Formal Language Selected Homework Chapter 2.2

1. Prove in detail the claim made in the previous section that if in a transition graph there is a walk labeled w, there must be some walk labeled w of length no more than $\Lambda + (1 + \Lambda)|w|$.

2. Find a dfa that accepts the language defined by the nfa in Figure 2.8.

3. Find a dfa that accepts the complement of the language defined by the nfa in Figure 2.8.

4. In Figure 2.9, find $\delta^*(q_0, 1011)$ and $\delta^*(q_1, 01)$.

5. In Figure 2.10, find $\delta^*(q_0, a)$ and $\delta^*(q_1, \lambda)$.

6. For the nfa in Figure 2.9, find $\delta^*(q_0, 1010)$ and $\delta^*(q_1, 00)$.

7. Design an nfa with no more than five states for the set $\{abab^n : n \geq 0\} \cup \{aba^n : n \geq 0\}$.

8. Construct an nfa with three states that accepts the language $\{ab, abc\}^*$.

9. Do you think Exercise 8 can be solved with fewer than three states?

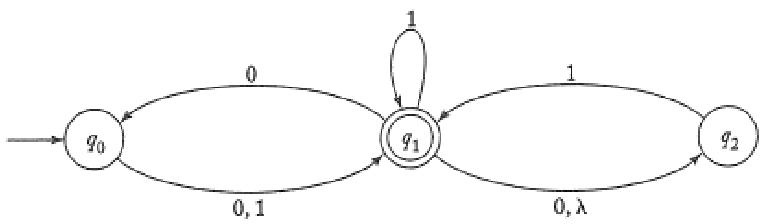
10. (a) Find an nfa with three states that accepts the language

$$L = \{a^n : n \geq 1\} \cup \left\{b^m a^k : m \geq 0, k \geq 0\right\}.$$

(b) Do you think the language in part (a) can be accepted by an nfa with fewer than three states?

11. Find an nfa with four states for $L = \{a^n : n \geq 0\} \cup \{b^n a : n \geq 1\}$.

12. Which of the strings 00, 01001, 10010, 000, 0000 are accepted by the following nfa?



13. What is the complement of the language accepted by the nfa in Figure 2.10?

14. Let L be the language accepted by the nfa in Figure 2.8. Find an nfa that accepts $L \cup \{a^5\}$.

15. Give a simple description of the language in Exercise 13.

16. Find an nfa that accepts $\{a\}^*$ and is such that if in its transition graph a single edge is removed (without any other changes), the resulting automaton accepts $\{a\}$.

17. Can Exercise 16 be solved using a dfa? If so, give the solution; if not, give convincing arguments for your conclusion.

18. Consider the following modification of Definition 2.6. An nfa with multiple initial states is defined by the quintuple

$$M = (Q, \Sigma, \delta, Q_0, F),$$

where $Q_0 \subseteq Q$ is a set of possible initial states. The language accepted by such an automaton is defined as

Show that for every nfa with multiple initial states there exists an nfa with a single initial state that accepts the same language.

19. Suppose that in Exercise 18 we made the restriction $Q_0 \cap F = \varnothing$. Would this affect the conclusion?

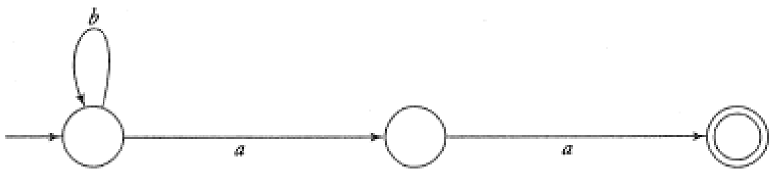
20. Use Definition 2.5 to show that for any nfa

$$\delta^* (q, wv) = \bigcup_{p \in \delta^* (q,w)} \delta^* (p, v),$$

for all $q \in Q$ and all $w, v \in \Sigma^*$.

21. An nfa in which (a) there are no λ-transitions, and (b) for all $q \in Q$ and all $a \in \Sigma$, $\delta (q, a)$ contains at most one element, is sometimes called an **incomplete** dfa. This is reasonable since the conditions make it such that there is never any choice of moves.

For $\Sigma = \{a, b\}$, convert the incomplete dfa below into a standard dfa.



22. Let L be a regular language on some alphabet Σ, and let $\Sigma_1 \subset \Sigma$ be a smaller alphabet. Consider L_1, the subset of L whose elements are made up only of symbols from Σ_1, that is,

$$L_1 = L \cap \Sigma_1^*.$$

Show that L_1 is also regular.