1-1 Comparison of running times

For each function $f(n)$ and time t in the following table, determine the largest size n of a problem that can be solved in time t, assuming that the algorithm to solve the problem takes $f(n)$ microseconds.

Answer

Considering the linear case where $f(n) = n$ and $1\mu = 10^{-6}$:

$$\begin{aligned}
&1 \text{ second} = 1.00 \times 10^{6}\mu \text{ second}\\
&1 \text{ minute} = 6.00 \times 10^{7}\mu \text{ second}\\
&1 \text{ hour} = 3.60 \times 10^{9}\mu \text{ second}\\
&1 \text{ day} = 8.64 \times 10^{10}\mu \text{ second}\\
&1 \text{ month} = 2.59 \times 10^{12}\mu \text{ second}\\
&1 \text{ year} = 3.15 \times 10^{13}\mu \text{ second}\\
&1 \text{ century} = 3.15 \times 10^{15}\mu \text{ second}
\end{aligned}$$

So, given that T is the time take in microseconds we just need to solve the equation for each given time t.

$$\begin{aligned}
&f(n) = \lg n \implies n = 2^{T}\\
&f(n) = \sqrt{n} \implies n = T^{2}\\
&f(n) = n \implies n = T\\
&f(n) = n \lg n \implies n = \lceil e^{W(T)} \rceil \text{ (Lambert W function)}\\
&f(n) = n^{2} \implies n = \lceil \sqrt{T} \rceil\\
&f(n) = n^{3} \implies n = \lceil \sqrt[3]{T} \rceil\\
&f(n) = 2^{n} \implies n = \lceil \lg T \rceil\\
&f(n) = n! \implies \text{iterating } n \text{ until } n! \leq T
\end{aligned}$$

So, two functions are hard to find the largest value of n which are $e^{W(T)}$ and $n!$. The first one you can use Wolfram Alpha[1] with the following expression:

solve $n : n \lg n = T$ (replace T with desired value).

The $n!$ I just implemented and tested for some values of n.

	1 second	1 minute	1 hour	1 day	1 month	1 year	1 century
$\lg n$	2^{10^6}	$2^{6.00\times10^7}$	$2^{3.60\times10^9}$	$2^{8.64\times10^{10}}$	$2^{2.59\times10^{12}}$	$2^{3.15\times10^{13}}$	$2^{3.15\times10^{15}}$
$\sqrt{n}$	1.00×10^{12}	3.60×10^{15}	1.30×10^{19}	7.46×10^{21}	6.72×10^{24}	9.95×10^{26}	9.95×10^{30}
n	1.00×10^{6}	6.00×10^{7}	3.60×10^{9}	8.64×10^{10}	2.59×10^{12}	3.15×10^{13}	3.15×10^{15}
$n \lg n$	6.27×10^{4}	2.80×10^{6}	1.33×10^{8}	2.76×10^{9}	7.18×10^{10}	7.97×10^{11}	6.85×10^{13}
n^2	1.00×10^{3}	7.75×10^{3}	6.00×10^{4}	2.94×10^{5}	1.61×10^{6}	5.62×10^{6}	5.62×10^{7}
n^3	1.00×10^{2}	3.91×10^{2}	1.53×10^{3}	4.42×10^{3}	1.37×10^{4}	3.16×10^{4}	1.47×10^{5}
2^n	19	25	31	36	41	44	51
$n!$	9	10	12	12	15	16	17

[1] http://www.wolframalpha.com/