methods such as cutting plane or cut-set inequalities or column generation [17]–[20]. These proposals are viable alternatives to the traditional branch-and bound method to solve capacitated network design¹ (it's worth noting that RFWA in VWP networks, i.e., without wavelength continuity constraint, is the same as a capacitated network design).

Nonetheless, all these approaches trade-off the possibility to achieve a relevant decrease in the computational times with the assurance to reach the optimal solution of the problem. In other words, all the previous approaches are typically able to return very good solution, but not to ensure that those solutions are the actual optimum of the problem. So, in the following, our objective will not be the proposal of an “ad hoc” method to solve

¹ Similar studies on capacitated network design associated to path restoration requirements have been carried on for the ATM networks [21].

RFWA problem with protection requirements (such as a new column generation approach or a new route formulation with constrained routing), but we will focus on a proper variable-aggregation mechanism in order to reduce the problem size without losing the assurance of optimality of our solution.

Undoubtedly the massive need of computational resources (i.e., time and memory occupation) represents the main obstacle to an efficient application of ILP in optical networks design. As we have discussed before, many techniques are able to overcome this limitation, but most of them results only in approximations of the actual problem optimum. On the other hand, the great advantage of ILP over heuristic methods is the ability to *guarantee* that the obtained solution is the absolute optimum value (e.g., for benchmarking purpose).

In the following we present an efficient flow-based ILP formulation developed to solve planning of WDM networks exploiting dedicated path protection as survivability strategy. This formulation achieves better performances in design problems with large offered traffic load compared to classical route and flow models. Although our model relaxes the traditional set of link-disjointness constraints, it is able to return the optimal values in all the realistic cases we have considered. Moreover, we propose a simple algorithm to verify if the obtained solution is the problem absolute optimum (i.e., it is not affected by approximations), so that a non-admissible solution could be identified and corrected.

III. ILP MODELS IN DEDICATED PATH PROTECTED NETWORKS

Let us consider a multifiber WDM network environment under static traffic (connection requests are expressed in terms of lightpaths), in which the number of wavelengths per fiber W is given *a priori*, while the number of fibers installed in each physical link are variables of the problem [22].

The definition of an ILP model in a WDM network with dedicated path protection is a well-known problem: To the usual set of constraints used in the unprotected network [2], we must add constraints deriving from the link disjointness condition required by dedicated path protection. These additional constraints can be easily set exploiting the traditional flow or route variables. This, however, will result in a computationally heavy representation of the problem, since we need to distinguish each connection requests in order to protect the requests one by one, thus involving a large number of variables.

In the rest of this section, we describe the flow and route formulations into details. For sake of brevity, we expressly consider here only the VWP case. Details about the extension of flow and route ILP models from VWP to WP case can be found in [2] and [8].

A. Flow Formulation

The physical topology is modeled by the graph $\mathcal{G} = \mathcal{G}(\mathcal{N}, \mathcal{A})$.² Physical links are represented by the undirected edges $l \in \mathcal{A}$ with $|\mathcal{A}| = L$, while the nodes $i \in \mathcal{N} = \{\infty, \epsilon, \dots, \mathcal{N}\}$, with $|\mathcal{N}| = N$, represent the OXCs. Each link is equipped with a

certain amount of unidirectional fibers in each of the two directions; fiber direction is conventionally identified by the binary variable k ($k = 0$ for forward direction, and $k = 1$ for backward direction).

Each source-destination node couple requiring connectivity (lightpaths) is associated to an index c . We refer to the source node as s_c and to destination as d_c ; the required traffic is v_c : If $v_c > 1$, we add an auxiliary index t having values between 1 and v_c .³ Connection requests are unidirectional. As far as dimensioning and resource allocation are concerned, it is not relevant to set a distinction between the working and the protection lightpath associated to the same connection request. Therefore we will refer to a 1 + 1 protected optical connection in terms of a link-disjoint couple of paths connecting the source node to the destination node.

Let us define all the variables involved in this protected flow formulation:

- $x_{l,k,c,t}$ is a boolean variable indicating whether a WDM channel on link l in direction k has been allocated to the t th connection requested by node couple c .
- $F_{l,k}$ is the number of fibers on link l in direction k .

The following additional symbols are also defined:

- (l, k) identifies the set of fibers of link l that are directed as indicated by k ; in the following we name (l, k) a “unidirectional link”;
- I_i^+ is the set of “unidirectional links” having the node i as one extreme and leaving the node; analogously, I_i^- is the set of “unidirectional links” having the node i as one extreme and pointing towards the node;
- (c, t) identifies a single connection request: c identifies the connection source-destination couple, while t identifies a particular connection request associated to the node couple c .

Now we can detail the flow formulation. The cost function to be minimized can be either the total fiber number

$$\min \sum_{(l,k)} F_{l,k}$$

or alternatively can contain an estimation of the cost $p_{l,k}$ of link (l, k) ($\sum_{(l,k)} p_{l,k} F_{l,k}$). We refer to this second metric as length metric, while the first is called hop metric.

The set of constraints is the following

$$\sum_{(l,k) \in I_i^+} x_{l,k,c,t} - \sum_{(l,k) \in I_i^-} x_{l,k,c,t} = \begin{cases} 2, & \text{if } i = s_c, \\ -2, & \text{if } i = d_c, \\ 0, & \text{otherwise,} \end{cases} \quad \forall i, (c, t); \quad (1)$$

$$\sum_{(c,t)} x_{l,k,c,t} \leq W F_{l,k}, \quad \forall (l, k); \quad (2)$$

$$\sum_k x_{l,k,c,t} \leq 1, \quad \forall l, (c, t); \quad (3)$$

$$x_{l,k,c,t} \text{ binary}, \quad \forall (l, k), (c, t); \quad (4)$$

$$F_{l,k} \text{ integer}, \quad \forall (l, k). \quad (5)$$

²All the following formulations require that the topology is at least 2-connected.

³Indices c and t could collapse in a single index directly associated to each single connection request, but this alternative notation is less intuitive. In the following we will use the former indexing for sake of clarity.

This formulation assigns a routing with respect to dedicated path protection strategy, as described in Section II. Constraint (1) is a solenoidality constraint. It corresponds to the following sequence. Let us consider the t th connection requested by node couple c . We express the flow conservation condition for each node i of the network, considering only traffic associated to connection (c, t) . This condition states that the total flow (c, t) leaving i must be equal to the total flow c incident on i . This equation is slightly modified in the source (destination) node of the connection request (c, t) , in which the outgoing (incoming) flow must be equal to 2. This is due to the fact that two lightpaths (working+spare) are associated to the connection request, according to the dedicated path protection technique. Constraints concerning dimensioning are a simple extensions of the corresponding constraints in the unprotected case. Constraint (2) ensures that the total number of WDM channels allocated to spare and working lightpaths on the unidirectional link (l, k) is bounded by the link capacity, given by the number of fibers $F_{l,k}$ multiplied by the number of wavelength W . Constraint (3) stems from link-disjointness condition: No more than one lightpath associated to connection request (c, t) can coexist on the same link, neither in opposite direction. From now on for sake of simplicity we refer to these formulations with the acronym flow formulation (FF).

B. Route Formulation

In order to apply route formulation, we have to carry out a preprocessing operation to prepare the set of route variables for the ILP optimization. To achieve a complete description of the problem, all the link-disjoint working-spare routes⁴ connecting source-destination nodes (at least those requiring traffic) have to be identified. This preprocessing time cannot be neglected: We will clarify this aspect in the last section.

Now, let us introduce the notation for route formulation. Let us consider a source-destination couple c and suppose we have precomputed all the n working-spare routes between this two nodes. The variable $r_{c,n}$ indicates how many protected connections are routed on the n th working-spare route between the node couple c . The subset $\mathcal{R}_{(\downarrow, \uparrow)}$ includes all the working-spare routes whose (either working or spare) path is routed on link (l, k) . The objective function is the same seen in the FF model. Let us analyze the constraints

$$\begin{aligned} \sum_n r_{c,n} &= v_c, & \forall c; & \quad (6) \\ \sum_{(c,n) \in \mathcal{R}_{(\downarrow, \uparrow)}} r_{c,n} &\leq W F_{l,k}, & \forall (l, k); & \quad (7) \\ r_{c,n} &\text{integer}, & \forall (c, n); & \\ F_{l,k} &\text{integer}, & \forall (l, k). & \end{aligned}$$

Constraint (6) ensures that the number of working-spare routes established between each source-destination couple c satisfies

⁴It is worth noting that an alternative approach to route formulation for DPP design is possible: It consists in assigning to the basic-route variable a single path, instead of working-spare couple. In this case, an explicit link-disjointness constraint, such as (3) in flow formulation, must be enforced and consequently the model become much more complex in terms of number of number of constraints.

offered load v_c . Constraint (7) ensures that number of fiber on link (l, k) can support working and spare traffic routed on this link. From now on for we refer to these formulations with the acronym route formulation (RF).

IV. SETTING A NECESSARY CONDITION: THE “MAX HALF” (MH) PER LINK

The two classical ILP models for DPP presented in the previous section are affected by relevant computational limitations. As for the route formulation, the number of admissible paths in a mesh network grows rapidly for increasing values of connectivity index and number of nodes; the route approach soon becomes unfeasible if we do not introduce constrained routing. On the other hand, the flow formulation is not scalable on the volume of offered traffic because the number of variables increases rapidly, especially is offered traffic matrix contains a high number of connections.

In the following we propose a variable aggregation method, called “max half” (MH) formulation, that is able to soothe the computational burden of the traditional flow formulation model: In brief, the variable number is reduced by aggregating all the flow variables associated to a given node couple, i.e., applying this aggregation

$$\sum_t x_{l,k,c,t} = x_{l,k,c}. \quad (8)$$

The use of the new aggregated variable $x_{l,k,c}$ allows a relevant saving on the number of variables, but in turn does not allow us to enforce the classical link-disjointness constraint in (3). So, in our MH proposal we enforce a new constraint, referred to as MH constraint, to obtain a link-disjoint routing even without an explicit link-disjoint constraint.

In the following we first discuss the MH rationale, then we present the MH formulation in both the versions for VWP and WP networks.

A. “Max Half” Rationale

First, let us discuss why it is possible to aggregate the flow variables according to (8) without losing the optimality of problem solution. Then, we will show the possible drawbacks of this approach and the way to avoid them.

When DPP routing is applied, for each unit of requested traffic by a node couple, two units of traffic are routed, namely, a working and a spare lightpath. Now, let us assume that a node couple requires k traffic units (i.e., k lightpaths): Consequently, $2k$ traffic units must be routed. The “Max Half” approach enforces the following simple constraint: No more than k units of traffic (i.e., half of the routed traffic) can concentrate on a single link. As a result, the routing obtained under the MH constraint has the non-trivial property to be able to deliver all the offered traffic in case of single link failure. This property stems from the fact that a link failure cannot waste more k units of traffic, since no more than k units of traffic are allowed to be routed on single link. In other words, if we route $2k$ connections under the MH constraint, at least k path always survive in the network even after a link failure.

However, it is crucial to note that enforcing the MH condition is not equivalent to enforce link-disjointness condition. More

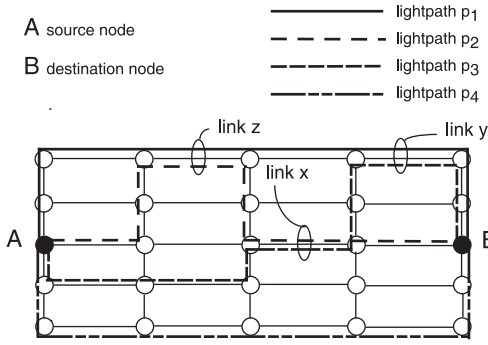


Fig. 1. This routing assignment satisfies the necessary condition enforced by “Max Half” formulation, but it does not satisfies link-disjointness.

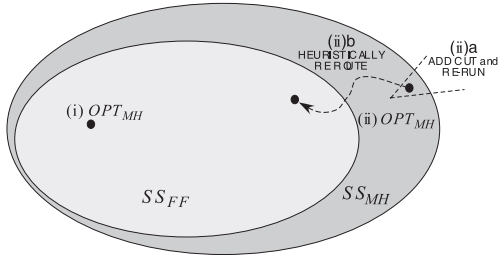


Fig. 2. FF solution space is a subset of MH solution space

precisely, MH condition is a *necessary condition* to provide a routing which satisfies link-disjointness between the working and spare lightpaths, *but not a sufficient condition*.

It is easy to demonstrate that the MH condition is necessary for link-disjointness: If *more* than k units of traffic flow along the same link, then it comes that it is not possible to have k working paths and k the spare paths that are link-disjoint.

Now, let us focus on the non-sufficiency of MH condition for link-disjointness: In Fig. 1, we show the case of a routing assignment which satisfies the MH (no more than two lightpaths on each link), but it does not verify the link-disjointness. Between nodes A and B we have routed 2 protected connection requests (i.e., 4 lightpaths in total, p_1 , p_2 , p_3 , and p_4). These lightpaths are routed so that a link failure does not preclude the delivery of at least 2 lightpaths, so MH condition is satisfied. But, although the MH condition is satisfied, it is not possible assign two working paths with two respective link-disjoint spare paths. As a matter of fact, p_1 is disjoint from p_4 only, and p_2 and p_3 are not disjoint. In conclusion, in Fig. 1, MH guarantees that a sufficient number of paths survive to deliver the requested traffic in case of link failure, but it does not guarantee two link-disjoint routing assignments, because it is not possible to identify unequivocally which are the working and the spare lightpaths. Despite this theoretical limitation, the MH approach remains interesting: Optimal-routing solutions, where situations analogous to that shown in Fig. 1 arise, are extremely unlikely.⁵ To confirm that these situations are extremely unlikely to occur, we have carried out optimization runs on two well-known case-study networks and on a set of 8-node networks with increasing values of connectivity index (see [2] for details on these 8-node networks), setting different values of W , and the MH model has always returned a link-disjoint routing assignment. Therefore

MH has allowed us to obtain optimal solutions of our RFWA problem on all the network cases under examination.

A Posteriori Test of Optimality

We can try to clarify the properties of MH model by referring to the relationship between MH and FF (or equivalently RF) solution space (SS). The solution space SS_{FF} of FF is a subset of SS_{MH} ($SS_{FF} \subseteq SS_{MH}$, see Fig. 2). Let us refer to the optimal solution obtained by MH and FF respectively with OPT_{FF} and OPT_{MH} . We can distinguish two alternative cases:

- (i) if $OPT_{MH} \in SS_{FF}$, the solution is the optimal one ($OPT_{MH} \equiv OPT_{FF}$);
- (ii) if $OPT_{MH} \in (SS_{MH} - SS_{FF})$ the solution is not admissible in SS_{FF} and we have to move the solution in the admissible field.

The solution found by MH model is admissible if the routing assignment allows us to unequivocally identify a link-disjoint subdivision of working and spare lightpaths. To verify the link-disjointness of the solution, we suggest to reduce this verification to the *maximum matching in a graph* [23]: For each node couple that requires connections, let us construct a graph, so that its nodes are associated to the paths resulting by optimization; two nodes are connected if they do not share any links. If there exists an optimum matching in the resulting graph (case i), then the routing satisfies the property of link-disjointness. Otherwise (case ii) the non-disjoint lightpaths must be rerouted, trying to minimize the additional needed capacity. To solve this problem we can heuristically reroute the non-disjoint connections, accepting the sub-optimality of this approach (case (ii)b). Alternatively, exploiting a more formal approach, we could add new “ad hoc” constraints in the MH model, so that the non-admissible solution is cut by the solution space and then re-run the optimization (case (ii)a).

B. “Max Half” Formulation

Let us analyze the constraints of the new model :

$$\sum_{(l,k) \in I_i^+} x_{l,k,c} - \sum_{(l,k) \in I_i^-} x_{l,k,c} = \begin{cases} 2v_c, & \text{if } i = s_c, \\ -2v_c, & \text{if } i = d_c, \\ 0, & \text{otherwise,} \end{cases} \quad \forall i, c; \quad (9)$$

$$\sum_c x_{l,k,c} \leq W F_{l,k}, \quad \forall (l, k); \quad (10)$$

$$\sum_k x_{l,k,c} \leq v_c, \quad \forall l, c; \quad (11)$$

$$x_{l,k,c} \text{ integer}, \quad \forall (l, k), c; \\ F_{l,k} \text{ integer}, \quad \forall (l, k).$$

Solenoidality constraint routes a doubled traffic; as in the previous formulation we refer with a unique variable to working and spare traffic (9). Constraints (10) do not change significantly

⁵Presumably, scenarios as in Fig. 1 are more likely to appear when the transmission resources are given as an input to the problem, not in a dimensioning case.

compared to unprotected case. Constraint (11) sets the necessary condition MH previously discussed: No more than half of the traffic (i.e., v_c because in the protected case the total traffic is $2v_c$) can flow on the same link in both backward or forward direction. In the following we refer to this model with the acronym MH.

We can now compare the complexity of the presented models, in terms of number of constraints and variables. The following notation will be used:

- N : Number of nodes;
- L : number of bidirectional links;
- W : Number of wavelengths per fiber;
- T : Total offered traffic (in terms of number of protected connections requests);
- C : Number of nodes couples requiring connections;
- R : Average number of working-spare paths (cycles) of a node couple.

Table 1 compares the complexity of FF, RF, and MH formulation. Let us now observe the difference between FF and MH, the flow based formulations. MH allows us to save variables and constraints in the order of T/C , that is to say that performance improvement will tend to be more significant for heavy traffic loads. In the national science foundation network (NSFNET) case study, the traffic matrix is characterized by $C = 108$ (node pairs requiring connections) and $T = 360$ (total connection requests), so the MH gain is approximately 3.3 on both variables and constraints number. The comparison with RF depends on R value and we will discuss it in the next section.

C. "Max Half" Formulation in WP Networks

The extension of the MH formulation to the WP case requires the introduction of a new index in the flow variables: $x_{l,k,c,\lambda}$ now indicates the number of WDM channels on wavelength λ which carry lightpaths associated with connections requested by the node couple c on the fibers of link (l, k) . Let us focus on the WP constraints:

$$\sum_{\lambda, (l,k) \in I_i^+} x_{l,k,c,\lambda} - \sum_{\lambda, (l,k) \in I_i^-} x_{l,k,c,\lambda} = \begin{cases} 2v_c, & \text{if } i = s_c, \\ -2v_c, & \text{if } i = d_c, \end{cases} \quad \forall i \in (s_c, d_c), c; \quad (12)$$

$$\sum_{(l,k) \in I_i^+} x_{l,k,c,\lambda} - \sum_{(l,k) \in I_i^-} x_{l,k,c,\lambda} = 0 \quad \forall i, c, \lambda; \quad i \neq s_c; \quad i \neq d_c; \quad (13)$$

$$\sum_c x_{l,k,c,\lambda} \leq F_{l,k} \quad \forall (l, k), \lambda; \quad (14)$$

$$\sum_{k,\lambda} x_{l,k,c,\lambda} \leq v_c \quad \forall l, c; \quad (15)$$

$$x_{l,k,c,\lambda} \text{ integer} \quad \forall (l, k), c, \lambda; \quad (16)$$

$$F_{l,k} \text{ integer} \quad \forall (l, k). \quad (17)$$

The solenoidality constraints in (1) are now split into constraints (12) and (13): Constraint (12) enforces that two lightpaths (working+spare) for each connection (c, t) must be routed

Table 1. Comparison on constraint and variable numbers between FF, RF, and MH formulations in VWP case.

formulation	variables	constraints
FF	$2L(T+1)$	$2L + T(N+L)$
MH	$2L(C+1)$	$2L + C(N+L)$
RF	$RC + 2L$	$C + 2L$

Table 2. Comparison on constraint and variable numbers between FF, RF, and MH formulation in WP case.

form.	variables	constraints
FF	$2L(1+TW)$	$T(2 + (N-2)W + L) + 2LW$
MH	$2L(1+CW)$	$C(2 + (N-2)W + L) + 2LW$
RF	$RCW + CW + 2L$	$C(W+1) + 2LW$

(not necessarily on the same wavelength), while constraint (13) imposes the flow conservation on each wavelength. As a result, the wavelength continuity constraint is automatically enforced. The capacity constraint (14) is also modified: The maximum number of connections routed over a single wavelength on a link must be less or equal than the number of allocated fibers. Finally, constraint (15) verifies that the working and the spare path of the same connection request are not routed on the same link. Table 2 reports the complexity of FF, RF, and MH formulation in the WP case.

V. CASE STUDY AND RESULT COMPARISON

In this section we present and discuss the results obtained by performing ILP optimization exploiting FF, RF, and MH formulation on case-study networks. Two well-known networks have been considered: The NSFNET and the european optical network (EON). Data regarding their physical topologies, were taken from [11] and [24], respectively. NSFNET has 14 nodes and 22 links, while EON has 19 nodes and 39 links. The static (symmetric) traffic matrices are derived from real traffic measurements which are reported in the same references and they comprise 360 and 1380 unidirectional connection requests for NSFNET and EON, respectively. The objective function to be minimized in all our optimizations is the number of fibers.

A. Performance Comparison among MH, FF, and RF Models

To solve the ILP problems we used the software tool CPLEX 6.5 based on the branch-and-bound method [25]. As hardware platform a workstation equipped with a 1 GHz processor was used. The available memory (physical RAM + swap) amounted to 460 MBs. This last parameter plays a fundamental role in performing our optimizations. The branch-and-bound algorithm progressively occupies memory with its data structure while it is running. When the optimal solution is found, the algorithm stops and the computational time and the final memory occupation can be measured. In some cases, however, all the available memory is filled up before the optimal solution can be found. In these cases CPLEX returns the best yet non-optimal branch-and-bound solution it has been able to find and forces the execution to quit. This cases are identified by the *out-of-memory* tag (OOM) and the computational time measures how long it has taken to fill up memory. We have clarified this particular aspect

Table 3. ILP variables and constraints for NSFNET and EON.

network/formul.	constraints	variables
NSFNET/FF	10856	13726
NSFNET/MH	3932	4796
NSFNET/RF	152	22468
EON/FF	67122	94104
EON/MH	19396	25980
EON/RF	420	$\approx 27 \cdot 10^6$

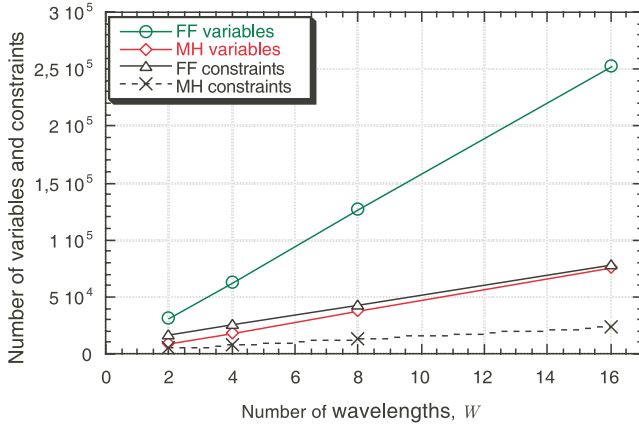


Fig. 3. ILP variables and constraints for NSFNET in the WP case.

of ILP to enable a clear understanding of the reported data.

Table 3 shows the number of variables and constraints that are involved in the ILP problem applied to the two networks in the VWP case. Data are taken from the parameters returned by CPLEX presolver. They clearly show the advantage of MH formulation on FF (in the order of T/C as foreseen). The advantage of MH on RF can be observed in the EON case, while in the NSFNET the complexity of the two models is comparable. RF based problems become intractable due to the exponential growth of admissible paths with mesh network complexity, while flow variables grow linearly with the link number. As a matter of fact the preprocessing time in the NSFNET case takes about a hour and half, while in the EON case we give up precalculating all the admissible paths after having computed all the routes between a single source-destination node couple obtaining about 80000 paths (i.e., variables) in 2 days of computational time (it is worth noting that the number of node couples requiring connections is 342). The number of variables reported in Table 3 is an estimation ($342 \cdot 80000$ variables).

As for the WP case, Fig. 3 displays the variables and constraints numbers as a function of the number of wavelengths per fiber W for FF and MH. The comparison between the curves shows the relevant savings on variables and constraints introduced by the MH formulation in both the networks and shows that savings increase linearly with the parameter W .

We have shown the advantage of FF versus MH in terms of variable and constraint numbers. It is important to see how much this advantage impacts on the actual computational performance of the ILP optimizations. Table 4 displays computational times and memory occupations concerning NSFNET optimization in the VWP case (s, m, and h stand for seconds, minutes, and hours, respectively, while MB stands for MBs). There is a great

Table 4. Computational time and memory occupation comparison between FF and MH formulations in NSFNET VWP network.

W	FF	MH	W	FF	MH
2	20 m	1.5 m	2	0.62 MB	0.26 MB
4	2 h	3 m	4	3 MB	0.54 MB
8	41.8 h	5.4 h	8	OOM	OOM
16	4.4 h	1.5 h	16	55 MB	73 MB
32	33.7 h	43 m	32	354 MB	20 MB
64	80 h	3.4 h	64	250 MB	30 MB

Table 5. Computational time comparison between FF and MH formulations in EON VWP network.

W	FF	MH
2	20 m	10 s
4	25.6 h	6.2 h
8	41.4 h	11.6 h
16	29.9 h	9.2 h
32	77.2 h	28.5 h
64	76 h	25 h

Table 6. Computational time and memory occupation comparison between FF and MH formulations in NSFNET WP network.

W	FF	MH	W	FF	MH
2	3 h	20 m	2	37 MB	15 MB
4	153 h	5.7 h	4	48.5 MB	5.2 MB

difference in terms of computational time, since we recall that $T/C = 3.3$ in NSFNET: e.g., with $W = 2$, 32 the computational time decreases from hours in FF case to minutes in MH case. Analogous considerations can be expressed from Table 5, which refers to EON.

As far as the WP case is concerned, we consider NSFNET with $W = 2, 4$: It is interesting to note that we have obtained the same results obtained in VWP case (minimizing the number of fiber we obtained 983 and 492 fibers, respectively). This confirms the scarce utility of wavelength conversion in a static environment, already emerged in other studies [12]. Again computational effort is significantly improved by means of MH (see Table 6). The main advantage of MH over FF model is the aggregation of traffic associated to a source-destination pair: This aggregation allows us to use integer variables instead of binary variables so reducing the number of variables by a factor T/C . So, in heavily loaded networks, with large values of T/C , the advantages of MH are more evident, in terms of computational time, memory occupation and quality of the solution found.⁶ In order to highlight the substantial performance difference between the two formulations, we have carried out a sequence of optimization runs on NSFNET by increasing the offered traffic load. The original traffic matrix, with $T/C = 3.3$, is multiplied by a factor γ , with $\gamma = 10, 20, 30$, and 40. The different behavior of FF and MH formulation on NSFNET with $W = 4$ is summarized in Table 7. Results confirm that MH model is independent of the traffic scaling: Optimization runs on NSFNET with $W = 2, 4$ show that MH is always able to find the optimal

⁶The two models may give different solutions only when the branch and bound algorithm doesn't terminate, i.e., when the entire memory saturates.

Table 7. Computational time and memory occupation comparison between FF, MH and RF formulation in NSFNET VWP network, $W = 4$.

γ	FF	MH	RF
10	38 m–200 MB	9 s–9 MB	15 s–11 MB
20	8 h–400 MB	5 s–6 MB	11 s–9 MB
30	Can't find integer solution	4 s–6 MB	6 s–7 MB
40	Can't read problem data	5 s–5 MB	15 s–9 MB

solution in less than 10 seconds, by occupying a limited amount of memory. Also with $\gamma = 100$, MH provides the same performance. On the contrary, the runs with the FF model confirm that the traffic scaling has a significant effect on problem complexity and consequently on FF performance: Execution times and memory occupations increase considerably. We need 200 MBs and some minutes when $\gamma = 10$, 400 MBs and some hours when $\gamma = 20$. When $\gamma = 30$, it becomes impossible to find any feasible integer solution; when $\gamma = 40$, it becomes impossible to read problem data.

For sake of completeness, an analogous analysis has been carried out using route formulation to show that also RF does not depend on traffic load. Anyway the efficiency comparison between RF and MH must be set from another point of view. Although RF complexity does not depend on traffic load (as seen for FF), its main drawback is related to the exponential relation between network dimension and the number of admissible paths pair between each node couple. For example upgrading network complexity from 14 nodes and 22 links of NSFNET to 19 nodes and 39 links of EON we have observed in Table 3 that the RF approach becomes intractable, while the MH approach provides good performance despite the increase in network dimension. Results for the WP case are not reported for sake of brevity.

B. Exploiting MH Model as a Benchmark

We have seen in the two case-study networks under dedicated path-protection that the MH model succeeds in finding optimum or at least values very close to optimal value and outperforms FF and RF models from a computational point of view. The MH model is thus used as a benchmark of the network design parameters obtained using a heuristic tool [26] developed by our research group.

Optimization runs in NSFNET and EON are carried out varying W from 2 to 64: The final values in NSFNET are optimal for each value of W except for $W = 8$, where the partial result returned after memory exhausting is characterized by a percent distance from optimum not greater than 1% (we use the *gap* parameter contained in CPLEX). In EON solution optimality is verified only with $W = 2$, but percent error of other solutions is lower than 1% for $W = 4, 8, 16$, than 1.6% for $W = 32$, than 2.6% for $W = 64$; these approximations seem to be acceptable and we suppose that a bigger availability of computational resources would have allowed us to demonstrate that the obtained integer solutions coincide with problems optima.

Figs. 4 and 5 compare the results provided by the heuristic tool with those given by the MH model, proving the heuristics efficiency. Thanks to MH modeling we have been able to benchmark our heuristic tool in the two case studies in a reasonable time. Indeed ILP performance for small networks can be con-

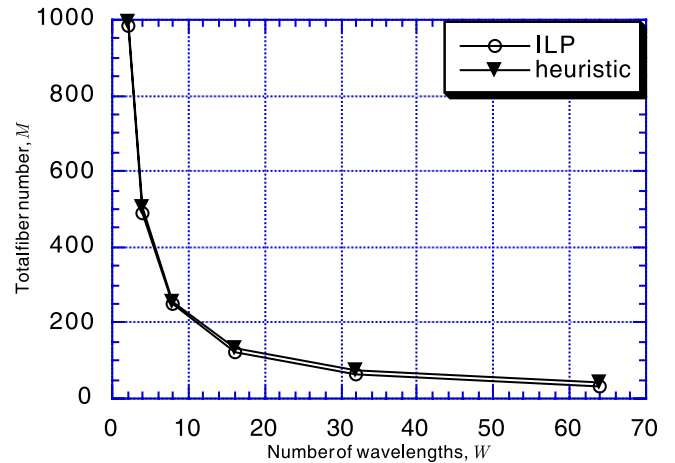


Fig. 4. Total fiber number in NSFNET network with dedicated path protection.

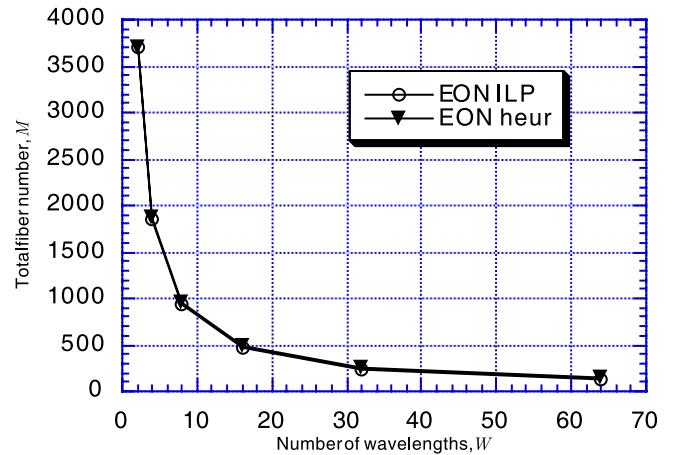


Fig. 5. Total fiber number in EON network with dedicated path protection.

sidered competitive with heuristic tool performance.

VI. CONCLUSIONS

We have considered the problem of designing and optimizing WDM resilient multifiber networks supporting unidirectional protected optical connections. We have presented the MH formulation, a novel approach to model the above problem with low computational complexity in the particular case of static traffic and dedicated path protection strategy. Thanks to the MH formulation, we are able to substantially reduce the multiplicity of both variables and constraints compared to the traditional flow formulation especially when traffic matrix is very large. A comparison has been carried out also with the well-known route formulation showing that it is outperformed by MH in large networks. By exploiting MH we thus obtain a substantial gain with respect to the traditional route and flow models achieving significantly lower computational times and memory occupations.

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