1D/2D Convolutional Neural Networks

Liang Liang

Cross-correlation / Convolution: a simple & powerful method to process 1D/2D/3D/N-D signals

ImageNet Classification with Deep Convolutional Neural Networks

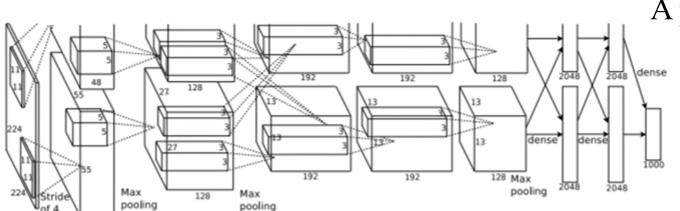
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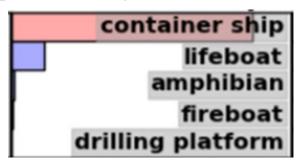
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container ship

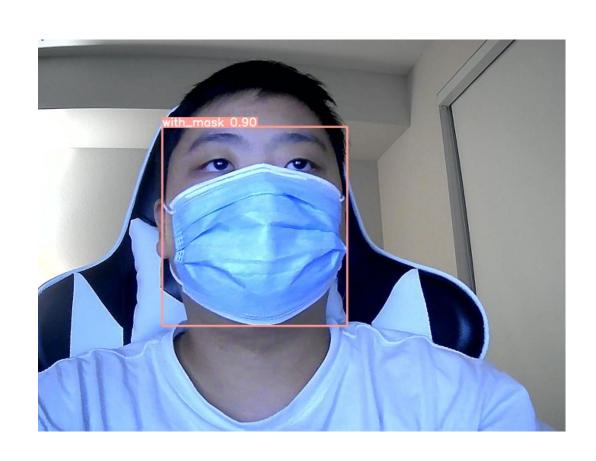


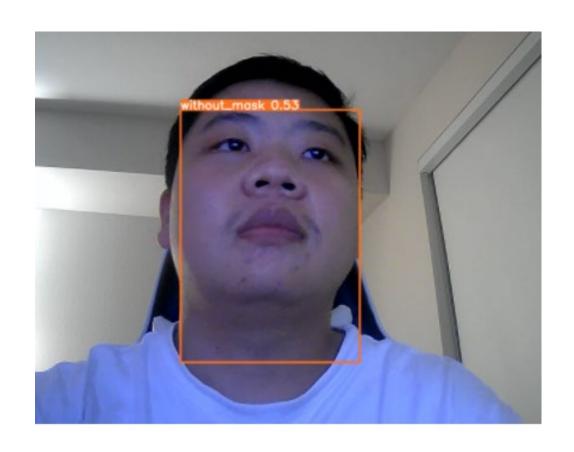
A probability distribution



In the field of machine learning, convolution is cross-correlation

Mask detection using a CNN





Student: Zipei Chen, CSC411

1D signal cross-correlation





kernel

$$|w_0| w_1 |w_2|$$



Input Signal x_0 x_1 x_2 x_3 x_4 x_5 x_6 x_7 x_8 x_9

$$w_0 | w_1 | w_2$$

$$y_0 = a \times w_0 + x_0 \times w_1 + x_1 \times w_2$$

a = 0 or x_0

Input Signal

$$\begin{vmatrix} x_0 & x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 & x_8 & x_9 \end{vmatrix}$$

$$|w_0| w_1 |w_2|$$

$$y_1 = x_0 \times w_0 + x_1 \times w_1 + x_2 \times w_2$$



Input Signal x_0 x_1 x_2 x_3 x_4 x_5 x_6 x_7 x_8 x_9 w_0 w_1 w_2

$$y_2 = x_1 \times w_0 + x_2 \times w_1 + x_3 \times w_2$$



Input Signal x_0 x_1 x_2 x_3 x_4 x_5 x_6 x_7 x_8 x_9 w_0 w_1 w_2

$$y_3 = x_2 \times w_0 + x_3 \times w_1 + x_4 \times w_2$$



Input Signal x_0 x_1 x_2 x_3 x_4 x_5 x_6 x_7 x_8 x_9 w_0 w_1 w_2

$$y_4 = x_3 \times w_0 + x_4 \times w_1 + x_5 \times w_2$$



Input Signal

$$\begin{vmatrix} x_0 & x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 & x_8 & x_9 \end{vmatrix}$$

 $|w_0| w_1 |w_2|$

$$y_5 = x_4 \times w_0 + x_5 \times w_1 + x_6 \times w_2$$



Input $x_0 \mid x_1 \mid x_2 \mid x_3 \mid x_4$ Signal

 $|w_0|w_1|w_2$

$$y_6 = x_5 \times w_0 + x_6 \times w_1 + x_7 \times w_2$$

Input Signal x_0 x_1 x_2 x_3 x_4 x_5 x_6 x_7 x_8 x_8

 $w_0 | w_1 | w_2$

$$y_7 = x_6 \times w_0 + x_7 \times w_1 + x_8 \times w_2$$

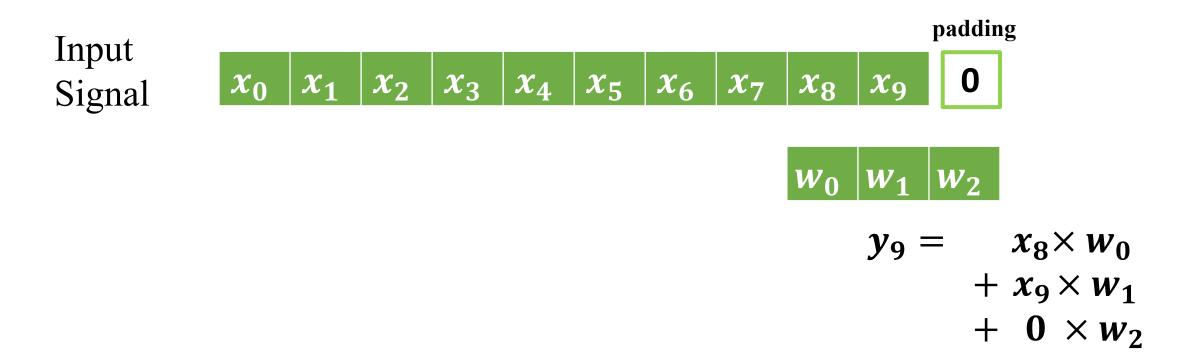




 $w_0 | w_1 | w_2 |$

$$y_8 = x_7 \times w_0 + x_8 \times w_1 + x_9 \times w_2$$





Output Signal

 $oxed{y_0 \ y_1 \ y_2 \ y_3 \ y_4 \ y_5 \ y_6 \ y_7 \ y_8 \ y_9}$

Moving Average: cross-correlation with a special kernel

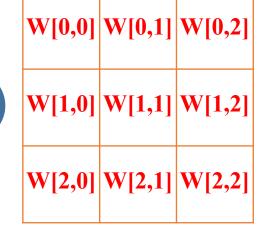
Input Signal
$$x_0$$
 x_1 x_2 x_3 x_4 x_5 x_6 x_7 x_8 x_9 kernel w_0 w_1 w_2 average $w_0 = w_1 = w_2 = 1/3$
$$y_1 = x_0 \times w_0 + x_1 \times w_1 + x_2 \times w_2 = (x_0 + x_1 + x_2)/3$$
 Processed Signal y_0 y_1 y_2 y_3 y_4 y_5 y_6 y_7 y_8 y_9

2D Convolution with Padding

image **A** (1 channel)

A[0,0]	A[0,1]	A[0,2]	A[0,3]	A[0,4]
A[1,0]	A[1,1]	A[1,2]	A[1,3]	A[1,4]
A[2,0]	A[2,1]	A[2,2]	A[2,3]	A[2,4]
A[3,0]	A[3,1]	A[3,2]	A[3,3]	A[3,4]
A[4,0]	A[4,1]	A[4,2]	A[4,3]	A[4,4]

1 kernel W



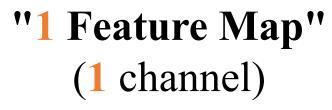


image **B**

B[0,0]	B[0,1]	B[0,2]	B[0,3]	B[0,4]
B[1,0]	B[1,1]	B[1,2]	B[1,3]	B[1,4]
B[2,0]	B[2,1]	B[2,2]	B[2,3]	B[2,4]
B[3,0]	B[3,1]	B[3,2]	B[3,3]	B[3,4]
B[4,0]	B[4,1]	B[4,2]	B[4,3]	B[4,4]

image A

image B

	A[0,1] W[0,1]		A[0,3]	A[0,4]
A[1,0]		A[1,2]	A[1,3]	A[1,4]
A[2,0]	A[2,1] W[2,1]	A[2,2]	A[2,3]	A[2,4]
		A[3,2]	A[3,3]	A[3,4]
A[4,0]	A[4,1]	A[4,2]	A[4,3]	A[4,4]

- Multiply the two numbers at each pixel location
- Take the sum of products

B[1,1]		

$$B[1,1] = W[0,0] \times A[0,0] + W[0,1] \times A[0,1] + W[0,2] \times A[0,2] + W[1,0] \times A[1,0] + W[1,1] \times A[1,1] + W[1,2] \times A[1,2] + W[2,0] \times A[2,0] + W[2,1] \times A[2,1] + W[2,2] \times A[2,2]$$

A[0,0]	A[0,1] W[0,0]	A[0,2] W[0,1]		A[0,4]
A[1,0]	A[1,1]		A[1,3]	A[1,4]
A[2,0]	A[2,1]		A[2,3]	A[2,4]
A[3,0]	A[3,1]			A[3,4]
A[4,0]	A[4,1]	A[4,2]	A[4,3]	A[4,4]

- Multiply the two numbers at each pixel location
- Take the sum of products

B[1,1]	B[1,2]	

$$B[1,2] = W[0,0] \times A[0,1] + W[0,1] \times A[0,2] + W[0,2] \times A[0,3] + W[1,0] \times A[1,1] + W[1,1] \times A[1,2] + W[1,2] \times A[1,3] + W[2,0] \times A[2,1] + W[2,1] \times A[2,2] + W[2,2] \times A[2,3]$$

A[0,0]	A[0,1]	A[0,2] W[0,0]	A[0,3] W[0,1]	
A[1,0]	A[1,1]	A[1,2]		A[1,4]
A[2,0]	A[2,1]	A[2,2]		A[2,4]
A[3,0]	A[3,1]	A[3,2]		
A[4,0]	A[4,1]	A[4,2]	A[4,3]	A[4,4]

- Multiply the two numbers at each pixel location
- Take the sum of products

B[1,1]	B[1,2]	B[1,3]	

$$B[1,3] = W[0,0] \times A[0,2] + W[0,1] \times A[0,3] + W[0,2] \times A[0,4] + W[1,0] \times A[1,2] + W[1,1] \times A[1,3] + W[1,2] \times A[1,4] + W[2,0] \times A[2,2] + W[2,1] \times A[2,3] + W[2,2] \times A[2,4]$$

A[0,0]	A[0,1]	A[0,2]	A[0,3]	A[0,4]
	A[1,1] W[0,1]	A[1,2] W[0,2]	A[1,3]	A[1,4]
A[2,0]	A[2,1] W[1,1]	A[2,2]	A[2,3]	A[2,4]
A[3,0]	A[3,1] W[2,1]	A[3,2]	A[3,3]	A[3,4]
		A[4,2]	A[4,3]	A[4,4]

- Multiply the two numbers at each pixel location
- Take the sum of products

B[1,1]	B[1,2]	B[1,3]	
B[2,1]			

$$B[2,1] = W[0,0] \times A[1,0] + W[0,1] \times A[1,1] + W[0,2] \times A[1,2] + W[1,0] \times A[2,0] + W[1,1] \times A[2,1] + W[1,2] \times A[2,2] + W[2,0] \times A[3,0] + W[2,1] \times A[3,1] + W[2,2] \times A[3,2]$$

A[0,0]	A[0,1]	A[0,2]	A[0,3]	A[0,4]
A[1,0]	A[1,1] W[0,0]	A[1,2] W[0,1]		A[1,4]
A[2,0]	A[2,1]		A[2,3]	A[2,4]
A[3,0]	A[3,1]		A[3,3]	A[3,4]
A[4,0]	A[4,1]			A[4,4]

- Multiply the two numbers at each pixel location
- Take the sum of products

B[1,1]	B[1,2]	B[1,3]	
B[2,1]	B[2,2]		

$$B[2,2] = W[0,0] \times A[1,1] + W[0,1] \times A[1,2] + W[0,2] \times A[1,3] + W[1,0] \times A[2,1] + W[1,1] \times A[2,2] + W[1,2] \times A[2,3] + W[2,0] \times A[3,1] + W[2,1] \times A[3,2] + W[2,2] \times A[3,3]$$

A[0,0]	A[0,1]	A[0,2]	A[0,3]	A[0,4]
A[1,0]	A[1,1]	A[1,2] W[0,0]	A[1,3] W[0,1]	
A[2,0]	A[2,1]	A[2,2]		A[2,4]
A[3,0]	A[3,1]	A[3,2]		A[3,4]
A[4,0]	A[4,1]	A[4,2]		

• Multiply the two numbers at each pixel location

• Take the sum of products

B[1,1]	B[1,2]	B[1,3]	
B[2,1]	B[2,2]	B[2,3]	

$$B[2,3] = W[0,0] \times A[1,2] + W[0,1] \times A[1,3] + W[0,2] \times A[1,4] + W[1,0] \times A[2,2] + W[1,1] \times A[2,3] + W[1,2] \times A[2,4] + W[2,0] \times A[3,2] + W[2,1] \times A[3,3] + W[2,2] \times A[3,4]$$

A[0,0]	A[0,1]	A[0,2]	A[0,3]	A[0,4]
A[1,0]	A[1,1]	A[1,2]	A[1,3]	A[1,4]
	A[2,1] W[0,1]	A[2,2] W[0,2]	A[2,3]	A[2,4]
A[3,0]	A[3,1] W[1,1]	A[3,2]	A[3,3]	A[3,4]
A[4,0]		A[4,2]	A[4,3]	A[4,4]

• Multiply the two numbers at each pixel location

• Take the sum of products

B[1,1]	B[1,2]	B[1,3]	
B[2,1]	B[2,2]	B[2,3]	
B[3,1]			

$$B[3,1] = W[0,0] \times A[2,0] + W[0,1] \times A[2,1] + W[0,2] \times A[2,2] \\ + W[1,0] \times A[3,0] + W[1,1] \times A[3,1] + W[1,2] \times A[3,2] \\ + W[2,0] \times A[4,0] + W[2,1] \times A[4,1] + W[2,2] \times A[4,2]$$

A[0,0]	A[0,1]	A[0,2]	A[0,3]	A[0,4]
A[1,0]	A[1,1]	A[1,2]	A[1,3]	A[1,4]
A[2,0]		A[2,2] W[0,1]	A[2,3] W[0,2]	A[2,4]
A[3,0]	A[3,1]		A[3,3]	A[3,4]
	A[4,1]	A[4,2]	A[4,3] W[2,2]	

- Multiply the two numbers at each pixel location
- Take the sum of products

B[1,1]	B[1,2]	B[1,3]	
B[2,1]	B[2,2]	B[2,3]	
B[3,1]	B[3,2]		

$$B[3,2] = W[0,0] \times A[2,1] + W[0,1] \times A[2,2] + W[0,2] \times A[2,3] + W[1,0] \times A[3,1] + W[1,1] \times A[3,2] + W[1,2] \times A[3,3] + W[2,0] \times A[4,1] + W[2,1] \times A[4,2] + W[2,2] \times A[4,3]$$

A[0,0]	A[0,1]	A[0,2]	A[0,3]	A[0,4]
A[1,0]	A[1,1]	A[1,2]	A[1,3]	A[1,4]
A[2,0]	A[2,1]	A[2,2] W[0,0]		A[2,4] W[0,2]
A[3,0]	A[3,1]	A[3,2]		A[3,4]
A[4,0]		A[4,2]	A[4,3]	

• Multiply the two numbers at

- each pixel location
- Take the sum of products

B[1,1]	B[1,2]	B[1,3]	
B[2,1]	B[2,2]	B[2,3]	
B[3,1]	B[3,2]	B[3,3]	

$$B[3,3] = W[0,0] \times A[2,2] + W[0,1] \times A[2,3] + W[0,2] \times A[2,4] + W[1,0] \times A[3,2] + W[1,1] \times A[3,3] + W[1,2] \times A[3,4] + W[2,0] \times A[4,2] + W[2,1] \times A[4,3] + W[2,2] \times A[4,4]$$

Padding

image **B** (output)

A[0,0]	A[0,1]	A[0,2]	A[0,3]	A[0,4]	Multiply the two numbers at
A[1,0]	A[1,1]	A[1,2]	A[1,3]	A[1,4]	each pixel locationTake the sum of products
A[2,0]	A[2,1]	A[2,2]	A[2,3]	A[2,4]	
A[3,0]	A[3,1]	A[3,2]		A[3,4] W[0,1]	A[3,5] $A[3,5] = 0$ or $A[3,4]$ $W[0,2]$
A[4,0]	A[4,1]	A[4,2]		A[4,4] W[1,1]	A[4,5] W[1,2]

	B[2,2] B[3,2]		
B[1,1]	B[1,2]	B[1,3]	

A[5,3] A[5,4] A[5,5] 'Pad' some extra elements to the boundary of the image A

$$B[4,4] = W[0,0] \times A[3,3] + W[0,1] \times A[3,4] + W[0,2] \times A[3,5] + W[1,0] \times A[4,3] + W[1,1] \times A[4,4] + W[1,2] \times A[4,5] + W[2,0] \times A[5,3] + W[2,1] \times A[5,4] + W[2,2] \times A[5,5]$$

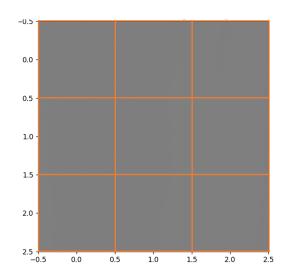
Run 2D_Image_Processing_Convolution.ipynb

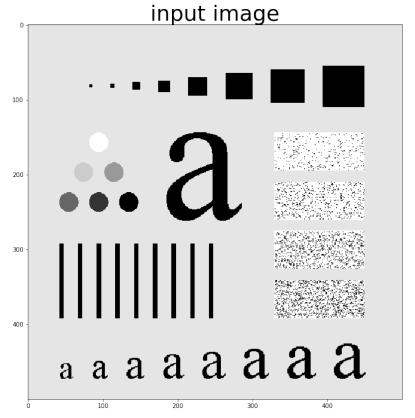
2D Moving Average Kernel

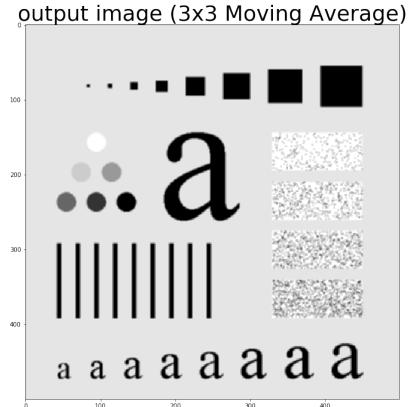
2D moving average kernel

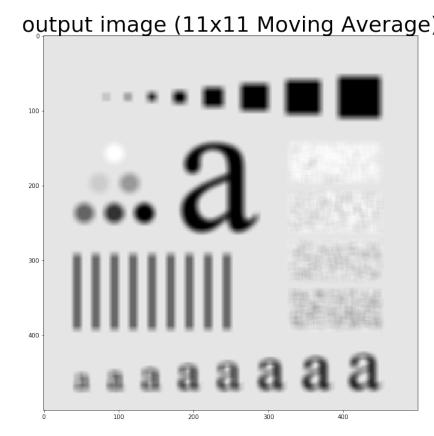
1/9	1/9	1/9
1/9	1/9	1/9
1/9	1/9	1/9

a 2D kernel is a small 2D image

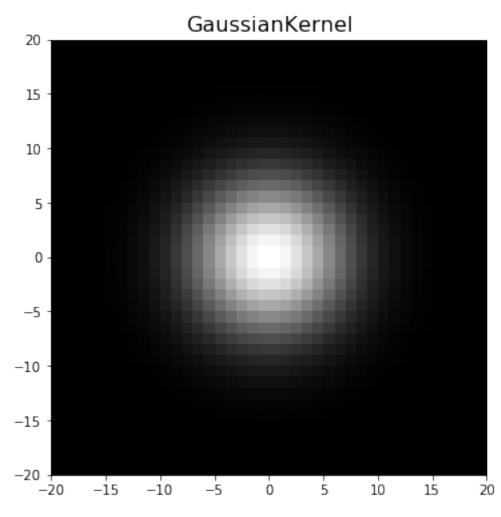








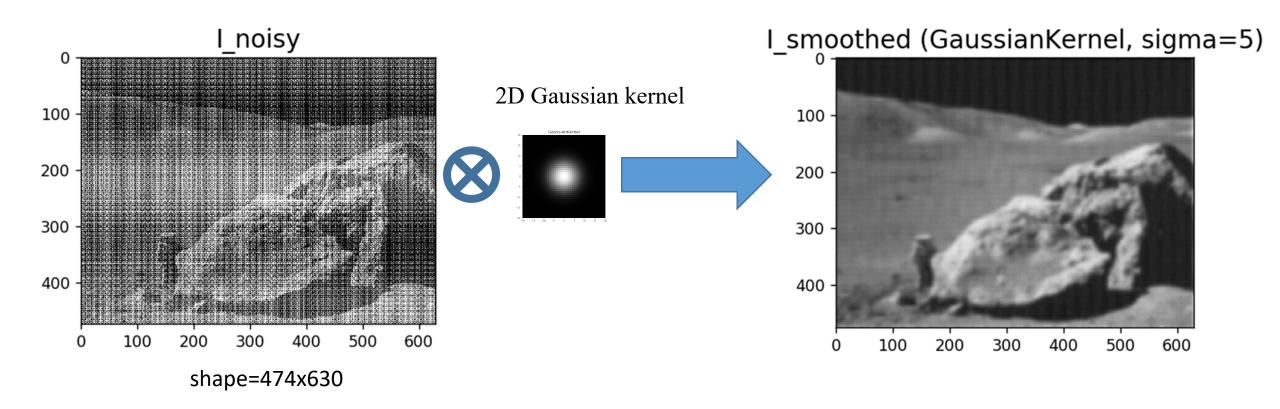
2D Gaussian Kernel for Image Denoising



$$q[i,j] = e^{\frac{-(i^2+j^2)}{2\sigma^2}}$$

The shape of the Kernel is (40, 40)

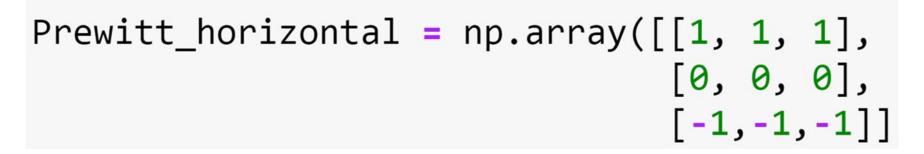
2D 'Convolution' for image processing



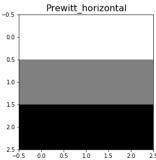
Convolution with a 2D Gaussian kernel to remove noises from the image

2D 'Convolution' for image processing

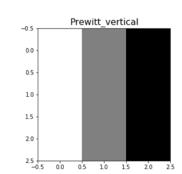
Convolution with 2D Prewitt kernels to detect object edges



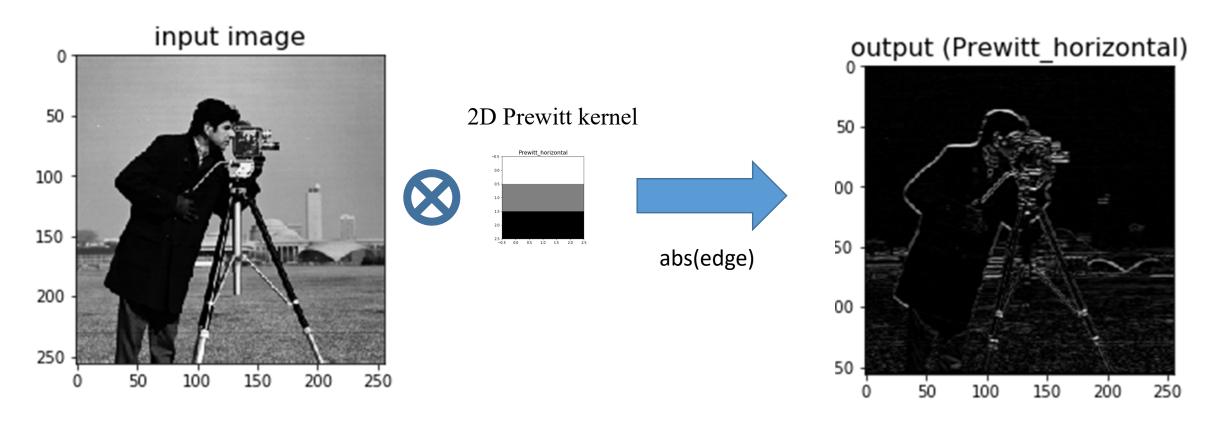
2D Prewitt kernel



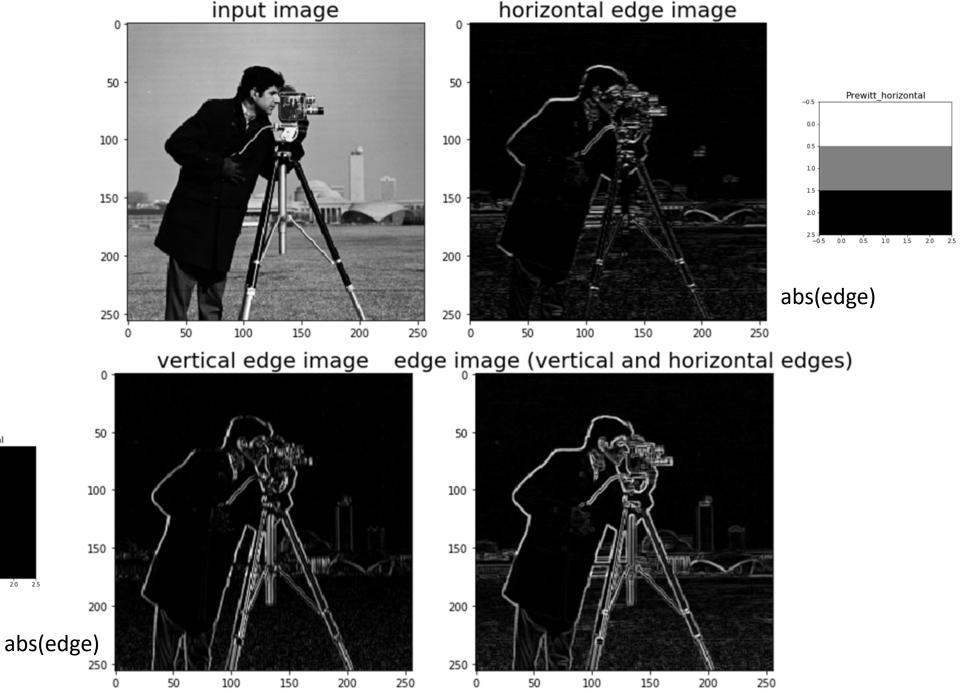
```
Prewitt_vertical = np.array([[1, 0, -1], [1, 0, -1], [1, 0, -1]],
```



2D 'Convolution' for image processing



Convolution with a 2D Prewitt kernel to detect object edges

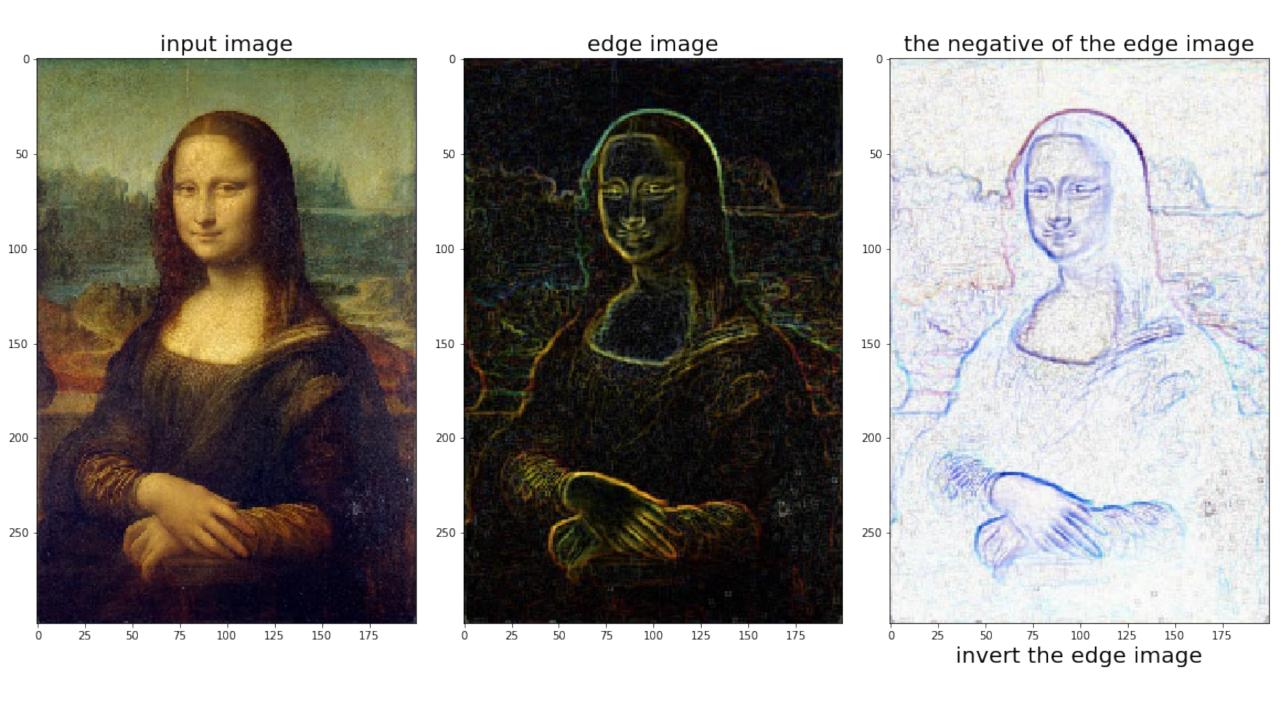


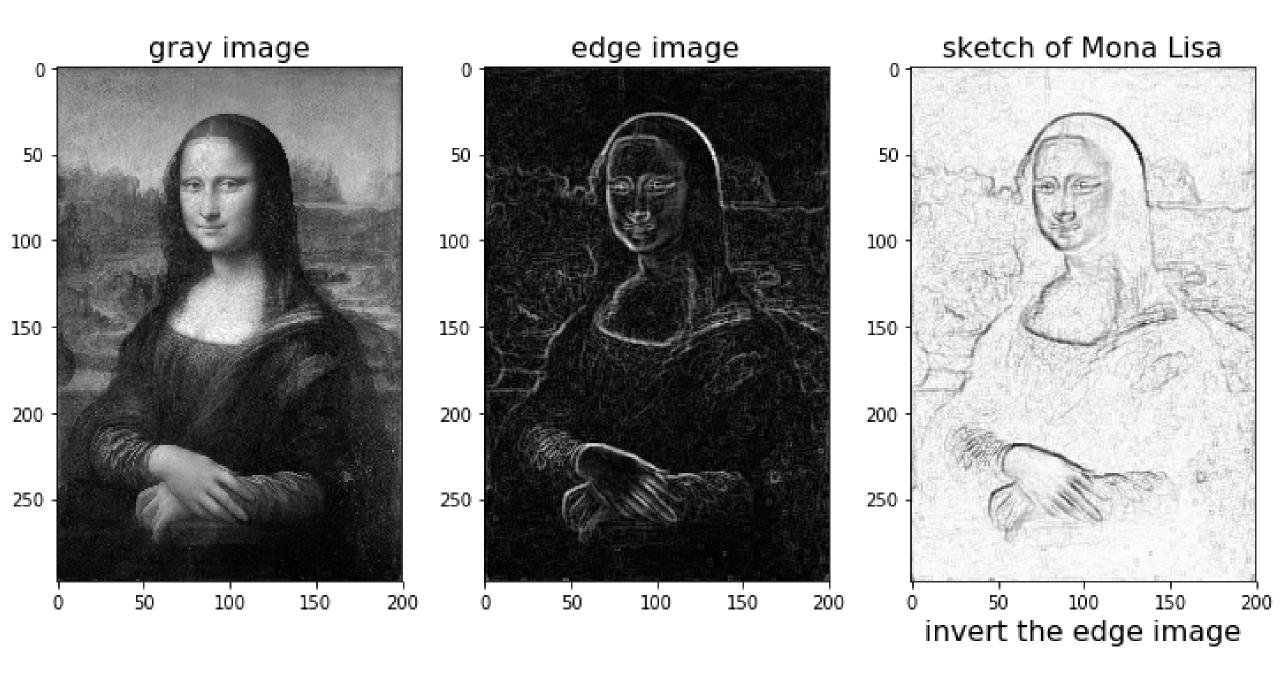
Prewitt vertical

0.5 10 15 20 25

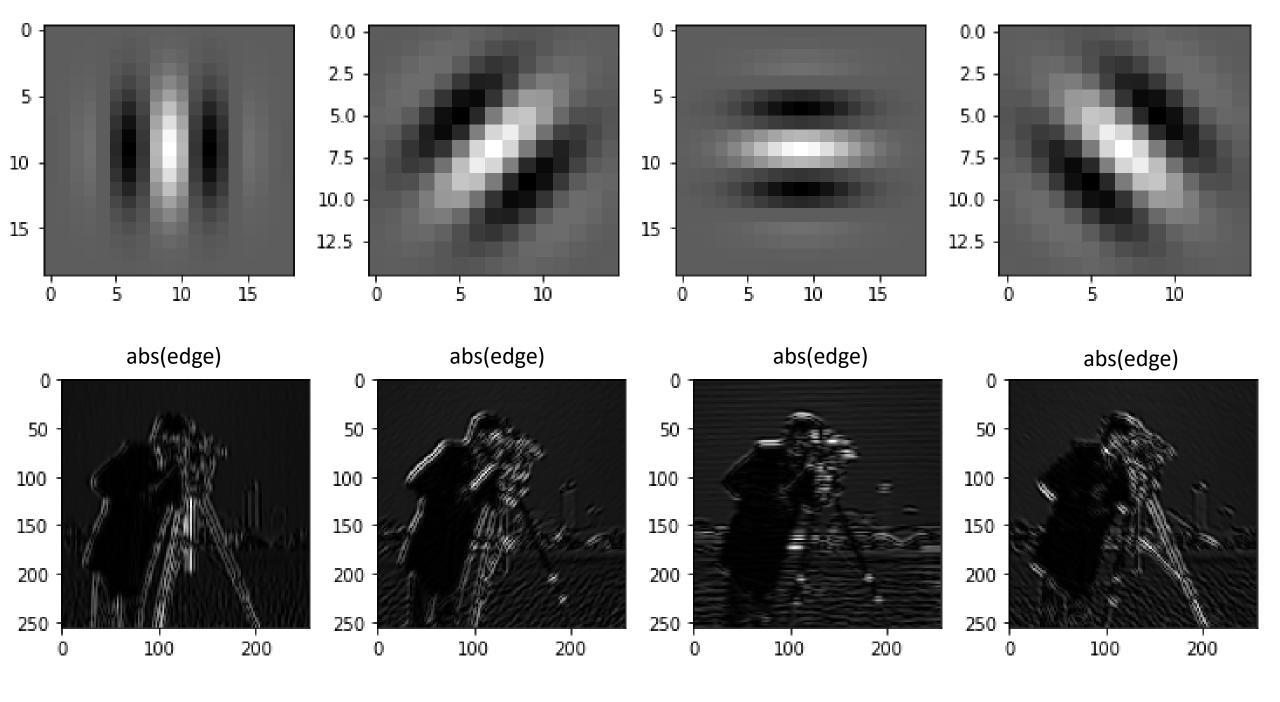
2.0

horizontal edge image + vertical edge image

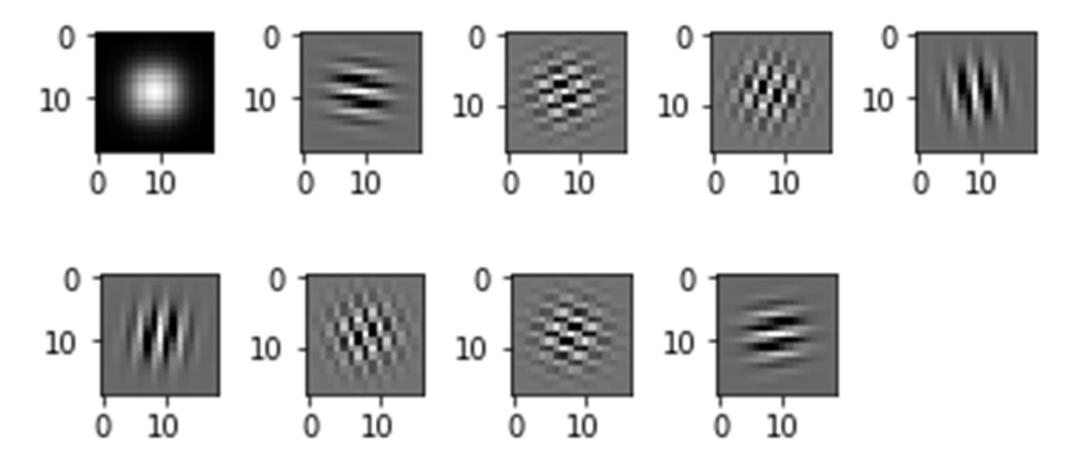




Run 2D_Image_Processing_Convolution_Gabor.ipynb



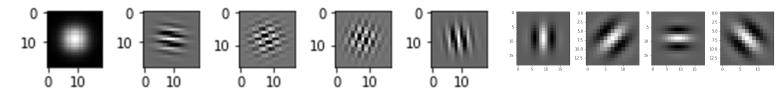
Gabor Kernels for Image Feature (texture) Detection



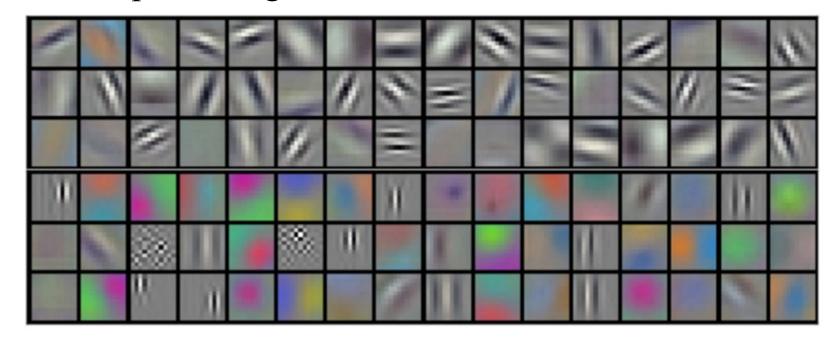
A Gabor kernel is a Gaussian function multiplied by a sin/cos wave https://web.archive.org/web/20180127125930/http://mplab.ucsd.edu/tutorials/gabor.pdf

Gabor Kernels vs Kernels Learned from Data

Gabor kernels are defined by math equations.



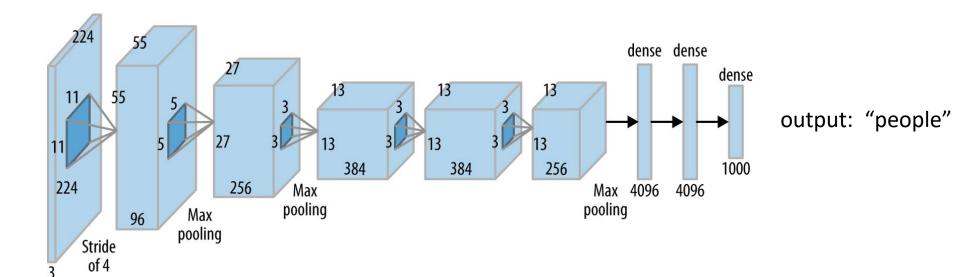
In deep learning, convolution kernels are learned from data



https://papers.nips.cc/paper/4824-imagenet-classification-with-deep-convolutional-neural-networks.pdf

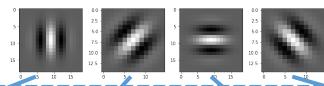
The Alexnet (2D CNN) for image classification



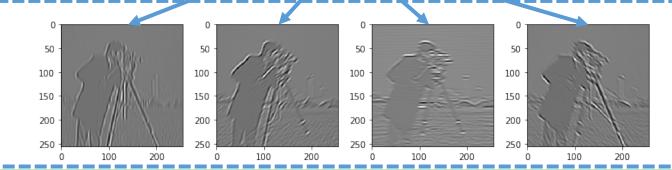


Explanation of the first layer of a convolutional neural network





Let's assume the first layer only has four kernels

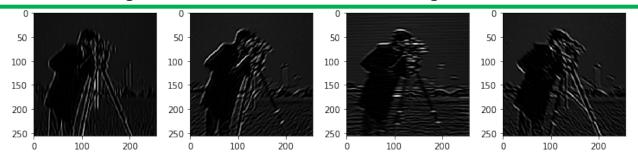


each edge image has positive pixels and negative pixels

Modify the value of every pixel using a nonlinear activation function:

Relu: f(p) = p if p > 0; f(p) = 0 if p < = 0

If the value of a pixel is less than 0, then set the pixel value to zero



store the four images in an array (tensor), each of the output images is called a feature map.

An Explanation of the Operations in the second layer of a convolutional neural network (CNN)

a 4-channel image (input to the second layer)

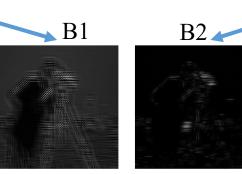


Let's assume the second layer has two kernels (each one has 4 channels)



Attention:

the kernels of the second layer may not look like Gabor kernels in many machine learning applications.



the physical sizes of kernels in the second layer are usually larger than the sizes of kernels in the first layer

N kernels are needed to generate N feature maps



Input Image (an array / tensor)



The First Layer of a CNN

4 kernels









4 Feature Maps => an array (tensor)









Downsized Feature Maps

The Second Layer of a CNN

2 kernels





2 Feature Maps => an array (tensor)

1998

Gradient-Based Learning Applied to Document Recognition

Yann LeCun, Léon Bottou, Yoshua Bengio, and Patrick Haffner

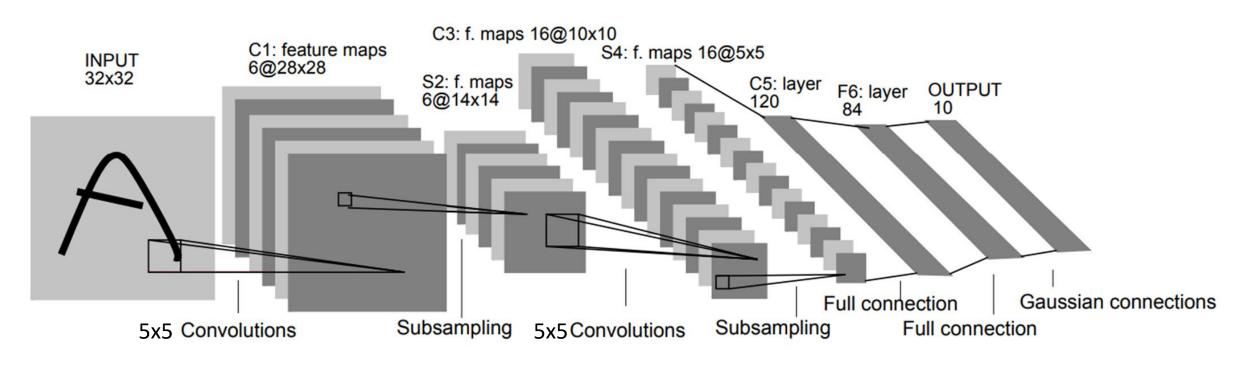


Fig. 2. Architecture of LeNet-5, a Convolutional Neural Network, here for digits recognition. Each plane is a feature map, i.e. a set of units whose weights are constrained to be identical.

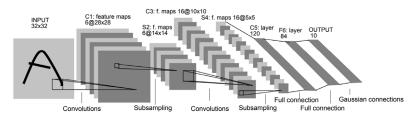


Fig. 2. Architecture of LeNet-5, a Convolutional Neural Network, here for digits recognition. Each plane is a feature map, i.e. a set of units whose weights are constrained to be identical.

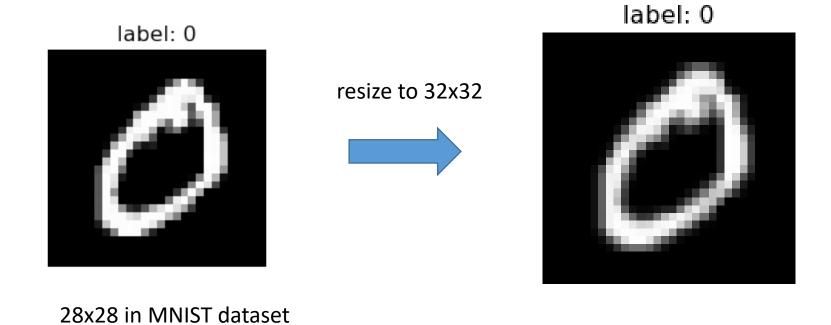
Keras

```
model = Sequential()
model.add(Conv2D(filters=6, kernel_size=(5,5), strides=(1,1), padding='valid', activation = 'relu', input_shape=(32,32,1)))
model.add(MaxPooling2D(pool_size=2))
model.add(Conv2D(filters=16, kernel_size=(5,5), strides=(1,1), padding='valid', activation = 'relu'))
model.add(MaxPooling2D(pool_size=2))
model.add(Conv2D(filters=120, kernel_size=(5,5), strides=(1,1), padding='valid', activation = 'relu'))
model.add(Flatten())
model.add(Dense(units=84, activation='relu'))
model.add(Dense(units=10, activation='relu'))
model.compile(loss='sparse_categorical_crossentropy', optimizer=SGD(lr=0.01, momentum=0.9), metrics=['accuracy'])
model.summary()
```

Model: "sequential"

Layer (type)	Output	Shape	Param #
conv2d (Conv2D)	(None,	28, 28, 6)	156
max_pooling2d (MaxPooling2D)	(None,	14, 14, 6)	0
conv2d_1 (Conv2D)	(None,	10, 10, 16)	2416
max_pooling2d_1 (MaxPooling2	(None,	5, 5, 16)	0
conv2d_2 (Conv2D)	(None,	1, 1, 120)	48120
flatten (Flatten)	(None,	120)	0
dense (Dense)	(None,	84)	10164
dense_1 (Dense)	(None,	10)	850

LeNet-5: input size is 32x32



1 from skimage.transform import resize

LeNet5_Keras.ipynb

https://adamharley.com/nn_vis/