Formal Language Selected Homework Chapter 5.2

2. Find an s-grammar for $L = \{a^n b^n : n \geq 1\}$.

6. Show that the following grammar is ambiguous.

$$\begin{aligned} S &\rightarrow AB|aaB, \\ A &\rightarrow a|Aa, \\ B &\rightarrow b. \end{aligned}$$

9. Show that a regular language cannot be inherently ambiguous.

14. Show that the grammar in Example 5.4 is ambiguous, but that the language denoted by it is not.

Example 5.4 Consider the grammar with productions

$$S \rightarrow aSb|SS|\lambda.$$

Sol

2. A solution is

$$S \to aA,\ A \to aAB|b,\ B \to b.$$

6. There are two leftmost derivations for $w = aab$.

$$S \Rightarrow aaB \Rightarrow aab,$$
$$S \Rightarrow AB \Rightarrow AaB \Rightarrow aaB \Rightarrow aab.$$

9. From the dfa for a regular language we can get a regular grammar by the method of Theorem 3.4. The grammar is an s-grammar except for $q_f \to \lambda$. But this rule does not create any ambiguity. Since the dfa never has a choice, there is never any choice in the production that can be applied.

p. 93 $\delta(q_i, a_j) = q_k$ $q_i \to a_j q_k$

$q_k \to \lambda$ if $q_k \in F$

Sol: **14.** Ambiguity of the grammar is obvious from the derivations

$$S \Rightarrow aSb \Rightarrow ab$$

$$S \Rightarrow SS \Rightarrow abS \Rightarrow ab.$$

An equivalent unambiguous grammar is

$$S \rightarrow A|\lambda$$

$$A \rightarrow aAb\,|ab|\,AA.$$

It is not easy to see that this grammar is unambiguous. To make it plausible, consider the two typical situations, $w = aabb$, which can only be derived by starting with $A \rightarrow aAb$, and $w = abab$, which can only be derived by starting with $A \rightarrow AA$. More complicated strings are built from these two situations, so they can be parsed only in one way.