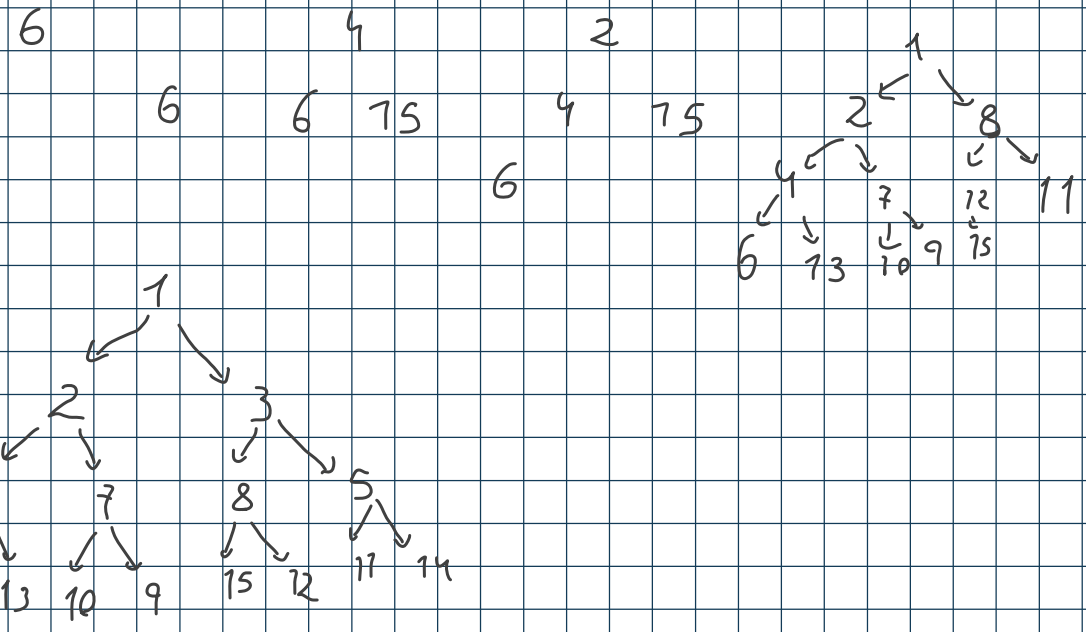
Heap

sábado, 8 de junio de 2024 16:06

Ejercicio 1

A partir de una heap inicialmente vacía, inserte de a uno los siguientes valores:

6, 4, 15, 2, 10, 11, 8, 1, 13, 7, 9, 12, 5, 3, 14



Array de heap

1	2	3	4	7	8	5	6	13	10	9	15	12	11	14

Ejercicio 2

a) ¿Cuántos elementos hay, al menos, en una heap de altura ***h***?

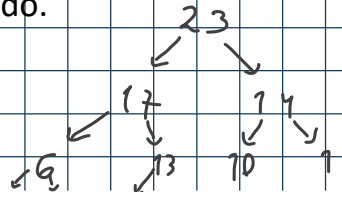
b) ¿Dónde se encuentra ubicado el elemento mínimo en una max-heap?

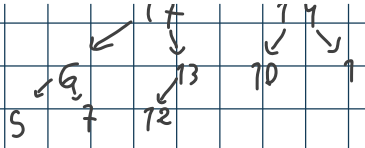
c) ¿El siguiente arreglo es una max-heap : [23, 17, 14, 6, 13, 10, 1, 5, 7, 12] ?

A_ Como una heap es un arbol completo de altura h, la cantidad minima de elementos es $(k^h + k - 2) / (k - 1)$

B_ Se encuentra almacenado en alguna de las hojas.

C_ No, no es una max heap ya que no cumple la propiedad de ordenamiento. El elemento 6 esta mal ordenado.





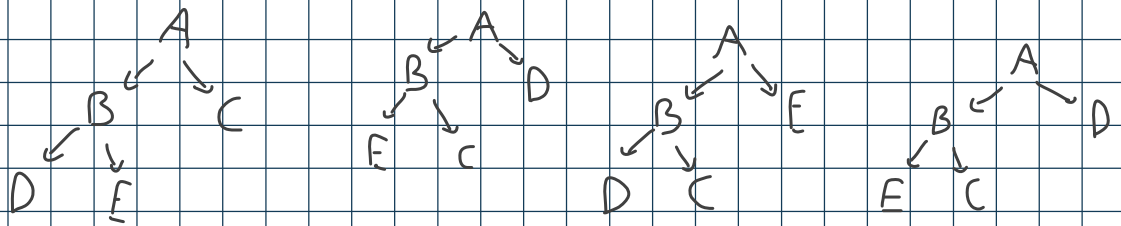
Ejercicio 3

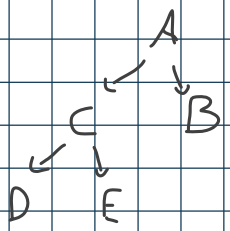
Dados los siguientes árboles, indique si representan una heap. Justifique su respuesta.

No es una heap, ya que ninguno de los 2 arboles cumple las propiedades estructurales ni de ordenamiento. El primero no cumple las propiedades estructurales(el arbol no se completa de izquierda a derecha) y el 2do no cumple la propiedad de orden.

Ejercicio 4

Dibuje todas las min-heaps posibles para este conjunto de claves: {A, B, C, D, E}



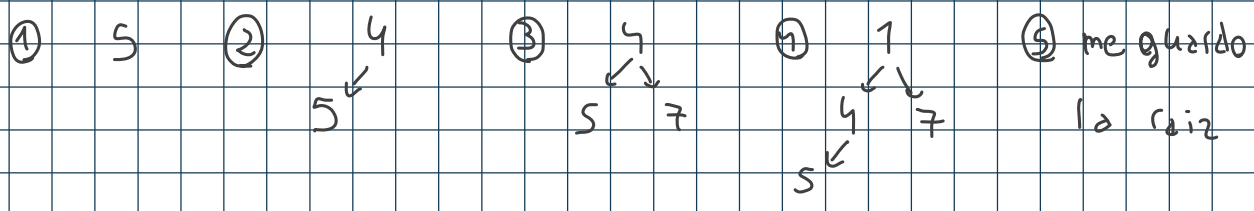


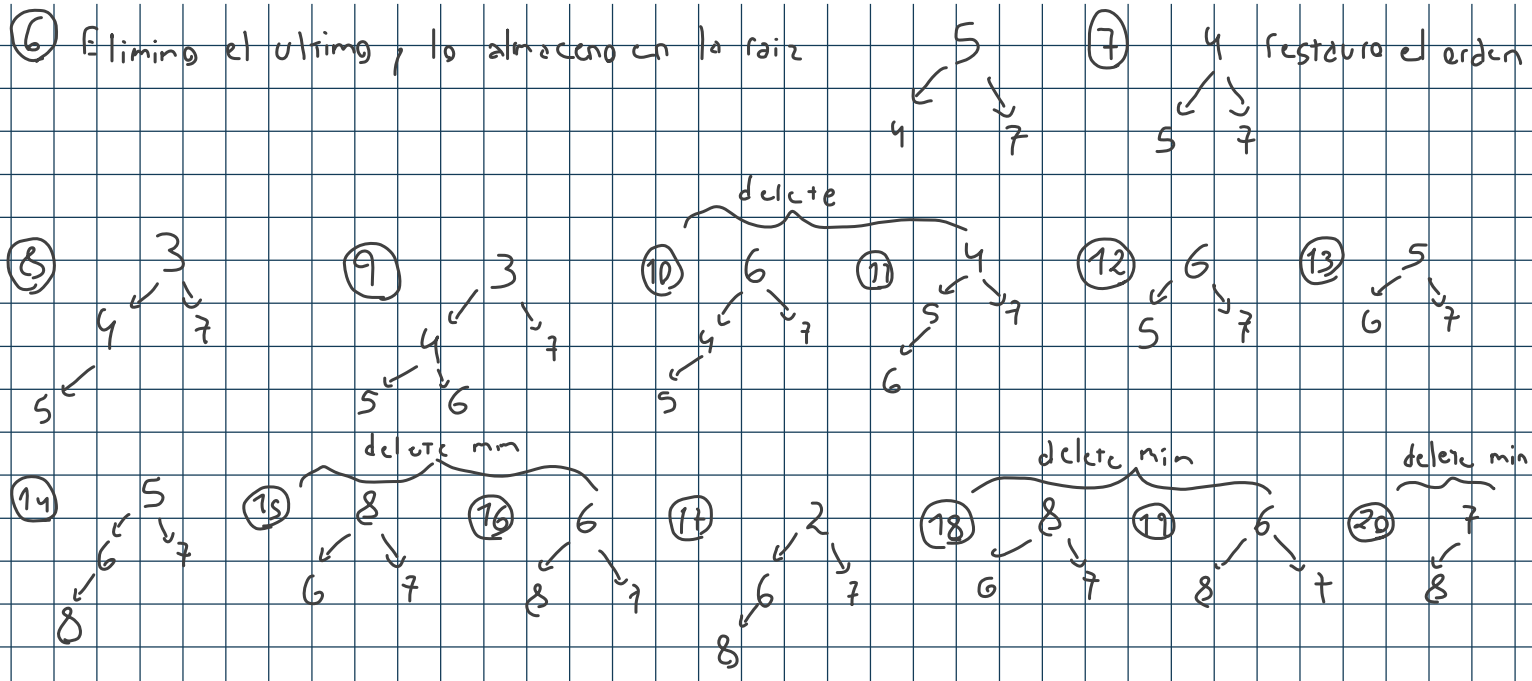
Me rindo 😞

Ejercicio 5

A partir de una min-heap inicialmente vacía, dibuje la evolución del estado de la heap al ejecutar las siguientes operaciones:

Insert(5), Insert(4), Insert(7), Insert(1), DeleteMin(), Insert(3), Insert(6), DeleteMin(), DeleteMin(), Insert(8), DeleteMin(), Insert(2), DeleteMin(), DeleteMin()

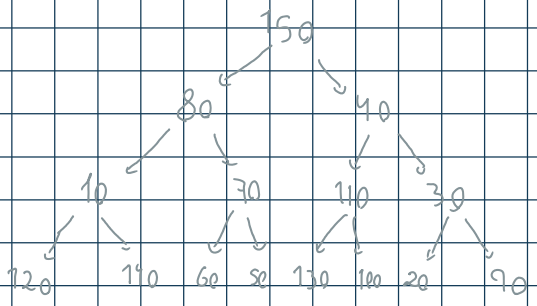


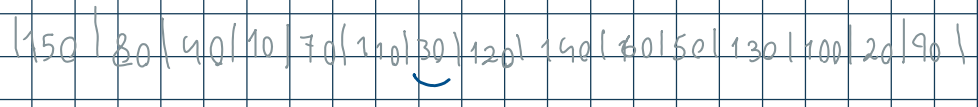


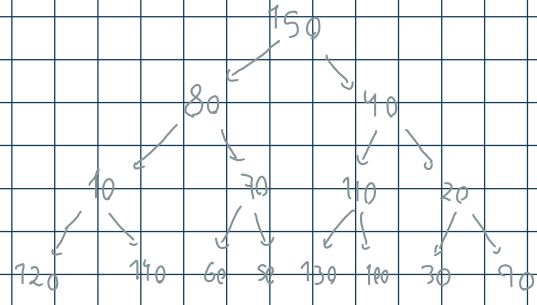
Ejercicio 6

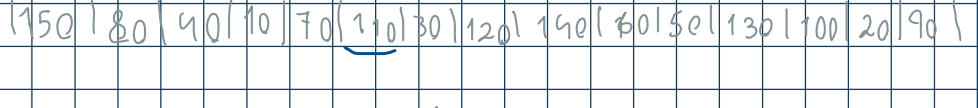
Aplique el algoritmo *BuildHeap*, para construir una min-heap en **tiempo lineal**, con los siguientes valores

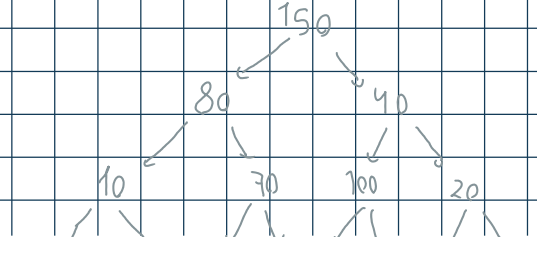
{150, 80, 40, 10, 70, 110, 30, 120, 140, 60, 50, 130, 100, 20, 90}

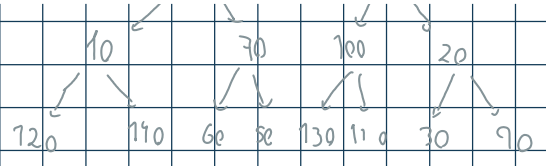


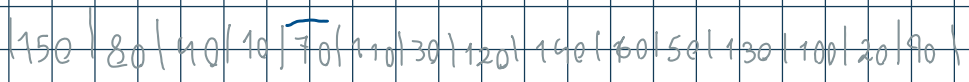


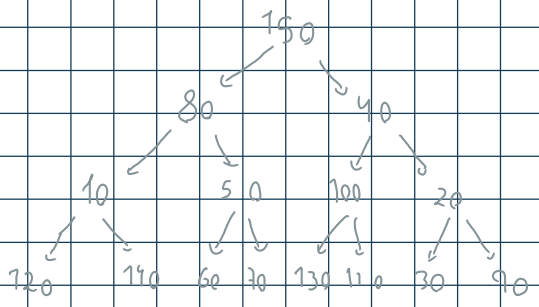


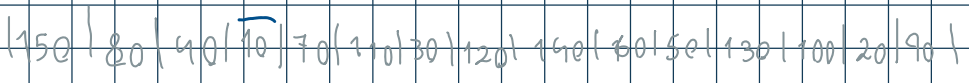


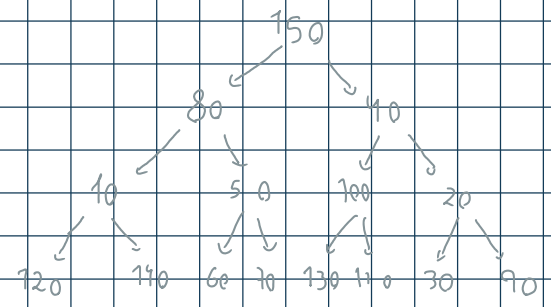


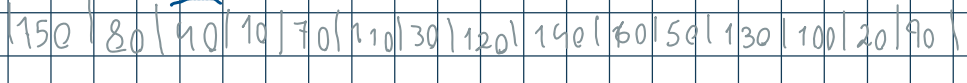


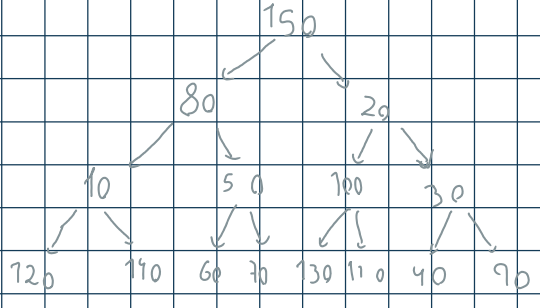


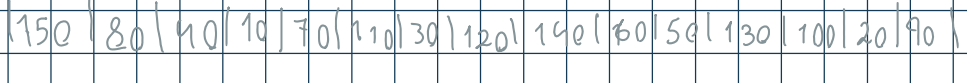


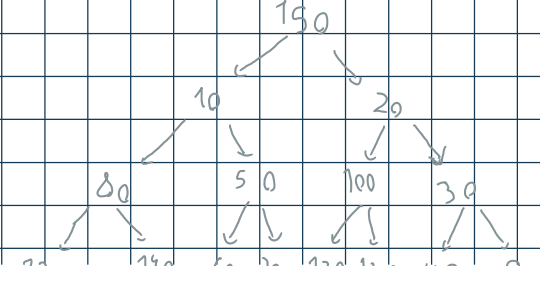


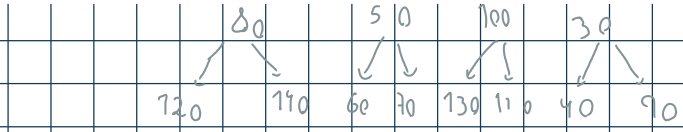

Queda igual

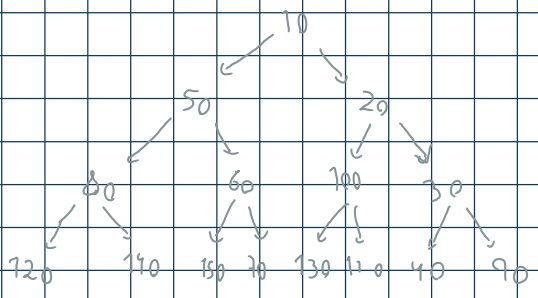










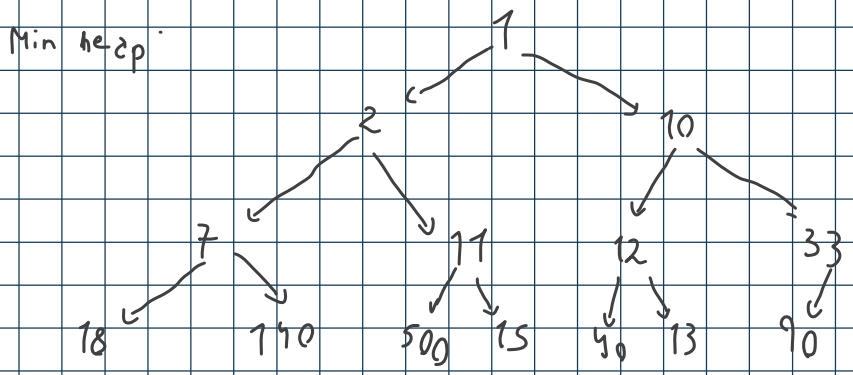


Ejercicio 7

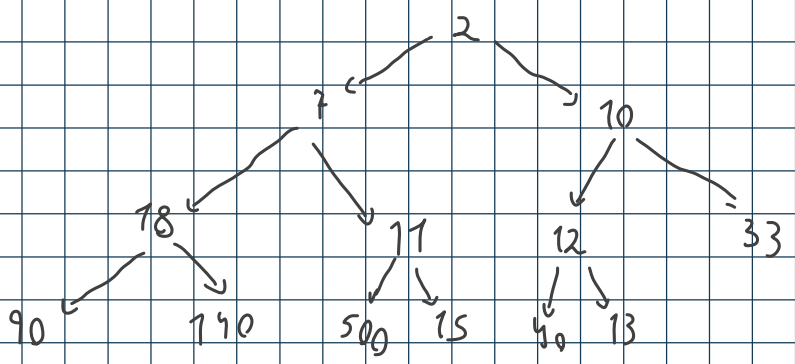
Aplique el algoritmo *HeapSort*, para ordenar descendentemente los siguientes elementos:

{15, 18, 40, 1, 7, 10, 33, 2, 140, 500, 11, 12, 13, 90}

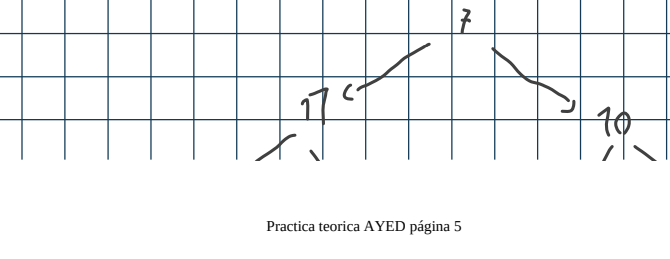
Muestre paso a paso la ejecución del algoritmo sobre los datos.

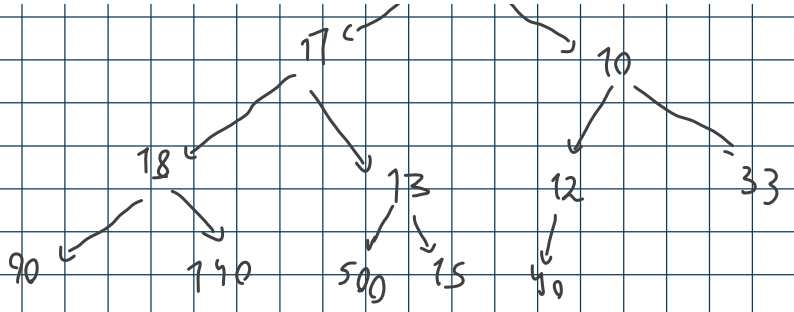


|1|2|10|7|11|12|33|18|140|500|15|40|13|90|

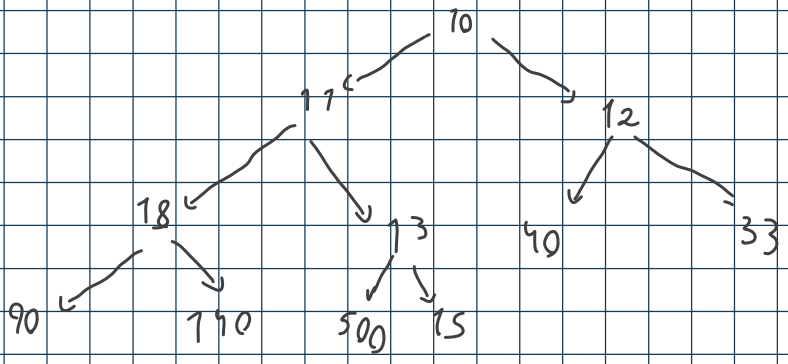


|1|2|10|7|11|12|33|18|140|500|15|40|13|90| 2 | 7



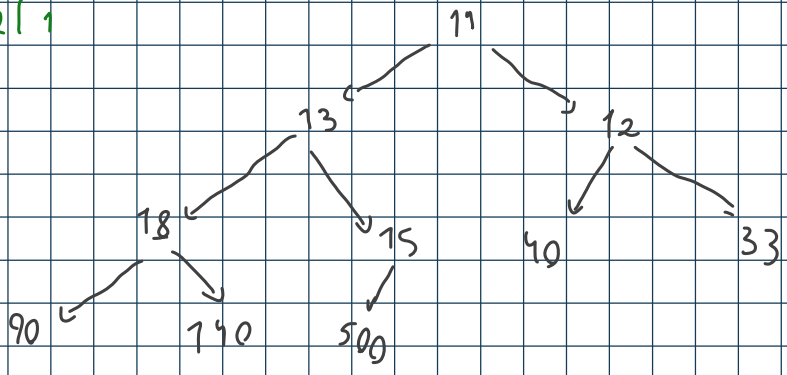


|1|2|10|7|11|12|33|18|140|500|15|40|13|90| 7|2|1



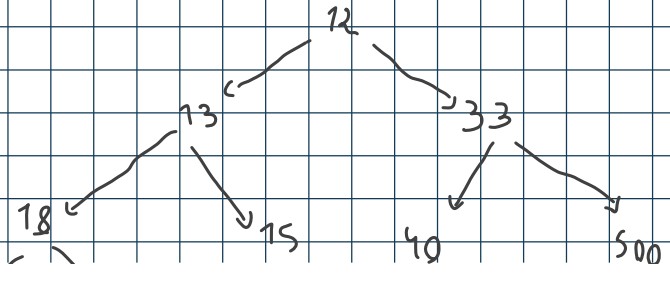
|1|2|10|7|11|12|33|18|140|500|15|40|13|90|

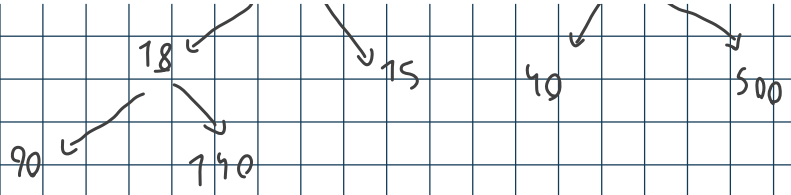
10|7|2|1



|1|2|10|7|11|12|33|18|140|500|15|40|13|90|

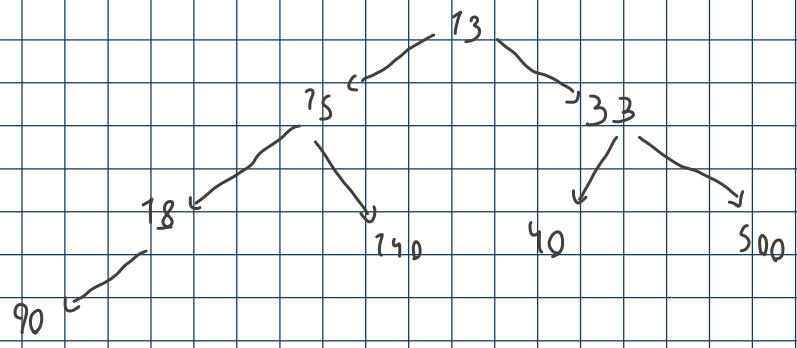
11|10|7|2|1





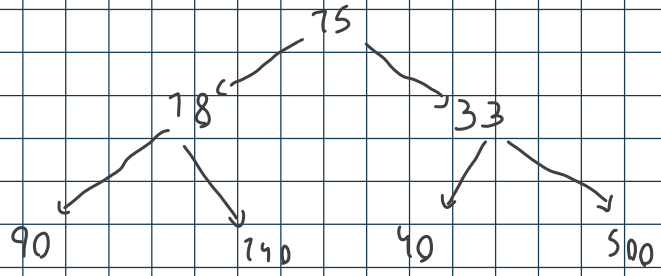
|1|2|10|7|11|12|33|18|140|500|15|40|1

12|11|10|7|2|1



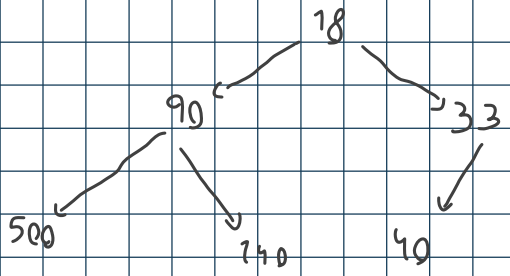
|1|2|10|7|11|12|33|18|140|500|15|40|1

13|12|11|10|7|2|1



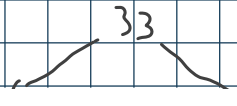
|1|2|10|7|11|12|33|18|140|500|15|40|1

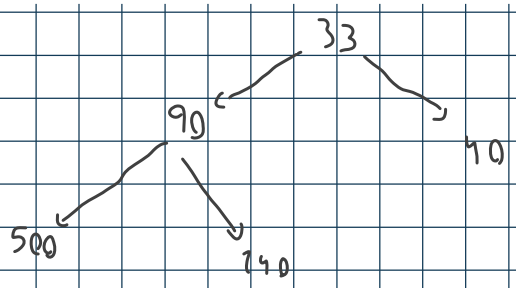
75|13|12|11|10|7|2|1



|1|2|10|7|11|12|33|18|140|500|15|40|1

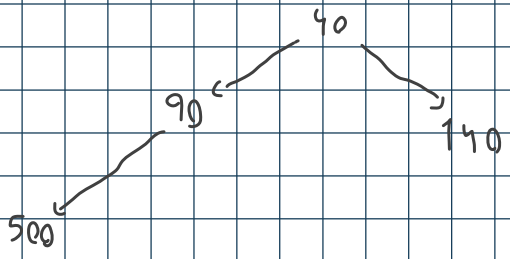
18|15|13|12|11|10|7|2|1





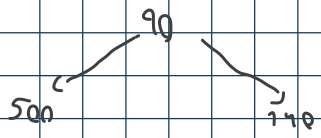
|1|2|10|7|11|12|33|18|140|500|15|40|1

33|18|15|13|12|11|10|7|2|1



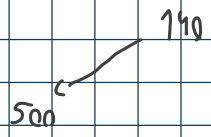
|1|2|10|7|11|12|33|18|140|500|15|40|1

40|33|18|15|13|12|11|10|7|2|1



|1|2|10|7|11|12|33|18|140|500|15|40|1

90|40|33|18|15|13|12|11|10|7|2|1



|1|2|10|7|11|12|33|18|140|500|15|40|1

140|90|40|33|18|15|13|12|11|10|7|2|1

500

500|140|90|40|33|18|15|13|12|11|10|7|2|1

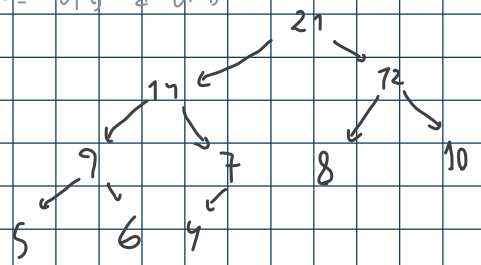
Ejercicio 8

Construir una max-heap binaria con los siguientes datos:

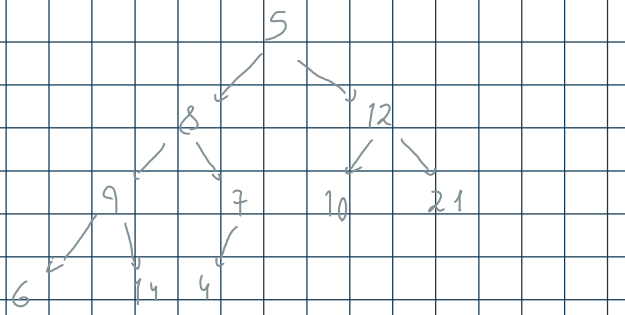
{5, 8, 12, 9, 7, 10, 21, 6, 14, 4}

a) Insertándolos de a uno
b) Usando el algoritmo BuildHeap

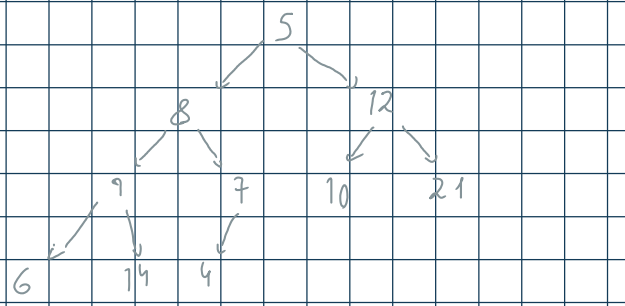
a_ Img a uno



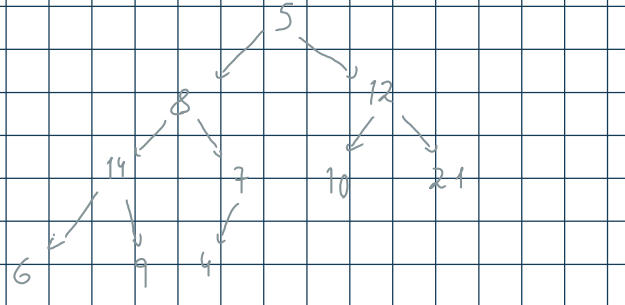
b_ BuildHeap



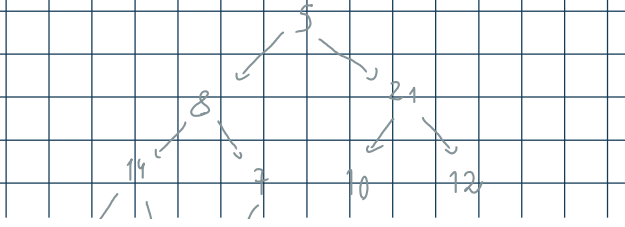
| 5 | 8 | 12 | 9 | 7 | 10 | 21 | 6 | 14 | 4 |

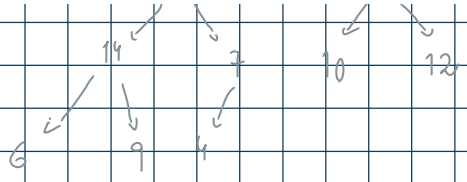


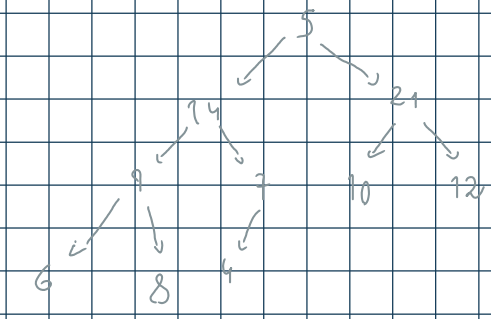
| 5 | 8 | 12 | 9 | 7 | 10 | 21 | 6 | 14 | 4 |

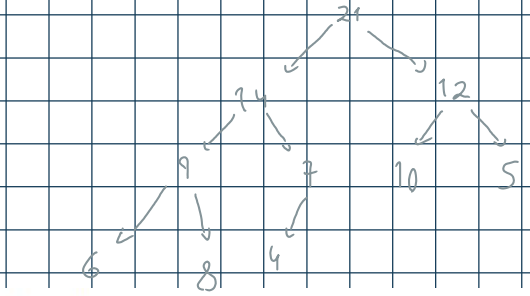


| 5 | 8 | 12 | 9 | 7 | 10 | 21 | 6 | 14 | 4 |







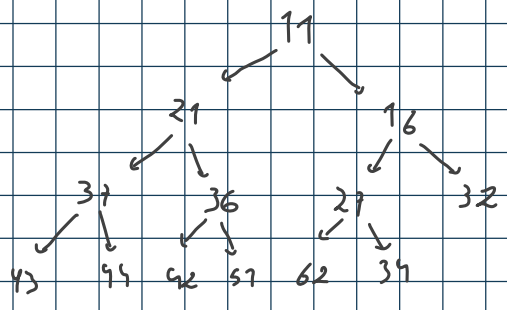


Ejercicio 9

Suponga que una heap que representa una cola de prioridades está almacenada en el arreglo A (se comienza de la posición A[1]). Si insertamos la clave 16, ¿en qué posición quedará?

i:	1	2	3	4	5	6	7	8	9	10	11	12
A[i]:	11	21	27	37	36	34	32	43	44	42	51	62

(a) A[2] (b) A[3] (c) A[6] (d) A[7] (e) A[12]



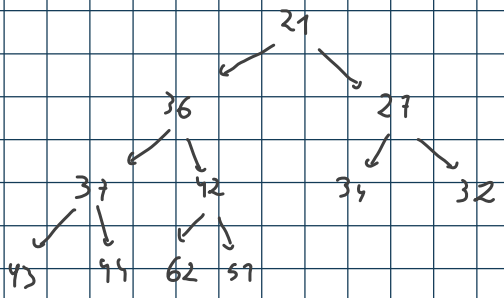
Opcion b = A[3]

Ejercicio 10

Suponga que una heap que representa una cola de prioridades está almacenada en el arreglo A (se comienza de la posición A[1]). Si aplica un delete-min, ¿en qué posición quedará la clave **62**?

i:	1	2	3	4	5	6	7	8	9	10	11	12
A[i]:	11	21	27	37	36	34	32	43	44	42	51	62

(a) A[1] (b) A[2] (c) A[10] (d) A[11] (e) A[12]



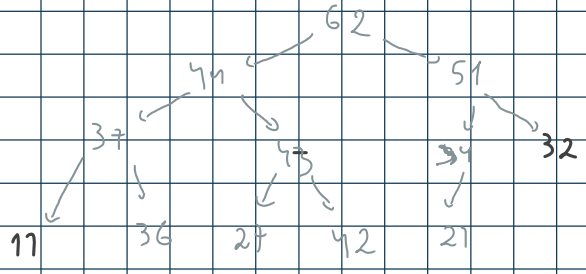
Opcion c = A[10]

Ejercicio 11

a) Ordenar en forma creciente los datos del ejercicio anterior, usando el algoritmo HeapSort.
b) ¿Cuáles serían los pasos a seguir si se quiere ordenar en forma decreciente?

Elementos: 11 21 27 37 36 34 32 43 44 42 51 62
Como me piden ordenar de forma creciente, primero realizo maxHeap.

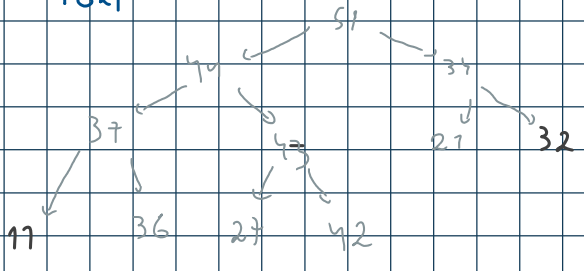
Maxheap:



Utilizo el algoritmo HeapSort

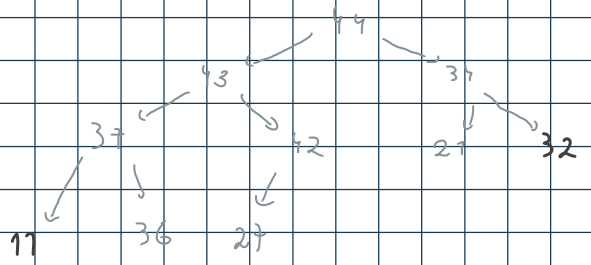
|62|44|51|37|43|34|32|11|36|27|42|21|

|62|



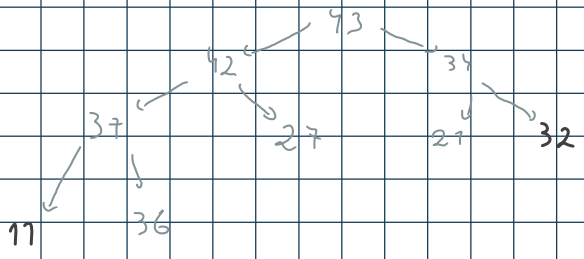
|62|44|51|37|43|34|32|11|36|27|42|21|

|51|62|



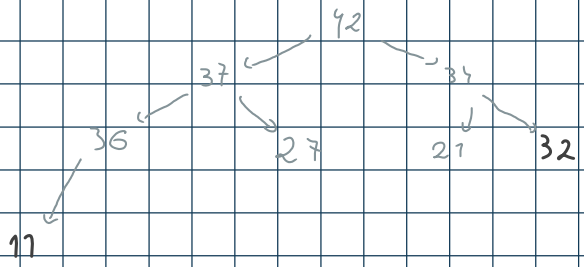
|62|44|51|37|43|34|32|11|36|27|42|21|

|44|51|62|



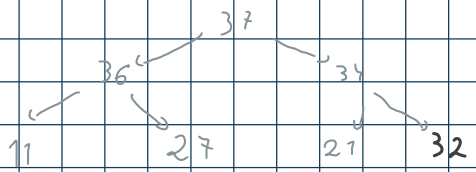
|62|44|51|37|43|34|32|11|36|27|42|21|

|43|44|51|62|



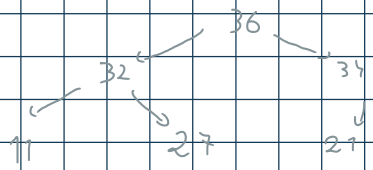
|62|44|51|37|43|34|32|11|36|27|42|21|

|42|43|44|51|62|



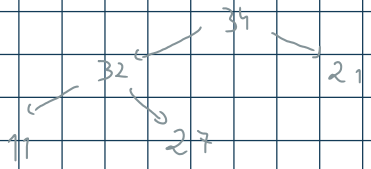
|62|44|51|37|43|34|32|11|36|27|42|21|

|37|42|43|44|51|62|



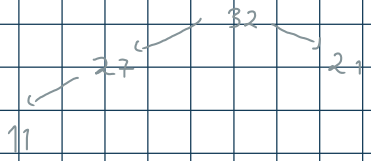
|62|44|51|37|43|34|32|11|36|27|42|21|

|36|37|42|43|44|51|62|



|62|44|51|37|43|34|32|11|36|27|42|21|

|34|36|37|42|43|44|51|62|



|62|44|51|37|43|34|32|11|36|27|42|21|

|32|34|36|37|42|43|44|51|62|

|62|44|51|37|43|34|32|11|36|27|42|21|

|27|32|34|36|37|42|43|44|51|62|

|62|44|51|37|43|34|32|11|36|27|42|21|

|27|27|32|34|36|37|42|43|44|51|62|

11

|62|44|51|37|43|34|32|11|36|27|42|21|

|11|27|27|32|34|36|37|42|43|44|51|62|

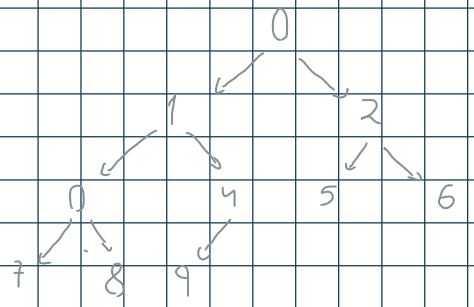
b_ Si se quiere ordenar de forma decreciente, primero deberiamos crear una min heap y luego aplicarle el algoritmo heap sort.

Ejercicio 12

¿Cuáles de los siguientes arreglos representan una max-heap, min-heap o ninguna de las dos?

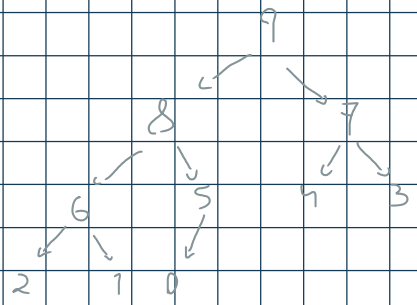
- arreglo 1: 0 1 2 0 4 5 6 7 8 9
- arreglo 2: 9 8 7 6 5 4 3 2 1 0
- arreglo 3: 5 5 5 6 6 6 6 7 7 1
- arreglo 4: 9 3 9 2 1 6 7 1 2 1
- arreglo 5: 8 7 6 1 2 3 4 2 1 2

1.



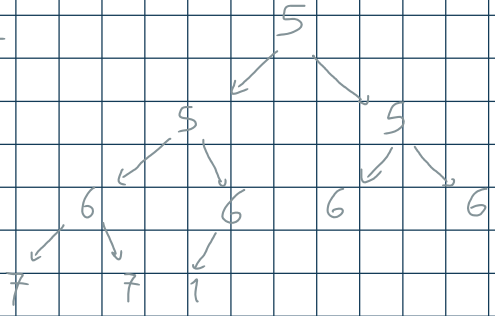
Ninguno de los dos.

2.



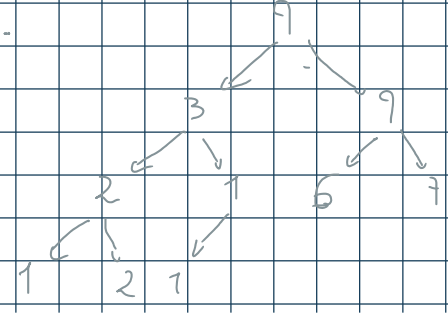
Max heap

3.



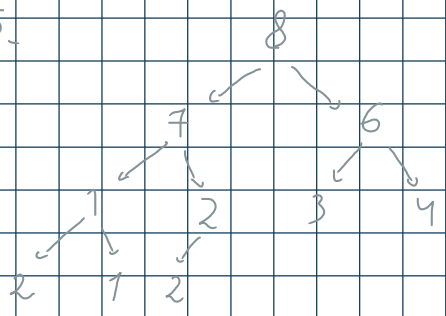
- Ninguno

4.



- Ninguno

5.



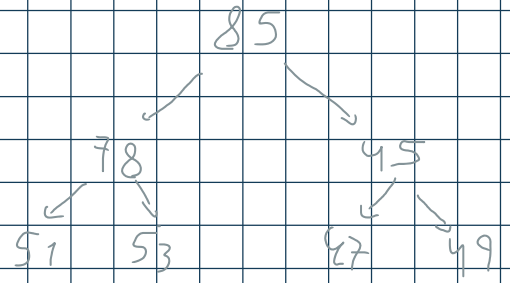
- Ninguno

Ejercicio 13

Un arreglo de 7 enteros se ordena ascendentemente usando el algoritmo *HeapSort*. Luego de la fase inicial del algoritmo (la construcción de la heap), ¿cuál de los siguientes es un posible orden del arreglo?

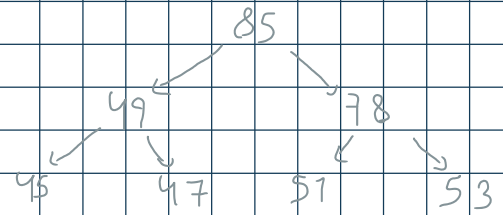
(a) 85 78 45 51 53 47 49
(b) 85 49 78 45 47 51 53
(c) 85 78 49 45 47 51 53
(d) 45 85 78 53 51 49 47
(e) 85 51 78 53 49 47 45

a.



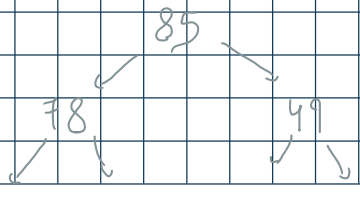
No cumple con maxHeap

b.



Cumple con maxHeap.

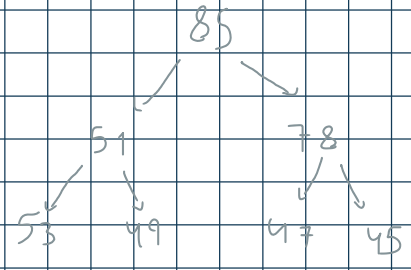
c.



No cumple con maxHeap.

d. No cumple con maxHeap.

e.



No cumple con maxHeap.

Opción b ✓.

Ejercicio 14

En una Heap, ¿para un elemento que está en la posición i su hijo derecho está en la posición……?

(a) | i/2 |
(b) 2*i
(c) 2*i + 1
(d) Ninguna de las anteriores

Opción c. 2*i+1

Ejercicio 15

¿Siempre se puede decir que un árbol binario lleno es una Heap?

(a) Sí (b) No

No debido a que no todo arbol lleno cumple con las propiedades de una heap. Para cumplir debe ser un arbol binario completo y cumplir con las propiedades estructurales y las propiedades de orden.

Ejercicio 16

La operación que agrega un elemento a la heap que tiene ***n*** elementos, en el peor caso es de ……

(a) O(n)
(b) O(n log n)
(c) O(log n)
(d) Ninguna de las otras opciones

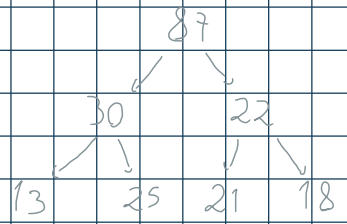
c. O(log n)

Ejercicio 17

Se construyó una Máx-Heap con las siguientes claves: 13, 21, 87, 30, 25, 22, 18. ¿Cuál de las siguientes opciones corresponde al resultado de realizar la construcción insertando las claves **una a una**?

(a) 87, 30, 25, 22, 21, 18, 13
(b) 87, 30, 22, 21, 25, 13, 18
(c) 87, 30, 25, 13, 22, 18, 21
(d) 87, 30, 22, 13, 25, 21, 18

Max heap.

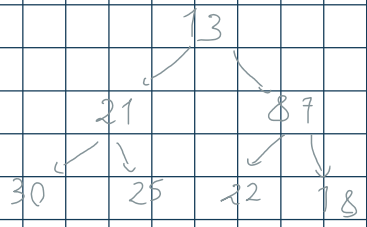


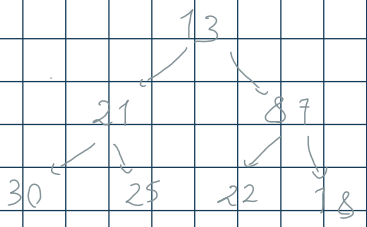
Opción d ✓.

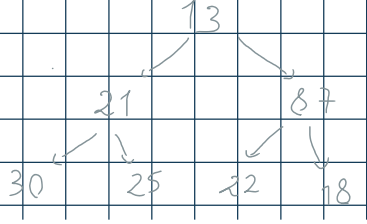
Ejercicio 18

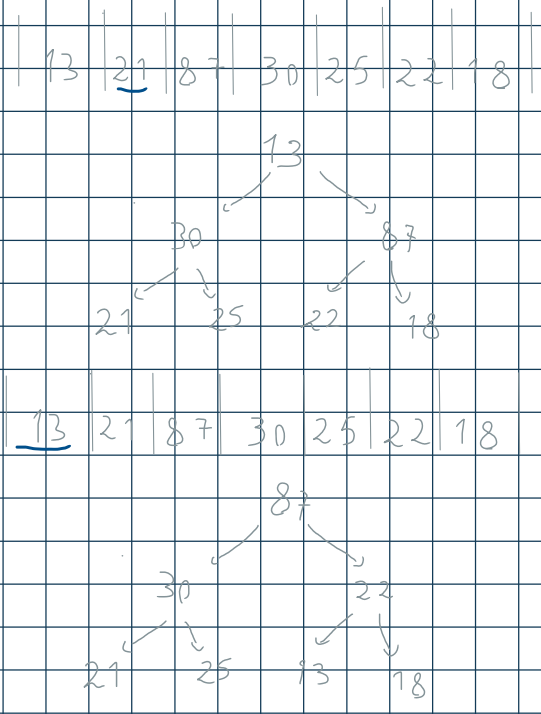
Se construyó una Máx-Heap con las siguientes claves: 13, 21, 87, 30, 25, 22, 18. ¿Cuál de las siguientes opciones corresponde al resultado de realizar la construcción aplicando el algoritmo **Build-Heap**?

(a) 87, 30, 25, 22, 21, 18, 13
(b) 87, 30, 22, 21, 25, 13, 18
(c) 87, 30, 25, 13, 22, 18, 21
(d) 87, 30, 22, 13, 25, 21, 18









Opcion b ✓

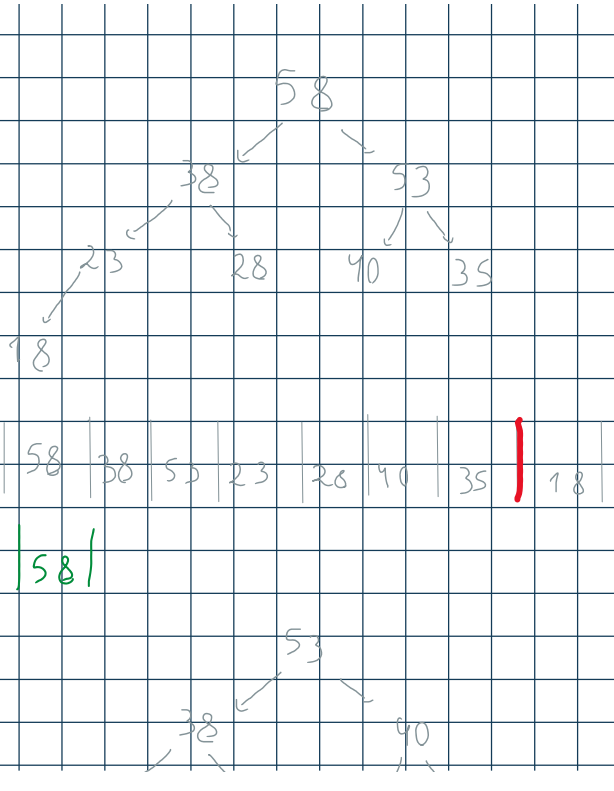
Ejercicio 19

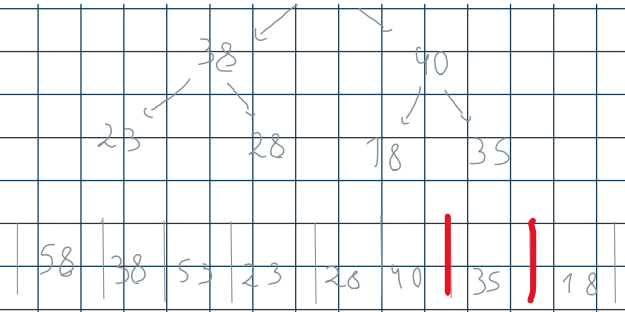
El algoritmo HeapSort consta de dos etapas:
1) se construye una heap y
2) se realizan los intercambios necesarios para dejar ordenados los datos.
Asuma que la heap ya está construida y es la siguiente:

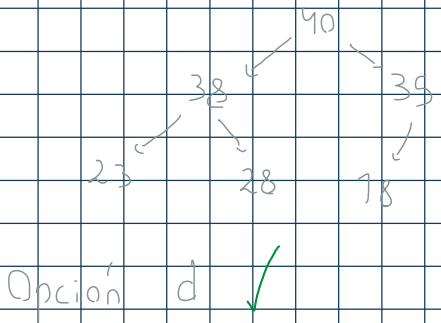
58 38 53 23 28 40 35 18

¿Cómo quedan los datos en el arreglo después de ejecutar sólo 2 pasos de la segunda etapa del Heapsort?

(a) 40 38 23 28 35 18 53 58
(b) 53 38 40 23 28 18 35 58
(c) 40 38 23 35 28 18 53 58
(d) 40 38 35 23 28 18 53 58







Ejercicio 20

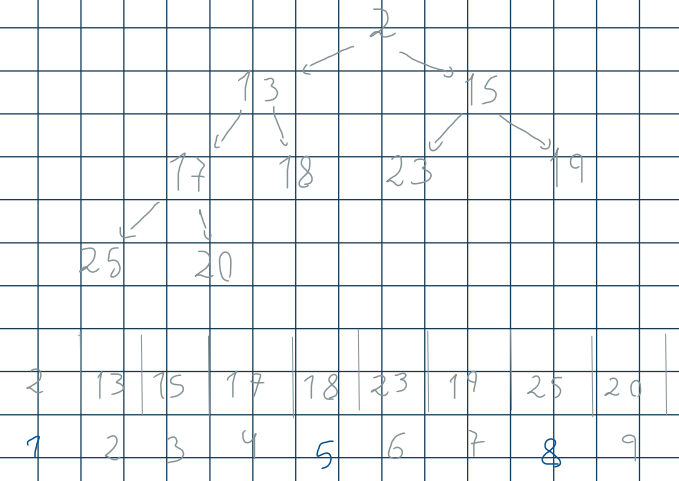
Dada la Min-Heap 3, 8, 5, 15, 10, 7, 19, 28, 16, 25, 12. ¿En qué **posición** está ubicado el hijo derecho de la clave 15?

(a) 7
(b) 8
(c) 9
(d) 10

En una heap el hijo derecho esta en la posicion 2*i+1 por lo tanto el hijo derecho de la clave 15 se encuentra en la 9 --> 2*4+1 = 9

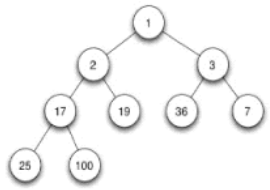
Ejercicio 21

Construya una min-heap con las siguientes claves: 15, 25, 23, 13, 18, 2, 19, 20, 17 insertándose una a una. Indique en qué posiciones quedaron ubicadas las claves: **2**, **18** y **25**.

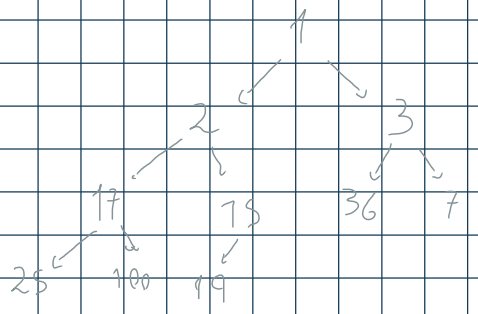


Ejercicio 22

Luego de insertar la clave **15** en la siguiente min-heap, ¿cuántas de las claves que ya estaban en la heap han mantenido su lugar (es decir, ocupan en la min-heap resultante la misma posición que ocupaban antes de la inserción)?



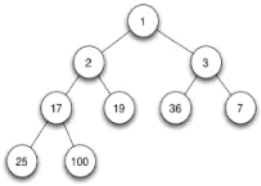
a) Ninguna
b) Seis
c) Ocho
d) Nueve



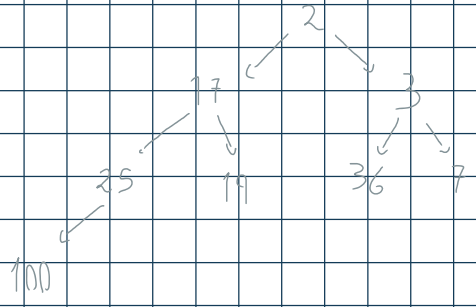
Opción c = Ocho

Ejercicio 23

Luego de una operación de borrado del mínimo en la siguiente min-heap, ¿cuántas claves han cambiado de lugar (es decir, ocupan en la min-heap resultante un lugar diferente al que ocupaban en la min-heap antes del borrado) ? (No contar la clave borrada, ya que no pertenece más a la heap)



a) Ninguno
b) Dos
c) Tres
d) Cuatro



Opcion d Cuatro