Two disks and a spring

1. The plant

Two disks, called disk 1 and disk 2, with inertia Θ_1 and Θ_2 respectively, are connected by a spring with constant c. Each disk as well as the spring itself is subject to friction (assume all frictions are proportional to the speed). The disk friction coefficients are d_1 and d2 and the spring friction coefficient is d. Each disk is actuated by a corresponding motor. The motor current-to-torque gains are k_{t1} and k_{t2}.

2. The space state model

The model of the system is given by the the following state-space representation

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{c}{\Theta_1} & -\frac{d_1+d}{\Theta_1} & -\frac{c}{\Theta_1} & -\frac{d}{\Theta_1} \\ 0 & 0 & 0 & 1 \\ -\frac{c}{\Theta_2} & -\frac{d}{\Theta_2} & -\frac{c}{\Theta_2} & -\frac{d_2+d}{\Theta_2} \end{bmatrix} \cdot x + \begin{bmatrix} 0 & 0 \\ \frac{k_{t1}}{\Theta_1} & 0 \\ 0 & 0 \\ 0 & \frac{k_{t2}}{\Theta_2} \end{bmatrix} \cdot \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

$$\begin{bmatrix} \varphi_1 \\ \varphi_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \cdot x + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

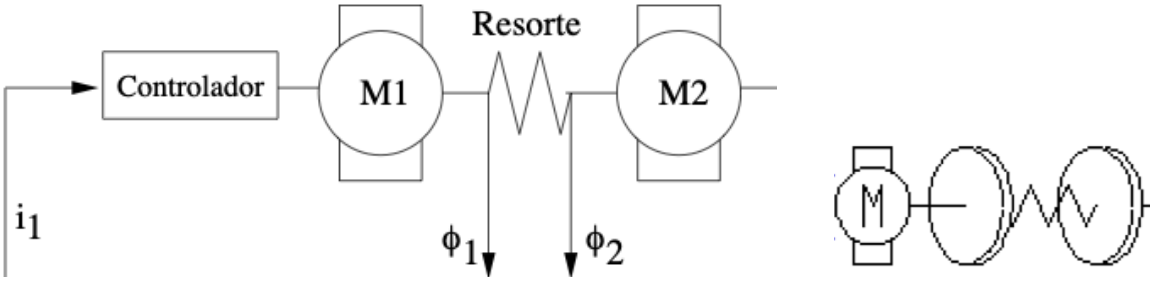
with state vector $[\phi, \omega, \phi_2, \omega_2]$

Consider the following parameters:

- Theta1 = 0.0019;
- Theta2 = 0.0019;
- kt1 = 3.6242e-005;
- kt2 = 2.05e-005;
- c = 0.2038;
- d = 0.0022;
- d1 = 4.7e-005;
- d2 = 4.7e-005 ;

3. Control criteria

The goal for the is to control the angle of the disk on the right by applying an appropriate torque to the disk on the left.



The design requirements are the following.

- Overshoot less than 6%
- Rise time less than 2 seconds
- Settling time less than 5 seconds
- Steady-state error less than 2%