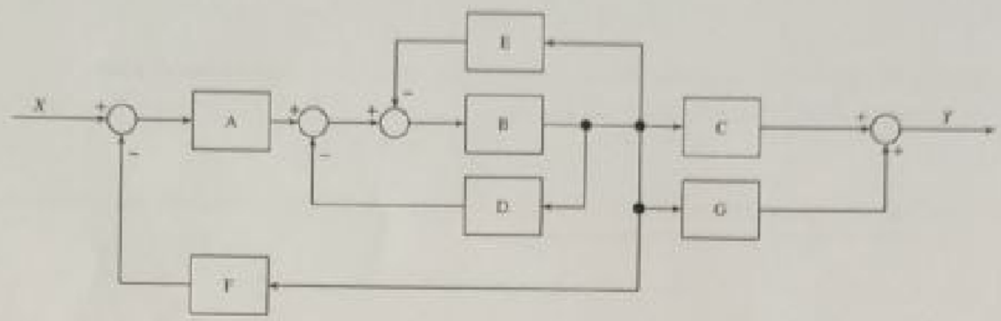
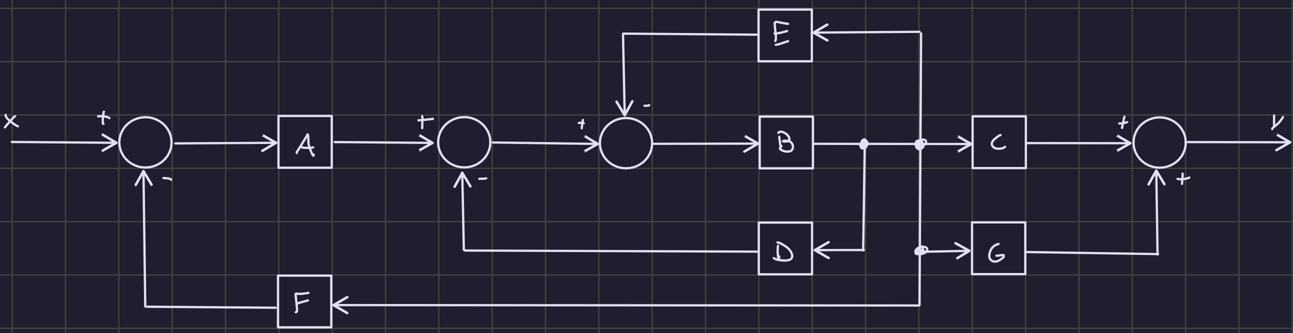
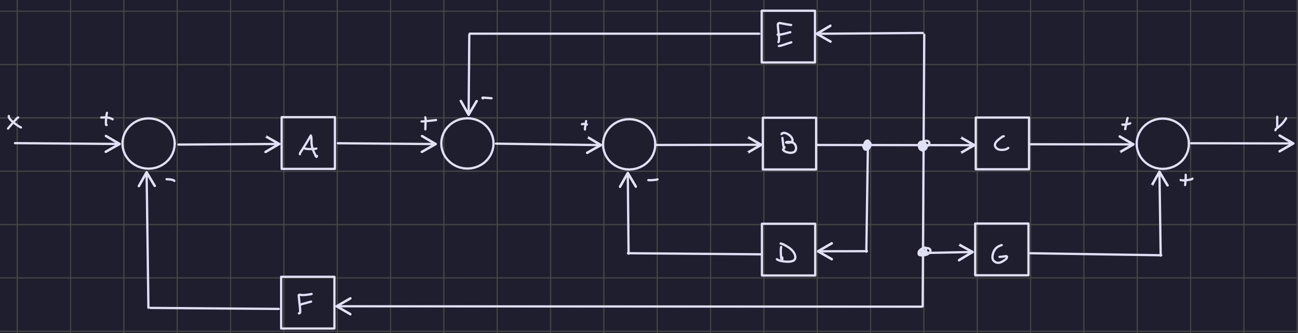
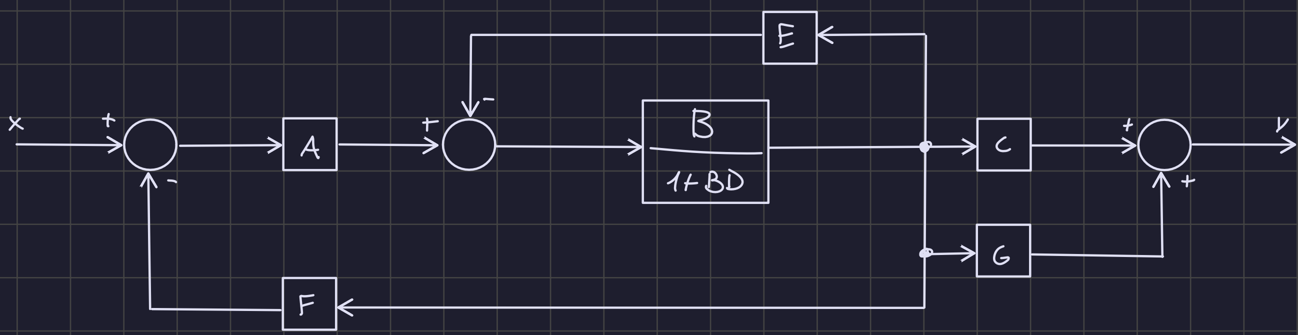
Esercizio 1 (20 punti)

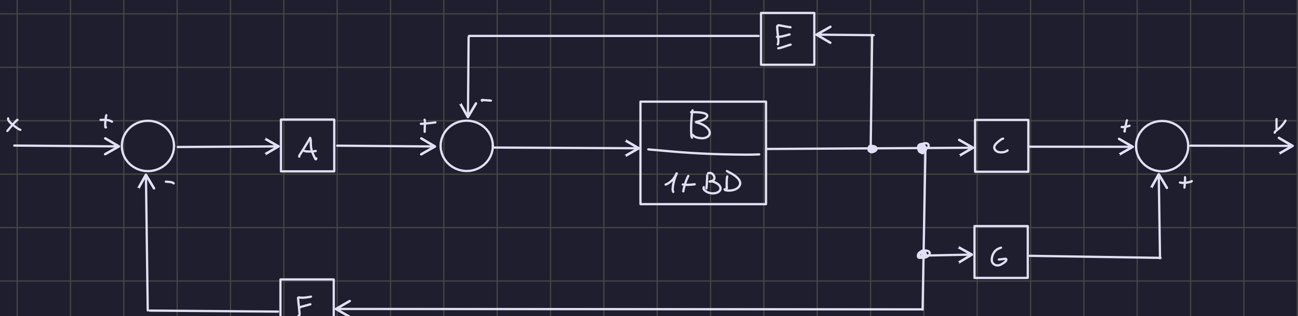
Calcolare la funzione di trasferimento del seguente schema a blocchi:



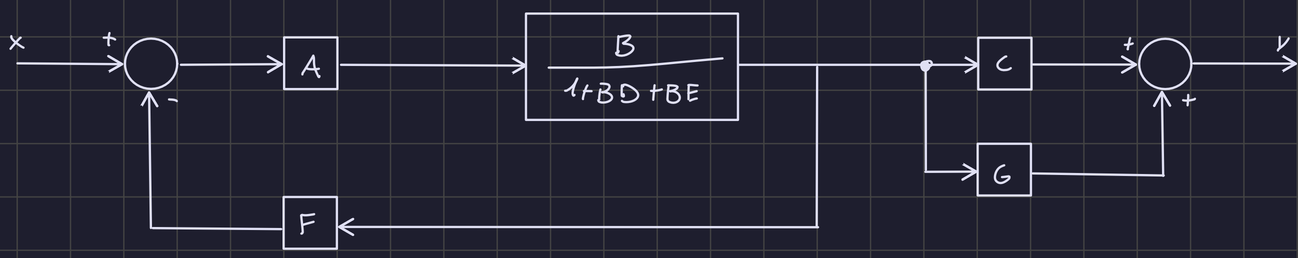


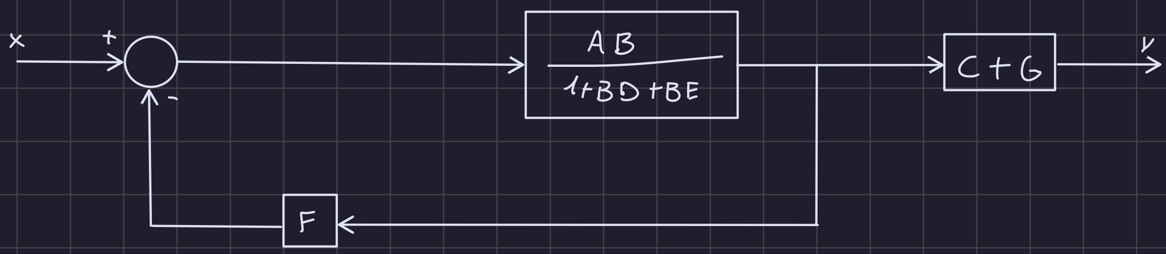




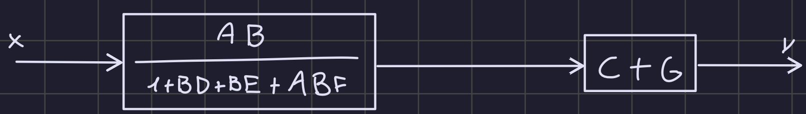


$$\frac{\frac{B}{1+BD}}{1+\frac{BE}{1+BD}} = \frac{\frac{B}{1+BD}}{\frac{1+BD+BE}{1+BD}} = \frac{B}{\cancel{1+BD}}\cdot\frac{\cancel{1+BD}}{1+BD+BE} = \frac{B}{1+BD+BE}$$





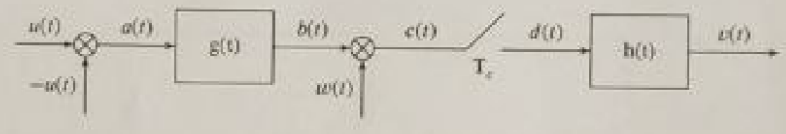
$$\frac{\frac{AB}{1+BD+BE}}{1+\frac{ABF}{1+BD+BE}} = \frac{\frac{AB}{1+BD+BE}}{\frac{1+BD+BE+ABF}{1+BD+BE}} = \frac{AB}{\cancel{1+BD+BE}}\cdot\frac{\cancel{1+BD+BE}}{1+BD+BE+ABF} = \frac{AB}{1+BD+BE+ABF}$$



$$y = \frac{AB(C+G)}{1+BD+BE+ABF}$$

Esercizio 3 (35 punti)

Dato il seguente schema a blocchi,



$u(t) = 4\cos(20\pi t)$ $\quad g(t) = -1200\,\mathrm{sinc}(60t) + 400\,\mathrm{sinc}(20t)$

$w(t) = 15\,\mathrm{sinc}(60t)$ $\quad h(t) = \frac{1}{60}\,\mathrm{sinc}(10t)$ $\quad T_c = \frac{1}{120}\,s$

(a) (5 punti) Si diano le trasformate di Fourier dei segnali e dei filtri dati.

(b) (25 punti) Si ricavi per via grafica lo spettro dell'uscita $v(t)$ e se ne dia l'espressione in tempo.

(c) (5 punti) Si dica se dal segnale $d(t)$ si può ricostruire il segnale $c(t)$ e se ne dia il motivo.

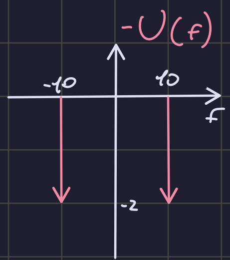
$$u(t) = 4\cos(2\pi \cdot 10t) \xrightarrow{F} U(f) = 2\,\delta(f-10) + 2\,\delta(f+10)$$

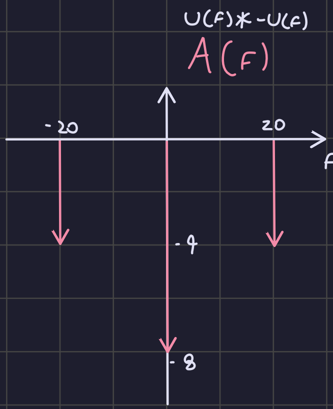
$$w(t) = \frac{1}{4} \cdot 60\,\mathrm{sinc}(60t) \xrightarrow{F} W(f) = \frac{1}{4}\,\Pi\left(\frac{f}{60}\right)$$

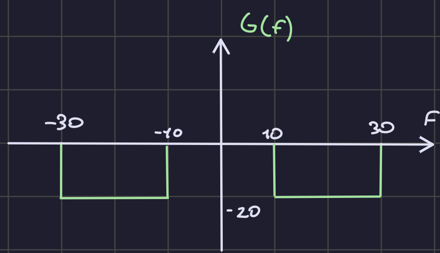
$$g(t) = -20 \cdot 60\,\mathrm{sinc}(60t) + 20 \cdot 20\,\mathrm{sinc}(20t) \xrightarrow{F} G(f) = -20\,\Pi\left(\frac{f}{60}\right) + 20\,\Pi\left(\frac{f}{20}\right)$$

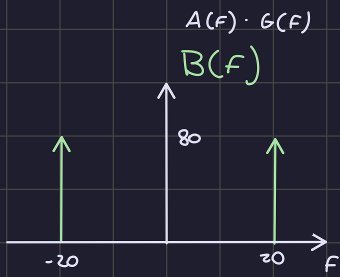
$$h(t) = \frac{1}{600}\,10\,\mathrm{sinc}(10t) \xrightarrow{F} H(f) = \frac{1}{600}\,\Pi\left(\frac{f}{10}\right)$$

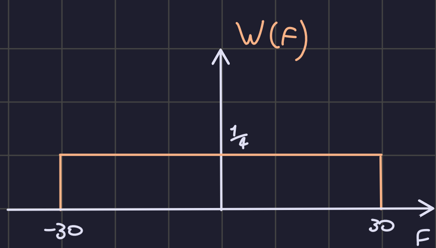
$$T_c = \frac{1}{120} \rightarrow f_c = \frac{1}{T_c} = 120\,Hz$$

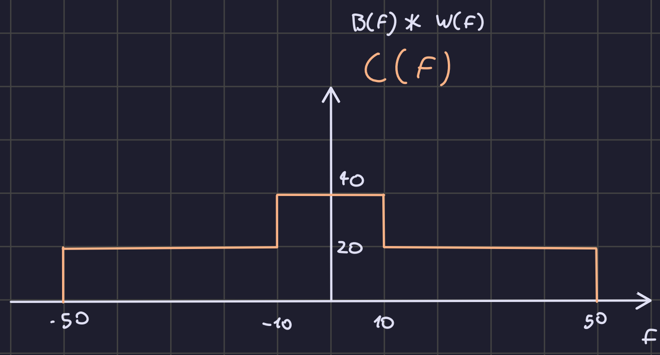


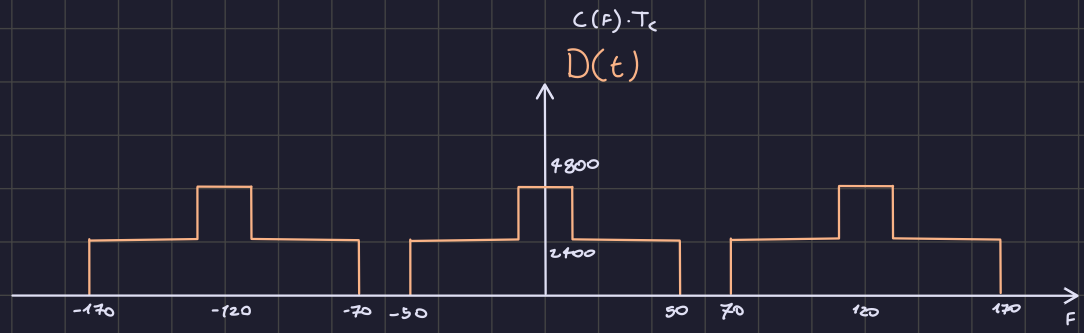


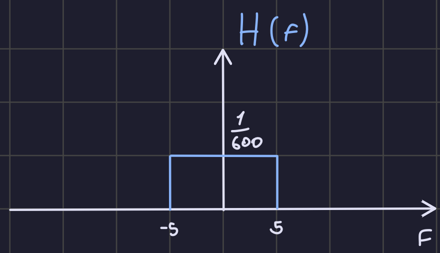


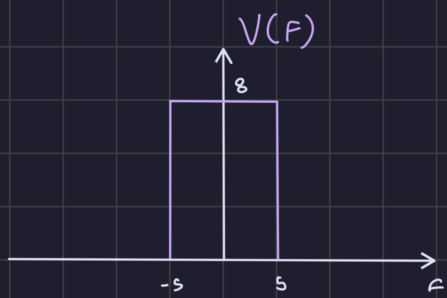












$$V(f) = 8\,\Pi\left(\frac{f}{10}\right)$$

$$\downarrow \mathcal{F}^{-1}$$

$$v(t) = 8 \cdot 10\,\mathrm{sinc}(10t) = 80\,\mathrm{sinc}(10t)$$

Esercizio 4 (15 punti)

Dato il sistema descritto dalla seguente funzione di trasferimento:

$$H(s) = \frac{5 \cdot (s-3)}{s^2 - 5s + 4}$$

(a) (5 punti) Discuterne la stabilità (asintotica e BIBO).

(b) (10 punti) Se ne dia l'antitrasformata di Laplace.

a) $$H(s) = \frac{5 \cdot (s-3)}{(s-1)(s-4)}$$

Il sistema è asintoticamente stabile se tutte le radici del polinomio caratteristico dell'uscita hanno parte reale negativa, in questo caso si ha $\lambda_1 = 1$ e $\lambda_2 = 4$, quindi il sistema non è asintoticamente stabile.
Il sistema è BIBO stabile se nella funzione di trasferimento le radici con parte reale negativa si semplificano, in questo caso non si semplificano quindi il sistema è instabile.

b) $$\frac{5s - 15}{(s-1)(s-4)} = \frac{A}{s-1} + \frac{B}{s-4} = \frac{s(A+B) - 4A - B}{(s-1)(s-4)}$$

$$\begin{cases} A+B=5 \\ -4A-B=-15 \end{cases} \rightarrow \begin{cases} A=5-B \\ -20-4B-B=-15 \end{cases} \rightarrow \begin{cases} A=5-B \\ -5B=5 \end{cases} \rightarrow \begin{cases} A=6 \\ B=-1 \end{cases}$$

$$H(s) = \frac{6}{s-1} - \frac{1}{s-4}$$

$$\Big\downarrow \mathcal{L}^{-1}$$

$$h(t) = \left(6e^{t} - e^{4t}\right)\delta_{-1}(t)$$

Esercizio 2 (30 punti)

Riportare la funzione in forma di Bode, disegnare i diagrammi di modulo e fase delle singole componenti elementari e il diagramma globale della seguente funzione di trasferimento:

$$G(s) = \frac{5s(s^4 - s^3)(s^2 - 5s + 25)}{s^6(s^2 + 2s - 3)(s+3)}.$$

$$G(s) = \frac{5s(s^4 - s^3)(s^2 - 5s + 25)}{s^6(s^2 + 2s - 3)(s+3)}$$

$$= -5 \frac{s^4(1-s)\, 25\left(1 - \frac{1}{5}s + \frac{1}{25}s^2\right)}{s^6(s+3)^2(s-1)}$$

$$= -125 \frac{s^4(1-s)\left(1 - \frac{1}{5}s + \frac{1}{25}s^2\right)}{s^6 \cdot 9\left(1 + \frac{1}{3}s\right)^2 \cdot (-1)(1+s)}$$

$$= \frac{125}{9} \frac{s^4(1-s)\left(1 - \frac{1}{5}s + \frac{1}{25}s^2\right)}{s^6\left(1 + \frac{1}{3}s\right)^2(1+s)}$$

$$K_b = \frac{125}{9} \approx 14$$

$Z_n = s^4$ $\quad Z_r = (1-s)^1$ $\quad Z_{cc} = \left(1 - \frac{1}{5}s + \frac{1}{25}s^2\right)$

$P_n = s^{-6}$ $\quad P_{r1} = \left(1 + \frac{1}{3}s\right)^{-2}$ $\quad P_{r2} = (1+s)^{-1}$

$A = K_b$

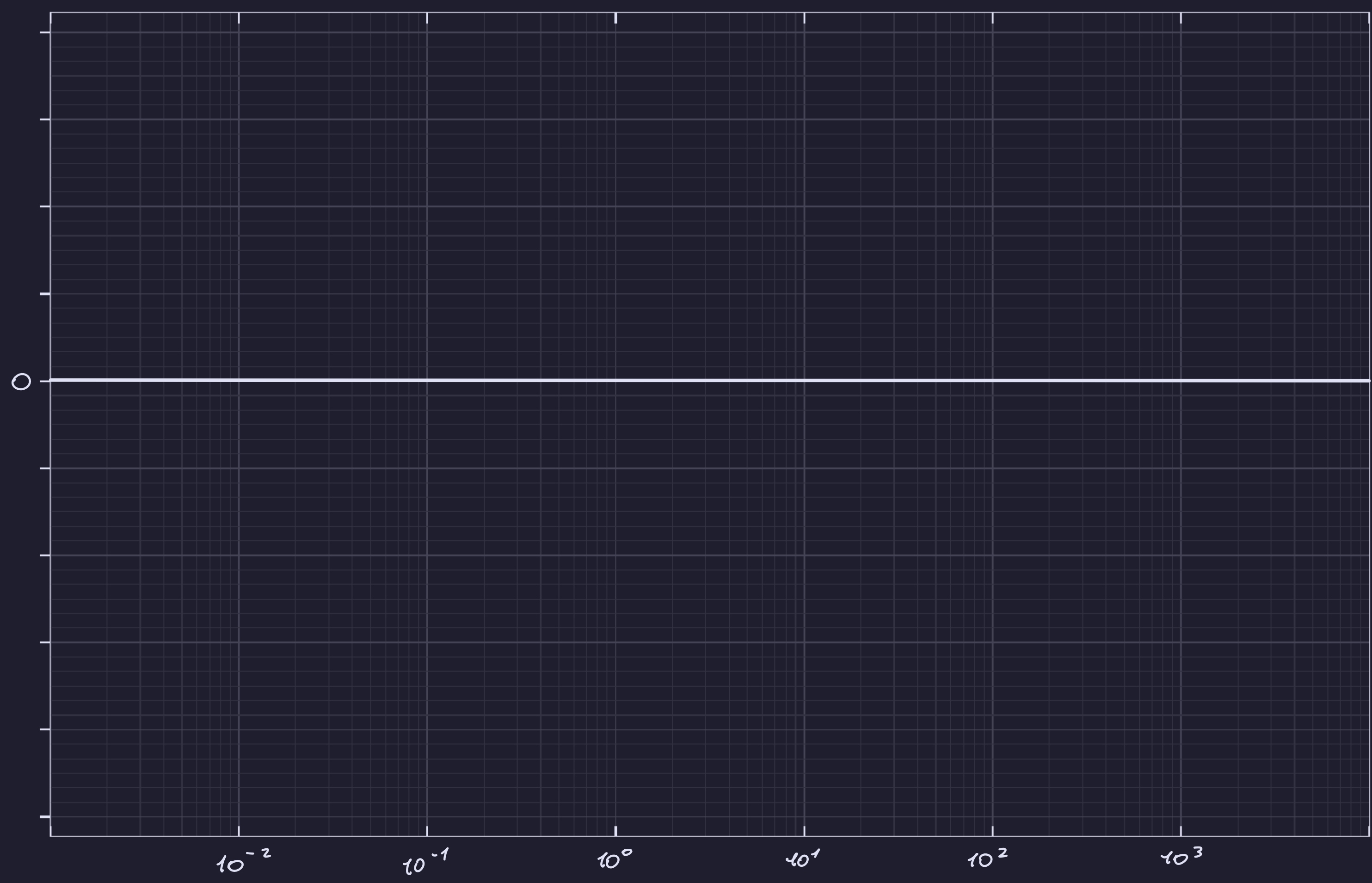
$\phi = \begin{cases} 0 & K_b > 0 \\ -180 & K_b < 0 \end{cases}$

Diagramma di Bode

$K_b = 14$

Ampiezza

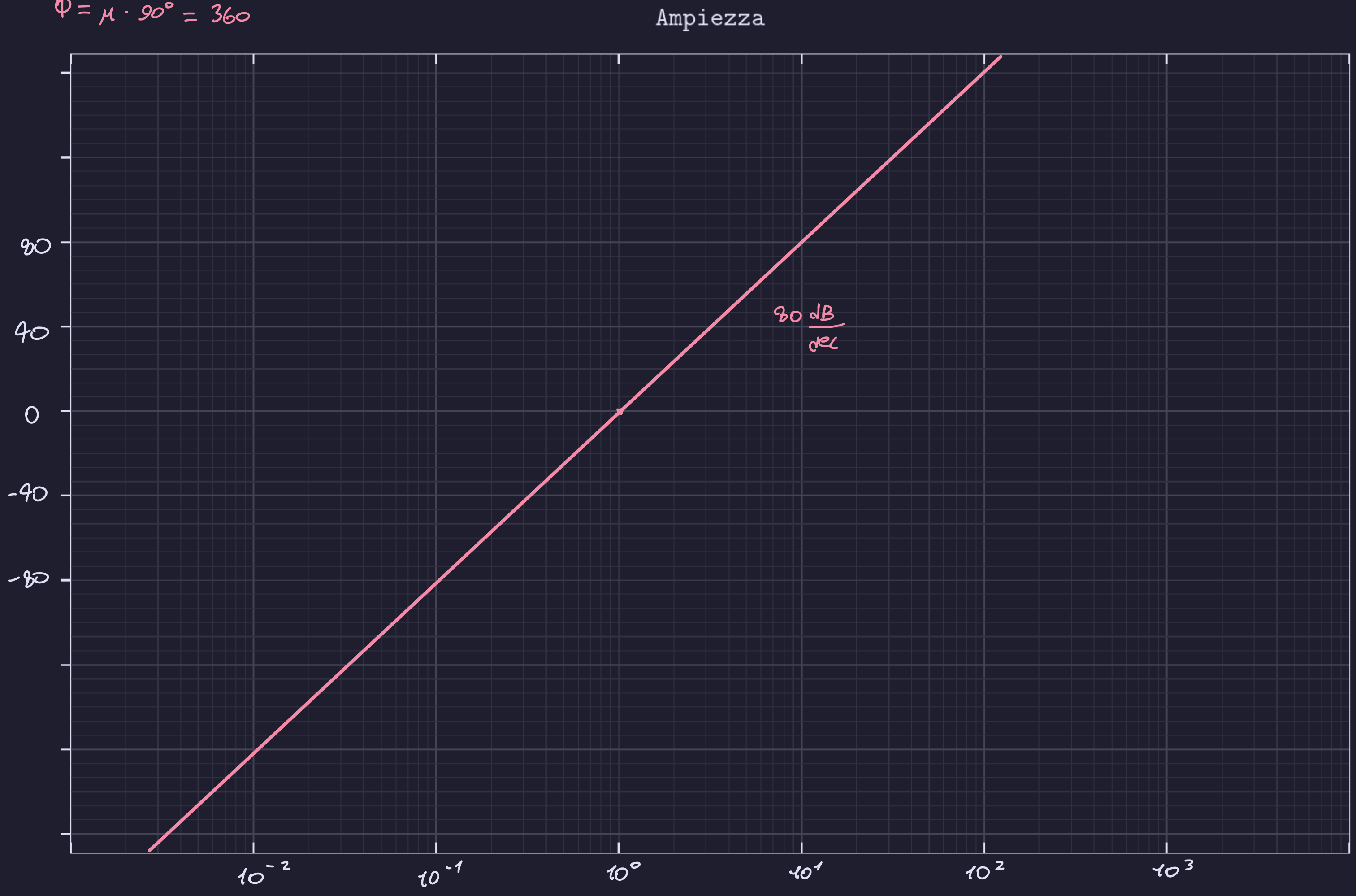
Fase

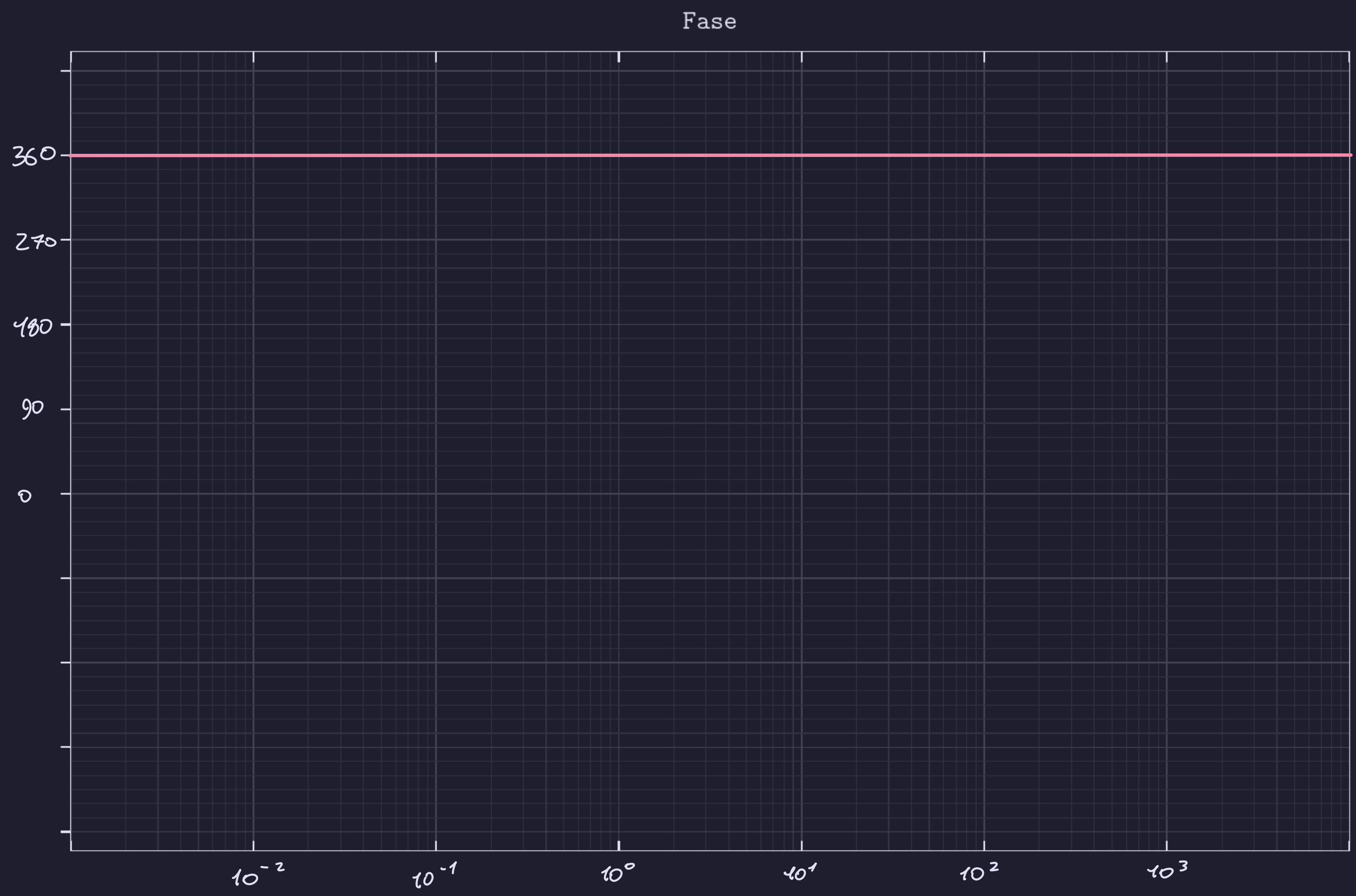


$A = 20\mu \frac{dB}{dec} = 20 \cdot 4 = 80 \frac{dB}{dec}$

$\phi = \mu \cdot 90° = 360$

Diagramma di Bode s^4





$$A = \begin{cases} 0 & \omega \le \omega_n \\ 20 \cdot 1 \frac{dB}{dec} & \omega > \omega_n = 20 \frac{dB}{dec} \end{cases}$$

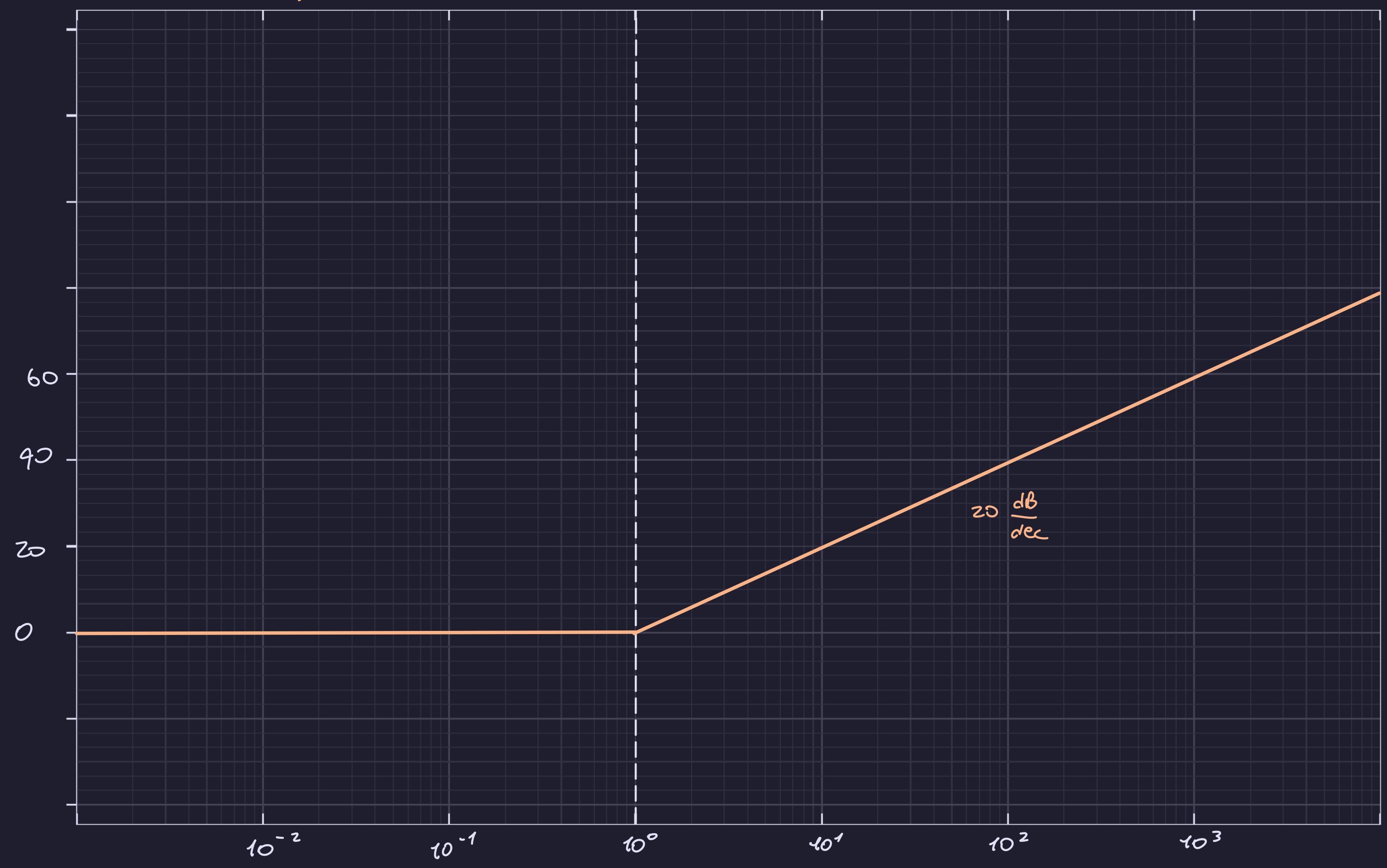
$$\phi = \begin{cases} 0 & \omega \le \omega_n \\ 1 \cdot 90 \cdot \text{segno}(\tau) = -90^\circ \end{cases}$$

$(1-s)^1$ $\quad \tau = -1$

$\omega_1 = \frac{1}{|\tau|} = 1$

Diagramma di Bode

Ampiezza



Fase

$$A = \begin{cases} 0 & w \leq w_n \\ 40\mu \frac{dB}{dec} & w > w_n \end{cases} = 40 \frac{dB}{dec}$$

$$\Phi = \begin{cases} 0 & w \leq w_n \\ \mu \cdot 180 \cdot \text{segno}(\xi) & w > w_n \end{cases} = -180°$$

Diagramma di Bode

$$\left(1 - \frac{1}{5}s + \frac{1}{25}s^2\right) \quad w_n = 5$$

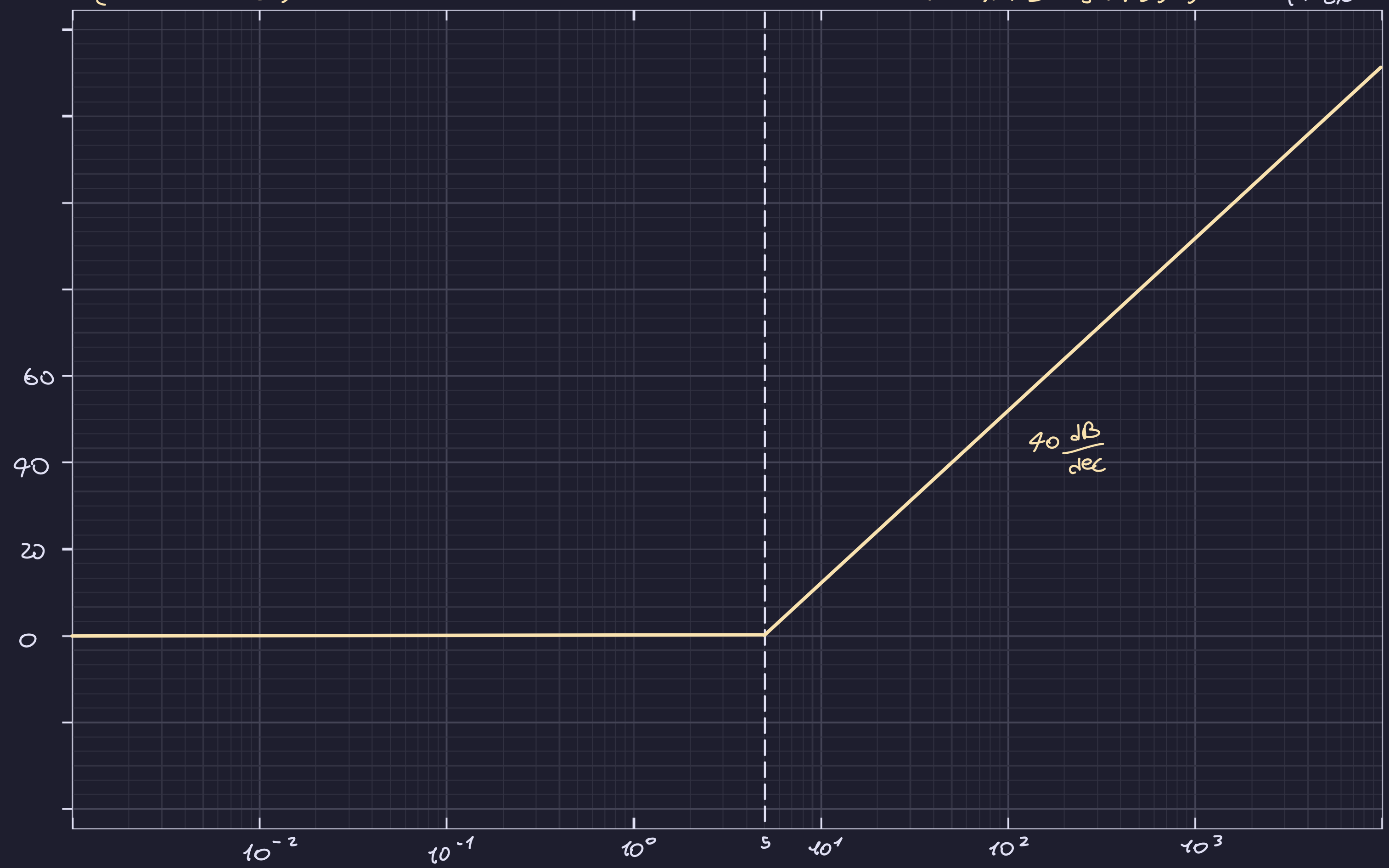
$$\frac{2\xi}{w_n} = -\frac{1}{w_n^2} \rightarrow \xi = -\frac{1}{10}$$

$$w_R = w_n \sqrt{1-2\xi^2} = 5$$

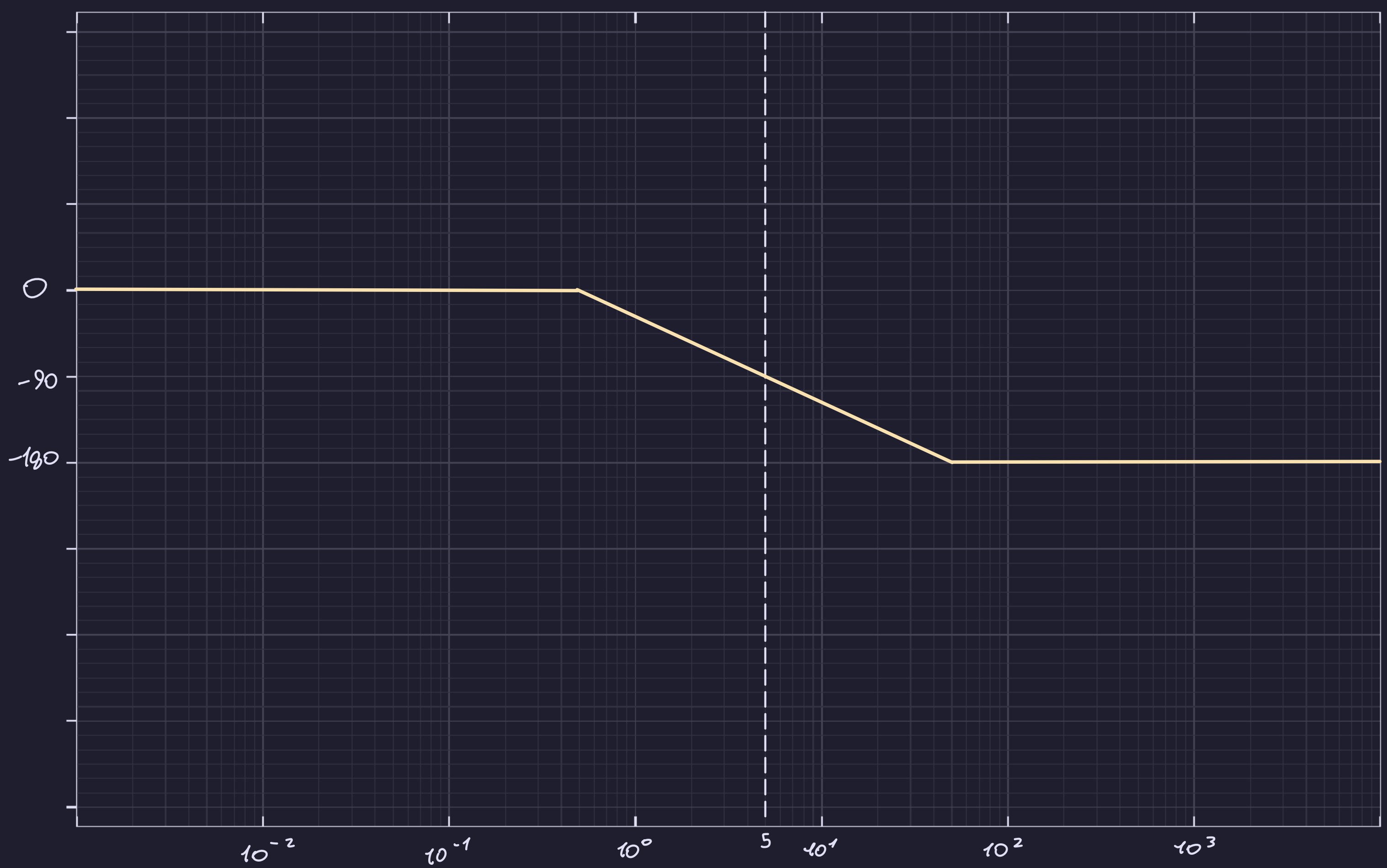
$$M_R = 20\mu \log\left(2\xi\sqrt{1-2\xi^2}\right) = -14$$

Non c'é picco

Ampiezza



Fase



$A = 20\,n \frac{dB}{dec} = -120 \frac{dB}{dec}$

$\phi = n \cdot 90 = -540$

Diagramma di Bode

s^{-6}

Ampiezza

Fase

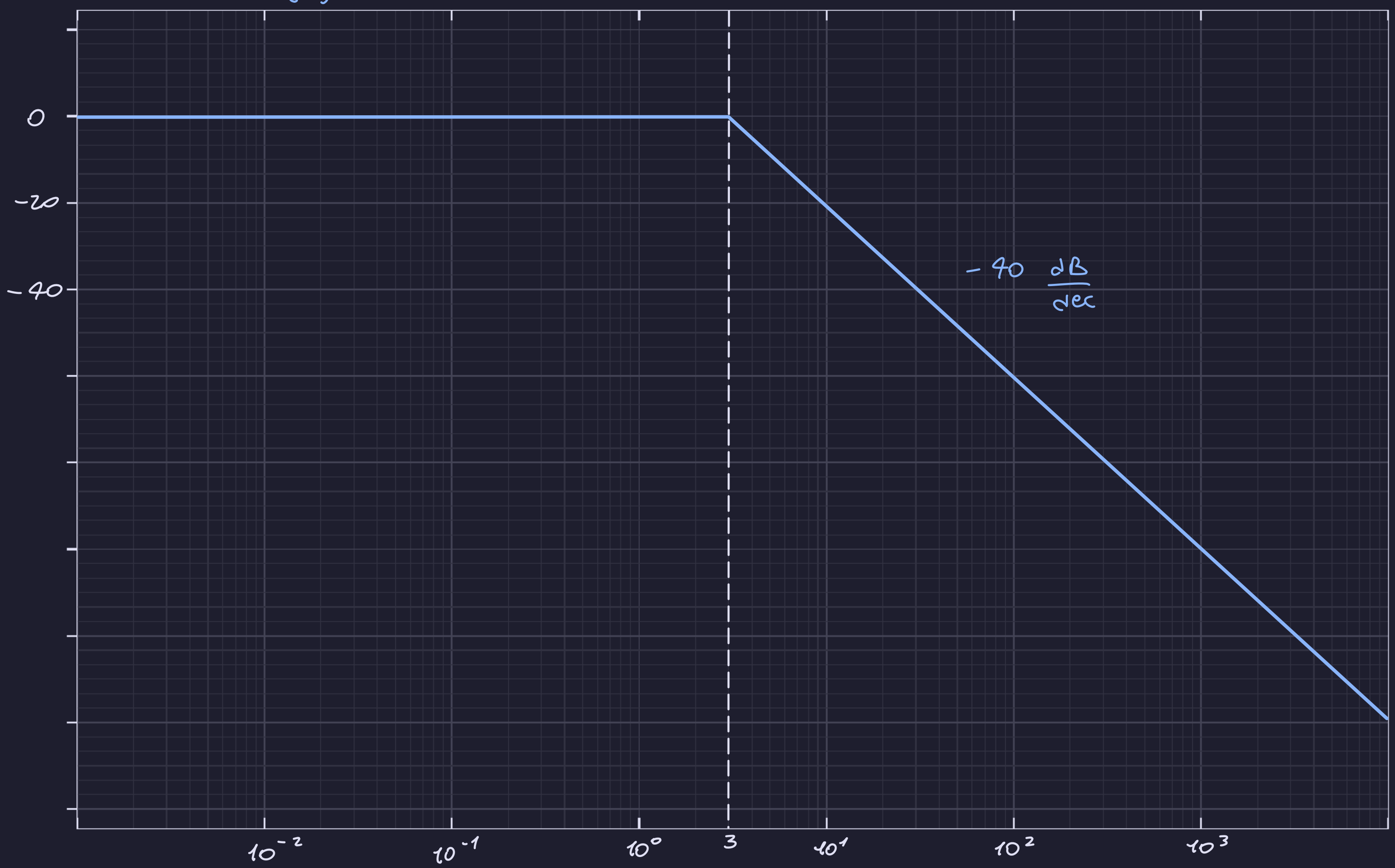
$$A = \begin{cases} 0 & w \le w_n \\ 20\,M \frac{dB}{dec} & w > w_n = -40 \frac{dB}{dec} \end{cases}$$

$$\phi = \begin{cases} 0 & w \le w_n \\ M \cdot 90 \cdot \text{segno}(\tau) & w > w_n = -180° \end{cases}$$

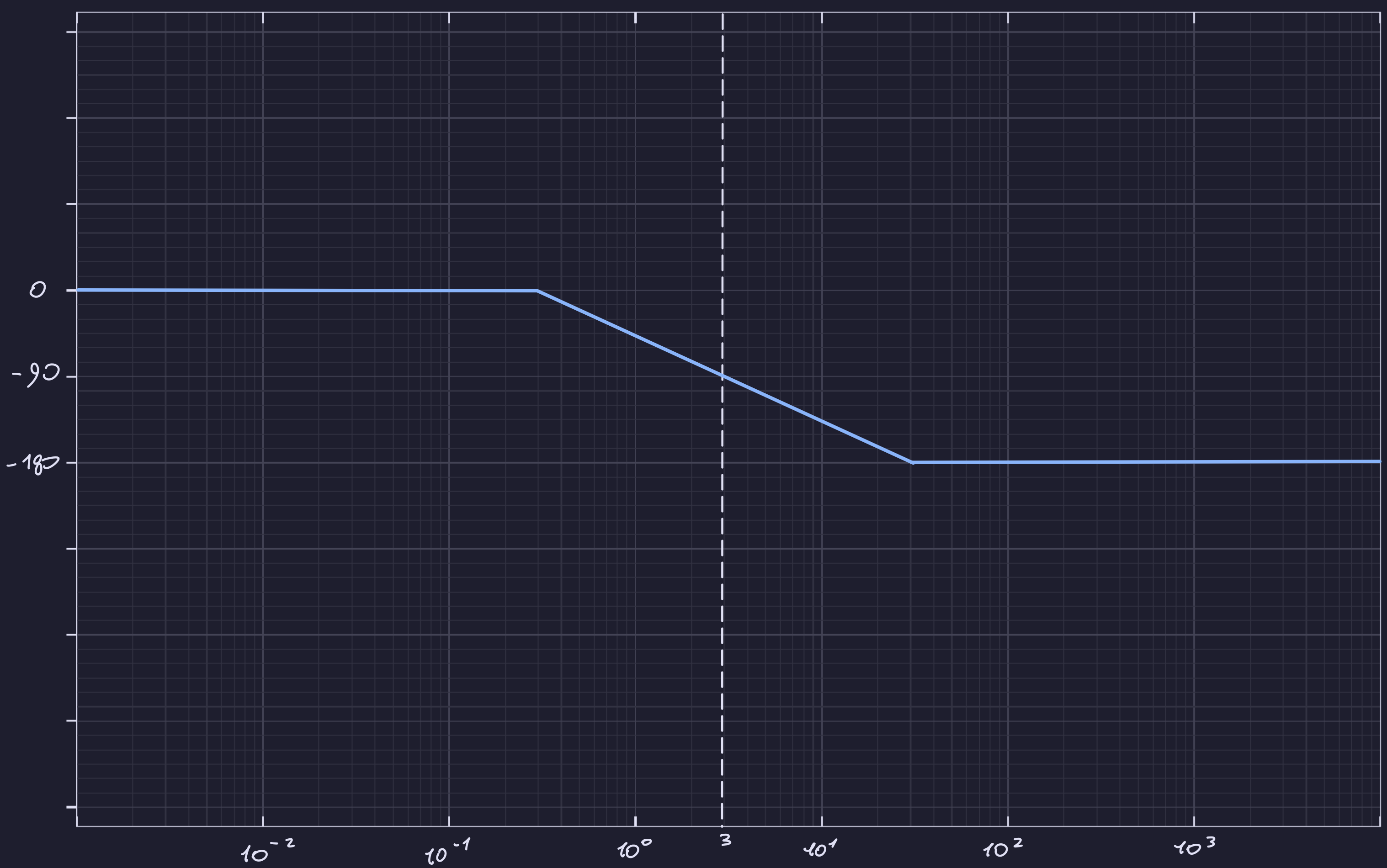
Diagramma di Bode

$$\left(1 + \frac{1}{3} s\right)^{-2} \qquad \tau = \frac{1}{3} \qquad w_n = \frac{1}{|\tau|} = 3$$

Ampiezza



Fase



$A = \begin{cases} 0 & \omega \le \omega_n \\ 20\mu \frac{dB}{dec} & \omega > \omega_n = -20 \frac{dB}{dec} \end{cases}$

$(1+s)^{-1}$ $\tau = 1$

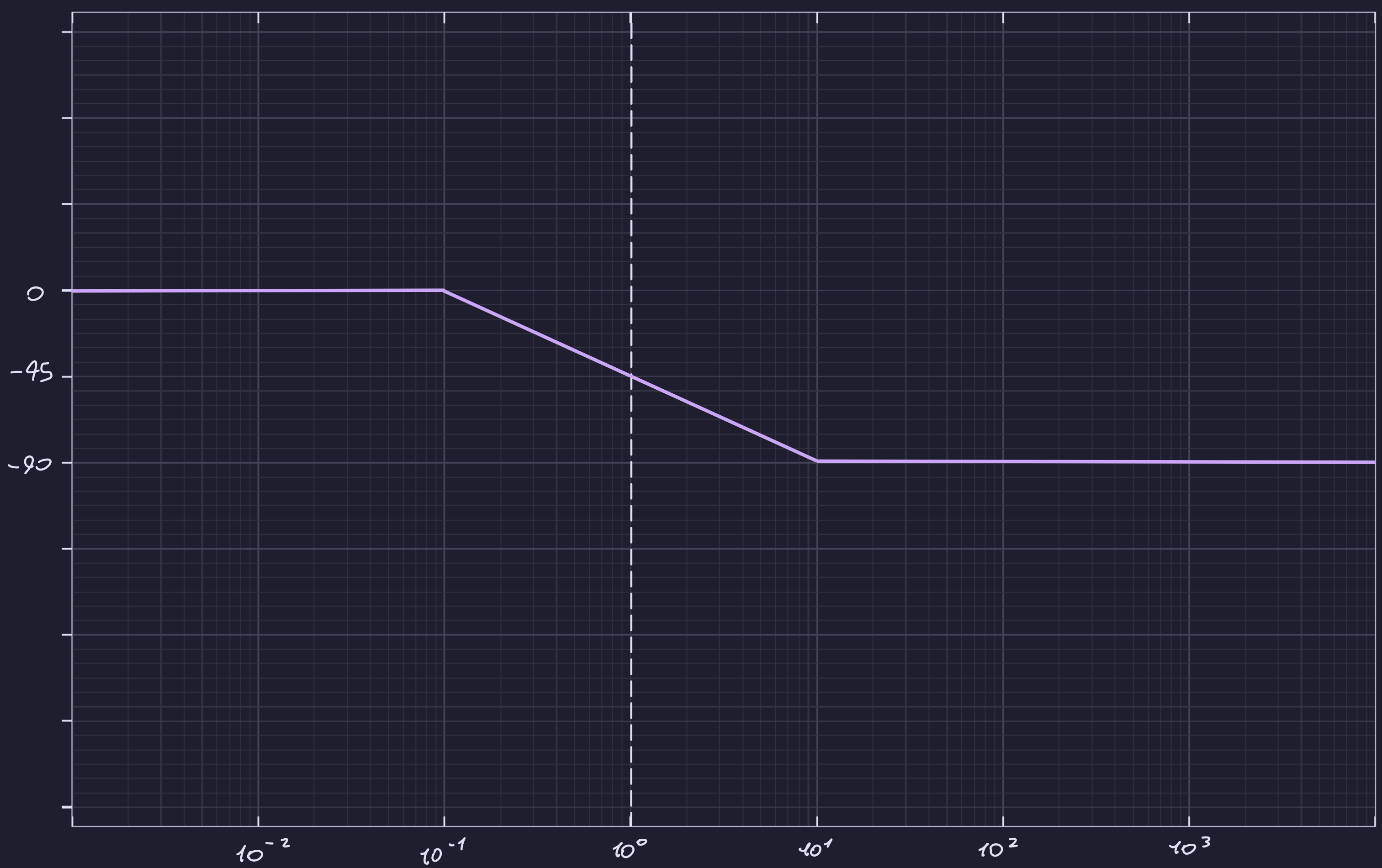
$\omega_n = \frac{1}{|\tau|} = 1$

$\phi = \begin{cases} 0 & \omega \le \omega_n \\ \mu\, 90 \cdot \text{segno}(\tau) & \omega > \omega_n = -90^\circ \end{cases}$

Diagramma di Bode

Ampiezza

Fase



Ampiezze

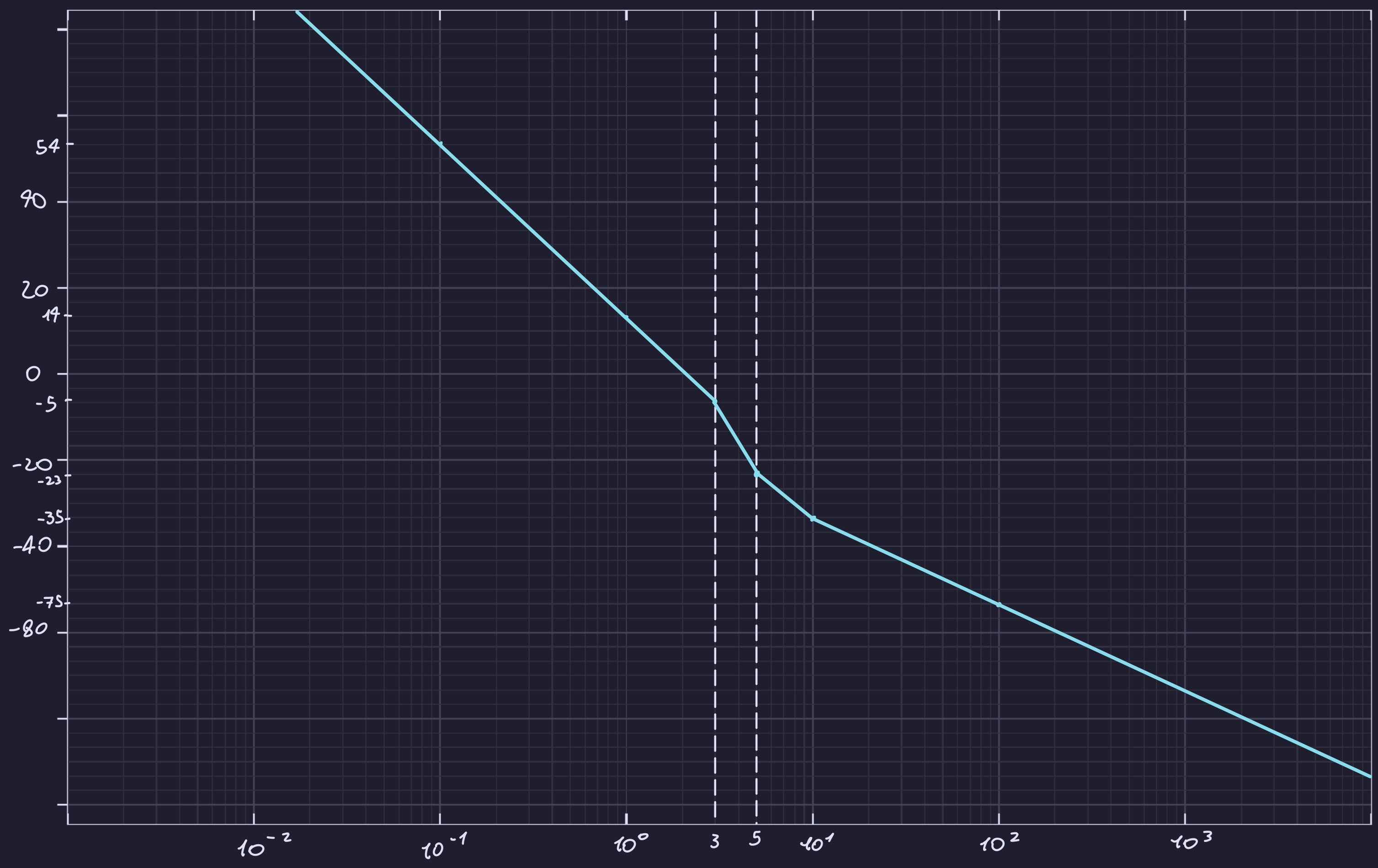
		10^{-1}	10^{0}	3	5	10^{1}	10^{2}	
	K_b	14	14	14	14	14	14	
$\mu=4$	Z_n	-80	0	38	56	80	160	$20\,\mu\log_{10}(\omega)$
$\tau=-1$, $\mu=1$	Z_r	0	0	10	14	20	40	$20\,\mu\log_{10}(\omega\cdot\lvert\tau\rvert)$
$\omega_n=5$, $\mu=1$	Z_{cc}	0	0	0	0	12	52	$40\,\mu\log_{10}\left(\frac{\omega}{\omega_n}\right)$
$\mu=-6$	P_n	120	0	-57	-84	-120	-240	
$\tau=\frac{1}{3}$, $\mu=-2$	P_{r1}	0	0	0	-9	-21	-61	
$\tau=1$, $\mu=-1$	P_{r2}	0	0	-10	-14	-20	-40	
	Totale	54	14	-5	-23	-35	-75	

Fasi

	10^{-1}	10^{0}	3	5	10^{1}	10^{2}
K_b	0	0	0	0	0	0
Z_n	360	360	360	360	360	360
Z_r	0	0	-90	-90	-90	-90
Z_{cc}	0	0	0	0	-180	-180
P_n	-540	-540	-540	-540	-540	-540
P_{r1}	0	0	0	-180	-180	-180
P_{r2}	0	0	-90	-90	-90	-90
Totale	-180	-180	-360	-540	-720	-720

Diagramma di Bode

Ampiezza



Fase