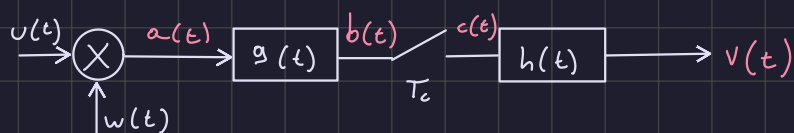


Esercizione Campionamento / Fourier

1)

Partendo dal seguente sistema a blocchi calcoliamo il segnale dopo ogni blocco



$$u(t) = 10 \cdot \sin(80\pi t)$$

$$w(t) = 8 \cdot \sin(40\pi t)$$

$$G(t) = 800 \cdot \text{sinc}^2(80t)$$

$$h(t) = 0.3 \cdot \text{sinc}(300t)$$

$$T_c = 5 \text{ ms}$$

Trasformiamo tutto nel dominio delle frequenze usando le trasformate di Fourier

$A \sin(2\pi F_0 t)$

$$u(t) = 10 \cdot \sin(80\pi t) = \underbrace{10}_A \cdot \sin(2\pi \cdot \underbrace{40}_{F_0} t) \xrightarrow{F} 5j(\delta(F-40) - \delta(F+40))$$

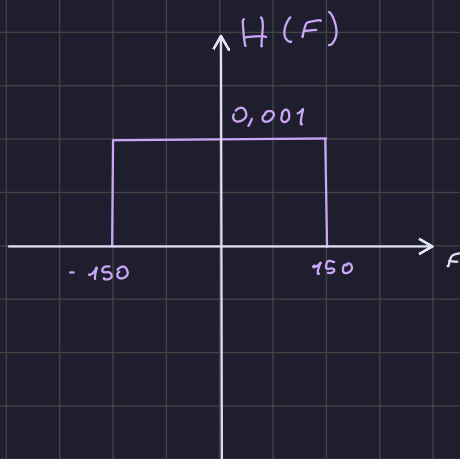
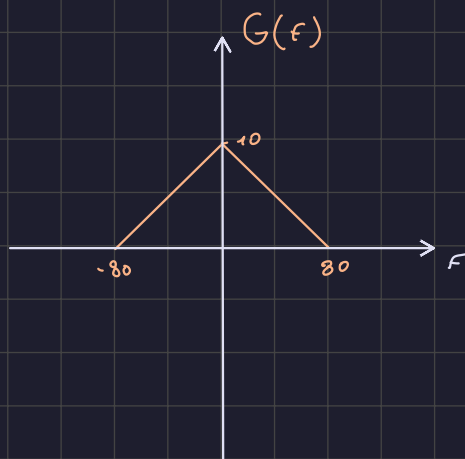
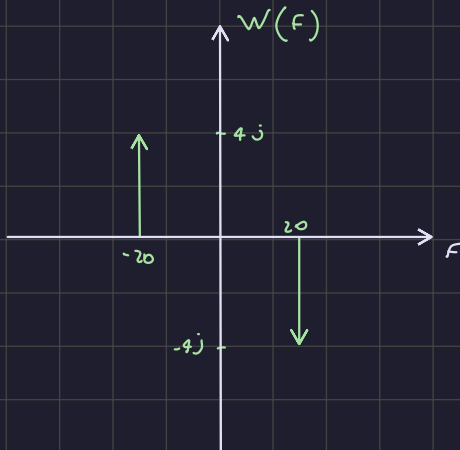
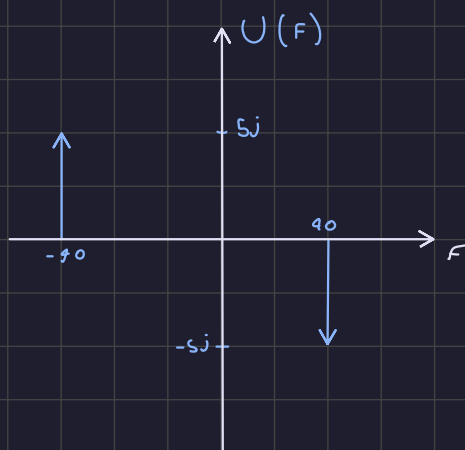
$$w(t) = 8 \cdot \sin(40\pi t) = 8 \cdot \sin(2\pi \cdot 20 t) \xrightarrow{F} 4j(\delta(F-20) - \delta(F+20))$$

$AT \text{sinc}^2(Tc)$ (Triangolo Δ)

$$G(t) = 800 \cdot \text{sinc}^2(80t) = \underbrace{10}_A \cdot \underbrace{80}_T \cdot \text{sinc}^2(\underbrace{80}_T t) \xrightarrow{F} 10 \Delta\left(\frac{F}{80}\right)$$

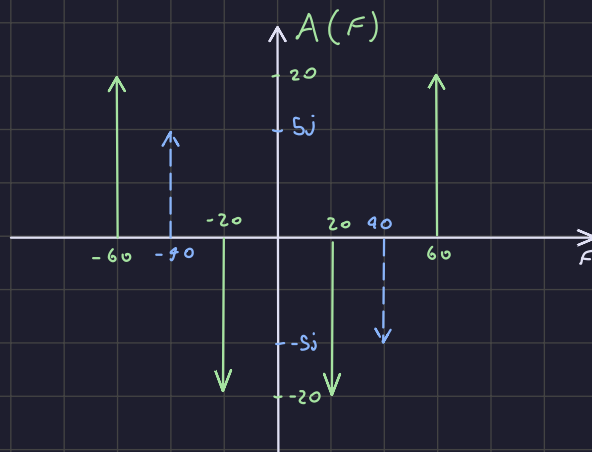
$AT \text{sinc}(Tc)$ (Box Π)

$$h(t) = 0.3 \cdot \text{sinc}(300t) = \frac{\underbrace{1}_A}{1000} \cdot \underbrace{300}_T \cdot \text{sinc}(\underbrace{300}_T t) \xrightarrow{F} \frac{1}{1000} \Pi\left(\frac{F}{300}\right)$$

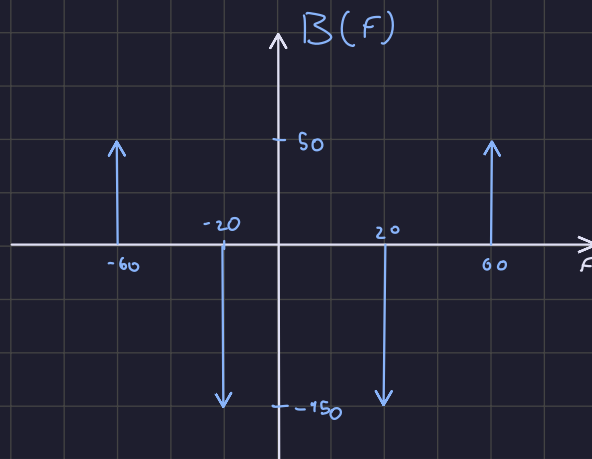


$$A(f) = U * W$$

Per fare la convoluzione si ribalta un segnale e si fa scorrere sull'altro replicandolo ogni volta che i segnali si sovrappongono facendo il prodotto delle ampiezze



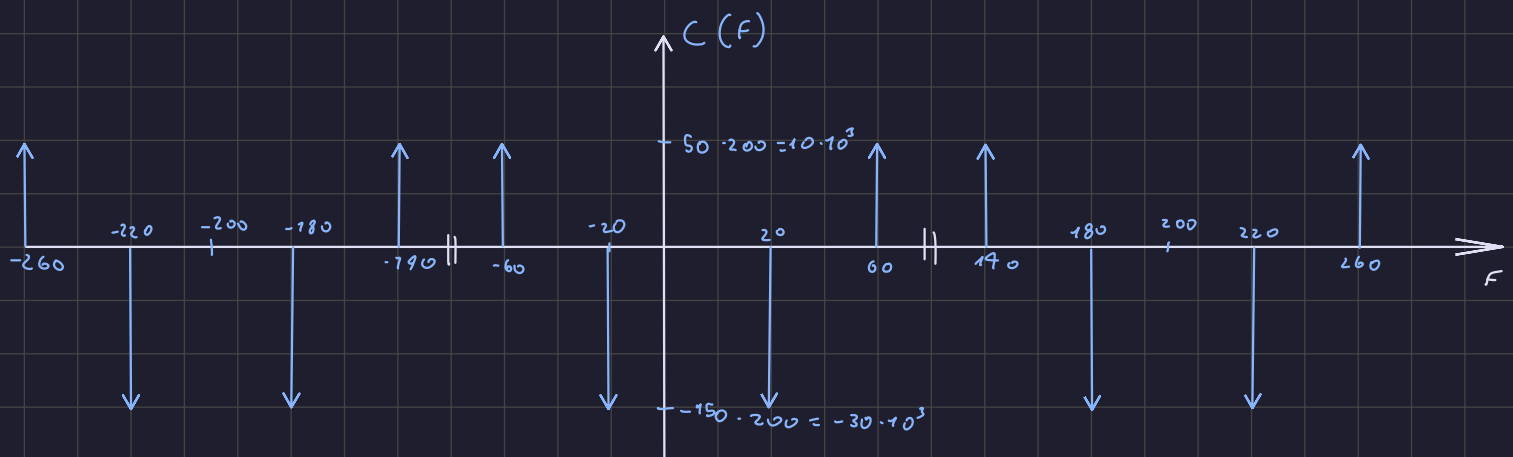
$$B(f) = A(f) \cdot G(f)$$



L'ampiezza è stata calcolata facendo la proporzione con il segnale triangolo

Per il campionamento si calcola la frequenza di campionamento (in secondi)

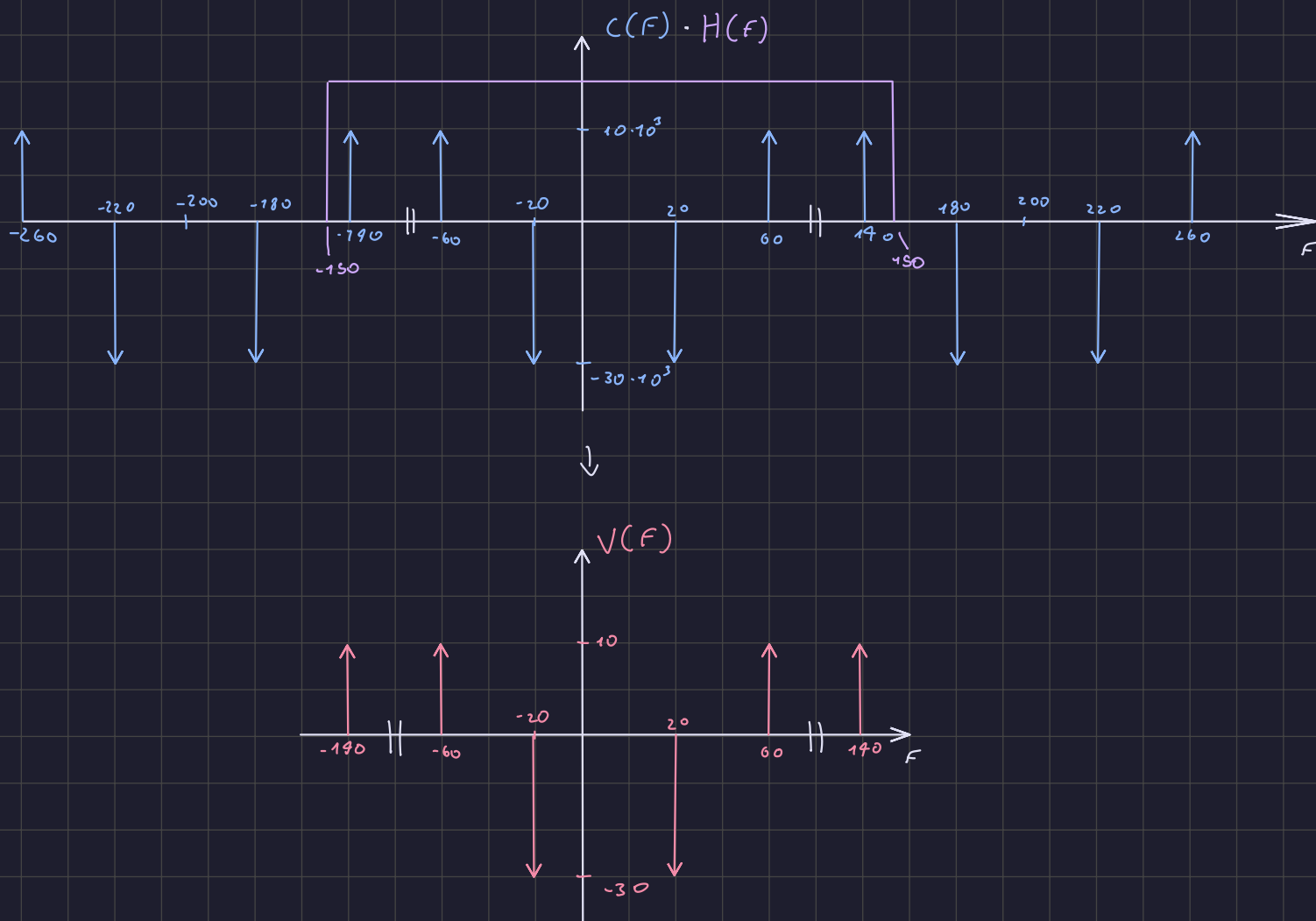
$$F_c = \frac{1}{T_c} = \frac{1}{0,005} = 200 \text{ Hz}$$



Si copia il segnale ogni $\pm k f_c$.

$$F_c > 2B \rightarrow 200 > 60 \cdot 2 \quad \checkmark$$

$$V(F) = H(F) \cdot C(F)$$



Antitrasformiamo l'uscita

$$V(F) = 10 \delta(F - 140) + 10 \delta(F - 60) - 30 \delta(F - 20) +$$

$$-30 \delta(F + 20) + 10 \delta(F + 60) + 10 \delta(F + 140)$$

$$= -30(\delta(F - 20) + \delta(F + 20)) +$$

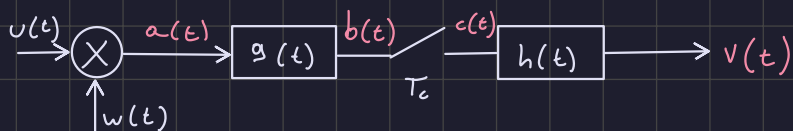
$$+ 10(\delta(F - 60) + \delta(F + 60)) +$$

$$+ 10(\delta(F - 140) + \delta(F + 140))$$

$\downarrow \mathcal{F}^{-1}$

$$v(t) = -60 \cos(40 \pi t) + 20 \cos(120 \pi t) + 20 \cos(280 \pi t)$$

2)



$$u(t) = \text{sinc}(2t) + 1.5$$

$$w(t) = 4 \cos(40\pi t)$$

$$g(t) = 4 \text{sinc}(40t)$$

$$h(t) = 12 \text{sinc}(12t)$$

$$T_c = 100 \text{ ms}$$

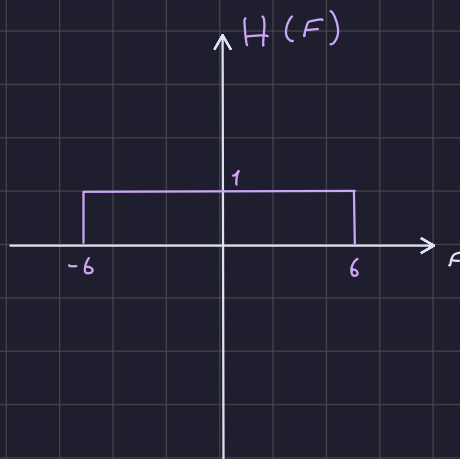
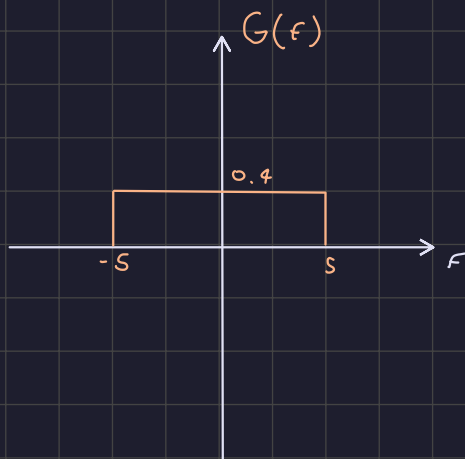
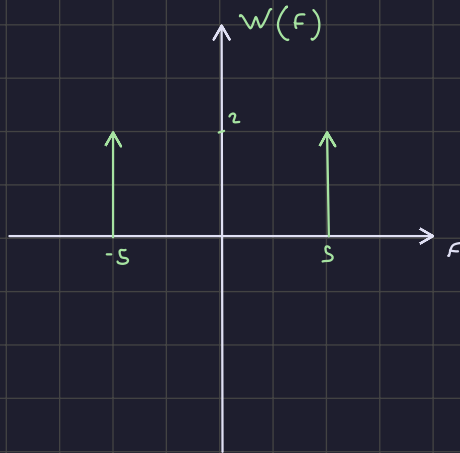
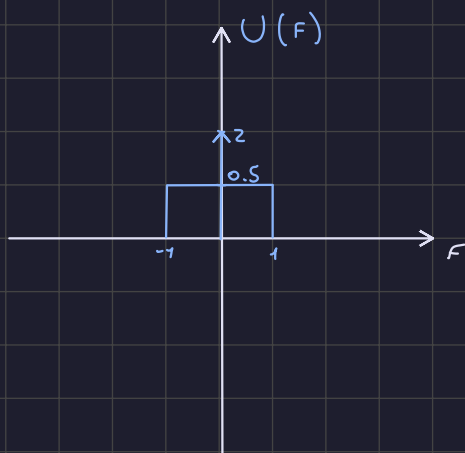
Trasformiamo tutto nel dominio delle frequenze usando le trasformate di Fourier

$$u(t) = \frac{1}{2} \cdot 2 \text{sinc}(2t) + 2 \xrightarrow{\mathcal{F}} \frac{1}{2} \Pi\left(\frac{f}{2}\right) + 2\delta(f)$$

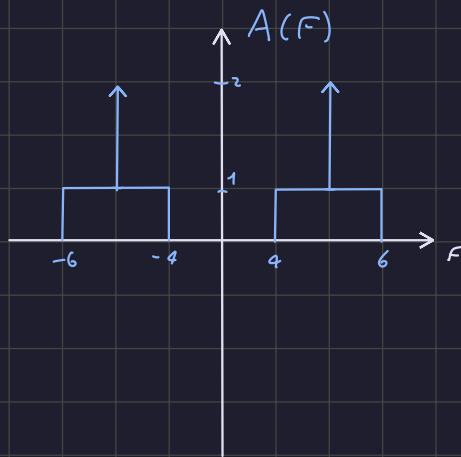
$$w(t) = 4 \cdot \cos(2\pi 5t) \xrightarrow{\mathcal{F}} 2(\delta(f-5) + \delta(f+5))$$

$$g(t) = 0.4 \cdot 10 \text{sinc}(40t) \xrightarrow{\mathcal{F}} 0.4 \Pi\left(\frac{f}{10}\right)$$

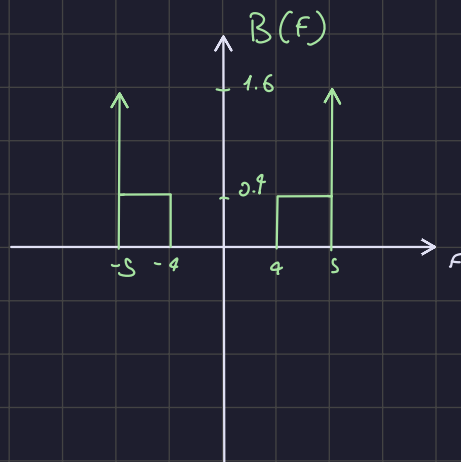
$$h(t) = 1 \cdot 12 \text{sinc}(12t) \xrightarrow{\mathcal{F}} \Pi\left(\frac{f}{12}\right)$$



$$A(f) = U(f) * W(f)$$

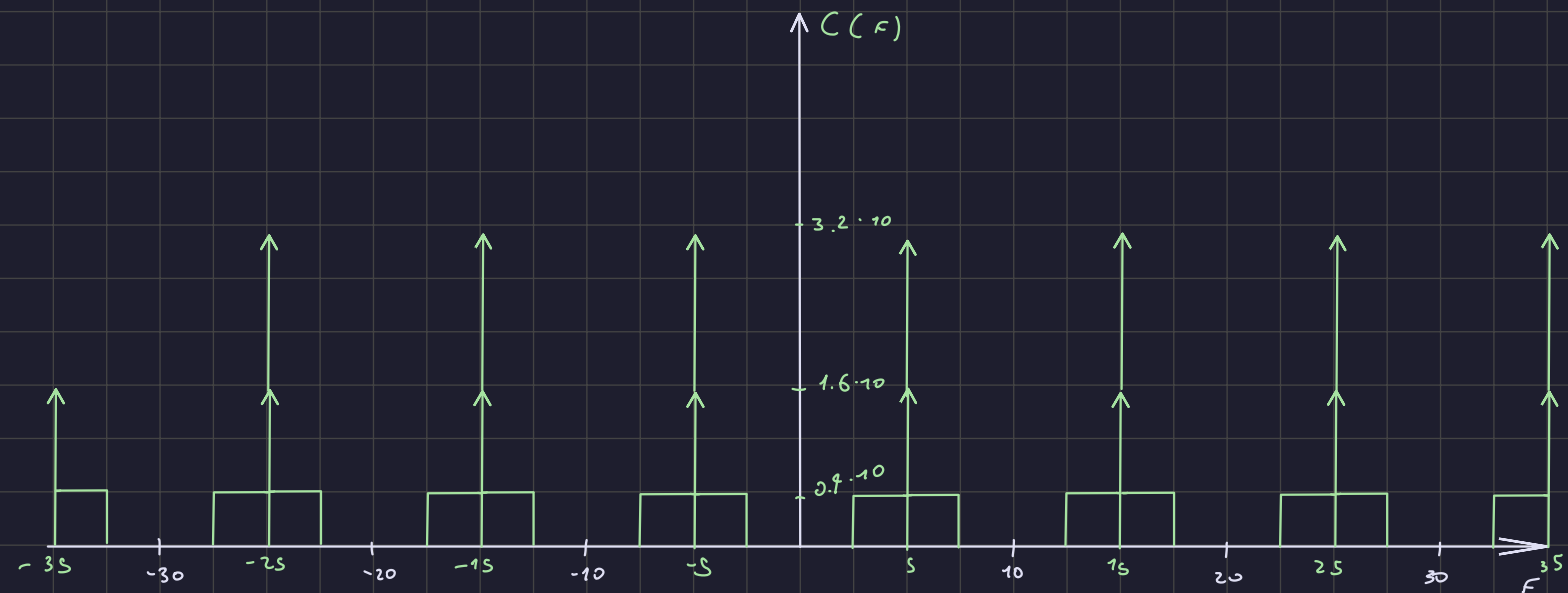


$$B(f) = A(f) \cdot G(f)$$



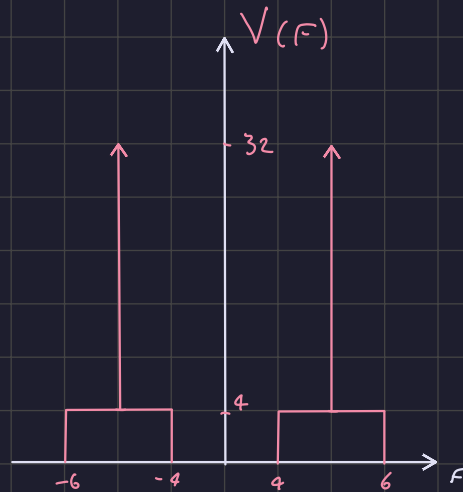
$$F_c = \frac{1}{T_c} = \frac{1}{0.1} = 10 \text{ Hz}$$

$F_c > 2B \rightarrow 10 > 10 \times$ Si verifichiamo aliasing



Nelle sovrapposizioni tra i segnali si fa la somma tra i 2

$$V(F) = C(F) \cdot H(F)$$



$$V(F) = 28 \delta(F-5) + 28 \delta(F+5) +$$

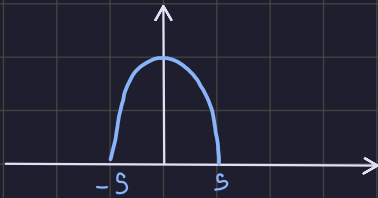
$$+ 4 \Pi\left(\frac{F}{2} - 5\right) + 4 \Pi\left(\frac{F}{2} + 5\right)$$

$$\downarrow \mathcal{F}^{-1}$$

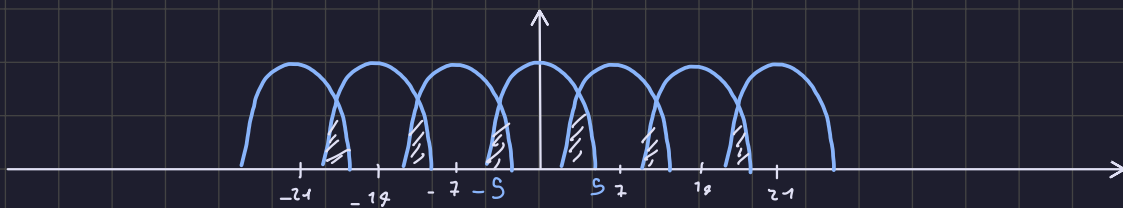
Traslazione nelle Frequenze

$$v(t) = 28 (\delta(F-5) + \delta(F+5)) + 8 \operatorname{sinc}(2t) e^{-10\pi t j} + 8 \operatorname{sinc}(2t) e^{10\pi t j}$$

3)



Campioniamo con $f_c = 7$



Aliasing