Formal Language Selected Homework Chapter 3.3

4. Construct right- and left-linear grammars for the language

$$L = \{a^n b^m : n \geq 2, m \geq 3\}.$$

Theorem 3.5 A language L is regular if and only if there exists a left-linear grammar G such that $L = L(G)$.

Proof: We only outline the main idea. Given any left-linear grammar with productions of the form

$$A \to Bv,$$

or

$$A \to v,$$

we construct from it a right-linear grammar $\widehat{G}$ by replacing every such production of G with

$$A \to v^R B,$$

or

$$A \to v^R,$$

respectively. A few examples will make it clear quickly that $L(G) = \left(L\left(\widehat{G}\right)\right)^R$. Next, we use Exercise 12, Section 2.3, which tells us that the reverse of any regular language is also regular. Since $\widehat{G}$ is right-linear, $L\left(\widehat{G}\right)$ is regular. But then so are $L\left(\left(\widehat{G}\right)\right)^R$ and $L(G)$. ■

8. In Theorem 3.5, prove that $L\left(\widehat{G}\right) = (L(G))^R$.

11. Find a regular grammar for the language $L = \{a^n b^m : n + m \text{ is even}\}$.

13. Find regular grammars for the following languages on $\{a, b\}$.

(a) $L = \{w : n_a(w) \text{ and } n_b(w) \text{ are both even}\}$.

(b) $L = \{w : (n_a(w) - n_b(w)) \bmod 3 = 1\}$.

17. Let $G_1 = (V_1, \Sigma, S_1, P_1)$ be right-linear and $G_2 = (V_2, \Sigma, S_2, P_2)$ be a left-linear grammar, and assume that V_1 and V_2 are disjoint. Consider the linear grammar $G = (\{S\} \cup V_1 \cup V_2, \Sigma, S, P)$, where S is not in $V_1 \cup V_2$ and $P = \{S \to S_1 | S_2\} \cup P_1 \cup P_2$. Show that $L(G)$ is regular.