

### Ejercicio 10

$$T(n) = \begin{cases} 2, & n = 1 \\ T(n-1) + n, & n \geq 2 \end{cases}$$

Buscamos un paso general

$$P1 = T(n-1) + n$$

$$P2 = T(n-2) + 2n$$

$$P3 = T(n-3) + 3n$$

$$P_i = T(n-i) + in$$

Buscamos caso base

$$n-i = 2$$

$$n = 2 + i$$

$$n - 2 = i$$

Reemplazamos por el caso base

$$P_{n-2} = T(n-(n-2)) + (n-2)n$$

$$= 2 + n^2 - 2n$$

$$= O(n^2)$$

$$T(n) = \begin{cases} 2, & n = 1 \\ T(n-1) + \frac{n}{2}, & n \geq 2 \end{cases}$$

Completar

$$T(n) = \begin{cases} 1 & n = 1 \\ 2T\left(\frac{n}{4}\right) + \sqrt{n}, & n \geq 2 \end{cases}$$

Completar

$$T(n) = \begin{cases} 1, n = 1 \\ 4T\left(\frac{n}{2}\right) + n^2, n \geq 2 \end{cases}$$

Buscamos un paso general

$$P_1 = 4T\left(\frac{n}{2}\right) + n^2$$

$$P_2 = 4[4T\left(\frac{n}{4}\right) + \left(\frac{n}{2}\right)^2] + n^2 = 16T\left(\frac{n}{4}\right) + 4\left(\frac{n}{2}\right)^2 + n^2 = 16T\left(\frac{n}{4}\right) + 2n^2$$

$$P_3 = 16[4T\left(\frac{n}{8}\right) + \left(\frac{n}{4}\right)^2] + 2n^2 = 64T\left(\frac{n}{8}\right) + 16\left(\frac{n}{4}\right)^2 + 2n^2 = 64T\left(\frac{n}{8}\right) + 3n^2$$

$$P_i = 4^i T\left(\frac{n}{2^i}\right) + i n^2$$

Reemplazar por el caso base

$$\frac{n}{2^i} = 1 \quad (2^i = 2^i)$$

$$n = 2^i$$

$$\log_2(n) = \log_2(2^i)$$

$$i = \log_2(n)$$

$$4^{\log_2(n)} T\left(\frac{n}{2^{\log_2(n)}}\right) + \log_2(n) n^2 = 4^{\log_2(n)} n^2 = (2^2)^{\log_2(n)} = (2^{\log_2(n)})^2 = n^2 + \log_2(n) n^2 = O(\log_2(n) n^2)$$