Formal Language Selected Homework Chapter 2.1

EXERCISES

1. Which of the strings 0001, 01001, 0000110 are accepted by the dfa in Figure 2.1?

2. For $\Sigma = \{a, b\}$, construct dfa's that accept the sets consisting of

   (a) all strings with exactly one a,

   (b) all strings with at least one a,

   (c) all strings with no more than three a's,

   (d) all strings with at least one a and exactly two b's,

   (e) all the strings with exactly two a's and more than two b's.

3. Show that if we change Figure 2.6, making q_3 a nonfinal state and making q_0, q_1, q_2 final states, the resulting dfa accepts $\overline{L}$.

4. Generalize the observation in the previous exercise. Specifically, show that if $M = (Q, \Sigma, \delta, q_0, F)$ and $\widehat{M} = (Q, \Sigma, \delta, q_0, Q - F)$ are two dfa's, then $\overline{L(M)} = L\left(\widehat{M}\right)$.

5. Give dfa's for the languages

   (a) $L = \{ab^5wb^2 : w \in \{a, b\}^*\}$,

   (b) $L = \{ab^na^m : n \geq 2, m \geq 3\}$,

   (c) $L = \{w_1abw_2 : w_1 \in \{a, b\}^*, w_2 \in \{a, b\}^*\}$,

   (d) $L = \{ba^n : n \geq 1, n \neq 5\}$.

6. With $\Sigma = \{a, b\}$, give a dfa for $L = \{w_1aw_2 : |w_1| \geq 3, |w_2| \leq 5\}$.

7. Find dfa's for the following languages on $\Sigma = \{a, b\}$.

   (a) $L = \{w : |w| \mod 3 = 0\}$.

   (b) $L = \{w : |w| \mod 5 \neq 0\}$.

   (c) $L = \{w : n_a(w) \mod 3 > 1\}$.

   (d) $L = \{w : n_a(w) \mod 3 > n_b(w) \mod 3\}$.

   (e) $L = \{w : (n_a(w) - n_b(w)) \mod 3 > 0\}$.

   (f) $L = \{w : (n_a(w) + 2n_b(w)) \mod 3 < 2\}$.

   (g) $L = \{w : |w| \mod 3 = 0, |w| \neq 6\}$.

★ 8. A run in a string is a substring of length at least two, as long as possible and consisting entirely of the same symbol. For instance, the string $abbbaab$ contains a run of b's of length three and a run of a's of length two. Find dfa's for the following languages on $\{a, b\}$.

   (a) $L = \{w : w$ contains no runs of length less than four$\}$.

   (b) $L = \{w :$ every run of a's has length either two or three$\}$.

(c) $L = \{w :$ there are at most two runs of a's of length three$\}$.

(d) $L = \{w :$ there are exactly two runs of a's of length $3\}$.

9. Consider the set of strings on $\{0, 1\}$ defined by the requirements below. For each, construct an accepting dfa.

(a) Every 00 is followed immediately by a 1. For example, the strings 101, 0010, 0010011001 are in the language, but 0001 and 00100 are not.

(b) All strings containing 00 but not 000.

(c) The leftmost symbol differs from the rightmost one.

(d) Every substring of four symbols has at most two 0's. For example, 001110 and 011001 are in the language, but 10010 is not since one of its substrings, 0010, contains three zeros.

(e) All strings of length five or more in which the fourth symbol from the right end is different from the leftmost symbol.

(f) All strings in which the leftmost two symbols and the rightmost two symbols are identical.

(g) All strings of length four or greater in which the leftmost three symbols are the same, but different from the rightmost symbol.

★ **10.** Construct a dfa that accepts strings on $\{0, 1\}$ if and only if the value of the string, interpreted as a binary representation of an integer, is zero modulo five. For example, 0101 and 1111, representing the integers 5 and 15, respectively, are to be accepted.

11. Show that the language $L = \{vwv : v, w \in \{a, b\}^*, |v| = 2\}$ is regular.

12. Show that $L = \{a^n : n \geq 4\}$ is regular.

13. Show that the language $L = \{a^n : n \geq 0, n \neq 4\}$ is regular.

14. Show that the language $L = \{a^n : n$ is either a multiple of three or a multiple of 5 is regular.

15. Show that the language $L = \{a^n : n$ is a multiple of three, but not a multiple of 5 is regular.

16. Show that the set of all real numbers in C is a regular language.

17. Show that if L is regular, so is $L - \{\lambda\}$.

18. Show that if L is regular, so is $L \cup \{a\}$, for all $a \in \Sigma$.

19. Use (2.1) and (2.2) to show that

$$\delta^* (q, wv) = \delta^* (\delta^* (q, w), v)$$

for all $w, v \in \Sigma^*$.

20. Let L be the language accepted by the automaton in Figure 2.2. Find a dfa that accepts L^2.

21. Let L be the language accepted by the automaton in Figure 2.2. Find a dfa for the language $L^2 - L$.

22. Let L be the language in Example 2.5. Show that L^* is regular.

23. Let G_M be the transition graph for some dfa M. Prove the following.

(a) If $L(M)$ is infinite, then G_M must have at least one cycle for which there is a path from the initial vertex to some vertex in the cycle and a path from some vertex in the cycle to some final vertex.

(b) If $L(M)$ is finite, then no such cycle exists.

24. Let us define an operation *truncate*, which removes the rightmost symbol from any string. For example, *truncate* $(aaaba)$ is $aaab$. The operation can be extended to languages by

$$truncate\,(L) = \{truncate\,(w) : w \in L\}.$$

Show how, given a dfa for any regular language L, one can construct a dfa for *truncate* (L). From this, prove that if L is a regular language not containing λ, then *truncate* (L) is also regular.

25. While the language accepted by a given dfa is unique, there are normally many dfa's that accept a language. Find a dfa with exactly six states that accepts the same language as the dfa in Figure 2.4.

26. Can you find a dfa with three states that accepts the language of the dfa in Figure 2.4? If not, can you give convincing arguments that no such dfa can exist?