Metaheuristics for Optimization

Series 4 : Ant System and Traveling Salesman Problem

Part 1 : Return no later than October 31, 2022 (16h00)

Part 2 : Return no later than November 7, 2022 (16h00)

The goal of this exercise is to implement the *Ant System* (AS) algorithm and use it to solve different instances of the *Traveling Salesman Problem* (TSP) we already encountered in the previous TP.

1 The AS algorithm

In the AS algorithm for the TSP problem, at each iteration t ($t = 1, \dots, t_{max}$), each ant k ($k = 1, \dots, m$) moves on a weighted graph that encodes the TSP problem and constructs a path of n nodes (i.e. cities).

In the construction of a solution, ants select consecutive cities to be visited through a stochastic mechanism. For each ant k which is in city i, the probability of going to city j in iteration t is given by :

$$p_{ij}^k(t) = \begin{cases} \frac{(\tau_{ij}(t))^\alpha (\eta_{ij})^\beta}{\sum_{l \in J} (\tau_{il}(t))^\alpha (\eta_{il})^\beta} & \text{if } j \in J \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

where J is the set of cities not yet visited by ant k , η_{ij} is the inverse of the distance[1] between cities i and j (i.e. $\eta_{ij} = \frac{1}{d_{ij}}$), $\tau_{ij}(t)$ is the intensity of the path between i and j at iteration t. The parameters α and β control the relative importance of the pheromone versus the heuristic information η_{ij}, respectively.

At each iteration, the pheromones are updated by all m ants that have built a solution. The pheromone τ_{ij} associated with the edge joining cities i and j is updated as following :

$$\tau_{ij}(t+1) = (1-\rho)\tau_{ij}(t) + \sum_{k=1}^{m} \Delta\tau_{ij}^k(t) \tag{2}$$

where ρ is the evaporation rate and $\Delta\tau_{ij}^k(t)$ is the quantity of pheromone laid on edge (i, j) by ant k :

$$\Delta\tau_{ij}^k(t) = \begin{cases} \frac{Q}{L^k(t)} & \text{if ant } k \text{ used edge } (i,j) \text{ in its tour} \\ 0 & \text{otherwise} \end{cases} \tag{3}$$

where Q is a constant, and L^k is the length of the tour constructed by ant k. Initially, a small quantity $\tau_0 \geq 0$ is assigned to each edge joining a pair of cities. The best path in terms of the length is returned as a final solution. The algorithm is summarized in the pseudo-code below.

1. The relative distances should be computed using the euclidean metric.

Algorithm 1

```
1: for all $t = 1, ..., t_{max}$ do
2:     for all ant $k = 1, ..., m$ do
3:         choose a city at random
4:         while there exists a city not visited do
5:             choose a city $j$ according to (1)
6:         end while
7:         mark a path according to (3)
8:     end for
9:     update all paths according to (2)
10:    Keep the best of solutions obtained at last iteration
11: end for
```

1.1 Choice of the parameters

The parameters α, β and ρ should have values 1, 5 and 0.1, respectively. The parameters Q and τ_0 have values L_{nn} and $\frac{1}{L_{nn}}$, respectively, where L_{nn} is a possible solution determined by using the nearest neighbour algorithm (see Section 2). You are free to assign the value to the m and t_{max} parameters, i.e. experiment with a few values of m and t_{max} and select the ones which give the best results. You are allowed to change the values of the above parameters as long as you mention it clearly in the discussion and argue on the way it improves the performance of the metaheuristic.

2 Work to do

In all of the experiments, assume that you start and end your path in the first city, i.e. the first city might be a warehouse from where the goods are delivered to all the facilities (it means that $c_{n+1} = c_1$ must hold in (4).

2.1 Part 1

Work on implementing the functions that will allow you to :
— Import & save data
— Compute the point-point distances
— Compute the fitness
— Get the probability to go to city j from i
— Perform the Greedy Algorithm
— Get the individual ant path (to parallelize later)

To start, you can take advantage of the given skeleton code.

2.2 Part 2

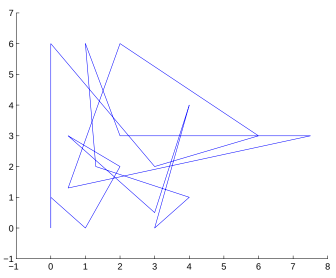
Complete the Ant System (AS) algorithm.
Run your AS algorithm on the two problems attached to this exercise [2] and report the solutions as in (4).

2. These problems represent facility placements (e.g. shops) in a city, where more facilities are in a center of a city, and less are in a suburb.

For the above problems analyze the influence of the m and t_{max} parameters to the quality of the solutions.

Compare the results with those obtained by using the simple greedy algorithm called *nearest neighbour algorithm*[3], repeated 10 times.

For the two TSP instantiations provide a visualization of the obtained optimal solutions by plotting the positions of the cities and the shortest paths obtained by the AS and the greedy algorithm. The following picture displays an example of such a visualization.



Moreover, randomly generate five TSP problems of size 50, 60, 80 and 100. For these problems, report and comment on the means and standard deviations of the execution times and lengths of the paths of the AS and greedy algorithm (the latter averaged over 10 runs).

Compare and discuss the performance of the Ant System (AS) and the Simulated Annealing (SA) algorithms applied on the TSP in terms of their solution quality and execution time.

Your code should return the solution of the TSP in the form of a sequence of cities corresponding to the shortest path and its length, i.e.

$$[c_1, \ldots, c_n] : \sum_{i=1}^{n} d(c_i, c_{i+1}) \tag{4}$$

where $c_1, \ldots, c_n$ are cities and d is a distance measure.

Make sure to comment on your results for :

1. The effect of varying t_{max}, and m (number of ants)
2. Greedy vs. Ant System
3. Performance of the Ant System (AS) and the Simulated Annealing (SA) algorithms applied on the TSP in terms of their solution quality and execution time.

In the comments, answer the following questions :

1. What is the Search Space for Ant System algorithm ?
   (given n cities, & m ants)
2. Can you describe the neighborhood in this case ?
3. Talk about the impact of parameters α, β, & ρ

3. This algorithm simply requires to pick a city randomly, then move to the nearest one, then to the nearest one that has not been visited yet, etc

3 Work to Return

For this series, submit your code, results, and comments, and upload them on moodle, by :

— Monday, October 31, 2022 at 16h00 for Part 1.

— Monday, November 7, 2022 at 16h00 for Part 2.

The use of python notebooks is highly recommended, to be able to include code, results, and comments in the same file. Graphs must include a title and a legend for the axes when needed.