Esercizio 1 (30 punti)

Si consideri il modello ingresso/uscita a tempo continuo descritto dalla sequente equazione differenziale:

$$2\frac{d^2v(t)}{dt^2} - 3\frac{dv(t)}{dt} - 2v(t) = 2\frac{du(t)}{dt} + u(t), \quad t \in \mathbb{R}_+.$$

a) Si studi la stabilitá asintotica e la stabilitá BIBO del sistema.

$$2s^2 - 3s - 2 = 0$$

$$s_{1,2} = \frac{3 \pm \sqrt{9+16}}{4} = \frac{3 \pm 5}{4} = \begin{cases} -\frac{1}{2} \\ 2 \end{cases}$$

$$s_1 = -\frac{1}{2} \qquad \mu_{1,2} = 1$$

$$s_2 = 2$$

Il sistema non è asintoticamente stabile perchè non tutte le radici hanno parte reale negativa. Per verificare se il sistema è BIBO stabile bisogna vedere se le radici con parte reale positiva si semplificano nella funzione di trasferimento, cioè quella funzione data dal rapporto del polinomio caratteristico dell'ingresso e dell'uscita:

$$H(s) = \frac{2s+1}{2s^2 - 3s - 2} = \frac{\cancel{2(s+\frac{1}{2})}}{\cancel{2(s+\frac{1}{2})}(s-2)}$$

Il sistema non è BIBO stabile perchè la radice con parte reale positiva non è stata semplificata.

b) Calcolare la risposta totale con Laplace:

$$v(0^-) = 4 \qquad u(t) = e^{-2t}\,\delta_{-1}(t)$$

$$v'(0^-) = -2$$

$$2v''(t) - 3v'(t) - 2v(t) = 2u'(t) + u(t)$$

$$\mathcal{L}[2v''(t) - 3v'(t) - 2v(t)] = \mathcal{L}[2u'(t) + u(t)]$$

$$2\mathcal{L}[v''(t)] = 2s^2V(s) - 2sv(0^-) - 2v'(0^-) = 2s^2V(s) - 8s + 4$$

$$-3\mathcal{L}[v'(t)] = -3sV(s) + 3v(0^-) = -3sV(s) + 12$$

$$-2\mathcal{L}[v(t)] = -2V(s)$$

$$\mathcal{L}[2u'(t) + u(t)] = 2sU(s) + U(s)$$

$$U(s) = \mathcal{L}[e^{-2t}\,\delta_{-1}(t)] = \frac{1}{s+2}$$

$$2s^2V(s) - 8s + 4 - 3sV(s) + 12 - 2V(s) = 2sU(s) + U(s)$$

$$(2s^2 - 3s - 2)V(s) - 8(s-2) = (2s+1)U(s)$$

$$V(s) = \frac{8\cancel{(s-2)}}{2(s+\frac{1}{2})\cancel{(s-2)}} + \frac{\cancel{2}\cancel{(s+\frac{1}{2})}}{\cancel{2}\cancel{(s+\frac{1}{2})}(s-2)(s+2)}$$

$$= \frac{8}{2(s+\frac{1}{2})} + \frac{1}{(s-2)(s+2)}$$

Fratti semplici:

$$\frac{2}{(s-2)(s+2)} = \frac{A}{s-2} + \frac{B}{s+2} = \frac{As+2A+Bs-2B}{(s-2)(s+2)} = \frac{s(A+B)+2A-2B}{(s-2)(s+2)}$$

$$\begin{cases} A+B=0 \\ 2A-2B=1 \end{cases} \to \begin{cases} A=-B \\ -4B=1 \end{cases} \to \begin{cases} A=-B \\ B=-\frac{1}{4} \end{cases} \to \begin{cases} A=\frac{1}{4} \\ B=-\frac{1}{4} \end{cases}$$

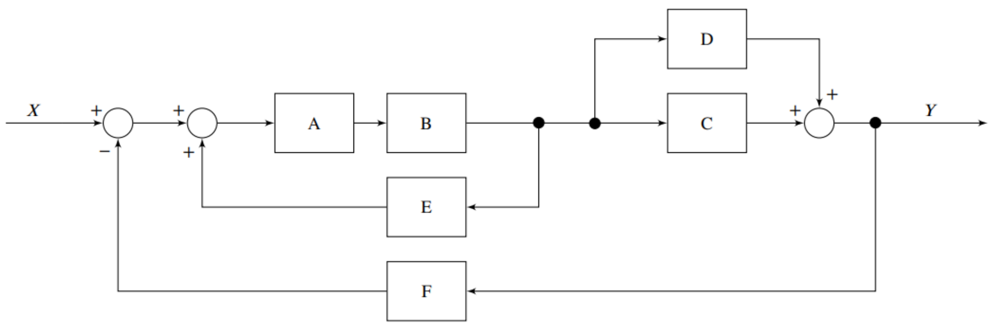
$$V(s) = \frac{1}{4}\,\frac{1}{s-2} - \frac{1}{4}\cdot\frac{1}{s+2} + \frac{4}{s+\frac{1}{2}}$$

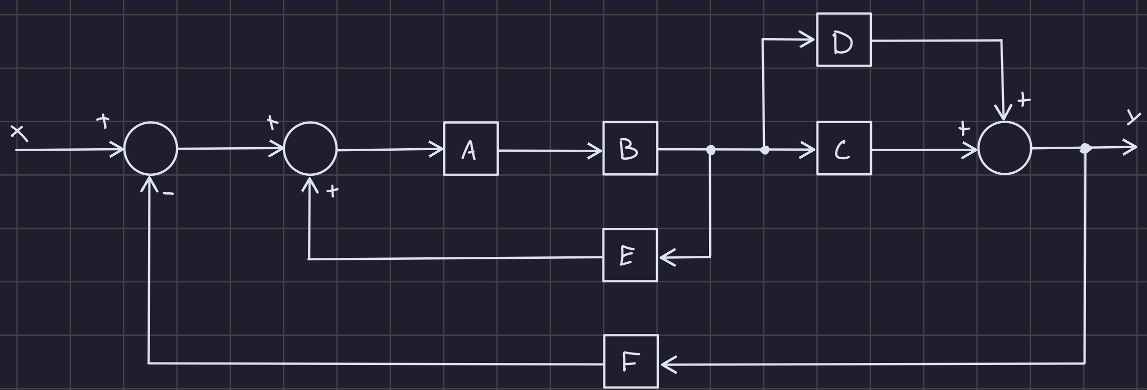
$$\downarrow \mathcal{L}^{-1}$$

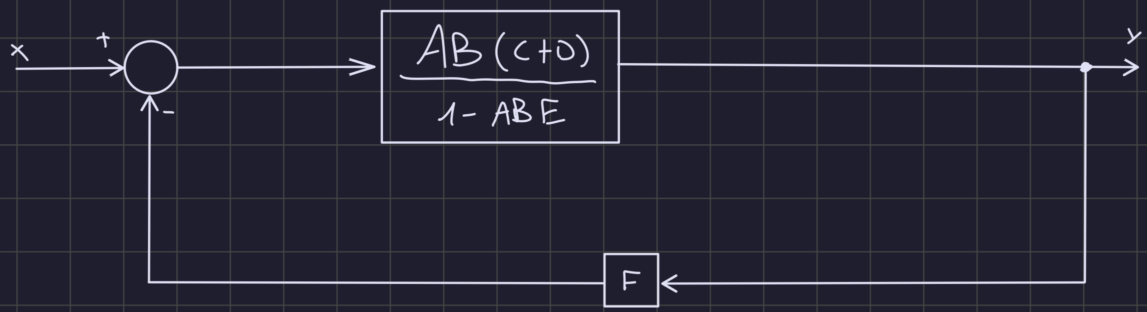
$$v(t) = \left(\frac{1}{4}e^{2t} - \frac{1}{4}e^{-2t} + 4e^{-\frac{1}{2}t}\right)\delta_{-1}(t)$$

Esercizio 2 (20 punti)

Calcolare la funzione di trasferimento del seguente schema a blocchi:







$$Y = \frac{\dfrac{AB(C+D)}{1-ABE}}{1+\dfrac{ABF(C+D)}{1-ABE}} = \frac{AB(C+D)}{\cancel{1-ABE}} \cdot \frac{\cancel{1-ABE}}{(1-ABE)+ABF(C+D)} = \frac{AB(C+D)}{(1-ABE)+ABF(C+D)}$$

Esercizio 3 (25 punti)

Riportare la funzione in forma di Bode, disegnare i diagrammi di modulo e fase delle singole componenti elementari e il diagramma globale della seguente funzione di trasferimento:

$$G(s) = \frac{240s^2}{2s^2 - 4s - 6}.$$

$$s_{1,2} = \frac{4 \pm \sqrt{16+48}}{4} = \frac{4 \pm 8}{4} = 1 \pm 2 = \begin{cases} 3 \\ -1 \end{cases}$$

$$G(s) = 240 \cdot \frac{s^2}{2(s+1)(s-3)}$$

$$= 120 \frac{s^2}{(1+s)\,-3(1-\frac{1}{3}s)}$$

$$= -40 \frac{s^2}{(1+s)(1-\frac{1}{3}s)}$$

$K_b = -40$ $\quad z_n = s^2$ $\quad P_{r1} = (1+s)^{-1}$

$P_{r2} = (1-\frac{1}{3}s)^{-1}$

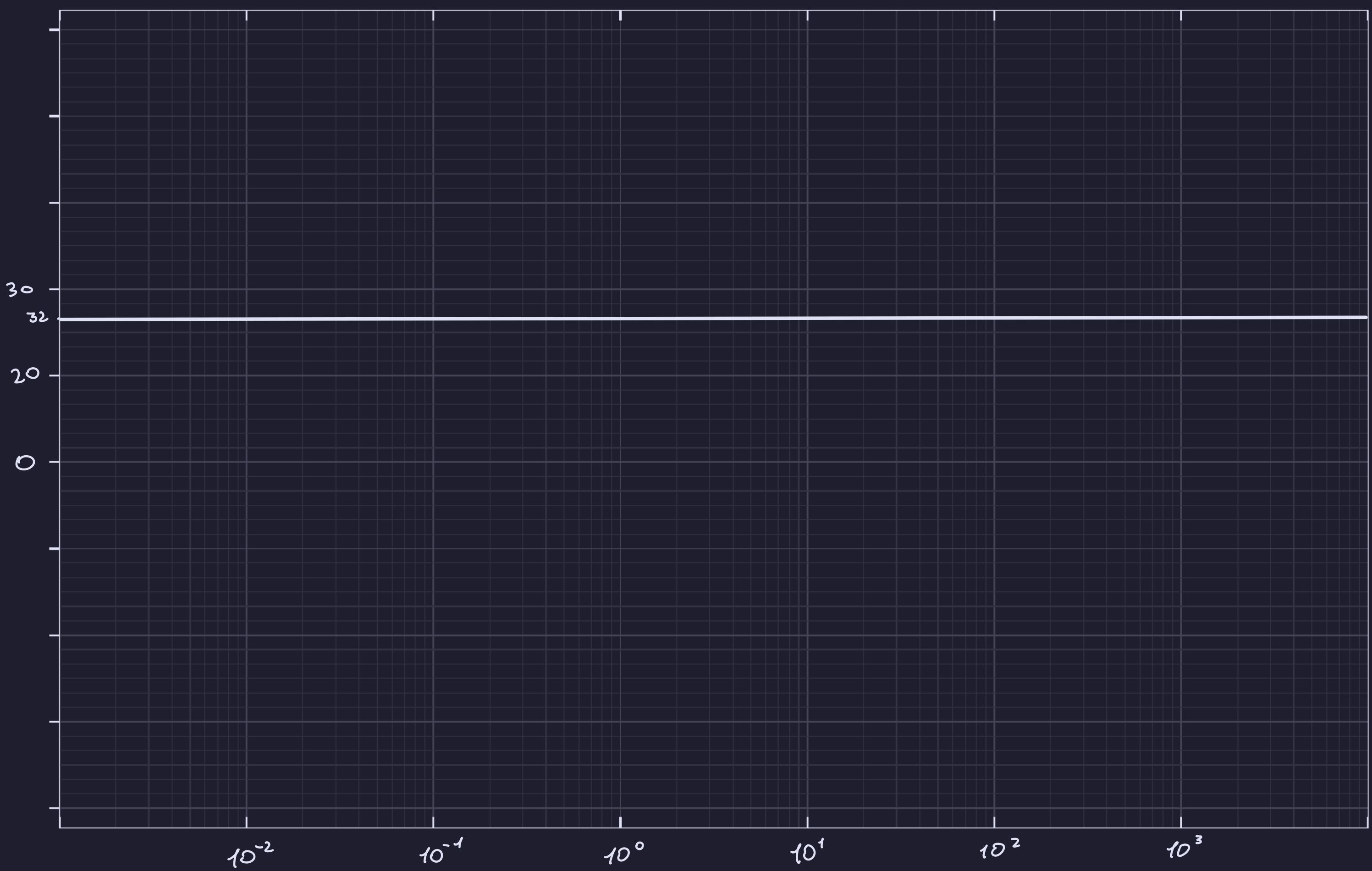
$A = 20\log_{10}(|-40|) \approx 32$

$Kb = -40$

$\Phi = \begin{cases} 0 & Kb \le 0 \\ -180 & Kb > 0 \end{cases}$

Diagramma di Bode

Ampiezza



Fase

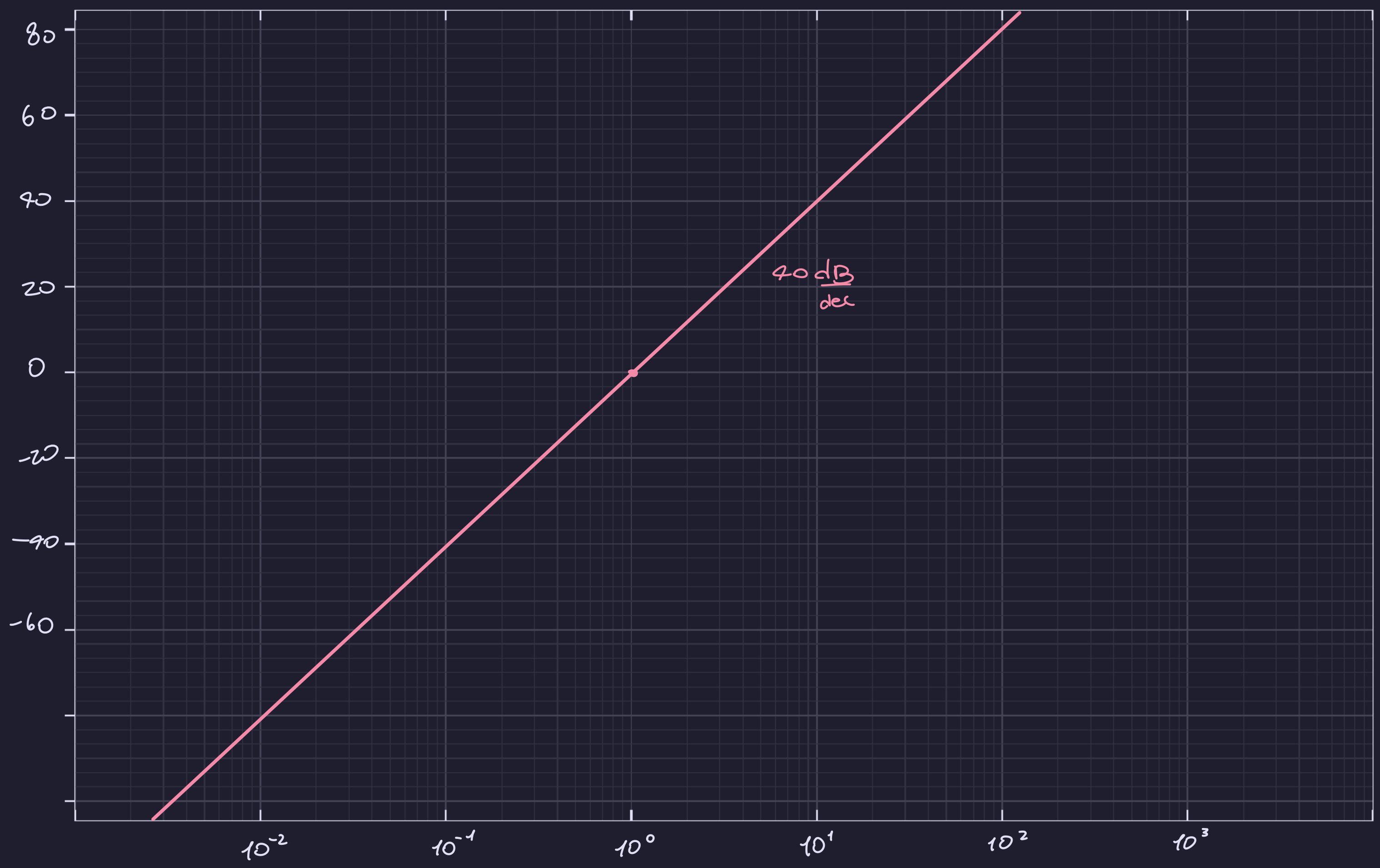
$A = 20\,\mu\,\frac{dB}{dec} = 40\,\frac{dB}{dec}$

$\phi = 90\,\mu = 180°$

s^2

Diagramma di Bode

Ampiezza



Fase

$$A = \begin{cases} 0 & \omega \le \omega_n \\ 20\mu \frac{dB}{dec} & \omega > \omega_n \end{cases} = -20 \frac{dB}{dec}$$

$$\phi = \begin{cases} 0 & \omega \le \omega_n \\ 90 \mu \, \text{segno}(\tau) & \omega \ge \omega_n \end{cases} = -90°$$

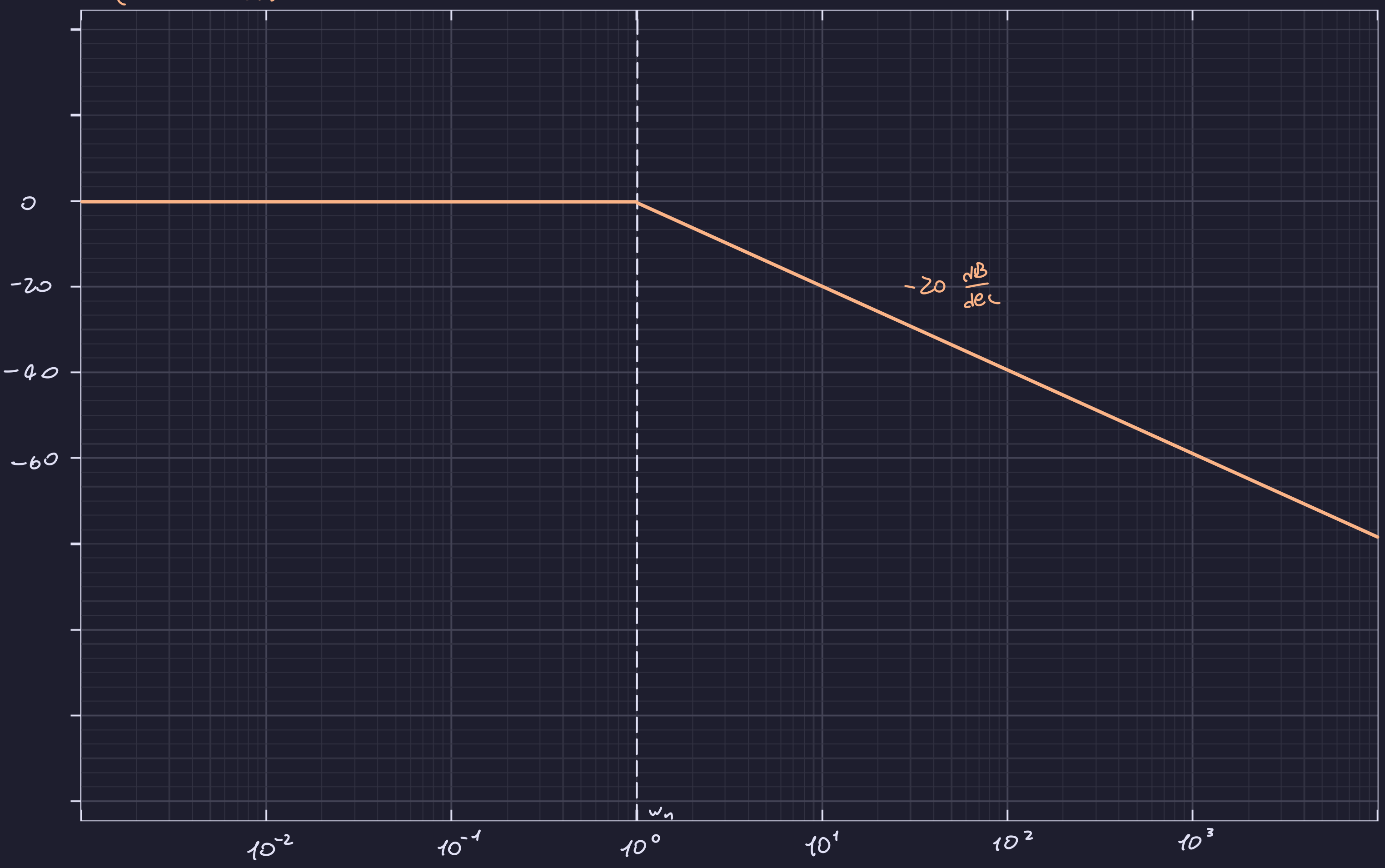
Diagramma di Bode

$(1+s)^{-1}$

$\tau = 1$

$\omega_n = \frac{1}{|\tau|} = 1$

Ampiezza



Fase

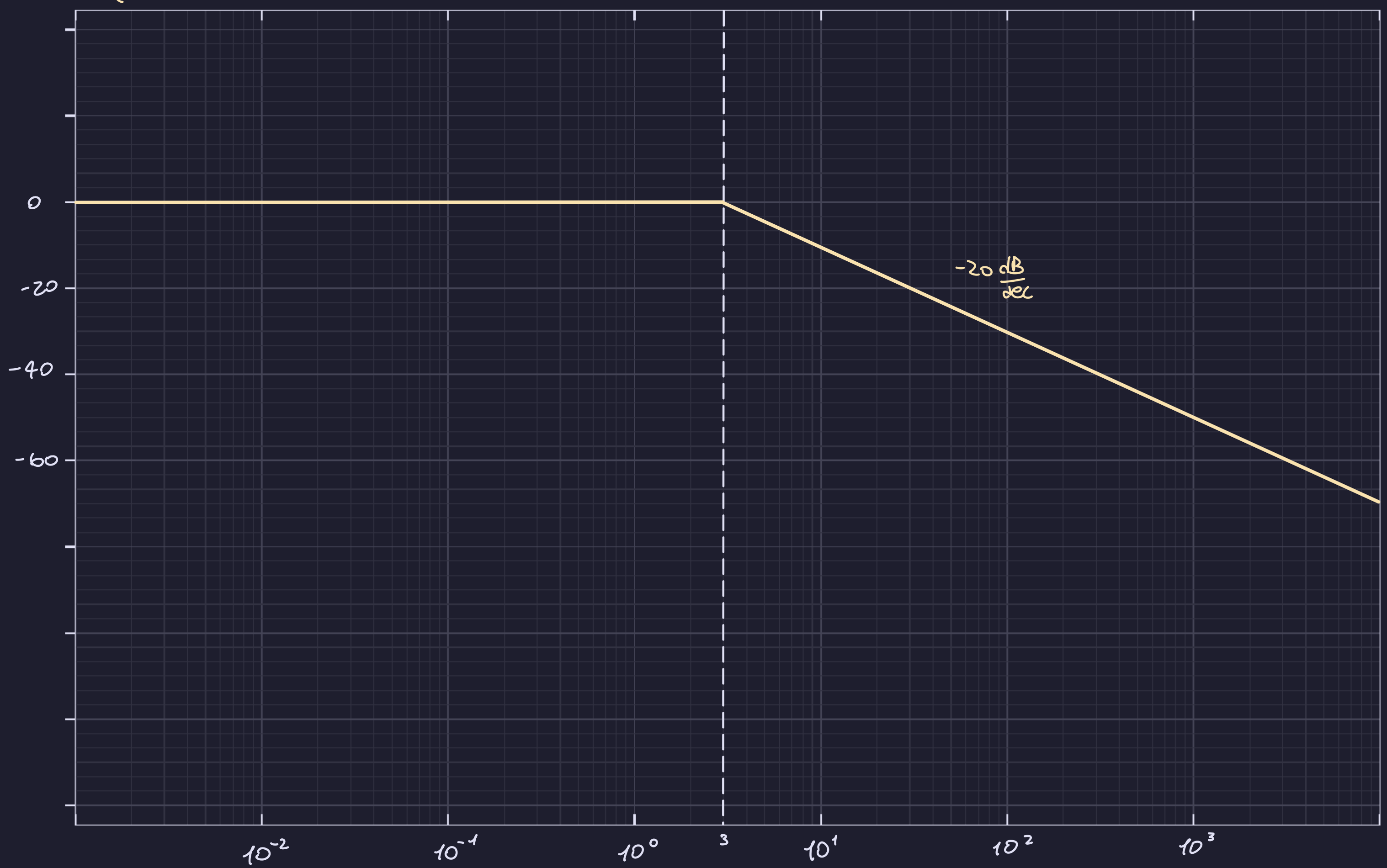
$$A = \begin{cases} 0 & \omega \le \omega_n \\ 20\,\mu \,\frac{dB}{dec} & \omega > \omega_n \end{cases} = -20\,\frac{dB}{dec}$$

$$\phi = \begin{cases} 0 & \omega \le \omega_n \\ 90\,\mu\,\text{segno}(\tau) & \omega \ge \omega_n \end{cases} = 90^\circ$$

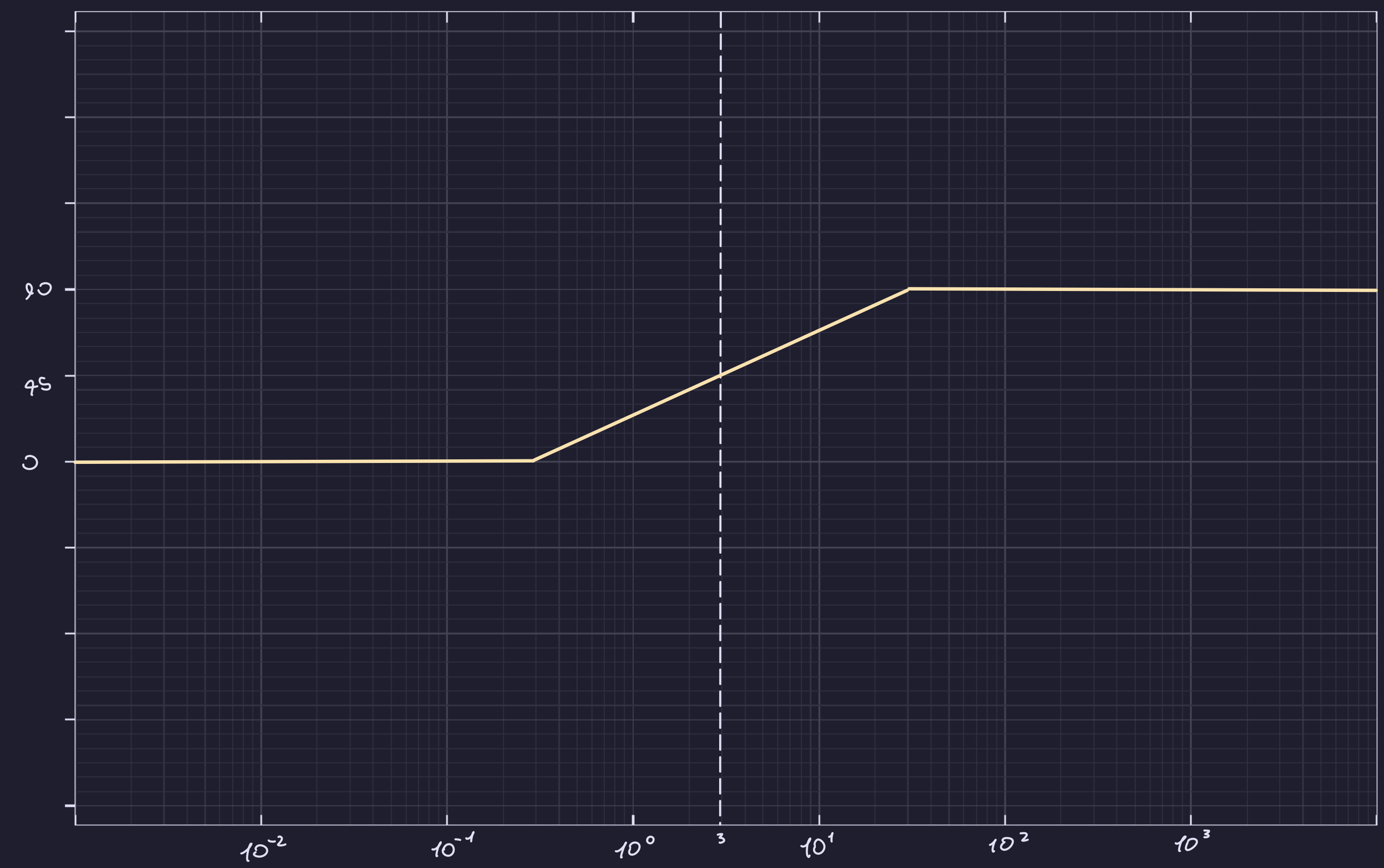
Diagramma di Bode

$$\left(1 - \frac{1}{3}s\right)^{-1} \qquad \tau = -\frac{1}{3} \qquad \omega_n = \frac{1}{|\tau|} = 3$$

Ampiezza



Fase



Ampiezze

	10^{-1}	10^{0}	3	10^{1}	
K_b	32	32	32	32	
Z_n	-40	0	19	40	$20 \cdot \mu \cdot \log_{10}(w)$
P_{r1}	0	0	-10	-20	$20 \cdot \mu \cdot \log_{10}(w\|\tau\|)$
P_{r2}	0	0	0	-10	$20 \cdot \mu \cdot \log_{10}(w\|\tau\|)$
Totale	-8	32	41	42	

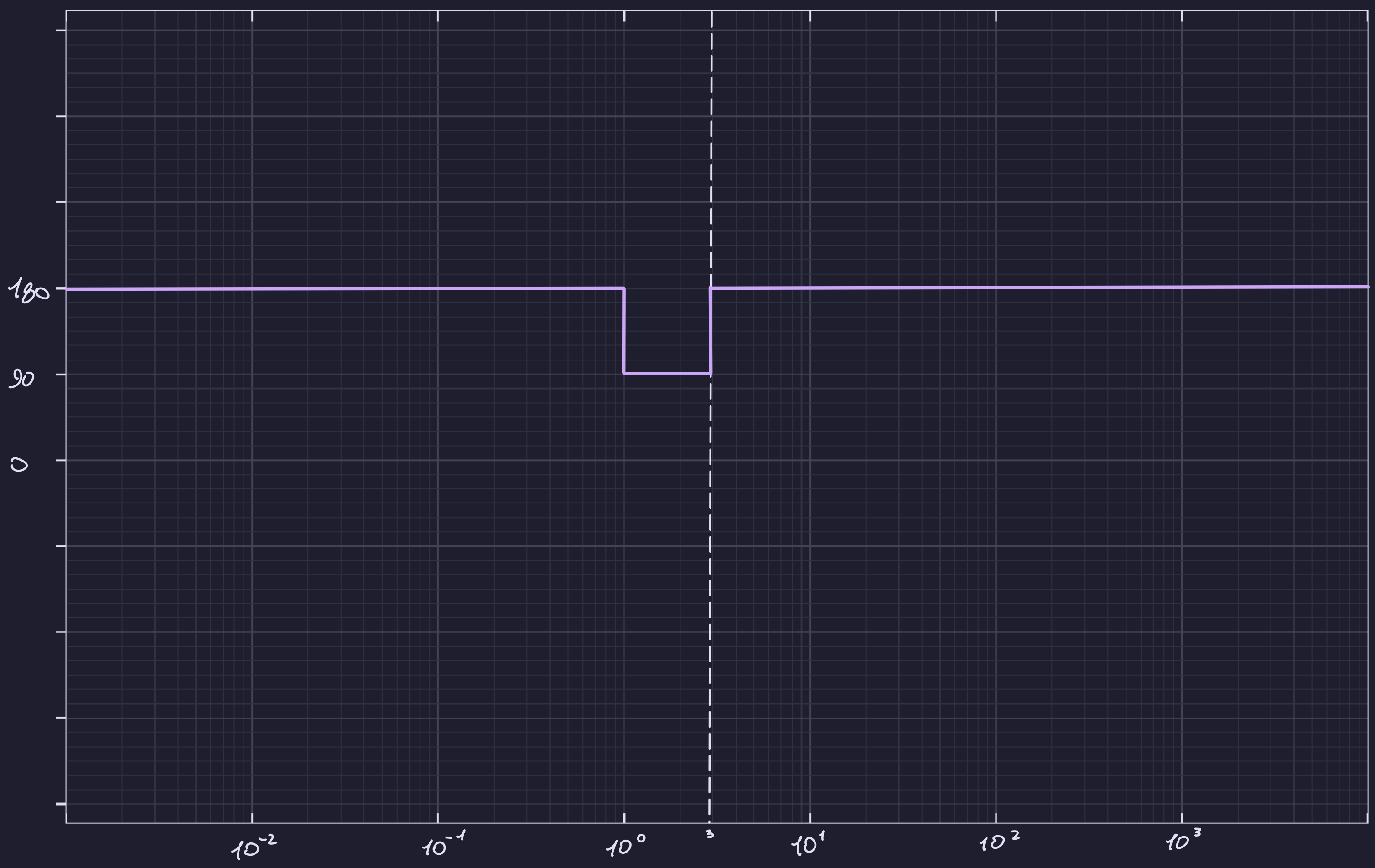
Fasi:

	10^{-1}	10^{0}	3	10^{1}
K_b	0	0	0	0
Z_n	180	180	180	180
P_{r1}	0	0	-90	-90
P_{r2}	0	0	0	90
Totale	180	180	90	180

Diagramma di Bode

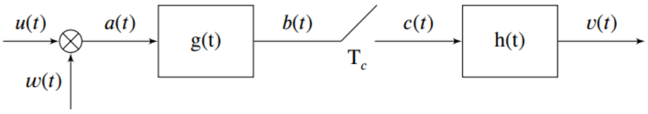
Ampiezza

Fase



Esercizio 4

Dato il seguente schema a blocchi,



$$u(t) = 2\cos(80\pi t) + 3 \qquad g(t) = 16\,\mathrm{sinc}(80t)$$
$$w(t) = 200\,\mathrm{sinc}(20t) \qquad h(t) = \frac{50}{27}\,\mathrm{sinc}(50t) \qquad T_c = \frac{1}{90}s$$

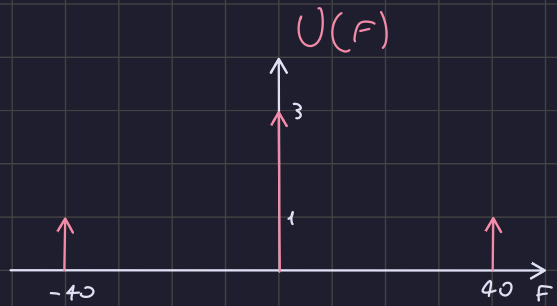
$u(t) = 2\cos(2\pi \cdot 40t) + 3 \quad \xrightarrow{F} \quad U(F) = \delta(F-40) + \delta(F+40) + 3\delta(F)$

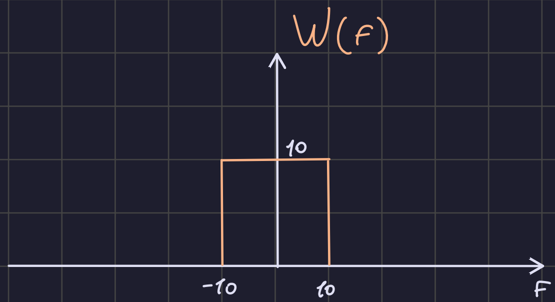
$w(t) = 10 \cdot 20\,\mathrm{sinc}(20t) \quad \xrightarrow{F} \quad W(F) = 10\,\Pi\left(\frac{F}{20}\right)$

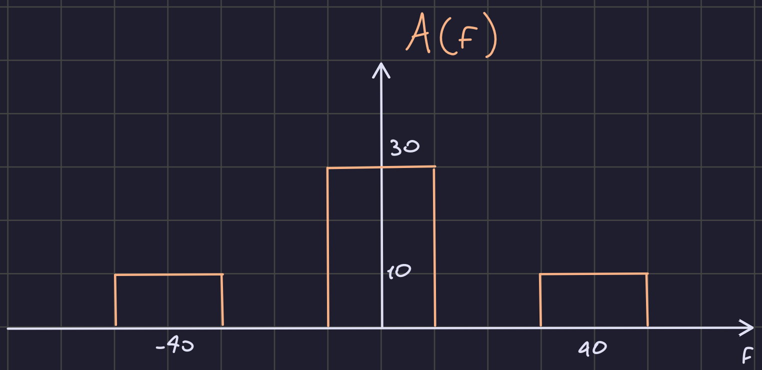
$g(t) = \frac{1}{5}\,80\,\mathrm{sinc}(80t) \quad \xrightarrow{F} \quad G(F) = \frac{1}{5}\,\Pi\left(\frac{F}{80}\right)$

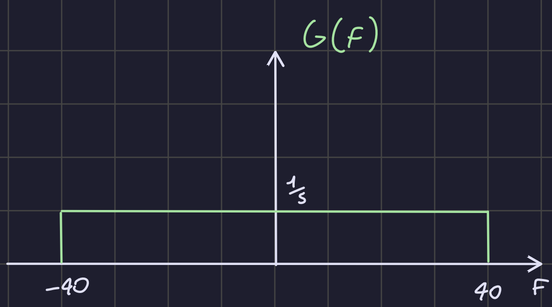
$h(t) = \frac{1}{27}\,50\,\mathrm{sinc}(50t) \quad \xrightarrow{F} \quad H(F) = \frac{1}{27}\,\Pi\left(\frac{F}{50}\right)$

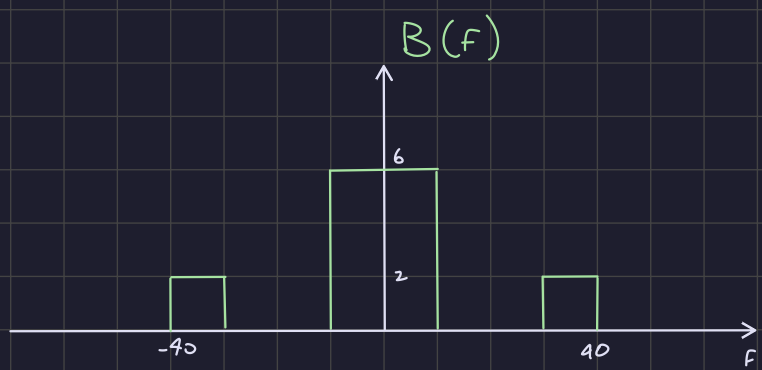
$T_c = \frac{1}{90}s \qquad f_c = \frac{1}{T_c} = 90\ Hz$

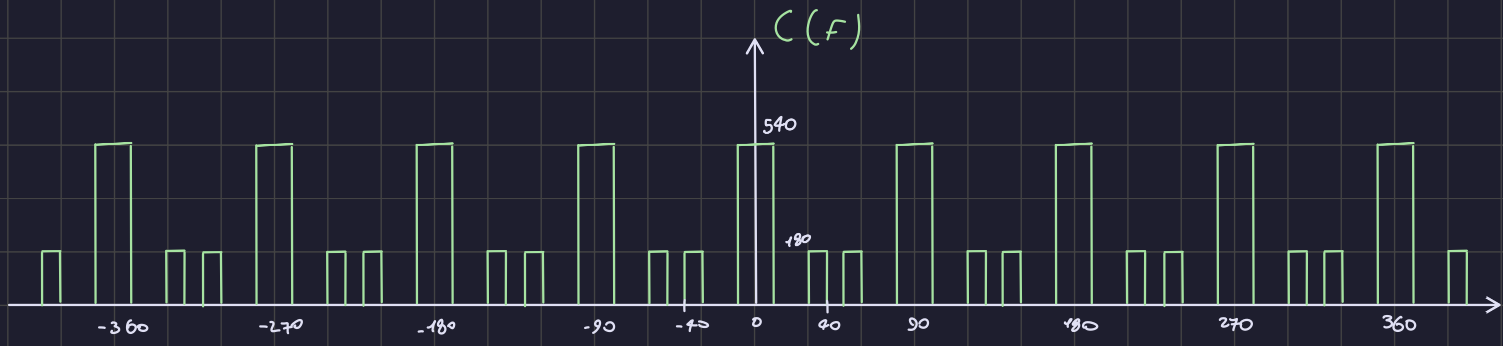


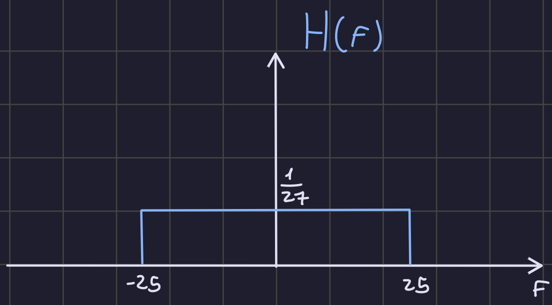


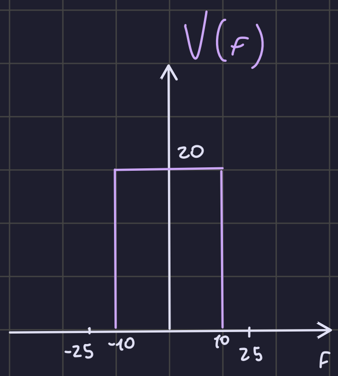












$$V(F) = 20\,\Pi\left(\frac{F}{20}\right)$$

$$\Big\downarrow \mathcal{F}^{-1}$$

$$v(t) = 20 \cdot 20 \,\text{sinc}(20t) = 400\,\text{sinc}(20t)$$

Non è presente aliasing, perchè abbiamo che f_c > 2B dove B = 40, quindi il segnale si può ricostruire.