Formal Language Selected Homework Chapter 6.1

8. Remove all unit-productions, all useless productions, and all λ-productions from the grammar

$$
\begin{aligned}
S &\to aA|aBB,\\
A &\to aaA|\lambda,\\
B &\to bB|bbC,\\
C &\to B.
\end{aligned}
$$

What language does this grammar generate?

13. Consider the grammar G with productions

$$
\begin{aligned}
S &\to A|B, \\
A &\to \lambda, \\
B &\to aBb, \\
B &\to b.
\end{aligned}
$$

Construct a grammar $\widehat{G}$ by applying the algorithm in Theorem 6.3 to G. What is the difference between $L(G)$ and $L(\widehat{G})$?

Sol

8. The only nullable variable is A, so removing λ-productions gives

$$
\begin{aligned}
S &\to aA\,|a|\,aBB,\\
A &\to aaA|aa,\\
B &\to bC|bbC,\\
C &\to B.
\end{aligned}
$$

$C \to B$ is the only unit-production and removing it results in

$$
\begin{aligned}
S &\to aA\,|a|\,aBB,\\
A &\to aaA|aa,\\
B &\to bC|bbC,\\
C &\to bC|bbC.
\end{aligned}
$$

Finally, B and C are useless, so we get

$$S \to aA|a,$$
$$A \to aaA|aa.$$

The language generated by this grammar is $L\left((aa)^* a\right)$.

13. $L(\widehat{G}) = L(G) - \{\lambda\}$.

$a(aa)^*$