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TUGAS 4

1) Hitung integral dibawah ini !

$$\int \frac{u \, du}{\sqrt{u-u^2}} = \int \left(\frac{1}{2\sqrt{u-u^2}} - \frac{1-2u}{2\sqrt{u-u^2}} \right) du$$

$$= \frac{1}{2} \int \frac{1}{2\sqrt{u-u^2}} + \frac{1}{2} \int \frac{2u-1}{2\sqrt{u-u^2}} \, du$$

Cabang kiri:

$$\int \frac{1}{2\sqrt{u-u^2}} = \int \frac{1}{\sqrt{\frac{1}{4}-\left(u-\frac{1}{2}\right)^2}} \, du$$

$$= \int \frac{1}{\sqrt{\frac{1}{4}-\left(\frac{2u-1}{2}\right)^2}} \, du$$

misal $2u-1 = u$

$$\frac{du}{du} = 2$$

$$du = \frac{du}{2}$$

$$= \int \frac{1}{2\sqrt{\frac{1}{4}-\frac{u^2}{4}}} \cdot du$$

$$= \int \frac{1}{\sqrt{1-u^2}} \, du$$

$$= \arcsin(u)$$

$$= \arcsin(2u-1) \ldots ①$$

Cabang kanan:

$$\int \frac{2u-1}{\sqrt{u-u^2}} \, du$$

misal : $u = u-u^2$

$$\frac{du}{du} = 1-2u$$

$$du = \frac{du}{1-2u}$$

$$= \int \frac{2u-1}{\sqrt{u}} \cdot \frac{du}{1-2u}$$

$$= -\int \frac{1}{\sqrt{u}} \, du$$

$$= -2\sqrt{u}$$

$$= -2\sqrt{u-u^2} \ldots ②$$

$$\frac{1}{2}\left[\arcsin(2u-1)\right] + \frac{1}{2}\left(-2\sqrt{u-u^2}\right) + C$$

② a) Hitung integral dari atas!

$$\int \left(\frac{1}{M-y} - \frac{1}{y}\right) dy = \int M \, dt$$

misal : $u = M-y$

$$\frac{du}{dy} = -1$$

$$du = -dy$$

$$\int -\frac{1}{u} \, du - \ln y = Mt + c$$

$$-\ln u - \ln y = Mt + c$$

$$-\ln(u \cdot y) = Mt + c$$

$$-\ln([M-y]+y]) + c = Mt + c$$

b) Jika $y(0) = 1$, nyatakan y sebagai fungsi t.

$$y(t) = t$$

$$-\ln((M-y)y) + e = Mt + e$$

$$e^{\ln((M-y)y)} = e^{-Mt}$$

$$|(M-y)y| = e^{-Mt} \qquad\qquad My - y^2 \geq 0$$

$$My - y^2 = e^{-Mt} \qquad\qquad y(y-M) \leq 0$$

$$y^2 - My = -e^{-Mt} \qquad\qquad y=0 \;\checkmark\; y=M$$

$$y^2 - My + \left(\frac{1}{4}m^2t - \frac{1}{4}mt\right) = -e^{-Mt} \qquad\qquad +\;[\!/\!/\!/\!/\!]\;+$$

$$\left(y - \frac{1}{2}M\right)^2 - \frac{1}{4}M^2 = -e^{-Mt} \qquad\qquad \underset{0\qquad M}{}$$

$$\left(y - \frac{1}{2}M\right)^2 = \frac{1}{4}M^2 - e^{-Mt} \qquad\qquad 0 \leq y \leq M$$

$$y = \frac{1}{2}M \pm \sqrt{\frac{1}{4}M^2 - e^{-Mt}}$$

Masukkan $y(0) = 1$

$$1 = \frac{1}{2}M \pm \sqrt{\frac{1}{4}M^2 - e^{-(0)M}}$$

$$\left(1 - \frac{1}{2}M\right)^2 = \frac{1}{4}M^2 - 1$$

$$1 - M + \frac{1}{4}M^2 = \frac{1}{4}M^2 - 1$$

$$\boxed{M = 2}$$

Maka, $y(t) = 1 \pm \sqrt{1 - e^{-2t}}$

c) $\lim\limits_{t\to\infty} 1\pm\sqrt{1-e^{-2t}} = \lim\limits_{t\to\infty} 1\pm\sqrt{1-\dfrac{1}{e^{2t}}}$

$$= 1\pm\sqrt{1-\dfrac{1}{e^{2\infty}}}$$

$$= 1\pm\sqrt{1-0}$$

$$= 1\pm\sqrt{1} \begin{cases} 1+1=2 \\ 1-1=0 \end{cases} \left.\begin{array}{l} \text{Nilai limit berada} \\ \text{pada 0 atau 2} \end{array}\right.$$

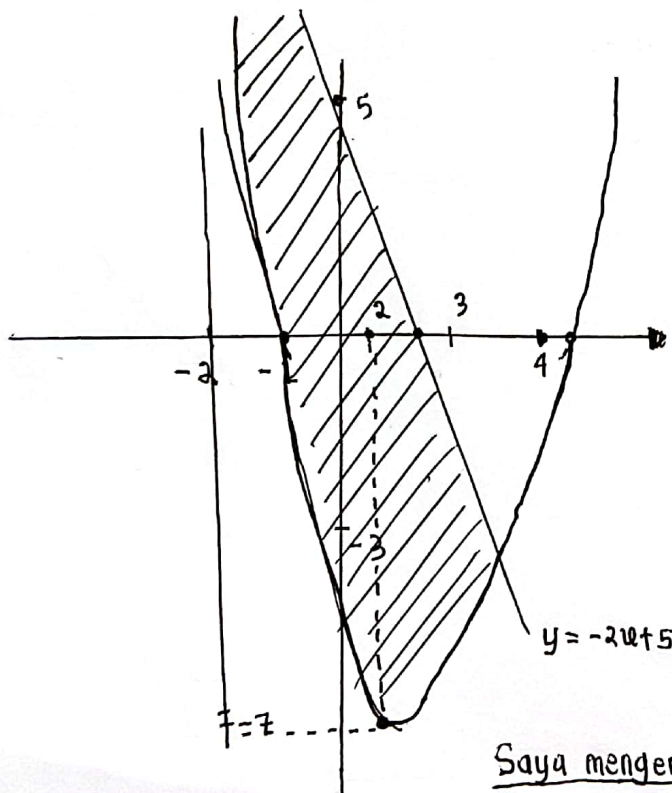Maka, fungsi diatas memiliki <u>limit</u>

(3) $y = x^2 - 4x - 3$

$$= \dfrac{4\pm\sqrt{16+12}}{2}$$

$$= \dfrac{4\pm\sqrt{28}}{2} = \dfrac{4\pm2\sqrt{7}}{2} = 2\pm\sqrt{7}$$

$x_p = -\dfrac{b}{2a} = \dfrac{4}{2} = \boxed{2}$

$y_p = (2)^2 - 4(2) - 3 = \boxed{-7}$

$x_p, y_p = (2, -7)$

$y = -2x+5$

$x=7$

* Metode kulit tabung

$$V = 2\pi\int_a^b x\, f(x)\, dx$$

$$V = 2\pi\int_{-2}^4 x\left[(-2x+5) - (x^2 - 4x - 3)\right] dx$$

$$V = 2\pi\int_{-2}^4 -x^3 + 2x^2 + 8x\, dx$$

$$V = 2\pi\left[-\dfrac{x^4}{4} + \dfrac{2}{3}x^3 + 4x^2\right]_{-2}^4$$

$$V = 2\pi\left[\left(-\dfrac{256}{4} + \dfrac{128}{3} + 64\right) - \left(-\dfrac{16}{4} - \dfrac{16}{3} + \dfrac{16}{4}\right)\right]$$

$$V = 2\pi\left(\dfrac{432}{126}\right)$$

$$V = 72\pi$$

Jadi, luas volume benda adalah 72π satuan atau ≈ 226 satuan.

<u>Saya mengerjakan tugas ini dengan tingkat kejujuran 80%</u>