UNIVERSITÉ DE GENÈVE
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Département d'informatique

Imagerie Numérique

Basics

TP Class N° 1

September 25, 2020

Notes: The details of algebraic calculations must be included in the report. If you do not use an Equation Editor (Word, Latex or similar), you can scan handwritten notes (readable!) and put them in the report.

Python *(2 points)*

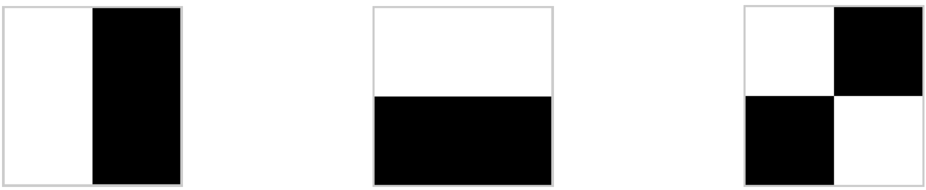
Exercise 1. *(0.33 points)* Read the image *lena.png*. Display the original image and its color components on the same plot (use *matplotlib.pyplot.subplot*). Explain the meaning of each color component.

Exercise 2. *(0.33 points)* Image generation.

(a) Generate a gradient image like the one represented in Figure 1.

Figure 1: Gradient image.

(b) Generate the images that are represented in the Figure 2a and 2b. Obtain image 2c by manipulating images 2a and 2b (you can use addition, subtraction, multiplication, boolean operators, etc.)



(a) (b) (c)

Figure 2

Exercise 3. *(0.33 points)* Read the image *lena.png* and convert it to a grayscale image (use *skimage.color.rgb2gray*). Crop Lena's face using coordinates: top left (x, y) = (150, 100) and right down (x, y) = (375, 380), where x and y correspond to the mathematical 2D standard : x is horizontal and y is vertical. Pay attention to how Python interprets horizontal and vertical directions. Perform the cropping in two ways:

(a) by using a function *skimage.util.crop*;
(b) by cropping the corresponding coordinates of the image matrix.

Display the obtained results.

Exercise 4. *(0.33 points)* Read the image *lena.png* and convert it to grayscale. Add a zero-mean white Gaussian noise $\mathcal{N}(0, \sigma^2)$ to the image into two ways:

(a) using matrix manipulations
Hint: use *numpy.random.normal*

(b) using the function *skimage.util.random_noise* with clip=True

Try $\sigma = 5,\ 25,\ 50$ and display the results. Describe how the value of σ impacts the resulting image. Explain why, for the same σ, the two methods yield different results.

Important: Pay attention to the fact that the σ values are given for an image whose dynamic range goes from 0 to 255 (8-bit encoding). If the dynamic range of the image is from 0 to 1 you need either to divide σ by 255 or to multiply the image pixel values by 255.

Exercise 5. *(0.33 points)* Read the image *lena.png* and convert it to grayscale. Add the so-called "salt & pepper" noise with density 0.0013, 0.031 and 0.113. Display the results. Compare them with the previous exercise

(a) from a visual point of view ;

(b) by calculating MSE between the original and the noisy versions.

Explain what kind of noise (Gaussian or "salt & pepper") is more difficult to remove from the visual point of view.

Hint:
- For the "salt & pepper" noise you can use *random_noise* from *skimage.util* package.
- For the *MSE* you can use *mean_squared_error* from *skimage.metrics* package.

Exercise 6. *(0.33 points)* Read the image *lena.png* and convert it to grayscale.

(a) Compute the global mean and the global variance of the image.

(b) Compute the local mean and variance of the image for a window size 5×5 with splitting steps 1 and 3. Display the obtained results as new (smaller) images. Explain the change in the image size and give an interpretation of the results, from an image processing point of view.

Hint: You can use *view_as_windows* from the *skimage.util.shape* package.

Linear algebra *(2 points)*

Exercise 7.

(a) *(0.125 points)* Compute:

$$4\begin{bmatrix} 2 \\ -3 \\ 4 \\ 1 \\ 0 \end{bmatrix} - 2\begin{bmatrix} 1 \\ 2 \\ -5 \\ 2 \\ 4 \end{bmatrix} + \begin{bmatrix} -1 \\ 3 \\ 0 \\ 1 \\ 2 \end{bmatrix}$$

(b) *(0.125 points)* Solve the given vector equation for x, or explain why no solution exists:

$$3\begin{bmatrix}1\\2\\-1\end{bmatrix}+4\begin{bmatrix}2\\0\\x\end{bmatrix}=\begin{bmatrix}11\\6\\17\end{bmatrix}$$

(c) *(0.125 points)* Solve the given vector equation for α, or explain why no solution exists:

$$\alpha\begin{bmatrix}1\\2\\-1\end{bmatrix}+4\begin{bmatrix}3\\4\\2\end{bmatrix}=\begin{bmatrix}-1\\0\\4\end{bmatrix}$$

(d) *(0.125 points)* Find α and β that solve the vector equation, or explain why no solution exists:

$$\alpha\begin{bmatrix}1\\2\end{bmatrix}+\beta\begin{bmatrix}1\\3\end{bmatrix}=\begin{bmatrix}5\\0\end{bmatrix}$$

(e) *(0.125 points)* Determine if the sets of vectors in the (a) and (b) are linearly independent or linearly dependent. When the set is linearly dependent, show a non-trivial relation of linear dependence.

(a)

$$\left\{\begin{bmatrix}1\\-2\\1\end{bmatrix},\begin{bmatrix}2\\-1\\3\end{bmatrix},\begin{bmatrix}1\\5\\0\end{bmatrix}\right\}$$

(b)

$$\left\{\begin{bmatrix}-1\\2\\4\\2\end{bmatrix},\begin{bmatrix}3\\3\\-1\\3\end{bmatrix},\begin{bmatrix}7\\3\\-6\\4\end{bmatrix}\right\}$$

(f) *(0.125 points)* Calculate the scalar product $\mathbf{a}\mathbf{b}^T$ between

$$\mathbf{a}=\begin{bmatrix}3\\-3\\1\end{bmatrix}\text{ and }\mathbf{b}=\begin{bmatrix}4\\9\\2\end{bmatrix}$$

(g) *(0.125 points)* Calculate the dot product $\mathbf{a}^T\mathbf{b}$. Do the vectors form an acute angle, right angle or obtuse angle?

$$\mathbf{a}=\begin{bmatrix}1\\2\\3\end{bmatrix}\text{ and }\mathbf{b}=\begin{bmatrix}4\\-5\\6\end{bmatrix}$$

(h) *(0.125 points)* If $\mathbf{a}=\begin{bmatrix}6\\-1\\3\end{bmatrix}$, for what value of c is the vector $\mathbf{b}=\begin{bmatrix}4\\c\\-2\end{bmatrix}$ perpendicular to $\mathbf{a}$?

(i) *(0.125 points)* Compute $A\mathbf{x}$, or explain why no solution exists:

$$A=\begin{bmatrix}1&2&3\\4&5&6\\7&8&9\\10&11&12\end{bmatrix}\text{ and }\mathbf{x}=\begin{bmatrix}-2\\1\\0\end{bmatrix}$$

(j) *(0.125 points)* Compute $A\mathbf{y}$, or explain why no solution exists:

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \\ 10 & 11 & 12 \end{bmatrix} \text{ and } \mathbf{y} = \begin{bmatrix} -3 \\ -3 \\ -1 \\ 0 \end{bmatrix}$$

(k) *(0.125 points)* Compute $B + C$, or explain why no solution exists:

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \text{ and } C = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$$

(l) *(0.03 points)* If it is true that for any matrices $B + C = C + B$?

(m) *(0.03 points)* If it is true that for any matrices $(A + B) + C = A + (B + C)$?

(n) *(0.03 points)* If it is true that for any matrices $(AB)C = A(BC)$?

(o) *(0.03 points)* If it is true that for any matrices $BC = CB$?

(p) *(0.125 points)* Compute BC, or explain why no solution exists:

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \text{ and } C = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$$

(q) *(0.125 points)* What is a matrix *rank*?
Find the rank of the matrix M

$$M = \begin{bmatrix} 2 & 1 & -4 \\ 3 & 5 & -7 \\ 4 & -5 & -6 \end{bmatrix}$$

(r) *(0.125 points)* Compute the determinant of the matrices:

$$A = \begin{bmatrix} 4 & 4 \\ 2 & -5 \end{bmatrix}, B = \begin{bmatrix} 1 & 1 & 2 \\ 2 & 3 & 1 \\ 3 & 4 & -5 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 & 0 & 3 \\ 2 & 1 & 0 & 1 \\ 3 & 0 & 5 & 4 \\ 0 & 3 & 2 & 2 \end{bmatrix}$$

(s) *(0.125 points)* Specify whether the matrix has an inverse **without** trying to compute the inverse

$$\begin{bmatrix} -1 & 1 & 1 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Probability and statistic *(2 points)*

Exercise 8.

(a) *(0.5 points)* Give the probability mass function for the following distributions:

- Bernoulli
- Binomial

(b) *(0.5 points)* Give the probability density function for the following distributions:

- Uniform
- Normal

(c) *(0.5 points)* For $A = \{1, 2\}$ and $B = \{2, 3\}$ draw a Venn diagram and find the following:

a) $A \cup B$
b) $A \cap B$
c) A^c (sometimes denotes as $\sim A$ or $\neg A$)
d) $A - B$ (sometimes denotes as $A \setminus B$)

(d) *(0.5 points)* For the probability table below calculate the following values

X	Y	$p_{XY}(x, y)$
0	0	$\frac{1}{4}$
0	1	0
1	0	$\frac{1}{4}$
1	1	$\frac{1}{2}$

a) $p_X(x)$
b) $p_Y(y)$
c) $p_{X|Y}(x|Y = 0)$
d) $p_{Y|X}(y|X = 1)$
e) $\mathbb{E}[X]$
f) $\mathbb{E}[X|Y = 0]$
g) $Var[X]$

Submission

Please archive your report and codes in "Name_Surname.zip" (replace "Name" and "Surname" with your real name), and upload to "`Assignments 2020-2021 /TP1: Basics`" on `https://moodle.unige.ch` before Thursday, October 8, 2020, 23:59 PM. Note, **the assessment is based not only on your code, but also on your report, which should include your answers to all questions and the experimental results.**