2-2 Correctness of bubblesort

a. In order to show that BUBBLESORT actually sorts, what else do we need to prove?

Answer

We need to prove that the initialization holds the invariant of BUBBLESORT.

b.

Answer

The loop invariant is always keep the larger value to the right.

Initialization We start proving that the invariant holds before the first iteration with $j = n$. Since, the $A[n+1]$ does not exist, we may conclude that $A[n]$ is smaller than its next value.

Maintenance For each iteration, we swaps the larger element to one position right, so we left with $A'[j-1] <= A'[j]$ holding the invariant of keeping the larger value to the right.

Termination We end with $j = i + 1$, since we swaps the current element with the previous one, we always endup with the smallest element on $A[i]$.

c.

Answer

The loop invariant is always keep the smallest elements to left

Initialization

Maintenance

Termination We finish with the subarray $A[i..n-1]$ elements sorted. Since the left subarray always holds the largest element, we don't need to sort the $A[n]$.

d. What is the worst-case running time of bubblesort? How does it compare to the running time of insertion sort?

Answer

The $T(n) = (n-1) + (n-2) + ... + 1 = \frac{n(n-1)}{2}$, so BUBBLESORT has $\Theta(n^2)$ the same as INSERTION-SORT.