

# The Variability of the Belgian Business Survey Indicator

Analysis and Predictive Power

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## **Abstract**

#### Abstract

The variance is, next to the mean, a very important information about a certain variable.

In the context of the business survey, the indicator has a variance that can be interpreted is the (dis)agreement among respondents. This piece of information will be looked at. The fact that the data is trichonomous, since there are only three possible answers to the questions of the survey, contains some interesting properties. The is a mean-variance relation and the variance is bounded between 0 and 1.

This Master Thesis explores the variance of the Belgian business survey.

Characteristics specific to the business survey

Panel surveys contain the information of the evolution of each participant. A measure is here proposed, the evolution of individual responses, that takes into account the responses changes of all the participants.

it's variance c

In a second part, the predictive power of the variance and the new indicator of evolution in individual responses is examined and it's found that, the variance of the indicator improves the fit of a linear model and its predictions

# Samenvatting

# Keywords

Business Surveys - Business Barometer - Trichotomous Observations - Survey Variance - Survey Volatility -

# List of Abbreviations

AIC Akaike's Information Criterion BIC Bayesian Information Criterion

BSI Business Survey Indicator/Barometer

Cor Correlation Cov Covariance

E Mean

ECB European Central Bank

EIR Evolution in individual Responses Eurostat The European Statistical Office

GDP Gross Domestic Product

INSEE Institut National de la Statistique et des Etudes Economiques (France)

LOCF Last Observation Carried Forward

MAE Mean Absolute Error

MAPE Mean Absolute Percentage Error MASE Mean Absolute Scaled Error MPE Mean Percentage Error

NBB The National Bank of Belgium

NBER The National Bureau of Economic Research (US)

NSI National Statistics Institutes RMSE Root Mean Squared Error

Var Variance YoY Year on Year

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# CHAPTER 1

## Introduction

Each month around 3000 companies reply to several questions of a survey that holds on one page and the questions have three possible answers.

The results are then summarised and deliver a surprisingly accurate picture of the present state of the Belgian economy.

The results are surprisingly accurate to capture the present state of the economy and make short term prediction.

A widespread method to predict the evolution of National Economies is the survey-based business indicator. Belgium have been collecting this indicator for more than 60 years. This long evolution

This Thesis is included in the continuity of a long tradition of papers proposing improvement and ways to add value to the Business Barometer (......) will propose ways to add information to the Belgian Business Barometer, that could also be applied to others

The business surveys are today

Add European dimension!

#### **Objectives**

This study explores the variability and the possible interest of taking it into account to make future predictions.

The variability will take two very different forms in this paper. First it will be measured with the variance of the indicator, as a measure of the (di)agreement among respondents.

The second variability measure will be called the evolution of individual responses (EIR) and will account for the changes in responses of the different participants. This is possible since the business survey is a panel survey where respondents usually participate in the survey for a long period.

#### Methodology

To make it possible to have a long period of time to analyse, the decision is made to take only unweighted indicators. The weighting procedure been very complex and prone to some companies influencing largely the indicator. Also the weights are only available for the period after 2008, here the dataset obtained goes from 1988 to 2018

Four questions are taken into account, since it are the four questions that are taken today in the calculation of the industry BSI. It has to be noted that before 2009, almost all the questions where taken into account for the calculation. In order to have the best comparability, only the four questions will be used for the whole period.

#### Plan of this Paper

This Master Thesis

Chapter 2 will introduce the business survey and the business survey indicator. Explaining the history, the objectives and the methodology of the business survey. The calculation of the different business survey indicators will then be presented. The method will be the same for the other indicators/variables, first the unweighted and weighted procedure will be explained when taking one question into account. Then the method of calculation taking more than one question into account will be presented.

Chapter 3 will present the variance of the business survey indicator. Starting from one questions variance when unweighted or weighted, it will then be enlarged to several questions taken into account.

Chapter 4 will present the indicator of the evolution of individual responses (EIR) while Chapter 5 will present its variance. The same procedure/method as for the BSI will be applied to the EIR.

Chapter 6 test for seasonal effects and explain the seasonal correction applied to the data. It will then discuss some potential issues/bias of the survey; non-responses, dropout and attrition.

In Chapter 7, descriptive statistics will be made of the different variables at hand and it will be looked into correlations among the different variables.

Chapter 8 applies linear regressions in order to see the potential interest of the evolution in individual responses and the different variances.

# The Business Survey Indicator

This chapter is a presentation of the Belgian business survey and the business survey indicator (BSI), also referred to as the business survey barometer or business confidence indicator/barometer.

First, a brief history of the business survey indicator will be presented. The second section will discuss the sampling method while the third section will discuss the objectives of the business survey, which are (1) understanding the short term evolution by sector of the Belgian economy, (2) nowcasting and (3) the analysis of business cycles. The next part will discuss the methodology regarding the questions and the weighting procedure(s). In the last section of the chapter, the calculation method of the business survey indicator will be presented.

# 2.1 History

The Belgian business survey celebrates this year its 65<sup>th</sup> anniversary. The survey was launched by the National Bank of Belgium in 1954, it was then part of the pioneers since only the United States (1930) and West Germany (1949) had a business survey at the time.

In 1972, the results were first synthesised in an indicator. The business survey barometer started by including only the industrial sector. It was then from 1970 on, small by small enlarged to other sectors: construction, trade, and services.

Over time, several improvements to the business survey were proposed and applied (1983, 1990 and 2009). The last improvements will be discussed in detail in section 2.3.

At the European level, it was in 1961 that the European Commission launched a harmonisation program of the business survey in the manufacturing industry. Since then, the sector coverage of the program has widened to account for the different sectors. The harmonisation program and the large implication of the EC, make it possible to compare BSI around the European Union. More information can be found in the "The Joint Harmonised EU Programme of Business and Consumer Surveys User Guide" European Commission (2016).

Over time, the business survey barometer got well known for being a very informative and useful indicator. In an article published in the Wall Street Journal titled "Euroland Discovers A Surprise Indicator: Belgian Confidence" (Rhoads, 1999), the BSI is described as an important and accurate measure of the evolution of the Belgian economy. It also suggested that it could be a good indicator for the European Union. This hypothesis was tested for the period between 1985-2000 in Vanhaelen et al. (2000) and the conclusion was very flattering for the BSI. It was shown that the BSI was indeed a leading indicator of the evolution of the European economy and could quite accurately forecast turning points. The explanation proposed by the authors is, first of all, that the Belgian economy, in itself, had some predictive power for the European Area for that period, since it is specialised in intermediate goods and is a very open economy. The other potential explanation pointed out, is the high representation of small and medium-sized enterprises in the business survey.

Today the business survey indicator is a well-known indicator of the Belgian economy, the indicator is on the homepage of the National Bank website and is used in several predictive models.

# 2.2 Objectives of the Business Survey

The main objective of the business survey barometer is to have a feeling of how the economy is now and how it will evolve in the short term. We will here decompose the main objectives of the business survey into three subjects; (1) the direct information of the Belgian economy that can be delivered by the global and sector specific BSI, (2) Nowcasting which refers to the now and short term prediction of the economy growth and (3) the long term analysis of the business cycles and the importance of identifying turning points.

The three objectives have in common the importance, the need, of only capturing the real evolution of the economy, without taking into account short term noise/variation as seasonal effects or bias that could erupt due to the survey method. This will be looked into in chapter 6 where seasonal effects, dropout, non-response, and attrition will be discussed.

# 2.2.1 National and Sector-Specific Short-Term Information

The business survey indicators are published at the end of each month (around the 21-25 of the month) and give a fast capture of the evolution of the Belgian economy over the past months. The data is available on stat.nbb.be and a press release is published on nbb.be. The press release contains a summary and interpretation of the BSI followed by graphs to show the evolution of the BSI over the 4-5 last years for the sector-specific and overall indicator. The public can use that information to have a snapshot of the economy, while other indicators, as GDP or unemployment, can take a very long time before being published.

# 2.2.2 Nowcasting

Also called "flash" estimation, nowcasting has increasingly gained importance in the last decade. it consists of the short term estimation of the economy, Usually GDP growth. It

is a fundamental approach since the business survey indicator is published monthly while other indicator's like GDP are published quarterly.

As can be seen in Figure 2.1, the lag between the observation and publication is even greater since the business survey indicator is published at the end of each month (around the 24-25<sup>th</sup> of each month), while the GDP is published with a lag of 3 to 4 weeks and is subject to revision.

If we look at Figure 2.1 we can see that at the end of the month of January, the GDP is published for the last quarter of the previous year, so the information is already out-dated of some weeks, when the business survey indicator is published for the month of January around the same date. After the end of January publication, observers of the economy have to wait for three months to have new information about the GDP, while each month, the BSI is published and available for everyone.

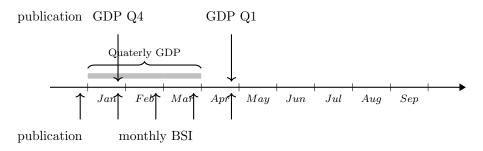


Figure 2.1: Timeline of the period and the publication of the business survey indicator (BSI) and the Gross Domestic Product (GDP)

Nowcasting became a catch-all word and includes a variety of predictive models as Linear regression, ARIMA models, State-Space models, Mixed-data sampling (MIDAS) regressions, Autoregressive Distributed Lag (ARDL) models and much more.

Indicators that can be used aside from the business survey are average weekly work hours, factory orders for goods, housing permits and stock prices index of consumer expectations, average weekly claims for unemployment insurance and the interest rate and more.

# 2.2.3 Business Cycles

In 1946, Mitchell and Burns defined a business cycle as a recursive fluctuations, affecting macroeconomics variables. Since then a lot of variables where used to model business cycles but it's commonly admitted that Growth Domestic Product (GDP) is the most important of them. A good measure of the growth of GDP is year on year GDP that is obtained as follow

YoY GDP = 
$$\frac{GDP_t - GDP_{t-12}}{GDP_{t-12}}$$
 (2.1)

Figure 2.2 shows a simplified version of business cycles theory when using as measure GDP or YoY GDP.

A very important question considering business cycles is their duration. Figure 2.2 can give the false impression that business cycles are all of the same lengths, this is, in real life, not so simple.

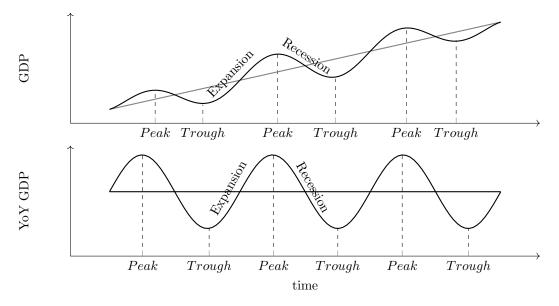


Figure 2.2: The Business Cycle theory of GDP and year on year GDP

Probably the first person to explore the duration of a business cycle was a French statistician, Juglar (1862), who set the business cycles to have a duration of 7 to 11 years. Mitchell and Burns (1946) proposed a minimum duration of 16-22 months and a maximum duration of 100-106 months. Lot of other propositions where done even though business cycles are rather empirically defined than theory based. Therefore let put the theory aside and look into real data. The example of the United States is interesting since the National Bureau of Economic Research (NBER) dated precisely and methodologically the turning points for the American economy. The empirical evidence that comes out of this work, is that the time from one economic peak to the next is on average 5 and a half years for the period 1945-2009.

We can see from Figure 2.3, which represent the different bottoms and peaks of business cycles identified by the NBER from 1975 to 2009, that there is no symmetry of the business cycles. Some business cycles are very short while others last more than ten years. We can notice that periods of economic growth, usually last longer than economical decrease.

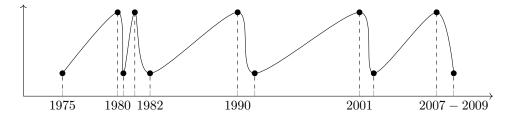


Figure 2.3: Business cycles from 1975 to 2009 of the American Economy according to the NBER<sup>1</sup>

<sup>&</sup>lt;sup>1</sup>data available at https://www.nber.org/cycles.html

# 2.3 Methodology of the Business Survey

Since 65 years of existence, the business survey was able to evolve without losing in long term comparability.

Three main methodology revisions since the launch of the business survey, in 1983, 1990 and 2009 De Greef and Van Nieuwenhuyze (2009). The most significant changes brought in 2009 where the number of questions taken into the calculation of the BSI, which was reduced to 3-4 questions, depending on the sector, for more simplicity and accuracy. Other improvements where the inclusion of the sector of services in the calculation of the global indicator and a new, simple method of smoothing the indicator.

## 2.3.1 Sampling Method

The Belgian business survey - as most of the business surveys around the world - has the particularity of not using random sampling. The selection of participants is quite complex and a lot of decisions are human, never is a statistical program or a random sampling system used to select new participants.

The selection of new participants is done by waves. When the department responsible for the business survey at the NBB decides that there aren't enough participants in a specific sector anymore, the recruitment of new participants is launched.

To find the new respondents, the first step is to decide for an optimal amount of new participants needed, regarding the different stratification of the sector. Each sector is composed of quite advanced trees of sub-sectors, sub-sub-sectors, and more. For example, the industry sector is divided into more than 300 sub-sectors / branches over 6 different levels.

It could be that some subdivision of a certain sector only has 1 or 2 respondents while it accounts for a significant part of the Belgian economy. The Department of the business survey of the NBB will then look into which company, from that specific part of the economy, could be a good fit as new respondent of the business survey, considering its activity, it's size, region, and other characteristics. As will be seen in section 2.3.3 more in detail, companies are weighted by there size (profit, number of employees, ...) and the size of the sector/branch they are part of. That information is crucial for selecting new participants. The procedure is quite complex and therefore contains a lot of human decisions.

Out of this process comes a list of potential new participants. This list is then sent to the Communication Department that makes contact with those potential new participants. Not always, but usually, a representative of the National Bank visits the new participant to explain the survey and have a contact. As a reward for participating in the survey, the companies receive privileged information. Each month they receive access to sub-sector indicators information that aren't publicly distributed. This can give them economical information regarding their specific sector of economic activity.

At the National Bank of Belgium, this procedure is usually referred to as prospecting, rather than selecting or sampling, since it is mostly based on recruiting new companies that will work/collaborate with them. New companies can be included in the survey outside of this procedure but this happens more occasionally.

An important side of the business survey is that people are staying as long as possible in the survey. From the moment companies are part of the survey, they stay in it until

they decide to leave, there are no participants removed from the survey by the National Bank, what can happen is that if participants don't answer for three months, contact will be taken with the company to see if they want to continue to participate.

There are resources put by the National Bank to make sure companies answer to the survey, and stay in it. This means that some companies are part of the survey for a very long time. To have an idea, if we look at the survey for the industry and trace respondents back to 30 years ago (1989), we can see that today, approximately one-third of the respondents were already in the survey in 1988.

From a statistical point of view, it can seem rather problematic to draw general conclusions over a population when not using random sampling. Without undermining one of the most important pillars of statistics, there are two main reasons why in this case, having a non-random selection of participants is not truly problematic.

The first reason is that the sampling method used is trying to represent as good as possible the population that it is representing. Therefore stratification is used at a quite advanced level as explained before. We could call this recruitment method: non-random stratification, as opposed to random stratification. Since it's not using sampling but takes into account the stratification of the population it's studying. The second reason, the most import one, is that the value of the business survey indicator doesn't have interest on its own. Indeed having a BSI equal to 0.5 or 0.1 doesn't mean much, what's important is the evolution of the indicator. If it was equal to 0.3 last month and this month it's equal to 0.5, it means that the economy is most probably growing and that we can forecast an increase of GDP over the month. On the other hand, if it's now equal to 0.1, it means a decrease in economic confidence among businesses and we can anticipate a deceleration or decline of the economy over the month.

#### 2.3.2 Questionnaire

The questionnaire exists in two languages, french and dutch (see Appendix page 54). It can be answered by mail, email, over the phone or by fax. It is divided into two part: (1) questions concerning current production and level of activity ("verloop en beoordeling") and (2) questions concerning predictions, expectation of the level of activity over the next three months ("vooruitzichten voor de volgende drie maanden").

The business survey indicator is also called the confidence indicator, since its a measure of how confidence companies are in the Belgian economy. Almost all the questions have only three possible answers that can be interpreted as a negative, neutral or positive answer.

In this paper, the answers and results of the industry business survey barometer will be used since 1988. Therefore it's very important to see if modifications were applied over time to the questionnaire. A questionnaire from 1990 can be seen in appendix (see page 55) and can be compared to a more recent version (see appendix page 54).

The layout was modified, and the phrasing of the questions changed over this long period. It was before asked in the first person while it's now phrased in the third person. Aside from those small changes, the survey kept the same questions and order. It would be interesting to have a closer look at the potential consequences of those changes over time. The layout, the phrasing and the method of answering can potentially influence the answers. Nevertheless, since it's not the subject of this paper, we will leave this study to future research. What we can say, is that the influence of those changes could be

limited since the respondents where mostly the same when the changes happened, so the interpretation they made from the questions could have a smaller impact.

The interest of this paper is the industry business survey indicator, which takes four questions into account (questions 18, 27, 32 and 33). The questions are translated into English on page 51 and are renumbered from 1 to 4 for simplicity. The two first questions relate to the current state of the company while the two others ask the participants there prediction for the next three months. The first question is about the stock of the company, the second question relates to the demand of there product, the third questions relate to the company's predictions of need of work forces and the last question ask their predictions regarding demand.

## 2.3.3 Weighting Procedure

The different companies participating in the business survey have all two weights; (1) according to the size of the company and (2) according to the size of the sector branch which the company is part of. The weight size is calculated based on the profit the company is making, the capital it's owning, the number of employees and other characteristics. The calculation is quite complex and is specific to each sector. For example, the companies having an industrial activity have a different calculation than a restaurant or a financial services company.

Since the characteristics taken into account during the calculation of the weight of each company aren't fixed, the weight of the companies is corrected periodically. Ones the new weight is calculated, the transition between the old and new weight is smoothened over a year. The difference between the new and old weights is divided by 12 and is then adapted each month for a year.

Regarding the globalisation weighting procedure, the National Bank of Belgium developed an elaborate division of the Belgian economic activity. This means that for example, the industry is subdivided into different sub-sectors, that they self contain sub-sectors that contain sub-sectors and so on for a total of six levels. Each division has a percentage according to its size in the economy. To obtain the weight of the lowest globalisation, the weight of each subdivision need to be multiply.

The procedure of weighting is then as follow

$$\omega_i = \frac{\text{weight company } i}{\sum \text{company weights within globalisation}} * \text{globalisation weight}$$
 (2.2)

where  $\sum_{i=1}^{n} \omega_i = 1$ . In other words, the specific weight for each company taking the two weighting procedures into account  $(\omega_i)$ , is obtained by dividing its weight coefficient by the total of weight coefficients within the lowest level of globalisation and multiply it with the weight of globalisation it's part of.

# 2.4 Calculation of the Business Survey Indicator

This section presents the method of calculation of the business survey indicator. The calculation in itself is rather standard, but the different ways to write it are important for the interpretation and a better understanding of the indicator and the following chapters.

We first present the calculation taking into account one question unweighted and then weighted. After we will present how different questions are combined together to obtain a business survey indicator.

#### 2.4.1 Unweighted Business Survey Indicator

The calculation of the unweighted indicator for a specific question at a specific time is the mean of the responses and can be written as follow;

$$E(X) = \frac{\sum_{i=1}^{n} x_i}{n}$$
 (2.3)

where  $x_i$  is the answer of the respondent i and can take value -1 (negative answer), 0 (neutral answer) and 1 (positive answer). n is the number of respondents.

Since  $x_i$  can only take three different values, we can decompose it into

$$E(X) = \frac{\sum_{i=1}^{n_{+}} x_{+i} + \sum_{i=1}^{n_{0}} x_{0i} + \sum_{i=1}^{n_{-}} x_{-i}}{n}$$
 (2.4)

where  $x_{+i}$ ,  $x_{0i}$  and  $x_{-i}$  are the positive (+), neutral (N) and negative (-) answers of the respondent i.

Since we know that  $\sum_{i=1}^{n} x_{0i} = 0$ ,  $x_{+i} = 1$  and  $x_{-i} = -1$  we can write

$$E(X) = \frac{n_{+}}{n} - \frac{n_{-}}{n} \tag{2.5}$$

Where  $n_+/n$  is the proportion of positive answers and  $n_-/n$  is the proportion of negative answer. We can write, for simplicity

$$E(X) = \pi_{+} - \pi_{-} \tag{2.6}$$

where  $\pi_+$  and  $\pi_-$  are the proportion of respondents answering positive and negative to the specific question.  $\pi$  was chosen as a symbol here since it can be interpreted as a probability: if we assume that all the respondents have the same probability of giving a certain answer,  $\pi$  is the probability that a respondent answers positive, negative or neutral to the question.

# 2.4.2 Weighted Business Survey Indicator

As described in section 2.3.3, each respondent has two different weights: one according to its size, one according to the size of the sector it's part of. Those weights are then combined and we end up with a specific weight  $\omega_i$ . We have then the following equation for the indicator

$$E(X) = \sum_{i=1}^{n} \omega_i x_i \quad \text{where } \sum_{i=1}^{n} \omega_i = 1$$
 (2.7)

 $x_i$  is the answer of the respondent i and can take values -1, 0 and 1.  $\omega_i$  is the weight of respondents i. The weights are standardised so their sum is equal to one.

As for the unweighted indicator, we can decompose the equation by the three possible answers with, in this case, their according weights.

$$E(X) = \sum_{i=1}^{n_+} \omega_i x_{+i} + \sum_{i=1}^{n_0} \omega_i x_{0i} + \sum_{i=1}^{n_-} \omega_i x_{-i}$$
 (2.8)

and again we know that  $\sum_{i=1}^{n} \omega_{0i} x_{0i} = 0$ ,  $x_{+i} = 1$  and  $x_{-i} = -1$  so we can write

$$E(X) = \sum_{i=1}^{n_{+}} \omega_{i} - \sum_{i=1}^{n_{-}} \omega_{i}$$
(2.9)

That we will write as follow

$$E(X) = \Omega_+ - \Omega_- \tag{2.10}$$

where  $\Omega_+$  and  $\Omega_-$  are the sum of weights of positive and negative respondents. In other words,  $\Omega_+$  and  $\Omega_-$  are the weighted proportion of respondents answering positive and negative.

 $\Omega$  is used here also in the probabilistic way as it can also be seen as the probability that a respondent answers positive, negative or neutral  $(\Omega_0)$  with  $\Omega_+ + \Omega_0 + \Omega_- = 1$ . Here we speak of weighted proportions/probabilities.

From Equation 2.6 and Equation 2.10 it can be seen that the weighted and unweighted indicators are bounded between -1 and 1. In the two cases, the indicator is the smallest if everyone has a negative answer, and is the largest when every answer is positive.

## 2.4.3 Take Different Questions Into Account

The previous calculations were specific to one question. The published indicators are usually taking different survey questions into account. For example, the industry indicator that we will be interested in is composed of four questions:

Industry BSI = 
$$\frac{E(X_{Q1}) + E(X_{Q2}) + E(X_{Q3}) + E(X_{Q4})}{4}$$
 (2.11)

where  $E(X_{Q1})$ ,  $E(X_{Q2})$ ,  $E(X_{Q3})$  and  $E(X_{Q4})$  are the different averages for question 18, 27, 32 and 33 (weighted or unweighted).

The equation can also be written as follow for the unweighted indicator<sup>2</sup>

Unweighted Industry BSI = 
$$\frac{\sum_{i=1}^{n} (x_{iQ1} + x_{iQ2} + x_{iQ3} + x_{iQ4})}{4n}$$
 (2.12)

and as follow for the weighted indicator<sup>3</sup>

Weighted Industry BSI = 
$$\frac{\sum_{i=1}^{n} \omega_i (x_{iQ1} + x_{iQ2} + x_{iQ3} + x_{iQ4})}{4}$$
 (2.13)

Another way to write the unweighted business survey indicator is as follow

<sup>&</sup>lt;sup>2</sup>Assuming the respondents that answered, answered to all the questions.

<sup>&</sup>lt;sup>3</sup>Assuming the respondents that answered, answered to all the questions and that the weighting  $(\omega_i)$  is the same across all the questions.

Unweighted industry BSI = 
$$\frac{1}{4} \left( \pi_{Q1,+} + \pi_{Q2,+} + \pi_{Q3,+} + \pi_{Q4,+} - \pi_{Q1,-} - \pi_{Q2,-} - \pi_{Q3,-} - \pi_{Q4,-} \right)$$
(2.14)

and regarding the weighted indicator, it can be written as

Weighted Industry BSI = 
$$\frac{1}{4} (\Omega_{Q1,+} + \Omega_{Q2,+} + \Omega_{Q3,+} + \Omega_{Q4,+} - \Omega_{Q1,-} - \Omega_{Q2,-} - \Omega_{Q3,-} - \Omega_{Q4,-})$$
 (2.15)

#### generalisation

The general formula for the weighted business survey indicator when taking several questions into account is

Unweighted BSI = 
$$\frac{\sum_{i=1}^{n} (x_{iQ1} + x_{iQ2} + \dots + x_{iQq})}{nq}$$

$$= \frac{1}{q} (\pi_{Q1,+} + \pi_{Q2,+} + \dots + \pi_{Qq,+} - \pi_{Q1,-} - \pi_{Q2,-} - \dots - \pi_{Qq,-})$$
(2.16)

While the formula for the weighted business survey indicator when taking several questions into account is

Weighted BSI = 
$$\frac{\sum_{i=1}^{n} \omega_{i} (x_{iQ1} + x_{iQ2} + \dots + x_{iQq})}{q}$$

$$= \frac{1}{q} (\Omega_{Q1,+} + \Omega_{Q2,+} + \dots + \Omega_{Qq,+} - \Omega_{Q1,-} - \Omega_{Q2,-} - \dots - \Omega_{Qq,-})$$
(2.18)

# CHAPTER 3

# The Variance of the Business Survey Indicator

The variance is, with the mean, one of the first tool for statisticians to study a certain variable. Next, to the mean, that is the average value of a certain variable, the variance is the measure of the dispersion. In the context of the business survey, the variance can be seen as "how much companies (dis)agree on the present state of the Belgian economy", a piece of important information that we can extract from the survey.

This chapter will present the calculation of the variance of the unweighted and weighted indicator for one question of the business survey. It will then be looked into its properties and specificities. The last section will present the method of calculation when different questions are taken into account.

# 3.1 Variance of the Unweighted Business Survey Indicator

The formula of the variance can be written as

$$Var(X) = E[(X - E(X))^{2}] = E(X^{2}) - E(X)^{2}$$
 (3.1)

In the case of one question of the business survey indicator, we decompose and develop the equation as follow

$$Var(X) = E(X^{2}) - E(X)^{2}$$

$$= \left(\frac{\sum_{i=1}^{n_{+}} x_{+i}^{2}}{n}\right) + \left(\frac{\sum_{i=1}^{n_{0}} x_{0i}^{2}}{n}\right) + \left(\frac{\sum_{i=1}^{n_{-}} x_{-i}^{2}}{n}\right) - E(X)^{2}$$

$$= \left(\frac{n_{+}}{n}\right) + \left(\frac{n_{-}}{n}\right) - E(X)^{2}$$
(3.2)

Since the positive answers take value 1 and negative answers value -1  $(x_{+i}^2 = x_{+i} = 1$  and  $x_{-i}^2 = |x_{-i}| = 1$ , and  $\left(\frac{\sum_{i=1}^{n_0} x_{0i}^2}{n}\right) = 0$ . The equation can further be simplified to

$$Var(X) = \pi_{+} + \pi_{-} - E(X)^{2}$$
(3.4)

Where  $\pi_{+}$  and  $\pi_{-}$  are the proportions of positive and negative answers.

In other words, the variance of the BSI is equal to the sum of the proportion of positive and negative answers, minus the squared indicator.

Since  $E(X) = \pi_+ - \pi_-$ , and  $\pi_+ + \pi_- = 1 - \pi_0$  (from  $\pi_+ + \pi_0 + \pi_- = 1$ ). It's possible to write the variance in several different ways;

$$Var(X) = \pi_{+} + \pi_{-} - E(X)^{2}$$

$$= \pi_{+} + \pi_{-} - (\pi_{+} - \pi_{-})^{2}$$

$$= 1 - \pi_{0} - E(X)^{2}$$
(3.5)
(3.6)

# 3.2 Variance of the Weighted Business Survey Indicator

We can now do the same for the weighted indicator. The equation is very similar to the variance of the unweighted variance;

$$\operatorname{Var}(X) = E(X^{2}) - E(X)^{2}$$

$$= \sum_{i=1}^{n_{+}} \omega_{i} x_{+i}^{2} + \sum_{i=1}^{n_{0}} \omega_{i} x_{0i}^{2} + \sum_{i=1}^{n_{-}} \omega_{i} x_{-i}^{2} - E(X)^{2}$$

$$= \sum_{i=1}^{n_{+}} \omega_{i} + \sum_{i=1}^{n_{-}} \omega_{i} - E(X)^{2}$$
(3.7)
$$= \sum_{i=1}^{n_{+}} \omega_{i} + \sum_{i=1}^{n_{-}} \omega_{i} - E(X)^{2}$$

As we did for the indicator, we will develop the equation by taking into account weighted proportion and write the equation the same way as for the unweighted variance. Again the variance can be written in different ways;

$$Var(X) = \Omega_{+} + \Omega_{-} - E(X)^{2}$$
(3.9)

$$= \Omega_{+} + \Omega_{-} - (\Omega_{+} - \Omega_{-})^{2} \tag{3.10}$$

$$= 1 - \Omega_0 - E(X)^2 \tag{3.11}$$

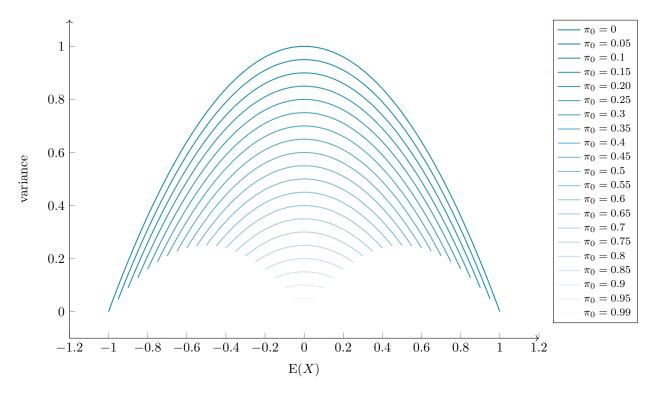


Figure 3.1: Plot of the possible values of the indicator (X axis) and variance (Y axis) for different values of  $\pi_0$ 

# 3.3 Properties

Based on the previous development of the equation of the variance of the indicator, we can make some observations.

First of all, the variance is bounded between 0 and 1. A variance can't be negative since it's a sum of squares, so the lower bound shouldn't surprise anyone. On the other hand, the upper bound is more unusual. An interesting approach is to take Equation 3.6 and see that  $\pi_0$  and  $E(X)^2$  can only take positive values,  $\pi_0$  because it's a proportion and  $E(X)^2$  since it's squared. Both variables have a minus sign in the equation, so the highest results are obtained when both variables are equal to zero. In other words, the highest variance is obtained when no respondent answers "neutral" and the BSI is equal to 0. This happens when there are as many negative as positive answers (50-50). This corresponds to the interpretation described before since it's the situation with the most disagreement among respondents. In the other hand, if all participants answer the same ("negative", "neutral" or "positive"), the variance is equal to zero. It corresponds to the situation where the agreement among respondents is the highest.

Another approach to better understand the variance of the business survey barometer, is to plot the different possible values of Var(X),  $\pi_0$  and E(X) from Equation 3.6 or Equation 3.11. The results can be seen in Figure 3.1. It's interesting to see from the plot that each E(X) can only have a certain amount of possible variance, in other words, there is a specific upper and lower bound for each indicator. For example, an indicator of 0.5 can only have a variance between 0.25 and 0.75. It's also interesting to notice that to know that with two of the three variables  $(Var(X), \pi_0 \text{ and } E(X))$ , it's very easy to calculate the third. This means that the three variables are related.

# 3.4 Take Different Questions Into Account

As already seen, the published indicator takes different questions into account. The combination of the variance of different questions is slightly more complex than the combination of different indicators since the questions are correlated, which means that covariance has to be taken into account.

The formula to combine different variances is the following

$$\operatorname{Var}\left(\sum_{i=1}^{q} X_{i}\right) = \sum_{i=1}^{q} \sum_{j=1}^{q} \operatorname{Cov}\left(X_{i}, X_{j}\right) = \sum_{i=1}^{q} \operatorname{Var}\left(X_{i}\right) + 2 \sum_{1 \le i < j \le q} \operatorname{Cov}\left(X_{i}, X_{j}\right) \quad (3.12)$$

In the case of combining the variances of the four different questions of the industry business survey, we have the following equation

$$\operatorname{Var}\left(\operatorname{Industry} \, \operatorname{BSI}\right) = \frac{1}{16} \left[ \operatorname{Var}(X_{Q1}) + \operatorname{Var}(X_{Q2}) + \operatorname{Var}(X_{Q3}) + \operatorname{Var}(X_{Q4}) + 2\operatorname{Cov}(X_{Q1}, X_{Q2}) + 2\operatorname{Cov}(X_{Q1}, X_{Q3}) + 2\operatorname{Cov}(X_{Q1}, X_{Q4}) + 2\operatorname{Cov}(X_{Q2}, X_{Q3}) + 2\operatorname{Cov}(X_{Q2}, X_{Q4}) + 2\operatorname{Cov}(X_{Q3}, X_{Q4}) \right]$$

$$(3.13)$$

The complexity of the formula encourages to rather calculate the indicator (taking all the questions into account), and then calculate the variance of that indicator. It can be written as follow

$$Var(BSI) = E[(BSI - E(BSI))^{2}] = E(BSI^{2}) - E(BSI)^{2}$$
(3.14)

$$\operatorname{Var}\left(\frac{X_{Q1} + X_{Q2} + X_{Q3} + X_{Q4}}{4}\right) = \frac{1}{16} \operatorname{Var}\left(\frac{\sum_{i=1}^{n} \left(x_{iQ1} + x_{iQ2} + x_{iQ3} + x_{iQ4}\right)}{n}\right)$$

$$= \frac{1}{16} \operatorname{Var}\left(\pi_{Q1,+} + \pi_{Q2,+} + \pi_{Q3,+} + \pi_{Q4,+} - \pi_{Q1,-} - \pi_{Q2,-} - \pi_{Q3,-} - \pi_{Q4,-}\right)$$
(3.15)

The generalisation of the previous equation can be written as follow for the variance of the unweighted indicator

Var (Unweighted BSI) = 
$$\frac{1}{q^2} \operatorname{Var} \left( \frac{\sum_{i=1}^n (x_{iQ1} + x_{iQ2} + \dots + x_{iQq})}{n} \right)$$
  
=  $\frac{1}{q^2} \operatorname{Var} \left( \pi_{Q1,+} + \pi_{Q2,+} + \dots + \pi_{Qq,+} - \pi_{Q1,-} - \pi_{Q2,-} - \dots - \pi_{Qq,-} \right)$ 
(3.16)

and as follow for the variance of the weighted indicator

Var (Weighted BSI) = 
$$\frac{1}{q^2} \text{Var} \left( \sum_{i=1}^n \omega_i \left( x_{iQ1} + x_{iQ2} + \dots + x_{iQq} \right) \right)$$
  
=  $\frac{1}{q^2} \text{Var} \left( \Omega_{Q1,+} + \Omega_{Q2,+} + \dots + \Omega_{Qq,+} - \Omega_{Q1,-} - \Omega_{Q2,-} - \dots - \Omega_{Qq,-} \right)$ 
(3.17)

# CHAPTER 4

# The Evolution of Individual Responses

In the same logic as for the variance, a proposition is done here of a method to extract more information out of the business survey. As explained in the first chapter, the business survey is answered by the same companies over time. Some new participants join and some companies leaving the survey, but the survey can be referred to and treated as a panel survey.

Is it possible to have more information by taking the evolution of the individual respondents into account? This chapter will address this question by applying a method proposed by Caron et al. (1996) that, by taking all the individual evolution of responses into account, offers a method to calculate an indicator that will be referred to as the indicator of the evolution of individual responses (EIR).

When only one period is taken into account, there are three possible answers; "negative", "neutral" and "positive". When we take two periods into account, a month (t) and the previous one (t-1) for example, there are nine possible situations as represented in Table 4.1.

			t	
		$x_{i-}$	$x_{i0}$	$x_{i+}$
	$x_{i-}$	$z_{i}$	$z_{i-0}$	$z_{i-+}$
t-1	$\begin{vmatrix} x_{i-} \\ x_{i0} \end{vmatrix}$	$z_{i0-}$	$z_{i00}$	$z_{i0+}$
	$x_{i+}$	$z_{i+-}$	$z_{i+0}$	$z_{i++}$

Table 4.1: Possible observations when taking t and t-1 into account

The same as for the business survey indicator, the evolution of individual responses take different values. In the case of the BSI,  $x_i$  can take value -1, 0 and 1. In the case of the EIR,  $z_i$  can take values -2, -1, 0, 1 and 2.

The EIR defers also from the BSI in the sense that it's a measure of change, so if the answer of a certain respondent is the same at a certain time and the previous answer,  $z_i = 0$ . On the other hand, if a certain participant changes his answer for a more positive answer it will take value 1, except if it's a radical change from a "negative" to a "positive" answer, then it will take value 2. Same the other way around, if it decreases it will take

value -1, except for a radical change from "positive" to "negative". We will here use z rather than x to make a clear distinction between the BSI and the EIR.

The calculation of  $z_i$  is the difference of  $x_i$  at t-1 and  $x_i$  at t

$$z_i = x_{i,t-1} - x_{i,t} (4.1)$$

the different possible values of  $z_i$  can be seen in Table 4.2.

$$t = \begin{bmatrix} x_{i-} & x_{i0} & x_{i+} \\ x_{i-} & z_{i--} = 0 & z_{i-0} = 1 & z_{i-+} = 2 \\ x_{i0} & z_{i0-} = -1 & z_{i00} = 0 & z_{i0+} = 1 \\ x_{i+} & z_{i+-} = -2 & z_{i+0} = -1 & z_{i++} = 0 \end{bmatrix}$$

Table 4.2: Values of the different observations of  $z_i$ 

The limitation of this indicator is that the previous answer is determinant of the possible results of the indicator. If the previous response was neutral (0), then it's possible to influence the EIR in a positive or negative way. On the other hand if the previous answer was negative or positive, it can only influence the EIR in the opposite way.

For those reasons, the EIR should be complementary to the BSI rather than replacing it or else.

# 4.1 Indicator of the Unweighted Evolution of Individual Responses

The indicator of the evolution of the individual responses can be obtained by taking the mean of the values, as defined in Table 4.2, the formula is then

$$E(Z) = \frac{\sum_{i=1}^{n} z_{i}}{n}$$

$$= \left(\frac{\sum_{i=1}^{n_{--}} z_{--i}}{n}\right) + \left(\frac{\sum_{i=1}^{n_{-0}} z_{-0i}}{n}\right) + \left(\frac{\sum_{i=1}^{n_{-+}} z_{-+i}}{n}\right) + \left(\frac{\sum_{i=1}^{n_{0-}} z_{0-i}}{n}\right) + \left(\frac{\sum_{i=1}^{n_{00}} z_{00i}}{n}\right)$$

$$+ \left(\frac{\sum_{i=1}^{n_{0+}} z_{0+i}}{n}\right) + \left(\frac{\sum_{i=1}^{n_{+-}} z_{+-i}}{n}\right) + \left(\frac{\sum_{i=1}^{n_{+0}} z_{+0i}}{n}\right) + \left(\frac{\sum_{i=1}^{n_{++}} z_{++i}}{n}\right)$$

$$(4.2)$$

The formula can further be simplified since  $z_{--} = z_{00} = z_{++} = 0$ .

$$E(Z) = \left(\frac{\sum_{i=1}^{n_{-0}} z_{-0i}}{n}\right) + \left(\frac{\sum_{i=1}^{n_{-+}} z_{-+i}}{n}\right) + \left(\frac{\sum_{i=1}^{n_{0-}} z_{0-i}}{n}\right) + \left(\frac{\sum_{i=1}^{n_{0+}} z_{0+i}}{n}\right) + \left(\frac{\sum_{i=1}^{n_{+-}} z_{+-i}}{n}\right) + \left(\frac{\sum_{i=1}^{n_{+0}} z_{+0i}}{n}\right)$$

$$= \left(\frac{n_{-0}}{n}\right) + 2\left(\frac{n_{-+}}{n}\right) - \left(\frac{n_{0-}}{n}\right) + \left(\frac{n_{0+}}{n}\right) - 2\left(\frac{n_{+-}}{n}\right) - \left(\frac{n_{+0}}{n}\right)$$

$$(4.4)$$

As for the indicator, it's possible here to have a formula with proportions.

$$E(Z) = \pi_{0+} + \pi_{-0} - \pi_{+0} - \pi_{0-} + 2\pi_{-+} - 2\pi_{+-}$$
(4.6)

where  $\pi$  is the proportion/probability of respondent answering negative(-), neutral (0) and positive (+) at t-1 and t.

The evolution of individual responses is equal to the difference between the proportion of respondents becoming more positive and the proportion of respondents becoming more negative. Where radical changes count double.

# 4.2 Indicator of the Weighted Evolution of Individual Responses

As for the business survey indicator, the evolution of individual responses can also be weighted. The calculation is very similar.

$$E(Z) = \sum_{i=1}^{n} \omega_{i} z_{i}$$

$$= \sum_{i=1}^{n_{--}} \omega_{i} z_{--i} + \sum_{i=1}^{n_{-0}} \omega_{i} z_{-0i} + \sum_{i=1}^{n_{-+}} \omega_{i} z_{-+i} + \sum_{i=1}^{n_{0-}} \omega_{i} z_{0-i} + \sum_{i=1}^{n_{00}} \omega_{i} z_{00i}$$

$$+ \sum_{i=1}^{n_{0+}} \omega_{i} z_{0+i} + \sum_{i=1}^{n_{+-}} \omega_{i} z_{+-i} + \sum_{i=1}^{n_{+0}} \omega_{i} z_{+0i} + \sum_{i=1}^{n_{+-}} \omega_{i} z_{++i}$$

$$= \sum_{i=1}^{n_{-0}} \omega_{i} + 2 \sum_{i=1}^{n_{-+}} \omega_{i} - \sum_{i=1}^{n_{0-}} \omega_{i} + \sum_{i=1}^{n_{0+}} \omega_{i} - \sum_{i=1}^{n_{+-}} \omega_{i} - \sum_{$$

The weighted EIR can, as for the unweighted EIR now be calculated with the following expression

$$E(Z) = \Omega_{0+} + \Omega_{-0} - \Omega_{+0} - \Omega_{0-} + 2\Omega_{-+} - 2\Omega_{+-}$$
(4.10)

where  $\Omega$ 's are the different sum of weights, or in other words, weighted proportions. The formula is very similar to the unweighted and can be interpreted the same way.

# 4.3 Generalisation for Different Period Lags

Changes in the economy are usually taking several months to influence all the companies. There can be some lag of effects on for example larger companies, of a very specific sector. The idea of only taking a certain month and the previous month can seem non-sufficient, it's relevant to take a larger period into account.

Different methods were explored to take more than two periods into account, but it was found to be flawed and very complex to interpret. We rather propose here a simple generalisation; use t-n rather than t-1. The answer at t-n is compared with the answer at time t, without taking the between answers into account.

The calculation is then the same for the unweighted as Equation 4.6 and for the weighted as Equation 4.10 except t-1 is replaced by t-n, the two periods taken into account are the actual month and the n<sup>th</sup> previous month.

# 4.4 Take Different Questions Into Account

Formula  $^1$ 

Industry EIR = 
$$\frac{\sum_{i=1}^{n} (z_{iQ1} + z_{iQ2} + \dots + z_{iQq})}{qn}$$
 (4.11)

#### Unweighted

Unweighted EIR 
$$= \frac{\sum_{i=1}^{n} (z_{iQ1} + z_{iQ2} + \dots + z_{iQq})}{qn}$$

$$= \pi_{Q1,0+} + \pi_{Q1,-0} - \pi_{Q1,+0} - \pi_{Q1,0-} + 2\pi_{Q1,-+} - 2\pi_{Q1,+-}$$

$$+ \pi_{Q2,0+} + \pi_{Q2,-0} - \pi_{Q2,+0} - \pi_{Q2,0-} + 2\pi_{Q2,-+} - 2\pi_{Q2,+-}$$

$$+ \dots$$

$$+ \pi_{Qq,0+} + \pi_{Qq,-0} - \pi_{Qq,+0} - \pi_{Qq,0-} + 2\pi_{Qq,-+} - 2\pi_{Qq,+-}$$

$$(4.12)$$

#### Weighted

Weighted EIR 
$$= \frac{\sum_{i=1}^{n} \omega_{i} \left( z_{iQ1}, + z_{iQ2} + \ldots + z_{iQq} \right)}{q}$$

$$= \Omega_{Q1,0+} + \Omega_{Q1,-0} - \Omega_{Q1,+0} - \Omega_{Q1,0-} + 2\Omega_{Q1,-+} - 2\Omega_{Q1,+-}$$

$$+ \Omega_{Q2,0+} + \Omega_{Q2,-0} - \Omega_{Q2,+0} - \Omega_{Q2,0-} + 2\Omega_{Q2,-+} - 2\Omega_{Q2,+-}$$

$$+ \ldots$$

$$+ \Omega_{Qq,0+} + \Omega_{Qq,-0} - \Omega_{Qq,+0} - \Omega_{Qq,0-} + 2\Omega_{Qq,-+} - 2\Omega_{Qq,+-}$$

$$(4.13)$$

<sup>&</sup>lt;sup>1</sup>Assuming the respondents that answered, answered to all the questions

# The Variance of the Evolution of Individual Responses

The evolution of individual responses has a variance that has some interesting interpretation. While the EIR is a measure of the direction companies change their answer(s), the variance of the EIR can be understood as the measure of changes in answers. From there, the variance of the EIR can be seen as the volatility of the indicator, in the sense that the variance of the EIR accounts for the dispersion of the difference in answers over two periods.

While the variance of the business survey indicator is the measure of disagreement among respondents, the variance of the evolution of individual responses is the measure of the magnitude of changes of respondents.

# 5.1 Variance of the Unweighted Evolution of Individual Responses

$$\operatorname{Var}(Z) = E\left(Z^{2}\right) - E\left(Z\right)^{2}$$

$$= \left(\frac{\sum_{i=1}^{n_{--}} z_{--i}^{2}}{n}\right) + \left(\frac{\sum_{i=1}^{n_{-0}} z_{-0i}^{2}}{n}\right) + \left(\frac{\sum_{i=1}^{n_{-+}} z_{-+i}^{2}}{n}\right) + \left(\frac{\sum_{i=1}^{n_{0-}} z_{0-i}^{2}}{n}\right)$$

$$+ \left(\frac{\sum_{i=1}^{n_{00}} z_{00i}^{2}}{n}\right) + \left(\frac{\sum_{i=1}^{n_{0+}} z_{0+i}^{2}}{n}\right) + \left(\frac{\sum_{i=1}^{n_{+-}} z_{+-i}^{2}}{n}\right)$$

$$+ \left(\frac{\sum_{i=1}^{n_{+0}} z_{+0i}^{2}}{n}\right) + \left(\frac{\sum_{i=1}^{n_{++}} z_{++i}^{2}}{n}\right) - \operatorname{E}(Z)^{2}$$

$$= \left(\frac{n_{-0}}{n}\right) + 4\left(\frac{n_{-+}}{n}\right) + \left(\frac{n_{0-}}{n}\right) + \left(\frac{n_{0+}}{n}\right) + 4\left(\frac{n_{+-}}{n}\right) + \left(\frac{n_{+0}}{n}\right) - \operatorname{E}(Z)^{2}$$

$$(5.2)$$

See Table 4.1 for the different values of z

$$Var(Z) = \pi_{0+} + \pi_{-0} + \pi_{+0} + \pi_{0-} + 4\pi_{-+} + 4\pi_{+-} - (\pi_{0+} + \pi_{-0} - \pi_{+0} - \pi_{0-} + 2\pi_{-+} - 2\pi_{+-})^{2}$$

$$(5.3)$$

$$= \pi_{0+} + \pi_{-0} + \pi_{+0} + \pi_{0-} + 4\pi_{-+} + 4\pi_{+-} - E(Z)^2$$
 (5.4)

$$= 1 - \pi_{++} - \pi_{00} - \pi_{--} + 3\pi_{+-} + 3\pi_{-+} - E(Z)^2$$
 (5.5)

In other words, the variance is ....

# 5.2 Variance of the Weighted Evolution of Individual Responses

$$Var(Z) = \Omega_{0+} + \Omega_{-0} + \Omega_{+0} + \Omega_{0-} + 4\Omega_{-+} + 4\Omega_{+-} - (\Omega_{0+} + \Omega_{-0} - \Omega_{+0} - \Omega_{0-} + 2\Omega_{-+} - 2\Omega_{+-})^{2}$$

$$(5.6)$$

$$= \Omega_{0+} + \Omega_{-0} + \Omega_{+0} + \Omega_{0-} + 4\Omega_{-+} + 4\Omega_{+-} - E(Z)^2$$
 (5.7)

$$= 1 - \Omega_{++} - \Omega_{00} - \Omega_{--} + 3\Omega_{+-} + 3\Omega_{-+} - E(Z)^2$$
 (5.8)

# 5.3 Properties

The quite similar to Var(BSI)

#### Property 1: the variance of Z is bounded between 0 and 2.5

As for the variance of the BSI, the variance of the EIR can't take any value. Again the variance can go as low as 0 but in this case can go up to 2.5. One method to obtain this is by calculating the following. In this case the situation where no participant has the same answer to the question

$$Var(Z) = 1 - \pi_{++} - \pi_{00} - \pi_{--} + 3\pi_{+-} + 3\pi_{-+} - E(Z)^{2}$$
$$= 1 - 0 - 0 - 0 + 3 * 0.5 + 3 * 0.5 - 0 = 2.5$$

There are here more explanatory, therefore it's not possible to plot is a was done for the variance of the BSI. Some interpretation can still be done

...

# 5.4 Take Different Questions Into Account

$$\operatorname{Var}\left(\sum_{i=1}^{q} Z_{i}\right) = \sum_{i=1}^{q} \sum_{j=1}^{q} \operatorname{Cov}\left(Z_{i}, Z_{j}\right) = \sum_{i=1}^{q} \operatorname{Var}\left(Z_{i}\right) + 2 \sum_{1 \le i < j \le q} \operatorname{Cov}\left(Z_{i}, Z_{j}\right)$$
(5.9)

$$\operatorname{Var}\left(\frac{Z_{Q1} + Z_{Q2} + Z_{Q3} + Z_{Q4}}{4}\right) = \frac{1}{16} \left[\operatorname{Var}(Z_{Q1}) + \operatorname{Var}(Z_{Q2}) + \operatorname{Var}(Z_{Q3}) + \operatorname{Var}(Z_{Q4}) + 2\operatorname{Cov}(Z_{Q1}, Z_{Q2}) + 2\operatorname{Cov}(Z_{Q1}, Z_{Q3}) + 2\operatorname{Cov}(Z_{Q1}, Z_{Q4}) + 2\operatorname{Cov}(Z_{Q2}, Z_{Q3}) + 2\operatorname{Cov}(Z_{Q2}, Z_{Q4}) + 2\operatorname{Cov}(Z_{Q3}, Z_{Q4})\right]$$

$$(5.10)$$

$$\operatorname{Var}\left(\frac{Z_{Q1} + Z_{Q2} + Z_{Q3} + Z_{Q4}}{4}\right) = \frac{1}{16} \operatorname{Var}\left(\frac{\sum_{i=1}^{n} (z_{iQ1} + z_{iQ2} + z_{iQ3} + z_{iQ4})}{n}\right)$$
(5.11)

The generalisation of the previous equation can be written as follow for the variance of the unweighted indicator

$$\operatorname{Var}\left(\operatorname{Unweighted}\,\mathrm{BSI}\right) = \frac{1}{q^2}\operatorname{Var}\left(\frac{\sum_{i=1}^n\left(z_{iQ1}+z_{iQ2}+\ldots+z_{iQq}\right)}{n}\right) \tag{5.12}$$

$$\operatorname{Var}\left(\operatorname{Weighted}\,\mathrm{BSI}\right) = \frac{1}{q^2}\operatorname{Var}\left(\sum_{i=1}^n \omega_i\left(z_{iQ1} + z_{iQ2} + \dots + z_{iQq}\right)\right) \tag{5.13}$$

# Non-Response, Dropout, Attrition and Seasonal Effects

Before modelling the data, it's important to look at the different effects that could mislead the outcome of the analysis. After eploring the data and doing a literature review, four main effects could mislead the results. Three are due to the survey and one to external effects; seasonal effects. We will first look into seasonal effects and apply a well know correction to the data to eliminate, as good as possible, the seasonal effects from the data. It will then be looked into the three survey based issues; non-response, dropout, and attrition.

As already mentioned before, for this analysis the data at hand is the unweighted indicators of the industry business survey, from 1988 to 2018.

Eight variables will be used for the analysis; the business survey indicator, the variance of the BSI, the evolution of individual responses and the variance of the EIR (three variations of the EIR and its variance; taking a 1, 2 or 3 months comparison)

#### 6.1 Seasonal Correction

The National Bank, before publishing the business survey indicator, applies an X11 seasonal correction. The literature about seasonal effects is very rich and variate. Without going too much into details, there are today two methods which are the most widely used and recommended which are X12/X13-ARIMA developed by the US Census Bureau and TRAMO/SEATS developed at the Bank of Spain.

The Department of Research and Development of the NBB developed JDemetra+, a statistical program which is recommended by the European Central Bank (ECB) and Eurostat for analysing seasonal effect and applying seasonal corrections.

JDemetra+ is used by the National Bank to apply seasonal corrections to the business survey indicator before publication. It will be used here, to test for seasonality and apply corrections to the data.

Tests for seasonality are summarised in Table 6.1. The results are clear, there is seasonality in all the variables.

The different variables are seasonal correction using RJDemetra, the R package based

Seasonality Test	BSI	Var(BSI)	EIR	Var(EIR)	EIR2	Var(EIR2)	EIR3	Var(EIR3)
Auto-corr. at seasonal lags	YES	YES	YES	YES	YES	YES	YES	YES
Friedman test	YES	YES	YES	YES	YES	YES	YES	YES
Kruskall-Wallis test	YES	YES	YES	YES	YES	YES	YES	YES
Spectral peaks	YES	YES	YES	?	YES	?	YES	YES
Periodogram	YES	YES	YES	YES	YES	YES	YES	YES
Seasonal dummies	YES	YES	YES	YES	YES	YES	YES	YES
Seasonal dummies (AMI)	YES	YES	YES	YES	YES	YES	YES	YES

Table 6.1: Seasonality Tests

on JDemetra+. The results are plotted in Figure 6.1. The method applied is called "RSA0" and is the simplest X12/X13-ARIMA method for seasonal correction since it's not applying log/level, outliers and calendar corrections. It was chosen since it's the closest to the X11 applied by the NBB to the business survey.

The S-I plots in appendix, see page 56, are interesting to look at to better understand the seasonal effect influencing the different variables. The business survey indicator is usually higher in August and September, and lowest in November and December. Seems like respondents are more optimist during and slightly after summer holidays, or is the economy doing better during those holidays. It's important to notice that year on year GDP is, by definition, seasonal corrected, its formula (Equation 2.1) eliminating all its seasonality. When running the seasonality tests, non of the test shows seasonality. Therefore its important, for the modelling of the data, that will be done in Chapter 9, to correct the different variables for seasonal effects. The S-I plot for the variance of the indicator shows that the variance is lowest in August and September while its peak is in October. In other words, respondents agree the most at the beginning of the year or the last month of summer holidays, while the largest disagreement is in October. The evolution of individual responses indicators (taking 1, 2 or 3 months lag into account) have a quite similar monthly behaviour as the business survey indicator. On the other hand, the variance of the EIR's seems to be less monthly influenced. The Var(EIR1) is smaller in June, while Var(EIR2) and Var(EIR3) are smaller during summer holidays.

# 6.2 Non-Response

The non-response of the business survey can be decomposed into two; non-participation to the survey and participants not responding to the survey.

Non-participation to the survey is difficult to study since not much trace is kept of the companies refusing to participate in the survey. As explained in the first chapter, the selection of participants uses stratification to account for the population at study. There is a bias that can come from the selection procedure since the survey is voluntary based, which means that if companies more prompt and motivated to participate are companies with a different profile compared to not motivated companies, there could be an issue. The potential bias would be limited since the business survey is a panel survey and is interpreted as such.

Non-response of participants is another issue. Since the participants, in this case, dont answer a certain month but continue the survey, it can also be called non-monotone missingness. The solution applied by the NBB in case of non-response, is to assume that the respondent would have answered the same as the previous month, the method

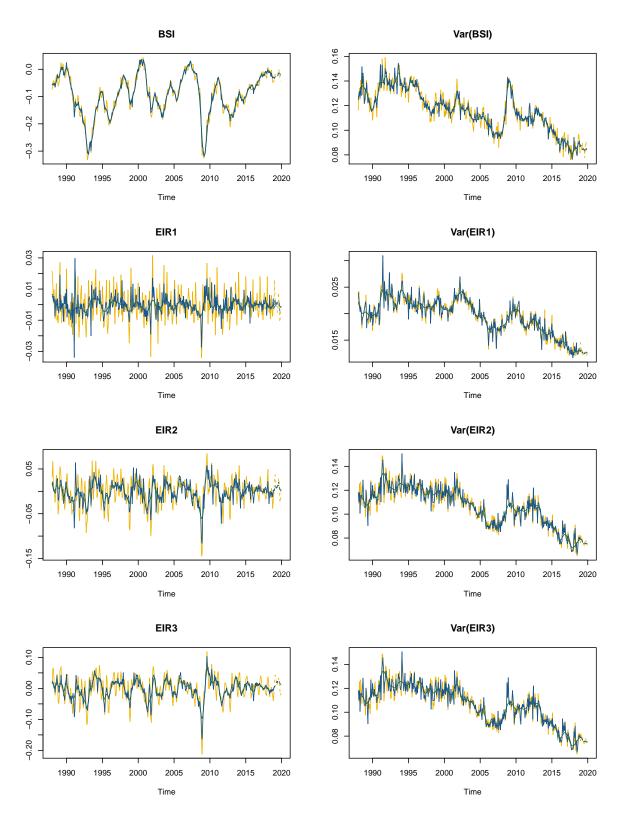


Figure 6.1: Plot of the industry business survey indicator (NS975), the indicator of the evolution of individual responses with the previous month (EIR1), two months (EIR2) and three months earlier (EIR3), with for each of them, their variance. The yellow lines are the raw data, the blue lines the seasonally corrected data and the green line is the trend of the variable.

is called "Last Observation Carried Forward" (LOCF). The theory regarding missing observations states three different ways non-response can be present. Responses can be missing completely at random (MCAR), missing at random (MAR), or missing not at random (MNAR). Missing completely at random is usually hard to assume while missing not at random can be hard to account for. In the case of the business survey, the LOCF technique assumes that observations are missing completely at random. This method is not the most efficient and unbiased but has the advantage of been very simple and have more data (compared to considering the missing observations as NA's).

There is a large effort made by the NBB to contact respondents and send them reminders to make sure that non-response is as small as possible. The response rate is usually around 95%, which is quite high for a survey, therefore it can be assumed that non-response is not very important in the business survey.

# 6.3 Dropout and Attrition

Dropout and Attrition are related to the structure and organisation of the survey. As explain in section 2.3.1, one's participants are recruited to participate in the business survey, they stay as long as they want. This brings two potential issues: companies who leave the survey create a bias (dropout) and companies change of answering behaviour over time (attrition).

#### 6.3.1 Dropout

The National Bank doesn't keep track from reasons why participants leave the survey. From discussions with the responsible persons for the survey at the NBB, the two mean reasons companies are leaving the survey are; (1) the company going bankrupt, acquired or merged and (2) the responsible person at the company leaves his job and the new contact person doesn't see the interest in participating in the survey. This is an issue since it means that it's a certain type of company that leaves the survey. If this type of profile has a different opinion or respond differently than the remaining companies this will create bias.

It could be argued that the bias is very diffused, due to the small number of companies leaving the survey each month. Again the fact that it's the evolution of the BSI that is important, means that we have a very small bias for each month (if we take a long period into account then the bias become larger) we are only comparing month to month evolution, and when the business survey is published, the 3-4 last year are showed.

It's not the subject of this work, so we will not dive more into this bias, but we would recommend having a closer look into this for future researchers.

#### 6.3.2 Attrition

Attrition, also called Panel Conditioning, is present when participants change there behaviour between different rounds of surveys. A very interesting master thesis was done about the Belgian labour force survey, where attrition was found to be significant Priyana Hardjawidjaksana (2019). The Belgian labour force survey was convenient to test for attrition since the survey is answered by participants exactly four times with a lag of six months. It was shown that indeed attrition was present in the survey.

In the case of the industry business survey, it's harder to test for it since we have only two major periods of recruitment for the period at interest (1988 - 2018); in the early 1990 and between around 2000 with some companies been recruited. Other sectors have the same issue.

Table 6.2: Correlation of time with different indicators

	GDP_year	BSI	Var(BSI)	EIR	Var(EIR)
Time	-0.339	0.133	-0.807	0.039	-0.705

An interesting approach to explore attrition and dropout is by looking at the correlations of the variances over time. In Table 6.2 it can be seen that there is a very large negative correlation between time and the variance of the BSI and the variance of the EIR. This can be interpreted as companies agreeing more and more over time and changing less and less there answers. Without making any conclusions, the variance of BSI and the variance of EIR seems to show the presence of dropout bias and/or attrition.

Same can be observed in Var(BSI) and Var(EIR) plots in Figure 6.1 where, aside of the peak of variance during the economic crises of 2008, there is a general tendency of decreasing of the variance of the BSI and the different variances of EIR after the beginning of the century, the last time there was a large recruitment done.

The variance shows here an interest, it can be used to better understand the survey and the behaviour of respondents. When looking at only the evolution of the business survey indicator, it's not possible to measure the vivacity of the survey.

It would be interesting to dive more into these issues, but this will be left for further research. In the context of this paper, it's important to notice the potential importance of attrition and dropout in the industry business survey and the importance of the variances to account for it.

# CHAPTER 7

## **Exploratory Analysis**

The previous chapter already did some of the exploratory analysis. Further observations of the different variables at hand will be done is a short descriptive statistics section. In the second part, correlations among the variables will be looked at, and the last part will focus on autocorrelation.

## 7.1 Descriptive Statistics

Statistic	N	Mean	St. Dev.	Min	Pctl(25)	Pctl(75)	Max
YoY GDP	124	1.936	1.634	-3.809	1.218	3.000	5.119
BSI	372	-0.094	0.075	-0.321	-0.141	-0.033	0.037
Var(BSI)	372	0.116	0.017	0.076	0.107	0.129	0.154
EIR1	372	-0.0003	0.006	-0.034	-0.004	0.003	0.030
Var(EIR1)	372	0.020	0.003	0.012	0.017	0.022	0.031
EIR2	372	-0.001	0.024	-0.116	-0.013	0.013	0.064
Var(EIR2)	372	0.107	0.016	0.067	0.095	0.119	0.151
EIR3	372	-0.002	0.031	-0.163	-0.018	0.018	0.103
Var(EIR3)	372	0.123	0.019	0.075	0.109	0.137	0.166

Table 7.1: Descriptive Statistics

The variables at hand are obtained from the industry business survey and take into account the four questions available on page 51, as explained in section 2.3.2. The different variables are unweighted, seasonally corrected (see section 6.1) and is obtained for 30 years, from 1988 to 2018. The year on year GDP calculation was explained from GDP byt the calculation in Equation 2.1 and is here obtained for the Belgian economy.

The different variables at hand are summarised in Table 7.1. The YoY GDP has only one third as many observations as the other variables since it's a quarterly measure, while the survey is monthly. The table shows the scale of the different variables, for example, the industry BSI, over 30 years, never went higher than 0.037 and never lower than -0.321.

The different variables are plotted in Figure 7.1, where it can be seen that the business survey indicator follows quite well the evolution of YoY GDP. The other variables, seem to react to major changes in GDP but are more volatile.

## 7.2 Correlation Analysis

Belgian industry claims 25% of the labour force in Belgium and was shown as a good indicator of the year to year GDP De Greef and Van Nieuwenhuyze (2009).

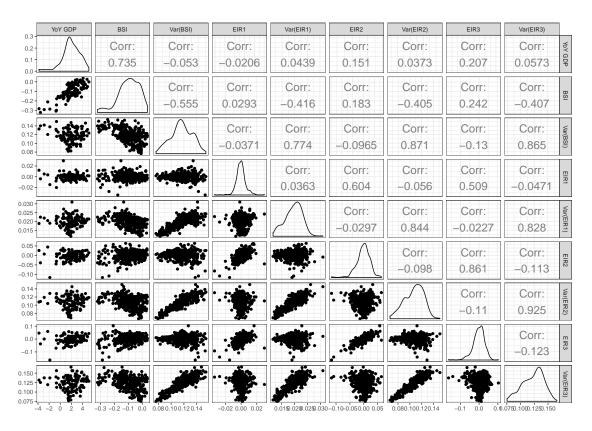


Figure 7.2: Correlation plot of the YoY GDP, the business survey indicator and its variance, and the evolution of individual responses for 1,2 and three months lag and its according variances

Figure 7.2 shows the correlation matrix and plots of the different variables at hand. Several observations can be done. First, it can be seen that the correlation between the business survey indicator and YoY GDP is very high while other variables have a rather small correlation with YoY GDP.

The correlation between the BSI and its variance is also high, this can be explained to some extent by the different observations done in section 3.3. The same can not be said about the correlation between the EIR and its variance since their correlation is small for the three different measures of EIR.

As could be foreseen, the correlations between EIR1, EIR2 and EIR3 and the correlations between Var(EIR1), Var(EIR2) and Var(EIR3) are high. There calculation and data used been quite close.

The correlation between the different types of variances is also high, which can mean that they contain quite similar information. With this definition, it's interesting to look

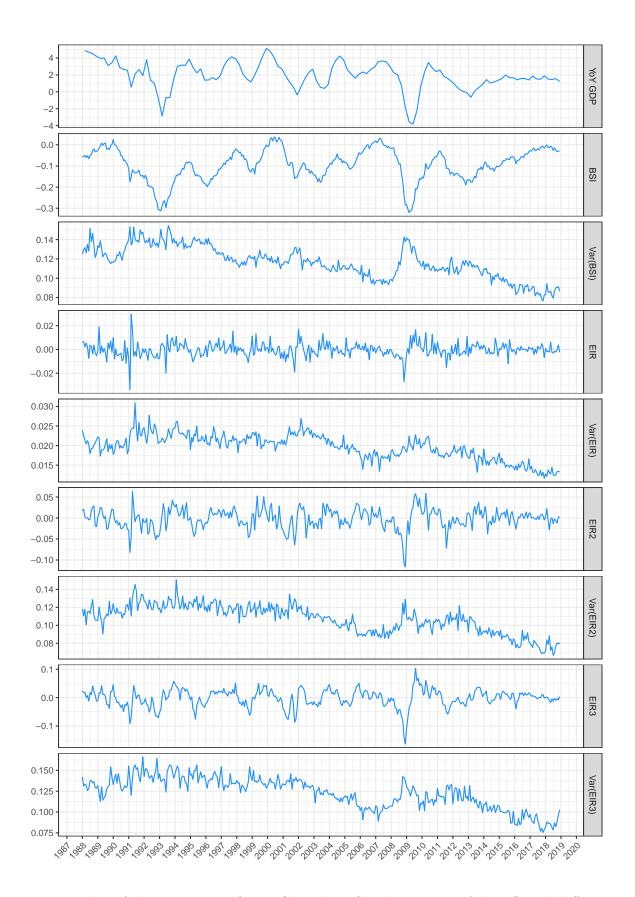


Figure 7.1: Plot of the time series of YoY GDP, the BSI, the variance of the BSI and different EIR's (taking 1, 2 or 3 months differences) and their variance for the period 1988 to 2018

at the correlation between the BSI and the EIR since it's almost equal to zero. This can be interpreted as the two variables containing different information.

#### 7.3 Autocorrelation

Autocorrelation, also known as serial correlation, is the correlation between an observation at time t and previous observations.

The method used here is to estimate the autocorrelation function and plot it, the result can be seen in Figure 7.3, where the correlation of a variable with its previous lags is shown. In the case of the YoY GDP, one lag (X axis of the plot) is three months since the variable is quarterly, while for the other variables the lag corresponds to one month. Several variables have high autocorrelation. The variances are highly correlated over time, as is the BSI. On the other hand, YoY GDP is slightly correlated with its three previous quarter values. For EIR1, the observations are not correlated at all with their previous observations. It can be observed that the autocorrelation increases by the number of months taking into account for the EIR.

Influence for modelling?

#### Autocorrelation of the YoY GDP 9.0 ACF 0.2 -0.2 15 0 5 10 20 Lag Autocorrelation of the BSI Autocorrelation of Var(BSI) 0.8 0.8 ACF ACF 9.0 9.4 0.0 0.0 10 20 5 10 15 20 25 15 Lag Lag Autocorrelation of the EIR1 Autocorrelation of Var(EIR1) 1.0 0.8 9.0 ACF ACF 0.4 0.2 0.0 -0.2 25 5 10 20 10 25 15 0 5 15 20 Lag Lag Autocorrelation of the EIR2 Autocorrelation of Var(EIR2) 1.0 0.8 9.0 0.2 0.0 -0.2 10 25 25 5 15 20 0 5 10 15 20 Lag Lag Autocorrelation of the EIR3 Autocorrelation of Var(EIR3) 0.8 9.0 ACF 9.4 -0.2 0.0 10 25 5 25 15 20 0 10 15 20 Lag Lag

Figure 7.3: Autocorrelation plots for YoY GDP, the BSI, Var(BSI), EIR1, Var(EIR1), EIR2, Var(EIR2), EIR3 and Var(EIR3)

## CHAPTER 8

#### **Linear Models**

This chapter uses a linear models to test the pertinence of the variables in the short term prediction of the evolution of GDP.

As mention in section 2.2.2, were nowcasting was discussed, there exists a large variety of different predictive models used in econometrics to predict the National Growth based on some explanatory variables. The National Bank of Belgium for example, developed a State-Space model available in JDemetra+ de Antonio Liedo (2014). Other well-known methods are ARIMA models, MIDAS and much more.

The interest of the research at hand, is to explore the utility of the variance of the business survey indicator, the evolution of individual responses and its variance.

To achieve this objective, it's better to use a model that account easily for the interest of each variable. The idea here is not to find the best model but rather to see if including the new indicators in a certain model will improve predictions. Therefore, the linear model is preferred.

This chapter will begin with proposing five different models, one simple model where only the BSI is used as regressor, and then four potential models that include other regressors. Several test statistics will be looked at and used to compare the different models and see if the new regressors added to the model, improve the model fit and the estimations. A method of model selection will then by applied to see if it decides to include the new regressors into the model aside from from the BSI. The last part will use out-of-sample estimation to check if the results from the previous tests are robust.

#### 8.1 Method

The quarterly year on year GDP is set in the last month of the quarter. This is the common way to go to take a reasonable approach and still have some predictive properties. Indeed if we look at Figure 8.1, it means that with the linear model it's possible to estimate quarterly YoY GDP one month before it's published. This means that the model will be estimated only with the indicators of the last month of each quarter. One the model is estimated, it's then possible to make predictions or estimations of YoY GDP for each

month.

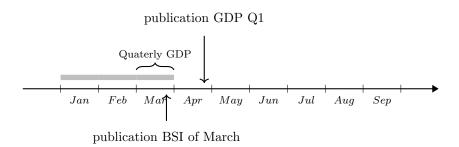


Figure 8.1: Timing of the observations

### 8.2 Linear Models

The linear regression can be written as follow

YoY GDP<sub>t</sub> = 
$$\beta_0 + \sum_{i=1}^{n} \beta_i X_t + \epsilon_t$$
 (8.1)

Where  $\beta_0$  is a constant,  $\beta_i$  are the different regression coefficients of the monthly predictors  $(X_t)$  and  $\epsilon_t$  is the error term.

This model is quite a good example of basic nowcasting, since it's modelling data that's happening at the same time, but since there is a lag of a month in the publication of GDP, it can be used to predict a month in advance, the published GDP.

It can also be used to estimate GDP monthly. Ones a linear model is obtained, based only on quarterly data, it can then be applied to all the available data, and a prediction can be done for each month. In real life, this means that at the end of January, when the results of GDP are published for the last semester of the previous year, the linear model can predict YoY growth for the month of January. Therefore it's not forecasting, since it doesn't predict the future, but predict what will be published

The first approach here is to propose five different models and compare them. The simplest model takes as unique regressor, the business survey indicator

YoY GDP<sub>t</sub> = 
$$\beta_0 + \beta_1 BSI_t + \epsilon_t$$
 (Model 1)

This will be the reference model. A model where only the BSI is used to predict GDP growth (YoY GDP). It will be compared to the following models.

A model that takes the business survey indicator and its variance as regressors

YoY GDP<sub>t</sub> = 
$$\beta_0 + \beta_1 BSI_t + \beta_2 Var(BSI)_t + \epsilon_t$$
 (Model 2)

And three different models where the three evolution of individual responses with their variances are each time added to the previous model

YoY GDP<sub>t</sub> = 
$$\beta_0 + \beta_1 BSI_t + \beta_2 Var(BSI)_t + \beta_3 EIR I_t + \beta_4 Var(EIR I)_t + \epsilon_t$$
 (Model 3)

YoY GDP<sub>t</sub> = 
$$\beta_0 + \beta_1 BSI_t + \beta_2 Var(BSI)_t + \beta_5 EIR 2_t + \beta_6 Var(EIR 2)_t + \epsilon_t$$
 (Model 4)

YoY GDP<sub>t</sub> = 
$$\beta_0 + \beta_1 BSI_t + \beta_2 Var(BSI)_t + \beta_7 EIR 3_t + \beta_8 Var(EIR 3)_t + \epsilon_t$$
 (Model 5)

Table 8.1: Linear Regression Results for the period 1988 to 2018

		]	Linear Regression		
		У	rear on year GDP		
	(Model 1)	(Model 2)	(Model 3)	(Model 4)	(Model 5)
Constant	3.429*** (0.160)	$-1.740^{***}$ (0.631)	$-1.821^{***}$ (0.615)	$-1.756^{***}$ $(0.635)$	$-1.729^{***}$ (0.633)
BSI	15.773*** (1.317)	21.443*** (1.252)	21.548*** (1.224)	21.310*** (1.306)	21.044*** (1.313)
Var(BSI)		49.102*** (5.869)	36.477*** (8.089)	40.631*** (11.895)	38.777*** (11.081)
EIR1			-28.718** (13.792)		
Var(EIR1)			78.555** (35.187)		
EIR2				-0.709 (3.878)	
Var(EIR2)				9.213 (11.207)	
EIR3					1.290 (2.610)
Var(EIR3)					9.401 (8.598)
Observations R <sup>2</sup> Adjusted R <sup>2</sup>	124 0.540 0.537	124 0.709 0.704	124 0.728 0.719	124 0.711 0.701	124 0.712 0.702
Resid. Std. Er. AIC BIC	1.112 382.292 390.753	0.889 327.699 338.980	0.866 $323.086$ $340.008$	0.894 330.907 347.829	0.891 330.293 347.215

Note:

\*p<0.1; \*\*p<0.05; \*\*\*p<0.01

The estimates can be seen in Table 8.1 for the five regressions. Some goodness of fit measures are also available. Those first results show the limited interest of EIR2 and EIR3 since the model with EIR1 and its variance (Equation Model 3) is better

performing according to all the results than the two other models (Equation Model 4 and Equation Model 5).

The other observation that can be done is that the interest of the variance in the prediction seems significant since adding, next to the information of the business survey, increases the Adjusted  $R^2$  by 0.151. The model is also better according to AIC and BIC results.

	RMSE	MAE	MPE	MAPE	MASE
Model 1	1.103	0.93	-27.657	86.696	0.764
Model 2	0.878	0.697	-10.923	61.185	0.573
Model 3	0.848	0.674	-12.455	55.698	0.554
Model 4	0.875	0.699	-12.757	62.707	0.574
$Model \ 5$	0.873	0.689	-9.83	59.311	0.566

Table 8.2: Accuracy Measures

Table 8.2 shows some more measures of goodness-of- fit; the Root Mean Squared Error (RMSE), the Mean Absolute Error (MAE), the Mean Percentage Error (MPE), Mean Absolute Percentage Error (MAPE) and Mean Absolute Scaled Error (MASE). All give similar result as before, Model 3 is performing best according to all the measures except in MPE.

The results for Model 2 are quite close to the results of Model 3.

#### Diebold-Mariano Test

A statistical test, called Diebold-Mariano test Diebold and Mariano (1995), is well known to compare forecast accuracy of different models.

The test is comparing the residuals of different models in test if a model is significantly better than the another one. It's modelling DM = 1.2727, Forecast horizon = 1, Loss function power = 2, p-value = 0.2055 alternative hypothesis: two.sided

p-values	Model 1	Model 2	Model 3	Model 4	Model 5
Model 1	1	5.126e-06	2.586e-06	6.352 e-06	5.602e-06
Model 2		1	0.2055	0.6836	0.5147
Model 3			1		
Model 4			0.1613	1	0.7876
Model 5			0.3017		1

Table 8.3: Diebold-Mariano tests for the models referred to in Table 8.1

#### **Model Selection**

Since there are a large among of different possible models when having eight different regressors, some procedure needs to be chosen to look for the best model.

The method applied here is a step procedure where AIC is used to select the best model. Three different selection algorithms will be applied; (1) forward, (2) backward and (3) both of the step() procedure in R. The three methods are all based on Akaike's information criterion (AIC) as a measure of goodness of fit. They differ in the sense that the first (1) starts from a model with no regressors and adds one by one the regressors

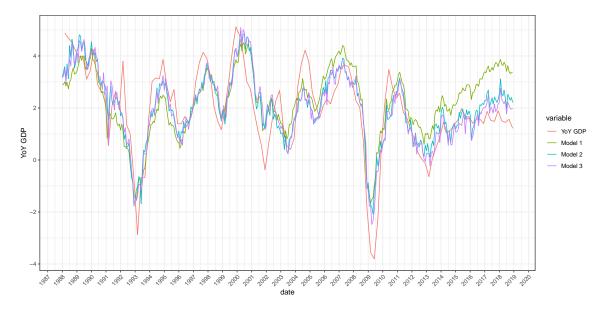


Figure 8.2: Plot of year on year GDP and the different estimation from model 1, 2 and 3

that improve the most the model fit, the second (2) does the opposite, it starts with the full model and takes out the variables that don't improve the model fit and the last (3) is a mixed of the two previous methods.

The results of the three different methods converge to the model already proposed refered to as Model 3, where BSI, Var(BSI), EIR1 and Var(EIR1) are the regressors that.

From the different results, it can already be stated that EIR1 and its variance are bringing more in the modelling of the YoY GDP than EIR2 and EIR3 with their variances.

It was also seen that the variance of the BSI improves the model quite largely and that EIR1 and its variance can also add some information.

Following those results, there are two good candidates; Equation Model 2 and Model 3. Those are the two models that will be further looked into and compared to the simplest model only including the BSI (Equation Model 1).

#### Out-of-Sample Performances

To know if the results from the linear model are robust, two out-of-sample estimation will be performed.

The first will use the period 1988 to 2000 to estimate the model, and its predictions for that period and for 1988 to 2018 will be compared using Diebold-Mariano test. The period was chose since it's only 12 years long, and will have to be accurate for the next 18 years. The situation it's representing, is if, since January, each model was used to make prediction, which would deliver the best results.

See if attrition and dropout, who could be assumed as been more present after 2000, since that was the last recruitment period (see graphs appendix)

The same will then be done for the period 1988 until 2012. The year 2012 was chosen after having a closer look at Figure 8.2, where it seems that most of the improvement brought by the models including more than only the BSI, is from 2012 on. Therefore it's interesting to see how the different models perform without taking that period into account.

	1988-2000				1988-2018	3
p-values	Model 1	Model 2	Model 1	Model 1	Model 2	Model 3
Model 1	1	0.2203	0.08282	1	1.867e-11	5.555e-11
Model 2		1	0.2886		1	0.000745
Model 3			1			1

Table 8.4: Diebold-Mariano tests for the models estimated with the data from 1988 to 2000, applied to the data from 1988-2000 and 1988-2018 with the estimates from Table 8.6

Table 8.4 shows that the models, estimated with the data from 1988 to 2000, there predictions aren't significantly different from each others, while if the model is applied to the whole period of 1988 to 2018, Model 3 is significantly better than Model 1 and Model 2.

The estimates and fitted plot can be seen on page 41. If exclusively the period 1988-2000 is taken into account, Model 2 and Model 3 are only slightly better than Model 1 according to most of the statistics. It can then be seen, from the plot, and even more from the Diebold-Mariano tests, that Model 2 and Model 3 outperform Model 3 after that period, even if the data used for the estimation of the model only uses the data available before 2000.

On the other hand, Table 8.5 shows the Diebold-Mariano test p-values for the model estimated with the data until 2012. The results for the period 1988-2012 and 1988-2018 are quite similar and come to the same conclusion, Model 2 and Model 3 outperform Model 1. While Model 3 is not significantly better than Model 2.

The estimates and fitted plot can be seen on page 42.

		1988-2012	2	1988-2018		
p-values	Model 1	Model 2	Model 1	Model 1	Model 2	Model 3
Model 1	1	0.00956	0.007979	1	1.269e-07	1.751e-07
Model 2		1	0.2813		1	0.09632
Model 3			1			1

Table 8.5: Diebold-Mariano tests for the models estimated with the data from 1988 to 2012, applied to the data from 1988-2012 and 1988-2018 with the estimates from Table 8.7

Table 8.6: Linear Regression results for the period 1988 to 2000

_	L	inear Regression	
	Ye	ear on Year GDP	
	(1)	(2)	(3)
Constant	4.437***	0.660	-0.519
	(0.211)	(1.899)	(2.041)
BSI	17.374***	19.212***	20.327***
	(1.547)	(1.758)	(1.813)
Var(BSI)		30.545*	31.371**
, ,		(15.269)	(14.992)
EIR1			-37.120**
			(17.899)
Var(EIR1)			53.535
(=====)			(52.433)
Observations	48	48	48
$\mathbb{R}^2$	0.733	0.754	0.779
Adjusted $R^2$	0.727	0.744	0.758
Residual Std. Error	0.851	0.825	0.801
AIC	124.714	122.625	121.622
BIC	130.327	130.109	132.849
Note:		*p<0.1; **p<0	.05; ***p<0.01

variable
— YoY GD
— Model 3
— Model 3
— Model 3

Figure 8.3: Plot of year on year GDP and the different estimation from model 1, 2 and 3 when estimated with the data from 1988 to 2000

Table 8.7: Linear Regression results for the period 1988 to 2012

_	L	inear Regression	
	Ye	ear on Year GDP	
	(1)	(2)	(3)
Constant	3.871***	-0.686	-1.349
	(0.171)	(1.054)	(1.102)
BSI	17.243***	20.939***	21.499***
	(1.341)	(1.491)	(1.478)
Var(BSI)		40.466***	33.997***
,		(9.256)	(10.062)
EIR1			-33.401**
			(16.292)
Var(EIR1)			71.077
			(44.449)
Observations	96	96	96
$\mathbb{R}^2$	0.637	0.699	0.718
Adjusted $R^2$	0.634	0.693	0.705
Residual Std. Error	1.064	0.974	0.954
AIC	288.325	272.383	270.247
BIC	296.018	282.641	285.633
Note:		*p<0.1; **p<0	.05; ***p<0.01

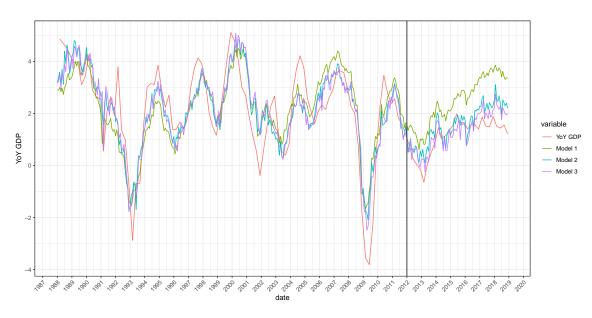


Figure 8.4: Plot of year on year GDP and the different estimation from model 1, 2 and 3 when estimated with the data from  $\bf 1988$  to  $\bf 2012$ 

# CHAPTER 9

### Conclusion

The variance of the business survey indicator showed some interesting interpretation and properties. The fact that the data at hand is trichonomous makes the calculation of the variance

The evolution of individual responses was developed and different calculation approaches were shown.

The variance of the EIR

Considering the predictive power of the different variables, it was seen that as expected the business survey indicator is a good predictor, and that its variance would have increase the prediction accuracy.

Those results have to be taken carefully since it was seen that the data at hand seems to be influenced by dropout bias and/or attrition. In this case the variance of the BSI and the variance of the EIR were very helpful to acknowledge and study the issue.

It was seen that if in 2000, it was decided to use linear models to do nowcasting of the YoY GDP, and that model would be used until this day, the model including the BSI, its variance, the EIR and its variance would have outperformed a model only using the BSI.

High correlation between var(BSI) and var(EIR) - people seems to change in the same direction.

# CHAPTER 10

#### Discussion

The Belgian industry business survey indicator is a very good indicator of the YoY GDP.

## Recruitment procedure and panel data

The recruitment procedure is not so usual for a survey. The particularity of the business survey, that has the ambition to capture the sentiment of companies considering the evolution is quite big, but it's realising it quite well.

not real sampling theory

## EIR that takes more periods into account

The whole discussion is whether this improvement of predictions when adding the variance of the indicator into the model, is caused by its predictive power or if it can be explained by the Attrition and/or dropout bias.

#### Limitations

Variance influence by drop-out, attrition, ...

## Improve the business survey

#### Change participants

From a statisticians point of view, a more sampling theory Including SRS or else would be more optimal

### Further Research

Considering non-response, some other technique dans Last Observation carried forwards could be replace by multiple imputations for more accuracy. This could be interesting to look at

Techniques to take out dropout and attrition bias out

MARKOV SWITCHING

More complex Nowcasting model with Space space models / MIDAS ....

Combine mixed models and Markov Chain for Panel Data (de Haan-Rietdijk et al., 2017)

State Space Model

Bayesian estimation Bialowolski et al.

## Bibliography

- Alcaniz, M., Costa, A., Luna, C., Rovira, C., and Guillén, M. (2006). Calculation of the Variance in Surveys of the Economic Climate. *SSRN Electronic Journal*.
- Bialowolski, P., Kuszewski, T., and Witkowski, B. Bayesian averaging of classical estimates in forecasting macroeconomic indicators with application of business survey data.
- Caron, N., Ravalet, P., and Sautory, O. (1996). Estimation de la précision d'un solde dans les enquêtes de conjoncture auprès des entreprises. Technical report.
- de Antonio Liedo, D. (2014). Nowcasting Belgium. SSRN Electronic Journal.
- De Greef, I. and Van Nieuwenhuyze, C. (2009). The National Bank of Belgium's new business survey indicator.
- de Haan-Rietdijk, S., Kuppens, P., Bergeman, C. S., Sheeber, L. B., Allen, N. B., and Hamaker, E. L. (2017). On the Use of Mixed Markov Models for Intensive Longitudinal Data. *Multivariate Behavioral Research*, 52(6):747–767.
- Diebold, F. X. and Mariano, R. S. (1995). Comparing Predictive Accuracy. *Journal of Business and Economic Statistics*, 13(3):12.
- European Commission, T. (2016). The Joint Harmonised EU Programme of Business and Consumer Surveys User Guide. page 56.
- Hlavac, M. (2018). stargazer: Well-Formatted Regression and Summary Statistics Tables.
- Juglar, C. .-. A. d. t. (1862). Des crises commerciales et de leur retour périodique en France, en Angleterre et aux États-Unis / par le Dr Clément Juglar...
- Mitchell, W. C. and Burns, A. F. (1946). Measuring Business Cycles. NBER.
- Priyana Hardjawidjaksana, I. (2019). Investigating Attrition in Belgian Labor Force Survey. PhD thesis, KU Leuven.
- Rhoads, C. (1999). Euroland Discovers A Surprise Indicator: Belgian Confidence. Wall Street Journal, Europe; Brussels, page 1.

Vanhaelen, J.-J., Dresse, L., and De Mulder, J. (2000). The Belgian Industrial Confidence Indicator: Leading Indicator of Economic Activity in the Euro Area? SSRN Electronic Journal.

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Appendix

# Questions taken into account for the calculation of the industry business survey

originally numbered question 18, 27, 32 and 33 (see page 54), for simplicity numbered here as 1, 2, 3 and 4. Note that the first question is interpreted in the opposite way from the three other. Having a higher stock than normal is considered a "negative" answer while lower stock is considered "positive".

#### Course and assessment

1.	Your current stock of this product will you consider, for the season, as: $\Box$ higher than normal (too high) $\Box$ normal (sufficient) $\Box$ lower than normal (too low)
2.	Your current aggregate order position for this product is what you consider to be: $\Box$ higher than normal $\Box$ normal $\Box$ lower than normal
Pro	spects for the next three months
3.	The personnel (workers and technicians) employed for the manufacture of this product will, according to you:  □ be expanded □ remain unchanged □ be reduced
4.	The demand of your customers for this product will, in your opinion:  □ be more important □ be equally important □ be less important as usual at that time of the year

# Specificity of question 3 and 4, are peoples predictions correct?

A discussion that needs Question 1 and 2 are Question 3 and 4 as

# Attrition

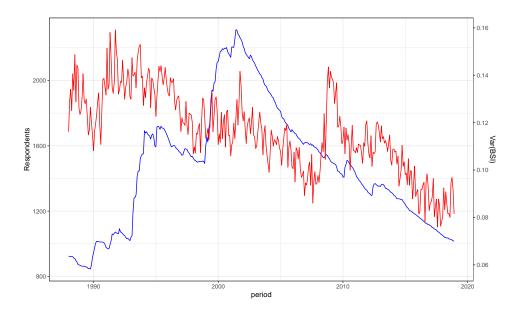


Figure 1: Caption

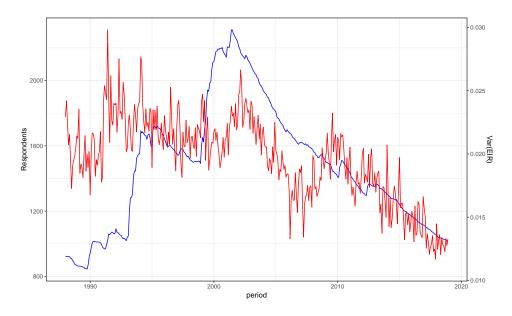


Figure 2: Caption

Table 1: Correlation Matrix of different lags of the BSI of question 3 with YoY GDP

	YoY GDP	BSI	lag1(BSI)	lag2(BSI)	lag3(BSI)	lag4(BSI)
YoY GDP	1	0.707	0.679	0.673	0.628	0.560
BSI		1	0.969	0.948	0.906	0.846
lag1(BSI)			1	0.975	0.940	0.892
lag2(BSI)				1	0.974	0.933
lag3(BSI)					1	0.969
lag4(BSI)						1

Table 2: Correlation Matrix of different lags of the BSI of question 4 with YoY GDP

	YoY GDP	BSI	lag1(BSI)	lag2(BSI)	lag3(BSI)	lag4(BSI)
YoY GDP	1	0.719	0.650	0.647	0.593	0.501
BSI		1	0.959	0.941	0.890	0.804
lag1(BSI)			1	0.970	0.928	0.863
lag2(BSI)				1	0.970	0.917
lag3(BSI)					1	0.959
lag4(BSI)						1

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Kruis per vraag, het vakje aan dat overeenstemt met uw antwoord en stuur per product uw antwoorden binnen de 10 dagen terug. U kunt gebruik maken van ons gratis faxnummer 0800 95 969 (enkel geldig binnen België) of van het faxnummer + 32 (0)2 221 31 07 (vanuit het buitenland).

Product: OKTOBER 2018

Gelieve enkel voor het hierboven vermelde product te antwoorden. Vermeld alle schommelingen, zelfs indien ze van geringe omvang zijn. Antwoord elke maand op alle vragen. Indien u in de beschouwde maand het product niet heeft geproduceerd (of geen bestellingen heeft ontvangen), vermeldt u "verminderd". Antwoord "onveranderd" gedurende de maand(en) dat deze toestand voortduurt totdat de productie herneemt. Uw antwoorden worden strikt vertrouwelijk behandeld.

	Verloop en beoordeling						
•	Uw productietempo voor dit product is in september 2018 t.o.v. augustus 2018:						
	15 1 versneld 5 onveranderd 9 vertraagd						
nden.	Hou geen rekening met schommelingen als gevolg van het maandelijks veranderlijke aantal werkdagen of betaald verlof.						
oorbeho	Tijdens de <u>afgelopen 3 maanden</u> was de trend van uw productie voor dit product:						
st <	16 1 stijgend 5 onveranderd 9 dalend						
ragenlij	Hou evenwel geen rekening met louter seizoengebonden schommelingen.						
• o	Uw verkoopprijzen voor dit product zijn in september 2018 t.o.v. augustus 2018:						
Ě	17 1 gestegen 5 onveranderd 9 gedaald						
ot gebi	Geef de tendens van uw prijzen aan op basis van uw contracten of uw aanbiedingen.						
chten t	Uw <u>huidige</u> voorraad van dit product beschouwt u, voor het seizoen, als:						
08 - Alle re	18 1 hoger dan 5 normaal 9 lager dan normaal (te hoog) 5 normaal (te laag)						
gië - 20(	Kruis "niet van toepassing" aan indien u nooit voorraad hebt van dit product. niet van toepassing						
Nationale Bank van België - 2008 - Alle rechten tot gebruik van de vragenijst voorbehouden	Bij het beantwoorden van de volgende twee vragen (22 en 26), mag u geen melding maken van de zuivere seizoenschommelingen die het verloop van de bestellingen gedurende de maand september 2018 kunnen hebben beïnvloed. De werkelijke tendens van de bestellingen moet dus tot uiting komen.						
National	Uw ontvangen bestellingen voor dit product vanwege de binnenlandse markt zijn in september 2018 t.o.v.	4					

22 1 vermeerderd 5 onveranderd 9 verminderd
Hou eveneens rekening met de van andere afdelingen van uw firma

ontvangen bestellingen en met loonwerk.

Uw <u>huidige</u> gezamenlijke orderpositie voor dit product beschouwt u als:
27 1 hoger dan 5 normaal 9 lager dan
Indien uw onderneming uitsluitend uit voorraad levert, dient u "orderpositie" op te vatten als "het peil van de vraag" naar dit product.
Indien u het <u>huidige</u> fabricatietempo voor dit product handhaaft, is uw activiteit nog verzekerd voor ongeveer:
28 maand(en) en/of
gedeelten van een maand.
Te ramen op basis van uw orderpositie of, bij gebrek hieraan, op basis van uw productieplannen.
Uw <u>huidige</u> positie inzake bestellingen uit het buitenland voor dit product beschouwt u als:
31 1 hoger dan 5 normaal 9 lager dan normaal
Indien uw onderneming uitsluitend uit voorraad levert, dient u uw buitenlandse orderpositie op te vatten als "het peil van de buitenlandse vraag" naar dit product.
Kruis "niet van toepassing" aan indien u dit niet van product nooit uitvoert. niet van toepassing
Vooruitzichten voor de volgende drie maanden
Het personeel (arbeiders en technici) tewerkgesteld voor de fabricatie van dit product zal volgens u:
32 1 worden 5 onveranderd 9 worden uitgebreid blijven verminderd
Het invoeren van gedeeltelijke werkloosheid dient als een vermindering van het personeel te worden beschouwd.
De vraag van uw klanten naar dit product zal volgens u:
33 1 belangrijker 5 even 9 minder

belangrijk

belangrijk

#### NATIONALE BANK VAN BELGIE Kruis het vakie aan dat overeenstemt met uw Departement Studiën antwoord en stuur één exemplaar van de vragen-CONJUNCTUURONDERZOEKINGEN lijst terug binnen de 10 dagen. de Berlaimontlaan 5 - 1000 BRUSSEL Het andere exemplaar is bestemd voor uw TEL. (02) 221 49 97 dossier. TELEFAX (02) 221 31 07 **NIJVERHEID** De geheimhouding van de antwoorden is gewaarborgd Het gedeelte onder de stippellijn terugsturen Beschouwde maand Produkt: 4100 22 ... ziin de bestellingen, vanwege de bin-Onze huidige positie inzake bestellin-Tijdens de beschouwde maand... nenlandse markt, voor dat produkt gen uit het buitenland, voor dat produkt, mag worden beschouwd als 15 ... is ons produktietempo voor dat produkt vermeerderd hoger dan normaal onveranderd gebleven versneld verminderd onveranderd gebleven t.o.v. de vorige maand. vertraagd Wij leveren dat produkt nooit voorbehouder op de binnenlandse markt t.o.v. de vorige maand. Vooruitzichten. (Geen rekening houden met de schommeling voortspruiten uit het van maand tot maant aantal werkdagen of die te wijten zijn aan be Tijdens de volgende drie maanden... zijn de bestellingen, die wij voor dat vragenlijst produkt inschreven bij de uitvoer ...zal, naar wij voorzien, het personeel 17 ... zijn de verkoopprijzen van dat produkt vermeerderd (arbeiders en technici) tewerkgesteld aan onveranderd gebieven de fabricatie van dat produkt aesteaen de vermeerderen verminderd onveranderd gebleven van gedaald t.o.v. de vorige maand. verminderen Wij voeren dat produkt nooit uit gebr t.o.v. de vorige maand. (Rekening houden met maakloonwerk.) o 27 Onze huidige gezamenlijke orderpositie ... zal, volgens onze inlichtingen, de vraag voor dat produkt mag worden beschouwd als hoger dan normaal belangrijker 18 Onze huidige voorraad van dat produkt moet worden beschouwd als Alfe normaal even belangriik lager dan normaal minder belangrijk 1990 hoger dan normaal (Indien uw onderneming uitsluitend uit voorraad levert, dient U « orderpositie » op te vatten als « het peil van de vraag » naar dat produkt.) zijn dan gewoonlijk gedurende die periode normaal van het jaar lager dan normaal Indien wij het huidige fabricatietempo voor Wij hebben nooit een voorraad van dat produkt handhaven is onze activiteit nog verzekerd voor ongeveer van dat produkt Bank ... zullen onze verkoopprijzen van dat produkt waarschijnlijk maand(en) of gedeelte stijgen onveranderd blijven dalen Vermeld alle schommelingen (van uw produktietempo, bestellingen, enz.) zelfs indien ze van geringe omvang zijn. Antwoord elke maand op alle vragen. Indien U gedurende een maand, in tegenstelling met de voorgaande maand, het bestudeerde produkt niet heeft geproduceerd (of geen bestellingen heeft ontvangen, enz.) dient U toch deze vraag te beantwoorden en vermeldt U « verminderd ». Antwoord « onveranderd » gedurende de maand(en) dat deze toestand voortduurt totdat de produktie (of de bestellingen, enz.) hernemen; op dat ogenblik, vermeldt U « vermeerderd ». Op de vraag betreffende de prijzen slechts antwoorden indien U gedurende de beschouwde maand werkelijk contracten afsloot of aanbiedingen

Figure 4: The Business Survey Questionnaire in Dutch for the Industrial Sector in 1990

Stuur uw antwoord terug vóór de 10° van de maand die volgt op de bestudeerde maand, zoniet brengt U de snelle mededeling van de resultaten in het gedrang.

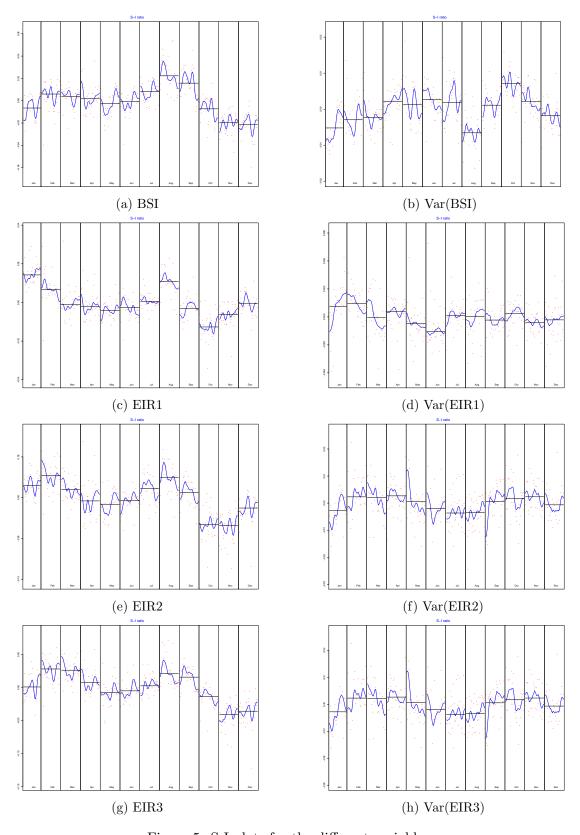


Figure 5: S-I plots for the different variables

		Code
		Oude

## R code for Seasonal Adjustments

Code for Seasonal Adjustments on https://github.com/fabricevb/Master-Thesis/blob/master/R Code/Seasonal Correction model data.R

## R code for Correlation Analysis

 $\label{lem:code} Code\ for\ the\ correlation\ analysis\ on\ https://github.com/fabricevb/Master-Thesis/blob/master/R\ Code/Correlations.R$ 

#### R code for Linear Models

 $\label{linear Models on https://github.com/fabricevbMaster-Thesis/blob/master/R Code/Linear Regression. R$ 

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