Methods and Heuristics for Learning and Optimization

Series 6: Genetic Algorithms and Function Minimization

Part 1: Return no later than November 28, 2022 (16h00)

Part 2: Return no later than December 5, 2022 (16h00)

1 Introduction

In this exercise we will introduce the concept of Genetic Algorithms. The task we will focus on is the minimization of the following function:

$$f(x,y) = -\left|\frac{1}{2}x\sin\left(\sqrt{|x|}\right)\right| - \left|y\sin\left(30\sqrt{\left|\frac{x}{y}\right|}\right)\right|$$

where $x, y \in [10, 1000] \cap \mathbb{N}$.

2 Instructions for the Genetic Algorithm

2.1 Individuals and Fitness

An individual of the evolving population must represent a solution of the optimization problem to be solved. You should code a solution as a binary sequence made up of two halves, representing x and y, respectively. Indicate the size of the search space in this case.

To compute the fitness, you need a mapping function between binary and real numbers. Assuming binary sequences (x and y) are composed of $m = 10$ bits, a good candidate for the mapping formula is

$$map : x \mapsto \frac{x}{2^m}(b-a) + a$$

since it ensures that $map(x) \in [a, b] \cap \mathbb{R}$. Starting from the above formula, propose a mapping function to minimize $f(x, y)$, i.e., for $x, y \in [10, 1000] \cap \mathbb{N}$.

2.2 Selection

You can choose a population of size $N = 100$, and pick a person via the 5-tournament selection method (repeat it N times to keep the population size constant).

2.3 Crossover

You should use the `One Point Crossover` with a `Mid-Break` policy and a probability p_c [1] (repeat it $N/2$ times to keep the population size constant).

[1] Crossover probability p_c can be set to 0.6 for instance.

2.4 Mutation

You should implement a mutation operator that with a small probability (p_m) will swap each element of an individual.

3 Work to do

3.1 Part 1

Work on implementing the functions that will allow you to:

- Evaluate function $f(x, y)$.
- Visualize the function to minimize in a 3D space to get an idea on how difficult is to find its global minimum, and localize its global minimum.
- Evaluate the mapping function.
- Generate a generation of N individuals, of k bit-strings each.
- Calculate the fitness of a population generation.
- Perform Selection Process.
- Perform Crossover Process.
- Perform Mutation Process.

To start, you can take advantage of the given skeleton code.

3.2 Part 2

1. Implement the GA Algorithm.
2. Test & Evaluate:
   You should experiment with 2 different probabilities of mutation p_m:
   - 0.01
   - 0.1

   and with the following crossover operators:
   - without crossover
   - `One Point Crossover` with a `Mid-Break` policy and a probability p_c

   You should run the algorithm (in each of the above 4 cases) several times by changing the seed of a random number generator.

   Measure and present:

(a) The best, average and standard deviation of fitness among the populations for $10^3, 10^4$, and 10^5 fitness evaluations. The number of fitness evaluations can be estimated by the product of the population size with the number of generations. These statistics should be computed over several runs of the genetic algorithm (e.g., 10 runs).

(b) The cumulative empirical probability to reach the following solution qualities (optimum, relative distance to optimum $\leq 1.0\%$, relative distance to optimum $\leq 2.5\%$) over the number of evaluations.

In the comments, answer the following questions:

1. What is the Search Space of our problem?

2. What are the roles of Selection, Crossover, & Mutation processes?

3. What does each of the 3 processes Selection, Crossover & Mutation favor : exploration/diversification or exploitation/intensification?

4 Work to Return

For this series, submit your code, results, and comments, and upload them on moodle, by :

- Monday, November 28, 2022 at 16h00 for Part 1.

- Monday, December 5, 2022 at 16h00 for Part 2.

The use of python notebooks is highly recommended, to be able to include code, results, and comments in the same file. Graphs must include a title and a legend for the axes when needed.