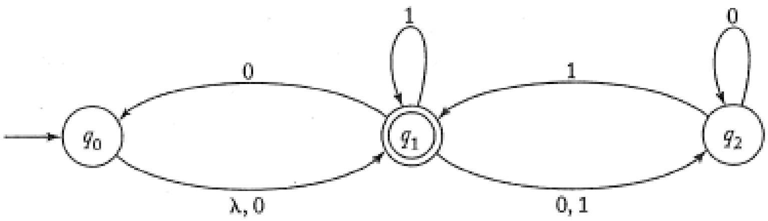
Formal Language Selected Homework Chapter 2.3

1. Use the construction of Theorem 2.2 to convert the nfa in Figure 2.10 to a dfa. Can you see a simpler answer more directly?

2. Convert the nfa in Exercise 12, Section 2.2, into an equivalent dfa.

3. Convert the following nfa into an equivalent dfa.



4. Carefully complete the arguments in the proof of Theorem 2.2. Show in detail that if the label of $\delta_D^*(q_0, w)$ contains q_f, then $\delta_N^*(q_0, w)$ also contains q_f.

5. Is it true that for any nfa $M = (Q, \Sigma, \delta, q_0, F)$ the complement of $L(M)$ is equal to the set $\{w \in \Sigma^* : \delta^*(q_0, w) \cap F = \varnothing\}$? If so, prove it. If not, give a counterexample.

6. Is it true that for every nfa $M = (Q, \Sigma, \delta, q_0, F)$ the complement of $L(M)$ is equal to the set $\{w \in \Sigma^* : \delta^*(q_0, w) \cap (Q - F) \neq \varnothing\}$? If so, prove it; if not, give a counterexample.

7. Prove that for every nfa with an arbitrary number of final states there is an equivalent nfa with only one final state. Can we make a similar claim for dfa's?

8. Find an nfa without λ-transitions and with a single final state that accepts the set $\{a\} \cup \{b^n : n \geq 1\}$.

9. Let L be a regular language that does not contain λ. Show that there exists an nfa without λ-transitions and with a single final state that accepts L.

10. Define a dfa with multiple initial states in an analogous way to the corresponding nfa in Exercise 18, Section 2.2. Does there always exist an equivalent dfa with a single initial state?

11. Prove that all finite languages are regular.

12. Show that if L is regular, so is L^R.

13. Give a simple verbal description of the language accepted by the dfa in Figure 2.16. Use this to find another dfa, equivalent to the given one, but with fewer states.

★14. Let L be any language. Define $even(w)$ as the string obtained by extracting from w the letters in even-numbered positions; that is, if

$$w = a_1 a_2 a_3 a_4 \ldots,$$

then

$$even(w) = a_2 a_4 \ldots.$$

Corresponding to this, we can define a language

$$even(L) = \{even(w) : w \in L\}.$$

Prove that if L is regular, so is $even(L)$.

15. From a language L we create a new language $chop2(L)$ by removing the two leftmost symbols of every string in L. Specifically,

$$chop2(L) = \{w : vw \in L, \text{ with } |v| = 2\}.$$

Show that if L is regular, then $chop2(L)$ is also regular.