Formal Language Selected Homework Chapter 4.1

2. Use the construction in Theorem 4.1 to find nfa's that accept

(a) $L((a+b)a^*) \cap L(baa^*)$.

7. The *nor* of two languages is

$$nor(L_1, L_2) = \{w : w \notin L_1 \text{ and } w \notin L_2\}.$$

Show that the family of regular languages is closed under the *nor* operation.

★12. Suppose we know that $L_1 \cup L_2$ is regular and that L_1 is finite. Can we conclude from this that L_2 is regular?

14. If L is a regular language, prove that the language $\{uv : u \in L, v \in L^R\}$ is also regular.

Sol.

2. (a) The construction is straightforward, but tedious. A dfa for $L((a+b)a^*)$ is given by

$$\delta(q_0, a) = q_1, \quad \delta(q_0, b) = q_1, \quad \delta(q_1, a) = q_1, \quad \delta(q_1, b) = q_t,$$

with q_t a trap state and final state q_1. A dfa for $L(baa^*)$ is given by

$$\begin{aligned} \delta(p_0, a) &= p_t, \delta(p_0, b) = p_1, \delta(p_1, a) = p_2, \\ \delta(p_1, b) &= p_t, \delta(p_2, a) = p_2, \delta(p_2, b) = p_t \end{aligned}$$

with final state p_2. From this we find

$$\begin{aligned} \delta((q_0, p_0), a) &= (q_1, p_t), \delta((q_0, p_0), b) = (q_1, p_1), \\ \delta((q_1, p_1), a) &= (q_1, p_2), \delta((q_1, p_2), a) = (q_1, p_2), \end{aligned}$$

etc. When we complete this construction, we see that the only final state is (q_1, p_2) and that $L((a+b)a^*) \cap L(baa^*) = L(baa^*)$.

Sol. **7.** Notice that

$$nor(L_1, L_2) = \overline{L_1 \cup L_2}.$$

The result then follows from closure under intersection and complementation.

Sol. **12.** The answer is yes. It can be obtained by starting from the set identity

$$L_2 = \left((L_1 \cup L_2) \cap \overline{L_1}\right) \cup (L_1 \cap L_2).$$

The key observation is that since L_1 is finite, $L_1 \cap L_2$ is finite and therefore regular for all L_2. The rest then follows easily from the known closures under union and complementation.

Sol. **14.** By closure under reversal, L^R is regular. The result then follows from closure under concatenation.

Prob.

16. Show that if the statement "If L_1 is regular and $L_1 \cup L_2$ is also regular, then L_2 must be regular" were true for all L_1 and L_2, then all languages would be regular.

18. The *head* of a language is the set of all prefixes of its strings, that is,

$$head\,(L) = \{x : xy \in L \text{ for some } y \in \Sigma^*\}.$$

Show that the family of regular languages is closed under this operation.

26. Let G_1 and G_2 be two regular grammars. Show how one can derive regular grammars for the languages

(a) $L(G_1) \cup L(G_2)$.

(b) $L(G_1)\, L(G_2)$.

(c) $L(G_1)^*$.

Sol. **16.** Use $L_1 = \Sigma^*$. Then, for any L_2, $L_1 \cup L_2 = \Sigma^*$, which is regular. The given statement would then imply that any L_2 is regular.

Sol. **18.** We can use the following construction. Find all states P such that there is a path from the initial vertex to some element of P, and from that element to a final state. Then make every element of P a final state.

Sol.

26. Suppose $G_1 = (V_1, T, S_1, P_1)$ and $G_2 = (V_2, T, S_2, P_2)$. Without loss of generality, we can assume that V_1 and V_2 are disjoint. Combine the two grammars and

(a) Make S the new start symbol and add productions $S \to S_1 | S_2$.

(b) In P_1, replace every production of the form $A \to x$, with $A \in V_1$ and $x \in T^*$, by $A \to xS_2$.

(c) In P_1, replace every production of the form $A \to x$, with $A \in V_1$, and $x \in T^*$, by $A \to xS_1$, $S_1 \to \lambda$.