Competition for Prominence

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Intermediaries linking retailers to buyers may influence consumers' purchasing decisions by assigning a prominent position to one retailer. In this paper, we study the business strategy employed by intermediaries (in particular, online marketplaces) of listing all the offers for a specific product available in the market and selecting a default, or prominent, retailer. As some consumers only consider the prominent offer, obtaining the default position translates into exclusive access to some consumers. We show that such a scheme can lead retailers to fiercely (endogenously) compete for prominence, significantly benefiting the intermediary by drastically mitigating the double marginalization problem otherwise present.

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1 Introduction

With the advent of the internet, retailers have become increasingly dependent on online intermediaries for selling their products. Worldwide, a few online marketplaces act as critical gateways for sellers to reach consumers and have increasingly been able to influence consumers' decisions and steer consumption. Relevant examples are recommender systems or assigning a prominent allocation to one or some sellers.

In the early 2000s, Amazon.com opened its marketplace to third-party sellers and introduced the 'buy-box' to the world. The novelty of the buy-box consists of grouping all listings associated with a specific product into a single webpage. One seller is chosen as the default seller (i.e., the seller in the buy box), whereas the other sellers are offered a less prominent position. This strategy was very successful and adopted by emergent marketplaces like Walmart in the United States, Otto in Germany, Fnac in France, and Bol.com in the Netherlands. Obtaining the default position in online marketplaces has become vital for retailers. For example, recent estimates suggest that as much as 80% of all purchases on Amazon are made through the buy-box (U.S. House Judiciary Committee, 2020). While the platforms do not fully disclose the criteria used by the algorithms, they often reveal some tips on how to obtain a prominent allocation. In the case of Amazon, very competitive prices, with free and fast shipping, excellent customer service, and high stock available, seem to be the way to succeed.[1]

In this paper, we unveil the rationale for why online platforms adopt a competition for prominence scheme and how the selection of a prominent retailer affects competition among retailers. We construct a model in which a monopolist platform connects consumers to two retailers selling identical products. However, retailers differ in the ancillary services they offer, influencing the consumers' purchasing experience: one of the retailers provides a better service than the rival one.[2] Consumers are of two types — actives and passives — and are heterogeneous in their willingness to pay for quality. Active consumers always compare all available offers and buy (if so) from the retailer that provides the highest utility. Passive consumers only consider the offer recommended by the intermediary and decide whether to buy or not. Upon observing prices, the prominent position is assigned to the retailer generating the highest returns to the intermediary.[3] This strategy, which maximizes the intermediary's profit, leads retailers to actively and fiercely compete for prominence by internalizing in their pricing strategies the interests of the intermediary.

Our first result is that the intermediary always finds it optimal to assign prominence to the

[1] In other words, the price-quality ratio of the overall retailing activity matters, and the retailer with the best quality-adjusted price offer is more likely to win the buy-box. For more details, visit amazon seller central hub.

[2] For example, faster delivery or a more generous return policy.

[3] Note that this case is equivalent to a case of no commitment on the allocation of prominence to one retailer. The intermediary makes its decision only based on its profit ex post, for given prices. Anticipating this, retailers set prices to obtain prominence if they find it optimal.

retailer that sets the lowest quality-adjusted price. Therefore, any undercutting strategy by the two retailers that aim to become prominent implies taking into account also their quality relative to the rival. Due to the quality gap between retailers, we show that the high-quality retailer can always obtain prominence. If the former competes for prominence, the low-quality retailer is permanently foreclosed.

Our second result is that, for a given commission fee, two equilibria may exist depending on the fraction of active consumers. If the share of actives is large enough, the high-quality retailer finds it optimal to focus on the active consumers only, letting the low-quality retailer cover a small portion of the passives. In this market-sharing equilibrium, both retailers obtain strictly positive profits. On the other hand, if the share of actives is sufficiently small, the high-quality retailer is incentivized to serve the large number of passive consumers. However, also the low-quality retailer has the incentive to attract these consumers as it is always foreclosed when the high-quality retailer is prominent. Therefore, both retailers have the incentive to become prominent, which entails a Bertrand-type downward pricing spiral, ultimately ending with the low-quality retailer setting a price equal to its marginal (commission) cost. Only the high-quality retailer obtains a strictly positive profit in this foreclosing equilibrium.

Our third result is that the intermediary can strategically choose the commission fee to induce its preferred outcome. We show that the intermediary always finds it optimal to set a commission fee that induces a foreclosing equilibrium. As a result, competition between retailers is intense, the low-quality retailer is foreclosed, the high-quality retailer sets a meager price (but still obtains a positive margin), and the total demand is maximized.

The market environment we analyze has a vertical structure that can potentially generate the double marginalization problem. With the competition for prominence scheme, the intermediary forces retailers to lower the prices by orchestrating a competition for the prominent display. As a result, the high-quality retailer becomes endogenously prominent and shares its profit with the intermediary. This strategy eliminates the double marginalization problem and allows replicating the multiproduct monopolist's market outcomes. Indeed, the total profit generated is the same one a multiproduct monopolist would get by directly setting a price for its high-quality (prominent) offer.

In an extension, we compare the competition for prominence scheme with a more traditional pay for prominence scheme similar to Armstrong and Zhou (2011). We consider a scenario in which the intermediary organizes a position auction. We show that while the high-quality retailer can always win prominence, this scheme suffers from the same double marginalization problem previously discussed. As a result, competition for prominence is always optimal for the intermediary.

We then endogenize the quality dimension by considering the case in which retailers are ex ante identical, and the intermediary offers a complementary service that upgrades the quality experience of retailers (e.g., the Fulfilled by Amazon — FBA— program). We show that the intermediary induces only one retailer to upgrade its quality. At equilibrium, the intermediary behaves as a multiproduct monopolist, extracting rents by triggering a competition between retailers and efficiently taxing the high-quality retailer via the upgrading program. This strategy reduces the retailers' surplus even though it increases the intermediary's profit and consumer surplus. Finally, we provide a discussion on the policy relevance of our analysis. Policymakers and antitrust authorities have expressed concerns regarding how platforms allocate prominence, especially when platforms offer to upgrade ancillary services that alter the likelihood of becoming prominent.[4]

The rest of the paper is organized as follows. In Section 2, we discuss the related literature and our contribution. In Section 3 we present the main ingredients of the model. In Section 4, we present the results of the baseline model. In Section 5 we endogenize the quality choice of retailers. In Section 6, we present concluding remarks and managerial implications.

2 Contribution to the literature

Our paper draws on the literature on prominence and platform intermediation. Prominence plays a non-trivial role and affects both purchasing decisions and competition among rival sellers.

Armstrong et al. (2009) study a setting in which prominence is exogenously-assigned to one firm, among $n \geq 2$ firms. In their framework, consumers first check the offer of the prominent seller, and if the offer is not satisfactory, they do a random sequential search among the non-prominent sellers. If sellers are symmetric, the prominent seller sets a low price, whereas non-prominent sellers serve unsatisfied customers with high prices. Prominence reduces consumer surplus while maximizing industry profits even though average search decreases. If sellers are asymmetric in their quality, only the high-quality becomes prominent and makes a strictly positive profit. This result resembles ours in which the intermediary finds it optimal to induce the high-quality retailer to compete for prominence. In such a case, there is a foreclosing equilibrium, with no profit for the low-quality retailer.

Moraga-González et al. (2021) also consider differentiated sellers. In their study, consumers are heterogeneous in their search costs and browse products simultaneously. In this case, the allocation of prominence can increase consumer surplus even though the prominent seller increases its price by focusing on the least elastic segment of the market, thereby letting non-prominent sellers price aggressively to serve those consumers searching for an alternative. In

[4]In 2020, the European Commission opened a second investigation into Amazon practices regarding its buy-box. In 2021, the Italian Competition Authority (AGCM) fined Amazon €1.128 billion for abusing its dominant position by inducing sellers to adopt its own logistics service - Fulfilment by Amazon (FBA) - to increase their chances to be displayed in the buy-box.

our paper, active consumers can costlessly compare alternative offers and buy (if they decide to do so) from the retailer that offers the best match. In contrast, passive consumers can only buy (if they decide to do so) from the prominent retailer. Contrary to Moraga-González et al. (2021), due to the competition for prominence scheme, the prominent retailer is incentivized to lower the price and not lose the least elastic segment of the market.[5]

The large majority of recent papers dealing with prominence allocation considers monetary compensation in exchange of a prominent display (Raskovich, 2007; Armstrong and Zhou, 2011; Inderst and Ottaviani, 2012; Teh and Wright, 2022; Krämer and Zierke, 2020; Bourreau and Gaudin, 2022; Long et al., 2022). Generally, the intermediary offers a prominent display against monetary compensation (e.g., auctions or direct payments). Obtaining prominence is costly for a seller, which might translate into a higher final price for consumers. For example, Armstrong and Zhou (2011) show that, by granting prominence to the highest bidder, sellers' marginal costs are artificially increased and raise overall retail prices at the benefit of the intermediary.[6] In Inderst and Ottaviani (2012), instead, the intermediary allocates prominence, balancing two tensions: the commission it receives from sellers and the reputation cost the intermediary incurs whenever it recommends a not-suitable product. Teh and Wright (2022), instead, study a setting in which the intermediary's profits depend both on the margin sellers leave to obtain prominence and the commission they pay for each item sold. Thus, sellers compete both in prices and in commissions to obtain prominence. The authors find that steering harms both firms and consumers, and fiercer competition exacerbates price inflation due to the higher incentive of being recommended. Long et al. (2022) analyze how an intermediary can use the bidding system for sponsoring sellers to infer their private quality information and improve matching. The platform can optimally design listings using the commission fee, and the relative weight of the quality information revealed via the sellers' bid. We differ from the studies mentioned above in that we consider a different prominence scheme that does not entail a direct payment to the intermediary. Sellers compete for prominence by internalizing the incentives of the intermediary. Using such non-pricing instruments can intensify sellers' competition and alleviate the double marginalization problem otherwise emerging. Ultimately, this strategy benefits consumers as they are exposed to a lower final price but might harm sellers relative to a case in which prominence is allocated via an auction.

Our analysis also relates to the literature that deals with heterogeneous consumers. The population of active/passive consumers we study is very similar to the taxonomy shoppers/captives introduced by Varian (1980). However, mixed equilibria results are present in these studies as

[5]More in line with the findings of Moraga-González et al. (2021), when considering a pay for prominence scheme, the prominent retailer has an incentive to raise prices. Indeed, what is critical is the timing of the decisions undertaken by the competing retailers.

[6]In their setting, as in ours, there are two groups of consumers: informed ones, who costlessly compares sellers' offers, and uniformed ones, who only consider the offer of the default vendor. Competition for prominence, however, overturns the result of prices. As the intermediary obtains a commission fee for each transaction, it is interested in allocating prominence to the retailers that lower prices and expand the total demand.

sellers offer homogeneous products.(Armstrong and Zhou, 2011; Ronayne and Taylor, 2021). Relying on a model with vertically differentiated products, with elastic participation in both the passive and the active segments, we can obtain an equilibrium in pure strategy.[7,8]

The closest paper to ours is by Bar-Isaac and Shelegia (2022). The authors compare the allocation of prominence via an auction and the allocation of prominence by algorithms. They consider a market with a share of consumers that follow the intermediary's recommendation, and retailers pay a proportional fee to the intermediary. The authors find that the timing of prominence allocation plays a crucial role and that prominence allocated via algorithms can alleviate the double marginalization problem. In our model, the timing of the moves is also crucial, and the main result is that a prominence allocation scheme can increase efficiency by removing the vertical externality. However, we differ in several dimensions. First, in our setting, the intermediary allocates prominence algorithmically upon observing prices, so retailers are strategic in their pricing decisions. This means that the intermediary does not directly dictate prices. In Bar-Isaac and Shelegia (2022), the intermediary dictates a price and grants the privileged position to the retailer that follows the instructed price. Second, the two papers differ in the way market power is microfounded. In ours, retailers have market power because of their vertical differentiation, which might lead to endogenous market segmentation. In theirs, instead, market power arises because of a matching value or cost difference. Third, throughout our analysis, we distinguish between active and passive consumers, the latter being the steerable ones. We obtain results in pure strategy. In their paper, the presence of two different groups of consumers might instead lead to mixed strategy equilibria.

3 The model

Consider an economy where two retailers can only sell a product via a monopolist intermediary in exchange for a per-unit commission fee. Retailers sell homogeneous products but are vertically differentiated in the purchasing experience offered to consumers. For instance, one retailer provides a better customer experience (e.g., available call center and easy return policy) or ensures faster delivery than the other. We denote the retailer that offers quality q_H by H, and its low-quality rival offering q_L by L , with $0 < q_L < q_H$. We assume that H and L have identical marginal production and service costs, normalized to 0.[9]

[7] Note that with a high marginal cost for the high-quality retailer, a mixed strategy equilibria might exist, as discussed in the Appendix.

[8] More broadly, our paper also relates to the stream of studies focusing on the problem of steering in platform's also acting as retailers (De Corniere and Taylor, 2019; Anderson and Bedre-Defolie, 2021; Etro, 2021; Anderson and Bedre-Defolie, 2022; Hagiu et al., 2022; Zennyo, 2022), how platforms govern their own platform ecosystem (Teh, 2022), as well as they influence intra-platform competition (Casner, 2020; Karle et al., 2020; Jeon et al., 2022). In our analysis, we also abstract away from the issue of self-preferencing, which has been studied extensively in recent years (Cure et al., 2022; Chen and Tsai, 2019; Lee and Musolff, 2021; Hunold et al., 2022).

[9] In the Appendix, we relax this assumption and allow the high-quality retailer to face a higher marginal service cost than the low-quality retailers. Service costs can account for logistics and the implementation of the return

Consumers can only buy products via the intermediary and have a heterogeneous taste for quality $\theta \in [\underline{\theta}, \overline{\theta}]$. We assume θ is uniformly distributed with support $[0, 1]$. We assume that there are two types of consumers. A share $\lambda \in (0, 1)$ are *active* consumers and their choice set contains all listings. The remaining share of consumers, $1 - \lambda$, are *passive* consumers and their choice set contains only the offer prominently displayed by the intermediary.[10] We assume that θ is independent and identically distributed across consumers.

An active consumer that buys a product from a retailer $i = \{H, L\}$ obtains the following utility

$$U_i^{\lambda}(p_i) = \theta q_i - p_i \qquad \forall i = H, L,$$

where the superscript λ denotes the consumer's type, and p_i is the price set by retailer i. An active consumer type-θ buys from retailer i if, and only if,

$$U_i^{\lambda}(p_i) \geq \max\{U_{-i}^{\lambda}(p_{-i}), 0\},$$

and we denote by $D_i^{\lambda}(p_i, p_{-i})$ the mass of active consumers that buy from retailer i.

A passive consumer that buys a product from a prominent retailer obtains the following utility

$$U^{1-\lambda}(p) = \theta q - p, \tag{1}$$

where the superscript $1 - \lambda$ denotes the consumer's type, q and p represent the quality and the price of the prominent retailer identified by the intermediary, respectively. A passive consumer type-θ buys from retailer i if, and only if,

$$U^{\lambda}(p) \geq 0,$$

and we denote by $D^{1-\lambda}(p)$ the mass of passive consumers that buy from the prominent retailer.

Retailers pay the intermediary a commission fee, denoted by w, for any transaction. We distinguish the settings in which either L or H are prominent by using the superscript L or H. If H receives the prominent position, the profits of the high and low-quality retailers are, respectively,

$$\begin{aligned}
\pi_H^H(p_H, p_L, w) &= \Big[\lambda D_H^{\lambda}(p_H, p_L) + (1 - \lambda) D_H^{1-\lambda}(p_H)\Big](p_H - w), \\
\pi_L^H(p_L, p_H, w) &= \Big[\lambda D_L^{\lambda}(p_L, p_H)\Big](p_L - w).
\end{aligned}$$

policy.

[10]This setting can be microfounded by assuming that active consumers costlessly compare all listings before making their purchasing decision whereas passive consumers have a sufficiently high search cost so that they always find it optimal to rely on the offer prominently displayed intermediary.

In this case, the profit of the intermediary is

$$\Pi^H(p_H, p_L, w) = w\Big[\lambda\Big(D_H^{\lambda}(p_H, p_L) + D_L^{\lambda}(p_L, p_H)\Big) + (1-\lambda)D_H^{1-\lambda}(p_H)\Big].$$

On the contrary, if L receives the prominent position, instead, the profits of the high and low-quality retailers are, respectively

$$\pi_H^L(p_H, p_L, w) = \Big[\lambda D_H^{\lambda}(p_H, p_L)\Big](p_H - w),$$
$$\pi_L^L(p_L, p_H, w) = \Big[\lambda D_L^{\lambda}(p_L, p_H) + (1-\lambda)D_L^{1-\lambda}(p_L)\Big](p_L - w).$$

In this case, the profit of the intermediary is

$$\Pi^L(p_H, p_L, w) = w\Big[\lambda\Big(D_H^{\lambda}(p_H, p_L) + D_L^{\lambda}(p_L, p_H)\Big) + (1-\lambda)D_L^{1-\lambda}(p_L)\Big].$$

We assume that the intermediary's profits are well-behaved, that is quasi-concave, continuous, and twice-differentiable.

We assume that the intermediary assigns the prominent position to the retailer that yields the largest monetary return for a given commission fee and given retail prices and that the allocation rule is general knowledge.[11] Specifically, the intermediary compares the profit obtained in the two scenarios in which the passive segment is exclusively assigned to either retailer. Therefore, the intermediary assigns prominence to the high-quality retailer if, and only if,

$$\Pi^H(p_H, p_L, w) \geq \Pi^L(p_H, p_L, w). \tag{2}$$

Otherwise, the intermediary assigns prominence to the low-quality retailer. As a tie-breaking rule, due to the superior quality offered by H, we assume that the intermediary assigns prominence to H in any condition of indifference.

The timing is the following:

- In period 1, the intermediary sets the commission fee w;
- In period 2, retailers compete in prices by setting respectively p_H and p_L;
- In period 3, the intermediary allocates prominence to the retailer that maximizes its total profit;
- In period 4, consumers in each group discover their type θ. Active consumers compare the two offers and decide whether to buy and from which retailer to buy (if so). Passive

[11] Note that, as discussed already, our results are equivalent to the case in which the intermediary does not commit to assign prominence based on a specific rule. As a result, as long as a buy box is present, retailers would anticipate that prominence will be assigned to the retailer that maximizes the intermediary's profit and will set prices so that they receive at equilibrium a prominent display if they find it optimal.

consumers decide whether to buy from the prominent retailer.

We solve the game by backward induction.

4 Analysis

In this Section, we present the analysis of the model by considering the fee w as exogenously-given. Furthermore, we consider the case in which w solves the maximization problem of the intermediary. Then, we discuss the welfare effects of the prominence allocation scheme hereby considered. Finally, we consider an alternative prominence scheme based on a direct monetary compensation and we compare the market outcomes with those resulting from a competition for prominence setting.

4.1 Analysis with an exogenous commission fee

We start by analyzing the demands of each retailer. We determine the active inframarginal and marginal consumers and, for the passives, the marginal consumer.

Regardless of the retailer receiving prominence, the mass of active consumers buying from L and H are,

$$D_L^{\lambda}(p_L, p_H) = \frac{p_H - p_L}{q_H - q_L} - \frac{p_L}{q_L}, \qquad D_H^{\lambda}(p_H, p_L) = 1 - \frac{p_H - p_L}{q_H - q_L}, \tag{3}$$

respectively. The mass of passive consumers that buy from the prominent retailer

$$D_L^{1-\lambda}(p_L) = 1 - \frac{p_L}{q_L}, \qquad D_H^{1-\lambda}(p_H) = 1 - \frac{p_H}{q_H}, \tag{4}$$

if either L or H are assigned prominence, respectively.

In the third stage, the intermediary allocates prominence to the retailer generating the largest return for a given commission fee. Anticipating the total realized demand in the two cases, the intermediary assigns prominence to H if the condition in (2) holds, that is, if and only if

$$\begin{gathered}\Pi^H(p_H, p_L, w) \equiv \left[\lambda\Big(1 - \frac{p_H - p_L}{q_H - q_L}\Big) + \lambda\Big(\frac{p_H - p_L}{q_H - q_L} - \frac{p_L}{q_L}\Big) + (1-\lambda)\Big(1 - \frac{p_H}{q_H}\Big)\right] w \geq \\ \left[\lambda\Big(1 - \frac{p_H - p_L}{q_H - q_L}\Big) + \lambda\Big(\frac{p_H - p_L}{q_H - q_L} - \frac{p_L}{q_L}\Big) + (1-\lambda)\Big(1 - \frac{p_L}{q_L}\Big)\right] w \equiv \Pi^L(p_H, p_L, w).\end{gathered} \tag{5}$$

Simplifying the above expression, it is possible to identify the condition on the prices for which the high-quality retailer obtains prominence. We report this result in the following lemma.

Lemma 1. *The intermediary assigns prominence to the high-quality retailer if*

$$p_H \leq \frac{q_H}{q_L} p_L. \tag{6}$$

Otherwise, prominence is assigned to the low-quality retailer.

Lemma 1 implies that it is sufficient for H to match the quality-adjusted price of L to maximize the intermediary's profit and be selected as the prominent retailer. An implication of the lemma is that the prominence rule induces retailers interested in becoming prominent to lower their prices.

Moreover, we also note that if (6) is satisfied with equality so that $p_H = \frac{q_H}{q_L} p_L$, H can receive prominence and also foreclose the rival. Specifically,

$$D_L^{\lambda}(p_L, p_H) = 0 \qquad \forall p_L \geq w.$$

In other words, it is sufficient for H, for a given price of L, to match the quality-adjusted price of the rival and steal all its potential customers. Instead, the low-quality retailer, conditional on being prominent, cannot foreclose H as the latter attracts active consumers due to its quality advantage. The following lemma summarizes this result.

Lemma 2. *If the high-quality retailer is prominent, the low-quality retailer is foreclosed. If the low-quality retailer is prominent, the two retailers compete for active consumers.*

Lemma 2 implies that if H finds it optimal to become prominent for any p_L, then it always forecloses L by setting $p_H = \frac{q_H}{q_L} p_L$. In this case, there is a foreclosure equilibrium. If H does not find it optimal to obtain prominence, then a market-sharing equilibrium arises.

In order to identify which equilibrium exists and under which conditions this is the case, we study the retailers' decisions in the pricing-setting stage. In the second stage of the game, retailers anticipate the allocation of prominence made by the intermediary and compete in prices. By Lemma 2, if H is prominent, L obtains zero demand and profits. Therefore, conditional on H being prominent, for any pair of prices (p_L, p_H), the low-quality retailer always has an incentive to undercut the quality-adjusted price of H and become prominent. This undercutting strategy can be blocked only in two scenarios: either because (i) L has reached its price floor, equal to the commission fee, and H finds it optimal to become prominent, thereby matching the quality-adjusted price of L with $p_H = \frac{q_H}{q_L} w$; or because (ii) H finds it optimal not to serve the passive consumers and to only focus on the active ones. In the latter scenario, L can set a price $p_L > w$, and both retailers simultaneously and non-cooperatively maximize their profits so that these are equal to:

$$p_H^L(w) = \arg\max_{p_H} \lambda D_H^{\lambda}(p_H, p_L)(p_H - w),$$

$$p_L^L(w) = \arg\max_{p_L} \left(\lambda D_L^\lambda(p_L, p_H) + (1-\lambda) D_L^{1-\lambda}(p_L)\right)(p_L - w),$$

subject to $p_L^L(w) < \frac{q_L}{q_H} p_H^L(w)$. Solving the system of equation, we get

$$\begin{aligned} p_H^L(w) &= \frac{2q_H^2 - (3-\lambda)q_H q_L + 3q_H w + (1-\lambda)q_L(q_L - 3w)}{4q_H - (4-3\lambda)q_L}, \\ p_L^L(w) &= \frac{2(q_H - q_L)(q_L + w) - \lambda q_L(q_H - q_L - 3w)}{4q_H - (4-3\lambda)q_L}, \end{aligned} \tag{7}$$

Note that $p_L^L(w) < \frac{q_L}{q_H} p_H^L(w)$ if and only if $q_L > 2w$ and $\lambda > \frac{q_L(q_L-3w)-2q_H w}{q_L(q_H+q_L-3w)} \equiv \hat{\lambda}$.[12] In other words, L can maximize profits and maintain a prominent allocation as long as its quality is sufficiently large and there is a large portion of active consumers. In all other cases, the low-quality retailer is never able to compete for prominence for any given pair of prices (p_L, p_H) and, as a result, H is prominent, and L is foreclosed. This discussion is summarized in the following lemma.

Lemma 3. *For a given commission fee w, a necessary condition for the low-quality retailer to obtain prominence by setting $p_L^L(w) < \frac{q_L}{q_H} p_H^L(w)$, conditional on H not competing for prominence is that the following conditions are jointly satisfied*

$$\lambda > \frac{2q_H w - q_L(q_L - 3w)}{q_L(3w - q_H - q_L)} \equiv \hat{\lambda}(w), \qquad \frac{q_L}{2} > w.$$

If the above conditions are not satisfied, L is foreclosed, and H is prominent.

In what follows, we study whether a parameter range exists in which H prefers to give up prominence. To this end, we provide an analysis at the limit.[13] Suppose the share of active consumers is slim so that $\lambda \to 0$. Then, most of the profits of the two retailers are made among the passive consumers. This scenario implies that reaching the passive consumers by obtaining prominence becomes more important for both. Then, H always has an incentive to serve the passive consumers. In order to avoid foreclosure, L always has an incentive to engage in the undercutting strategy. Therefore, L reaches its lowest possible price, equivalent to w, whereas H becomes prominent by matching the quality-adjusted price of L. In turn, this strategy blocks further deviation and forecloses the low-quality rival.[14]

Suppose now the share of active consumers is considerable so that $\lambda \to 1$. Then, most of the profits of the two retailers are made among the active consumers. This scenario implies that the incentive of H to compete for prominence decreases, whereas the incentive of L to obtain prominence remains essential to avoid foreclosure. As a result, no deviation takes place from L

[12]To see why, note that $p_L^L = \frac{q_L}{q_H} p_H^L$ at $\lambda = \hat{\lambda}$, which is positive if and only if $q_L > 2w$.
[13]Additional details can be provided upon request to the authors.
[14]Also note that at $\lambda = 0$, one of the two conditions in Lemma 3 is violated.

and, provided that H gives up prominence, a market-sharing equilibrium in which both retailers are active and set $p_H > w$ and $p_L > w$ exists.

By continuity, the above discussion implies that there exists a critical threshold $\tilde{\lambda}(w)$ for which H is indifferent between giving up prominence or undercutting the rival. The following lemma provides conditions on the equilibrium that emerges in the market for a given commission fee.

Lemma 4. *For any given commission fee w, H has an incentive to obtain (resp. give up) prominence prominence if $\lambda < \tilde{\lambda}(w)$ (resp. $\lambda > \tilde{\lambda}(w)$).*

Lemma 3 and 4 together imply that if $\lambda \leq \max\{\tilde{\lambda}(w), \hat{\lambda}(w)\}$, L is never prominent either because H finds it optimal to obtain prominence or because L is never able to sustain competition for prominence. Therefore, the equilibrium prices are $p_H^H = \frac{q_H}{q_L} w$ a $p_L^H = w$. The profits of the two retailers are, respectively,

$$\pi_H^H(p_H^H, p_L^H, w) = \frac{(q_H - q_L)(q_L - w)w}{q_L^2}, \qquad \pi_L^H(p_L^H, p_H^H, w) = 0.$$

Importantly, the low-quality retailer is foreclosed, and the platform profits only through the sales generated by the high-quality retailer in the two market segments. Specifically, the intermediary obtains

$$\Pi^H(p_H^H, p_L^H, w) = \frac{(q_L - w)w}{q_L}.$$

On the contrary, if $\lambda \in [\max\{\tilde{\lambda}(w), \hat{\lambda}(w)\}, 1]$, H finds it optimal to give up the active consumers and L's best response price allows it to obtain prominence. In this case, the low-quality retailer is not foreclosed and makes profits competing with H for the actives and being a default monopolist for the passive consumers. The equilibrium prices are in (7) and the associated profits of the retailers are, respectively, given by

$$\begin{aligned} \pi_H^L(p_H^L, p_L^L, w) &= \frac{\lambda(q_H - q_L)[(1-\lambda)q_L - 2q_H + w]^2}{[4q_H - (4-3\lambda)q_L]^2}, \\ \pi_L^L(p_L^L, p_H^L, w) &= \frac{(q_H - q_L)[q_H - (1-\lambda)q_L][2w - (2-\lambda)q_L]^2}{q_L[4q_H - (4-3\lambda)q_L]^2}. \end{aligned} \tag{8}$$

In this case, the intermediary collects the per-unit commission fee from both retailers and obtains

$$\Pi^L(p_H^L, p_L^L, w) = \frac{\lambda q_L w(q_H + 2q_L - 3w) + 2w(q_H - q_L)(q_L - w)}{q_L[4q_H - (4-3\lambda)q_L]}.$$

We summarize this discussion in the following proposition.

Proposition 1. *For any given commission, if $\lambda < \max\{\tilde{\lambda}(w), \hat{\lambda}(w)\}$ there exists a unique equilibrium with the high-quality retailer being prominent in which $p_H^H(w) = q_H/q_L w$ and*

$p_L^H(w) = w$. If $\lambda \geq [\max\{\tilde{\lambda}(w), \hat{\lambda}(w)\}, 1]$ there exists a unique market-sharing equilibrium with the low-quality retailer being prominent $p_H^L > w$ and $p_L^L > w$.

The above proposition states that a market-sharing equilibrium is only possible for a given commission fee if the share of active consumers is sufficiently large. Put differently. Suppose the number of consumers that rely on the buy-box to make their purchasing decisions is large. In that case, there is only one equilibrium in which the low-quality retailer is foreclosed, and all transactions occur with the high-quality retailer. We end up in such a situation either because the high-quality prefers to price aggressively to become prominent or because the low-quality retailer cannot induce a market-sharing equilibrium when the high-quality retailer is willing to give up prominence. The market-sharing equilibrium occurs, for a given fee, only when the population of consumers that rely on the buy-box is sufficiently small. In this equilibrium, the prices are the ones that would occur in a traditional duopoly model with vertical differentiation.

4.2 Endogenous commission fee

So far, we have considered the optimal choice of the intermediary assuming the commission fee to be exogenous. In what follows, we endogenize the decision about the commission fee and study whether and how the intermediary can induce one of the two equilibria in Proposition 1 in the first stage of the game. The intermediary sets w to maximize the following profit

$$\Pi(p_H, p_L, w) = w \times \begin{cases} \left[\lambda D_H^{\lambda}(p_H^H, p_L^H, w) + (1-\lambda) D_H^{1-\lambda}(p_H^H, w)\right] & \text{if } \lambda < \min\{\tilde{\lambda}(w), 1\}, \\ \left[\lambda D_H^{\lambda}(p_H^L, p_L^L, w) + \lambda D_L^{\lambda}(p_H^L, p_L^L, w) + (1-\lambda) D_L^{1-\lambda}(p_L^L, w)\right] & \text{if } \lambda \geq \min\{\tilde{\lambda}(w), 1\}. \end{cases}$$

If $\lambda < \min\{\tilde{\lambda}(w), 1\}$, from the first-order condition with respect to w, we obtain

$$w^H = \frac{q_L}{2},$$

and the associated equilibrium profit is

$$\Pi^H(p_H^H, p_L^H, w^H) = \frac{q_L}{4}.$$

If $\lambda \geq \min\{\tilde{\lambda}(w), 1\}$, from the first-order condition ith respect to w, we obtain

$$w^L = \frac{q_L[q_H(2+\lambda) - 2(1-\lambda)q_L]}{4(q_H - q_L) + 6\lambda q_L},$$

and the associated equilibrium profit is

$$\Pi^L(p_H^L, p_L^L, w^L) = \frac{q_L[(2+\lambda)q_H - 2(1-\lambda)q_L]^2}{4[4q_H - (4-3\lambda)q_L][2q_H - (2-3\lambda)q_L]}.$$

Evaluating $\tilde{\lambda}$ at $w = w^H$

$$\tilde{\lambda}(w)\Big|_{w=w^H} = 1,$$

meaning that if the intermediary sets the commission fee that is the optimal condition on H being prominent, the parameter space in which H finds it optimal to compete for prominence is $\lambda \in [0, 1)$. In order to verify whether the platform has such an incentive, we compare the intermediary's profit in the two cases. We obtain that

$$\Pi^H(p_H^H, p_L^H, w^H) > \Pi^L(p_H^L, p_L^L, w^L).$$

The above expression indicates that the intermediary's profit is always higher when it assigns prominence to H. As a result, when the commission fee is endogenized, a unique subgame perfect Nash equilibrium exists, with H being prominent and L being foreclosed. The following proposition summarizes the above discussion.

Proposition 2. *There exists a unique equilibrium in which the intermediary sets a commission fee equal to* $w^H = q_L/2$ *and, for any* $\lambda \in (0, 1)$, *the high-quality retailer is prominent, and the low-quality retailer is foreclosed.*

In the following Subsection, we discuss how the market structure chosen by the intermediary affects efficiency and welfare.

4.3 Competition for prominence and efficiency

In what follows, we compare the welfare effects of the competition for prominence setting, as defined in our model, with those arising in a classic vertical chain and to the case in which there is a multiproduct monopolist. In the former case, there is a typical vertical externality which is inefficient. In the latter case, the intermediary maximizes profits directly controlling the price of two offers in the marketplace.

Classic vertical chain setting. The first comparison we provide is for the case in which there is a classic vertical chain, and the intermediary does not segment the market between the two types of consumers. Therefore, suppose the intermediary cannot steer consumers, and all consumers are active (i.e., $\lambda = 1$). Due to vertical differentiation, both retailers sell a strictly positive amount of products, with high-type consumers going to the high-quality retailer, medium-type consumers going to the low-quality retailer, and the remaining consumers deciding not to buy. The market structure resembles a classical vertical chain setting, in which the intermediary acts as the upstream firm providing an intermediation service to the two competing and differentiated downstream retailers. A typical inefficiency arising in these markets comes from the double marginalization problem, which implies that the equilibrium price of the two retailers is above the monopoly price. The high prices cause a loss of consumer welfare. In turn, the reduced demand lowers retailers' surplus. Competition for prominence, and the possibility to steer a share of consumers, mitigate this problem and therefore raises welfare. Nevertheless, there are distributive issues as benefits from higher total welfare should be weighed against the market foreclosure of the low-quality retailer occurring when there is competition for prominence. However, in the Appendix, we show that competition for prominence maximizes total welfare, and the benefits obtained by the high-quality retailer more than compensate for the losses of the low-quality one.

Multiproduct monopolist. The second comparison we provide concerns the case in which the intermediary is a multiproduct monopolist proposing low-quality and high-quality offers and directly controlling their prices. Differently from the baseline model, in this setting, the intermediary does not charge any commission fee as it is vertically integrated with the retailers. Moreover, by default, there is no vertical externality, implying that its strategy is the most profitable. Regardless of the offer being made prominent, the intermediary always sets the following monopoly prices $p_H^M = \frac{q_H}{2}$ and $p_L^M = \frac{q_L}{2}$ for each retailer, where the superscript M indicates the multiproduct setting. As prices and total demands are the same in either prominence allocation, the intermediary can boost its profits by assigning prominence to the high-quality offer. Thus, the intermediary steers all consumption to the high-quality offers, and there is no market for the low-quality offer.[15] Comparing the profit of the intermediary in the competition for prominence and in the multiproduct monopolist settings, we have $\Pi^M(p_H^M, p_L^M) \equiv \frac{q_H}{4} > \frac{q_L}{4} \equiv \Pi^H(p_H^H, p_L^H, w^H)$

whereas the total industry profits and consumer welfare in the two cases are identical:

$$\Pi^H(p_H^H, p_L^H, w^H) + \pi_H^H(p_H^H, p_L^H, w^H) = \frac{q_H}{4} \equiv \Pi^M(p_H^M, p_L^M)$$

$$CS^H(p_H^H, p_L^H) \equiv \frac{q_H}{8} \equiv CS^M(p_H^M, p_L^M)$$

The presence of two types of consumers and the possibility of steering the passive consumers allows the intermediary to make the competition between retailers much fiercer. This way, the intermediary removes the vertical externality that is present otherwise. This result also suggests that while the intermediary cannot directly control the strategy of the two retailers, it can strategically design its marketplace to influence the prices set by the retailers by making them compete for a prominent position. In other words, competition for prominence allows the intermediary to replicate the behavior of a multiproduct monopolist without directly controlling the two offers.

The following Subsection discusses an alternative prominence allocation scheme that the inter-

[15] Contrary to Mussa and Rosen (1978), the presence of high- and low-quality offers does not imply quality discrimination among consumers with different tastes for quality. Indeed, at equilibrium, none of them finds it optimal to purchase the low-quality offer.

mediary could employ. We compare its market outcomes to the ones we obtain under competition for prominence.

4.4 Alternative prominence allocation: pay for prominence

In this Section, we consider the pay for prominence scheme, which requires the default retailer to pay the intermediary to obtain the prominent position. In order to microfound the payment between the retailer and the intermediary, we assume that the intermediary launches an auction for prominence, and the retailer that submits the highest bid is awarded prominence and pays the amount bid by the second bidder. The timing is therefore modified as follows

- In period 1, the intermediary sets the commission fee w;
- In period 2, retailers submit their bids simultaneously, and the highest bidder wins prominence;
- In period 3, retailers compete in prices by setting respectively p_H and p_L;
- In period 4, consumers in each group discover their type θ. Active consumers compare the two offers and decide whether to buy and from which retailer to buy (if so). Passive consumers decide whether to buy from a prominent retailer.

The above timing also unveils a critical difference between the baseline model with competition for prominence and the case pay for prominence scheme. In the former case, the intermediary does not commit to assigning prominence before the competition occurs. In the latter, the intermediary assigns prominence to the highest bidder before the competition occurs, and the intermediary's choice is visible to both retailers.

The gross profits of retailers H and L in this setting are identical to those in the baseline model. Also in this case, we distinguish the settings in which either L or H are prominent by using the superscript L or H. If H receives the prominent position, the profits of the high and low-quality retailers are, respectively,

$$\begin{aligned}\pi_H^H(p_H, p_L, w) &= \Big[\lambda D_H^\lambda(p_H, p_L) + (1-\lambda) D_H^{1-\lambda}(p_H)\Big](p_H - w),\\ \pi_L^H(p_L, p_H, w) &= \Big[\lambda D_L^\lambda(p_L, p_H)\Big](p_L - w),\end{aligned}$$

If L receives the prominent position, instead, the profits of the high and low-quality retailers are, respectively.

$$\begin{aligned}\pi_H^L(p_H, p_L, w) &= \Big[\lambda D_H^\lambda(p_H, p_L)\Big](p_H - w),\\ \pi_L^L(p_L, p_H, w) &= \Big[\lambda D_L^\lambda(p_L, p_H) + (1-\lambda) D_L^{1-\lambda}(p_L)\Big](p_L - w).\end{aligned}$$

The net profit of the retailer i that submits the highest bid and pays a monetary compensation to the intermediary to obtain prominence, which we denote by $\tilde{\pi}_i^i$, is equal to the gross profit of that retailer net of the payment to the intermediary, which we denote by b_i. Specifically, $\tilde{\pi}_i^i(p_i, p_{-i}) = \pi_i^i(.) - b_i$.

Note that we solve the model by backward induction. The latest stage of the game is the same as in the baseline model. In period 3, retailers make their pricing decisions. Importantly, what changes now is that retailers make their pricing decisions knowing they have been awarded prominence. The equilibrium prices in the continuation game in which H is prominent are

$$p_H^H = \arg\max_{p_H} \left(\lambda D_H^\lambda(p_H, p_L) + (1-\lambda) D_H^{1-\lambda}(p_H)\right)(p_H - w) - b_H,$$

$$p_L^H = \arg\max_{p_L} \lambda D_L^\lambda(p_L, p_H)(p_L - w),$$

and the equilibrium prices in the continuation game in which L is prominent are

$$p_H^L = \arg\max_{p_H} \lambda D_H^\lambda(p_H, p_L)(p_H - w),$$

$$p_L^L = \arg\max_{p_L} \left(\lambda D_L^\lambda(p_L, p_H) + (1-\lambda) D_L^{1-\lambda}(p_L)\right)(p_L - w) - b_L.$$

Having determined the optimal prices in the two subgames — the one in which L is prominent and the one in which H is prominent — we move to period 2. We determine the bids that each retailer submits in the position auction. In order to do so, we first compute each retailer's marginal benefit from being prominent, equivalent to the difference in the profits obtained by the retailers when receiving prominence and the profit obtained when not receiving prominence.

Denoting WTP_i the willingness to pay for prominence of retailer i, for $i = L, H$, we get

$$\begin{aligned} WTP_L :=& \pi_L^L(p_L^L, p_H^L, w) - \pi_L^H(p_L^H, p_H^H, w), \\ WTP_H :=& \pi_H^H(p_H^H, p_L^H, w) - \pi_H^L(p_H^L, p_L^L, w). \end{aligned}$$

Comparing the two in the Appendix, we observe that the following inequality always holds

$$\pi_L^L(p_L^L, p_H^L, w) + \pi_H^L(p_H^L, p_L^L, w) < \pi_H^H(p_H^H, p_L^H, w) + \pi_L^H(p_L^H, p_H^H, w),$$

which indicates that total retailers' profits, for given w, are always larger when the high-quality retailer is prominent.[16] This implies that $WTP_H > WTP_L$, that is, due to its superior quality, the high-quality retailer has more to gain than the rival thanks to a prominent display, and therefore it has a higher willingness to pay relative to the low-quality retailer. The following lemma presents this comparison.

[16]Note that this is consistent with the result obtained with the multiproduct monopolist.

Lemma 5. *For a given commission fee, the high-quality retailer is more willing to pay than the low-quality retailer.*

The above lemma also suggests that if the intermediary awards prominence via a second sealed-bid auction, the high-quality retailer is always in the condition to outbid the low-quality retailer in light of its higher willingness to pay. Specifically, in an auction, H can secure prominence by bidding slightly above the willingness-to-pay of the low-quality retailer; therefore, $b_H = WTP_L$. This result is summarized in the following proposition.

Proposition 3. *For a given commission fee, with a pay-for-prominence scheme, the intermediary always awards prominence to the high-quality retailer and the optimal bid is* $b_H = WTP_L$.

In the first stage of the game, the intermediary anticipates that prominence will always be awarded to the high-quality retailer. As a result, it sets the commission fee such that

$$w^H = \arg\max_w w\Big[\lambda D_H^\lambda(p_H^H, p_L^H, w) + \lambda D_L^\lambda(p_L^H, p_H^H, w) + (1-\lambda) D_H^{1-\lambda}(p_H^H, w)\Big] + b_H(w).$$

We now compare the market outcomes between the competition for prominence (superscript CP) and the pay-for-prominence schemes (superscript PP). Details are reported in the Appendix.

We start comparing the optimal fee set by the intermediary, which is always lower under competition for prominence, $w^{CP} < w^{PP}$. The lower commission fee implies a large total demand $TD^{CP} > TD^{PP}$. Despite the lower margin on each transaction, we show that competition for prominence always leads to a higher profit for the intermediary. The reason is that the pay for prominence case is akin to the classic vertical chain setting discussed in the previous Subsection (4.3), allowing the intermediary to control final prices relative to pay for prominence better. In the latter case, changing the commission fee also leads to a change in the optimal bid of the high-quality retailer. The monetary compensation received by the intermediary for the prominent display does not compensate for the loss of revenues from reduced demand. More importantly, pay-for-prominence requires a commitment of the intermediary to assign prominence to the highest bidder before prices are set, which generates a double marginalization problem that competition for prominence would remove. Therefore, the commission fee set by the intermediary is inefficiently high, and the total demand captured is lower than the competition for prominence setting, as prices are also above the monopoly level. This discussion shows that the intermediary would always find it optimal to assign prominence by adequately designing a contest and letting retailers compete when the alternative is an auction-based prominence allocation. This discussion is summarized in the following proposition.

Proposition 4. *An intermediary strictly prefers employing a competition for prominence scheme over a monetary-based prominence allocation scheme.*

5 Quality upgrade

In the baseline model, we started our analysis by considering a setting with retailers exogenously vertically differentiated. As they sell the same product, the prominence problem we analyze is particularly relevant when there is asymmetry among retailers. In this Section, we endogenize the quality decision of retailers by considering a setting in which the intermediary offers homogeneous retailers the possibility to purchase a quality upgrade. The quality upgrade is akin to marketplaces offering services that allow for faster logistics, generous return policies, and 24/7 customer services. As the marketplaces can exploit large economies of scale, we suppose that these services proposed by the intermediary are of larger quality than those the retailers may have access to. Therefore, in this Section, we study a scenario in which the two symmetric retailers offer the consumers ancillary services of quality q_L. The intermediary offers the retailers the possibility to purchase its ancillary services of higher quality q_H, with $q_H > q_L$, for a fixed upgrading fee F. The upgrading decision of retailers takes place prior to the pricing stage.

The timing of the game is modified as follows.

- In period 1, the intermediary sets the commission fee w and the fixed upgrading fee F.
- In period 2, retailers decide whether to upgrade their quality by purchasing it from the intermediary. This decision is public.
- In period 3, retailers set their prices simultaneously and non-cooperatively.
- In period 4, the intermediary allocates prominence to the retailer that maximizes its total profit.
- In period 5, consumers in each group discover their type θ. Active consumers compare the two offers and decide whether to buy and from which retailer to buy (if so). Passive consumers decide whether to buy from a prominent retailer.

We solve the game by backward induction. With respect to the baseline model, the following cases can occur.

(i) **No upgrade**. Both retailers do not find it optimal to upgrade their quality and are homogeneous in their intrinsic quality and the ancillary quality provided. As a result, product homogeneity implies Bertrand-type of competition, with a unique equilibrium determined by the retailers' price floor, consisting of the per-unit fee w. At equilibrium, both retailers have the same chance of obtaining a prominent allocation and earning a profit equal to zero.

(ii) **Joint upgrade**. Retailers are ex ante and ex post symmetric. Therefore, a Bertrand-type competition occurs between homogeneous retailers leading to a unique equilibrium in which both retailers price their products at a value equal to the commission fee and have equal chances to obtain prominence. Because the upgrading fee F is sunk, retailers have negative profits for any $F > 0$.

(iii) **Asymmetric upgrade**. Only one retailer upgrades its quality, creating a case in which intermediary-driven vertical differentiation exists. This case is equivalent to the one emerging in the baseline model. The only difference is that the retailer upgrading its quality has to pay the upgrading fee of F to the intermediary before the competition occurs. Therefore, as in the baseline model, for a given commission fee, depending on $\lambda(w)$, either a market-sharing or a foreclosing equilibrium can arise.

Independently of whether the asymmetric cases lead to a foreclosing equilibrium or a market-sharing equilibrium, the game with quality upgrades can be represented in the following normal form:

		Retailer -i	
		U	NU
Retailer i	U	$\pi_H^B - F, \pi_H^B - F$	$\pi_H - F, \pi_L$
	NU	$\pi_L, \pi_H - F$	π_L^B, π_L^B

Table 1: Upgrading decision stage.

where the superscript B indicates the profits resulting from Bertrand's competition. For any w, in case of a joint upgrade, (U, U), profits are negative, whereas, in case of no upgrade, (NU, NU), profits are 0. Therefore, it is strictly dominant for retailers to be in an asymmetric scenario, (U, NU) or (NU, U). The reason is that, for a given commission fee, a foreclosing equilibrium implies zero profits for the low-quality retailer, whereas the market-sharing equilibrium always implies positive profits. Independently of the value of λ, solving the normal form game yields only asymmetric equilibria, with only one of the two retailers finding it optimal to upgrade quality.[17]

We can now solve for the optimal commission and upgrading fees conditional on an asymmetric outcome to occur at equilibrium. As in the baseline model, the intermediary sets in the first stage $w^H = \frac{q_L}{2}$ and induces H to compete for prominence as $\tilde{\lambda}(w^H) = 1$, whereas the optimal upgrading fee solves the incentive-compatibility constrain of the retailer for which $\pi_H^H(p_H^H, p_L^H, w^H, F) - F \geq \pi_L^L(p_L^H, p_H^H, w^H)$. As at equilibrium, the constraint binds

$$F = \pi_H^H(p_H^H, p_L^H, w^H, F) - \pi_L^L(p_L^H, p_H^H, w^H) = \frac{q_H - q_L}{4},$$

[17]Note that if F were not sunk, retailers would obtain a profit equal to zero in the joint upgrade scenario, meaning that retailers would be indifferent between upgrading or not. This would result in a multiplicity of equilibria, (U, U),(U, NU) or (NU, U). As the intermediary always extracts monopoly rents in any of the three equilibria, it would be indifferent between either outcome and would not solve the coordination game.

the linear upgrading fee allows the intermediary to extract the remaining surplus of the high-quality retailer. As a result, the profit of the intermediary is

$$\Pi^H(F, p_H^H, p_L^H, w^H) = \frac{q_H}{4},$$

which represents the highest profit obtainable. In other words, by allowing retailers that are ex ante homogeneous to become ex post heterogeneous and letting them compete for prominence, the intermediary can fully extract all the rents in the market.

Proposition 5. *The intermediary can replicate the profit level of the multiproduct monopolist setting by combining the competition for prominence scheme with the offering of ancillary services that enhance retailers' quality.*

5.1 Discussion

We analyzed the possibility for the intermediary to sell a quality upgrade of the ancillary services to the retailers to enhance consumers' purchasing experience. This business strategy is, in principle, efficient in terms of total welfare. However, we have shown that the distribution of rents only favors the intermediary, and the efficiency argument holds if the services of the intermediary are not offered below costs and are either the only ones available or have the best quality-price ratio offer in the market.

A survey reported in a recent investigation by the Italian Competition Authority (AGCM) against Amazon (case A528)[18] shows that 42% of retailers on Amazon's Italian marketplace using FBA would stop paying for the fulfillment program if the latter would only consist in its logistics services. While some associated services are highly appreciated (i.e., 87% of retailers use FBA because customers enjoy fast shipping and generous return policies, and 80% because logistics are integrated with the platform), a vast majority of sellers consider the FBA program to be vital because of the higher visibility they obtain (85%), better ratings (71%),[19] and higher likelihood of winning the buy-box (57%).[20]

These answers show that there are some benefits associated to the FBA program that both retailers and consumers enjoy. However, the efficiency argument may not hold. For instance, almost half of the retailers would prefer to use third-party operators to fulfill an order. Moreover, in Amazon's reply to the investigation, it seems that the FBA service is provided, at most, at

[18] The press release is available here: https://en.agcm.it/en/media/press-releases/2021/12/A528. The full investigation document is only available in Italian (https://www.agcm.it/dotcmsdoc/allegati-news/A528_chiusura%20istruttoria.pdf).

[19] Note that Amazon deletes negative reviews related to the delivery experience when the product is shipped using its logistics. Therefore, the retailer using FBA does not suffer any backlash from logistic problems. See https://www.promarket.org/2019/11/06/how-amazon-rigs-its-shopping-algorithm/.

[20] AGCM – case A528, para. 316-317.

cost.[21] Indeed, depending on the entity of these costs, it may not be efficient for the intermediary to offer these high-quality ancillary services from a total welfare perspective.

6 Conclusions

Our analysis provides insights into the optimal prominence scheme that intermediaries can implement. We show that orchestrating a contest for the default position fosters competition and induces retailers to set low prices and expand demand, thereby benefiting both the intermediary and consumers. Competition for prominence is a powerful tool to eliminate double marginalization in markets with a vertical structure. Alternative prominence allocation schemes that, for example, rely on monetary instruments (e.g., position auctions) are always dominated by competition for prominence. This is because they rely on the intermediary's commitment to grant prominence to a retailer, which, in turn, relaxes competition.

These results present a rationale for why large retail companies, such as Walmart in the United States, or Fnac in France, have started adopting a platform business model, allowing third-party sellers to operate on their marketplace. These platforms collect all listings associated with a specific product on one page and make one of them prominent via their version of the buy-box. This evidence may suggest that new entrants know that competition for prominence scheme is crucial to success in this industry.

Our study shows that competition for prominence maximizes social welfare, but this result is achieved with the foreclosure of the low-quality seller. Indeed, there are critical redistributive effects. Moreover, our analysis is static, and welfare might not be maximized if competition becomes unsustainable in the long run. For example, in response to the investigation on Amazon's buy-box opened by the European Commission in 2020, Amazon offered to reduce the buy-box's prominent role by adding a second buy-box that displays an alternative offer with a similar purchasing experience.[22] Such a measure may slightly lower the fierce competition for prominence.

Finally, our analysis also microfounds the vertical differentiation between two identical offers and how this can be determined by the decision of the intermediary to offer ancillary services. This has become an emergent practice in the industry as well.[23] Our analysis shows that if an intermediary can steer passive consumers using a prominent display and offers the possibility to upgrade the quality of ancillary services, it extracts the largest surplus from retailers. This strategy is the most profitable for the intermediary as it removes the double marginalization while maximizing consumer surplus.

[21] AGCM – case A528, para. 727.

[22] See: https://tcrn.ch/3O1v2Dz

[23] For example, Walmart offers the Walmart Fulfillment Services: https://marketplace.walmart.com/walmart-fulfillment-services/.

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Appendix

Proof of Lemma 1

Consider (5). Simplifying it, we obtain that $\Pi^H \geq \Pi^L$ if

$$(1-\lambda)\Big(1-\frac{p_H}{q_H}\Big) \geq (1-\lambda)\Big(1-\frac{p_L}{q_L}\Big).$$

Dividing both sides by $1-\lambda$, we get that $\Pi^H \geq \Pi^L$ if

$$1-\frac{p_H}{q_H} \geq 1-\frac{p_L}{q_L}.$$

Rearranging the above expression, we conclude that $\Pi^H \geq \Pi^L$ if, and only if,

$$p_H \leq \frac{q_H}{q_L} p_L,$$

for any p_H and p_L.

Proof of Lemma 2

The first part of the proof follows from (6). Suppose that H is prominent. It can set two prices: either it sets $p_H = \frac{q_H}{q_L} p_L$ so that (6) binds, or it sets $p_H < \frac{q_H}{q_L} p_L$. Suppose $p_H = \frac{q_H}{q_L} p_L$. In this case, L's demand conditional on H being prominent is

$$\begin{aligned} D_L^\lambda(p_L, p_H) =& \frac{\frac{q_H}{q_L} p_L - p_L}{q_H - q_L} - \frac{p_L}{q_L} \\ =& \frac{p_L}{q_L} - \frac{p_L}{q_L} \\ =& 0, \end{aligned}$$

which implies foreclosure of L for any $p_L \geq w$. Because L is foreclosed for $p_H = \frac{q_H}{q_L} p_L$, it is also foreclosed for any $p_H < \frac{q_H}{q_L} p_L$. As a result, the equilibrium price set by H conditional on being prominent is $p_H = \frac{q_H}{q_L} p_L$, and $D_L^\lambda(p_L, p_H) = 0$.

The second part of the proof is as follows. Suppose L is prominent: in this case, H is foreclosed (i.e., $D_H^\lambda = 0$) if, and only, if

$$p_L \leq p_H - q_H + q_L.$$

Suppose L sets its lowest price $p_L = w$. Then, foreclosure of H occurs if and only if

$$w \leq p_H - q_H + q_L,$$

or alternatively if $p_H \geq w + q_H - q_L$.

Suppose also H sets the lowest possible price $p_H = w$. Then, H is foreclosed if and only if $q_L \geq q_H$, which contradicts the assumption that $q_H > q_L$. As a result, if L is awarded prominence it can never foreclose H. This concludes the proof.

Proof of Lemma 3

Consider the case in which L is prominent. H chooses p_H to maximize the following program

$$\max_{p_H} \pi_H^L(p_H, p_L) \equiv \lambda\Big(1 - \frac{p_H - p_L}{q_H - q_L}\Big)(p_H - w) \qquad \text{s.t.} \qquad p_H > p_L \frac{q_H}{q_L},$$

whereas the program of L is to choose p_L to maximize the following

$$\max_{p_L} \pi_L^L(p_L, p_H) \equiv \Big[\lambda\Big(\frac{p_H - p_L}{q_H - q_L} - \frac{p_L}{q_L}\Big) + (1-\lambda)\Big(1 - \frac{p_L}{q_L}\Big)\Big](p_L - w) \quad \text{s.t.} \quad p_H > p_L \frac{q_H}{q_L}.$$

From the first-order conditions we get

$$\frac{\partial \pi_L^L(p_L, p_H)}{\partial p_L} = 0 \Longleftrightarrow p_L^L(p_H) = \frac{\lambda q_L(p_H - (q_H - q_L) + w) + (q_H - q_L)(q_L + w)}{2(q_H - (1-\lambda)q_L)},$$
$$\frac{\partial \pi_H^L(p_H, p_L)}{\partial p_H} = 0 \Longleftrightarrow p_H^L(p_L) = \frac{1}{2}(p_L + q_H - q_L + w).$$

Therefore, the optimal prices are equal to

$$p_L^L(w) = \frac{\lambda q_L(3w - q_H + q_L) + 2(q_H - q_L)(q_L + w)}{4q_H - (4 - 3\lambda)q_L},$$
$$p_H^L(w) = \frac{2q_H^2 + (\lambda - 3)q_H q_L + 3q_H w - (\lambda - 1)q_L(q_L - 3w)}{4q_H - (4 - 3\lambda)q_L}.$$

However, as $p_H > p_L \frac{q_H}{q_L}$ for L to be prominent, we should verify whether the above pair of prices satisfy such a condition.

Solving for $p_L^L \leq \frac{q_L}{q_H} p_H^L$, we get that this is possible only if $\lambda \geq \frac{q_L(q_L - 3w) - 2q_H w}{q_L(q_H + q_L - 3w)}$. Denoting $\hat{\lambda} \equiv \frac{q_L(q_L - 3w) - 2q_H w}{q_L(q_H + q_L - 3w)}$, as $\lambda \in [0,1]$, it also must be the case that $q_L > 2w$ for $\hat{\lambda} \in [0,1]$.

As a result, a necessary condition for L to be prominent is that the following two conditions jointly hold

$$\lambda \geq \hat{\lambda}, \qquad q_L > 2w.$$

If the above conditions are not satisfied, then H is prominent and, as a result, L is foreclosed by Lemma 2.

Proof of Lemma 4

The proof immediately follows from the discussion in the main text. Details on the exact values of $\tilde{\lambda}(w)$ can be provided upon request to the authors, as it presents polynomials of different degrees.

Proof of Proposition 1

In this proof, we formalize conditions on the equilibrium emerging for different values of λ.

We follow the following strategy. First, we start from L being prominent and identify conditions under which H finds it optimal to deviate and obtain prominence. Then, if H finds it optimal to deviate, we identify the equilibrium prices and profits in this subgame.

Suppose L is able to guarantee a market-sharing equilibrium, which therefore implies that conditions in Lemma 3 hold. Equilibrium prices are given by (7) and profits are as in (8). In order to verify if this is an equilibrium, we must verify that (i) L does not have any incentive to deviate and give up prominence, and (ii) H does not have any incentive to deviate and obtain prominence. As giving up prominence implies that L is foreclosed by Lemma 2, L has an never an incentive to deviate. Therefore, we only have to verify whether, and under which conditions, H has an incentive to deviate.

Suppose H deviates and obtains prominence. This means setting at least as equal to $p_L^L\, q_H/q_L$ so that it satisfies the prominence allocation constraint, i.e., $p_H^d = p_L^L\, q_H/q_L$. Using $p_L^L = \frac{\lambda q_L(-q_H+q_L+3w)+2(q_H-q_L)(q_L+w)}{4q_H+(3\lambda-4)q_L}$, previously identified, the deviation price is

$$p_H^d(w) = \frac{\lambda q_H q_L[3w-(q_H-q_L)]+2q_H(q_H-q_L)(q_L+w)}{q_L[4q_H-(4-3\lambda)q_L]},$$

and the associated deviation profit is

$$\pi_H^d(w) = \frac{(q_H-q_L)[2q_H(q_L+w)-\lambda q_H q_L-(4-3\lambda)q_L w][\lambda q_L(q_H+2q_L-3w)+2(q_H-q_L)(q_L-w)]}{q_L^2[4q_H-(4-3\lambda)q_L]^2}.$$

Comparing H's deviation profit with the candidate equilibrium profit, we can identify a critical value below which (resp. above which) H has an incentive to (resp. not to) deviate

$$\pi_H^d(w) > (<) \pi_H^L(w) \implies \lambda < (>) \tilde{\lambda}(w),$$

with $0 < \tilde{\lambda}(w) \leq 1$ representing a threshold value of λ (which is not reported for tractability issues).

Using this result together with the condition on $\hat{\lambda}$ in Lemma 3, for any $\lambda > \max\{\hat{\lambda}(w), \tilde{\lambda}(w)\}$

there is a market sharing equilibrium as H has no incentive to compete for prominence. In this subgame, prices are given by (7) and profits are as in (8).

It remains now to verify optimal prices in the remaining parameter space. when either H has an incentive to obtain prominence (i.e., $\lambda < \tilde{\lambda}(w)$) or L has no ability to obtain prominence $\lambda < \hat{\lambda}(w)$. As in this parameter range, L is foreclosed, there is always an incentive for L to undercut H and obtain strictly positive profits by winning prominence. This translates into a Bertrand-like spiral that only stops when $p_L = w$. Two cases can arise at this stage.

(i) H obtains prominence and meets the (binding) constraint $p_H \leq p_L \frac{q_H}{q_L}$. Thus, $p_H = \frac{q_H}{q_L} w$. In this case,

$$\pi_H = \frac{(q_L - w)}{q_L^2}(q_H - q_L)\lambda w.$$

(ii) H gives up prominence and plays its best-response to $p_L = w$, i.e., $p_H = \frac{1}{2}(p_L + (q_H - q_L) + w) = \frac{q_H - q_L}{2} + w$, thus $p_H > p_L \frac{q_H}{q_L}$. In this case

$$\pi_H = \frac{1}{4}(q_H - q_L)\lambda,$$

which is independent on w.

Thus, H prefers to have prominence provided that

$$\frac{(q_L - w)}{q_L^2}(q_H - q_L)\lambda w \geq \frac{1}{4}(q_H - q_L)\lambda$$
$$\implies w \leq \frac{q_L}{2}.$$

However, the case in which $\lambda < \hat{\lambda}(w)$ implies that $w < \frac{q_L}{2}$, therefore the above condition is always satisfied.[24]

Therefore, for any given commission, if $\lambda < \max\{\tilde{\lambda}(w), \hat{\lambda}(w)\}$ there exists a unique equilibrium with the high-quality retailer being prominent in which $p_H^H(w) = w\ q_H/q_L$ and $p_L^H(w) = w$. If $\lambda \geq [\max\{\tilde{\lambda}(w), \hat{\lambda}(w)\}, 1]$ there exists a unique market-sharing equilibrium with the low-quality retailer being prominent $p_H^L > w$ and $p_L^L > w$. This completes the proof.

Proof of Proposition 2

This proof immediately follows from the main text.

[24] Note that the maximum level of the commission fee that would induce H to compete for prominence that the intermediary can set is exactly equal to $\frac{q_L}{2}$, which is the price that a multiproduct monopolist would set for its low-quality offer.

Classic vertical chain setting

Consider a classic vertical chain and assume that $\lambda = 1$. H chooses p_H to maximize the following program

$$\max_{p_H} \pi_H(p_H, p_L) \equiv \Big(1 - \frac{p_H - p_L}{q_H - q_L}\Big)(p_H - w),$$

whereas the program of L is to choose p_L to maximize the following

$$\max_{p_L} \pi_L^L(p_L, p_H) \equiv \Big(\frac{p_H - p_L}{q_H - q_L} - \frac{p_L}{q_L}\Big)(p_L - w).$$

From the first-order conditions of retailers' profits with respect to p_H and p_L respectively, we get

$$\begin{aligned}
\frac{\partial \pi_L(p_L, p_H, w)}{\partial p_L} = 0 &\iff p_L(p_H, w) = \frac{p_H q_L - 2p_L q_H + q_H w}{q_L(q_H - q_L)},\\
\frac{\partial \pi_H(p_H, p_L, w)}{\partial p_H} = 0 &\iff p_H(p_L, w) = \frac{q_H - q_L + p_L - 2p_H + w}{q_H - q_L}.
\end{aligned}$$

The optimal prices are determined as follows

$$\begin{aligned}
p_L(w) &= \frac{q_H(q_L + 2w) + q_L(w - q_L)}{4q_H - q_L},\\
p_H(w) &= \frac{q_H(2q_H - 2q_L + 3w)}{4q_H - q_L}.
\end{aligned}$$

The profit of the intermediary is

$$\Pi(w) = \frac{w(3q_H q_L - w(2q_H + q_L))}{q_L(4q_H - q_L)}.$$

In the first stage of the game the intermediary maximizes its profit — identified by the above expression - with respect to w, which yields the following optimal commission fee

$$w = \frac{3q_H q_L}{2(2q_H + q_L)}.$$

The intermediary, at equilibrium, obtains

$$\Pi = \frac{9q_H^2 q_L}{4(8q_H^2 + 2q_H q_L - q_L^2)}.$$

Welfare comparison. We compute the consumer surplus and total industry profit in the classic vertical chain setting. The consumer surplus is given by:

$$\begin{aligned} CS &= \int_{\frac{p_H - p_L}{q_H - q_L}}^{1} (\theta q_H - p_H)\, d\theta + \int_{\frac{p_L}{q_L}}^{\frac{p_H - p_L}{q_H - q_L}} (\theta q_L - p_L)\, d\theta \\ &= \frac{q_H^2 \left(64 q_H^3 - 12 q_H^2 q_L + 21 q_H q_L^2 + 8 q_L^3\right)}{8(q_L - 4q_H)^2 (2q_H + q_L)^2}. \end{aligned}$$

The total industry profit is the sum of the profit of the intermediary and of those of the retailers, i.e.,

$$\Pi + \pi_H + \pi_L = \frac{q_H \left(64 q_H^4 + 28 q_H^3 q_L - 9 q_H^2 q_L^2 + 2 q_H q_L^3 - 4 q_L^4\right)}{4(q_L - 4q_H)^2 (2q_H + q_L)^2}.$$

Total welfare, denoted as the sum of the industry profit and consumer surplus, is

$$W = CS + \Pi + \pi_H + \pi_L = \frac{q_H \left(192 q_H^4 + 44 q_H^3 q_L + 3 q_H^2 q_L^2 + 12 q_H q_L^3 - 8 q_L^4\right)}{8(q_L - 4q_H)^2 (2q_H + q_L)^2}.$$

With a slight abuse of notation, denote the competition for prominence case with the superscript CP. Consumers surplus, at equilibrium values, is given by

$$CS^{CP} = \lambda \int_{\frac{p_H - p_L}{q_H - q_L}}^{1} (\theta q_H - p_H)\, d\theta + (1 - \lambda) \int_{\frac{p_H}{q_H}}^{1} (\theta q_H - p_H)\, d\theta = \frac{q_H}{8}.$$

The total industry profit is the sum of the profit of the intermediary and of those of the retailers. Denoting them with the superscript CP

$$\Pi^{CP} + \pi_H^{CP} = \frac{q_H}{4}.$$

Total welfare, denoted as the sum of the industry profit and consumer surplus, is

$$W^{CP} = CS^{CP} + \Pi^{CP} + \pi_H^{CP} = \frac{3}{8} q_H.$$

By comparison, we observe the following:

$$\begin{aligned} CS^{CP} &> CS, \\ \Pi^{CP} + \pi_H^{CP} &> \Pi + \pi_H + \pi_L, \\ W^{CP} &> W, \end{aligned}$$

for any $q_H > q_L$.

Multiproduct monopolist

Consider a multiproduct monopolist that has two offers and compare the profit obtained when prominence is given to the high-quality offer and the profit obtained when prominence is given to the low-quality offer.

Suppose the low-quality offer is prominent. Then, the multiproduct monopolist chooses p_L and p_H to maximize the following

$$\max_{p_L,p_H} \Pi^L \equiv \lambda\Big(1 - \frac{p_H - p_L}{q_H - q_L}\Big)p_H + \Big[\lambda\Big(\frac{p_H - p_L}{q_H - q_L} - \frac{p_L}{q_L}\Big) + (1-\lambda)\Big(1 - \frac{p_L}{q_L}\Big)\Big]p_L.$$

From the first-order conditions, we obtain

$$p_L^L = \frac{q_L}{2}, \quad p_H^L = \frac{q_H}{2}.$$

and the associated profit is

$$\Pi^L = \frac{\lambda q_H + q_L(1-\lambda)}{4}.$$

Suppose the high-quality offer is prominent. Then, the multiproduct monopolist chooses p_L and p_H to maximize the following

$$\max_{p_L,p_H} \Pi^H \equiv \Big[\lambda\Big(1 - \frac{p_H - p_L}{q_H - q_L}\Big) + (1-\lambda)\Big(1 - \frac{p_H}{q_H}\Big)\Big]p_H + \lambda\Big(\frac{p_H - p_L}{q_H - q_L} - \frac{p_L}{q_L}\Big)p_L.$$

From the first-order conditions, we obtain

$$p_L^H = \frac{q_L}{2}, \quad p_H^H = \frac{q_H}{2}.$$

and the associated profit is

$$\Pi^H = \frac{q_H}{4}.$$

Comparing the profits in the two cases, it is immediate that We then compare the profits in the two scenarios and find that

$$\Pi^H \equiv \frac{q_H}{4} > \frac{\lambda q_H + q_L(1-\lambda)}{4} \equiv \Pi^L,$$

as $q_H > q_L$ by assumption.

Welfare comparison. With a slight abuse of notation, we denote the competition for prominence case with the superscript CP, and the multiproduct monopolist case with the superscript M. Consumers surplus, at equilibrium values, is given by

$$CS^M = \lambda \int_{\frac{p_H - p_L}{q_H - q_L}}^{1} (\theta q_H - p_H)\, d\theta + (1-\lambda) \int_{\frac{p_H}{q_H}}^{1} (\theta q_H - p_H)\, d\theta = \frac{q_H}{8}.$$

In this case, the total industry profit is equal to the profit of the vertically integrated intermediary.

$$\Pi^M = \frac{q_H}{4}.$$

Total welfare, denoted as the sum of the industry profit and consumer surplus, is

$$W^M = CS^M + \Pi^M = \frac{3}{8} q_H.$$

Comparing competition for prominence with the result under a multiproduct monopolist shows that competition for prominence is unambiguously welfare superior. Specifically

$$\begin{aligned} CS^{CP} &= CS^M, \\ \Pi^{CP} + \pi_H^{CP} &= \Pi^M, \\ W^{CP} &= W^M, \end{aligned}$$

for any $q_H > q_L$. This completes the proof.

Proof of Lemma 5

In this proof, we compare the willingness-to-pay for prominence of the high-quality and the low-quality retailers.

First, we start identifying the profit obtained by the retailers conditional on L being prominent, which are given, respectively. by

$$\begin{aligned} \pi_L^L(p_L, p_H, w) &= \Big[\lambda D_L^\lambda(p_L, p_H) + (1-\lambda) D_L^{1-\lambda}(p_L)\Big](p_L - w), \\ \pi_H^L(p_H, p_L, w) &= \Big[\lambda D_H^\lambda(p_H, p_L)\Big](p_H - w). \end{aligned}$$

From the first-order conditions with respect to p_H and p_L we get

$$\begin{aligned} \frac{\partial \pi_L^L(p_L, p_H, w)}{\partial p_L} = 0 &\iff p_L^L(p_H, w) = \frac{\lambda(p_H - 2p_L - q_H + q_L + w)}{q_H - q_L} + \frac{q_L - 2p_L + w}{q_L}, \\ \frac{\partial \pi_H^L(p_H, p_L, w)}{\partial p_H} = 0 &\iff p_H^L(p_L, w) = \frac{\lambda(q_H - q_L + p_L - 2p_H + w)}{q_H - q_L}. \end{aligned}$$

The optimal prices, conditional on L being prominent, are

$$\begin{aligned} p_L^L(w) &= \frac{\lambda q_L(3w - q_H + q_L) + 2(q_H - q_L)(q_L + w)}{4q_H - (4 - 3\lambda)q_L}, \\ p_H^L(w) &= \frac{2q_H^2 - (3-\lambda)q_H q_L + 3q_H w + (1-\lambda)q_L(q_L - 3w)}{4q_H - (4 - 3\lambda)q_L}, \end{aligned}$$

and the associated profits are

$$\pi_L^L(w) = \frac{(q_H - q_L)[q_H - (1-\lambda)q_L][2w - (2-\lambda)q_L]^2}{q_L[4q_H - (4-3\lambda)q_L]^2},$$
$$\pi_H^L(w) = \frac{\lambda(q_H - q_L)[(1-\lambda)q_L - 2q_H + w]^2}{[4q_H - (4-3\lambda)q_L]^2}.$$

Then, consider the profit obtained by the retailers conditional on H being prominent, which are given, respectively. by

$$\pi_L^H(p_L, p_H, w) = \Big[\lambda D_L^\lambda(p_L, p_H)\Big](p_L - w),$$
$$\pi_H^H(p_H, p_L, w) = \Big[\lambda D_H^\lambda(p_H, p_L) + (1-\lambda) D_H^{1-\lambda}(p_H)\Big](p_H - w).$$

From the first-order conditions with respect to p_H and p_L we get

$$\frac{\partial \pi_L^H(p_L, p_H, w)}{\partial p_L} = 0 \Longleftrightarrow p_L^L(p_H, w) = \frac{\lambda(p_H q_L - 2p_L q_H + q_H w)}{q_L(q_H - q_L)},$$
$$\frac{\partial \pi_H^H(p_H, p_L, w)}{\partial p_H} = 0 \Longleftrightarrow p_H^L(p_L, w) = \frac{\frac{\lambda(q_L(w - 2p_H) + p_L q_H)}{q_H - q_L} - 2p_H + q_H + w}{q_H}.$$

The optimal prices are

$$p_L^H(w) = \frac{\lambda q_L w(2q_H + q_L) + (q_H - q_L)(q_H(q_L + 2w) + q_L w)}{q_H[4q_H - (4-3\lambda)q_L]},$$
$$p_H^H(w) = \frac{\lambda w(q_H + 2q_L) + 2(q_H - q_L)(q_H + w)}{4q_H - (4-3\lambda)q_L},$$

and the associated profits are

$$\pi_L^H(w) = \frac{\lambda(q_H - q_L)[q_H(q_L - 2w) + (1-\lambda)q_L w]^2}{q_H q_L[4q_H - (4-3\lambda)q_L]^2},$$
$$\pi_H^H(w) = \frac{(q_H - q_L)(q_H - (1-\lambda)q_L)(2q_H - (2-\lambda)w)^2}{q_H[4q_H - (4-3\lambda)q_L]^2}.$$

Denoting WTP_i the willingness to pay for prominence of retailer i, for $i = L, H$, we get

$$\begin{aligned} WTP_L :=& \pi_L^L(p_L^L, p_H^L, w) - \pi_L^H(p_L^H, p_H^H, w) \\ =& \frac{(q_H - q_L)\Big\{[q_H - (1-\lambda)q_L][2w - (2-\lambda)q_L]^2 - \frac{\lambda[q_H(q_L - 2w) + (1-\lambda)q_L w]^2}{q_H}\Big\}}{q_L[4q_H - (4-3\lambda)q_L]^2}, \\ WTP_H :=& \pi_H^H(p_H^H, p_L^H, w) - \pi_H^L(p_H^L, p_L^L, w) \\ =& \frac{(q_H - q_L)\Big\{\frac{[q_H - (1-\lambda)q_L][2q_H - (2-\lambda)w]^2}{q_H} - \lambda[(1-\lambda)q_L - 2q_H + w]^2\Big\}}{[4q_H - (4-3\lambda)q_L]^2}. \end{aligned}$$

Comparing the two, we observe that

$$WTP_H - WTP_L = \frac{(1-\lambda)(q_H - q_L)\{4q_H^2 - (8-5\lambda)q_H q_L + [4-\lambda(5-2\lambda)]q_L^2]\}\left(q_H q_L - w^2\right)}{q_H q_L[4q_H - (4-3\lambda)q_L]^2}.$$

As $\lambda \in (0,1)$, $q_H > q_L$, the above expression is positive as long as $q_H q_L > w^2$ (which holds at equilibrium as $q_H > q_L > w$), the sign of the above expression is the same as the sign of

$$4q_H^2 - (8-5\lambda)q_H q_L + [4-\lambda(5-2\lambda)]q_L^2,$$

which always positive in the parameter range considered. Therefore, $WTP_H - WTP_L > 0$.

Proof of Proposition 3

As the WTP of H is larger than that of L, the former can simply win the auction by bidding the willingness-to-pay of the latter and become prominent. As a result

$$b_H = WTP_L.$$

This concludes the proof.

Proof of Proposition 4

The profit of the intermediary conditional on giving prominence to the high-quality retailer is the sum of the profit obtained from the commission fee and the bid of the high-quality retailer. Simplifying, this is equal to

$$\begin{aligned}\Pi(w,b_H) =& \frac{1}{q_H q_L[4q_H - (4-3\lambda)q_L]^2} \\ & \Big\{q_H^3\left(-(4-\lambda)(1-\lambda)q_L^2 + 12\lambda q_L w + 4(1-3\lambda)w^2\right) + \\ & q_H^2 q_L\left(-(1-\lambda)[(5-\lambda)\lambda - 8]q_L^2 - 3\lambda(4-3\lambda)q_L w + 2[7(2-\lambda)\lambda - 8]w^2\right) + \\ & q_H q_L^2\left(-[4(5-\lambda)\lambda^2 - 35\lambda + 20]w^2 + (2-\lambda)^2(1-\lambda)q_L^2\right) - (1-\lambda)[8-\lambda(11-4\lambda)]q_L^3 w^2\Big\}.\end{aligned}$$

Solving for the optimal commission fee, we obtain

$$w = \frac{3\lambda q_H^2 q_L(4q_H - (4-3\lambda)q_L)}{2(1-\lambda)[8-\lambda(11-4\lambda)]q_L^3 - 8(1-3\lambda)q_H^3 - 4[7(2-\lambda)\lambda - 8]q_H^2 q_L - 2[4(5-\lambda)\lambda^2 - 35\lambda + 20]q_H q_L^2},$$

from which we determine the associated profit of the intermediary.

Denoting the profit of the platform with Π^{PP} and comparing it with the profit obtained under competition for prominence in Section 4.2, denoted by Π^{CP}, we obtain the following

$$\Pi^{CP} - \Pi^{PP} = \frac{1}{108}$$
$$\Bigg\{ - \frac{243\lambda^2 q_H^3 q_L}{4(3\lambda-1)q_H^3 + 2(7(\lambda-2)\lambda+8)q_H^2 q_L + (\lambda(4(\lambda-5)\lambda+35)-20)q_H q_L^2 - (\lambda-1)(\lambda(4\lambda-11)+8)q_L^3}$$
$$- \frac{12q_H(q_H-q_L)(8q_H+q_L)}{q_L(4q_H+(3\lambda-4)q_L)} + 12\lambda(q_L-q_H) + \frac{16(q_L-4q_H)(q_L-q_H)^3}{q_L(4q_H+(3\lambda-4)q_L)^2} + \frac{4(q_H-q_L)(5q_H+7q_L)}{q_L} + 27q_L \Bigg\}.$$

which is always positive when restricting attention to the case in which an interior solution always exists.

Quality Upgrade

In this Section, we present the proof for the results presented in Section 5. The game is solved by backward induction and we identify the equilibrium profits of the retailers in the four subgames.

No upgrade. If both retailers do not find it optimal to upgrade their quality, they are perfectly homogeneous. The profit of a representative retailer, denoted by $i = 1, 2$ is

$$\pi_i(p_i, p_{-i}, w) = \Big(\lambda D_i^{\lambda} + (1-\lambda)D_i^{1-\lambda}\Big)(p_i - w).$$

Because of perfect symmetry, retailers compete à la Bertrand, setting prices equal to the commission fee. At equilibrium, the profit of retailer i is

$$\pi_i = 0 \qquad \forall i = 1, 2.$$

Joint upgrade. In this case, retailers are ex ante and ex post symmetric. Therefore, Bertrand-type competition occurs. The profit of a representative retailer, denoted by $i = 1, 2$ is

$$\pi_i(p_i, p_{-i}, w, F) = \Big(\lambda D_i^{\lambda} + (1-\lambda)D_i^{1-\lambda}\Big)(p_i - w) - F,$$

with $F > 0$ the upgrading fee. Because of perfect symmetry, retailers compete à la Bertrand, setting prices equal to the commission fee. At equilibrium, the profit of retailer i is

$$\pi_i(F) = -F \qquad \forall i = 1, 2.$$

Asymmetric upgrade. In this case, retailers are ex ante symmetric, but asymmetric ex post: one retailer upgrades, while the other does not. This scenario is equivalent to the baseline model, except from the added upgrading stage. From Lemma 3, for given fees, if $\lambda < \tilde{\lambda}(w)$, H has an incentive to obtain prominence.

For a given w, if $\lambda < \tilde{\lambda}(w)$ the equilibrium prices are

$$p_H^H = \frac{q_H}{q_L} w, \qquad p_L^H = w.$$

The profit of the retailers are

$$\pi_H^H(p_H^H, p_L^H, w, F) = \frac{(q_H - q_L)(q_L - w)w}{q_L^2} - F, \qquad \pi_L^H(p_L^H, p_H^H, w) = 0.$$

If $\lambda > \tilde{\lambda}(w)$, H prefers to give up prominence. Equilibrium prices are

$$\begin{aligned} p_H^L(w) &= \frac{2q_H^2 - (3-\lambda)q_H q_L + 3q_H w + (1-\lambda)q_L(q_L - 3w)}{4q_H - (4-3\lambda)q_L}, \\ p_L^L(w) &= \frac{2(q_H - q_L)(q_L + w) - \lambda q_L(q_H - q_L - 3w)}{4q_H - (4-3\lambda)q_L}, \end{aligned}$$

The profit of the retailers are

$$\begin{aligned} \pi_H^L(p_H^L, p_L^L, w, F) &= \frac{\lambda(q_H - q_L)[(1-\lambda)q_L - 2q_H + w]^2}{[4q_H - (4-3\lambda)q_L]^2} - F, \\ \pi_L^L(p_L^L, p_H^L, w) &= \frac{(q_H - q_L)[q_H - (1-\lambda)q_L][2w - (2-\lambda)q_L]^2}{q_L[4q_H - (4-3\lambda)q_L]^2}. \end{aligned}$$

In the upgrading stage, we distinguish therefore between two cases, depending on λ, and we rewrite Table 1.

If $\lambda < \tilde{\lambda}(w)$, each retailer solves the normal form game in Table 2

		Retailer -i: U	Retailer -i: NU
Retailer i	U	$-F, -F$	$\pi_H^H - F, 0$
	NU	$0, \pi_H^H - F$	$0, 0$

Table 2: Upgrading decision ($\lambda < \tilde{\lambda}(w)$).

If $\lambda > \tilde{\lambda}(w)$, each retailer solves the normal form game in Table 3

		Retailer -i: U	Retailer -i: NU
Retailer i	U	$-F, -F$	$\pi_H^L - F, \pi_L^L$
	NU	$\pi_L^L, \pi_H^L - F$	$0, 0$

Table 3: Upgrading decision ($\lambda > \tilde{\lambda}(w)$).

Solving the games in the two cases, regardless of λ, only asymmetric equilibria (U, NU) and (NU, U) arise. To see why, consider that if retailer i upgrades, retailer $-i$ does not upgrade, as

$0 > -F$. If retailer i does not upgrade, retailer $-i$ upgrades, as $\pi_H > 0$. if retailer i does not upgrade, retailer $-i$ upgrades, as $\pi_H > 0$. Due to symmetry, therefore, only the asymmetric equilibria are possile.

We can now move to the stage in which the intermediary sets the commission fee w and the upgrading fee F in the two subagmes. From Proposition 2, the intermediary chooses w^H and $\tilde{\lambda}(w)|_{w=w^H} = 1$, which implies that only the foreclosing equilibrium is subgame perfect. With respect to the upgrading fee is linear, the intermediary can set it equal to the difference between the profit of the upgraded retailer and the profit of the retailer that does not upgrade, with the latter being equal to $\pi_L^L(p_L^H, p_H^H, w^H) = 0$. Therefore

$$F = \pi_H^H(p_H^H, p_L^H, w^H) - \pi_L^L(p_L^H, p_H^H, w^H, F) = \frac{q_H - q_L}{4}.$$

The intermediary extracts all the rents from the market and obtains

$$\Pi^H = \frac{q_H}{4},$$

which is equivalent to the profit of a multiproduct monopolist as in Subsection 4.3.

Asymmetric marginal cost

An important assumption in our baseline model is that both retailers face a common marginal cost, which was normalized to zero. In this Section, we relax this assumption and discuss how the presence of asymmetric marginal cost alters the market outcome. Specifically, we assume that the high-quality retailer has a higher marginal cost than the low-quality retailer. Denoting c_H and c_L, respectively, the marginal cost of H and L, and normalizing the latter to 0, we assume $c_H > c_L = 0$.

This assumption has an important effect on changing how retailers compete. The reason is that the price floor of H is now $p_H = c_H + w$, whereas the price floor of L is $p_L = w$.

Conditional on H being prominent, the profits of the retailers are, respectively,

$$\begin{aligned}
\pi_H^H(p_H, p_L, w) &= \Big[\lambda D_H^{\lambda}(p_H, p_L) + (1-\lambda) D_H^{1-\lambda}(p_H)\Big](p_H - w - c_H), \\
\pi_L^H(p_L, p_H, w) &= \Big[\lambda D_L^{\lambda}(p_L, p_H)\Big](p_L - w).
\end{aligned}$$

Conditional on L being prominent, that is for $\lambda \geq \tilde{\lambda}(w)$ are, respectively

$$\begin{aligned}
\pi_H^L(p_H, p_L, w) &= \Big[\lambda D_H^{\lambda}(p_H, p_L)\Big](p_H - w - c_H). \\
\pi_L^L(p_L, p_H, w) &= \Big[\lambda D_L^{\lambda}(p_L, p_H) + (1-\lambda) D_L^{1-\lambda}(p_L)\Big](p_L - w).
\end{aligned}$$

For any $\lambda < \tilde{\lambda}(w)$, we distinguish between two alternative scenarios, which depend on the magnitude of the marginal cost. In the first one, c_H is sufficiently low (and the baseline model is a degenerate case), and H can always match the quality-adjusted price of L and win prominence. In the second one, c_H is sufficiently high, and H cannot obtain prominence as the low-quality retailer can always undercut the high-quality one.

Using H and L's price floors together with the prominence allocation rule in (6), we can identify the cut-off value below (resp. above), which H is always able to undercut L and maximize the profit of the intermediary if

$$c_H \leq (>) \Big(\frac{q_H}{q_L} - 1 \Big) w. \tag{9}$$

If (9) holds, therefore, the analysis in our baseline model holds qualitatively, and the low-quality retailer is always foreclosed at equilibrium (see Proposition 2).

If (9) does not hold, it is not always possible for H to lower its price to win prominence. As in the baseline model, L always has an incentive to obtain prominence whenever H obtains prominence. Otherwise, it is foreclosed. Therefore, the equilibrium outcome depends on the incentives of the high-quality seller in this case.

Differently from the baseline model, if H has an incentive to obtain prominence, there is now no equilibrium in pure strategy exists.

Too see why, consider the case in which H reaches its lowest price, $p_H = w + c_H$, and L sets $p_L = p_H \; q_L/q_H - \varepsilon$ (with $\varepsilon \to 0$). In this case, L gains prominence and positive profit, whereas H obtains zero profits. Since the loss of prominence for the high-quality retailer does not imply losing the active consumers due to vertical differentiation (see Lemma 2), H always has an incentive to raise its price and monetize the actives.

Consider now the case in which L sets $p_L = (w + c_H) \; q_L/q_H - \varepsilon$ (with $\varepsilon \to 0$), which implies that L certainly obtains prominence. As H can always serve the active consumers if L is prominent (by Lemma 2), it maximizes its profit subject to $p_L = (w + c_H) \; q_L/q_H - \varepsilon$. From the first-order condition with respect to p_H, we obtain the following candidate equilibrium price

$$p_H = \frac{c_H(q_H + q_L) + w(q_H + q_L) + q_H(q_H - q_L)}{2q_H}.$$

We notice that (p_L, p_H) here defined is not equilibrium because L can always raise its price just below the prominence threshold level, i.e., $p_L \approx p_H \; q_L/q_H - \varepsilon$.

This analysis suggests that if (9) does not hold, there is never an equilibrium in pure strategy. Therefore, only a mixed strategy equilibrium exists.