$$F(\omega) = \int_{-\infty}^{+\infty} f(t)\, e^{-j\omega t}\, dt$$

$$\mathcal{F}\{f(t \pm t_0)\} = \int_{-\infty}^{+\infty} f(t \pm t_0)\, e^{-j\omega t}\, dt = \left\{ \begin{array}{l} t \pm t_0 = a \\ t = a \mp t_0 \\ dt = da \end{array} \right\}$$

$$= \int_{-\infty}^{+\infty} f(a)\, e^{-j\omega(a \mp t_0)}\, da =$$

$$= \int_{-\infty}^{+\infty} f(a)\, e^{-j\omega a}\, \underbrace{e^{-(\mp t_0) j\omega}}_{\text{doesn't depend on } a}\, da =$$

$$= e^{\pm j\omega t_0} \underbrace{\int_{-\infty}^{+\infty} f(a)\, e^{-j\omega a}\, da}_{= F(\omega)} =$$

$$= \boxed{F(\omega) \cdot e^{\pm j\omega t_0}}$$

1. $F(\omega) \cdot e^{\pm j\omega t_0} = \left(|F(\omega)| \cdot e^{j\varphi(\omega)} \right) \cdot e^{\pm j\omega t_0} =$

$$= |F(\omega)| \cdot e^{j(\varphi(\omega) \pm \omega t_0)}$$ => shift affects only phase part.

2. $F(\omega) \cdot e^{\pm j\omega t_0} = \left\{ e^{j\theta} = \cos(\theta t) + j \sin(\theta t) \right\} =$

$$= F(\omega) \cdot \left(\cos(\pm \omega t_0) + j \sin(\pm \omega t_0) \right) =$$

$$= F(\omega) \cdot \cos(\omega t_0) \pm j F(\omega) \sin(\omega t_0) =$$

real : $F_{real}(\omega) \cdot \cos(\omega t_0)$

imag : $F_{imag}(\omega) \cdot \sin(\omega t_0)$

shift affects both real and imag parts