Esercitazione sistemi discreti

1) $v(k) - 5v(k-1) + 4v(k-2) = u(k)$

$$CI = \begin{cases} v(-1) = \frac{1}{4} \\ v(-2) = -\frac{1}{2} \end{cases}$$

a) Risposta libera

Risolviamo il polinomio caratteristico

$$\lambda^0 - 5\lambda^{-1} + 4\lambda^{-2} = 0$$

$$\downarrow \cdot z^2$$

$$\lambda^2 - 5\lambda + 4 = 0$$

$$\lambda_{1,2} = \frac{5 \pm \sqrt{25-16}}{2} = \frac{5 \pm 3}{2} = \begin{cases} 1 \\ 4 \end{cases}$$

$$\lambda_1 = 1$$

$$\lambda_2 = 4$$

$$v_l(k) = C_1 \cdot 1^k + C_2 \cdot 4^k$$

Troviamo l'equazione specifica utilizzando le condizioni iniziali

$$\begin{cases} v_l(-1) = C_1 \cdot 1^{-1} + C_2 \cdot 4^{-1} \\ v_l(-2) = C_1 \cdot 1^{-2} + C_2 \cdot 4^{-2} \end{cases}$$

$$\downarrow$$

$$\begin{cases} C_1 + C_2 \cdot \frac{1}{4} = \frac{1}{4} \\ C_1 + C_2 \cdot \frac{1}{16} = -\frac{1}{2} \end{cases} \quad \begin{cases} C_1 = \frac{1}{4} - C_2 \cdot \frac{1}{4} = \frac{1-C_2}{4} \\ \frac{1-C_2}{4} + \frac{C_2}{16} = -\frac{1}{2} \end{cases} \quad \begin{cases} C_1 = \frac{1-C_2}{4} \\ \frac{4-4C_2+C_2}{16} = -\frac{1}{2} \end{cases}$$

$$\begin{cases} C_1 = \frac{1-C_2}{4} \\ \frac{4}{16} - \frac{3}{16}C_2 = -\frac{1}{2} \end{cases} \quad \begin{cases} C_1 = \frac{1-C_2}{4} \\ -\frac{3}{16}C_2 = -\frac{1}{2} - \frac{4}{16} \end{cases} \quad \begin{cases} C_1 = \frac{1-C_2}{4} \\ C_2 = \left(-\frac{1}{2} - \frac{4}{16}\right)\left(-\frac{16}{3}\right) \end{cases}$$

$$\begin{cases} c_1 = \frac{1-c_2}{4} \\ c_2 = 4 \end{cases} \qquad \begin{cases} c_1 = -\frac{3}{4} \\ c_2 = 4 \end{cases}$$

$$v_L(k) = -\frac{3}{4} + 4 \cdot 4^k = -\frac{3}{4} + 4^{k+1}$$

b) Risposta libera in frequenza

$$v(k) - 5v(k-1) + 4v(k-2) = 0$$

$$V_L(z) - 5\left(z^{-1}V_L(z) + v(-1)\right) + 4\left(z^{-2}V_L(z) + z^{-1}v(-1) + v(-2)\right) = 0$$

Moltiplichiamo tutto per z^h

$$z^2 V(z) - 5zV_L(z) - z^2 \cdot \frac{5}{4} + 4V_L(z) + z - 2z^2 = 0$$

Raccogliamo per Vl(z)

$$V_L(z)(z^2 - 5z + 4) - z^2 \cdot \frac{5}{4} - 2z^2 + z = 0$$

$$V_L(z)(z^2 - 5z + 4) - \left(\frac{13}{4}z^2 - z\right) = 0$$

$$V_L(z)(z^2 - 5z + 4) = \frac{13}{4}z^2 - z$$

$$V_L(z) = \frac{\frac{13}{4}z^2 - z}{z^2 - 5z + 4} = \frac{\frac{13}{4}z^2 - z}{(z-1)(z-4)}$$

Bisogna ora antitrasformare, quindi si passa ai fratti semplici

$$\frac{\frac{13}{4}z^2 - z}{(z-1)(z-4)} = \frac{A \cdot z}{z-1} + \frac{B \cdot z}{z-4}$$

Abbiamo z in più quindi si deve passare a $\tilde{V}$

$$\tilde{V} = \frac{V_L(z)}{z} = \frac{\cancel{z}\left(\frac{13}{4}z - 1\right)}{(z-1)(z-4)} \cdot \frac{1}{\cancel{z}} = \frac{\frac{13}{4}z - 1}{(z-1)(z-4)}$$

$$\frac{\frac{13}{4}z - 1}{(z-1)(z-4)} = \frac{A}{z-1} + \frac{B}{z-4} = \frac{z(A+B) - 4A - B}{(z-1)(z-4)}$$

$$\begin{cases} A+B=\frac{13}{4} \\ -4A-B=-1 \end{cases} \quad \begin{cases} A+1-4A=\frac{13}{4} \\ B=1-4A \end{cases} \quad \begin{cases} 3A=-\frac{9}{4} \\ B=1-4A \end{cases} \quad \begin{cases} A=-\frac{3}{4} \\ B=1-4A \end{cases}$$

$$\begin{cases} A=-\frac{3}{4} \\ B=1+\cancel{4}\frac{3}{\cancel{4}} \end{cases} \quad \begin{cases} A=-\frac{3}{4} \\ B=4 \end{cases}$$

$$\tilde{V}_c(z)=-\frac{3}{4}\left(\frac{1}{z-1}\right)+4\left(\frac{1}{z-4}\right)$$

$$V_c(z)=\tilde{V}_c(z)\,z=-\frac{3}{4}\left(\frac{z}{z-1}\right)+4\left(\frac{z}{z-4}\right)$$

$$\big\downarrow z^{-1}$$

$$v_c(z)=-\frac{3}{4}+4^{k+1}$$