C1 - Assignment 1 Report: Sparse Matrices.

Student Number: 1894945

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Abstract

1 Introduction

1.1 Well-posed, direct problems

The problems that will be addressed in the following are assumed to be always representable in the form:

$$F(x, d) = 0 \tag{1}$$

where x represents the unknown, d the set of data from which the solution depends on and F the functional relation between x and d. Such types of problem are called *direct problems* ([1]).
If the problem admits a unique solution[1] x that depends continuously on the data d, then the problem is said to be *well-posed* or *stable*. Whenever the aforementioned properties are not satisfied, the problem is said to be *ill-posed*.

1.2 Numerical Methods

In the following, it will always be assumed that problem 1 is well-posed. A numerical method for the approximate solution of the aforementioned equation consists in a sequence of approximate problems:

$$F_n(x_n, d_n) = 0 \quad n \geq 1 \tag{2}$$

with the underlying expectation that $x_n \to x$ as $n \to \infty$, i.e. the approximate solution converges to the exact one.

Definition 1. The numerical method 2 is convergent iff

$$\forall \epsilon > 0,\ \exists n_\epsilon,\ \exists \delta(n_\epsilon) \quad | \quad \forall n > n_\epsilon,\ \forall \delta d_n : ||x(d) - x_n(d + \delta d_n)|| < \epsilon$$

where d_n is an admissible datum for the n^{th} approximate problem, δd_n a perturbation of d_n, $x_n(d + \delta d_n)$ the corresponding solution of it and $x(d)$ the solution for corresponding exact problem.

2 Problem setup

2.1 Performed tests

3 Conclusive remarks

[1] In this case d is said to be admissible for 1.

References

[1] A. Quateroni, R. Sacco, F. Saleri; *Numerical Mathematics*, Vol.37, Springer Verlag, (2007).