Competition for Prominence

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This paper examines the role of online intermediaries in influencing consumers' purchasing decisions by giving preferential treatment to certain retailers. We investigate the incentives of intermediaries to assign a default position to either a low- or high-quality retailer. For a given fee, the intermediary is more likely to assign a default position to the high-quality (respectively, low-quality) retailer if a significant (resp., relatively small) proportion of consumers are influenced by their recommendation. If the intermediary sets the fee, both retailers fiercely compete for prominence awarded to the high-quality retailer. We then compare this competition for prominence model with a pay for prominence auction and find that the latter is socially desirable, being more beneficial for consumers and retailers while detrimental to the intermediary.

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1 Introduction

Worldwide, a few online marketplaces act as critical gateways for sellers to reach consumers and have increasingly been able to influence consumers' and sellers' decisions and steer consumption. Relevant examples are recommender systems or the allocation of a prominent position to one (or a few) seller(s). A major example is given by Amazon— the largest marketplace in the world— which groups all listings associated with a specific product on a single webpage. Among the available retailers, the platform chooses one as the default retailer, showcased in the "buy-box", whereas the other sellers are offered a less prominent position.[1] In this paper, we label this prominence allocation strategy the "competition for prominence scheme". Obtaining the default position in online marketplaces has become vital for retailers, e.g., as much as 80% of all transactions on Amazon take place via the buy-box (U.S. House Judiciary Committee, 2020). Although platforms do not fully disclose the criteria they use to allocate prominence, they often reveal some tips on obtaining it. In the case of Amazon, very competitive prices, with free and fast shipping, excellent customer service, and high stock available, seem to be the way to succeed.[2]

In this paper, we study how competition for prominence, if implemented by an online platform, affects retailers' competition and consumer surplus. We then compare this prominence allocation scheme to alternative ones and study its desirability for retailers and consumers.

We consider a model in which a monopolist platform connects consumers to two vertically differentiated retailers. Having in mind Amazon's example, retailers may sell the same product but differ in their ancillary services, influencing the consumers' purchasing experience: one of the retailers provides a better service than the rival one.[3] For its intermediation service, the platform collects a commission fee for each realized sale. Consumers are of two types, actives and passives, and are heterogeneous in their willingness to pay for quality. Active consumers always compare all offers and buy from the retailer that provides the highest utility. Passive consumers only consider the offer the intermediary recommends and decide whether to buy. The intermediary assigns a prominent position to either seller upon observing prices.

We show that the intermediary finds it optimal to assign a prominent position to the retailer with the lowest price adjusted by the quality. Anticipating the strategy of the intermediary, retailers can compete fiercely to be awarded the default position. For a given fee, the low-quality retailer always has the incentive to get prominence. Differently, the high-quality retailer has the incentive to obtain prominence only if the share of passive consumers is sufficiently large. The

[1] A similar default position allocation is adopted by several marketplaces nowadays, e.g., Walmart in the United States, Otto in Germany, Fnac in France, and Bol.com in the Netherlands.

[2] Specifically, the price-quality ratio of the overall retailing activity seems to matter and the retailer with the best quality-adjusted price offer is more likely to win the buy-box. For more details, see the *Amazon Seller Central Hub*.

[3] Notable examples are a next-day delivery or a more generous return policy.

rationale is that, due to vertical differentiation, the high-quality retailer always obtains a share of the active consumers, and reaching the passive consumers implies competing fiercely with the low-quality retailer. Facing the usual trade-off between volume and margins, the high-quality retailer can profitably give up competition for prominence if the share of passive consumers is low and, instead, compete for prominence if the share of these consumers is large. Depending on the high-quality retailer's incentives, for a given fee two possible (alternative) equilibria can emerge. If the share of passive consumers is low, there is *market-sharing equilibrium*, where the low-quality retailer is prominent (because the high-quality retailer is not interested in the default position), competition is softened and both retailers obtain strictly positive profits. In the *excluding equilibrium*, competition is intense, the high-quality retailer is prominent, and the low-quality retailer has no sales.

We then examine how an online intermediary can influence the market outcomes of two retailers by endogenizing the commission fee. Our findings reveal that the intermediary prefers to set a commission fee that leads to an excluding equilibrium, where the high-quality retailer becomes prominent by lowering its price and serves both passive and active consumers, while the low-quality retailer is left with no demand. Consequently, the high-quality retailer becomes endogenously prominent and shares its profit with the intermediary. This strategy eliminates the double marginalization problem and produces market outcomes equivalent to a market structure where a multiproduct monopolist directly sets the price for the two offers.

To study the welfare implications of the competition for prominence scheme, we compare it with a more traditional prominence scheme based on monetary compensation in the spirit of Armstrong and Zhou (2011). Under this scheme, the intermediary makes revenues only via this payment and retailers compete for the prominent position through an auction, with the high-quality retailer always emerging as the winner. However, its exclusive access to the passive consumers coexists with a more softened competition in the active segment of the market where the low-quality retailer serves those consumers with a lower willingness to pay for ancillary services. Moreover, in this scheme, the intermediary is constrained by the active presence of the low-quality retailer to fully extract rents from the high-quality rival because the latter would just bid enough to outcompete its rival. Because prominence is awarded without retailers internalizing in their pricing decisions the interests of the intermediary, this prominence allocation scheme is suboptimal to the intermediary and will never arise in equilibrium in the absence of regulation.

To highlight the welfare effects of the two prominence schemes, it is worth noting that the pay for prominence scheme generates important differences compared to the competition for prominence setting. Firstly, equilibrium prices under the pay for prominence scheme are below monopolistic levels, leading to increased total demand and higher consumer surplus. Secondly, the active presence of the low-quality retailer in the market under pay for prominence contributes to increasing total retailer surplus, compared to the competition for prominence setting. Additionally, the high-quality retailer retains more surplus due to the position auction. Combining these effects, we find that the gains for consumers and retailers under the pay for prominence scheme exceed the intermediary's losses. This result has important implications for managers and policymakers, particularly in the context of ongoing antitrust proceedings such as the investigations against Amazon by the UK Competition and Market Authority (CMA) and the Italian antitrust authority (AGCM).[4]

Finally, we endogenize the quality of the retailers. We assume that retailers are ex ante identical but can buy quality-enhancing complementary services from the intermediary (e.g., Fulfilled by Amazon program). We show that, in equilibrium, only one retailer finds it optimal to upgrade its quality. Moreover, using a two-part tariff consisting in the upgrading fixed fee and the commission fee, the intermediary can obtain the same profit that a multiproduct monopolist would obtain.

The paper is structured as follows. Section 2 provides a review of the literature on prominence allocation in online platforms and highlights our contributions. Section 3 presents the model and its key features. In Section 4, we analyze the equilibrium allocation of prominence for both exogenous and endogenous commission fees. Section 5 compares the competition for prominence scheme with the pay for prominence allocation scheme, examining their welfare implications. In Section 6, we study the intermediary's quality-upgrading programs and their impact on equilibrium outcomes. Finally, Section 7 summarizes our findings and concludes with implications for policy and managerial decisions.

2 Related Literature

This paper relates to the literature on intermediaries that, by organizing transactions on their marketplaces, decide whether and how they can make some sellers more prominent. A general finding in this literature is that prominence plays a non-trivial role and affects both purchasing decisions and competition among rival sellers.

Armstrong, Vickers, and Zhou (2009) and Moraga-González, Sándor, and Wildenbeest (2021) study a setting in which prominence is *exogenously* assigned by an intermediary to a seller and consumers have search costs. The authors abstract from the intermediary's interests and analyze the welfare effects of prominence. Armstrong et al. (2009) examine the effects of prominence in the context of sequential search, showing that the prominent seller sets a lower price and the non-prominent ones that serve unsatisfied consumers with a higher price. They also find that, relative to the random search case, prominence maximizes industry profits but

[4]See for example the investigations against Amazon of the UK Competition and Market Authority (CMA) and the Italian antitrust authority (AGCM).

reduces consumer surplus, although prominence reduces the average search cost. In Moraga-González et al. (2021), consumers browse sellers' offers simultaneously. The authors find that prominence can increase consumer surplus even though the prominent seller increases its price by focusing on the least elastic segment of the market, thereby letting non-prominent sellers price aggressively to serve those consumers searching for an alternative. We differ from these studies in that the intermediary endogenously allocates prominence as it grants more visibility in order to maximizing its profits.

Many recent papers dealing with prominence allocation consider monetary compensation (e.g., auctions or direct payments) in exchange for a prominent display (Armstrong & Zhou, 2011; Bourreau & Gaudin, 2022; Inderst & Ottaviani, 2012; Krämer & Zierke, 2020; Long, Jerath, & Sarvary, 2022; Raskovich, 2007; Teh & Wright, 2022). Obtaining prominence is costly for a seller, which might translate into a higher final price for consumers. For example, Armstrong and Zhou (2011) show that, by granting prominence to the highest bidder, sellers' marginal costs are artificially increased and raise overall retail prices at the benefit of the intermediary.[5] In Inderst and Ottaviani (2012), instead, the intermediary allocates prominence, balancing two tensions: the commission it receives from sellers and the reputation cost the intermediary incurs whenever it recommends a not-suitable product. Teh and Wright (2022), instead, study a setting in which the intermediary's profits depend both on the margin sellers leave to obtain prominence and the commission they pay for each item sold. Thus, sellers compete both in prices and in commissions to obtain prominence. The authors find that steering harms both firms and consumers and fiercer competition exacerbates price inflation due to the higher incentive of being recommended. Long et al. (2022) analyze how an intermediary can use the bidding system for sponsoring sellers to infer their private quality information and improve matching. The platform can optimally design listings using the commission fee and the relative weight of the quality information revealed via the sellers' bid. We differ from the studies mentioned above in that we consider a different prominence scheme that does not entail a direct payment to the intermediary. By organizing a competition for prominence, the intermediary drives retailers to internalize the impact of their pricing decision on total demand (and not just on their demand), as the intermediary seeks to maximize the commissions it collects on all units sold. Using such non-pricing instruments can intensify sellers' competition and alleviate the double marginalization problem otherwise emerging. Ultimately, this strategy benefits consumers as they are exposed to a lower final price but might harm sellers relative to a case in which prominence is allocated via an auction.

Our analysis also relates to the search literature that deals with heterogeneous consumers. The population of active/passive consumers we study is very similar to the taxonomy shop-

[5] In their setting, as in ours, there are two groups of consumers: informed ones, who costlessly compares sellers' offers, and uniformed ones, who only consider the offer of the default vendor. Competition for prominence, however, overturns the result of prices. As the intermediary obtains a commission fee for each transaction, it is interested in allocating prominence to the retailers that lower prices and expand the total demand.

pers/captives introduced by Varian (1980). However, mixed equilibria results exist in these studies because sellers offer homogeneous products (Armstrong & Vickers, 2022; Armstrong & Zhou, 2011; Johnen & Ronayne, 2021; Ronayne & Taylor, 2021). Relying on a model with vertically differentiated products, with elastic participation in both the passive and the active segments, we can obtain an equilibrium in pure strategy.[6,7]

The papers most closely related to ours are Dinerstein, Einav, Levin, and Sundaresan (2018) and Bar-Isaac and Shelegia (2022). In Dinerstein et al. (2018), the authors develop a simple theoretical model in which an online intermediary optimally determines the probability of a retailer being displayed prominently to consumers. Similar to our setting, the intermediary considers both the product's intrinsic quality and, depending on the platform's design, its price. The authors then use their model to estimate, in a structural framework, the welfare gains associated with changes in the platform design of eBay. In contrast to their work, we focus on the impact of competition for prominence schemes on vendors' strategies and compare the market outcomes under this prominence allocation with those arising in alternative schemes.

More related to the spirit of our paper, Bar-Isaac and Shelegia (2022) compare the intermediary's advantages of allocating prominence via an auction or via an algorithm.[8] In a scenario with perfectly symmetric sellers and only passive consumers, the authors show that the intermediary is indifferent between the two steering methods. Then, they show that the intermediary prefers algorithmic steering in a scenario in which retailers have market power. Algorithmic steering is also preferred when the intermediary connects multiple separated markets in which consumers are either passive or active. In contrast, our study provides a unified and analytically tractable analysis that combines the presence of active and passive consumers with a single retail market in which sellers may enjoy market power because of their heterogeneous quality. Whereas our primary focus is to show the impact of prominence allocation on competition and welfare, we also show that the intermediary refrains from using a position auction, although it would benefit consumers and retailers and, therefore, be desirable for all its users.

[6] Note that with a high marginal cost for the high-quality retailer, a mixed strategy equilibria might exist, as discussed in the Appendix.

[7] More broadly, our paper also relates to the stream of studies focusing on the problem of steering in the platform's also acting as retailers (S. Anderson & Bedre-Defolie, 2022; S. P. Anderson & Bedre-Defolie, 2021; de Cornière & Taylor, 2019; Etro, 2021; Hagiu, Teh, & Wright, 2022; Zennyo, 2022), how platforms govern their ecosystem (Teh, 2022), as well as how they influence intra-platform competition (Casner, 2020; Jeon, Lefouili, & Madio, 2022; Karle, Peitz, & Reisinger, 2020). In our analysis, we abstract away from the issue of self-preferencing, which has been studied extensively in recent years (Chen & Tsai, 2023; Cure, Hunold, Kesler, Laitenberger, & Larrieu, 2022; Hunold, Laitenberger, & Thébaudin, 2022; Lee & Musolff, 2021).

[8] As in our prominence allocation rule, their algorithm guides the intermediary in assigning prominence in order to maximize profits.

3 Model Setup

Two retailers sell a product via a monopolist intermediary and pay a per-unit commission fee w to the latter. We assume that retailers are vertically differentiated in the purchasing experience offered to consumers. For instance, one retailer provides a better customer experience than the other (e.g., available call center and generous return policy, or faster delivery). For simplicity, we refer to the high-quality and the low-quality retailers as H and L, respectively, and the associated quality is q_H and q_L, with $0 < q_L < q_H$. We assume that H and L have identical marginal production and service costs, which are normalized to 0.[9]

We consider a market in which consumers can only purchase products through the intermediary and have varying preferences for increases in quality, represented by a parameter $\theta \in [\underline{\theta}, \overline{\theta}]$ that is uniformly distributed in the range $[0, 1]$. Consumers have unit demand and their population is divided into two segments: *active* consumers who account for a share $\lambda \in (0, 1)$ of the total population and have access to all product listings, and *passive* consumers who account for the remaining share $1 - \lambda$ and only consider the product that is prominently displayed by the intermediary.[10] We assume θ to be independently and identically distributed across both segments of consumers.[11]

An active consumer that buys a product from retailer i at the price p_i obtains the following utility:

$$U_i^{\lambda}(p_i) = \theta q_i - p_i \qquad \forall i = H, L,$$

whereas a passive consumer that buys a product from the prominent retailer obtains:

$$U^{1-\lambda}(p) = \theta q - p, \tag{1}$$

where q and p represent, respectively, the quality and the price of the prominent retailer identified by the intermediary.

The intermediary imposes a fixed commission fee w on retailers for every transaction that occurs. We denote $D_i^{\lambda}(p_i, p_j, w)$ the demand of a retailer i from the active consumers and $D_H^{1-\lambda}(p_H)$ (resp. $D_L^{1-\lambda}(p_L)$) the demand of the high-quality (resp. low-quality) retailer from passive consumers if made prominent. The profits of the retailers can be expressed as follows. If retailer H is prominently displayed, then the profits of the high-quality and low-quality retailers

[9]In the Appendix, we allow for the possibility that the high-quality retailer faces a higher marginal service cost due, e.g., to logistics and the implementation of the return policy.

[10]This segmentation can be motivated by assuming that active consumers have a low cost of searching and compare all product listings before making a purchase decision, whereas passive consumers have a higher search cost and are more likely to rely on the prominent offer.

[11]We assume that both active and passive consumers, on average, value quality improvements in the same way. Allowing for non-independent distributions of θ and consumer segments would make the model intractable.

are given by:

$$\pi_H^H(p_H, p_L, w) = \Big[\lambda D_H^\lambda(p_H, p_L) + (1-\lambda) D_H^{1-\lambda}(p_H)\Big](p_H - w),$$
$$\pi_L^H(p_L, p_H, w) = \Big[\lambda D_L^\lambda(p_L, p_H)\Big](p_L - w).$$

In this case, the profit of the intermediary is:

$$\Pi^H(p_H, p_L, w) = w\Big[\lambda\Big(D_H^\lambda(p_H, p_L) + D_L^\lambda(p_L, p_H)\Big) + (1-\lambda) D_H^{1-\lambda}(p_H)\Big].$$

If L is prominent instead, the profits of the high- and low-quality retailers are given by:

$$\pi_H^L(p_H, p_L, w) = \Big[\lambda D_H^\lambda(p_H, p_L)\Big](p_H - w),$$
$$\pi_L^L(p_L, p_H, w) = \Big[\lambda D_L^\lambda(p_L, p_H) + (1-\lambda) D_L^{1-\lambda}(p_L)\Big](p_L - w).$$

The profit of the intermediary is:

$$\Pi^L(p_H, p_L, w) = w\Big[\lambda\Big(D_H^\lambda(p_H, p_L) + D_L^\lambda(p_L, p_H)\Big) + (1-\lambda) D_L^{1-\lambda}(p_L)\Big].$$

The timing of the game is as follows. In the first stage, the intermediary sets the commission fee w. In the second stage, retailers engage in price competition, setting their prices p_H and p_L, respectively. In stage 3, the intermediary allocates prominence to the retailer that maximizes its total profit. In the final stage, active consumers compare the offers of the two retailers and decide whether to buy and from which retailer, whereas passive consumers decide whether to buy from the prominent retailer. We solve the game for its subgame-perfect equilibrium.

4 Analysis

4.1 Analysis for a given commission fee

We start by analyzing the consumer's decision in the last stage of the game. An active consumer type-θ buys from retailer i if, and only if, $U_i^\lambda(p_i) \geq \max\{U_{\text{-}i}^\lambda(p_{\text{-}i}), 0\}$. We denote $\overline{\theta}(p_L, p_H)$ the consumer who is indifferent between buying from the high- or the low-quality retailer, i.e., $\overline{\theta}(p_L, p_H)q_H - p_H = \overline{\theta}(p_L, p_H)q_L - p_L$. Similarly, we define $\underline{\theta}(p_L, p_H)$ the consumer who is indifferent between buying from the low-quality retailer or not buying, i.e., $\underline{\theta}(p_L, p_H)q_L - p_L = 0$. As long as $q_L/p_L > q_H/p_H$, all consumers of type $\overline{\theta} > \theta > \underline{\theta}$ buy from the low-quality retailer, whereas all consumers of type $\theta > \overline{\theta}$ buy from the high-quality retailer. The masses of active consumers buying from L and H are then equal to, respectively:

$$D_L^{\lambda}(p_L, p_H) = \max\left\{0, \frac{p_H - p_L}{q_H - q_L} - \frac{p_L}{q_L}\right\}, \qquad D_H^{\lambda}(p_H, p_L) = \min\left\{1 - \frac{p_H - p_L}{q_H - q_L}, 1 - \frac{p_H}{q_H}\right\}. \quad (2)$$

A passive consumer type-θ buys from retailer i if, and only if, $U^{\lambda}(p) \geq 0$. Therefore, the mass of passive consumers that buy from the prominent retailer is either:

$$D_L^{1-\lambda}(p_L) = 1 - \frac{p_L}{q_L}, \quad \text{or} \qquad D_H^{1-\lambda}(p_H) = 1 - \frac{p_H}{q_H}, \quad (3)$$

depending on whether L or H are prominent.

In the third stage, the intermediary allocates prominence to the retailer that, conditional on receiving a prominent display, would generate the highest revenues for the intermediary. This implies maximizing the total demand. Moreover, we assume, as a tie-breaking rule, that in any case of indifference the intermediary assigns prominence to the high-quality retailer.[12] Importantly, the intermediary considers the retailers' prices as given when making this decision. To derive the prominence allocation rule, we first determine the demand functions of H and L for given prices.

Let us first consider the active segment. If $\frac{p_H}{q_H} > \frac{p_L}{q_L}$, then $\frac{p_H - p_L}{q_H - q_L} > \frac{p_L}{q_L}$ and, therefore, demands are given by:

$$D_L^{\lambda}(p_L, p_H) = \frac{p_H - p_L}{q_H - q_L} - \frac{p_L}{q_L}, \qquad D_H^{\lambda}(p_H, p_L) = 1 - \frac{p_H - p_L}{q_H - q_L}.$$

Otherwise, if $\frac{p_H}{q_H} \leq \frac{p_L}{q_L}$, then $\frac{p_H - p_L}{q_H - q_L} \leq \frac{p_L}{q_L}$ and, therefore, the demands are given by:

$$D_L^{\lambda}(p_L, p_H) = 0, \qquad D_H^{\lambda}(p_H, p_L) = 1 - \frac{p_H}{q_H}.$$

Let us consider now the passive segment. If H is prominent, demands are given by:

$$D_L^{1-\lambda}(p_L, p_H) = 0, \qquad D_H^{1-\lambda}(p_H, p_L) = 1 - \frac{p_H}{q_H}.$$

On the contrary, if L is prominent, demands are given by:

$$D_L^{1-\lambda}(p_L, p_H) = 1 - \frac{p_L}{q_L}, \qquad D_H^{1-\lambda}(p_H, p_L) = 0.$$

We can now examine the profit function of the intermediary in each scenario. If $\frac{p_H}{q_H} > \frac{p_L}{q_L}$, the

[12]Note that this assumption can be treated as equilibrium refinement because it allows us to have a unique equilibrium. Relaxing this assumption would just imply the presence of multiple equilibria in which, conditional on the high-quality retailer being prominent, the low-quality retailer obtains zero demand and the high-quality retailer sets $p_H = q_H/q_L p_L$ for any $p_L \geq w$.

intermediary obtains the following profits by assigning prominence to L and H, respectively:

$$\Pi^L(p_H, p_L, w) = \Big(1 - \frac{p_L}{q_L}\Big)w,$$
$$\Pi^H(p_H, p_L, w) = \lambda\Big(1 - \frac{p_L}{q_L}\Big)w + (1-\lambda)\Big(1 - \frac{p_H}{q_H}\Big)w.$$

and prominence is awarded to L if:

$$\begin{aligned}\Pi^H(p_H, p_L, w) - \Pi^L(p_H, p_L, w) &= \lambda\Big(1 - \frac{p_L}{q_L}\Big)w + (1-\lambda)\Big(1 - \frac{p_H}{q_H}\Big)w - \Big(1 - \frac{p_L}{q_L}\Big)w \\ &= w(1-\lambda)\Big(\frac{p_L}{q_L} + \frac{p_H}{q_H}\Big) < 0.\end{aligned}$$

If $\frac{p_H}{q_H} \le \frac{p_L}{q_L}$, the intermediary obtains the following profits by assigning prominence to L and H, respectively:

$$\Pi^L(p_H, p_L, w) = \lambda\Big(1 - \frac{p_H}{q_H}\Big)w + (1-\lambda)\Big(1 - \frac{p_L}{q_L}\Big)w,$$
$$\Pi^H(p_H, p_L, w) = \Big(1 - \frac{p_H}{q_H}\Big)w.$$

and prominence is awarded to H if:

$$\begin{aligned}\Pi^H(p_H, p_L, w) - \Pi^L(p_H, p_L, w) &= \Big(1 - \frac{p_L}{q_L}\Big)w - \Big[\lambda\Big(1 - \frac{p_H}{q_H}\Big)w + (1-\lambda)\Big(1 - \frac{p_L}{q_L}\Big)w\Big] \\ &= w(1-\lambda)\Big(\frac{p_L}{q_L} + \frac{p_H}{q_H}\Big) \ge 0.\end{aligned}$$

Simplifying the above expressions, the following lemma presents the conditions for which prominence is assigned to the high-quality or low-quality retailer. From the above expression, we can conclude the following.

Lemma 1. *The intermediary assigns prominence to the high-quality (resp. low-quality) retailer if:*

$$p_H \le (>) \frac{q_H}{q_L} p_L. \tag{4}$$

Lemma 1 means that the retailer with the lowest quality-adjusted price is granted prominence. Therefore, if the high-quality retailer wants to be prominent, it must match the quality-adjusted price of its low-quality competitor. This pricing strategy maximizes the intermediary's profit. One important implication of this lemma is that the competition for prominence scheme leads to more competition in the market, as it induces retailers to lower their prices, for a given unit fee, in order to be granted prominence.[13]

One implication of (4) is that, as we have already shown, the high-quality retailer can eliminate its competitor from the market by setting its price to $p_H = \frac{q_H}{q_L} p_L$. In other words, $D_L^\lambda(p_L, p_H)\Big|_{p_H = \frac{q_H}{q_L} p_L} = 0$ for any $p_L \geq w$. On the contrary, the low-quality retailer can never exclude its competitor in this way. Because of its quality advantage, the high-quality retailer can attract some active consumers even if it loses the passive segment. The following lemma summarizes this finding.

Lemma 2. *If the high-quality retailer is prominent, the low-quality retailer leaves the market (excluding equilibrium). If the low-quality retailer is prominent, the two retailers compete for the active consumers (market-sharing equilibrium).*

To determine the pricing strategies employed by the retailers and identify the conditions under which the market will reach an excluding equilibrium or a market-sharing equilibrium, we proceed to the second stage of the game.

In this stage, the retailers anticipate the allocation of prominence made by the intermediary and compete in prices. As shown in Lemma 2, if the high-quality retailer (H) is prominent, the low-quality retailer (L) obtains zero demand and profits. Therefore, given that H is prominent, the low-quality retailer always has the incentive to undercut the quality-adjusted price of H and become prominent, regardless of the price pair (p_L, p_H).

However, this undercutting strategy can be blocked in two scenarios. Firstly, if L reaches its price floor, equal to the commission fee, and H finds it optimal to become prominent, the equilibrium price pair will be $p_H = \frac{q_H}{q_L} w$. Secondly, if H finds it optimal not to serve the passive consumers, thereby focusing only on the active ones, the market will reach a market-sharing equilibrium. In this case, L and H's optimal prices are given by:

$$p_H^L(w) := \arg\max_{p_H} \lambda D_H^\lambda(p_H, p_L)(p_H - w),$$

$$p_L^L(w) := \arg\max_{p_L} \Big(\lambda D_L^\lambda(p_L, p_H) + (1-\lambda) D_L^{1-\lambda}(p_L)\Big)(p_L - w),$$

subject to $p_L^L(w) < \frac{q_L}{q_H} p_H^L(w)$. In the Appendix, we demonstrate that $p_L^L(w) < \frac{q_L}{q_H} p_H^L(w)$ is not possible when $\lambda = 0$, but it is possible when $\lambda = 1$ if and only if $q_L > 2w$. Notably, the condition

[13] Note that considering a different commission fee, such as an ad-valorem fee, would imply a different prominence allocation rule. Specifically, the intermediary would account for the revenues on its intensive and extensive margins from both retailers and the rule will ultimately be the following:

$$\Big(1 - \frac{p_H}{q_H}\Big) p_H \geq \Big(1 - \frac{p_L}{q_L}\Big) p_L.$$

However, in our setting with competing vertically differentiated retailers, the model would not be solved either with pure or mixed strategies. A simplified setting with homogeneous retailers would instead deliver results as in the spirit of Bar-Isaac and Shelegia (2022).

for a prominent allocation is linear in λ. Thus, by continuity, we establish that if $q_L > 2w$, there exists a critical threshold $\hat{\lambda}$ such that the low-quality retailer is never prominent with the price pair $(p_L^L(w), p_H^L(w))$ when $\lambda < \hat{\lambda}$, but may be prominent (depending on the strategy of the high-quality retailer) with the price pair $(p_L^L(w), p_H^L(w))$ if $\lambda > \hat{\lambda}$. The following lemma summarizes these findings.

Lemma 3. *For a given fee, a necessary condition for a market-sharing equilibrium to exist is that $q_L > 2w$ and $\lambda > \hat{\lambda}$.*

In the next paragraphs, we study the high-quality retailer's incentives to obtain the default position. First, it is immediate that if the low-quality retailer competes for prominence by undercutting the rival's offer, the high-quality retailer does it as well, thereby matching the quality-adjusted price of the low-quality retailer. As a result, the Bertrand-like spiral ultimately leads to the following price set by H: $p_H = wq_H/q_L$. This scenario occurs either because the low-quality retailer has a sufficiently low quality (i.e., $q_L \leq 2w$) or because the share of the passive consumers is sufficiently large (i.e., $\lambda \leq \hat{\lambda}$).

Second, if the low-quality retailer obtains prominence by setting $p_L^L(w)$, its rival has two options: it can either forfeit the passive segment or undercut the low-quality retailer by deviating from $p_H^L(w)$.

Specifically, if λ is sufficiently close to one, the passive segment has a relatively low value for both firms. Therefore, revenues made from the passive segment are limited. In such a scenario, firm H has lower incentive to compete for prominence because it can still make substantial profits by allowing the rival firm to serve the passive segment. Since firm L always finds it optimal to retain the passive segment, neither firm has the incentive to deviate from the proposed equilibrium prices.

On the other hand, if λ is small enough (and very close to zero), it is crucial for both parties to reach passive consumers and gain prominence. Tedious computation also shows that there exists a critical threshold, denoted as $\tilde{\lambda} \in (0, 1]$, above (resp. below) which the high-quality retailer finds it optimal not to compete for prominence and let the low-quality retailer serve the passive segment.

The following lemma summarizes this result.

Lemma 4. *For a given commission fee, a necessary condition for a market-sharing equilibrium to exist is that $\lambda > \tilde{\lambda}$.*

Based on the results in Lemmas 3 and 4, we can now establish the equilibrium for a given commission fee by considering the different intervals we have identified.

When $q_L \leq 2w$, the low-quality retailer always has an incentive to undercut the high-quality retailer, leading to a Bertrand spiral that ultimately results in the equilibrium pair of prices $(p_L, p_H) = (w, wq_H/q_L)$. In this case, there exists a unique excluding equilibrium, where the intermediary allocates prominence to the high-quality retailer, and the low-quality retailer does not serve any customer.

If $q_L > 2w$, there are different subcases to consider, which we discuss separately. First, if $\lambda < \min\{\hat{\lambda}, \tilde{\lambda}\}$, the passive segment is large enough for each retailer to consider giving it up. In this case, fierce competition for being the default retailer leads to the Bertrand spiral and the resulting equilibrium prices are $(p_L, p_H) = (w, wq_H/q_L)$.

Second, if $\lambda < \max\{\hat{\lambda}, \tilde{\lambda}\}$, we need to determine which of the two critical thresholds is higher. If $\tilde{\lambda} < \hat{\lambda}$, the low-quality retailer is unable to secure a prominent position by setting $p_L^L(w)$ and instead resorts to undercutting the high-quality retailer. This leads to an excluding equilibrium. Conversely, if $\hat{\lambda} < \tilde{\lambda}$, the high-quality retailer has an incentive to be prominent and therefore undercuts the quality-adjusted price of the low-quality retailer, triggering the Bertrand spiral. In this case, the equilibrium pair of prices is $(p_L, p_H) = (w, wq_H/q_L)$ if $\lambda < \max\{\hat{\lambda}, \tilde{\lambda}\}$.

Third, if $\lambda > \max\{\hat{\lambda}, \tilde{\lambda}\}$, it is optimal for the high-quality retailer to give up the passive segment, whereas the low-quality retailer keeps it exclusively. This results in the equilibrium pair of prices $(p_L, p_H) = (p_L^L(w), p_H^L(w))$ and a market-sharing equilibrium emerges. We summarize this discussion in the following proposition.

Proposition 1. *If $q_L > 2w$ and $\lambda > \max\{\hat{\lambda}, \tilde{\lambda}\}$, then the equilibrium is such that the low-quality retailer is prominent while the high-quality retailer serves part of the active consumers ("market-sharing equilibrium"). Otherwise, the equilibrium is such that the high-quality retailer is prominent and serves all consumers ("excluding equilibrium").*

4.2 Endogenous commission fee

So far, we have considered the strategies of the retailers for a given commission fee. We will now turn our attention to the first stage of the game and analyze the optimal commission fee for the intermediary.

Dropping the arguments of the demands for the sake of exposition, the intermediary sets w to maximize the following profit. If $\lambda \geq \max\{\hat{\lambda}, \tilde{\lambda}\}$ and $q_L > 2w$, the profit to be maximized is:

$$\begin{aligned}\Pi(p_L(w), p_H(w), w) &= w\Big[\lambda\Big(D_H^{\lambda}(\cdot) + D_L^{\lambda}(\cdot)\Big) + (1-\lambda)D_L^{1-\lambda}(\cdot)\Big] \\ &= w\frac{\lambda q_L(q_H + 2q_L - 3w) + 2(q_H - q_L)(q_L - w)}{q_L(4q_H - (4-3\lambda)q_L)}.\end{aligned}$$

Otherwise, the profit to be maximized is:

$$\begin{aligned}\Pi(p_L(w), p_H(w), w) =& w\left[\lambda D_H^{\lambda}(\cdot) + (1-\lambda) D_H^{1-\lambda}(\cdot)\right] \\ &= w\frac{(q_L - w)}{q_L}.\end{aligned}$$

In the Appendix, we show that the optimal commission fee that the intermediary would set conditional on prominence to be allocated to the low-quality retailer is:

$$w^L = \frac{q_L[q_H(2+\lambda) - 2(1-\lambda)q_L]}{4(q_H - q_L) + 6\lambda q_L}.$$

which, however, does not satisfy the constraint that $q_L > 2w^L$. As a result, the intermediary always finds it optimal to set a commission fee, which we denote at equilibrium as w_H, such that the high-quality retailer is prominent. The optimal commission fee is set at $w^H = q_L/2$ and, therefore, only an excluding equilibrium is subgame perfect. The high-quality retailer obtains:

$$\pi_H^H(p_H^H(w^H), p_L^H(w^H), w^H) = \frac{q_H - q_L}{4},$$

whereas the intermediary obtains:

$$\Pi^H(p_L^H(w^H), p_H^H(w^H), w^H) = \frac{q_L}{4}.$$

Therefore, we conclude the following:

Proposition 2. *In equilibrium, the intermediary sets $w^H = q_L/2$. The high-quality retailer is prominent and the low-quality retailer serves no customers.*

4.3 Competition for prominence and efficiency

To better understand the results generated by a competition for prominence scheme, we compare the profit of the platform and its welfare effects to those arising in a vertical chain setting and in that of a multiproduct monopolist.

Vertical chain. The first comparison we make is with the classic vertical chain setting, where the intermediary does not segment the market between the two types of consumers, and it acts as an upstream firm offering intermediation services to two competing downstream retailers. In this scenario, we assume that the intermediary cannot steer consumers such that all consumers are active (i.e., $\lambda = 1$). Due to vertical differentiation, both retailers sell a positive amount of products, with high-θ consumers buying from the high-quality retailer, intermediate-θ consumers buying from the low-quality retailer, and low-θ consumers not buying at all.

Unsurprisingly, this setting exhibits a vertical externality: the double marginalization problem causes both retailers to set prices above the monopoly price, which reduces downstream demand and consumer surplus. Our analysis of the competition for prominence setting suggests that the ability to steer a portion of consumers and segment the market can alleviate the inefficiency resulting from the vertical externality. Consequently, this leads to an increase in welfare.

Multiproduct monopolist. In the second comparison, we examine the case where the intermediary is a multiproduct monopolist that proposes both low- and high-quality offers, and controls their prices directly. In this setup, the intermediary is vertically integrated with the retailers and does not charge any commission fee, unlike the baseline model. The monopolist sets the same monopoly prices $p_H^M = \frac{q_H}{2}$ and $p_L^M = \frac{q_L}{2}$ for each retailer, regardless of which offer is made prominent. As prices and total demands are the same for either prominence allocation, the multiproduct monopolist can maximize its profit by assigning prominence to the high-quality offer. Therefore, the monopolist directs all consumption to the high-quality offer, and there is no demand for the low-quality offer.[14]

Comparing the equilibrium profit of the multiproduct monopolist and of the intermediary conditional on inducing a prominent high-quality retailer, we have $\Pi^M(p_H^M, p_L^M) \equiv \frac{q_H}{4} > \frac{q_L}{4} \equiv \Pi^H(p_H^H(w^H), p_L^H(w^H), w^H)$ Moreover, comparing total industry profits and consumer surplus in the two cases respectively, we identify an equivalence result:

$$\begin{aligned} \Pi^H(p_H^H(w^H), p_L^H(w^H), w^H) + \pi_H^H(p_H^H, p_L^H, w^H) = \frac{q_H}{4} \equiv \Pi^M(p_H^M, p_L^M), \\ CS^H(p_H^H(w^H), p_L^H(w^H)) \equiv \frac{q_H}{8} \equiv CS^M(p_H^M, p_L^M). \end{aligned} \tag{5}$$

The above analysis highlights that although the intermediary does not have direct control over the strategies of the two retailers, it can still shape the marketplace strategically to influence the prices set by the retailers. This outcome effectively replicates the behavior of a multiproduct monopolist, which controls the prices of the offers it provides. By using competition for prominence, the intermediary can create a situation where the retailers compete for a prominent position, ultimately resulting in prices and profits that mimic those of a multiproduct monopolist.

5 Alternative scheme: Pay for prominence

In this section, we propose an alternative prominence allocation scheme for the intermediary. Instead of allocating prominence based on the number of purchases, we consider a payment-based scheme where retailers can purchase prominence. Specifically, we assume that the inter-

[14] Note that the presence of high- and low-quality offers does not imply quality discrimination among consumers with different tastes for quality, as at equilibrium, none of them finds it optimal to purchase the low-quality offer. This is in contrast to the model by Mussa and Rosen (1978).

mediary conducts an auction for prominence, and the retailer with the highest bid is awarded prominence and pays the amount bid by the second-highest bidder. The timing of the model is modified as follows: In the first stage, retailers submit their bids simultaneously, and the highest bidder wins prominence. In the second stage, retailers compete in prices by setting p_H and p_L. In the third period, consumers decide whether to purchase and from which retailer.

This alternative scheme differs from the baseline model in two main ways. First, the prominence allocation decision is made before competition occurs, and the intermediary assigns prominence to the highest bidder. This decision is visible to both retailers, and prices reflect the allocation of prominence. Second, the intermediary solely relies on the auction to monetize its intermediation service. Therefore, we assume $w = 0$ in our analysis.[15]

If H is made prominent, the gross profits of the high- and low-quality retailers are, respectively:

$$\pi_H^H(p_H, p_L) = \Big[\lambda D_H^\lambda(p_H, p_L) + (1-\lambda) D_H^{1-\lambda}(p_H)\Big] p_H,$$
$$\pi_L^H(p_L, p_H) = \Big[\lambda D_L^\lambda(p_L, p_H)\Big] p_L.$$

If L is made prominent, instead, the gross profits of the high- and low-quality retailers are, respectively:

$$\pi_H^L(p_H, p_L) = \Big[\lambda D_H^\lambda(p_H, p_L)\Big] p_H,$$
$$\pi_L^L(p_L, p_H) = \Big[\lambda D_L^\lambda(p_L, p_H) + (1-\lambda) D_L^{1-\lambda}(p_L)\Big] p_L.$$

The net profit of the retailer i that wins the auction and pays a monetary compensation to the intermediary to obtain prominence, denoted by $\tilde{\pi}_i^i$, is equivalent to the gross profit of the same retailer, net of the payment made to the intermediary, which we refer to as b_i. Specifically, $\tilde{\pi}_i^i(p_i, p_{-i}) = \pi_i^i(.) - b_i$.

In stage 2, retailers decide prices. The equilibrium prices in the continuation game in which H is prominent are:

$$p_H^H := \arg\max_{p_H} \Big(\lambda D_H^\lambda(p_H, p_L) + (1-\lambda) D_H^{1-\lambda}(p_H)\Big) p_H - b_H,$$

$$p_L^H := \arg\max_{p_L} \lambda D_L^\lambda(p_L, p_H) p_L.$$

The equilibrium prices in the continuation game in which L is prominent are:

$$p_H^L := \arg\max_{p_H} \lambda D_H^\lambda(p_H, p_L) p_H,$$

[15] Note that one can show that it is not in the intermediary's interest to set a commission fee $w \neq 0$. To see why, differentiating the intermediary's profit with respect to w and evaluating it at $w = 0$, yields a negative value. Therefore, $w^\star = 0$.

$$p_L^L := \arg\max_{p_L} \Big(\lambda D_L^\lambda(p_L, p_H) + (1-\lambda) D_L^{1-\lambda}(p_L)\Big) p_L - b_L.$$

In stage 1, we determine the bids that each retailer submits in the position auction. In order to do so, we first compute each retailer's marginal benefit from being prominent, which is equivalent to the difference in the profits obtained by the retailers when receiving prominence and the profit obtained when not receiving prominence.

Denoting WTP_i the willingness to pay for prominence of retailer i, for $i = L, H$, we get:

$$\begin{aligned} WTP_L &:= \pi_L^L(p_L^L, p_H^L) - \pi_L^H(p_L^H, p_H^H), \\ WTP_H &:= \pi_H^H(p_H^H, p_L^H) - \pi_H^L(p_H^L, p_L^L). \end{aligned}$$

Comparing the two in the Appendix, we observe that the following inequality always holds:

$$\pi_L^L(p_L^L, p_H^L) + \pi_H^L(p_H^L, p_L^L) < \pi_H^H(p_H^H, p_L^H) + \pi_L^H(p_L^H, p_H^H),$$

which indicates that total retailers' profits are always larger when the high-quality retailer is prominent. This implies that $WTP_H > WTP_L$, that is, due to its superior quality, the high-quality retailer has more to gain than the rival because of its prominent position. We conclude the following.

Lemma 5. *The willingness to pay for the prominent position of the high-quality retailer is higher than the willingness to pay of the low-quality retailer.*

This result indicates that if the intermediary chooses to allocate prominence through a second sealed-bid auction, the high-quality retailer will always be able to outbid the low-quality retailer due to its higher willingness to pay. In other words, in an auction, the high-quality retailer can secure prominence by bidding just above the willingness to pay of the low-quality retailer. This proposition is summarized below.

Proposition 3. *With a pay for prominence scheme, the intermediary always awards prominence to the high-quality retailer and the optimal bid is $b_H = WTP_L$.*

We compare the market outcomes of the competition for prominence (denoted by CP) and the pay for prominence schemes (denoted by PP). It is important to note that assigning prominence ex ante, along with vertical differentiation, leads to a softened competition, where the low-quality retailer is not excluded. Thus, the intermediary can only extract an amount equivalent to the willingness to pay for the prominence of the low-quality retailer through the fee. Unlike the competition for prominence scheme, in the pay for prominence the two retailers compete in the market, leading to prices below the monopoly level, resulting in increased market demand.

We show in the Appendix that $\Pi^{PP} < \Pi^{CP}$. Therefore, the auction does not allow the intermediary to extract as much rent as in the competition for prominence, and if the intermediary had to choose between the two schemes, it would prefer a competition for prominence one.

Proposition 4. *The intermediary always finds it optimal to adopt a competition for prominence scheme.*

In order to verify which scheme is welfare-enhancing, we now compare the aggregate retailer surplus and consumer surplus in the two cases. First, recall that under competition for prominence, when the fee is endogenized, the low-quality retailer is excluded from the market whereas the high-quality retailer obtains $(q_H - q_L)/4$. Under pay for prominence, both retailers ultimately end up with some strictly positive surplus. Therefore, the low-quality retailer undoubtedly benefits from a pay for prominence scheme. Moreover, we show in the Appendix that also the high-quality retailer is better-off under pay for prominence as $\pi_H^{PP} > \pi_H^{CP}$, for $q_H > q_L$. Indeed, the position auction relaxes competition between retailers, and the active presence of the low-quality retailer limits the amount of surplus the intermediary can extract. Therefore, the high-quality retailer can keep a larger share of revenues than under competition for prominence. Nevertheless, total industry profit is higher under competition for prominence. As we showed in the previous section, this scheme allows the intermediary to replicate the market outcomes of a multiproduct monopolist.

Second, we compare consumer surplus in the two scenarios. Recall that in the competition for prominence, the intermediary induces retailers to fiercely compete in prices, which allows it to impose an almost-monopoly fee to the high-quality retailer. Ultimately, consumers must pay a monopolistic price of $q_H/2$. Differently, under pay for prominence, both retailers compete in the active segment and prices are below the monopoly level, which implies a higher total demand in the market and a higher consumer surplus.

Finally, the social desirability of a scheme over another depends on whether the gains for consumers offset the losses in total industry profit. In the Appendix, we show that the total welfare is always higher under a pay for prominence scheme. However, in a laissez-faire regime the intermediary finds it optimal to introduce a competition for prominence scheme. These results provide important implications for possible regulation of intermediaries' prominence allocation schemes.

Proposition 5. *Pay for prominence is socially desirable as it benefits retailers and consumers relative to competition for prominence.*

6 Endogenous quality

In this section, we endogenize quality, which has so far been considered exogenously given. We assume that retailers are initially homogeneous in both the products they offer and their retailing services. However, similar to Amazon, we introduce the concept of ancillary services provided by the intermediary. Retailers have the option to purchase a quality upgrade from the intermediary at a fixed fee $F > 0$. These upgrades can include improved logistics and fulfillment services, enrollment in generous return policies, or access to 24/7 customer service. Furthermore, the intermediary has the ability to provide these services at a higher quality level denoted as q_H, which is higher than the quality level that independent retailers can offer, denoted as $q_L < q_H$.

The model is modified as follows to endogenize quality. In period 1, the intermediary sets the fixed upgrading fee F. Retailers, which are ex ante homogeneous, have the option to purchase a quality upgrade independently from the intermediary at the fixed upgrading fee F. In period 2, all retailers simultaneously and independently decide whether to purchase the quality upgrade from the intermediary. This decision is public knowledge, which aligns with Amazon's practice of providing information on whether a retailer is part of its Fulfillment by Amazon (FBA) program or classified as Fulfillment by Merchants (FBM) with independent logistics. In period 3, retailers simultaneously set prices, and, in period 4, the intermediary allocates prominence to a retailer based on the prominence allocation rule.

Unlike the baseline setting with exogenous quality, we have now the following cases:

(i) **No upgrade** (NU, NU). Both retailers do not purchase the quality upgrade. Because retailers stick with their ex ante homogeneous quality, their prices are the canonical Bertrand prices: the unique equilibrium is determined by the retailers' price floor, which in our setting is the per-unit fee w charged by the intermediary.

(ii) **Joint upgrade** (U, U). Both retailers purchase the quality upgrade. Therefore, retailers are ex ante and ex post homogeneous and, as a result, there is Bertrand competition. As in the previous scenario, retailers set a price equal to w and obtain zero profits.

(iii) **Asymmetric upgrade** (U, NU), (NU, U). Only one retailer purchase the quality upgrade. This case is consistent with the baseline setting in which there is vertical differentiation between retailers. However, the retailer with the higher quality incurs a fixed upgrading cost F. We denote the profits of the upgrading and non-upgrading retailers as π_H and π_L, respectively. Note that these profits, gross of the upgrading fee, are equivalent to those in the baseline setting for a given fee. Depending on the value of λ, and in particular whether it is below or above the threshold $\tilde{\lambda}$ in Lemma 4, the pair (π_H, π_L) can be consistent with either a market-sharing or an excluding equilibrium.

Independently of whether the asymmetric cases lead to an excluding equilibrium or a market-sharing equilibrium, the game in Period 2 can be represented in the normal form, as depicted in Figure (1).

		Retailer $-i$	
		U	NU
Retailer i	U	$0-F, 0-F$	$\pi_H - F, \pi_L$
	NU	$\pi_L, \pi_H - F$	$0, 0$

Figure 1: Upgrading decision stage.

It is straightforward to see that because the symmetric scenarios will drive profits to zero (or below zero), for any w, it is strictly dominant for retailers to be in an asymmetric scenario, (U, NU) or (NU, U).[16] Formally, let us first consider the case in which there is an excluding equilibrium in the asymmetric scenario, that is $\lambda > \tilde{\lambda}$ (in Lemma 4). Then, the high-quality retailer makes strictly positive profits for any $F < \pi_H$ and the low-quality retailer makes zero profit. As a result, the asymmetric outcome is strictly dominant for the high-quality retailer and, at worst, weakly dominant for the low-quality retailer.

Now, let us consider the case where there is a market-sharing equilibrium in the asymmetric scenario, that is, $\lambda < \tilde{\lambda}$ (as stated in Lemma 4). In this case, the high-quality retailer earns strictly positive profits for any $F < \pi_H$, and the low-quality retailer also earns strictly positive profits because it obtains a default position. As a result, it is strictly dominant for both retailers to be in the asymmetric outcome, with only one of the two retailers finding it optimal to upgrade quality. This finding implies that in equilibrium, there are two asymmetric scenarios that involve vertical differentiation between retailers, thereby providing a microfoundation for our baseline model. We summarize this result in the following proposition.

Proposition 6. *Suppose quality is endogenous and retailers can purchase a quality upgrade from q_L to q_H paying a fee $F > 0$ to the intermediary. For a given commission fee w, there is a multiplicity of equilibria with two asymmetric upgrading scenarios.*

We can now solve for the optimal commission and upgrading fees given the asymmetric upgrading. As shown in Proposition 2, the intermediary sets in the first stage $w^H = \frac{q_L}{2}$ and induces an excluding equilibrium in which prominence is awarded to the high-quality retailer. In this scenario, the retailer that purchases the upgrade obtains prominence.

Moreover, the intermediary sets the upgrading fee F that is incentive-compatible, that is:

$$\pi_H^H(p_H^H, p_L^H, w^H, F) - F \geq \pi_L^L(p_L^H, p_H^H, w^H),$$

[16] Note that if F were not sunk, retailers would obtain a profit equal to zero in the joint upgrade scenario, meaning that retailers would be indifferent between upgrading or not. This would result in a multiplicity of equilibria, (U, U),(U, NU) or (NU, U). As the intermediary always extracts monopoly rents in any of the three equilibria, it would be indifferent between either outcome and would not solve the coordination game.

where $\pi_L^L(p_L^H, p_H^H, w^H) = 0$ because in the excluding equilibrium the low-quality retailer does not operate in the market (Lemma 2) and $\pi_H^H(p_H^H, p_L^H, w^H, F) = \frac{q_H - q_L}{4}$, at the optimal fee w_H^H. Therefore, the optimal upgrading fee, denoted as $F^\star$, is simply:

$$F = \pi_H^H(p_H^H, p_L^H, w^H, F) - \pi_L^H(p_L^H, p_H^H, w^H) = \frac{q_H - q_L}{4},$$

and allows the intermediary to fully extract the remaining surplus of the high-quality retailer. As a result, the profit of the intermediary is:

$$\Pi^H(F^\star, p_H^H, p_L^H, w^H) = \frac{q_H}{4},$$

which is the same as the profit of a multiproduct monopolist (see (5)). This result is summarized in the following proposition.

Proposition 7. *Suppose quality is endogenous and retailers can purchase a quality upgrade from q_L to q_H paying a fee $F > 0$ to the intermediary. By setting $F^\star$, the intermediary replicates the profit level of a multiproduct monopolist, and consumer surplus increases relative to a scenario in which there is no quality upgrade, but all retailers are left with no surplus.*

The above result has important policy implications and welfare effects. First, the fact that intermediary can replicate the multiproduct monopolist's profit by using a two-part tariff ensures the largest rent extraction from retailers. Importantly, retailers' surplus remains unchanged compared to a situation in which the intermediary does not offer a quality upgrade and, therefore, both retailers are of low quality. In such a case, due to Bertrand-like competition, retailers would ultimately receive zero surplus.

It is worth noticing that the redistribution of surplus from the retailers to the intermediary in the case of a quality upgrade is beneficial to consumers. This is because the equilibrium prices are set equal to the fee set by the intermediary, and thus, equal to $p_L = w^L = q_L/2$. In the asymmetric upgrading scenario, all consumers would pay a higher price, $p_H^\star = q_H/2$, and purchase from the high-quality retailer. Because the gains from higher quality more than compensate for the losses from a higher price, the net effect is positive, resulting in higher consumer surplus.[17] Therefore, the combination of a competition for prominence scheme and the endogenous quality upgrading program has a positive effect for the intermediary and consumers, whereas retailers' surplus remains unchanged.

[17] Note that

$$CS_{(NU,NU)} \equiv \int_{\frac{p_L}{q_L}}^{1} (\theta q_L - p_L)\, d\theta = \frac{q_L}{8} < \frac{q_H}{8} = \int_{\frac{p_H}{q_H}}^{1} (\theta q_H - p_H)\, d\theta \equiv CS_{(U,NU),(NU,U)},$$

for any $q_H > q_L$.

7 Conclusions

Our analysis provides insights into the optimal prominence scheme that intermediaries can implement. Competition for prominence serves as a potent mechanism for addressing the issue of double marginalization, a prevalent problem in vertical markets where intermediaries and retailers add markups to the final price. By introducing a contest for the default position, retailers are encouraged to compete, leading to lower overall consumer prices. Our analysis also shows that alternative prominence allocation schemes, such as position auctions, relying on monetary instruments, are suboptimal compared to competition for prominence. These schemes require the intermediary to commit to granting prominence to a particular retailer, leading to a relaxation of competition. In contrast, the contest for the default position is an open competition that encourages all retailers to compete on price and quality.

Our findings provide a compelling explanation for why major retail companies, such as Walmart in the United States or FNAC in France, have started implementing a platform business model that allows third-party sellers to operate on their marketplace. These platforms gather all product listings in one location and highlight one through their version of the buy-box, which represents the default option for consumers. In addition, our research suggests that new entrants in this industry recognize the significance of competition for prominence as a critical factor in achieving success.

However, our analysis also reveals that whether the commission fee is considered as given or is chosen contingently by the intermediary, a default position can be assigned to either the low-quality or high-quality retailer. In cases where the commission fee is considered as given, for example, due to long-run commitments by the intermediary or external regulation, competition for prominence can lead to an outcome where low-quality retailers are promoted to passive consumers. This scenario occurs when the share of passive consumers is not significant enough to intensify competition between the two retailers. Therefore, the intermediary promotes the low-quality retailer to attract passive consumers and prevent them from choosing a competitor.

On the other hand, a large passive market segment stimulates competition. Ultimately, there is an excluding equilibrium in which the high-quality retailer obtains prominence and serves all consumers that join the marketplace. If the commission fee is endogenized, the intermediary can induce the latter scenario, thereby leading to an excluding equilibrium where prominence is granted to the high-quality retailer.

Our analysis has also revealed that while competition for prominence represents the most profitable prominence allocation scheme for an intermediary, it is not socially desirable. We demonstrate that an alternative auction-based scheme, where prominence is allocated to the highest bidder, can increase social welfare, benefiting retailers and consumers, even though it may be detrimental to the intermediary's profits. This finding suggests that introducing default positions that incorporate the interests of the intermediary in retailers' strategies can have distortive effects relative to alternative prominence allocation schemes.

Moreover, the presence of a second buy-box, as proposed by Amazon in its settlement with the European Commission, may not necessarily solve the problem of surplus extraction that can result from the assignment of a buy-box. The reason is that the criteria used by the intermediary may still stimulate fierce competition between retailers and monopolistic rent extraction by the intermediary through its fee. Therefore, careful consideration must be given to the design of prominence allocation schemes to ensure they do not have negative welfare effects.

Finally, our analysis sheds light on how the vertical differentiation between two identical offers can be determined by the decision of the intermediary to offer ancillary services, which has become an emerging practice in the industry.[18] We find that if an intermediary can steer passive consumers using a prominent display and offers the possibility to upgrade the quality of ancillary services, it can extract the largest surplus from retailers. This strategy is the most profitable for the intermediary because it removes the double marginalization problem and increases consumer surplus. However, it comes at the cost of leaving both retailers without surplus.

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Appendix

Proof of Lemma 1

This proof immediately follows from the main text.

Proof of Lemma 2

The first part of the proof follows from (4). Suppose that H is prominent. It can set two prices: either it sets $p_H = \frac{q_H}{q_L} p_L$ so that (4) binds, or it sets $p_H < \frac{q_H}{q_L} p_L$ and the constraint in (4) is slack. Suppose $p_H = \frac{q_H}{q_L} p_L$. In this case, L's demand conditional on H being prominent is:

$$\begin{aligned} D_L^\lambda(p_L, p_H) =& \frac{\frac{q_H}{q_L} p_L - p_L}{q_H - q_L} - \frac{p_L}{q_L} \\ =& \frac{p_L}{q_L} - \frac{p_L}{q_L} \\ =& 0, \end{aligned}$$

which implies the exclusion of L for any $p_L \geq w$. Because L leaves the market for $p_H = \frac{q_H}{q_L} p_L$, it is also excluded for any $p_H < \frac{q_H}{q_L} p_L$. As a result, the equilibrium price set by H conditional on being prominent is $p_H = \frac{q_H}{q_L} p_L$, and $D_L^\lambda(p_L, p_H) = 0$.

The second part of the proof is as follows. Suppose L is prominent: in this case, H is out of the market (i.e., $D_H^\lambda = 0$) if, and only, if:

$$p_L \leq p_H - q_H + q_L.$$

Suppose L sets its lowest price $p_L = w$. Then, exclusion of H occurs if and only if:

$$w \leq p_H - q_H + q_L,$$

or alternatively if $p_H \geq w + q_H - q_L$.

Suppose also H sets the lowest possible price $p_H = w$. Then, H is out of the market if and only if $q_L \geq q_H$, which contradicts the assumption that $q_H > q_L$. As a result, if L is awarded prominence it cannot exclude H from the marketplace. This concludes the proof.

Proof of Lemma 3

Consider the case in which L is prominent. H chooses p_H to maximize the following program:

$$\max_{p_H} \pi_H^L(p_H, p_L) \equiv \lambda\Big(1 - \frac{p_H - p_L}{q_H - q_L}\Big)(p_H - w) \qquad \text{s.t.} \qquad p_H > p_L \frac{q_H}{q_L},$$

whereas L chooses p_L to maximize the following:

$$\max_{p_L} \pi_L^L(p_L, p_H) \equiv \Big[\lambda\Big(\frac{p_H - p_L}{q_H - q_L} - \frac{p_L}{q_L}\Big) + (1-\lambda)\Big(1 - \frac{p_L}{q_L}\Big)\Big](p_L - w) \quad \text{s.t.} \quad p_H > p_L \frac{q_H}{q_L}.$$

Differentiating profits with respect to prices yields:

$$\begin{aligned}
\frac{\partial \pi_L^L(p_L, p_H)}{\partial p_L} = 0 &\Longleftrightarrow p_L^L(p_H) = \frac{\lambda q_L(p_H - (q_H - q_L) + w) + (q_H - q_L)(q_L + w)}{2(q_H - (1-\lambda)q_L)}, \\
\frac{\partial \pi_H^L(p_H, p_L)}{\partial p_H} = 0 &\Longleftrightarrow p_H^L(p_L) = \frac{1}{2}(p_L + q_H - q_L + w).
\end{aligned}$$

For a given w, we then have:

$$\begin{aligned}
p_L^L(w) &= \frac{\lambda q_L(3w - q_H + q_L) + 2(q_H - q_L)(q_L + w)}{4q_H - (4 - 3\lambda)q_L}, \\
p_H^L(w) &= \frac{2q_H^2 - (3-\lambda)q_H q_L + 3q_H w + (1-\lambda)q_L(q_L - 3w)}{4q_H - (4 - 3\lambda)q_L}.
\end{aligned} \tag{6}$$

Because $p_H > p_L \frac{q_H}{q_L}$ for L to be prominent, we should verify whether the above pair of prices satisfy such a condition. Specifically, multiplying both prices by $4q_H - (4-3\lambda)q_L$, then $p_H(w) > p_L(w)\frac{q_H}{q_L}$ if:

$$\begin{aligned}
q_L\Big(2q_H^2 - (3-\lambda)q_H q_L + 3q_H w + (1-\lambda)q_L(q_L - 3w)\Big) > \\
\Big(\lambda q_L(3w - q_H + q_L) + 2(q_H - q_L)(q_L + w)\Big)q_H.
\end{aligned} \tag{7}$$

Simplifying, we have $p_H(w) > p_L(w)\frac{q_H}{q_L}$ if:

$$\frac{(q_H - q_L)(q_L(\lambda(q_H + q_L - 3w) - q_L + 3w) - 2q_H w)}{q_L} > 0.$$

Note that at $\lambda = 0$, $p_H(w) > p_L(w)\frac{q_H}{q_L}$ if:

$$-\frac{(q_H - q_L)\,(2q_H w + q_L^2 - 3q_L w)}{q_L} > 0,$$

which is never satisfied.

At $\lambda = 1$, $p_H(w) > p_L(w)\frac{q_H}{q_L}$ if:

$$\frac{q_H(q_H - q_L)(q_L - 2w)}{q_L} > 0,$$

which is always satisfied for any $q_L > 2w$.

Finally, as the condition in (7) is linear and increasing in λ, by continuity there exists a critical threshold of λ, which we denote as $\hat{\lambda} \in [0, 1]$ such that for $\lambda < \hat{\lambda}$, the low-quality retailer is never prominent, whereas for $\lambda > \hat{\lambda}$ the low-quality retailer is prominent. Formally, for any $q_L > 2w$, the critical value is:

$$\hat{\lambda} \equiv \frac{q_L(q_L - 3w) - 2q_H w}{q_L(q_H + q_L - 3w)}.$$

Proof of Lemma 4

In this proof, we formalize conditions on the equilibrium emerging for different values of λ.

We follow the following strategy. First, we start from L being prominent and identify conditions under which H finds it optimal to deviate and obtain prominence. Then, if H finds it optimal to deviate, we identify the equilibrium prices and profits in this subgame.

Suppose L is able to guarantee a market-sharing equilibrium, which therefore implies that conditions in Lemma 3 hold. Equilibrium prices are given by (6) and profits are:

$$\pi_L^L(p_L^L, p_H^L, w) = \frac{(q_H - q_L)[q_H - (1 - \lambda)q_L][2w - (2 - \lambda)q_L]^2}{q_L[4q_H - (4 - 3\lambda)q_L]^2},$$
$$\pi_H^L(p_H^L, p_L^L, w) = \frac{\lambda(q_H - q_L)[(1 - \lambda)q_L - 2q_H + w]^2}{[4q_H - (4 - 3\lambda)q_L]^2}.$$

Suppose H deviates and obtains prominence. This means setting at least as equal to $p_L^L\, q_H/q_L$ so that it satisfies the prominence allocation constraint, i.e., $p_H^d = p_L^L\, q_H/q_L$. Using p_L^L in (6), the deviation price is:

$$p_H^d(w) = \frac{\lambda q_H q_L[3w - (q_H - q_L)] + 2q_H(q_H - q_L)(q_L + w)}{q_L[4q_H - (4 - 3\lambda)q_L]}.$$

The associated deviation profit is:

$$\pi_H^d(w) = \frac{(q_H - q_L)[2q_H(q_L + w) - \lambda q_H q_L - (4 - 3\lambda)q_L w][\lambda q_L(q_H + 2q_L - 3w) + 2(q_H - q_L)(q_L - w)]}{q_L^2[4q_H - (4 - 3\lambda)q_L]^2}.$$

We now proceed by comparing H's deviation profit with the candidate equilibrium profit.

Formally, $\pi_H^d(w) > \pi_H^L(w)$ if:

$$\frac{(q_H - q_L)[2q_H(q_L + w) - \lambda q_H q_L - (4 - 3\lambda)q_L w][\lambda q_L(q_H + 2q_L - 3w) + 2(q_H - q_L)(q_L - w)]}{q_L^2[4q_H - (4 - 3\lambda)q_L]^2} > \frac{\lambda(q_H - q_L)[(1 - \lambda)q_L - 2q_H + w]^2}{[4q_H - (4 - 3\lambda)q_L]^2}. \tag{8}$$

which is equivalent to the following condition:

$$[2q_H(q_L + w) - \lambda q_H q_L - (4 - 3\lambda)q_L w][\lambda q_L(q_H + 2q_L - 3w) + 2(q_H - q_L)(q_L - w)] > q_L^2\lambda[(1 - \lambda)q_L - 2q_H + w]^2. \tag{9}$$

Note that at $\lambda = 1$, (9) is equal to:

$$-q_H^2(q_H - q_L)(q_L - 2w)^2 > 0.$$

which is never satisfied. Moreover, tedious computation and inspection (whose analysis is available upon request to the authors) show that there exists a threshold value, which we denote as $\tilde{\lambda} \in [0, 1]$ such that (8) holds if $\lambda < \tilde{\lambda}$ and does not hold otherwise.

Proof of Proposition 1

This proof immediately follows from the main text.

Proof of Proposition 2

Consider first the case in which L is prominent, which arises if $\lambda \geq \max\{\hat{\lambda}, \tilde{\lambda}\}$ and $q_L > 2w$. In this case, the profit of the intermediary is:

$$\Pi^L(p_L^L(w), p_H^L(w), w) = \frac{\lambda q_L w(q_H + 2q_L - 3w) + 2w(q_H - q_L)(q_L - w)}{q_L(4q_H - (4 - 3\lambda)q_L)}.$$

Differentiating it with respect to w, and rearranging, yields:

$$w = \frac{q_L[q_H(2 + \lambda) - 2(1 - \lambda)q_L]}{4(q_H - q_L) + 6\lambda q_L}.$$

However, we note that the necessary condition that $q_L > 2w$ never holds as:

$$
\begin{aligned}
q_L - 2\frac{q_L[q_H(2+\lambda) - 2(1-\lambda)q_L]}{4(q_H - q_L) + 6\lambda q_L} \\
&= 1 - \frac{[q_H(2+\lambda) - 2(1-\lambda)q_L]}{2(q_H - q_L) + 3\lambda q_L} \\
&= 2(q_H - q_L) + 3\lambda q_L - [q_H(2+\lambda) - 2(1-\lambda)q_L] \\
&= \lambda(q_L - q_H) < 0.
\end{aligned}
$$

Therefore, it is never the case that the intermediary induces an equilibrium in which L is prominent.

It is now immediate to determine the optimal commission fee that leads to an excluding equilibrium. In this case, H is prominent and the profit of the intermediary is:

$$\Pi^H(p_L^H(w), p_H^H(w), w) = \frac{w(q_L - w)}{q_L}.$$

Differentiating it with respect to w, and rearranging it, yields:

$$w = \frac{q_L}{2}.$$

For ease of exposition, denoting this value as w^H, the associated equilibrium profit of the intermediary is:

$$\Pi^H(p_H^H(w^H), p_L^H(w^H), w^H) = \frac{q_L}{4}.$$

Dropping the arguments for brevity, consumer surplus, at equilibrium values, is given by:

$$CS = \lambda \int_{\frac{p_H - p_L}{q_H - q_L}}^{1} (\theta q_H - p_H)\, d\theta + (1-\lambda) \int_{\frac{p_H}{q_H}}^{1} (\theta q_H - p_H)\, d\theta = \frac{q_H}{8}. \tag{10}$$

The total industry profit is:

$$\Pi + \pi_H = \frac{q_H}{4}. \tag{11}$$

Total welfare, which is defined as the sum of the industry profit and consumer surplus, is:

$$W = CS^{CP} + \Pi + \pi_H = \frac{3}{8} q_H. \tag{12}$$

Proofs of Section 4.3

Classic vertical chain

Consider a classic vertical chain and assume that $\lambda = 1$. H chooses p_H to maximize the following program:

$$\max_{p_H} \pi_H(p_H, p_L) \equiv \left(1 - \frac{p_H - p_L}{q_H - q_L}\right)(p_H - w),$$

whereas the program of L is to choose p_L to maximize the following:

$$\max_{p_L} \pi_L(p_L, p_H) \equiv \left(\frac{p_H - p_L}{q_H - q_L} - \frac{p_L}{q_L}\right)(p_L - w).$$

Differentiating retailers' profits with respect to their prices, we get:

$$\frac{\partial \pi_L(p_L, p_H, w)}{\partial p_L} = 0 \Longleftrightarrow p_L(p_H, w) = \frac{p_H q_L - 2p_L q_H + q_H w}{q_L(q_H - q_L)},$$
$$\frac{\partial \pi_H(p_H, p_L, w)}{\partial p_H} = 0 \Longleftrightarrow p_H(p_L, w) = \frac{q_H - q_L + p_L - 2p_H + w}{q_H - q_L}.$$

The optimal prices, which we denote as $p_L(w)$ and $p_H(w)$, are as follows:

$$p_L(w) = \frac{q_H(q_L + 2w) + q_L(w - q_L)}{4q_H - q_L},$$
$$p_H(w) = \frac{q_H(2q_H - 2q_L + 3w)}{4q_H - q_L}.$$

The profit of the intermediary is:

$$\Pi(p_H(w), p_L(w), w) = \frac{w(3q_H q_L - w(2q_H + q_L))}{q_L(4q_H - q_L)}.$$

Differentiating it with respect to w and rearranging, and denoting the optimal commission fee as $w^\star$, we have:

$$w^\star = \frac{3q_H q_L}{2(2q_H + q_L)}.$$

The intermediary, at equilibrium, obtains:

$$\Pi(p_H(w^\star), p_L(w^\star), w^\star) = \frac{9q_H^2 q_L}{4(8q_H^2 + 2q_H q_L - q_L^2)}.$$

We compute the consumer surplus and total industry profit in the classic vertical chain setting. For the sake of exposition, we drop the arguments.

The consumer surplus, which we denote as CS is given by:

$$\begin{aligned} CS(p_H(w^\star), p_L(w^\star)) &= \int_{\frac{p_H-p_L}{q_H-q_L}}^{1} (\theta q_H - p_H)\, d\theta + \int_{\frac{p_L}{q_L}}^{\frac{p_H-p_L}{q_H-q_L}} (\theta q_L - p_L)\, d\theta \\ &= \frac{q_H^2\left(64q_H^3 - 12q_H^2 q_L + 21 q_H q_L^2 + 8q_L^3\right)}{8(q_L - 4q_H)^2(2q_H + q_L)^2}. \end{aligned}$$

The total industry profit — the sum of the profit of the intermediary and of those of the retailers — is given by:

$$\Pi(\cdot) + \pi_H(\cdot) + \pi_L(\cdot) = \frac{q_H\left(64q_H^4 + 28q_H^3 q_L - 9q_H^2 q_L^2 + 2q_H q_L^3 - 4q_L^4\right)}{4(q_L - 4q_H)^2(2q_H + q_L)^2}.$$

For ease of exposition, let us drop the arguments $(\cdot)$. The total welfare, denoted as W, is:

$$W(\cdot) = CS(\cdot) + \Pi(\cdot) + \pi_H(\cdot) + \pi_L(\cdot) = \frac{q_H\left(192q_H^4 + 44q_H^3 q_L + 3q_H^2 q_L^2 + 12q_H q_L^3 - 8q_L^4\right)}{8(q_L - 4q_H)^2(2q_H + q_L)^2}.$$

Welfare comparison. With a slight abuse of notation, let us denote the classic vertical chain case with the superscript VC, and the competition for prominence case with the superscript CP, where consumer surplus is given by (10), industry profits by (11) and total welfare by (12). We observe the following:

$$CS^{CP} - CS^{VC} = \frac{3q_H}{8} - \frac{q_H^2\left(64q_H^3 - 12q_H^2 q_L + 21q_H q_L^2 + 8q_L^3\right)}{8(q_L - 4q_H)^2(2q_H + q_L)^2},$$

which has the same sign as the sign of:

$$\begin{aligned} &3q_H(q_L - 4q_H)^2(2q_H + q_L)^2 - q_H^2\left(64q_H^3 - 12q_H^2 q_L + 21q_H q_L^2 + 8q_L^3\right) \\ =&q_H\left(1472q_H^4 + 780q_H^3 q_L - 309q_H^2 q_L^2 - 104q_H q_L^3 + 24q_L^4\right), \end{aligned}$$

which is positive for any $q_H > q_L$. Moreover, the difference in total industry profits is given by:

$$\Pi^{CP} + \pi_H^{CP} - (\Pi^{VC} + \pi_H^{VC} + \pi_L^{VC}) = \frac{q_H - q_L}{4} - \frac{q_H(q_H - q_L)\left(64q_H^3 + 20q_H^2 q_L - 7q_H q_L^2 + 4q_L^3\right)}{4(q_L - 4q_H)^2(2q_H + q_L)^2},$$

which has the same sign as the sign of:

$$\begin{aligned} &(q_H - q_L)(q_L - 4q_H)^2(2q_H + q_L)^2 - q_H(q_H - q_L)\left(64q_H^3 + 20q_H^2 q_L - 7q_H q_L^2 + 4q_L^3\right) \\ =&q_L(q_H - q_L)^2\left(12q_H^2 + 7q_H q_L - q_L^2\right), \end{aligned}$$

which is positive for any $q_H > q_L$. A fortiori, given that industry profits and consumer surplus are higher under competition for prominence than under a vertical chain.

Multiproduct monopolist

Consider a multiproduct monopolist that has two offers and compare the profit obtained when prominence is given to the high-quality offer and the profit obtained when prominence is given to the low-quality offer.

Suppose the low-quality offer is prominent. Then, the multiproduct monopolist chooses p_L and p_H to maximize the following:

$$\max_{p_L,p_H} \Pi^L(p_H,p_L) \equiv \lambda\Big(1-\frac{p_H-p_L}{q_H-q_L}\Big)p_H+\Big[\lambda\Big(\frac{p_H-p_L}{q_H-q_L}-\frac{p_L}{q_L}\Big)+(1-\lambda)\Big(1-\frac{p_L}{q_L}\Big)\Big]p_L.$$

Differentiating with respect to prices and solving for the optimal prices, which we denote by p_L^L and p_H^L, we obtain:

$$p_L^L=\frac{q_L}{2},\quad p_H^L=\frac{q_H}{2},$$

and the associated profit is:

$$\Pi^L(p_L^L,p_H^L)=\frac{\lambda q_H}{4}+\frac{(1-\lambda)q_L}{4}.$$

Suppose the high-quality offer is prominent. Then, the multiproduct monopolist chooses p_L and p_H to maximize the following:

$$\max_{p_L,p_H} \Pi^H(p_L,p_H) \equiv \Big[\lambda\Big(1-\frac{p_H-p_L}{q_H-q_L}\Big)+(1-\lambda)\Big(1-\frac{p_H}{q_H}\Big)\Big]p_H+\lambda\Big(\frac{p_H-p_L}{q_H-q_L}-\frac{p_L}{q_L}\Big)p_L.$$

Differentiating with respect to prices and solving for the optimal prices, which we denote by p_L^H and p_H^H, we obtain:

$$p_L^H=\frac{q_L}{2},\quad p_H^H=\frac{q_H}{2},$$

and the associated profit is:

$$\Pi^H(p_L^H,p_H^H)=\frac{q_H}{4}.$$

Comparing the profits in the two cases, it follows that:

$$\Pi^H(p_L^H,p_H^H)\equiv\frac{q_H}{4}>\frac{\lambda q_H+q_L(1-\lambda)}{4}\equiv\Pi^L(p_L^L,p_H^L),$$

because $q_H>q_L$.

Consumer surplus, at equilibrium values, is given by:

$$CS^M=\lambda\int_{\frac{p_H-p_L}{q_H-q_L}}^{1}(\theta q_H-p_H)\,d\theta+(1-\lambda)\int_{\frac{p_H}{q_H}}^{1}(\theta q_H-p_H)\,d\theta=\frac{q_H}{8},$$

which is the same as in (10). The total industry profit is equal to the profit of the vertically integrated intermediary, which is the same as in (11). Total welfare, denoted as the sum of the industry profit and consumer surplus, is:

$$W^M = CS^M + \Pi^M = \frac{3}{8}q_H,$$

which is the same as in (12).

We conclude that there is an equivalence result between a multiproduct monopolist and competition for prominence.

Proof of Lemma 5

In this proof, we compare the willingness to pay for prominence of two retailers.

First, the profit obtained by the retailers conditional on L being prominent, are given by:

$$\begin{aligned}
\pi_L^L(p_L, p_H) &= \Big[\lambda D_L^\lambda(p_L, p_H) + (1-\lambda) D_L^{1-\lambda}(p_L)\Big] p_L - b_L, \\
\pi_H^L(p_H, p_L) &= \Big[\lambda D_H^\lambda(p_H, p_L)\Big] p_H.
\end{aligned}$$

From the first-order conditions with respect to p_H and p_L we get:

$$\begin{aligned}
\frac{\partial \pi_L^L(p_L, p_H)}{\partial p_L} &= 0 \Longleftrightarrow p_L^L(p_H) = \frac{q_L(\lambda p_H - \lambda q_H + q_H + (\lambda-1)q_L)}{2(q_H + (\lambda-1)q_L)}, \\
\frac{\partial \pi_H^L(p_H, p_L)}{\partial p_H} &= 0 \Longleftrightarrow p_H^L(p_L) = \frac{1}{2}(p_L + q_H - q_L).
\end{aligned}$$

The optimal prices, conditional on L being prominent, are:

$$\begin{aligned}
p_L^L &= -\frac{(\lambda-2)q_L(q_H - q_L)}{4q_H + (3\lambda-4)q_L}, \\
p_H^L &= \frac{(q_H - q_L)(2q_H + (\lambda-1)q_L)}{4q_H + (3\lambda-4)q_L},
\end{aligned}$$

and the associated profits are:

$$\begin{aligned}
\pi_L^L &= \frac{(\lambda-2)^2 q_L(q_H - q_L)(q_H + (\lambda-1)q_L)}{(4q_H + (3\lambda-4)q_L)^2}, \\
\pi_H^L &= \frac{\lambda(q_H - q_L)(2q_H + (\lambda-1)q_L)^2}{(4q_H + (3\lambda-4)q_L)^2}.
\end{aligned}$$

Then, the profit obtained by the retailers conditional on H being prominent, are given by:

$$\begin{aligned}
\pi_L^H(p_L, p_H) &= \Big[\lambda D_L^\lambda(p_L, p_H)\Big] p_L, \\
\pi_H^H(p_H, p_L) &= \Big[\lambda D_H^\lambda(p_H, p_L) + (1-\lambda) D_H^{1-\lambda}(p_H)\Big] p_H - b_H.
\end{aligned}$$

From the first-order conditions with respect to p_H and p_L we get:

$$\begin{aligned}
\frac{\partial \pi_L^H(p_L, p_H)}{\partial p_L} &= 0 \Longleftrightarrow p_L^L(p_H) = \frac{p_H q_L}{2 q_H}, \\
\frac{\partial \pi_H^H(p_H, p_L)}{\partial p_H} &= 0 \Longleftrightarrow p_H^L(p_L) = \frac{q_H(\lambda p_L + q_H - q_L)}{2(q_H - (1-\lambda) q_L)}.
\end{aligned}$$

The optimal prices are:

$$\begin{aligned}
p_L^H &= \frac{q_L(q_H - q_L)}{4q_H - (4-3\lambda) q_L}, \\
p_H^H &= \frac{2q_H(q_H - q_L)}{4q_H - (4-3\lambda) q_L},
\end{aligned}$$

and the associated profits are:

$$\begin{aligned}
\pi_L^H &= \frac{\lambda q_H q_L (q_H - q_L)}{(4q_H - (4-3\lambda) q_L)^2}, \\
\pi_H^H &= \frac{4q_H(q_H - q_L)(q_H + (\lambda - 1) q_L)}{(4q_H - (4-3\lambda) q_L)^2}.
\end{aligned}$$

Denoting WTP_i the willingness to pay for prominence of retailer i, for $i = L, H$, we get:

$$\begin{aligned}
WTP_L :=& \frac{(1-\lambda) q_L (q_H - q_L)((4-\lambda)(q_H + \lambda q_L) - 4q_L)}{(4q_H - (4-3\lambda) q_L)^2}, \\
WTP_H :=& \frac{(1-\lambda)(q_H - q_L)\left(4(1-\lambda) q_H q_L + (1-\lambda)\lambda q_L^2 - 4q_H^2\right)}{(4q_H - (4-3\lambda) q_L)^2}.
\end{aligned}$$

Comparing the two, we observe that:

$$WTP_H - WTP_L = \frac{(1-\lambda)(q_H - q_L)\left(4q_H^2 + (5\lambda - 8) q_H q_L + (\lambda(2\lambda - 5) + 4) q_L^2\right)}{(4q_H + (3\lambda - 4) q_L)^2}.$$

The above expression is positive as long as $q_H > q_L$ and $1 > \lambda > 0$, which is always the case. Therefore, $WTP_H - WTP_L > 0$.

Proof of Proposition 3

As the WTP of H is larger than that of L, the former can simply win the auction by bidding the willingness to pay of the latter and become prominent. As a result:

$$b_H = WTP_L.$$

This concludes the proof.

Proof of Proposition 4

We denote the profit of the intermediary using a pay for prominence scheme with Π^{PP}. These are simply given by:

$$\Pi^{PP} = b_H = \frac{(1-\lambda)q_L(q_H - q_L)((4-\lambda)(q_H + \lambda q_L) - 4q_L)}{(4q_H - (4-3\lambda)q_L)^2}.$$

We compare the profit above with the one obtained with the competition for prominence scheme:

$$\Pi^{PP} - \Pi^{CP} = \frac{(1-\lambda)q_L(q_H - q_L)((4-\lambda)(q_H + \lambda q_L) - 4q_L)}{(4q_H - (4-3\lambda)q_L)^2} - \frac{q_L}{4},$$

because $w = 0$. To determine the sign of the difference, we need to verify the sign of the resulting numerator:

$$\lambda q_L \left(4q_H^2(\lambda - 5) + 4q_H q_L \left(\lambda^2 - 6\lambda + 7\right) + q_L^2 \left(-4\lambda^2 + 11\lambda - 8\right)\right).$$

Tedious computation shows that the above expression is negative for $q_H > q_L$ and $1 > \lambda > 0$. Therefore, the intermediary generates more profit using a competition for prominence scheme compared to a position auction.

Proof of Proposition 5

We now compare the welfare effects of the two prominence schemes. Consumer surplus under the paying for prominence scheme is given by:

$$\begin{aligned} CS^{PP} &= \lambda \int_{\frac{p_H - p_L}{q_H - q_L}}^{1} (\theta q_H - p_H)\, d\theta + \lambda \int_{\frac{p_L}{q_L}}^{\frac{p_H - p_L}{q_H - q_L}} (\theta q_L - p_L)\, d\theta + (1-\lambda) \int_{\frac{p_H}{q_H}}^{1} (\theta q_H - p_H)\, d\theta \\ &= \frac{q_H(q_H - (1-\lambda)q_L)(4q_H - (4-9\lambda)q_L)}{2(4q_H - (4-3\lambda)q_L)^2}. \end{aligned}$$

Consumer surplus under the competition for prominence scheme is $CS^{CP} = q_H/8$. Comparing consumer surplus in the two alternatives, we observe that:

$$CS^{PP} - CS^{CP} = \frac{q_H(q_H - (1-\lambda)q_L)(4q_H - (4-9\lambda)q_L)}{2(4q_H - (4-3\lambda)q_L)^2} - \frac{q_H}{8},$$

which is determined by the sign of the following term:

$$\begin{aligned} &8(q_H(q_H + (\lambda-1)q_L)(4q_H + (9\lambda-4)q_L)) - q_H\left(2(4q_H + (3\lambda-4)q_L)^2\right) \\ =&2\lambda q_H q_L(28q_H - (28-27\lambda)q_L), \end{aligned}$$

which is positive for $q_H > q_L$.

Clearly, the low-quality retailer is worse-off under competition for prominence as $\pi_L^{PP} > 0 = \pi_L^{CP}$. Comparing the profit of the high-quality retailer in the two scenarios we observe that:

$$\pi_H^{PP} - \pi_H^{CP} = \frac{\lambda q_L(q_H - q_L)(4(3-\lambda)q_H - (8-\lambda(11-4\lambda))q_L)}{4(4q_H - (4-3\lambda)q_L)^2},$$

which is determined by the sign of the following term:

$$4(3-\lambda)q_H - (8-\lambda(11-4\lambda))q_L,$$

which is positive for $q_H > q_L$.

The total industry profit is equal to the sum of the profit of the vertical intermediary, and the net profits of the two retailers:

$$\Pi^{PP} + \pi_H^{PP} + \pi_L^{PP} = \frac{q_H(q_H - q_L)(4q_H - (4-5\lambda)q_L)}{(4q_H - (4-3\lambda)q_L)^2}.$$

The total industry profit under the competition for prominence scheme is $\Pi^{CP} + \pi_H^{CP} = q_H/4$. We study the sign of the difference:

$$\Pi^{PP} + \pi_H^{PP} + \pi_L^{PP} - (\Pi^{CP} + \pi_H^{CP}) = \frac{q_H(q_H - q_L)(4q_H - (4-5\lambda)q_L)}{(4q_H - (4-3\lambda)q_L)^2} - \frac{q_H}{4},$$

which is determined by the sign of the following term:

$$\begin{aligned} &4(q_H(q_H - q_L)(4q_H + (5\lambda-4)q_L)) - q_H(4q_H + (3\lambda-4)q_L)^2 \\ &= \lambda q_H q_L((4-9\lambda)q_L - 4q_H), \end{aligned}$$

which is negative for $q_H > q_L$.

Total welfare, denoted as the sum of the industry profit and consumer surplus, is:

$$W^{PP} = CS^{PP} + \Pi^{PP} + \pi_H^{PP} + \pi_L^{PP} = \frac{q_H\left(23\lambda q_L(q_H - q_L) + 12(q_H - q_L)^2 + 9\lambda^2 q_L^2\right)}{2(4q_H - (4 - 3\lambda)q_L)^2}.$$

The total welfare under the competition for prominence scheme is $W^{CP} = 3q_H/8$. We study the sign of the difference:

$$W^{PP} - W^{CP} = \frac{q_H\left(23\lambda q_L(q_H - q_L) + 12(q_H - q_L)^2 + 9\lambda^2 q_L^2\right)}{2(4q_H - (4 - 3\lambda)q_L)^2} - \frac{3q_H}{8},$$

which has the same sign as the sign of the following term:

$$\begin{aligned}
&8\left(q_H\left(23\lambda q_L(q_H - q_L) + 12(q_H - q_L)^2 + 9\lambda^2 q_L^2\right)\right) - 3q_H\left(2(4q_H + (3\lambda - 4)q_L)^2\right) \\
&= 2\lambda q_H q_L(20q_H - (20 - 9\lambda)q_L),
\end{aligned}$$

which is positive for $q_H > q_L$.

Proof of Proposition 6

The game is solved by backward induction and we identify the equilibrium profits of the retailers in the four subgames.

No upgrade. Retailers are homogeneous and the profit of retailer $i = 1, 2$ is:

$$\pi_{L_i}(p_i, p_{-i}, w) = \left(\lambda D_i^{\lambda} + (1 - \lambda)D_i^{1-\lambda}\right)(p_i - w).$$

Because of perfect symmetry, retailers compete à la Bertrand, setting prices equal to the commission fee. At equilibrium, the profit of retailer i is:

$$\pi_L^B \equiv \pi_{L_i}^B = \pi_{L_{-i}}^B = 0 \qquad \forall i = 1, 2,$$

where, with abuse of notation, B indicates Bertrand competition.

Joint upgrade. Retailers are ex ante and ex post homogeneous. Therefore, Bertrand-type competition occurs. The profit of a representative retailer, denoted by $i = 1, 2$ is:

$$\pi_{H_i}(p_i, p_{-i}, w, F) = \left(\lambda D_i^{\lambda} + (1 - \lambda)D_i^{1-\lambda}\right)(p_i - w) - F,$$

with $F > 0$ the upgrading fee. Because of perfect symmetry, retailers compete à la Bertrand, setting prices equal to the commission fee. At equilibrium, the profit of retailer i is:

$$\pi_H^B(F) \equiv \pi_{H_i}^B(F) = \pi_{H_{-i}}^B(F) = -F \qquad \forall i = 1, 2,$$

where, with abuse of notation, B indicates Bertrand competition.

Asymmetric upgrade. In this case, retailers are ex ante symmetric, but asymmetric ex post: one retailer upgrades, while the other does not. This scenario is equivalent to the baseline model, except from the added upgrading stage. From Lemma 3, for given fees, if $\lambda < \tilde{\lambda}(w)$, H has an incentive to obtain prominence.

For a given w, if $\lambda < \tilde{\lambda}(w)$ the equilibrium prices are:

$$p_H^H = \frac{q_H}{q_L} w, \qquad p_L^H = w.$$

The profit of the retailers are

$$\pi_H^H(p_H^H, p_L^H, w, F) = \frac{(q_H - q_L)(q_L - w)w}{q_L^2} - F, \qquad \pi_L^H(p_L^H, p_H^H, w) = 0.$$

If $\lambda > \tilde{\lambda}(w)$, H prefers to give up prominence. Equilibrium prices are:

$$p_H^L(w) = \frac{2q_H^2 - (3-\lambda)q_H q_L + 3q_H w + (1-\lambda)q_L(q_L - 3w)}{4q_H - (4-3\lambda)q_L},$$
$$p_L^L(w) = \frac{2(q_H - q_L)(q_L + w) - \lambda q_L(q_H - q_L - 3w)}{4q_H - (4-3\lambda)q_L}.$$

The profit of the retailers are:

$$\pi_H^L(p_H^L, p_L^L, w, F) = \frac{\lambda(q_H - q_L)[(1-\lambda)q_L - 2q_H + w]^2}{[4q_H - (4-3\lambda)q_L]^2} - F,$$
$$\pi_L^L(p_L^L, p_H^L, w) = \frac{(q_H - q_L)[q_H - (1-\lambda)q_L][2w - (2-\lambda)q_L]^2}{q_L[4q_H - (4-3\lambda)q_L]^2}.$$

In the upgrading stage, we distinguish therefore between two cases, depending on λ, and we rewrite Table 1.

If $\lambda < \tilde{\lambda}(w)$, each retailer solves the normal form game in Table 1:

		Retailer -i	
		U	NU
Retailer i	U	$-F, -F$	$\pi_H^H - F, 0$
	NU	$0, \pi_H^H - F$	$0, 0$

Table 1: Upgrading decision ($\lambda < \tilde{\lambda}(w)$).

If $\lambda > \tilde{\lambda}(w)$, each retailer solves the normal form game in Table 2:

		Retailer $-i$	
		U	NU
Retailer i	U	$-F, -F$	$\pi_H^L - F, \pi_L^L$
	NU	$\pi_L^L, \pi_H^L - F$	$0, 0$

Table 2: Upgrading decision ($\lambda > \tilde{\lambda}(w)$).

Solving the games in the two cases, regardless of λ, only asymmetric equilibria (U, NU) and (NU, U) arise. To see why, consider that if retailer i upgrades, retailer $-i$ does not upgrade, as $0 > -F$. If retailer i does not upgrade, retailer $-i$ upgrades, as $\pi_H > 0$. if retailer i does not upgrade, retailer $-i$ upgrades, as $\pi_H > 0$. Due to symmetry, therefore, only the asymmetric equilibria are possible.

Proof of Proposition 7

The proof immediately follows from the discussion in the main text.

Asymmetric marginal cost

An important assumption in our baseline model is that both retailers face a common marginal cost, which was normalized to zero. In this Section, we relax this assumption and discuss how the presence of asymmetric marginal cost alters the market outcome. Specifically, we assume that the high-quality retailer has a higher marginal cost than the low-quality retailer. Denoting c_H and c_L, respectively, the marginal cost of H and L, and normalizing the latter to 0, we assume $c_H > c_L = 0$.

This assumption has an important effect on changing how retailers compete. The reason is that the price floor of H is now $p_H = c_H + w$, whereas the price floor of L is $p_L = w$.

Conditional on H being prominent, the profits of the retailers are, respectively:

$$\pi_H^H(p_H, p_L, w) = \left[\lambda D_H^{\lambda}(p_H, p_L) + (1-\lambda) D_H^{1-\lambda}(p_H)\right](p_H - w - c_H),$$
$$\pi_L^H(p_L, p_H, w) = \left[\lambda D_L^{\lambda}(p_L, p_H)\right](p_L - w).$$

Conditional on L being prominent, that is for $\lambda \geq \tilde{\lambda}(w)$ are, respectively:

$$\pi_H^L(p_H, p_L, w) = \left[\lambda D_H^{\lambda}(p_H, p_L)\right](p_H - w - c_H),$$
$$\pi_L^L(p_L, p_H, w) = \left[\lambda D_L^{\lambda}(p_L, p_H) + (1-\lambda) D_L^{1-\lambda}(p_L)\right](p_L - w).$$

For any $\lambda < \tilde{\lambda}(w)$, we distinguish between two alternative scenarios, which depend on the magnitude of the marginal cost. In the first one, c_H is sufficiently low (and the baseline model is a degenerate case), and H can always match the quality-adjusted price of L and win prominence. In the second one, c_H is sufficiently high, and H cannot obtain prominence as the low-quality retailer can always undercut the high-quality one.

Using H and L's price floors together with the prominence allocation rule in (4), we can identify the cut-off value below (resp. above), which H is always able to undercut L and maximize the profit of the intermediary if:

$$c_H \leq (>) \Big(\frac{q_H}{q_L} - 1 \Big) w. \tag{13}$$

If (13) holds, therefore, the analysis in our baseline model holds qualitatively, and the low-quality retailer is out of the market at equilibrium (see Proposition 2).

If (13) does not hold, it is not always possible for H to lower its price to win prominence. As in the baseline model, L always has an incentive to obtain prominence whenever H obtains prominence. Otherwise, it is excluded. Therefore, the equilibrium outcome depends on the incentives of the high-quality seller in this case.

Differently from the baseline model, if H has an incentive to obtain prominence, there is now no equilibrium in pure strategy exists.

Too see why, consider the case in which H reaches its lowest price, $p_H = w + c_H$, and L sets $p_L = p_H \; q_L/q_H - \varepsilon$ (with $\varepsilon \to 0$). In this case, L gains prominence and positive profit, whereas H obtains zero profits. Since the loss of prominence for the high-quality retailer does not imply losing the active consumers due to vertical differentiation (see Lemma 2), H always has an incentive to raise its price and monetize the actives.

Consider now the case in which L sets $p_L = (w + c_H) \; q_L/q_H - \varepsilon$ (with $\varepsilon \to 0$), which implies that L certainly obtains prominence. As H can always serve the active consumers if L is prominent (by Lemma 2), it maximizes its profit subject to $p_L = (w + c_H) \; q_L/q_H - \varepsilon$. From the first-order condition with respect to p_H, we obtain the following candidate equilibrium price

$$p_H = \frac{c_H(q_H + q_L) + w(q_H + q_L) + q_H(q_H - q_L)}{2q_H}.$$

We notice that (p_L, p_H) here defined is not equilibrium because L can always raise its price just below the prominence threshold level, i.e., $p_L \approx p_H \; q_L/q_H - \varepsilon$.

This analysis suggests that if (13) does not hold, there is never an equilibrium in pure strategy. Therefore, only a mixed strategy equilibrium exists.