1-1 Comparison of running times

For each function $f(n)$ and time t in the following table, determine the largest size n of a problem that can be solved in time t, assuming that the algorithm to solve the problem takes $f(n)$ microseconds.

Answer

Considering the linear case where $f(n) = n$ and $1\mu = 10^{-6}$:

1 second $= 1.00 \times 10^6 \mu$ second
1 minute $= 6.00 \times 10^7 \mu$ second
1 hour $= 3.60 \times 10^9 \mu$ second
1 day $= 8.64 \times 10^{10} \mu$ second
1 month $= 2.59 \times 10^{12} \mu$ second
1 year $= 3.15 \times 10^{13} \mu$ second
1 century $= 3.15 \times 10^{15} \mu$ second

So, given that T is the time take in microseconds we just need to solve the equation for each given time t.

$f(n) = \lg n \implies n = 2^T$
$f(n) = \sqrt{n} \implies n = T^2$
$f(n) = n \implies n = T$
$f(n) = n \lg n \implies n = e^{W(T)}$ (Lambert W function)
$f(n) = n^2 \implies n = \sqrt{T}$
$f(n) = n^3 \implies n = \sqrt[3]{T}$
$f(n) = 2^n \implies n = \lg T$
$f(n) = n! \implies$ iterating n until $n! \leq T$

So, two functions are hard to find the largest value of n which are $e^{W(T)}$ and $n!$. The first one you can use Wolfram Alpha[1] with the following expression:

solve $n : n \lg n = T$ (replace T with desired value).

The $n!$ I just implemented and tested for some values of n.

	1 second	1 minute	1 hour	1 day	1 month	1 year	1 century
$\lg n$	$2^{6.27\times10^4}$	$2^{2.80\times10^6}$	$2^{1.33\times10^8}$	$2^{2.76\times10^9}$	$2^{7.18\times10^{10}}$	$2^{7.97\times10^{11}}$	$2^{6.85\times10^{13}}$
$\sqrt{n}$	1.00×10^{12}	3.60×10^{15}	1.30×10^{19}	7.46×10^{21}	6.72×10^{24}	9.95×10^{26}	9.95×10^{30}
n	1.00×10^6	6.00×10^7	3.60×10^9	8.64×10^{10}	2.59×10^{12}	3.15×10^{13}	3.15×10^{15}
$n \lg n$	6.27×10^4	2.80×10^6	1.33×10^8	2.76×10^9	7.18×10^{10}	7.97×10^{11}	6.85×10^{13}
n^2	1.00×10^3	7.75×10^3	6.00×10^4	2.94×10^5	1.61×10^6	5.62×10^6	5.62×10^7
n^3	1.00×10^2	3.91×10^2	1.53×10^3	4.42×10^3	1.37×10^4	3.16×10^4	1.47×10^5
2^n	19.9	25.8	31.7	36.3	41.2	44.8	51.5
$n!$	9	10	12	12	15	16	17

[1] http://www.wolframalpha.com/