Formal Language Selected Homework Chapter 3.3

4. Construct right- and left-linear grammars for the language

$$L = \{a^n b^m : n \geq 2, m \geq 3\}.$$

~~**8.**~~ In ~~Theorem 3.5, prove that $L(\widehat{G}) = (L(G))$~~^{R}. (skip)

11. Find a regular grammar for the language $L = \{a^n b^m : n + m \text{ is even}\}$.

13. Find regular grammars for the following languages on $\{a, b\}$.

(a) $L = \{w : n_a(w) \text{ and } n_b(w) \text{ are both even}\}$.

(b) $L = \{w : (n_a(w) - n_b(w)) \bmod 3 = 1\}$.

17. Let $G_1 = (V_1, \Sigma, S_1, P_1)$ be right-linear and $G_2 = (V_2, \Sigma, S_2, P_2)$ be a left-linear grammar, and assume that V_1 and V_2 are disjoint. Consider the linear grammar $G = (\{S\} \cup V_1 \cup V_2, \Sigma, S, P)$, where S is not in $V_1 \cup V_2$ and $P = \{S \to S_1 | S_2\} \cup P_1 \cup P_2$. Show that $L(G)$ is regular.

Sol. **4.** Right linear grammar:

$$S \longrightarrow aaA$$
$$A \longrightarrow aA|B$$
$$B \longrightarrow bbbC$$
$$C \longrightarrow bC|\lambda$$

Left linear grammar:

$$S \longrightarrow Abbb$$
$$A \longrightarrow Ab|B$$
$$B \longrightarrow Caa$$
$$C \longrightarrow Ca|\lambda$$

11. Split this into two cases: (i) n and m are both even and (ii) n and m are both odd. The solution then falls out easily, with

$$S \to aaS|A$$
$$A \to bbA|\lambda$$

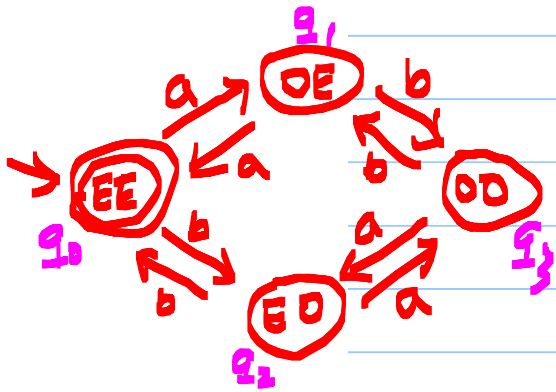
(ii) $S' \to aaS'|aB$

$B \to bbB|b$

taking care of case (i). $\therefore$ Initial variable $S_0 \to S|S'$

13. (a) First construct a dfa for L. This is straightforward and gives transitions such as

$$\delta(q_0, a) = q_1, \delta(q_0, b) = q_2,$$
$$\delta(q_1, a) = q_0, \delta(q_1, b) = q_3,$$
$$\delta(q_2, a) = q_3, \delta(q_2, b) = q_0,$$
$$\delta(q_3, a) = q_2, \delta(q_3, b) = q_1,$$

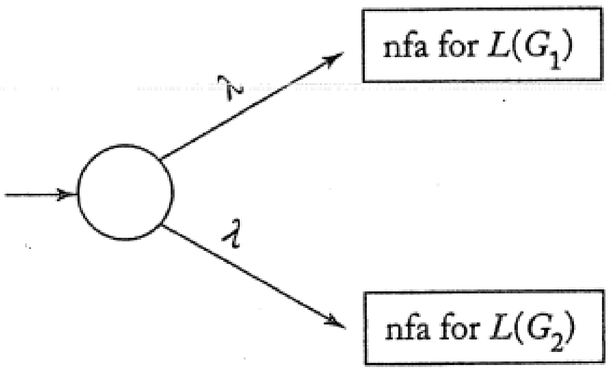


with q_0 the initial and final state. Then the construction of Theorem 3.4 gives the answer

$$\begin{aligned} q_0 &\to aq_1 \,|bq_2|\, \lambda, \\ q_1 &\to bq_3|aq_0, \\ q_2 &\to aq_3|bq_0, \\ q_3 &\to aq_2|bq_1. \end{aligned}$$

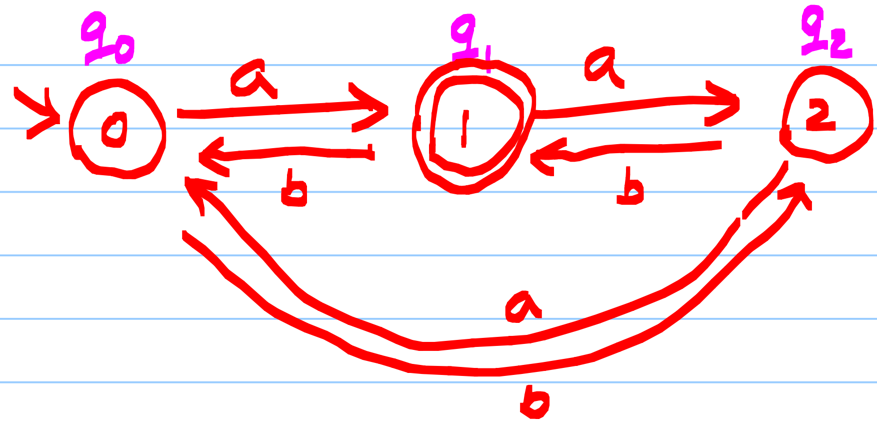
(b) see next page.

17. Obviously, $L(G_1)$ is regular, as is $L(G_2)$. We can show that their union is also regular by constructing the following dfa.



The condition that V_1 and V_2 should be disjoint is essential so that the two nfa's are distinct.

(3.(b)



$\delta(q_0, a) = q_1$

$\delta(q_1, a) = q_2$

$\delta(q_2, a) = q_0$

$\delta(q_0, b) = q_2$

$\delta(q_2, b) = q_1$

$\delta(q_1, b) = q_0$

$$\Rightarrow \quad q_0 \rightarrow a q_1 \mid b q_2$$

$$q_1 \rightarrow a q_2 \mid b q_0 \mid \lambda$$

$$q_2 \rightarrow a q_0 \mid b q_1$$