Formal Language Selected Homework Chapter 4.3

2. Prove the following generalization of the pumping lemma, which includes Theorem 4.8 as well as Exercise 1 as special cases.

If L is regular, then there exists an m, such that the following holds for every sufficiently long $w \in L$ and every one of its decompositions $w = u_1 v u_2$, with $u_1, u_2 \in \Sigma^*$, $|v| \geq m$. The middle string v can be written as $v = xyz$, with $|xy| \leq m$, $|y| \geq 1$, such that $u_1 x y^i z u_2 \in L$ for all $i = 0, 1, 2, \ldots$.

4. Prove that the following languages are not regular.

(a) $L = \{a^n b^l a^k : k \geq n + l\}$.

(b) $L = \{a^n b^l a^k : k \neq n + l\}$.

(c) $L = \{a^n b^l a^k : n = l \text{ or } l \neq k\}$.

(d) $L = \{a^n b^l : n \leq l\}$.

(e) $L = \{w : n_a(w) \neq n_b(w)\}$.

5. Determine whether or not the following languages on $\Sigma = \{a\}$ are regular.

(a) $L = \{a^n : n \geq 2, \text{ is a prime number}\}$.

(b) $L = \{a^n : n \text{ is not a prime number}\}$.

(c) $L = \{a^n : n = k^3 \text{ for some } k \geq 0\}$.

(d) $L = \{a^n : n = 2^k \text{ for some } k \geq 0\}$.

14. Prove or disprove the following statement: If L_1 and L_2 are nonregular languages, then $L_1 \cup L_2$ is also nonregular.

15. Consider the languages below. For each, make a conjecture whether or not it is regular. Then prove your conjecture.

(a) $L = \{a^n b^l a^k : n + l + k > 5\}$.

(b) $L = \{a^n b^l a^k : n > 5, l > 3, k \leq l\}$.

(c) $L = \{a^n b^l : n/l \text{ is an integer}\}$.

(d) $L = \{a^n b^l : n + l \text{ is a prime number}\}$.

(e) $L = \{a^n b^l : n \leq l \leq 2n\}$.

17. Let L_1 and L_2 be regular languages. Is the language $L = \{w : w \in L_1, w^R \in L_2\}$ necessarily regular?

19. Are the following languages regular?

(a) $L = \{uww^R v : u, v, w \in \{a, b\}^+\}$.

★ (b) $L = \{uww^R v : u, v, w \in \{a, b\}^+, |u| \geq |v|\}$.

21. Let P be an infinite but countable set, and associate with each $p \in P$ a language L_p. The smallest set containing every L_p is the union over the infinite set P; it will be denoted by $\cup_{p \in P} L_P$. Show by example that the family of regular languages is not closed under infinite union.

25. In the chain code language in Exercise 24, Section 3.1, let L be the set of all $w \in \{u, r, l, d\}^*$ that describe rectangles. Show that L is not a regular language.