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Linear and non-linear propagation of electron plasma waves in quantum plasma

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Using one-dimensional quantum hydrodynamic model, the linear and non-linear characteristics of electron plasma waves have been studied both analytically and numerically in a two-component unmagnetized dense quantum plasma with streaming motion. It is shown that quantum effect modifies the linear dispersion character of the electron plasma waves and streaming effect makes it possible the excitation of two distinct modes. To describe the non-linear behaviour, Korteweg de Vries equation is derived by using the standard reductive perturbation technique and incorporating quantum-mechanical effects. It is shown that depending on some critical values of the quantum diffraction parameter both rarefactive and compressive type of solitons can exist in the model plasma. The structure of the solitary waves is shown to be significantly affected by the quantum plasma parameters and streaming motion.

Keywords: Quantum plasma, Electron plasma waves, Quantum hydrodynamic model, Korteweg de Vries equation

1 Introduction

In recent years, the different aspects of non-linear wave propagation in quantum plasma have been studied. Traditional plasma physics has mainly focused on regimes characterized by high temperature and low density where quantum effects have virtually no impact. But in plasmas where the density is quite high and the temperature is very low thermal de Broglie wavelength of electrons may become comparable to the inter-particle distances. The condition is well satisfied in metals, semiconductors and laser produced plasmas. In such situations, quantum effects become important due to overlapping of the wavefunctions of the neighbouring particles. In quantum plasmas,where the electron thermal energy is much smaller than their Fermi energy the statistical behaviour of plasma particles should be described by Fermi-Dirac statistics and not by the classical Boltzmann statistics. Such quantum plasma may be found in a variety of environment such as metal nanostructures[1], astrophysical system[2], ultra small electronic devices[3,4], biophotonics[5], cool vibes[6], neutron stars[7], laser produced plasmas[8], quantum wells and quantum diodes[9,10]. The inclusion of quantum-mechanical effects in plasma requires new mathematical formulation or a suitable modification of the formulations used in classical situations.

Quantum effects in plasmas are usually studied with the help of two well-known formulations, viz. the Wigner-Poisson and the Schrödinger-Poisson formulations. The Wigner-Poisson model is often used to study the quantum kinetic behaviour of plasma. The Schrödinger-Poisson model describes the hydrodynamic behaviour of plasma particles in quantum scales. It can be considered as the quantum analog of the fluid model of traditional plasma. The quantum hydrodynamic (QHD) model is derived by taking velocity-space moments of the Wigner equations as in the classical fluid model. This model consists of a set of equations describing the transport of charge, momentum and energy in a charged particle system interacting through a self-consistent electrostatic potential. The QHD model generalizes the fluid model for plasma with the inclusion of a quantum correction term also known as the Bohm potential[11]. The Bohm potential term appropriately describes negative differential resistance in resonant tunneling diodes. It is based on resonant tunneling which is a quantum phenomenon and does not occur in classical transport model. The QHD model incorporates quantum statistical effects through an equation of state pertaining to a zero-temperature Fermi gas. The quantum corrections give rise to new aspects of purely quantum origin in the collective behaviour of plasma at quantum scale. For example, it may lead to the generation of new wave modes in plasma[12]. The weakness of the model lies in its inability to take into account kinetic effects like Landau damping driven by resonant wave-particle interaction. Because of simplicity, straight forward approach and numerical efficiency, the QHD model has been widely used by several reseachers[13-20] in dealing with different aspects of linear and non-linear wave propagation in unmagnetized quantum plasmas. For example, using the QHD model Haas *et al.*[13] have studied the important role of quantum diffraction in linear as well as non-linear regimes for the propagation of ion acoustic waves. Gardner and Ringhofer[14] have studied the electron-hole dynamics in semiconductors. Using the same model, Shukla and Eliasson[15] have studied the dynamics and formation of dark soliton and vortices in quantum plasma. The same model has also been used to study the Korteweg deVries (KdV) solitary wave structure for ion acoustic waves[16,17], electron-acoustic waves[18], dust-acoustic waves[19,20] and dust ion-acoustic waves[16]. It has also been used to study the modulational instability of electron plasma waves in a quantum plasma[21]. Streaming motion of plasma particles may become important in a variety of situations e.g. in some space-plasma phenomena, laser plasma interactions, in solar atmosphere and interstellar space.

A survey of the available literature indicates that the solitary wave structure for electron plasma waves has been studied by a few researchers[22-25] for classical bounded plasma. Such a study has not yet been made for quantum plasma. The objective of the present paper is to study the influence of quantum mechanical effects and streaming motion on the linear and non-linear properties of electron plasma waves using the QHD model in unmagnetized, collisionless, ultracold electron-ion quantum plasma with streaming motion. It is shown that the quantum effects and streaming motion can significantly influence the formation and properties of solitary wave structure.

2 Basic Equations

We consider an unmagnetized collisionless two-component dense quantum plasma consisting of electrons and ions. In order to study electron plasma waves, we assume the ions to form a uniform neutralizing background while electrons move with certain non-relativistic streaming velocity. The set of one dimensional QHD equations governing the dynamics of the electron plasma waves in such a model plasma is given by:

$$\frac{\partial n_e}{\partial t}+\frac{\partial (n_e u_e)}{\partial x}=0 \qquad \ldots(1)$$

$$\left(\frac{\partial}{\partial t}+u_e\frac{\partial}{\partial x}\right)u=\frac{e}{m_e}\frac{\partial\phi}{\partial x}-\frac{1}{m_e n_e}\frac{\partial p_e}{\partial x}+\frac{\hbar^2}{2m_e^2}\frac{\partial}{\partial x}\left[\frac{1}{\sqrt{n_e}}\frac{\partial^2\sqrt{n_e}}{\partial x^2}\right] \qquad \ldots(2)$$

$$\frac{\partial^2\phi}{\partial x^2}=4\pi e\left(n_e-n_0\right) \qquad \ldots(3)$$

where u_e, m_e and p_e are the fluid velocity, mass and pressure of the electrons, respectively, e the magnitude of electronic charge, n_e the number density of electrons, n_0 the equilibrium number density of electrons or ions, $\hbar$ the Planck's constant divided by 2π and ϕ is the electrostatic wave potential.

The pressure term p_e for electrons is obtained by assuming that the electrons behave as a one-dimensional Fermi gas at zero temperature and therefore, the pressure law[26] is:

$$p_e=\frac{m_e V_{Fe}^2}{3n_0^3}n_e^3 \qquad \ldots(4)$$

where $V_{Fe}=\sqrt{2k_B T_{Fe}/m_e}$ is the Fermi speed of electrons, T_{Fe} the Fermi temperature and k_B the Boltzmann constant and n_e is the number density with the equilibrium value n_0.

We now use the following normalization:

$x\rightarrow x\omega_{pe}/V_{Fe}$, $t\rightarrow t\omega_{pe}$, $\phi\rightarrow e\phi/2k_B T_{Fe}$, $n_e\rightarrow n_e/n_0$ and $u_e\rightarrow u_e/V_{Fe}$

where $\omega_{pe}=\sqrt{4\pi n_0 e^2/m_e}$ is the electron plasma oscillation frequency. The normalized form of Eqs (1)-(3) is the following:

$$\frac{\partial n_e}{\partial t}+\frac{\partial (n_e u_e)}{\partial x}=0 \qquad \ldots(5)$$

$$\left(\frac{\partial}{\partial t}+u_e\frac{\partial}{\partial x}\right)u_e=\frac{\partial\phi}{\partial x}-n_e\frac{\partial n_e}{\partial x}+\frac{H^2}{2}\frac{\partial}{\partial x}\left[\frac{1}{\sqrt{n_e}}\frac{\partial^2\sqrt{n_e}}{\partial x^2}\right] \qquad \ldots(6)$$

$$\frac{\partial^2\phi}{\partial x^2}=\left(n_e-n_i\right) \qquad \ldots(7)$$

where $H=\hbar\omega_{pe}/2\,k_B T_{Fe}$ is a non-dimensional quantum parameter proportional to the quantum diffraction and is equal to the ratio between the plasmon energy $\hbar\omega_{pe}$ (energy of an elementary excitation associated with an electron plasma wave) and the Fermi energy $k_B T_{Fe}$. The Eqs(5)-(7) constitute the basic set of quantum hydrodynamic equations to be used in the investigation of non-linear propagation of electron plasma waves in a quantum plasma.

Quantum effects are included in the model through the second and third terms on the RHS of Eq. (6). The second term in the RHS of Eq. (6) includes quantum statistical effect. It is included in the model through the equation of state Eq. (4). The third term in the RHS of Eq. (6) arises due to quantum correction of density fluctuations and this type of quantum effect is called quantum diffraction or Bohm potential.

3 Linear Dispersion Characteristics

In order to investigate the non-linear behaviour of electron plasma waves, we make the following perturbation expansions for the field quantities n_e, u_e and ϕ about their equilibrium values:

$$\begin{bmatrix} n_e \\ u_e \\ \phi \end{bmatrix} = \begin{bmatrix} 1 \\ u_0 \\ 0 \end{bmatrix} + \varepsilon \begin{bmatrix} n_e^{(1)} \\ u_e^{(1)} \\ \phi^{(1)} \end{bmatrix} + \varepsilon^2 \begin{bmatrix} n_e^{(2)} \\ u_e^{(2)} \\ \phi^{(2)} \end{bmatrix} + ... \quad ...(8)$$

Now assuming that all the field variables are varying as $\exp[i(kx-\omega t)]$, we get for normalized wave frequency ω and wavenumber k, the following dispersion relation of electron plasma waves in quantum plasma including quantum effects:

$$(\omega - ku_0)^2 = 1 + k^2 + (H^2k^4)/4 \quad ...(9)$$

Eq.(9) is a quadratic equation in ω^2 and has two solutions for ω^2 as :

$$\omega_1 = ku_0 + \sqrt{1 + k^2 + (H^2k^4)/4} \quad ...(10)$$

$$\omega_2 = ku_0 - \sqrt{1 + k^2 + (H^2k^4)/4} \quad ...(11)$$

Thus in the presence of streaming motion, we have two distinct modes of electron plasma wave. The former mode has a phase velocity greater than the later mode. So they may be called 'fast mode' and 'slow mode', respectively. The slow mode can exist only for the regime:

$$ku_0 > \sqrt{1 + k^2 + (H^2k^4)/4} \quad ...(12)$$

In this paper, we shall consider the non-linear behaviour of the fast mode only. For a given k, the frequency of oscillation of the fast mode increases with H and u_0 (Fig. 1). Thus, it is important to study the effects of streaming motion on quantum electron plasma waves. In the absence of streaming motion (u_0=0), the slow mode does not exist at all and the fast mode reduces to :

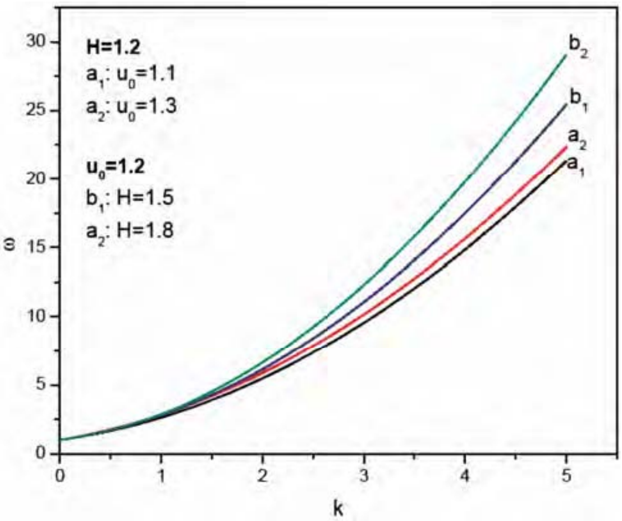

Fig. 1 — Normalized dispersion characteristics of electron plasma wave for different values of quantum diffraction parameter and streaming velocity

$$\omega = \sqrt{1 + k^2 + (H^2k^4)/4} \quad ...(13)$$

In the dimensional form:

$$\omega = \sqrt{\omega_{pe}^2 + k^2V_{Fe}^2 + H^2k^4V_{Fe}^4/4\omega_{pe}^2} \quad ...(14)$$

It represents the usual electron plasma wave associated with the inertia of electrons as well as a force caused by the quantum correlations of the electron number density fluctuations that balance the electrostatic force. The frequency ω of the mode for a given k, increases with H. It means that the quantum effects enhance the wave frequency. In the absence of the quantum diffraction (i.e.H=0) the dispersion relation given in Eq. (14) reduces to the well-known dispersion relation for classical electron plasma waves. It may be noted that in the frequency range where $\omega >> \omega_{pe}$ the dispersion relation given in Eq. (13) reduces to the form:

$$\omega = k + H^2k^3/8 \quad ...(15)$$

4 KdV Equation

In order to derive the desired KdV equation describing the non-linear behaviour of electron plasma waves, we use the standard reductive perturbation technique. We introduce the usual stretching of the space and time variables:

$$\xi = \varepsilon^{1/2}(x - Vt) \text{ and } \tau = \varepsilon^{3/2}t \quad ...(16)$$

where V is a constant representing the phase velocity of the wave and ε is a smallness parameter measuring the dispersion and nonlinear effects.

Eqs (5)-(7) are written in terms of the stretched coordinates ξ and τ and then the perturbation expansions given in Eq.(8) are substituted. Solving the lowest order equations with the boundary conditions that $n_e^{(1)}$, $u_e^{(1)}$ and $\phi^{(1)} \to 0$ as $|\xi| \to \infty$, the following solutions are obtained:

$$u_e^{(1)} = \frac{(V-u_0)}{1-(V-u_0)^2}\phi^{(1)} \text{ and}$$

$$n_e^{(1)} = \frac{1}{1-(V-u_0)^2}\phi^{(1)} \quad \ldots(17)$$

where $\alpha_e = \frac{(V-u_0)}{1-(V-u_0)^2}$ and

$$\beta_e = \frac{1}{1-(V-u_0)^2} \quad \ldots(18)$$

Going for the next higher order terms in ε, after a few algebraic steps the desired KdV equation is obtained:

$$\frac{\partial\phi}{\partial\tau} + A\phi\frac{\partial\phi}{\partial\xi} + B\frac{\partial^3\phi}{\partial\xi^3} = 0 \quad \ldots(19)$$

in which

$$A = \left[\alpha_e^2 + \beta_e^2 + 2(V-u_0)\alpha_e\beta_e\right] / \left[\alpha_e + (V-u_0)\beta_e\right]$$

$$B = \left[1-(V-u_0)^2 - \left(H^2\beta_e\right)/4\right] / \left[\alpha_e + (V-u_0)\beta_e\right] \quad \ldots(20)$$

α_e, β_e are as defined earlier in Eqs (18).

It should be noted that the coefficient B of the dispersive term in KdV Eq. (19), depends both on the quantum diffraction parameter H and streaming velocity u_0 whereas the coefficient A of the non-linear term depends on u_0 but is independent of H. Thus, the non-linear and dispersive terms get modified by the quantum and streaming effects.

To find the steady state solution of Eq. (19), we transform the independent variables ξ and τ into one variable $\eta = \xi - M\tau$ where M is the normalized constant speed of the wave frame. Applying the boundary conditions that as $\eta \to \pm\infty$; $\phi, \frac{\partial\phi}{\partial\eta}, \frac{\partial^2\phi}{\partial^2\eta} \to 0$ the possible stationary solution of Eq. (19) is obtained as:

$$\phi = \phi_m \sec h^2(\eta/\Delta) \quad \ldots(21)$$

where the amplitude ϕ_m and width Δ of the soliton are given by:

$$\phi_m = 3M / A \quad \ldots(22)$$

$$\text{and } \Delta = \sqrt{4B/M} \quad \ldots(23)$$

The solitary wave structure is formed due to a delicate balance between the dispersive effect and non-linear effect. Relative strength of these two effects determines the characteristic of such solitary wave structure. The coefficients A and B thus play a crucial role in determining the solitary wave structure. From Eq. (20), it is found that these coefficients get modified by quantum effects and streaming motion. So it is important to study the dependence of these coefficients on quantum diffraction parameter H and streaming velocity u_0. The coefficient A is independent of H but the coefficient B depends interestingly on H. Numerical calculations show that the value of B decreases as H is increased from zero and it becomes zero for $H=2$.

5 Results and Discussion

Dependence of the solitary wave structure, its amplitude and width on different plasma parameters and streaming motion are studied numerically. It is found that in all physically acceptable situations with $H < 2$ and $M > 0$ only compressive solitary wave structure is obtained; its amplitude does not depend on the quantum parameter H, but its width decreases significantly with the increase of H (Fig. 2). Similar result was obtained by Mushtaq and Khan[27] for ion-acoustic waves. With increase in streaming velocity u_0 the amplitude of this compressive soliton decreases and its width increases (Fig. 3). It is to be noted that the dispersion coefficient B vanishes at $H=2$ (say H_c). This critical value of H destroys the KdV evolution equation and no solitary wave excitation can occur for this critical case. No soliton solution is possible for $H > H_c$ with velocity $M > 0$. In this case, periodic oscillatory solution is obtained. However, we find that for $H > H_c$ formation of solitary wave structure is possible only for wave frames moving opposite to the direction of propagation of the wave, i.e. for $M < 0$. In this case, rarefactive soliton is obtained (Figs 4 and 5). From Fig. 4, it is clear that the amplitude of the rarefactive soliton is independent of H but unlike compressive soliton its width increases with increase

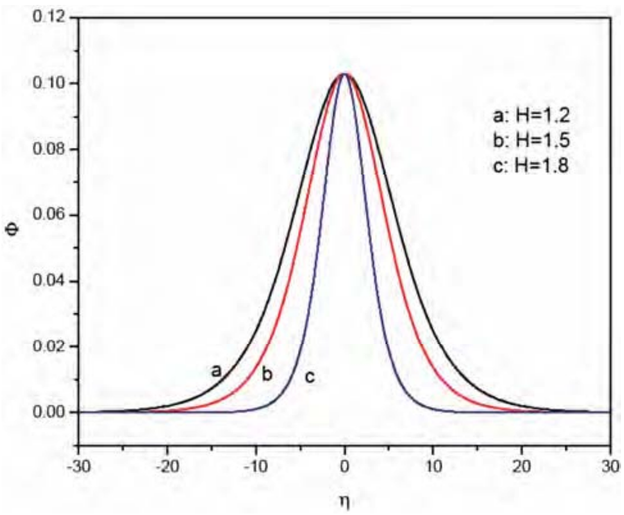

Fig. 2 — Compressive solitons for different quantum diffraction parameter (H) with u_0=1.2, V=1.5 and M=0.1

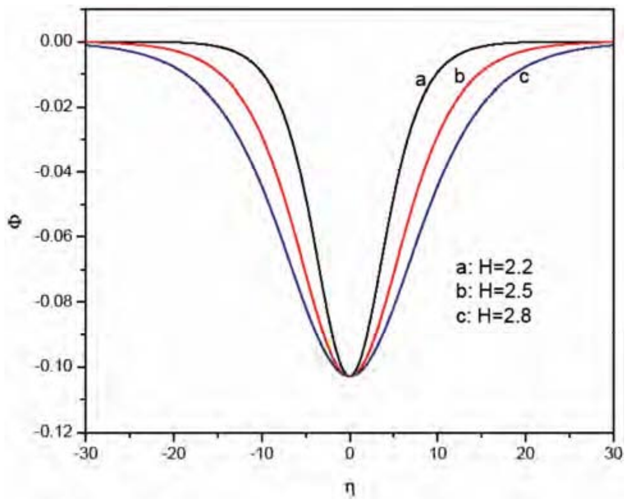

Fig. 4 — Rarefactive solitons for different values quantum diffraction parameter (H) with V=1.5, u_0 =1.2 and M= −0.1

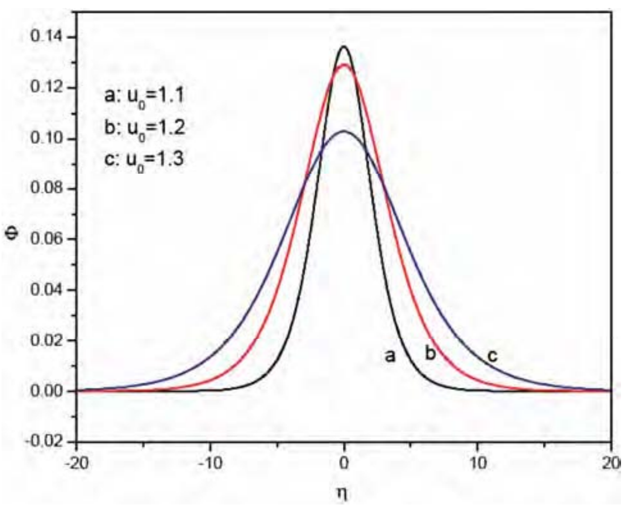

Fig. 3 — Compressive solitons for different values of streaming velocity (u_0) with H=1.5, V=1.5 and M=0.1

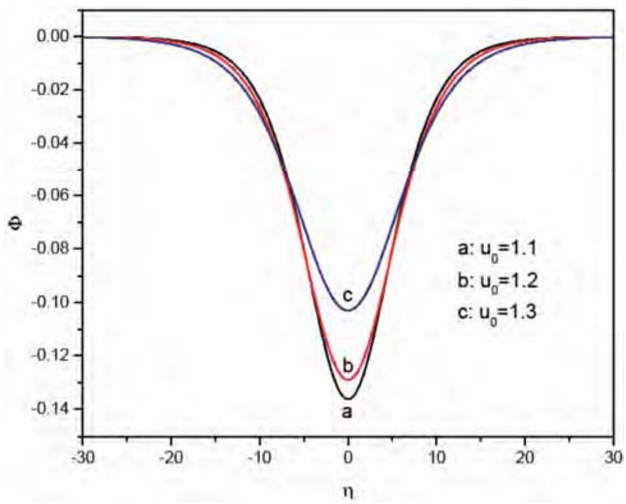

Fig. 5 — Rarefactive solitons for different values of streaming velocity (u_0) with H=2.5, V=1.5 and M= −0.1

in H. Figure 5 shows that the rarefactive soliton becomes shorter with increase in u_0 and its width is not much sensitive to the variation in u_0.

The role of quantum effects and streaming motion on the linear and non-linear properties of electron plasma waves in an unmagnetized quantum plasma with streaming motion have been investigated. Modified linear dispersion relation is obtained for electron plasma waves, which in the absence of quantum effect and streaming motion, reduces to the usual dispersion relation of electron plasma waves. To study the non-linear behaviour, we have derived the KdV equation and obtained its stationary localized solutions. From analytical and numerical analysis, it is shown that the formation of both compressive and rarefactive solitary wave structure is possible in dense quantum plasma with immobile ions and streaming electrons. There exists a critical value of the quantum diffraction parameter below which compressive soliton and above which a rarefactive soliton formation are possible. The quantum diffraction parameter and streaming velocity are shown to significantly affect the amplitude and width of the soliton. The results presented in this paper for electron plasma waves are new and have not been reported so far. The present results may be useful for understanding the origin of electrostatic fluctuations and associated phenomena in dense electron-ion quantum plasma such as can be found in metal nanostructure, intense laser-solid plasma experiments as well as in some astrophysical environment.

Finally we would like to point out that we have considered non-relativistic quantum plasma. But there are many environments where both the relativistic and quantum effects are important. To study such a system the QHD model needs modification.

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