Formal Language Selected Homework Chapter 5.1

7. Find context-free grammars for the following languages (with $n \geq 0$, $m \geq 0$).

(a) $L = \{a^n b^m : n \leq m + 3\}$.

(b) $L = \{a^n b^m : n \neq m - 1\}$.

(c) $L = \{a^n b^m : n \neq 2m\}$.

(d) $L = \{a^n b^m : 2n \leq m \leq 3n\}$.

(e) $L = \{w \in \{a, b\}^* : n_a(w) \neq n_b(w)\}$.

(f) $L = \{w \in \{a, b\}^* : n_a(v) \geq n_b(v)$, where v is any prefix of $w\}$.

8. Find context-free grammars for the following languages (with $n \geq 0$, $m \geq 0, k \geq 0$).

(a) $L = \{a^n b^m c^k : n = m$ or $m \leq k\}$.

(b) $L = \{a^n b^m c^k : n = m$ or $m \neq k\}$.

(c) $L = \{a^n b^m c^k : k = n + m\}$.

(d) $L = \{a^n b^m c^k : n + 2m = k\}$.

(e) $L = \{a^n b^m c^k : k = |n - m|\}$.

(f) $L = \{w \in \{a, b, c\}^* : n_a(w) + n_b(w) \neq n_c(w)\}$.

(g) $L = \{a^n b^m c^k, k \neq n + m\}$.

(h) $L = \{a^n b^n c^k : k \geq 3\}$.

13. Let $L = \{a^n b^n : n \geq 0\}$.

(a) Show that L^2 is context-free.

(b) Show that L^k is context-free for any given $k \geq 1$.

20. Consider the grammar with productions

$$S \rightarrow aaB,$$
$$A \rightarrow bBb|\lambda,$$
$$B \rightarrow Aa.$$

Show that the string *aabbabba* is not in the language generated by this grammar.

23. Find a context-free grammar for the set of all regular expressions on the alphabet $\{a, b\}$.