Conformal Prediction Analysis

FLAVIA SCARFÒ – FABIO TRIPODI

Formal description of Adaptive Conformal Prediction

- **Goal**: Create adaptive prediction intervals $\hat{C}_t$ with valid marginal coverage *1 -* α.
- **Key method**: Dynamically update α_t using:

$$\alpha_{t+1} = \alpha_t + \gamma(\alpha - err_t)$$

$$or, alternatively:$$

$$\alpha_{t+1} = \alpha_t + \gamma(\alpha - \sum_{s=1}^{t} w_s err_s)$$

- **Importance**: Handles non-exchangeable data and shifts in distribution to maintain robust prediction sets.

Standard CP

A standard (non-adaptive) CP strategy with both simple (non-normalized) and studentized or normalized absolute residual scores

Defining the Functions: Key components to implement CP methods for volatility forecasting.

Key Parameters:

Lookback Window: 1250 past observations for quantile calculation.

Alpha Level: α = 0.05 for 95% coverage.

Models:

GARCH(1,1): Captures time-varying volatility through conditional variance.

EGARCH(1,1): Adds asymmetry for modeling leverage effects in returns.

Outputs:

Binary Error Sequence (errSeq): Indicates if the interval covers the observed value.

Coverage Performance: Evaluates empirical coverage relative to target $1 - \alpha$.

Adaptive CP Simple

An adaptive CP strategy with both simple (non-normalized) and studentized or normalized absolute residual scores

Eq. (2) in Section 2.2: since the assumption of exchangeability is relaxed, α is dynamically updated with this equation:

$$\alpha_{t+1} = \alpha_t + \gamma(\alpha - err_t).$$

γ: Adaptation rate (step size). Its choice gives a trade-off between adaptability and stability.

err_t: Binary indicator for interval coverage error.

Outputs:

Binary Error Sequence (errSeq): Indicates if the interval covers the observed value.

Coverage Performance: Evaluates empirical coverage relative to target $1 - \alpha$.

Alpha Sequence: adjusted α over time.

Adaptive CP Momentum

An adaptive CP strategy with studentized or normalized absolute residual scores

Eq. (3) in Section 2.2: Here, α is updated dynamically using a new equation that incorporates momentum-based weights:

$$\alpha_{t+1} = \alpha_t + \gamma(\alpha - \textstyle\sum_{s=1}^{t} w_s err_s).$$

New parameter: historical errors are assigned weights using an exponential decay factor (*momentumBW*):

$$w_s := \frac{0.95^{t-s}}{\sum_{s'=1}^{t} 0.95^{t-s'}}$$

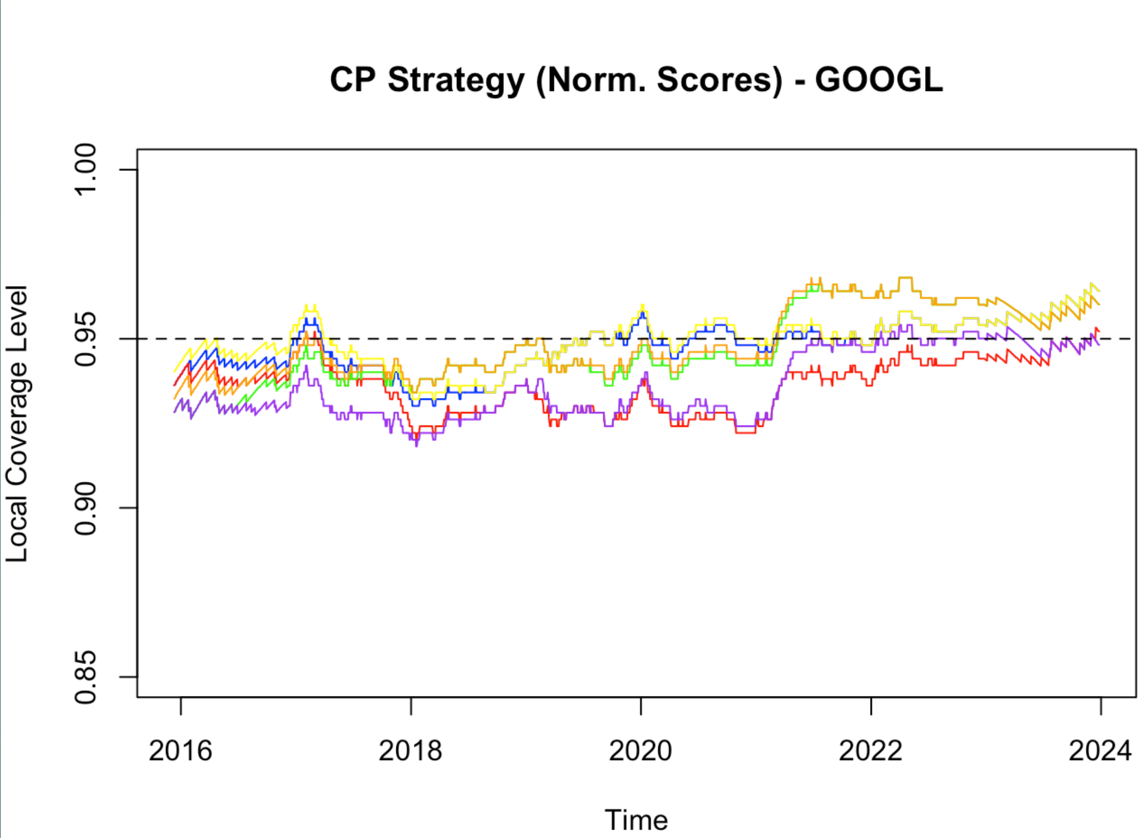
Outputs:

Binary Error Sequence (errSeq): Indicates if the interval covers the observed value.

Coverage Performance: Evaluates empirical coverage relative to target $1 - \alpha$.

Alpha Sequence: adjusted α over time.

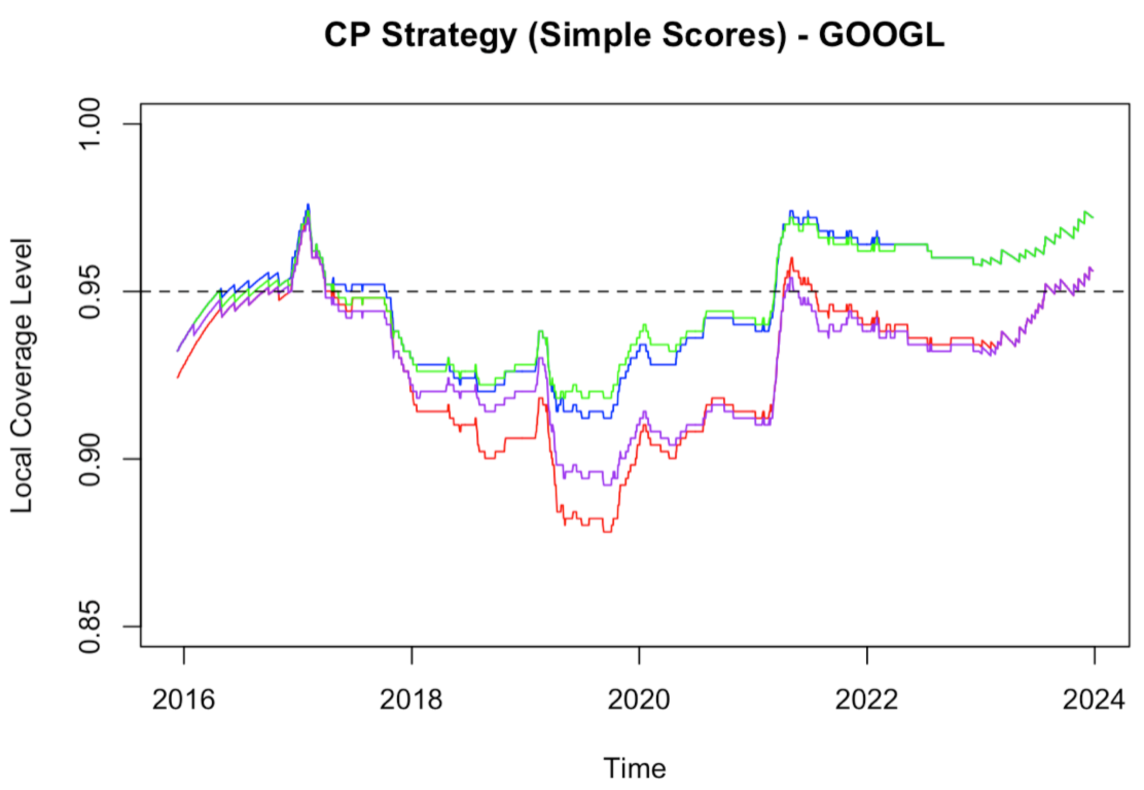
Comparison of CP strategies using normalized scores



- Upload Google data;
- Define the function to compute local coverage data;
- Compare different strategies:
  - blue: Adaptive GARCH
  - red: Standard GARCH
  - green: Adaptive EGARCH
  - purple: Standard EGARCH
  - yellow: Adaptive GARCH with momentum
  - orange: Adaptive EGARCH with momentum

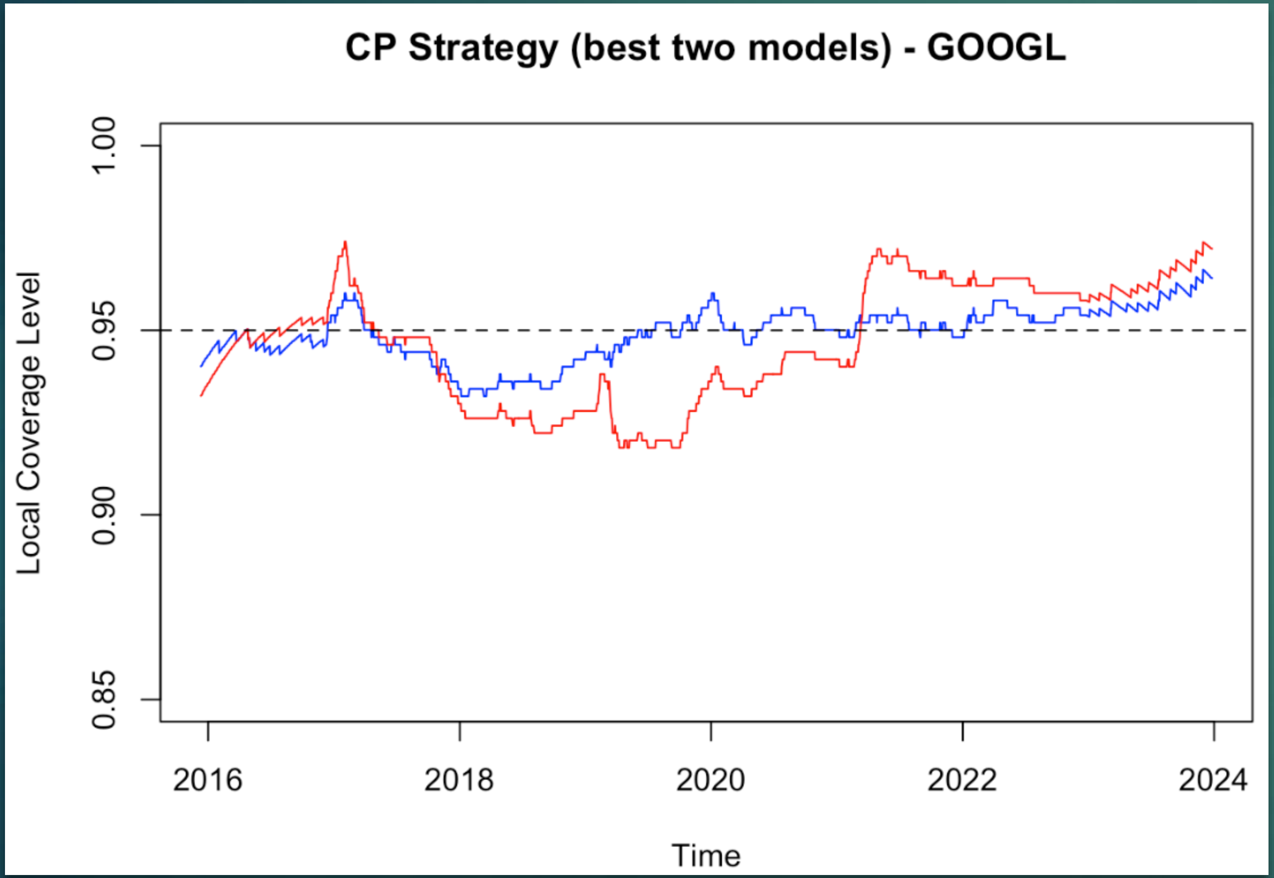
The best performing strategy is the Adaptive CP momentum with GARCH(1,1) (yellow line).

Comparison of CP strategies using Simple Scores



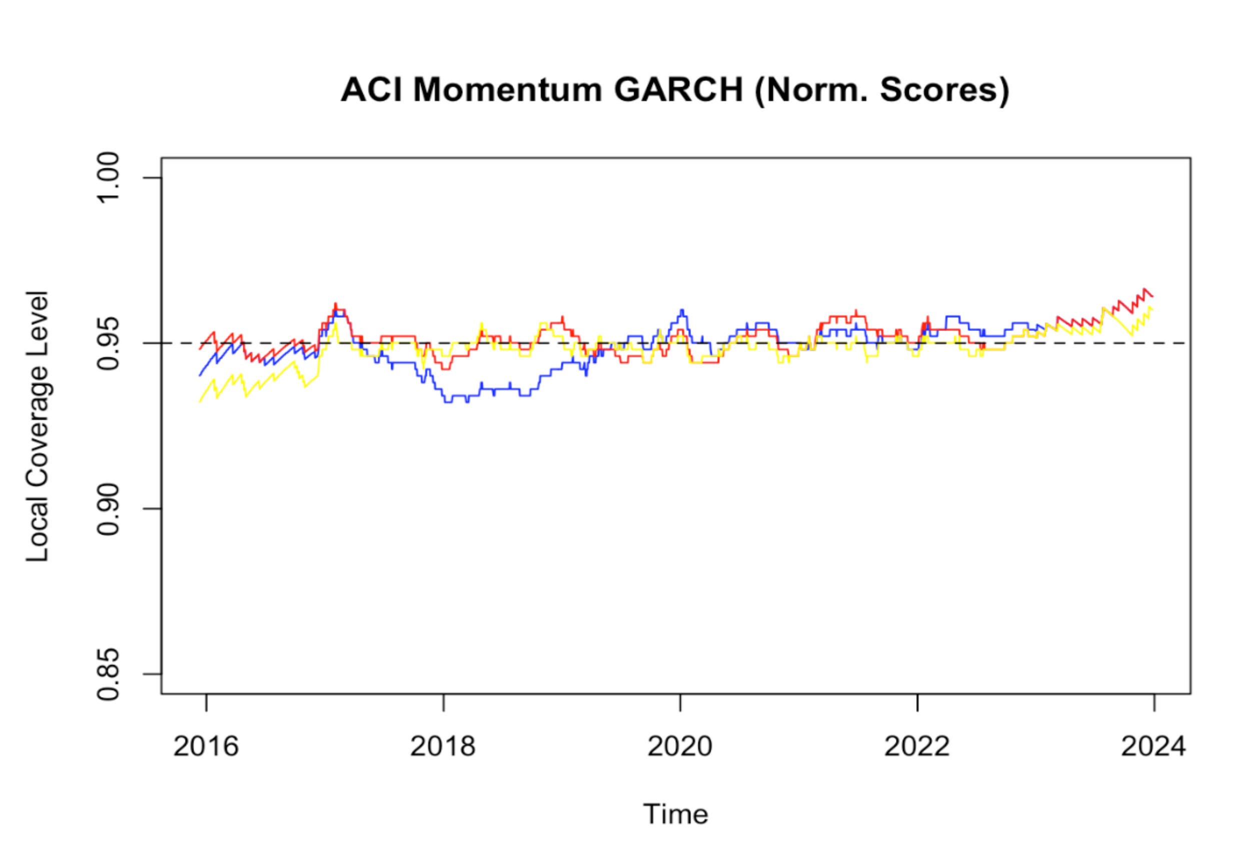
- Compare different strategies:
  - blue: Adaptive GARCH
  - red: Standard GARCH
  - green: Adaptive EGARCH
  - purple: Standard EGARCH
- The best performing strategy this time is EGARCH model with Adaptive strategy simple, highlighted in green.

Comparison of the Two Best Models: Adaptive Momentum GARCH vs. Adaptive EGARCH



- Compare two different strategies:
  - red: Adaptive EGARCH with simple scores
  - blue: Adaptive Momentum GARCH with Normalized scores
- The overall best performing strategy is the Adaptive Momentum GARCH with Normalized scores (blue).

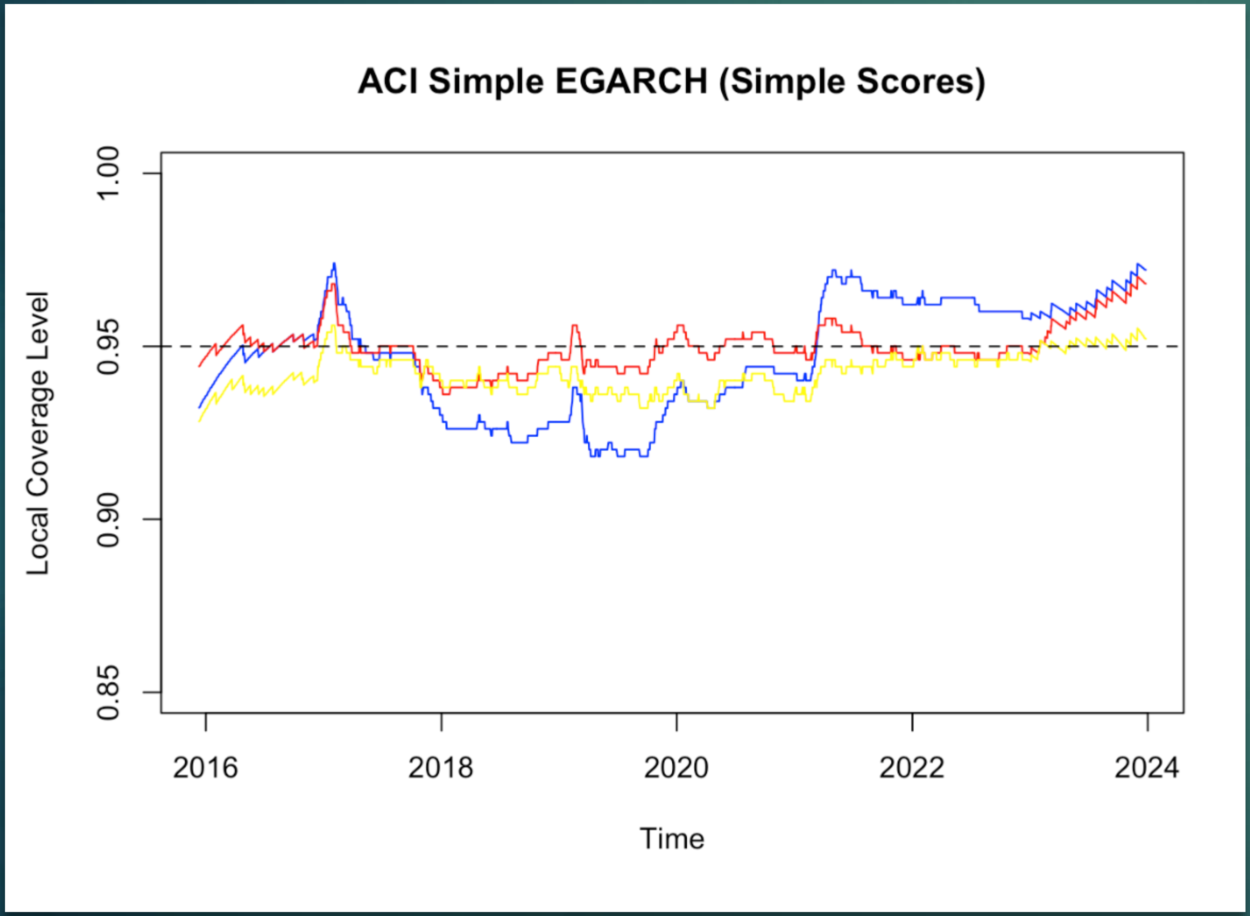
Comparison of ACI Momentum GARCH with Normalized Scores Across Different Gamma Values



- Adaptive CP Momentum with GARCH model and normalized scores with gamma values equal to {0.001, 0.01, 0.05}
  - Blue: $\gamma_1 = 0.001$
  - Red: $\gamma_2 = 0.01$
  - Yellow: $\gamma 3 = 0.05$

The best is the red line with $\gamma = 0.01$

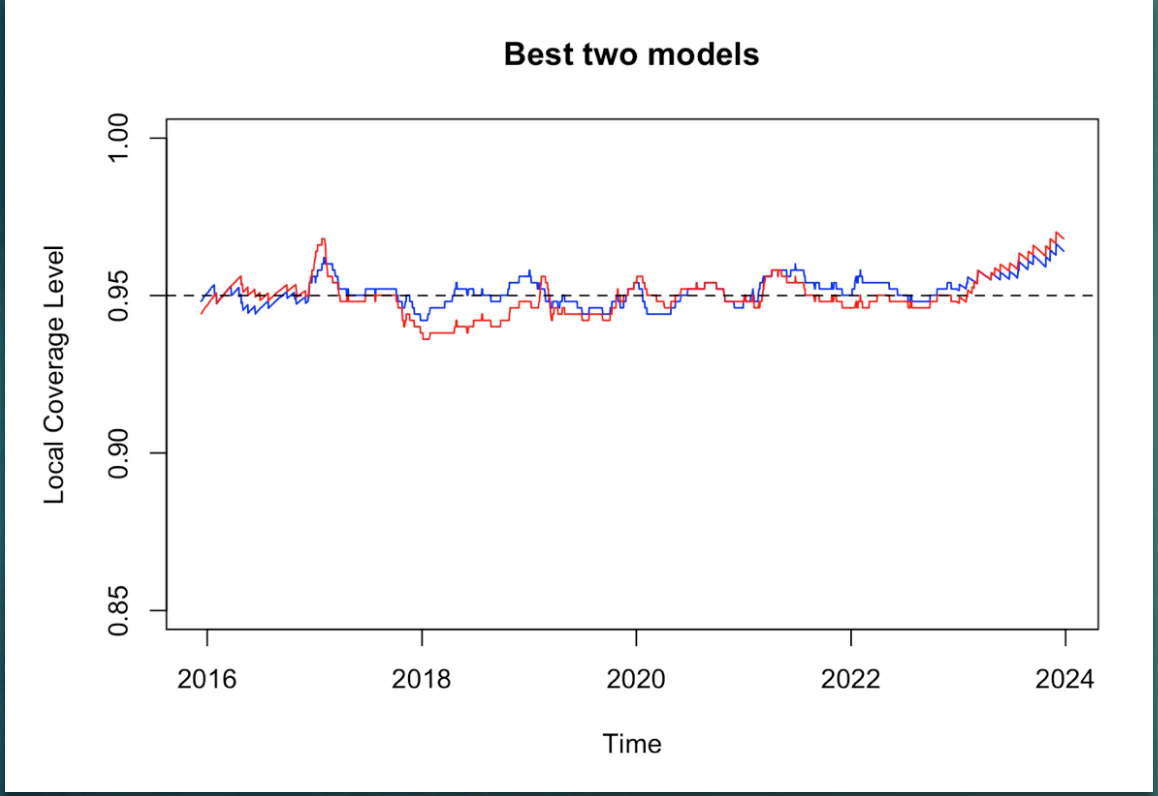
Comparison of ACI Simple EGARCH with Simple Scores Across Different Gamma Values



- Adaptive CP with EGARCH model and simple scores with gamma values equal to {0.001, 0.01, 0.05}
  - Blue: $\gamma_1 = 0.001$
  - Red: $\gamma_2 = 0.01$
  - Yellow: $\gamma3 = 0.05$

The best is the red line with $\gamma = 0.01$

Comparison of the Two Best Models: ACI Momentum GARCH vs ACI Simple EGARCH



- ACI Momentum GARCH with Normalized Scores: the blue line is the best performance
- ACI Simple EGARCH with Simple Scores: the red line.

Formal description of AgACI

- **Purpose of AgACI**: Eliminates the need for manual selection of the γ parameter, making the method parameter-free.
- **Aggregation Mechanism:** Aggregates predictions from k-ACI models (experts), each initialized with a different γ. Combines predictions using the Bayesian Online Aggregation (BOA) method.
- **Weighting System:** Weights are assigned dynamically based on past performance using the pinball loss function. Better-performing models receive higher weights, while poorly performing models are minimized.
- **Adaptive Intervals:** Predictive intervals are computed dynamically by weighting bounds across all experts.

Why GARCH instead of Quantile Regression in AgACI?

- Continuity with prior results:
  - Exercise 1 identified GARCH as the best-performing model.
  - Using GARCH in AgACI ensures consistency and enables a direct comparison with ACI Momentum.
- Focus on volatility:
  - GARCH is designed to model volatility in financial data.
  - It adapts on clustering volatility.

Function AgACI Volatility Forecasting

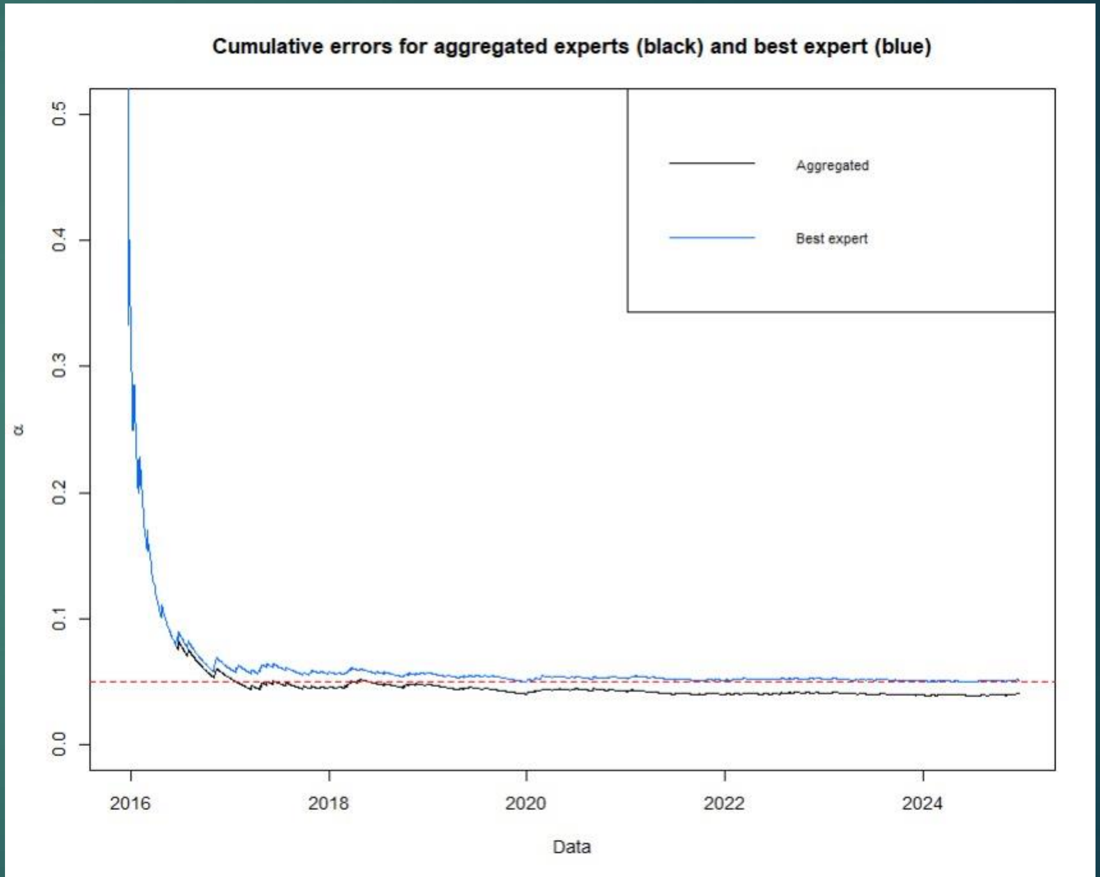
- Combines GARCH(1,1) forecasting with:
  - Adaptive Conformal Inference (ACI)
  - Bayesian Online Aggregation (BOA).
- K experts with k different γ: {0.001, 0.004, 0.007, 0.01, 0.013}

- **Outputs**:
  - *aggregatedalpha*: Final aggregated alpha values.
  - *err*: Error sequence for aggregated predictions.
  - *weights_df*: Weights assigned to experts by BOA.
  - *garchForecastVec*: Forecasted variances.
  - *alphaSeq*: Alpha updates for each expert.
- **Key Advantage**:
  - Enhances prediction accuracy by dynamically combining multiple expert models.

Comparison of Aggregated Experts vs Best Expert

- Cumulative errors of:
  - Aggregated model: black line
  - Best-performing expert ($\gamma_4 = 0.01$): blue line

The AgACI aggregated strategy minimizes the error, but the best-performing expert is nearer to the 95% threshold.



Comparison of Best Model (Part I) vs AgACI

- Local coverage level of:
  - Best Model (Part I): blue line
  - AgACI Aggregate: red line
- The best (contestably) performing model is the Best Model from Part I, which is the Adaptive Momentum GARCH with normalized scores.