Formal Language Selected Homework Chapter 4.1

- 2. Use the construction in Theorem 4.1 to find nfa's that accept
 - (a) $L((a+b)a^*) \cap L(baa^*)$.
- 7. The nor of two languages is

$$nor(L_1, L_2) = \{w : w \notin L_1 \text{ and } w \notin L_2\}.$$

Show that the family of regular languages is closed under the nor operation.

- ***12.** Suppose we know that $L_1 \cup L_2$ is regular and that L_1 is finite. Can we conclude from this that L_2 is regular?
- 14. If L is a regular language, prove that the language $\{uv : u \in L, v \in L^R\}$ is also regular.



2. (a) The construction is straightforward, but tedious. A dfa for $L\left((a+b)\,a^*\right)$ is given by

$$\delta(q_0, a) = q_1, \quad \delta(q_0, b) = q_1, \quad \delta(q_1, a) = q_1, \quad \delta(q_1, b) = q_t,$$

with q_t a trap state and final state q_1 . A dfa for $L(baa^*)$ is given by

$$\delta(p_0, a) = p_t, \delta(p_0, b) = p_1, \delta(p_1, a) = p_2,$$

 $\delta(p_1, b) = p_t, \delta(p_2, a) = p_2, \delta(p_2, b) = p_t$

with final state p_2 . From this we find

$$\delta((q_0, p_0), a) = (q_1, p_t), \delta((q_0, p_0), b) = (q_1, p_1),$$

$$\delta((q_1, p_1), a) = (q_1, p_2), \delta((q_1, p_2), a) = (q_1, p_2),$$



etc. When we complete this construction, we see that the only final state is (q_1, p_2) and that $L((a + b) a^*) \cap L(baa^*) = L(baa^*)$.

7. Notice that

$$nor\left(L_{1},L_{2}\right)=\overline{L_{1}\cup L_{2}}.$$

The result then follows from closure under intersection and complementation.



12. The answer is yes. It can be obtained by starting from the set identity

$$L_2 = \left((L_1 \cup L_2) \cap \overline{L_1} \right) \cup (L_1 \cap L_2).$$

The key observation is that since L_1 is finite, $L_1 \cap L_2$ is finite and therefore regular for all L_2 . The rest then follows easily from the known closures under union and complementation.



14. By closure under reversal, L^R is regular. The result then follows from closure under concatenation.

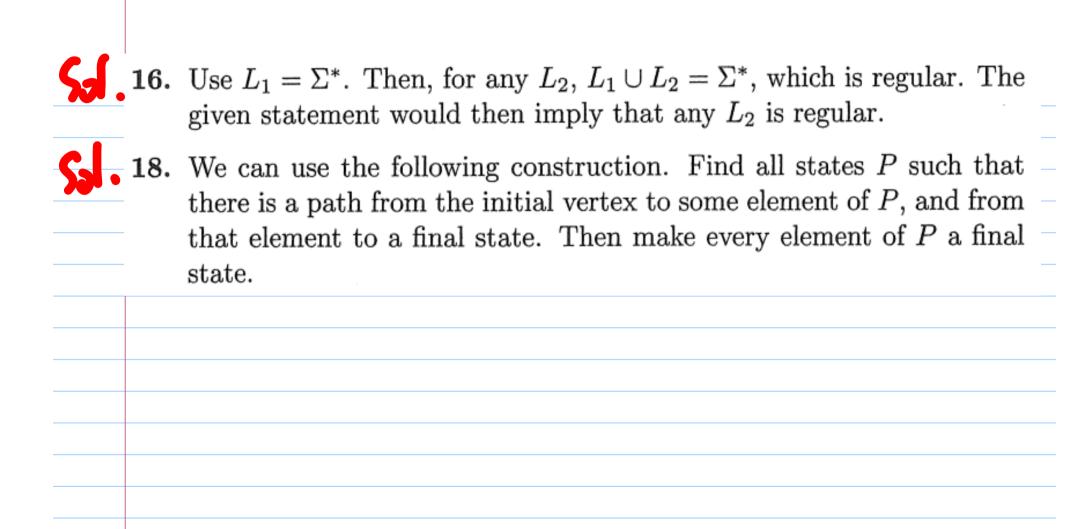


- Show that if the statement "If L_1 is regular and $L_1 \cup L_2$ is also regular, then L_2 must be regular" were true for all L_1 and L_2 , then all languages would be regular.
- 18. The head of a language is the set of all prefixes of its strings, that is,

$$head(L) = \{x : xy \in L \text{ for some } y \in \Sigma^*\}.$$

Show that the family of regular languages is closed under this operation.

- 26. Let G_1 and G_2 be two regular grammars. Show how one can derive regular grammars for the languages
 - (a) $L(G_1) \cup L(G_2)$.
 - (b) $L(G_1)L(G_2)$.
 - (c) $L(G_1)^*$.





- **26.** Suppose $G_1 = (V_1, T, S_1, P_1)$ and $G_2 = (V_2, T, S_2, P_2)$. Without loss of generality, we can assume that V_1 and V_2 are disjoint. Combine the two grammars and
 - (a) Make S the new start symbol and add productions $S \to S_1 | S_2$.
 - (b) In P_1 , replace every production of the form $A \to x$, with $A \in V_1$ and $x \in T^*$, by $A \to xS_2$.
 - (c) In P_1 , replace every production of the form $A \to x$, with $A \in V_1$, and $x \in T^*$, by $A \to xS_1, S_1 \to \lambda$.