2-4 Inversions

a. List the five inversions of the array $\langle 2, 3, 8, 6, 1 \rangle$

Answer

$\{(1, 2), (1, 3), (1, 4), (2, 3), (3, 4)\}$

b. What array with elements from the set $\{1, 2, ..., n\}$ has the most inversions? How many does it have?

Answer

It is the sorted array from $[1 \,..\, n]$ which have all permutations possible in the collection that give us $(n-1)+(n-2)+(n-3)+...+1$ which is the summation of $n-1$ numbers, which gives us $n(n-1)/2$ inversions.

c. What is the relationship between the running time of insertion sort and the number of inversions in the input array? Justify your answer.

Answer

The number of inversions can be related to the swaps made on insertion sort. So, given a reversed sorted array, the number of swaps will be proportional to the number of inversion on sorted array.

d. Give an algorithm that determines the number of inversions in any permutation on n elements in $\Theta(n \lg n)$ worst-case time. (Hint: Modify merge sort.)

Answer

The basic idea is to count every swap made on $MERGE-SORT$. And then we subtract from summation of $n-1$ with the number of swaps made.

$INVERSIONS(A) = \sum_{i=0}^{n} i -$ number of swaps in $MERGESORT$