TEKNIK INTEGRASI

MATEMATIKA LANJUT
Fakultas Teknologi
Maju dan Multidisiplin

Outline

1. Integrasi Fungsi Trigonometri
2. Integral Tak Wajar
3. Integrasi Numerik

1. INTEGRASI FUNGSI TRIGONOMETRI

Integrasi Fungsi Trigonometri

5 Tipe integrasi fungsi trigonometri:

1. $\int \sin^n x \, dx$ dan $\int \cos^n x \, dx$
2. $\int \sin^m x \, . \cos^n x \, dx$
3. $\int \sin mx . \cos nx \, dx$, $\int \sin mx . \sin nx \, dx$, dan $\int \cos mx . \cos nx \, dx$
4. $\int \tan^n x \, dx$ dan $\int \cot^n x \, dx$
5. $\int \tan^m x \, . \sec^n x \, dx$ dan $\int \cot^m x \, . \csc^n x \, dx$

Tipe 1 : $\int \sin^n x\, dx$ dan $\int \cos^n x\, dx$

- Untuk **n gasal** → $\sin^2 x + \cos^2 x = 1$

$$\sin x\, dx = -d(\cos x)$$

$$\cos x\, dx = d(\sin x)\ ;$$

Contoh:

a. $$\int \cos^3 x\, dx = \int \cos^2 x \cdot \cos x\, dx = \int (1 - \sin^2 x) \cos x\, dx$$

$$= \int (1 - u^2)\, du = u - \tfrac{1}{3}u^3 + C$$

$$= \sin x - \tfrac{1}{3}\sin^3 x + C$$

Tipe 1 : $\int \sin^n x\, dx$ dan $\int \cos^n x\, dx$

Contoh:

b. $$\int \sin^5 x\, dx = \int \sin^4 x \sin x\, dx$$

$$= \int (1 - \cos^2 x)^2 \sin x\, dx$$

$$= \int (1 - 2\cos^2 x + \cos^4 x) \sin x\, dx$$

$$= -\int (1 - 2\cos^2 x + \cos^4 x)(-\sin x\, dx)$$

$$= -\cos x + \tfrac{2}{3}\cos^3 x - \tfrac{1}{5}\cos^5 x + C$$

Tipe 1: $\int \sin^n x\, dx$ dan $\int \cos^n x\, dx$ (lanjutan)

- Untuk **n genap** → $\sin^2 x = \frac{1-\cos 2x}{2}$ dan $\cos^2 x = \frac{1+\cos 2x}{2}$

Contoh:

a. $$\int \sin^2 x\, dx = \int \frac{1-\cos 2x}{2}\, dx$$

$$= \frac{1}{2}\int dx - \frac{1}{4}\int (\cos 2x)(2\, dx)$$

$$= \frac{1}{2}x - \frac{1}{4}\sin 2x + C$$

Tipe 1: $\int \sin^n x\, dx$ dan $\int \cos^n x\, dx$ (lanjutan)

Contoh:

b. $$\int \cos^4 x\, dx = \int \left(\frac{1 + \cos 2x}{2}\right)^2 dx$$

$$= \frac{1}{4}\int (1 + 2\cos 2x + \cos^2 2x)\, dx$$

$$= \frac{1}{4}\int dx + \frac{1}{4}\int (\cos 2x)(2)\, dx + \frac{1}{8}\int (1 + \cos 4x)\, dx$$

$$= \frac{3}{8}\int dx + \frac{1}{4}\int \cos 2x(2\, dx) + \frac{1}{32}\int \cos 4x(4\, dx)$$

$$= \frac{3}{8}x + \frac{1}{4}\sin 2x + \frac{1}{32}\sin 4x + C$$

Tipe 2 : $\int \sin^m x \,.\cos^n x \, dx$

- Untuk ***m*** atau ***n*** **gasal** → $\sin^2 x + \cos^2 x = 1$

$$\sin x \, dx = -\, d(\cos x)$$

$$\cos x \, dx = \; d(\sin x)$$

Contoh: $\int \sin^3 x \cos^{-4} x \, dx = \int (1 - \cos^2 x)(\cos^{-4} x)(\sin x) \, dx$

$$= -\int (\cos^{-4} x - \cos^{-2} x)(-\sin x \, dx) \; = -\int \left(\cos^{-4} x - \cos^{-2} x\right) \; d(\cos x)$$

$$= -\left[\frac{(\cos x)^{-3}}{-3} - \frac{(\cos x)^{-1}}{-1}\right] + C$$

$$= \frac{1}{3}\sec^3 x - \sec x + C$$

Tipe 2 : $\int \sin^m x \,.\cos^n x \, dx$ (lanjutan)

- Untuk ***m*** dan ***n*** **genap** → $\sin^2 x = \frac{1-\cos 2x}{2}$ dan $\cos^2 x = \frac{1+\cos 2x}{2}$

Contoh:

$$\int \sin^2 x \cos^4 x \, dx = \int \left(\frac{1-\cos 2x}{2}\right)\left(\frac{1+\cos 2x}{2}\right)^2 dx$$

$$= \frac{1}{8}\int (1 + \cos 2x - \cos^2 2x - \cos^3 2x)\, dx$$

$$= \frac{1}{8}\int \left[1 + \cos 2x - \frac{1}{2}(1 + \cos 4x) - (1 - \sin^2 2x)\cos 2x\right] dx$$

$$= \frac{1}{8}\int \left[\frac{1}{2} - \frac{1}{2}\cos 4x + \sin^2 2x \cos 2x\right] dx$$

$$= \frac{1}{8}\left[\int \frac{1}{2} dx - \frac{1}{8}\int \cos 4x(4\, dx) + \frac{1}{2}\int \sin^2 2x(2\cos 2x\, dx)\right]$$

$$= \frac{1}{8}\left[\frac{1}{2}x - \frac{1}{8}\sin 4x + \frac{1}{6}\sin^3 2x\right] + C$$

Tipe 3 : $\int \sin mx.\cos nx \ dx$, $\int \sin mx.\sin nx \ dx$, dan $\int \cos mx.\cos nx \ dx$

Ingat:

1. $\sin mx \cos nx = \frac{1}{2}[\sin(m+n)x + \sin(m-n)x]$
2. $\sin mx \sin nx = -\frac{1}{2}[\cos(m+n)x - \cos(m-n)x]$
3. $\cos mx \cos nx = \frac{1}{2}[\cos(m+n)x + \cos(m-n)x]$

Contoh:

a. $\int \sin 4x \cos 5x \, dx = \int \frac{1}{2}[\sin(-x) + \sin 9x] \, dx$

$= \frac{1}{2}\int (-\sin x + \sin 9x) \, dx$

$= \frac{1}{2}(\cos x - \frac{1}{9}\cos 9x) + C$

Tipe 3 : $\int \sin mx.\cos nx \ dx$, $\int \sin mx.\sin nx \ dx$, dan $\int \cos mx.\cos nx \ dx$

Contoh:

b. $$\int \sin 2x \sin 5x \ dx = -\frac{1}{2}\int [\cos(2+5)x + \cos(2-5)\,x] \ dx$$

$$= -\frac{1}{2}\int [\cos 7x + \cos 3x] \ dx$$

$$= -\frac{1}{14}\sin 7\,x + \frac{1}{6}\sin 3x + C$$

c. $$\int \cos 3x \cos 4x \ dx = \frac{1}{2}\int [\cos(3+4)x + \cos(3-4)\,x] \ dx$$

$$= \frac{1}{2}\int [\cos 7x + \cos x] \ dx$$

$$= \frac{1}{14}\sin 7\,x + \frac{1}{2}\sin x + C$$

Latihan A

Tentukan nilai integral berikut.

1. $\int \sin^3 x \cos x \, dx$
2. $\int \sin^3 x \cos^2 x \, dx$
3. $\int \cos^3 2x \sin^5 2x \, dx$
4. $\int_0^{\pi/2} \sqrt{\cos x} \sin^3 x \, dx$
5. $\int \sin^3 x \, dx$
6. $\int \cos^3 2x \, dx$
7. $\int_0^{\pi} \cos^2 \frac{x}{2} \, dx$
8. $\int \cos^4 x \, dx$
9. $\int_0^{1} \sin^4 \pi x \, dx$
10. $\int \sin^2 2x \cos^4 2x \, dx$
11. $\int \sin^2\left(\frac{x}{2}\right)\cos^2\left(\frac{x}{2}\right) dx$
12. $\int \cos^4 x \sin^4 x \, dx$
13. $\int_0^{\pi} \sin^2 x \cos^4 x \, dx$
14. $\int \sin^6 u \, du$

Latihan B

Tentukan nilai integral berikut.

1. $\int \sin^3 x \cos^2 x \, dx$

2. $\int \sin^6 x \cos^3 x \, dx$

3. $\int_{\pi/2}^{3\pi/4} \sin^5 x \cos^3 x \, dx$

4. $\int_0^{\pi/2} \cos^5 x \, dx$

5. $\int \sin^2(\pi x) \cos^5(\pi x) \, dx$

6. $\int \frac{\sin^3(\sqrt{x})}{\sqrt{x}} \, dx$

7. $\int_0^{\pi/2} \cos^2\theta \, d\theta$

8. $\int_0^{\pi/2} \sin^2(2\theta) \, d\theta$

9. $\int_0^{\pi} \sin^4(3t) \, dt$

10. $\int_0^{\pi} \cos^6\theta \, d\theta$

11. $\int (1 + \cos\theta)^2 \, d\theta$

12. $\int x \cos^2 x \, dx$

13. $\int_0^{\pi/2} \sin^2 x \cos^2 x \, dx$

14. $\int_0^{\pi} \sin^2 t \cos^4 t \, dt$

15. $\int \frac{\cos^5\alpha}{\sqrt{\sin\alpha}} \, d\alpha$

16. $\int \cos\theta \cos^5(\sin\theta) \, d\theta$

17. $\int \cos^2 x \tan^3 x \, dx$

18. $\int \cot^5\theta \sin^4\theta \, d\theta$

19. $\int \frac{\cos x + \sin 2x}{\sin x} \, dx$

20. $\int \cos^2 x \sin 2x \, dx$

END