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LETTER TO THE EDITOR

Dynamics of forest fires as a directed percolation model

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Abstract. Forest fires under directional constraints, such as wind or local topography, generalise the bond percolation problem of the symmetrical fires. We analyse the mass M and the radius of gyration R_g of the burning tree clusters as a function of time. $M \sim t^{\alpha}$ and $R_g \sim t^{\beta}$, with α and β being the fractal exponents.

Forest fires are a very descriptive model for two-dimensional percolation [1] and as such have found their way into a recent textbook [2]. A particularly picturesque version of the problem consists in taking a two-dimensional regular lattice and igniting one starting site. Then the fire may propagate to nearest neighbours with given probabilities. For simplicity one starts with a discrete-time model [1] for which in a given time interval each ignited tree burns completely. Then, at each given time one has burnt-out trees, burning trees, warm trees (these are unburnt nearest neighbours of the burning ones) and the remaining unburnt trees. After a time interval the burning trees are burnt-out and at this time a subset of the warm trees gets ignited. For each tree of this subset we determine through a random number whether it is ignited or not. Notice that a tree not ignited in this step may become a warm tree again and be ignited at a later stage. As is obvious, forest fires are a special model for general epidemic processes [3, 4], and the applications range from the spreading of diseases to the kinetics of formation of branched polymers [5].

Particularly interesting in the forest fire problem is the dynamical aspect, i.e. the fact that the dynamical pattern develops in *time*. This feature enhances the richness of the picture, since all aspects common to random walks in restricted geometries [6], such as persistence lengths, anisotropies [7] and processes in continuous time [8, 9] may enter. Also, forest fires show interesting bond and site percolation aspects [10]. As we will show in the following, the inclusion of the anisotropy (i.e. wind) already leads to dynamical patterns with no obvious static percolative counterpart.

To show this we assign direction-dependent probabilities for the ignition, and have as a special case the isotropic situation. We found it convenient to work on the square lattice, but we have also used a triangular geometry for comparison to reference [1]. We have taken lattices of 1000×1000 sites, on which, for each set of parameters, 5000 different realisations of the fire starting from the origin were simulated. Each realisation stopped either by its own attrition or because the rim of the lattice was reached. We analysed the onset of percolation, i.e. the possibly unlimited spread of the fire in terms of two criteria: (*a*) to reach the rim in half of all realisations and (*b*) the fractal (time-scaling) form of the burnt-out patterns.

Consider first the isotropic case. We note that for the fully developed burnt-out cluster all internal bonds and all bonds connecting the cluster to its surroundings have been questioned exactly *once*, from the burning to the warm trees, and have propagated (or not) the fire with probability p (resp. $1-p$). Since for the final pattern the order in which this questioning took place is immaterial, the set of clusters obtained by either burning isotropically or by bond percolation arguments are identical. Hence the critical probabilities are the same in the two problems. Therefore the isotropic fire on the square lattice has a critical value for ignition $p_c = 0.5$. Both criteria (*a*) and (*b*) display this value nicely. Out of 5000 realisations the rim was reached 2173 times for $p = 0.499$, 2451 times for $p = 0.5$ and 2738 times for $p = 0.501$. As an example for a larger deviation from p_c, for $p = 0.49$ the rim was reached only 1049 times.

The situation is also borne out by the study of the scaling relations of the ensemble averaged mass M of the burnt-out clusters (total number of trees) and by their radius of gyration R_g, as a function of t. One assumes scaling at percolation; thus

$$M(t) \equiv \left\langle \sum 1 \right\rangle \sim t^{\alpha} \tag{1}$$

and

$$R_g^2(t) \equiv \left\langle \sum r^2 / \sum 1 \right\rangle - \left\langle \left(\sum r / \sum 1 \right)^2 \right\rangle \sim t^{2\beta} \tag{2}$$

where t is the number of elapsed time intervals, $\langle\ \rangle$ is the configurational average and the sums Σ extend over the burnt-out sites at t.

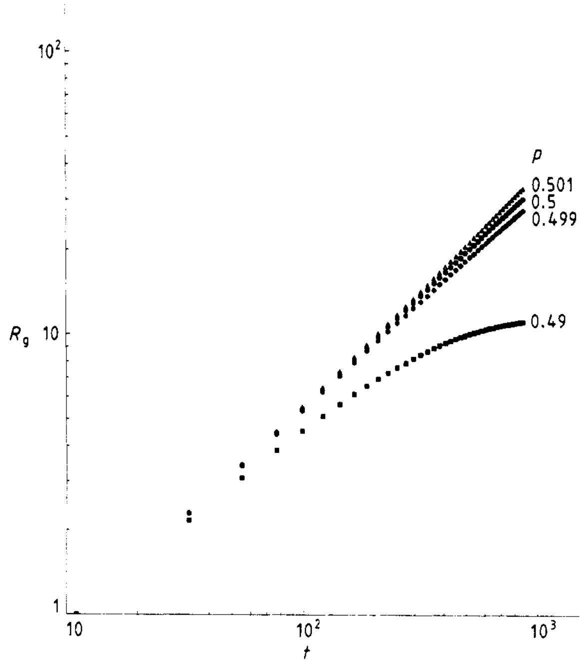

Figure 1. Scaling of the radius of gyration, R_g, with time on a square lattice near the percolation threshold. The curves correspond to $p = 0.49$, 0.499, 0.5 and 0.501, as indicated. For display purposes, all R_g are normalised to their values at $t = 11$.

In figure 1 the temporal behaviour of R_g is displayed as a log-log plot. Care has been taken to consider only such times at which the rim was not yet reached in *any* realisation, just to ensure that *no* finite-size effects occur. As is evident, the scaling behaviour is excellent for $p=0.5$ and deviations from linearity are already felt at $p=0.49$ and $p=0.51$. Thus *temporal* scaling is a very good indicator for p_c in the symmetric case. We find at $p_c=0.5$ as fractal parameters in (1) and (2), $\alpha=1.57$ and $\beta=0.78$.

To make the connection with the results of [1], we have also investigated the situation on the triangular lattice. Out of 5000 trials the rim was reached in 2384 cases for $p=0.3470$, in 2463 cases for $p=0.3473$ and in 2554 cases for $p=0.3476$. The agreement with the exactly known value for bond percolation on the triangular lattice, $p_c=2\sin(\pi/18)=0.3472$, is really very good. As a comparison, for the approximate value $p=\frac{1}{3}=0.333$ the rim was never reached in 5000 realisations. For M and R_g the findings for the triangular lattice around $p_c=0.3473$ closely reproduce those for the square lattice around $p_c=0.5$ and we find $\alpha=1.59$ and $\beta=0.79$. In both cases we find that the ratio α/β is around 2, different from the static fractal dimension which is about 1.9. We also note that our α value is substantially lower than the $\hat{d}$ parameters reported for bond percolation [11], $\hat{d}\sim 1.675$ when the chemical distance, and not the true time development, are used to measure M.

A possible reason for this discrepancy is the different time evolution considered here and in [11]. There a cluster is given and the sites are assigned a time corresponding to the shortest path (chemical distance) from an origin. In our case, this is a lower bound for the actual ignition time, since here it may take several attempts until igniting a tree. Our patterns, viewed as a function of time, are thus less dense.

Forest fires are much influenced by wind. To simulate this aspect on the square lattice, we assign different ignition probabilities to the forward, lateral and backward directions, an aspect related to directed percolation [12, 13]. In the figures the forward direction is taken to the right. Since in the isotropic case at percolation these two values add to unity, we took as probabilities to ignite in forward (resp. backward) directions p_F (resp. $1-p_F$) and chose $p_F=0.6$, 0.7, 0.8 and 0.9. We are now able to search for critical behaviour by varying the probabilities p_L in the two lateral directions, which are taken equal.

Table 1. Critical probabilities p_L for forest fires with wind.

Forward p_F	Backward $1-p_F$	Lateral p_L^c
0.5	0.5	0.5
0.6	0.4	0.459
0.7	0.3	0.385
0.8	0.2	0.287
0.9	0.1	0.160

In table 1 we present the critical p_L values obtained by simulation, where we use criterion (a), to reach the rim of the lattice in half of all cases. As expected intuitively, an increase in anisotropy reduces the value for p_L or, expressed differently, for preassigned lateral ignition probabilities increasing the strength of the wind renders a previously subcritical situation critical.

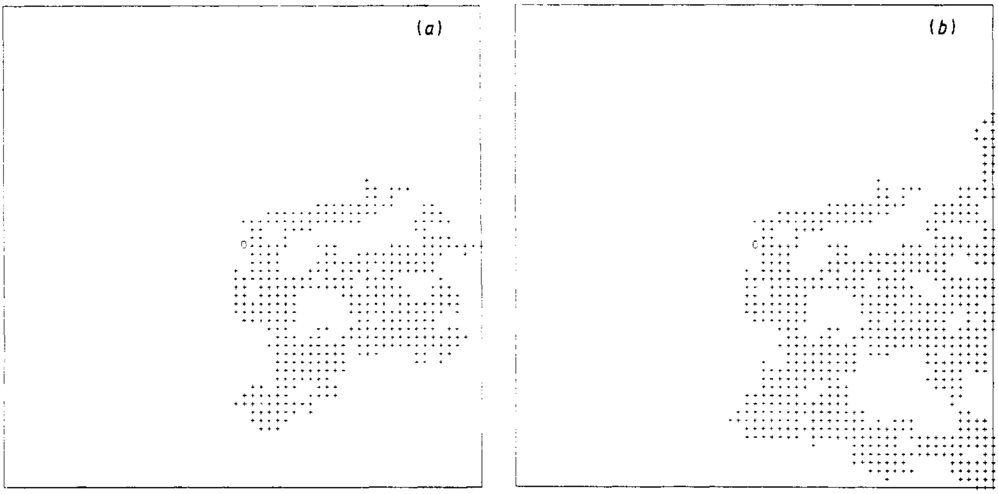

Figure 2. (a) Spreading of fire at the percolation threshold $p_F = 0.7$. The forward direction is to the right and the origin is indicated. The picture is a cut-out of the 1000×1000 matrix after reaching the border of the indicated 59×59 submatrix. (b) Same realisation as in (a), at a later time, when the fire reaches the 1000×1000 rim.

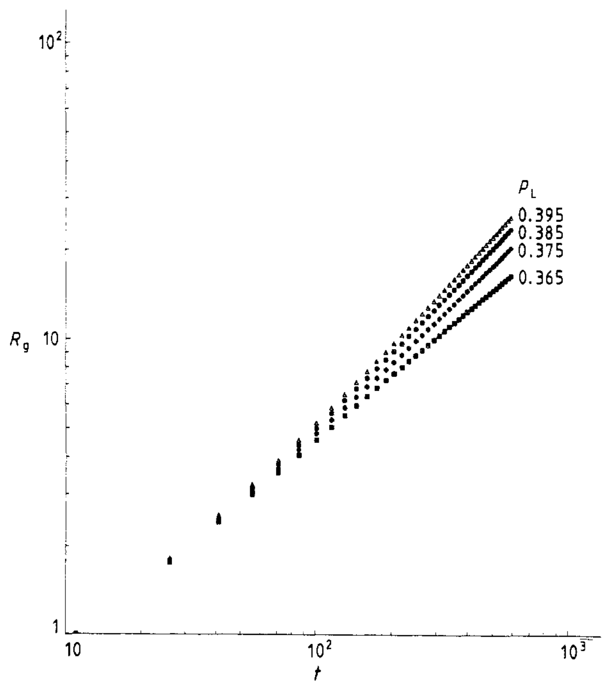

Figure 3. Same as in figure 1 for the anisotropic situation $p_F = 0.7$ around the critical threshold. The lateral ignition probabilities are $p_L = 0.365$, 0.375, 0.385 and 0.395, as indicated.

A point to be mentioned is that several realisations of fires using the same parameters may lead to widely distinct patterns. One may think that strong winds will act very directionally (and thus lead to rather regular shapes). Indeed, already for $p_F = 0.6$ the forward rim was reached *first* in all 5000 realisations. However, the fires do not necessarily propagate as conical forms. A not too unusual pattern of the fire for $p_F = 0.7$ is given in figures 2(a) and (b), which show the situation after crossing the borderline of the 59×59 lattice and after reaching the rim of the total lattice, respectively.

To round out the picture we have also analysed the scaling behaviour for M and R_g with t. We again find scaling, albeit at some lower lateral p_L values than the ones inferred from the criterion to reach the rim. Thus for $p_F = 0.6$ the p_L value at which R_g scales is 0.446 and for $p_F = 0.7$ it is 0.370. To display how well temporal scaling is obeyed, we present in figure 3 the situation for $p_F = 0.7$, with p_L varying between 0.365 and 0.395. At $p_L = 0.370$ we find as fractal parameters $\alpha = 1.26$ and $\beta = 0.79$. As a final remark we note that, distinct from the symmetrical spread of the fire, the asymmetrical situation is not readily amenable to the previous bond percolation argument, since now even assuming directed bonds, the *order* in which the two sites of each bond are questioned matters.

In summary, we have analysed the forest fire problem both in the presence and in the absence of anisotropies (wind). In all cases the patterns obtained scale with time only for ignition probabilities very close to the critical ones. Due to its dynamical facets the problem may be enriched by the inclusion of further aspects, such as continuous times.

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