Competition for Prominence

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This paper examines the role of online intermediaries in influencing consumers' purchasing decisions by giving preferential treatment to certain retailers. We investigate the incentives of intermediaries to employ a "competition for prominence scheme", analyzing the impact on retailer strategies and consumer surplus. We show that, for a given fee, three alternative equilibria can emerge, two of which result in one retailer making no sale. We also find that intermediaries have a strategic interest in inducing intense competition between retailers by selecting one excluding equilibrium where double marginalization issues are mitigated and overall demand is maximized. We show that in equilibrium, the interests of the intermediary and the consumers are fully aligned. Finally, this scheme is always more profitable than auction-based alternatives and allows the intermediary to replicate multiproduct monopolistic outcomes.

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1 Introduction

The global landscape of online marketplaces has evolved to encompass a handful of dominant platforms that play a pivotal role in connecting sellers with consumers, significantly influencing their decisions and consumption patterns. Key mechanisms, such as recommender systems and the allocation of prominent positions to sellers, have emerged as powerful tools wielded by these platforms. A major example is given by Amazon— the largest marketplace in the world— which groups all listings associated with a specific product on a single webpage. Among the available retailers, the platform designates one as the default retailer, which appears prominently in the so-called "buy-box", whereas the other sellers are relegated to less prominent positions.[1] Obtaining the default position in online marketplaces has become vital for retailers, with as much as 80% of all Amazon transactions occurring through the buy-box. (U.S. House Judiciary Committee, 2020).

In this paper, we refer to this prominence allocation strategy as the "competition for prominence scheme". We delve into the dynamics and the implications of the competition for prominence scheme when implemented by online intermediaries, particularly examining its impact on retailers' competition and strategies as well as consumer surplus. While platforms often do not fully disclose their precise criteria for prominence allocation, they occasionally offer insights on how retailers can enhance their chances of obtaining this coveted position. For instance, Amazon seemingly prioritizes competitive prices, fast shipping, and excellent customer service, which can be considered as "quality" measures .[2]

We consider a model in which a monopolist platform connects consumers to two vertically differentiated retailers. Taking inspiration from Amazon's approach, in our model the retailers offer the same product but differ in the "quality" of their ancillary services (e.g., next-day delivery or a more generous return policy), creating vertical differentiation from a consumer's perspective. We analyze the case in which one retailer provides a superior service compared to the other. For ease of exposition, we will refer to the retailer with the superior service as the high-quality retailer and its counterpart as the low-quality retailer. For its intermediation service, the platform collects a per-unit fee for each realized sale. Consumers are of two types, actives and passives, and their willingness to pay for quality is heterogeneous. Active consumers always compare all offers and buy from the retailer that provides the highest utility. In contrast, passive consumers only consider the recommended offer and decide whether to buy it. The intermediary determines which retailer to show prominently based on the observed prices.

We show that the intermediary finds it optimal to assign a prominent position to the retailer with the lowest quality-adjusted price to boost consumer demand and the associated fees. An-

[1] Comparable default prominence allocation schemes are now adopted by several other marketplaces, including Walmart in the United States, Otto in Germany, Fnac in France, and Bol.com in the Netherlands.
[2] For more details, see the *Amazon Seller Central Hub*.

ticipating the allocation rule of the intermediary, retailers decide whether and how to compete for the prominent display. If the high-quality retailer is awarded prominence, the low-quality retailer always fails to make sales. Consequently, the low-quality retailer always has incentives to seek prominence for any given fee. On the other hand, the high-quality retailer is motivated to obtain prominence only if the share of passive consumers is sufficiently large. The rationale is that, due to vertical differentiation, the high-quality retailer naturally captures a portion of the active consumers, and reaching the passive consumers implies competing fiercely with the low-quality retailer. Facing the usual trade-off between volume and margins, the high-quality retailer can profitably give up prominence if the share of passive consumers is low and, instead, compete for prominence if the share of these consumers is large. Depending on the high-quality retailer's incentives, three alternative equilibria exist for a given fee.

If the share of passive consumers is large and the quality of the low-quality retailer is high enough, the high-quality retailer can give up the passive segment without losing much and let the low-quality retailer serve it, thereby relaxing competition. In this scenario, the low-quality retailer is an active competitor, and in equilibrium, there is a partially segmented market, softened competition, and both retailers earn positive profits. We refer to this outcome as the *market sharing equilibrium*.

Otherwise, when the share of passive consumers is small, or the quality of the low-quality retailer is low enough, the high-quality retailer cannot afford to abandon the passive consumers. In these situations, we identify two alternative *excluding equilibria*, in which the low-quality retailer fails to make any sales. In the first excluding equilibrium, which occurs if the intermediary's commission fee is sufficiently low, the high-quality retailer becomes prominent by pricing aggressively, resulting in a low equilibrium price. In the second excluding equilibrium, which arises if the commission fee of the intermediary is sufficiently high, the high-quality retailer can maintain prominence even by setting a monopoly price, as the low-quality retailer lacks the ability to compete effectively.

To determine which of the three equilibria is subgame perfect, we examine the incentives of the intermediary in setting the commission fee. The intermediary's interest lies in mitigating the double marginalization problem that arises when the two retailers compete by segmenting the market and softening competition. Additionally, it seeks to avoid the double marginalization problem that occurs when the high-quality retailer sets a monopoly price. Therefore, the intermediary finds it optimal to induce the first excluding equilibrium by allocating prominence to the high-quality retailer while stimulating a Bertrand-like competition that expands consumer demand in both segments of the market. Moreover, we find that the market sharing equilibrium is never socially desirable, whereas the two excluding equilibria would be socially desirable in the parameter range in which they emerge. This result shows that the interests of the intermediary are fully aligned with those of consumers. Remarkably, the competition for prominence strategy eliminates the double marginalization problem and yields market outcomes equivalent to a market structure where a multiproduct monopolist directly sets prices for the two offers.

We extend the model to consider the case in which two ex ante symmetric retailers have the option to directly acquire a quality upgrade (such as enhanced delivery services) from the intermediary for a fixed fee. This case resembles the service provided by Amazon known as the Fulfilled by Amazon program. Under this program, retailers can store their products in Amazon's warehouse and benefit from the marketplace's fast logistics network. This example provides a microfoundation for our baseline model, where vertical differentiation exists between retailers. We demonstrate that this extended scenario leads to the emergence of two symmetric equilibria characterized by asymmetric quality upgrades.

Finally, we compare the competition for prominence allocation scheme with an alternative auction-based prominence allocation scheme. In this scenario, the intermediary assigns prominence to the retailer bidding the highest fee. This case mirrors, for example, sponsorship programs made available by online marketplaces such as eBay, with its "Promoted Listings" where the highest bidder obtains the highest visibility. We show that an auction-based prominence allocation scheme, whereby the intermediary charges a per-unit fee and a fixed fee for assigning prominence, softens competition and exacerbates double marginalization. While this scheme always makes both retailers sell products, their prices are above the monopolistic level and reduce demand. As a result, an intermediary always finds it optimal to assign prominence via a competitive scheme rather than via an auction. We also show that while competition for prominence benefits consumers because their demand is expanded due to lower final prices, total retailers' surplus may be lower under competition for prominence than under an auction-based allocation.

The paper is structured as follows. Section 2 reviews the literature on prominence allocation in online platforms and highlights our contributions. Section 3 presents the model and its key features. Section 4 analyzes the equilibrium allocation of prominence for both exogenous and endogenous commission fees. Section 5 presents two main extensions. Finally, Section 6 summarizes our findings and provides concluding remarks. Proofs are relegated to the Appendix.

2 Related Literature

This paper relates to the literature on intermediaries that, by organizing transactions on their marketplaces, decide whether and how they can make some sellers more prominent. A general finding in this literature is that prominence plays a critical role and affects both purchasing decisions and competition among rival sellers.

Armstrong, Vickers, and Zhou (2009) and Moraga-González, Sándor, and Wildenbeest (2021) study a setting in which prominence is *exogenously* assigned by an intermediary to a seller and consumers have search costs. The authors abstract from the intermediary's interests and analyze the welfare effects of prominence. Armstrong et al. (2009) examine the effects of prominence in the context of sequential search, showing that the prominent seller sets a lower price and the non-prominent ones that serve unsatisfied consumers with a higher price. They also find that, relative to the random search case, prominence maximizes industry profits but reduces consumer surplus, although prominence reduces the average search cost. In Moraga-González et al. (2021), consumers browse sellers' offers simultaneously. The authors find that prominence can increase consumer surplus even though the prominent seller increases its price by focusing on the least elastic segment of the market, thereby letting non-prominent sellers price aggressively to serve those consumers searching for an alternative. We differ from these studies in that the intermediary endogenously allocates prominence as it grants more visibility in order to maximizing its profits.

A more recent stream of the literature has focused on prominence allocation as a result of payments (e.g., auctions or direct payments) (Armstrong & Zhou, 2011; Bourreau & Gaudin, 2022; Ciotti, 2023; Inderst & Ottaviani, 2012; Krämer & Zierke, 2020; Long, Jerath, & Sarvary, 2022; Raskovich, 2007; Teh & Wright, 2022). Armstrong and Zhou (2011) show that, by granting prominence to the highest bidder, sellers' marginal costs are artificially increased and so the overall retail prices at the benefit of the intermediary.[3] In Inderst and Ottaviani (2012), the intermediary allocates prominence, balancing two tensions: the commission it receives from sellers and the reputation cost the intermediary incurs whenever it recommends a non-suitable product. Teh and Wright (2022), study a setting in which the intermediary's profits depend both on the margin sellers leave to obtain prominence and the commission they pay for each item sold. Thus, sellers compete both in prices and in commissions to obtain prominence. The authors find that steering harms both firms and consumers, and fiercer competition exacerbates price inflation due to the higher incentive of being recommended. Long et al. (2022) analyze how an intermediary can use the bidding system for sponsoring sellers to infer their private quality information and improve matching. Unlike this literature, the intermediary does not allocate prominence through auctions or direct payments in our setting. We show that it is in the interest of the intermediary to maximize the number of transactions by employing a non-monetary approach where prominence serves as a tool to foster competition among retailers.

Our analysis also relates to the search literature that deals with heterogeneous consumers. The population of active/passive consumers we study share some similarities with the taxonomy shoppers/captives introduced by Varian (1980). Mixed equilibria results generally arise in these studies because of the presence of captive consumers and sellers offering homogeneous products (Armstrong & Vickers, 2022; Armstrong & Zhou, 2011; Johnen & Ronayne, 2021; Ronayne &

[3] In their setting, as in ours, there are two groups of consumers: informed ones, who costlessly compares sellers' offers, and uniformed ones, who only consider the offer of the default vendor. However, in our setting, prominence lowers retailers' prices. As the intermediary obtains a commission fee for each transaction, it is interested in allocating prominence to the retailers that lower prices and expand the total demand.

Taylor, 2021).[4] In our model, passives are not locked into a single firm as the captives. Instead, their consumption set is bounded by the recommendation of the intermediary. This difference plays an important role, as retailers do not have a fallback line if competition for the passive segment of the market is too intense. Therefore, when both retailers compete for prominence, the pricing strategies follow a downward Bertrand spiral, and retailers cannot profitably deviate by setting a higher price. As a result, the market dynamic we study leads to an equilibrium in pure strategy.[5,6]

The papers most closely related to our research are Dinerstein, Einav, Levin, and Sundaresan (2018) and Bar-Isaac and Shelegia (2022). In Dinerstein et al. (2018), the authors present a theoretical model where an online intermediary optimizes the probability of displaying a retailer prominently to consumers. Like our study, they consider both the product's intrinsic quality and, depending on the platform's design, its price as factors influencing prominence. The authors then use their model in a structural framework to estimate the welfare gains resulting from changes in eBay's platform design. In contrast to their work, our research focuses on the impact of competition for prominence schemes on retailers' strategies. We specifically analyze how the allocation of prominence affects market outcomes and compare this prominence scheme with alternative allocation methods. In contrast to their work, we focus on the impact of competition for prominence schemes on vendors' strategies and compare the market outcomes under this prominence allocation with those arising in alternative schemes.

More related to the spirit of our paper, Bar-Isaac and Shelegia (2022) compare the intermediary's advantages of allocating prominence via an auction or via an algorithm.[7] The authors study different separate scenarios and show that depending on their assumptions (e.g., symmetric/asymmetric sellers, homogeneous/heterogeneous consumers, single/multiple markets), the intermediary may prefer either steering method. Our paper, instead, focuses on how a competition for prominence scheme affects the retailers' strategies and welfare for both an exogenous and endogenous commission fee. We show that a richer set of alternative equilibria can emerge when considering the commission fee as given. Moreover, our approach allows us to consider all the different sources of heterogeneity, which the authors instead study separately,

[4] An exception is given by Myatt and Ronayne (2023). The authors show the existence of pure strategy equilibrium in a sequential game where firms sell homogeneous products but have asymmetric marginal costs.

[5] Note that with a high marginal cost for the high-quality retailer, a mixed strategy equilibria might exist, as discussed in the Appendix.

[6] More broadly, our paper also relates to the stream of studies focusing on the problem of steering in the platform's also acting as retailers (Anderson & Bedre-Defolie, 2021, 2022; de Cornière & Taylor, 2019; Etro, 2021; Hagiu, Teh, & Wright, 2022; Zennyo, 2022), how platforms govern their ecosystem (Teh, 2022), as well as how they influence intra-platform competition (Casner, 2020; Jeon, Lefouili, & Madio, 2022; Karle, Peitz, & Reisinger, 2020). In our analysis, we abstract away from the issue of self-preferencing, which has been studied extensively in recent years (Chen & Tsai, 2023; Cure, Hunold, Kesler, Laitenberger, & Larrieu, 2022; Hunold, Laitenberger, & Thébaudin, 2022; Lee & Musolff, 2021).

[7] As in our prominence allocation rule, their algorithm guides the intermediary in assigning prominence in order to maximize profits. However, in their setting, the algorithm assigns prominence to the retailer setting the price closest to the monopoly price, as the intermediary can fully extract the retailers' revenues through an ad valorem fee.

and find pure strategy equilibria. Finally, in an extension, we also compare the two steering methods. Similarly to Bar-Isaac and Shelegia (2022), we show that auction-based auctions generate and exacerbate double marginalization problems. However, by taking into account the different source of heterogeneity in the market, we show that the intermediary never prefers an auction-based prominence allocation approach.

3 Model Setup

We consider a market in which two retailers sell a product via a monopolist intermediary and pay a fixed and symmetric per-unit commission fee w to the latter.[8] We assume that retailers are differentiated in the (exogenous) quality of their ancillary services. For instance, one retailer provides a better customer experience than the other (e.g., a generous return policy or faster delivery). For ease of exposition, throughout the paper we refer to the retailer with the superior service as the high-quality retailer and the retailer with the inferior service as the low-quality retailer. Moreover, with a slight abuse of notation, we often refer to them as H and L, respectively, and the associated quality is q_H and q_L, with $0 < q_L < q_H$.

In our analysis, we also assume that the difference in the quality of the ancillary services of the two retailers is not too high, i.e., $q_H < 2q_L$. This assumption ensures that the low-quality retailer is a potential competitor of the high-quality retailer.[9] We assume that H and L have identical marginal production and service costs, which are normalized to 0.[10]

We consider a market in which consumers can only purchase products through the intermediary and have varying preferences for increases in quality, represented by a parameter $\theta \in [\underline{\theta}, \overline{\theta}]$ that is uniformly distributed in the range $[0, 1]$. Consumers have unit demand and their population is divided into two segments: *active* consumers who account for a share $\lambda \in (0, 1)$ of the total population and have access to all product listings, and *passive* consumers who account for the remaining share $1 - \lambda$ and only consider the product that the intermediary prominently displays.[11] We assume θ to be independently and identically distributed across both segments of consumers.

An active consumer that buys a product from retailer $i = \{L, H\}$ at the price p_i obtains the

[8] We discuss asymmetric commission fees at the end of Section 4.3.

[9] Relaxing this assumption would make the low-quality retailer unable to compete with the high-quality one, thereby creating a typical vertical chain. We abstract from this less interesting case in our analysis by assuming $q_H < 2q_L$.

[10] In the Appendix, we allow for the possibility that the high-quality retailer faces a higher marginal service cost due, e.g., to logistics and the implementation of the return policy. In this case, we discuss that our results do not hold qualitatively.

[11] This segmentation can be motivated by assuming that active consumers have a low cost of searching and compare all product listings before making a purchase decision, whereas passive consumers have a higher search cost and are more likely to rely on the prominent offer.

following utility:

$$U_i^\lambda(p_i) = \theta q_i - p_i \qquad \forall i = H, L,$$

whereas a passive consumer that buys a product from the prominent retailer obtains:

$$U^{1-\lambda}(p) = \theta q - p,$$

where q and p represent, respectively, the quality and the price of the prominent retailer identified by the intermediary, i.e., $p = p_L$ and $q = q_L$ if the low-quality retailer is prominence, otherwise $p = p_H$ and $q = q_H$.

The intermediary imposes a fixed commission fee w on retailers for every transaction that occurs. We denote as $D_i^\lambda(p_i, p_j, w)$ the demand of a retailer i from the active consumers and as $D_H^{1-\lambda}(p_H)$ (resp. $D_L^{1-\lambda}(p_L)$) the demand of the high-quality (resp. low-quality) retailer from passive consumers if made prominent. If the high-quality retailer is prominently displayed, the profits of the high- and low-quality retailers are given by:

$$\begin{aligned}
\pi_H^H(p_H, p_L, w) &= \Big[\lambda D_H^\lambda(p_H, p_L) + (1-\lambda) D_H^{1-\lambda}(p_H)\Big](p_H - w),\\
\pi_L^H(p_L, p_H, w) &= \Big[\lambda D_L^\lambda(p_L, p_H)\Big](p_L - w),
\end{aligned}$$

and the profit of the intermediary is:

$$\Pi^H(p_H, p_L, w) = w\Big[\lambda\Big(D_H^\lambda(p_H, p_L) + D_L^\lambda(p_L, p_H)\Big) + (1-\lambda) D_H^{1-\lambda}(p_H)\Big].$$

If the low-quality retailer is prominently displayed instead, the profits of the high- and low-quality retailers are given by:

$$\begin{aligned}
\pi_H^L(p_H, p_L, w) &= \Big[\lambda D_H^\lambda(p_H, p_L)\Big](p_H - w),\\
\pi_L^L(p_L, p_H, w) &= \Big[\lambda D_L^\lambda(p_L, p_H) + (1-\lambda) D_L^{1-\lambda}(p_L)\Big](p_L - w),
\end{aligned}$$

and the profit of the intermediary is:

$$\Pi^L(p_H, p_L, w) = w\Big[\lambda\Big(D_H^\lambda(p_H, p_L) + D_L^\lambda(p_L, p_H)\Big) + (1-\lambda) D_L^{1-\lambda}(p_L)\Big].$$

The timing of the game is as follows. In the first stage, the intermediary sets the commission fee w. In the second stage, retailers compete by setting prices p_H and p_L, respectively. In the third stage, the intermediary allocates prominence to the retailer that maximizes its total profit. In the final stage, active consumers compare the offers of the two retailers and decide whether to buy and from which retailer, whereas passive consumers decide whether to buy from the prominent retailer. Our solution concept is subgame perfect Nash equilibrium.

4 Analysis

In this section, we present the analysis of the baseline model. First, we solve the model for a given commission fee w and study the best replies of the two retailers. Second, we endogenize the commission fee and solve the first stage of the game.

4.1 Analysis for a given commission fee

We start by analyzing the consumer's decision in the last stage of the game. An active consumer type-θ buys from retailer i if, and only if, $U_i^\lambda(p_i) \geq \max\{U_{\text{-}i}^\lambda(p_{\text{-}i}), 0\}$. We denote as $\overline{\theta}(p_L, p_H)$ the consumer who is indifferent between buying from the high- or the low-quality retailer, i.e., $\overline{\theta}(p_L, p_H)q_H - p_H = \overline{\theta}(p_L, p_H)q_L - p_L$. Similarly, we denote as $\underline{\theta}(p_L, p_H)$ the consumer who is indifferent between buying from the low-quality retailer or not buying, i.e., $\underline{\theta}(p_L, p_H)q_L - p_L = 0$. Note that for $q_L/p_L > q_H/p_H$, all consumers of type $\overline{\theta} > \theta > \underline{\theta}$ buy from the low-quality retailer, whereas all consumers of type $\theta > \overline{\theta}$ buy from the high-quality retailer. The masses of active consumers buying from the two retailers are:

$$D_L^\lambda(p_L, p_H) = \max\left\{0, \frac{p_H - p_L}{q_H - q_L} - \frac{p_L}{q_L}\right\}, \qquad D_H^\lambda(p_H, p_L) = \min\left\{1 - \frac{p_H - p_L}{q_H - q_L}, 1 - \frac{p_H}{q_H}\right\}. \quad (1)$$

A passive consumer type-θ buys from retailer i if, and only if, $U^\lambda(p) \geq 0$. Therefore, the mass of passive consumers that buy from the prominent retailer is either:

$$D_L^{1-\lambda}(p_L) = 1 - \frac{p_L}{q_L}, \quad \text{or} \qquad D_H^{1-\lambda}(p_H) = 1 - \frac{p_H}{q_H}, \quad (2)$$

depending on whether the low-quality retailer or the high-quality retailer is shown prominently.

In the third stage, the intermediary allocates prominence to the retailer that, conditional on receiving a prominent display, maximizes the profit for the intermediary. Since we consider a per-unit fee, this is equivalent to maximizing the number of transactions and, therefore, the consumer demand. As a tie-breaking rule, we assume that, in any case of indifference in assigning prominence to either retailer, the intermediary assigns it to the high-quality retailer.

To derive the prominence allocation rule, we first determine the demands of the two retailers in each market segment. Let us first consider the active segment. If $p_H/q_H > p_L/q_L$, then $(p_H - p_L)/(q_H - q_L) > p_L/q_L$ and, therefore, demands are given by:

$$D_L^\lambda(p_L, p_H) = \frac{p_H - p_L}{q_H - q_L} - \frac{p_L}{q_L}, \qquad D_H^\lambda(p_H, p_L) = 1 - \frac{p_H - p_L}{q_H - q_L}.$$

Otherwise, if $\frac{p_H}{q_H} \leq \frac{p_L}{q_L}$, then $\frac{p_H - p_L}{q_H - q_L} \leq \frac{p_L}{q_L}$ and, therefore, the demands are given by:

$$D_L^{\lambda}(p_L, p_H) = 0, \qquad D_H^{\lambda}(p_H, p_L) = 1 - \frac{p_H}{q_H}.$$

Let us consider now the passive segment. If H is prominent, demands are given by:

$$D_L^{1-\lambda}(p_L, p_H) = 0, \qquad D_H^{1-\lambda}(p_H, p_L) = 1 - \frac{p_H}{q_H}.$$

On the contrary, if L is prominent, demands are given by:

$$D_L^{1-\lambda}(p_L, p_H) = 1 - \frac{p_L}{q_L}, \qquad D_H^{1-\lambda}(p_H, p_L) = 0.$$

We can now examine the profit function of the intermediary in each scenario. If $p_H/q_H > p_L/q_L$, the intermediary obtains the following profits by assigning prominence to L and H, respectively:

$$\begin{aligned}
\Pi^L(p_H, p_L, w) &= \left(1 - \frac{p_L}{q_L}\right)w, \\
\Pi^H(p_H, p_L, w) &= \lambda\left(1 - \frac{p_L}{q_L}\right)w + (1-\lambda)\left(1 - \frac{p_H}{q_H}\right)w.
\end{aligned}$$

and prominence is awarded to L if:

$$\begin{aligned}
\Pi^H(p_H, p_L, w) - \Pi^L(p_H, p_L, w) &= \lambda\left(1 - \frac{p_L}{q_L}\right)w + (1-\lambda)\left(1 - \frac{p_H}{q_H}\right)w - \left(1 - \frac{p_L}{q_L}\right)w \\
&= w(1-\lambda)\left(\frac{p_L}{q_L} + \frac{p_H}{q_H}\right) < 0.
\end{aligned}$$

If $p_H/q_H \leq p_L/q_L$, the intermediary obtains the following profits by assigning prominence to L and H, respectively:

$$\begin{aligned}
\Pi^L(p_H, p_L, w) &= \lambda\left(1 - \frac{p_H}{q_H}\right)w + (1-\lambda)\left(1 - \frac{p_L}{q_L}\right)w, \\
\Pi^H(p_H, p_L, w) &= \left(1 - \frac{p_H}{q_H}\right)w.
\end{aligned}$$

and prominence is awarded to H if:

$$\begin{aligned}
\Pi^H(p_H, p_L, w) - \Pi^L(p_H, p_L, w) &= \left(1 - \frac{p_L}{q_L}\right)w - \left[\lambda\left(1 - \frac{p_H}{q_H}\right)w + (1-\lambda)\left(1 - \frac{p_L}{q_L}\right)w\right] \\
&= w(1-\lambda)\left(\frac{p_L}{q_L} + \frac{p_H}{q_H}\right) \geq 0.
\end{aligned}$$

Simplifying the above expressions, the following lemma presents the conditions for which prominence is assigned to the high-quality or low-quality retailer.

Lemma 1. *The intermediary assigns prominence to the high-quality (resp. low-quality) retailer if:*

$$\frac{p_H}{q_H} \leq (>) \frac{p_L}{q_L}. \tag{3}$$

Lemma 1 means that the retailer with the lowest quality-adjusted price is granted prominence. Therefore, if the high-quality retailer wants to be prominent, it must match the quality-adjusted price of its low-quality competitor. One important implication of this lemma is that the competition for prominence scheme leads to more competition in the market, as it induces retailers to lower their prices, for a given unit fee, in order to be granted prominence.[12]

One implication of (3) is that, as we have already shown, the high-quality retailer can eliminate its competitor from the market by setting its price to $p_H = p_L q_H / q_L$. In other words, evaluating the demand of the low-quality retailer at this price, we observe that $D_L^\lambda(p_L, p_H) = 0$, for any $p_L \geq w$. On the contrary, the low-quality retailer can never exclude its competitor. This is because the quality advantage makes the high-quality retailer attract some active consumers even when losing access to the passive segment. The following lemma summarizes this finding.

Lemma 2. *If the high-quality retailer is prominent, the low-quality retailer makes no sales (excluding equilibrium). If the low-quality retailer is prominent, both retailers make sales (market sharing equilibrium).*

To define the retailers' pricing strategies and identify the emerging equilibrium(a), we determine the retailers' best replies in the second stage of the game.

Retailers anticipate the allocation of prominence made by the intermediary and compete in prices. As shown in Lemma 2, if the high-quality retailer is prominent, the low-quality retailer makes no sales. Therefore, conditional on the high-quality retailer setting a price that makes it prominent, the low-quality retailer always has the incentive to undercut the quality-adjusted price of its rival. This undercutting strategy can be blocked in two scenarios. Firstly, if L reaches its price floor, equal to the commission fee, and H finds it optimal to become prominent. Secondly, if H finds it optimal not to serve the passive consumers, thereby focusing only on the active ones, the market will reach a market sharing equilibrium.

In what follows we report and discuss the best-responses of the two retailers. For ease of exposition, we use $\hat{p}_H$, $\hat{p}_L$ and $\tilde{p}_L$ (which are defined in the Appendix), where $\tilde{p}_L > \hat{p}_L > w$,

[12] Note that considering a different commission fee, such as an ad-valorem fee, would imply a different prominence allocation rule, and significantly different retailers' strategies. Specifically, the intermediary would account for the profits on its intensive and extensive margins from both retailers. On the other hand, retailers willing to obtain prominence would need to set prices close to the monopoly level. However, this price choice would allow the rival to easily capture the active segment by undercutting the quality-adjusted price of the prominent firm. Depending on the quality difference and the size of the active segment, the prominent retailer may not be willing to let go the actives. Therefore, this would create a setting similar to Varian (1980), in which equilibria in pure strategies do not exist.

to represent those threshold values for which the best replies are feasible given the prominence allocation rule. The best reply of the low-quality retailer is:

$$p_L(p_H) = \begin{cases} \frac{\lambda q_L(p_H+w-(q_H-q_L))+(q_H-q_L)(q_L+w)}{2(q_H-(1-\lambda)q_L)}, & \text{if } \; p_H \geq \hat{p}_H \\ p_H\frac{q_L}{q_H} - \varepsilon, & \text{if } \; w\frac{q_L}{q_H} < p_H < \hat{p}_H \\ w, & \text{if } \; p_H \leq w\frac{q_H}{q_L} \end{cases} \tag{4}$$

Specifically, the high-quality retailer loses prominence if its price is sufficiently high, i.e., $p_H \geq \hat{p}_H$. Therefore, the price set by the low-quality retailer ensures a market sharing equilibrium, i.e., it is equal to the one identified in the first line of (4). On the other hand, if the price set by the high-quality retailer is intermediate, i.e., $w\frac{q_L}{q_H} < p_H < \hat{p}_H$, the low-quality retailer can obtain prominence only by undercutting the high-quality retailer by an $\varepsilon \to 0$, resulting in the following best reply: $p_L(p_H) = p_H q_L/q_H - \varepsilon$. Finally, suppose the price of the high-quality retailer is sufficiently low, i.e., $p_H \leq wq_H/q_L$. In that case, it is not possible for the low-quality retailer to undercut the rival and become prominent. In the latter case, the best this retailer can do is to set a price equal to its marginal cost — the fee set by the intermediary w — and get zero profit.

The best reply of the high-quality retailer is:

$$p_H(p_L) = \begin{cases} \frac{1}{2}(p_L + q_H - q_L + w), & \text{if } \; p_L < \tilde{p}_L \\ p_L\frac{q_H}{q_L}, & \text{if } \; \tilde{p}_L \leq p_L < \hat{p}_L \\ \frac{q_H+w}{2}, & \text{if } \; p_L \geq \hat{p}_L \end{cases} \tag{5}$$

If the price set by its low-quality rival is sufficiently low, i.e., less than $\tilde{p}_L$, the high-quality retailer gives up on prominence, and a market sharing outcome can emerge with the high-quality retailer setting $p_H = (1/2)(p_L + q_H - q_L + w)$. If the low-quality sets an intermediate price, the high-quality retailer can obtain prominence by just matching the rival's quality-adjusted price, i.e., $p_H(p_L) = p_L q_H/q_L$. Finally, if the price of the low-quality retailer is high enough, the high-quality retailer can secure prominence by setting $p_H = (q_H + w)/2$.

In the Appendix, we show that the market sharing equilibrium can be sustained with the following prices only under some circumstances. Formally,

$$\begin{aligned} p_L(w) &= \frac{\lambda q_L(3w - q_H + q_L) + 2(q_H - q_L)(q_L + w)}{4q_H - (4 - 3\lambda)q_L}, \\ p_H(w) &= \frac{2q_H^2 - (3 - \lambda)q_H q_L + 3q_H w + (1 - \lambda)q_L(q_L - 3w)}{4q_H - (4 - 3\lambda)q_L}. \end{aligned} \tag{6}$$

under the conditions that $p_L < \tilde{p}_L$ and $p_H \geq \hat{p}_H$ (first line in Equation 4). This equilibrium is possible if and only if the following two conditions jointly hold:

$$(a) \quad q_L > 2w, \qquad (b) \quad \tilde{\lambda} \equiv \frac{2q_H w + q_L^2 - 3q_L w}{q_H q_L + q_L^2 - 3q_L w} < \lambda < 1. \tag{7}$$

In other words, when the share of active consumers is significant (represented by $\lambda > \tilde{\lambda}$), the size of the passive segment is small, and the value attached to obtaining prominence becomes relatively low for both retailers. In this situation, the high-quality retailer is less motivated to aggressively compete for prominence as it can still achieve substantial profits by allowing its competitor to cater to the passive segment. As a result, the low-quality retailer serves part of the active and passive segments, whereas the high-quality retailer only serves the (large) active segment without any further deviation. This condition leads to a market sharing equilibrium, where both retailers coexist without engaging in intense competition for the passive consumers.

Figure 1 illustrates the existence of market sharing equilibrium for a given w in the case in which $\lambda > \tilde{\lambda}$ and $q_L > 2w$. In this scenario, the equilibrium results from the intersection of the blue line — the best reply of the low-quality retailer — and the orange line — the best reply of the high-quality retailer.

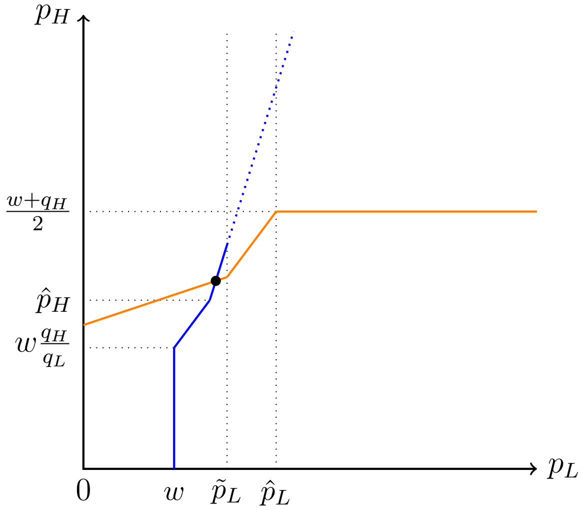

Figure 1: Market sharing equilibrium ($w = \frac{1}{5}$, $q_L = \frac{3}{4}$, $q_H = \frac{3}{2}$, $\lambda = \frac{5}{7} > \tilde{\lambda} = \frac{19}{33}$).

If conditions in (7) are not satisfied, the high-quality retailer is prominent, and the low-quality retailer is excluded from the market. In this situation, two possible alternative equilibria can arise. The first equilibrium arises because of the Bertrand-like downward spiral of prices (illustrated in Figure 2). This equilibrium involves the high-quality retailer setting a very low price, precisely $p_H = wq_H/q_L$, to match the low-quality retailer's quality-price ratio to retain the prominent position and attract all the active consumers. The second equilibrium(illustrated in Figure 3) involves the high-quality retailer setting a monopoly price, as indicated in the third line of Equation (5). In the Appendix, we show that the first equilibrium emerges when the price of the low-quality retailer is still relatively low, which, in turn, is the case if the commission fee w is sufficiently small, i.e., $w \leq (q_H q_L)/(2q_H - q_L)$. The excluding equilibrium prices in this scenario are $p_H = wq_L/q_L$ and $p_L = w$, respectively. The second equilibrium emerges instead when the fee is sufficiently large, which leads to a higher price for the low-quality retailer. In this case, the excluding equilibrium prices are $p_H = (q_H + w)/2$ and $p_L = w$, respectively.

Figures 2 and 3 illustrate the existence of the two types of excluding equilibria. We restrict attention to the case in which $q_L < 2w$. For relatively low values of w (Figure 2), the excluding equilibrium is given by the intersection of the two best replies, which occurs as a consequence of a Bertrand downward spiral of prices. For relatively high values of w (Figure 3), instead, the excluding equilibrium is given by the crossing of the two best replies, and it occurs with the high-quality retailer setting a monopoly price. We summarize this discussion in the following proposition.

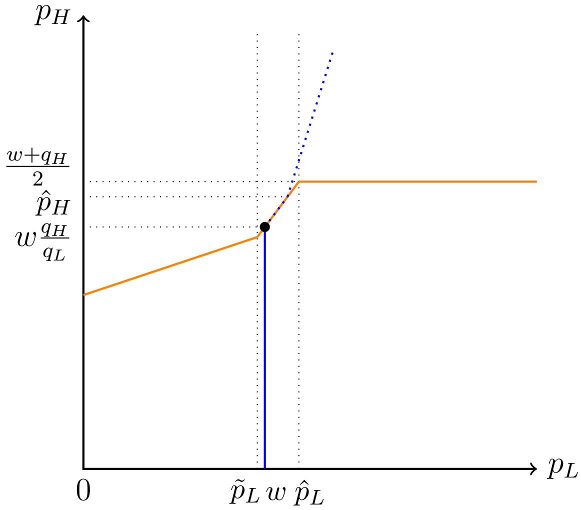

Figure 2: Excluding equilibrium with a low w ($w = \frac{2}{5}$, $q_L = \frac{3}{4}$, $q_H = \frac{3}{2}$, $\lambda = \frac{5}{7}$).

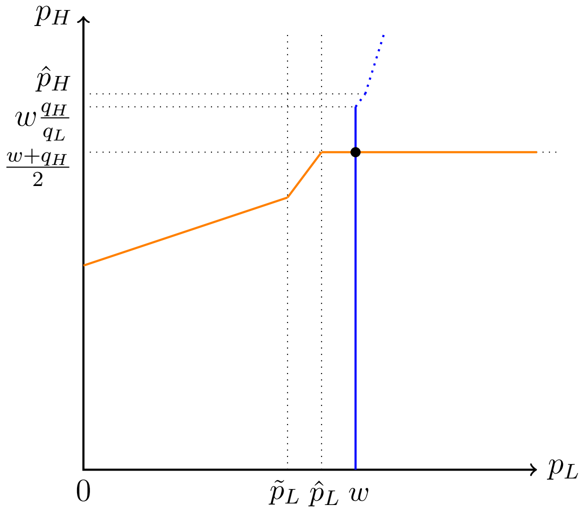

Figure 3: Excluding equilibrium with a high w ($w = \frac{3}{5}$, $q_L = \frac{3}{4}$, $q_H = \frac{3}{2}$, $\lambda = \frac{5}{7}$).

Proposition 1. *If $q_L > 2w$ and $\lambda > \tilde{\lambda}$, there exists a unique equilibrium in which the low-quality retailer is prominent whereas the high-quality retailer serves part of the active consumers ("market sharing equilibrium"); prices are given by* (6)*. Otherwise, there is a unique equilibrium ("excluding equilibrium") such that the high-quality retailer is prominent and serves all consumers with a price of $p_H = wq_L/q_L$ if $w \leq (q_H q_L)/(2q_H - q_L)$ or with a price of $p_H = (q_H + w)/2$ if $w > (q_H q_L)/(2q_H - q_L)$, whereas the low-quality retailer sets $p_L = w$.*

A graphical summary of Proposition 1 is illustrated in Figure 4, which shows the equilibrium outcomes as a function of the size of the active segment, λ (on the y-axis), and the exogenous commission fee w (on the x-axis). The dark blue area represents the parameter constellation in which a market sharing equilibrium occurs. In contrast, the mild blue and the light blue areas represent those where excluding equilibria arise. The orange line represents the function of $\tilde{\lambda}$ for a given w.

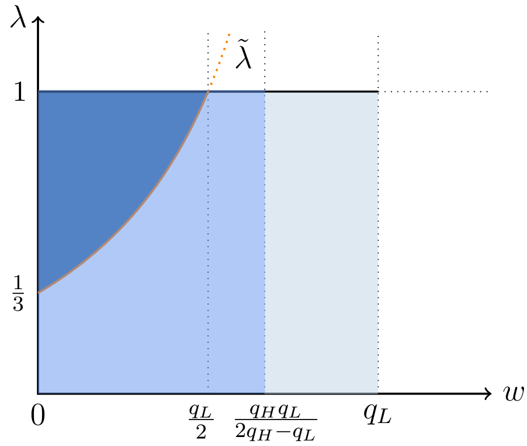

Figure 4: Summary of the equilibria as a function of λ and w ($q_L = \frac{3}{4}$, $q_H = \frac{3}{2}$).

Note that these results hold for any given commission fee, which can rationalize scenarios in which fees are a long-run commitment of the platform and, therefore, not adjusted depending on each product type. Alternatively, one can think about a marketplace with multiple product categories and homogeneous commission fees across categories. In this scenario, the intermediary is limited in adjusting the fee depending on the competition for a given product.

4.2 Welfare effects

Now, we can investigate whether the different equilibria maximize social welfare compared to the other cases within the parameter range where they exist. We designate the three equilibria as follows: the market sharing equilibrium is denoted by the superscript L, the first excluding equilibrium is denoted by $H1$, and $H2$ denotes the second excluding equilibrium.

We start by comparing the profit of the intermediary in the market sharing and the first excluding equilibrium. Note that, in this section, the intermediary plays a passive role and cannot induce any of the equilibria identified by optimally setting the fee. First, we note that:

$$\begin{aligned}\Pi^{L}(\cdot) \equiv & \frac{\lambda q_L w(q_H + 2q_L - 3w) + 2w(q_H - q_L)(q_L - w)}{q_L(4q_H - (4 - 3\lambda)q_L)} < w\frac{(q_L - w)}{q_L} \equiv \Pi^{H1}(\cdot) \\ \Leftrightarrow & \frac{w(q_H - q_L)((\lambda - 2)q_L + 2w)}{q_L(4q_H + (3\lambda - 4)q_L)} < 0 \\ \Leftrightarrow & 2w - q_L(2 - \lambda) < 0,\end{aligned} \tag{8}$$

which is always the case given that $q_L > 2w$ is a necessary condition for the market sharing equilibrium to exist. Therefore, if a market sharing equilibrium arises, the intermediary is worse off relative to the first excluding equilibrium, meaning that the market sharing equilibrium is never the most profitable choice for the intermediary in the parameter range in which this equilibrium occurs. Now, we compare the profit of the intermediary in the two excluding equilibria and show that:

$$\begin{aligned}\Pi^{H2}(\cdot) \equiv & w - \frac{w(q_H - w)}{2q_H} > w\frac{(q_L - w)}{q_L} \equiv \Pi^{H1}(\cdot) \\ \Leftrightarrow & w > \frac{q_H q_L}{2q_H - q_L},\end{aligned} \tag{9}$$

and the opposite otherwise. Based on Proposition (1), the second (resp. first) excluding equilibrium occurs if $w > (\leq)(q_H q_L)/(2q_H - q_L)$. Therefore, if any excluding equilibrium arises, for a given fee, it will always maximize the intermediary's profit.

As demonstrated with Lemma (1), the intermediary's prominence allocation strategy maximizes consumer demand. Consequently, the interests of the intermediary are inherently aligned with those of the consumers. As a result, the market sharing equilibrium is also never desirable for consumers, whereas the two excluding equilibria are desirable when they emerge. In essence, the market sharing equilibrium leads to suboptimal outcomes for consumers, as it fails to fully exploit competitive pricing and the benefits of differentiation between retailers. On the other hand, the two excluding equilibria, despite potentially restricting the choices for consumers, enhance market efficiency and lower prices, leading to more favorable consumer outcomes overall.

As of the retailers, it is clear that the low-quality retailer would never find it optimal to be in any of the excluding equilibria, whereas the high-quality retailer would find it optimal to be in the second excluding equilibrium because of the monopoly-like price being set. We compare welfare in the relevant parameter ranges in the Appendix and conclude the following.

Proposition 2. *For a given fee, the market sharing equilibrium is never socially desirable. The excluding equilibria are socially desirable in the parameter ranges they emerge.*

4.3 Endogenous commission fee

So far, we have considered the retailers' strategies for a given commission fee. We now turn our attention to the first stage of the game and analyze the optimal commission fee for the intermediary. From (8), it is immediate that the intermediary would never prefer to induce a market sharing equilibrium. Therefore, we are left to compare the intermediary's profit in the two excluding equilibria when the commission fee is optimally set.

In the first excluding equilibrium, the optimal fee is $w^{H1} = q_L/2$, and the associated equilibrium profit of the intermediary is $\Pi^{H1}(\cdot) = q_L/4$.

In the second excluding equilibrium, the profit of the intermediary is $\Pi(p_L(w), p_H(w), w) = w - w(q_H + w)/2q_H$, and the optimal fee is $w^{H2} = q_H/2$. The associated equilibrium profit of the intermediary is $\Pi^{H2}(\cdot) = q_H/8$.

We can observe that in the two cases, the fees are greater or equal than $q_L/2$. Therefore, the intermediary can naturally avoid a market sharing equilibrium by optimally setting the commission fee in either excluding case. Because of (8), we can therefore state the following lemma.

Lemma 3. *The intermediary always finds it optimal not to induce the emergence of a market sharing equilibrium.*

Comparing the intermediary's profit in the two excluding equilibria at their optimal fee, we obtain $\Pi^{H2}(\cdot) \equiv q_H/8 < q_L/4 \equiv \Pi^{H1}(\cdot)$ because $q_L > 2q_H$ by assumption. Therefore, the intermediary finds it optimal to induce the first excluding equilibrium in which the high-quality retailer competes fiercely for prominence, and the low-quality retailer cannot make any sales. We summarize this result with the following proposition.

Proposition 3. *Under the assumption that $q_L > 2q_H$, the equilibrium pair of prices $(p_L, p_H) = (w, wq_H/q_L)$ is subgame perfect. In equilibrium, the intermediary imposes a fee equal to $w = q_L/2$, which induces the high-quality to set a price equal to $p_H = wq_H/q_L$, gain prominence, and consequently, serve the entire market.*

Proposition 3 shows that, in equilibrium, the intermediary always finds it optimal to trigger an intense competition between the two retailers, thereby inducing a Bertrand-like spiral. This approach mitigates the double marginalization problem that would otherwise arise in the second excluding equilibrium.

This finding depends on the assumption that $q_L > 2q_H$, which ensures that the low-quality retailer can compete fiercely in the market. Note that this condition results from the fact that $q_L \geq w$, where w represents the highest fee set by the intermediary ($w^{H2} = q_H/2$). This condition holds if and only if $q_L > 2q_H$. Notably, failure to meet this condition would lead to a significant quality differential between the two retailers, prompting the intermediary to have an ex ante incentive to exclude the low-quality retailer by inducing the second excluding equilibrium. In such a scenario, the market structure would resemble a classic vertical chain, exacerbating the double marginalization problem. It is important to notice that this outcome would be equivalent to one in which the intermediary excludes the low-quality retailer, for example, as a result of an initial screening. In such a case, no competition for prominence would take place.

In equilibrium, therefore, the profit of the intermediary and of the retailers are;

$$\Pi^{H1}(\cdot) = \frac{q_L}{4}, \qquad \pi_H^{H1}(\cdot) = \frac{q_H - q_L}{4}, \qquad \pi_L^{H1}(\cdot) = 0, \tag{10}$$

whereas consumer surplus, denoted as $CS^{H1}(\cdot)$, and social welfare, denoted as $W^{H1}(\cdot)$, are:

$$CS^{H1}(\cdot) = \frac{1}{8}q_H, \qquad W^{H1}(\cdot) = \frac{3}{8}q_H. \tag{11}$$

It is worth highlighting that the intermediary, through its strategic selection of the equilibrium via the choice of the fee, achieves identical total industry profit, consumer surplus, and welfare to that of a multiproduct monopolist — a hypothetical entity controlling the entire distribution and pricing of products. To understand why, consider the following game in which a monopolist sells both high and low-quality offers. Since the intermediary also distributes its product, no commission fee is paid, meaning that $w = 0$. In the first stage of the game, the monopolist sets prices for both offers and chooses which one to display prominently, whereas in the second stage, consumers make their purchasing decisions.

The multiproduct monopolist has the following profit function if its low-quality offer is prominent:

$$\Pi^M(p_H, p_L) \equiv \lambda\Big(1 - \frac{p_H - p_L}{q_H - q_L}\Big)p_H + \Big[\lambda\Big(\frac{p_H - p_L}{q_H - q_L} - \frac{p_L}{q_L}\Big) + (1-\lambda)\Big(1 - \frac{p_L}{q_L}\Big)\Big]p_L,$$

and the following profit function if its high-quality offer is prominent:

$$\Pi^M(p_L, p_H) \equiv \Big[\lambda\Big(1 - \frac{p_H - p_L}{q_H - q_L}\Big) + (1-\lambda)\Big(1 - \frac{p_H}{q_H}\Big)\Big]p_H + \lambda\Big(\frac{p_H - p_L}{q_H - q_L} - \frac{p_L}{q_L}\Big)p_L.$$

Regardless of which offer is made prominent, the multiproduct monopolist would choose the following pair of monopoly prices for the two products $(p_H^M, p_L^M) = (q_H/2, q_L/2)$. Intuitively, given prices, the monopolist would maximize total demand by assigning prominence to its high-quality offer. As in the baseline model, this pair of prices would crowd out demand for the low-quality retailer.[13] Interestingly, the profit of the intermediary at equilibrium is equal to the total industry profit of the baseline model (i.e., $\Pi^{H1}(\cdot) + \pi_H^{H1}(\cdot)$):

$$\Pi^M(p_H^M, p_L^M) = \frac{q_H}{4}.$$

Consumer surplus and welfare are given by:

$$CS^M(p_H^M, p_L^M) = \frac{q_H}{8}, \qquad W^M(p_H^M, p_L^M) = \frac{3}{8}q_H, \tag{12}$$

which are identical to (11). We can conclude the following.

Proposition 4. *The market outcome of a multiproduct monopolist is equivalent to the one emerging in equilibrium under competition for prominence.*

This proposition shows that the intermediary can still influence the prices set by the retailers in a way that replicates the behavior of a multiproduct monopolist despite lacking direct control over the strategies of the two retailers. Yet, the intermediary does not extract the entire surplus from the high-quality retailer, which could be possible only via price discrimination strategies. One possibility is to allow retailers that are ex ante symmetric to become asymmetric in their quality via ancillary services and extract the remaining surplus of the high-quality retailer via a fixed fee. We discuss this in the next section. Alternatively, the intermediary may set two different per-unit fees, i.e., w_L and w_H. These fees would induce the intermediary to always prefer the first excluding equilibrium and obtain monopoly profits by setting $w_L = q_L/2$ and $w_H = q_H/2$. This practice, however, would ring the bells of antitrust authorities. The unjustified discriminatory treatments among retailers would violate art. 102 of the Treaty of Functioning of the European Union.

5 Extensions

In this section, we explore two extensions of our model. Firstly, we consider a scenario where retailers are initially homogeneous, offering identical products and retailing services, but can upgrade their quality by purchasing ancillary services from the intermediary. Secondly, we present an alternative prominence allocation scheme for the intermediary, wherein retailers have the option to purchase prominence.

[13]Note that the presence of high- and low-quality offers does not imply quality discrimination among consumers with different tastes for quality, as at equilibrium, none of them finds it optimal to purchase the low-quality offer. This is in contrast to the model by Mussa and Rosen (1978).

5.1 Endogenous asymmetric quality

One way to microfound asymmetric quality between retailers is by considering that the intermediary offers ancillary services. This is common in many online marketplaces, where ancillary services are offered to enhance the consumer experience. These services may include improved logistics and fulfillment, enrollment in generous return policies, or access to round-the-clock customer service. For instance, in the case of Amazon, retailers can choose to enroll in the Fulfillment by Amazon (FBA) program by paying a fee or, alternatively, use their logistics independently (Fulfillment by Merchants).

In our modified setting, retailers are initially homogeneous with quality q_L. However, they can purchase a quality upgrade from the intermediary at a fixed fee of $F > 0$, which elevates their quality to q_H. In the first stage, the intermediary determines the fixed upgrading fee F and the commission fee w. In the second stage, retailers independently decide whether to purchase the quality upgrade from the intermediary. Then, in the third stage, retailers simultaneously set their prices, and finally, the intermediary allocates prominence to a retailer based on the established prominence allocation rule.

The following cases can now emerge.

(i) **No upgrade** (NU, NU). Both retailers do not purchase the quality upgrade. Because retailers stick with their ex ante homogeneous quality, their prices are the canonical Bertrand prices: the unique equilibrium is determined by the retailers' price floor, which in our setting is the per-unit fee w charged by the intermediary.

(ii) **Joint upgrade** (U, U). Both retailers purchase the quality upgrade. Therefore, retailers are ex ante and ex post homogeneous and, as a result, there is Bertrand competition. As in the previous scenario, retailers set a price equal to w and obtain zero profits.

(iii) **Asymmetric upgrade** (U, NU), (NU, U). Only one retailer purchase the quality upgrade. This case is consistent with the baseline setting in which there is vertical differentiation between retailers. However, the retailer with the higher quality incurs a fixed upgrading cost F. We denote the profits of the upgrading and non-upgrading retailers as π_H and π_L, respectively. Note that these profits, gross of the upgrading fee, are equivalent to those in the baseline setting for a given fee.

Independently of whether the asymmetric cases lead to one of the two excluding equilibria or a market sharing equilibrium, the game in Period 2 can be represented in the normal form, as depicted in Figure 5.

The symmetric scenarios, characterized by joint upgrade (U, U) or no upgrade (NU, NU), would drive profits to zero for any w. Hence, it becomes (weakly) dominant for retailers to adopt an asymmetric scenario, such as (U, NU) or (NU, U), where one retailer chooses to

		Retailer $-i$	
		U	NU
Retailer i	U	$0-F, 0-F$	$\pi_H - F, \pi_L$
	NU	$\pi_L, \pi_H - F$	$0, 0$

Figure 5: Upgrading decision stage.

upgrade while the other does not. As a result, a coordination problem arises, leading to the emergence of asymmetric upgrading as the equilibrium outcome. This finding implies that, in the equilibrium, there exist two asymmetric scenarios that involve vertical differentiation between retailers. These scenarios, where one retailer offers a higher quality product due to the upgrade, and the other sticks to the baseline quality, provide a microfoundation for our baseline model's assumptions[14]

Hereafter, the analysis is identical to the baseline model in which retailers are exogenously asymmetric in quality. As shown in Proposition 3, the intermediary sets in the first stage $w^{H1} = q_L/2$ and induces the first excluding equilibrium in which prominence is awarded to the high-quality retailer and Bertrand-like competition is triggered. In this scenario, the retailer that purchases the upgrade obtains prominence. Because the intermediary can use the fixed fee to extract surplus, it is intuitive that it would set it to an incentive-compatible level, i.e., $F = \pi_H^{H1}(p_H^{H1}, p_L^{H1}, w^{H1}, F) - \pi_L^{H1}(p_L^{H1}, p_H^{H1}, w^{H1}) = (q_H - q_L)/4$. With this fee, the intermediary can fully extract the surplus of the high-quality retailer and obtain the same profit that it would obtain if it were a multiproduct monopolist, i.e., $\Pi^{H1}(F^{\star}, p_H^{H1}, p_L^{H1}, w^{H1}) = q_H/4$ which is the same as in (12). We summarize the results in the following proposition.

Proposition 5. *An intermediary can extract the entirety of retailers' surplus and reach the profit level of a multiproduct monopolist by offering a quality upgrade and triggering competition for prominence.*

The above result has important policy implications and welfare effects. First, the fact that the intermediary can replicate the multiproduct monopolist's profit by using a two-part tariff ensures the largest rent extraction from retailers. Notably, retailers' surplus remains the same compared to when retailers are of low quality, and the intermediary does not offer a quality upgrade. Indeed, in such a case, retailers would ultimately receive zero surplus due to Bertrand-like competition.

Note that the redistribution of surplus from the retailers to the intermediary in the quality upgrade scenario benefits consumers. If retailers are of low quality and the intermediary does

[14]Note that a mixed strategy equilibrium exists in which retailers mix between purchasing or not purchasing the upgrade. However, this equilibrium is unstable and can be broken if retailers are allowed to communicate or make the upgrade decision non-simultaneously. Therefore, we will not consider the mixed strategy equilibrium in our analysis.

not offer a quality upgrade program, consumers would have to pay a price of $p_L = w = q_L/2$. In contrast, when the asymmetric upgrading scenario arises, purchasing consumers would pay a higher price, $p_H = q_H/2$, for a better service. The net effect is positive for consumers as the higher quality more than compensates the monetary losses, resulting in a higher consumer surplus:

$$CS_{(NU,NU)} \equiv \int_{\frac{p_L}{q_L}}^{1} (\theta q_L - p_L)\, d\theta = \frac{q_L}{8} < \frac{q_H}{8} = \int_{\frac{p_H}{q_H}}^{1} (\theta q_H - p_H)\, d\theta \equiv CS_{(U,NU),(NU,U)},$$

for any $q_H > q_L$. Therefore, competition for prominence and the endogenous quality upgrading program positively affect the intermediary and consumers, whereas retailers' surplus remains unchanged.

5.2 Alternative scheme: Auction-based prominence

In many e-commerce and online marketplaces, various strategies for product prominence allocation have emerged, each designed to enhance visibility and drive sales. One key approach is the one presented in the baseline model in Section 4, which reflects the main features of Amazon's buy-box. Another widely adopted business strategy involves assigning the most prominent position through an auction process. In this scenario, the retailer placing the highest bid is granted prominence. A notable example of a marketplace in the retailing sector that uses this approach is eBay. The auction allows retailers to gain a prominent display, thereby attracting a significant portion of clicks.[15]

In this section, we compare the prominence allocation scheme of Section 4 and an auction-based alternative. To this end, we develop a model in which the intermediary continues to monetize its intermediation service with the per-unit fee, w, but also makes profits directly from the assignment of prominence via a second-price sealed bid auction. In this auction, a retailer secures prominence by bidding the highest (fixed) fee and paying an amount equivalent to the second-highest bid. Naturally, this alternative prominence scheme changes the timing of the game relative to the baseline model. Unlike in the competition for prominence setting, retailers now compete in prices after observing which retailer has received prominence. These differences mirror real-world dynamics, as retailers adjust their prices knowing whether their listing will be promoted. Note that this change of timing plays a crucial role in the retailers' pricing strategies: as competition occurs upon observing the intermediary's prominence allocation choice, retailers do not have incentives to compete fiercely. In combination with the quality difference among retailers, competition is softened compared to the competition for prominence case, leading to double marginalization.

[15] There is empirical evidence that a better ranking matters (Ursu, 2018) and that a large share of searches happens passively, where consumers merely react to online ads (Ursu, Simonov, & An, 2022).

The timing in this scenario is now the following. In the first stage, retailers submit their bids simultaneously. Then, the intermediary assigns prominence to the bidder submitting the highest offer and optimally chooses w. In the second stage, having observed which retailer obtains prominence, retailers compete in prices by setting p_H and p_L. Finally, consumers make their purchasing decisions, whereby passive consumers can only decide between acquiring the sponsored product or not buying.

If the high-quality retailer is prominent, the gross profits of the high- and low-quality retailers are, respectively:

$$\begin{aligned}\pi_H^H(p_H,p_L) &= \Big[\lambda D_H^\lambda(p_H,p_L) + (1-\lambda)D_H^{1-\lambda}(p_H)\Big](p_H - w),\\ \pi_L^H(p_L,p_H) &= \Big[\lambda D_L^\lambda(p_L,p_H)\Big](p_L - w).\end{aligned}$$

Instead, if the low-quality retailer is prominent, the gross profits of the retailers are:

$$\begin{aligned}\pi_H^L(p_H,p_L) &= \Big[\lambda D_H^\lambda(p_H,p_L)\Big](p_H - w),\\ \pi_L^L(p_L,p_H) &= \Big[\lambda D_L^\lambda(p_L,p_H) + (1-\lambda)D_L^{1-\lambda}(p_L)\Big](p_L - w).\end{aligned}$$

The net profit of the retailer i that wins the auction and pays a monetary compensation to the intermediary to obtain prominence, denoted by $\tilde{\pi}_i^i$, is equivalent to the gross profit of the same retailer, net of the payment made to the intermediary, which we refer to as b_i. Specifically, $\tilde{\pi}_i^i(p_i,p_{-i}) = \pi_i^i(.) - b_i$.

In stage 2, retailers decide prices. The equilibrium prices in the continuation game in which the high-quality retailer is prominent are:

$$p_H^H := \arg\max_{p_H}\Big(\lambda D_H^\lambda(p_H,p_L) + (1-\lambda)D_H^{1-\lambda}(p_H)\Big)(p_H - w) - b_H,$$

$$p_L^H := \arg\max_{p_L}\lambda D_L^\lambda(p_L,p_H)(p_L - w).$$

The equilibrium prices in the continuation game in which the low-quality retailer is prominent are:

$$p_H^L := \arg\max_{p_H}\lambda D_H^\lambda(p_H,p_L)(p_H - w),$$

$$p_L^L := \arg\max_{p_L}\Big(\lambda D_L^\lambda(p_L,p_H) + (1-\lambda)D_L^{1-\lambda}(p_L)\Big)(p_L - w) - b_L.$$

In stage 1, we determine the bids that each retailer submits in the auction. We first compute each retailer's marginal benefit from being prominent, equivalent to the difference in the profits obtained by the retailers when receiving prominence and the profit obtained when not.

Denoting WTP_i the willingness to pay for prominence of retailer i, we get:

$$\begin{aligned}WTP_L :=& \pi_L^L(p_L^L, p_H^L) - \pi_L^H(p_L^H, p_H^H),\\ WTP_H :=& \pi_H^H(p_H^H, p_L^H) - \pi_H^L(p_H^L, p_L^L).\end{aligned}$$

Comparing the two willingness-to-pay in the Appendix, we observe that $WTP_H > WTP_L$, that is, due to its superior quality, the high-quality retailer has more to gain than the rival because of its prominent position. We conclude the following.

Lemma 4. *In the auction-based prominence allocation, the high-quality retailer submits the highest bid and is awarded prominence. Both retailers make strictly positive sales and profits.*

We can now compare the market outcomes of the competition for prominence and the auction-based prominence scheme (denoted by A). In the Appendix, we identify the optimal commission fee w and calculate the resulting equilibrium profit of the intermediary. Comparing profits of the intermediary in the auction-based prominence allocation scheme with those emerging in the first excluding equilibrium (10), we find that $\Pi^A < \Pi^{H1}$. In other words, the intermediary always makes more profits under competition for prominence as this scheme drives retailers to lower their prices, mitigating double marginalization. Instead, an auction-based system would soften competition and avoid the exclusion of the low-quality retailer. In the latter case, the commission fee w generates a substantial double marginalization problem, which leads prices above the monopoly level that, in turn, reduce consumers' demand. Interestingly, the fee collected by the intermediary via the auction does not compensate for the lower profits generated by the per-transaction fee w. As a result, an auction-based prominence is sub-optimal to the intermediary relative to competition for prominence. It is also worth noting that in the competition for prominence scheme, the interest of the intermediary and those of the consumers are fully aligned since the intermediary aims to maximize total transactions. In the auction-based system, this is no longer the case as the intermediary takes into account both consumer demand and the fee collected from the prominent retailer, which makes the intermediary also account for the interests of the retailers. In turn, prices are higher in this case because of the softening of the competition and the double marginalization ultimately hurts consumers. A fortiori, consumer surplus is higher under competition for prominence than under an auction-based prominence scheme. We can therefore conclude with the following proposition.

Proposition 6. *Competition for prominence is more profitable for an intermediary and generates a higher consumer surplus than an auction-based prominence allocation.*

It remains to compare how retailers perform under this allocation scheme relative to the competition for prominence. Note that it is immediate that the low-quality retailer is better off in the auction setting as it makes sales, even without obtaining prominence. The high-quality retailer, instead, faces a trade-off. Under competition for prominence, the high-quality retailer benefits from the absence of double marginalization and higher demand, but the intense competition drives prices down. Under the auction, the high-quality retailer benefits from the softened competition, but fees reduce its revenues.

Due to tractability reasons, we can only prove the existence of this trade-off numerically. Assuming without loss of generality the following value for the low-quality retailer $q_L = 5/4$, Figure 6 illustrates the areas in which the retailers' profits are higher under competition for prominence (i.e., the orange area) and auction-based prominence (i.e., the blue area), respectively.

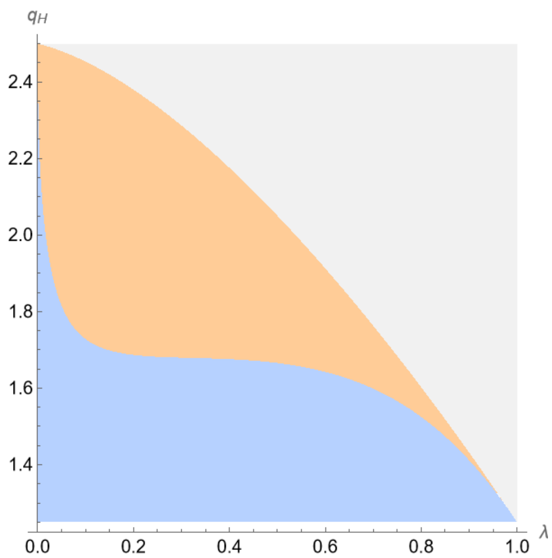

Figure 6: Profit comparison of the high-quality retailer in the two prominence allocation schemes. ($q_L = 5/4$)

Figure 6 shows that the high-quality retailer can be better off with either prominence schemes depending on the parameter range. In particular, its profits are higher in an auction-based system when a small quality differential and/or the number of active consumers is very large. The intuition is that in this scenario, the two retailers are almost symmetric, and there is little gain for competing fiercely for the prominent allocation. In turn, the high-quality can secure its prominence allocation under an auction-based prominence by paying a very small fee while benefiting from a softened competition. On the contrary, as the number of passive consumers gets larger, attracting them becomes important for both retailers, increasing their willingness to pay for prominence (and the associated fee). If the quality differential gets larger as well, then the high-quality retailer can obtain higher profits under competition for prominence since, due to the high quality, its price does not decrease too much. As a result, the high-quality retailer is better off under competition for prominence for a large share of passive consumers and a large quality differential.

6 Conclusions

This paper provides insights into the optimal prominence scheme that intermediaries can implement. Competition for prominence emerges as a powerful mechanism to tackle the issue of double marginalization and benefit consumers by inducing lower equilibrium prices.

Our analysis shows that the low- or high-quality retailer can obtain prominence depending on whether the commission fee is treated as given or chosen contingently by the intermediary. If the commission fee is considered as given, which could be due to long-run commitments or external regulation, competition for prominence can result in a scenario where low-quality retailers are promoted to passive consumers. This situation arises when the share of passive consumers is not significant enough to intensify competition between the two retailers, and the quality of the low-quality retailer is not deemed "too low". Consequently, competition softens, and the intermediary promotes the low-quality retailer to passive consumers. In all other cases, competition between the two retailers intensifies, as both retailers cannot afford to overlook the passive segment. Such intense competition can lead to two alternative equilibria, where the low-quality retailer makes no sale. If the commission fee is low, the Bertrand-like competition culminates with the high-quality retailer setting a low price. On the other hand, if the commission fee is high, the competition concludes with the high-quality retailer setting a high (monopoly-like) price.

However, when the intermediary can promptly adjust the fee based on market circumstances, it will strategically induce the excluding equilibrium where prices are low and competition is intense, as this outcome allows the intermediary to replicate the market outcomes of a multiproduct monopolist. In this equilibrium, the low-quality retailer serves no consumers but plays a crucial role in compelling the high-quality retailer to lower its price and expand demand. In turn, this strategy benefits consumers by providing access to a lower-priced high-quality product while optimizing the intermediary's overall profits.

Our analysis also shows that alternative prominence allocation schemes, such as auctions, are suboptimal for the intermediary and consumers compared to competition for prominence. Auction schemes require the intermediary to commit to granting prominence to a particular retailer, leading to a relaxation of competition, which lowers demand. However, retailers' total surplus can be higher in an auction-based prominence scheme. This result suggests that all welfare effects should be taken into account by policymakers and authorities when intervening against steering mechanisms, and a possible trade-off between the two types of users (i.e., final consumers and business users) may ultimately be present. Indeed, our research highlights the need for careful deliberation in designing prominence allocation mechanisms to ensure they do not result in negative welfare effects. It is essential to strike a balance that promotes competition and efficiency while safeguarding the interests of all stakeholders involved.

In addition, this paper provides a rationale for why major retail companies, such as Walmart in the United States or FNAC in France, have started implementing a platform business model that allows third-party sellers to operate on their marketplace. These platforms gather all product listings in one location and highlight one through their version of the buy-box, representing consumers' default option. This trend suggests that new entrants in this industry acknowledge the importance of competition for prominence as a critical factor in achieving success. Furthermore, our analysis sheds light on how the vertical differentiation between two identical offers can be determined by the decision of the intermediary to offer ancillary services, which has also become an emerging practice in the industry.[16] We find that if an intermediary can steer passive consumers using a prominent display and offers the possibility to upgrade the quality of ancillary services, it can extract the largest surplus from retailers. This strategy is the most profitable for the intermediary because it removes double marginalization and increases consumer surplus. However, it comes at the cost of leaving both retailers without surplus.

Recently, Amazon attracted the attention of antitrust authorities for possible suspected anti-competitive practices related to the buy-box. At the end of 2022, Amazon settled the European Commission's investigations by offering, among other commitments, to display a second competing offer to the Buy Box winner.[17] Our findings suggest that the commitment proposed may only partially address the surplus extraction issue that can arise from the assignment of a buy-box. The criteria used by the intermediary may still stimulate fierce competition among retailers and allow for monopolistic rent extraction through its fee structure. Hence, thoughtful consideration and thorough evaluation are necessary to devise prominence allocation schemes to optimize market outcomes and maximize welfare for all participants.

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[16] For example, Walmart offers the Walmart Fulfillment Services: https://marketplace.walmart.com/walmart-fulfillment-services/.

[17] See: https://ec.europa.eu/commission/presscorner/detail/en/ip_22_7777.

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Appendix

Proof of Lemma 1

This proof immediately follows from the main text.

Proof of Lemma 2

The first part of the proof follows from (3). Suppose that H is prominent. It can set two prices: either it sets $p_H = p_L q_H/q_L$ so that (3) binds, or it sets $p_H < p_L q_H/q_L$ and the constraint in (3) is slack. Suppose $p_H = p_L q_H/q_L$. In this case, L's demand conditional on H being prominent is:

$$\begin{aligned} D_L^\lambda(p_L, p_H) =& \frac{\frac{q_H}{q_L}p_L - p_L}{q_H - q_L} - \frac{p_L}{q_L} \\ =& \frac{p_L}{q_L} - \frac{p_L}{q_L} \\ =& 0, \end{aligned}$$

which implies the exclusion of L for any $p_L \geq w$. Because L leaves the market for $p_H = p_L q_H/q_L$, it is also excluded for any $p_H < p_L q_H/q_L$. As a result, the equilibrium price set by H conditional on being prominent is $p_H = p_L q_H/q_L$, and $D_L^\lambda(p_L, p_H) = 0$.

The second part of the proof is as follows. Suppose L is prominent: in this case, H is out of the market (i.e., $D_H^\lambda = 0$) if, and only, if:

$$p_L \leq p_H - q_H + q_L.$$

Suppose L sets its lowest price $p_L = w$. Then, exclusion of H occurs if and only if:

$$w \leq p_H - q_H + q_L,$$

or alternatively if $p_H \geq w + q_H - q_L$.

Suppose also H sets the lowest possible price $p_H = w$. Then, H is out of the market if and only if $q_L \geq q_H$, which contradicts the assumption that $q_H > q_L$. As a result, if L is awarded prominence it cannot exclude H from the marketplace. This concludes the proof.

Proof of Proposition 1

To determine the pricing strategies employed by the retailers, we first identify the best reply of the two retailers for a given prominence allocation.

Low-quality-retailer. If the low-quality retailer is prominent, L chooses p_L to maximize the following program:

$$\max_{p_L} \pi_L^L(p_L, p_H) \equiv \left[\lambda\left(\frac{p_H - p_L}{q_H - q_L} - \frac{p_L}{q_L}\right) + (1-\lambda)\left(1 - \frac{p_L}{q_L}\right)\right](p_L - w).$$

The low-quality retailer's price is determined by solving the first-order condition of the profit with respect to p_L, which yields

$$\frac{\partial \pi_L^L(p_L, p_H)}{\partial p_L} = \frac{\lambda(p_H - 2p_L - q_H + q_L + w)}{q_H - q_L} + \frac{q_L - 2p_L + w}{q_L} = 0.$$

Solving for $p_L(p_H)$ gives:

$$p_L(p_H) = \frac{\lambda q_L(p_H - q_H + q_L + w) + (q_H - q_L)(q_L + w)}{2(q_H - (1-\lambda)q_L)}. \tag{13}$$

Note that this price should respect the following two constraints: (i) $p_L < p_H q_L/q_H$, which guarantees that the prominence is allocated to the low-quality firm (Lemma 1); (ii) $p_L \geq w$, which guarantees a non-negative profit. We verify these conditions in what follows:

(i) $p_L(p_H) < p_H q_L/q_H$ if and only if:

$$\begin{aligned} p_L(p_H) \equiv& \frac{\lambda q_L(p_H - q_H + q_L + w) + (q_H - q_L)(q_L + w)}{2(q_H - (1-\lambda)q_L)} < p_H \frac{q_L}{q_H}, \\ \Leftrightarrow \; p_H & \; \frac{q_H(q_H((1-\lambda)q_L + w) - (1-\lambda)q_L(q_L + w))}{q_L((2-\lambda)q_H - 2(1-\lambda)q_L)} \equiv \hat{p}_H. \end{aligned}$$

(ii) $p_L(p_H) \geq w$ if and only if:

$$\begin{aligned} p_L(p_H) \equiv& \frac{\lambda q_L(p_H - q_H + q_L + w) + (q_H - q_L)(q_L + w)}{2(q_H - (1-\lambda)q_L)} \geq w, \\ \Leftrightarrow \; p_H \geq & \; \frac{(q_H - q_L)(w - q_L)}{\lambda q_L} + q_H - q_L + w \equiv \tilde{p}_H. \end{aligned}$$

Note that comparing $\tilde{p}_H$ with $\hat{p}_H$, we observe that $\hat{p}_H - \tilde{p}_H > 0$, i.e.:

$$\hat{p}_H - \tilde{p}_H = \frac{2(1-\lambda)(q_H - q_L)(q_L - w)(q_H - (1-\lambda)q_L)}{\lambda q_L((2-\lambda)q_H) - 2(1-\lambda)q_L} > 0,$$

meaning that constraints (i) and (ii) are satisfied as long as $p_H \geq \hat{p}_H$.

From Lemma 2, we know that for $p_L \geq p_H q_L/q_H$, the low-quality retailer has zero overall demand and the high-quality serves the entire market. Therefore, for any price that H sets between $\hat{p_H}$ and wq_L/q_H, L has an incentive to undercut its rival by an $\epsilon > 0$ such that the prominence allocation rule favors the low-quality retailer. Instead, for $p_H \leq wq_H/q_L$, the low-quality retailer can only set its minimum price of $p_L = w$.

The best-responses of L is thus determined as follows:

$$p_L(p_H) = \begin{cases} \frac{\lambda q_L(p_H - q_H + q_L + w) + (q_H - q_L)(q_L + w)}{2(q_H - (1-\lambda) q_L)}, & \text{if } \ p_H \geq \hat{p}_H \\ p_H \frac{q_L}{q_H} - \varepsilon, & \text{if } \ w\frac{q_L}{q_H} < p_H < \hat{p}_H \\ w, & \text{if } \ p_H \leq w\frac{q_H}{q_L} \end{cases} \tag{14}$$

High-quality-retailer. If the low-quality retailer is prominent, H chooses p_H to maximize the following program:

$$\max_{p_H} \pi_H(p_H, p_L) \equiv \lambda\Big(1 - \frac{p_H - p_L}{q_H - q_L}\Big)(p_H - w).$$

The high-quality retailer's price is determined by solving the first-order condition of the profit with respect to p_H, which yields:

$$\frac{\partial \pi_H(p_H, p_L)}{\partial p_H} = \frac{\lambda(p_L - 2p_H + q_H - q_L + w)}{q_H - q_L} = 0.$$

Solving for $p_H(p_L)$ gives:

$$p_H(p_L) = \frac{1}{2}(p_L + q_H - q_L + w). \tag{15}$$

Note that this price is feasible subject to the following constraint: $p_H > p_L q_H/q_L$, which guarantees that the prominence is allocated to the low-quality firm (Lemma 1). Therefore, $p_H(p_L) > p_L q_H/q_L$ if and only if

$$\begin{aligned} p_H(p_L) \equiv& \frac{1}{2}(p_L + q_H - q_L + w) > p_L \frac{q_H}{q_L} \\ \Leftrightarrow \ \ p_L <& \frac{q_H q_L - q_L^2 + q_L w}{2q_H - q_L} \equiv \tilde{p}_L. \end{aligned}$$

From Lemma 2, we know that for $p_H \leq p_L q_H/q_L$, the high-quality serves the entire market. Therefore, if the price of L is sufficiently low, H can match its rival and obtain prominence by setting $p_H = p_L q_H/q_L$. However, if the price set by the low-quality retailer is sufficiently high, H can set the monopoly price of $(q_H + w)/2$, which is obtained by maximizing the following program:

$$\max_{p_H} \pi_H^H(p_H, p_L) \equiv \Big(1 - \frac{p_H}{q_H}\Big)(p_H - w).$$

Indeed, $p_H(p_L) \leq p_L q_H/q_L$ if and only if

$$\begin{aligned} p_H(p_L) \equiv& \frac{q_H + w}{2} \leq p_L q_H/q_L, \\ \Leftrightarrow \ \ p_L \geq& \frac{q_H q_L + q_L w}{2q_H} \equiv \hat{p}_L. \end{aligned}$$

Note that comparing $\tilde{p}_L$ with $\hat{p}_L$, we observe that $\hat{p}_L - \tilde{p}_L > 0$, i.e.,

$$\hat{p}_L - \tilde{p}_L = \frac{q_H q_L^2 - q_L^2 w}{4q_H^2 - 2q_H q_L} > 0.$$

The best-responses of H is thus determined as follows:

$$p_H(p_L) = \begin{cases} \frac{1}{2}(p_L + q_H - q_L + w), & \text{if } \ p_L < \tilde{p}_L \\ p_L \frac{q_H}{q_L}, & \text{if } \ \hat{p}_L > p_L \geq \tilde{p}_L \\ \frac{q_H + w}{2}, & \text{if } \ p_L \geq \hat{p}_L \end{cases} \tag{16}$$

Equilibrium analysis. Having determined the best replies of the two firms, we can now identify the equilibrium prices for a given w. Specifically, we verify under which conditions the two alternative equilibria in Lemma 2 exists.

- **Market sharing equilibrium.** Solving the system of first order conditions in (13) and (15) simultaneously, we have:

$$\begin{aligned} p_L(w) &= \frac{\lambda q_L(3w - q_H + q_L) + 2(q_H - q_L)(q_L + w)}{4q_H - (4 - 3\lambda)q_L}, \\ p_H(w) &= \frac{2q_H^2 - (3 - \lambda)q_H q_L + 3q_H w + (1 - \lambda)q_L(q_L - 3w)}{4q_H - (4 - 3\lambda)q_L}. \end{aligned} \tag{17}$$

  We can now ex post verify whether $p_L < \tilde{p}_L$ and $p_H \geq \hat{p}_H$ (first line in (14)).

  (i) $p_L < \tilde{p}_L$ if and only $p_L(w)$ in (17) satisfies the following:

$$\begin{aligned} & p_L(w) \equiv \frac{2q_H^2 - (3 - \lambda)q_H q_L + 3q_H w + (1 - \lambda)q_L(q_L - 3w)}{4q_H - (4 - 3\lambda)q_L} < \frac{q_H q_L + q_L w}{2q_H} \equiv \tilde{p}_L \\ \Leftrightarrow \quad & -\frac{2(q_H - q_L)\left(\lambda q_H q_L - 2q_H w + (\lambda - 1)q_L^2 - 3(\lambda - 1)q_L w\right)}{(2q_H - q_L)(4q_H + (3\lambda - 4)q_L)} < 0 \\ \Leftrightarrow \quad & \lambda q_H q_L - 2q_H w + (\lambda - 1)q_L^2 - 3(\lambda - 1)q_L w > 0. \end{aligned} \tag{18}$$

  which is the case if and only if the following two conditions are jointly satisfied:

$$(a) \ \ q_L > 2w, \qquad (b) \ \ \tilde{\lambda} \equiv \frac{2q_H w + q_L^2 - 3q_L w}{q_H q_L + q_L^2 - 3q_L w} < \lambda < 1.$$

(ii) $p_H \geq \hat{p}_H$ if and only $p_H(w)$ in (17) satisfies the following:

$$
\begin{aligned}
&p_H(w) \equiv \frac{2q_H^2 - (3-\lambda)q_H q_L + 3q_H w + (1-\lambda)q_L(q_L - 3w)}{4q_H - (4-3\lambda)q_L} \geq \\
&\frac{q_H(q_H((1-\lambda)q_L + w) - (1-\lambda)q_L(q_L + w))}{q_L((2-\lambda)q_H - 2(1-\lambda)q_L)} \equiv \hat{p}_H \\
&\Leftrightarrow \frac{2(q_H - q_L)(q_H + (\lambda - 1)q_L)\left(\lambda q_H q_L - 2q_H w - (1-\lambda)q_L^2 - 3(\lambda-1)q_L w\right)}{q_L(2(\lambda-1)q_L - (\lambda-2)q_H)(4q_H + (3\lambda - 4)q_L)} > 0 \\
&\Leftrightarrow \lambda q_H q_L - 2q_H w + (\lambda-1)q_L^2 - 3(\lambda-1)q_L w > 0.
\end{aligned}
$$

where the last line is the same as the last line in (18). Therefore, $p_L < \tilde{p}_L$ and $p_H \geq \hat{p}_H$ are jointly satisfied if and only if the following two conditions are jointly satisfied:

$$
(a)\;\; q_L > 2w, \qquad (b)\;\; \tilde{\lambda} \equiv \frac{2q_H w + q_L^2 - 3q_L w}{q_H q_L + q_L^2 - 3q_L w} < \lambda < 1.
$$

Therefore, there exists a market sharing equilibrium if and only if $q_L > 2w$ and $\lambda \in (\tilde{\lambda}, 1]$.

- **Excluding equilibria.** The equilibria exist if either $q_L \leq 2w$ or if $q_L > 2w$ and $\lambda \leq \tilde{\lambda}$. If either are verified, the best-response of L is always to set $p_L = w$ whereas H has two choices: (i) set $p_H = (q_H + w)/2$ or, alternatively, (ii) $p_H q_L/q_H$.

  (i) First, we note that $p_H = (q_H + w)/2$ can be a best-response to $p_L = w$ if and only if $p_H \leq w q_L/q_H$ (third line in (14)), namely if:

  $$
  \begin{aligned}
  \frac{q_H + w}{2} &\leq w\frac{q_H}{q_L} \\
  \Leftrightarrow w &\geq \frac{q_H q_L}{2q_H - q_L}.
  \end{aligned}
  $$

  Therefore, H sets a monopoly price if w is large enough. Otherwise, the next equilibrium is feasible.

  (ii) If $w < (q_H q_L)/(2q_H - q_L)$, H responds to $p_L = w$ by setting $p_H(p_L) = p_L q_L/q_H$, that is $p_H = w q_L/q_H$, which a fortiori satisfies the constraint for which $p_L = w$ (third line in (14)).

Thus, we conclude the following. For any given w, we have the following three equilibria:

1. if $q_L > 2w$ and $\lambda \in (\tilde{\lambda}, 1]$, there is a unique market sharing equilibrium and prices are as in (17);
2. otherwise, there is a unique excluding equilibrium and prices are

   2.1.) $p_L = w$ and $p_H = w q_L/q_H$ if $w \leq (q_H q_L)/(2q_H - q_L)$, or

   2.2.) $p_L = w$ and $p_H = (q_H + w)/2$ if $w > (q_H q_L)/(2q_H - q_L)$.

This concludes the proof.

Proof of Lemma 3

This proof immediately follows from the discussion in the main text.

Proof of Proposition 2

Let us consider social welfare, denoted as the sum of the retailers' and the intermediary profits and consumer surplus.

Market sharing equilibrium. Using prices in (17), retailers' profits are given by:

$$\pi_L^L(\cdot) = \frac{(q_H - q_L)(q_H - (1-\lambda)q_L)(2w - (2-\lambda)q_L)^2}{q_L(4q_H - (4-3\lambda)q_L)^2},$$
$$\pi_H^L(\cdot) = \frac{\lambda(q_H - q_L)(q_L + w - 2q_H - \lambda q_L)^2}{(4q_H - (4-3\lambda)q_L)^2}.$$

Consumer surplus is given by:

$$\begin{aligned} CS^L(\cdot) &= \lambda \int_{\frac{p_H - p_L}{q_H - q_L}}^{1} (\theta q_H - p_H)\, d\theta + \lambda \int_{\frac{p_L}{q_L}}^{\frac{p_H - p_L}{q_H - q_L}} (\theta q_L - p_L)\, d\theta + (1-\lambda) \int_{\frac{p_L}{q_L}}^{1} (\theta q_L - p_L)\, d\theta \\ &= \frac{(q_H - (1-\lambda)q_L)\left(\lambda q_L \left(4q_H^2 - 8q_H w + 5q_L^2 - 10 q_L w + 9w^2\right)\right)}{2q_L(4q_H - (4-3\lambda)q_L)^2} \\ &+ \frac{(q_H - (1-\lambda)q_L)\left(\lambda^2 q_L^2 (q_H - q_L) + 4(q_H - q_L)(q_L - w)^2\right)}{2q_L(4q_H - (4-3\lambda)q_L)^2}. \end{aligned}$$

Therefore, social welfare is given by:

$$\begin{aligned} W^L(\cdot) &= \Pi^L(\cdot) + \pi_L^L(\cdot) + \pi_H^L(\cdot) + CS^L(\cdot) \\ &= \frac{5\lambda^3 q_L^3 (q_H - q_L) + 4(q_H - q_L)^2 (q_L - w)(3q_L + w)}{2q_L(4q_H - (4-3\lambda)q_L)^2} \\ &+ \frac{\lambda q_L (q_H - q_L)\left(12q_H^2 - 16 q_H q_L + 27 q_L^2 - 10 q_L w - 13 w^2\right)}{2q_L(4q_H - (4-3\lambda)q_L)^2} \\ &+ \frac{\lambda^2 q_L^2 \left(15 q_H^2 + 2q_H(w - 13q_L) + 20q_L^2 - 2q_L w - 9w^2\right)}{2q_L(4q_H - (4-3\lambda)q_L)^2}. \end{aligned} \tag{19}$$

First excluding equilibrium. Since prices are $p_L = w$ and $p_H = wq_H/q_L$, the low-quality retailer makes no sales, the high-quality retailer obtains $\pi_H^{H1}(\cdot) = w(q_H - q_L)(q_L - w)/q_L^2$, and the intermediary's profit is the same as in (8). Consumer surplus is given by:

$$CS^{H1}(\cdot) = \int_{\frac{p_H}{q_H}}^{1} (\theta q_H - p_H)\, d\theta = \frac{q_H (q_L - w)^2}{2q_L^2},$$

whereas social welfare is:

$$W^{H1}(\cdot) = \Pi^{H1}(\cdot) + \pi_H^{H1}(\cdot) + CS^{H1}(\cdot) = \frac{1}{2}q_H \left(1 - \frac{w^2}{q_L^2}\right). \tag{20}$$

Second excluding equilibrium. Since prices are $p_L = w$ and $p_H = (w + q_H)/2$, the low-quality retailer makes no sales, the high-quality retailer obtains $\pi_H^{H2}(\cdot) = (q_H - w)^2/(4q_H)$, and the intermediary's profit is the same as in (9). Consumer surplus is:

$$CS^{H2}(\cdot) = \int_{\frac{p_H}{q_H}}^{1} (\theta q_H - p_H)\, d\theta = \frac{(q_H - w)^2}{8q_H},$$

whereas social welfare is:

$$W^{H2}(\cdot) = \Pi^{H2}(\cdot) + \pi_H^{H2}(\cdot) + CS^{H2}(\cdot) = \frac{(q_H - w)(3q_H + w)}{8q_H}. \tag{21}$$

Comparison. We now proceed with comparing total welfare in each scenario in the relevant parameter ranges.

Comparing social welfare in (19) (market sharing equilibrium) with the one in (20) (first excluding equilibrium), we have:

$$\begin{aligned} W^L(\cdot) - W^{H1}(\cdot) = \frac{q_H - q_L}{2q_L^2(4q_H - (4 - 3\lambda)q_L)^2}\Big[&4q_H^2 \left(4w^2 - (4 - 3\lambda)q_L^2\right) \\ &+ (1 - \lambda)q_L^2 \left(2(4 - \lambda)q_L w - \left((12 - 5(3 - \lambda)\lambda)q_L^2\right) + (4 - 9\lambda)w^2\right) \\ &+ q_H q_L \left((28 - 5\lambda(8 - 3\lambda))q_L^2 - 8q_L w - 4(5 - 6\lambda)w^2\right)\Big] < 0, \end{aligned}$$

if $q_L > 2w$ and $\tilde{\lambda} < \lambda < 1$, that is the parameter range in which the market sharing equilibrium occurs.[18] As a result, a market sharing equilibrium is never socially desirable in its parameter range as it is dominated by the first excluding equilibrium.

Then, comparing welfare in the first excluding equilibrium in (20) with the one in the second excluding equilibrium in (21), we observe that:

$$\begin{aligned} W^{H1}(\cdot) \equiv \frac{1}{2}q_H \left(1 - \frac{w^2}{q_L^2}\right) &> \frac{(q_H - w)(3q_H + w)}{8q_H} \equiv W^{H2}(\cdot) \\ \Leftrightarrow \frac{1}{8}\left(q_H - \frac{4q_H w^2}{q_L^2} + \frac{w^2}{q_H} + 2w\right) &> 0 \\ \Leftrightarrow w &< \frac{q_H q_L}{2q_H - q_L}, \end{aligned}$$

and the opposite otherwise. Therefore, in the parameter range in which the first (resp. second)

[18] Formally, this is verified using *Wolfram Mathematica*.

excluding equilibria arise, the first (resp. second) excluding equilibrium is the socially desirable one.

Thus, we prove that whereas the market sharing equilibrium is never socially desirable, the two excluding equilibria are socially desirable in the parameter range in which they arise.

Proof of Proposition 3

Suppose we are in the first excluding equilibrium where $p_H^{H1} = wq_H/q_L$. This case exists if and only if the equilibrium fee w is less or equal than $(q_H q_L)/(2q_H - q_L)$. For a given w, the profit of the intermediary is:

$$\Pi^{H1}(\cdot) = \frac{w(q_L - w)}{q_L}$$

Differentiating it with respect to w we obtain the following equilibrium fee $w = q_L/2$, and the associated equilibrium profit of the intermediary is $\Pi^{H1}(\cdot) = q_L/4$.

Suppose now that we are in the second excluding equilibrium where $p_H^{H2} = (q_H + w)/2$. This case exists if and only if the equilibrium fee w is more than $(q_H q_L)/(2q_H - q_L)$. For a given w, the profit of the intermediary is:

$$\Pi^{H2}(\cdot) = w - \frac{w(q_H + w)}{2q_H}$$

Differentiating it with respect to w we obtain the following equilibrium fee $w = q_H/2$, and the associated profit is $\Pi^{H2}(\cdot) = q_H/8$.

Comparing profits in the different scenarios, we observe that:

$$\Pi^{H1}(\cdot) - \Pi^{H2}(\cdot) = \frac{q_L}{4} - \frac{q_H}{8} > 0$$

which holds as long as $q_L > 2q_H$. Thus, we have proved that the intermediary sets a fee equal to $w = q_L/2$ and the first excluding equilibrium is subgame perfect.

Welfare In equilibrium, dropping the arguments for brevity, consumer surplus is given by:

$$CS^{H1}(\cdot) = \lambda \int_{\frac{p_H - p_L}{q_H - q_L}}^{1} (\theta q_H - p_H)\, d\theta + (1-\lambda) \int_{\frac{p_H}{q_H}}^{1} (\theta q_H - p_H)\, d\theta = \frac{q_H}{8}. \tag{22}$$

The total industry profit is:

$$\Pi^{H1}(\cdot) + \pi_H^{H1}(\cdot) = \frac{q_H}{4}. \tag{23}$$

Total welfare, which is defined as the sum of the industry profit and consumer surplus, is:

$$W^{H1}(\cdot) = CS^{H1}(\cdot) + \Pi^{H1}(\cdot) + \pi_H^{H1}(\cdot) = \frac{3}{8} q_H. \tag{24}$$

Proof of Proposition 4

This proof immediately follows from the discussion in the main text. We report here the comparison of the profit of the multiproduct monopolist in the two prominence allocation rules to show that it always prefers to make its high-quality offer prominent.

If the low-quality offer is prominent, the multiproduct monopolist obtains:

$$\Pi^L(p_L^L, p_H^L) = \frac{\lambda q_H}{4} + \frac{(1-\lambda)q_L}{4}.$$

If the highquality offer is prominent, the multiproduct monopolist obtains:

$$\Pi^H(p_L^H, p_H^H) = \frac{q_H}{4}.$$

Comparing the two profits, it immediately follows that:

$$\Pi^H(p_L^H, p_H^H) \equiv \frac{q_H}{4} > \frac{\lambda q_H + q_L(1-\lambda)}{4} \equiv \Pi^L(p_L^L, p_H^L),$$

because $q_H > q_L$.

Proof of Proposition 5

It immediately follows from the discussion in the main text.

Proof of Lemma 4

For any given w, the profits of the retailers conditional on L being prominent, are given by:

$$\begin{aligned}
\pi_L^L(p_L, p_H) &= \Big[\lambda D_L^{\lambda}(p_L, p_H) + (1-\lambda) D_L^{1-\lambda}(p_L)\Big](p_L - w) - b_L, \\
\pi_H^L(p_H, p_L) &= \lambda D_H^{\lambda}(p_H, p_L)(p_H - w).
\end{aligned}$$

Differentiating them with respect to p_L and p_H, respectively, yields:

$$\begin{aligned}
\frac{\partial \pi_L^L(p_L, p_H)}{\partial p_L} &= 0 \Longleftrightarrow p_L^L(p_H) = \frac{\lambda q_L(p_H - q_H + q_L + w) + (q_H - q_L)(q_L + w)}{2(q_H - (1-\lambda)q_L)}, \\
\frac{\partial \pi_H^L(p_H, p_L)}{\partial p_H} &= 0 \Longleftrightarrow p_H^L(p_L) = \frac{1}{2}(p_L + q_H - q_L + w).
\end{aligned}$$

Solving simultaneously, equilibrium prices are given by:

$$p_L^L = \frac{\lambda q_L(q_L - q_H + 3w) + 2(q_H - q_L)(q_L + w)}{4q_H - (4 - 3\lambda)q_L},$$
$$p_H^L = \frac{2q_H^2 - (3 - \lambda)q_H q_L + 3q_H w + (1 - \lambda)q_L(q_L - 3w)}{4q_H - (4 - 3\lambda)q_L},$$

and the associated profits are:

$$\pi_L^L = \frac{(q_H - q_L)(q_H - (1 - \lambda)q_L)(2w - (2 - \lambda)q_L +)^2}{q_L(4q_H - (4 - 3\lambda)q_L)^2},$$
$$\pi_H^L = \frac{\lambda(q_H - q_L)(w - 2q_H + (1 - \lambda)q_L)^2}{(4q_H - (4 - 3\lambda)q_L)^2}.$$

For any given w, the profits of the retailers conditional on H being prominent, are given by

$$\pi_L^H(p_L, p_H) = \Big[\lambda D_L^\lambda(p_L, p_H)\Big](p_L - w),$$
$$\pi_H^H(p_H, p_L) = \Big[\lambda D_H^\lambda(p_H, p_L) + (1 - \lambda)D_H^{1-\lambda}(p_H)\Big](p_H - w) - b_H.$$

Differentiating them with respect to p_L and p_H, respectively, yields:

$$\frac{\partial \pi_L^H(p_L, p_H)}{\partial p_L} = 0 \Longleftrightarrow p_L^L(p_H) = \frac{p_H q_L + q_H w}{2q_H},$$
$$\frac{\partial \pi_H^H(p_H, p_L)}{\partial p_H} = 0 \Longleftrightarrow p_H^L(p_L) = \frac{\lambda(p_L q_H + q_L w) + (q_H - q_L)(q_H + w)}{2(q_H - (1 - \lambda)q_L)}.$$

Solving simultaneously, equilibrium prices are given by:

$$p_L^H = \frac{\lambda q_L w(2q_H + q_L) + (q_H - q_L)(q_H(q_L + 2w) + q_L w)}{q_H(4q_H - (4 - 3\lambda)q_L)},$$
$$p_H^H = \frac{\lambda w(q_H + 2q_L) + 2(q_H - q_L)(q_H + w)}{4q_H - (4 - 3\lambda)q_L},$$

and the associated profits are:

$$\pi_L^H = \frac{\lambda(q_H - q_L)(q_H(q_L - 2w) + (1 - \lambda)q_L w)^2}{q_H q_L(4q_H + (4 - 3\lambda)q_L)^2},$$
$$\pi_H^H = \frac{(q_H - q_L)(q_H - (1 - \lambda)q_L)(2q_H - (2 - \lambda)w)^2}{q_H(4q_H + (3\lambda - 4)q_L)^2}.$$

Denoting WTP_i the willingness to pay for prominence of retailer i, for $i = L, H$, we get:

$$
\begin{aligned}
WTP_L :=& \frac{(q_H - q_L)\left((q_H - (1-\lambda)q_L)(2w - (2-\lambda)q_L +)^2 - \frac{\lambda(q_H(q_L - 2w) + (1-\lambda)q_L w)^2}{q_H}\right)}{q_L(4q_H - (4-3\lambda)q_L)^2}, \\
WTP_H :=& \frac{(q_H - q_L)\left(\frac{(q_H - (1-\lambda)q_L)(2q_H - (2-\lambda)w)^2}{q_H} - \lambda(w - 2q_H + (1-\lambda)q_L)^2\right)}{(4q_H - (4-3\lambda)q_L)^2}.
\end{aligned}
$$

Comparing the two willingness-to-pay, we have $WTP_H - WTP_L > 0$ if

$$
\begin{aligned}
&\frac{(1-\lambda)(q_H - q_L)\left(4q_H^2 - (8-5\lambda)q_H q_L + (4 - \lambda(5-2\lambda))q_L^2\right)(q_H q_L - w^2)}{q_H q_L (4q_H - (4-3\lambda)q_L)^2} > 0 \\
\Leftrightarrow\ &\left(4q_H^2 - (8-5\lambda)q_H q_L + (4 - \lambda(5-2\lambda))q_L^2\right) > 0
\end{aligned}
\tag{25}
$$

which is positive since the above term is positive at $\lambda = 0$ and $\lambda = 1$, and is monotonically increasing in λ. Therefore, $WTP_H - WTP_L > 0$.

Proof of Proposition 6

Since we consider a sealed-bid second price auction, following the result established in (25), the fee paid by the prominent retailer is given by:

$$
b_H = WTP_L.
$$

The intermediary's profit when awarding prominence to the high-quality retailer is:

$$
\begin{aligned}
\Pi^H(\cdot) =& \frac{1}{q_H q_L (4q_H - (4-3\lambda)q_L)^2} \\
&\Big\{(q_H^3\left((4-\lambda)(1-\lambda)q_L^2 + 12\lambda q_L w + 4(1-3\lambda)w^2\right) - (1-\lambda)(8 - \lambda(11-4\lambda))q_L^3 w^2) \\
&+ q_H^2 q_L\left(-\left((1-\lambda)(8-(5-\lambda)\lambda)q_L^2\right) - 3\lambda(4-3\lambda)q_L w - 2(8 - 7(2-\lambda)\lambda)w^2\right) \\
&+ q_H q_L^2\left((1-\lambda)(\lambda-2)^2 q_L^2 + ((4(5-\lambda)\lambda - 35)\lambda + 20)w^2\right)\Big\}.
\end{aligned}
$$

Differentiating it with respect to w and solving the related first-order condition to zero yields

$$
w^A = \frac{3\lambda q_H^2 q_L (4q_H - (4-3\lambda)q_L)}{\Omega + 2(1-\lambda)(8 - \lambda(11-4\lambda))q_L^3 - 2(20 - \lambda(35 - 4(5-\lambda)\lambda))q_H q_L^2},
$$

where $\Omega \equiv 4(8 - 7(2-\lambda)\lambda)q_H^2 q_L - 8(1-3\lambda)q_H^3$.

The equilibrium profit of the intermediary under the auction-based prominence is:

$$
\begin{aligned}
\Pi^A(\cdot) = \frac{1}{108}\Big[&12\lambda(q_H - q_L) + \frac{4(q_L - q_H)(5q_H + 7q_L)}{q_L} \\
&- \frac{16(q_L - 4q_H)(q_L - q_H)^3}{q_L(4q_H - (4-3\lambda)q_L)^2} + \frac{12q_H(q_H - q_L)(8q_H + q_L)}{q_L(4q_H + (4-3\lambda)q_L)}\Big] \\
&+ \frac{243\lambda^2 q_H^3 q_L}{108}\Big[2(8 - 7(2-\lambda)\lambda)q_H^2 q_L - 4(1-3\lambda)q_H^3 \\
&+ (1-\lambda)(8 - \lambda(11-4\lambda))q_L^3 - (20 - \lambda(35 - 4(5-\lambda)\lambda))q_H q_L^2\Big]^{-1}
\end{aligned}
\tag{26}
$$

Although technically untractable, it is possible to show that $\Pi^A < \Pi^{H1}$ always, where Π^{H1} is given by (10).[19]

Asymmetric marginal cost

An important assumption in our baseline model is that both retailers face a common marginal cost, normalized to zero. In this section, we relax this assumption and discuss how asymmetric marginal cost alters the market outcome. Specifically, we assume that the high-quality retailer has a higher marginal cost than the low-quality retailer. Denoting c_H and c_L, respectively, the marginal cost of H and L, and normalizing the latter to 0, we assume $c_H > c_L = 0$.

This assumption has an important effect on changing how retailers compete. The reason is that the price floor of H is now $p_H = c_H + w$, whereas the price floor of L is $p_L = w$.

Conditional on H being prominent, the profits of the retailers are, respectively:

$$
\begin{aligned}
\pi_H^H(p_H, p_L, w) &= \Big[\lambda D_H^\lambda(p_H, p_L) + (1-\lambda)D_H^{1-\lambda}(p_H)\Big](p_H - w - c_H), \\
\pi_L^H(p_L, p_H, w) &= \Big[\lambda D_L^\lambda(p_L, p_H)\Big](p_L - w).
\end{aligned}
$$

Conditional on L being prominent, that is for $\lambda \geq \tilde{\lambda}(w)$ are, respectively:

$$
\begin{aligned}
\pi_H^L(p_H, p_L, w) &= \Big[\lambda D_H^\lambda(p_H, p_L)\Big](p_H - w - c_H), \\
\pi_L^L(p_L, p_H, w) &= \Big[\lambda D_L^\lambda(p_L, p_H) + (1-\lambda)D_L^{1-\lambda}(p_L)\Big](p_L - w).
\end{aligned}
$$

For any $\lambda < \tilde{\lambda}(w)$, we distinguish between two alternative scenarios, which depend on the magnitude of the marginal cost. In the first one, c_H is sufficiently low (and the baseline model is a degenerate case), and H can always match the quality-adjusted price of L and win

[19] Formally, this is verified using *Wolfram Mathematica*. The proof is available at the following link: https://fabriziociotti.github.io/fciotti/competition-for-prominence-mathematica.rar. A similar proof is also available for what concerns the analysis of consumer surplus and retailers' profits in the two scenarios.

prominence. In the second one, c_H is sufficiently high, and H cannot obtain prominence as the low-quality retailer can always undercut the high-quality one.

Using H and L's price floors together with the prominence allocation rule in (3), we can identify the cut-off value below (resp. above), which H is always able to undercut L and maximize the profit of the intermediary if:

$$c_H \leq (>) \Big(\frac{q_H}{q_L} - 1\Big) w. \tag{27}$$

If (27) holds, therefore, the analysis in our baseline model holds qualitatively, and the low-quality retailer is out of the market at equilibrium (see Proposition 3).

If (27) does not hold, it is not always possible for H to lower its price to win prominence. As in the baseline model, L is always incentivized to obtain prominence whenever H obtains prominence. Otherwise, it is excluded. Therefore, the equilibrium outcome depends on the incentives of the high-quality seller in this case.

Unlike the baseline model, no equilibrium in pure strategy exists if H has the incentive to obtain prominence.

Too see why, consider the case in which H reaches its lowest price, $p_H = w + c_H$, and L sets $p_L = p_H \; q_L/q_H - \varepsilon$ (with $\varepsilon \to 0$). In this case, L gains prominence and positive profit, whereas H obtains zero profits. Since the loss of prominence for the high-quality retailer does not imply losing the active consumers due to vertical differentiation (see Lemma 2), H is always incentivized to raise its price and monetize the actives.

Consider now the case in which L sets $p_L = (w + c_H) \; q_L/q_H - \varepsilon$ (with $\varepsilon \to 0$), which implies that L certainly obtains prominence. As H can always serve the active consumers if L is prominent (by Lemma 2), it maximizes its profit subject to $p_L = (w + c_H) \; q_L/q_H - \varepsilon$. From the first-order condition with respect to p_H, we obtain the following candidate equilibrium price

$$p_H = \frac{c_H(q_H + q_L) + w(q_H + q_L) + q_H(q_H - q_L)}{2q_H}.$$

We notice that (p_L, p_H) here defined is not equilibrium because L can always raise its price just below the prominence threshold level, i.e., $p_L \approx p_H \; q_L/q_H - \varepsilon$.

This analysis suggests that if (27) does not hold, there is never an equilibrium in pure strategy. Therefore, only a mixed strategy equilibrium exists.