

Activities

Activity 1.3

$$A = \begin{bmatrix} 2 & 1 & 7 & 8 \\ 0 & -2 & 5 & -1 \\ 4 & 9 & 3 & 0 \end{bmatrix}$$

$$a_{32} = 9$$

Activity 1.4

The matrix below is the only one that is diagonal

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

Activity 1.11

The matrices $A = \begin{bmatrix} 2 & 1 & 3 \\ 1 & 2 & 1 \end{bmatrix}$ $B = \begin{bmatrix} 3 & 1 \\ 1 & 0 \\ 1 & 1 \end{bmatrix}$

Matrix A has the dimensions 2×3 and matrix B has the dimensions 3×2 , therefore matrix AB will have the dimensions 2×2 and the matrix BA will have the dimensions 3×3 .

Multiply the matrices to find the two product matrices, AB and BA .

$$AB = A * B = \begin{bmatrix} 2 & 1 & 3 \\ 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ 1 & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 10 & 5 \\ 6 & 2 \end{bmatrix}$$

$$BA = B * A = \begin{bmatrix} 3 & 1 \\ 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 & 3 \\ 1 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 5 & 10 \\ 2 & 1 & 3 \\ 3 & 3 & 4 \end{bmatrix}$$

In [11]:

```
import numpy as np
print("AB:")
print(np.matrix([[2, 1, 3], [1, 2, 1]])*np.matrix([[3, 1], [1, 0], [1, 1]]))
print("BA:")
print(np.matrix([[3,1],[1,0],[1,1]])*np.matrix([[2,1,3],[1,2,1]]))
```

```
AB:
[[10  5]
 [ 6  2]]
BA:
[[ 7  5 10]
 [ 2  1  3]
 [ 3  3  4]]
```

Activity 1.12

Show that $AB \neq BA$ by multiplying AB and BA and comparing them with each other. $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} B = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

$$AB = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ 3 & 7 \end{bmatrix} \quad BA = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 4 & 6 \\ 3 & 4 \end{bmatrix}$$

In [13]:

```
import numpy as np
print("AB:")
print(np.matrix([[1, 2], [3, 4]])*np.matrix([[1,1],[0,1]]))
print("BA:")
print(np.matrix([[1,1],[0,1]])*np.matrix([[1,2],[3,4]]))
```

```
AB:
[[1 3]
 [3 7]]
BA:
[[4 6]
 [3 4]]
```

Activity 1.32

Fill in the missing numbers if the Matrix A is symmetric:

$$A = \begin{bmatrix} 1 & 4 & x \\ x & 2 & x \\ 5 & x & 3 \end{bmatrix} = \begin{bmatrix} 1 & x & 5 \\ 4 & 2 & -1 \\ x & x & 3 \end{bmatrix}$$

Exercises

Exercise 1.1

Given the matrices: $A = \begin{bmatrix} 1 & 3 & 5 \\ -1 & 1 & 0 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 1 & 1 \\ 1 & 1 & -1 \end{bmatrix}$ $C = \begin{bmatrix} 1 & 1 \\ 3 & 2 \\ -1 & 4 \end{bmatrix}$ $\mathbf{d} = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$ which of the following matrix expressions are defined?

Compute those which are defined.

a. $A\mathbf{d}$ b. $AB + C$ c. $A + C^T$ d. $C^T C$ e. BC f. $\mathbf{d}^T B$ g. $C\mathbf{d}$ h. $\mathbf{d}^T \mathbf{d}$ i. $\mathbf{d}\mathbf{d}^T$

a) Not defined.

b) Not defined.

$$\text{c) } A + C^T = \begin{bmatrix} 1 & 3 & 5 \\ -1 & 1 & 0 \end{bmatrix} + \begin{bmatrix} 1 & 3 & -1 \\ 1 & 2 & 4 \end{bmatrix} = \begin{bmatrix} 2 & 6 & 4 \\ 0 & 3 & 4 \end{bmatrix}$$

$$\text{d) } C^T C = \begin{bmatrix} 1 & 3 & -1 \\ 1 & 2 & 4 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 3 & 2 \\ -1 & 4 \end{bmatrix} = \begin{bmatrix} 11 & 3 \\ 3 & 21 \end{bmatrix}$$

$$\text{e) } BC = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 1 & 1 \\ 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 3 & 2 \\ -1 & 4 \end{bmatrix} = \begin{bmatrix} 0 & 5 \\ 4 & 8 \\ 5 & -1 \end{bmatrix}$$

$$\text{f) } \mathbf{d}^T B = \begin{bmatrix} 2 & -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ 2 & 1 & 1 \\ 1 & 1 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$$

g) Not defined.

$$\text{h) } \mathbf{d}^T \mathbf{d} = \begin{bmatrix} 2 & -1 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 6 \end{bmatrix}$$

$$\text{i) } \mathbf{d} \mathbf{d}^T = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix} \begin{bmatrix} 2 & -1 & 1 \end{bmatrix} = \begin{bmatrix} 4 & -2 & 1 \\ -2 & 1 & -1 \\ 2 & 1 & 1 \end{bmatrix}$$

Problems

Problem 1.5

A square matrix M is said to be *skew symmetric* if $M = -M^T$. Given that the 3×3 matrix $A = (a_{ij})$ is symmetric and the 3×3 matrix $B = (b_{ij})$ is skew symmetric, find the missing entries in the following matrices:

$$A = \begin{bmatrix} 1 & -4 & 7 \\ -4 & 6 & 0 \\ 7 & 0 & 2 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & -3 & 5 \\ 3 & 0 & 2 \\ -5 & -2 & 0 \end{bmatrix}$$

In []: