Homework 3

DDL: March 25th

Question 1

Questions: The square matrix A of dimension $N*N$ is said to be symmetric if $A = A'$ (A' is the transpose of A). Determine whether the following square matrices are symmetric.

$$
\begin{aligned}
&(a)\begin{vmatrix} 1 & -6 & 3 \\ -6 & 2 & 0 \\ 3 & 0 & 4 \end{vmatrix} \qquad\qquad (b)\begin{vmatrix} 9 & -6 & 3 \\ -6 & 2 & 5 \\ 3 & 6 & 4 \end{vmatrix} \\
&(c) A = [a_{ij}]_{N*N} \;\; where\ a_{ij} = \begin{cases} ij & i = j \\ i + ij - j & i \neq j \end{cases} \\
&(d) A = [a_{ij}]_{N*N} \;\; where\ a_{ij} = \begin{cases} 2ij & i = j \\ 2i - ij + 2j & i \neq j \end{cases}
\end{aligned} \tag{1}
$$

Question 2

Questions: Let A be an $M*N$ matrix and X an $N*1$ matrix.

(a) How many multiplications are needed to calculate AX ?

(b) How many additions are needed to calculate AX?

Question 3

Questions: Show that back substitution requires N divisions, $(N^2 - N)/2$ multiplications, and $(N^2 - N)/2$ additions or subtractions. *Hint.* You can use the formula

$$\sum_{k=1}^{M} k = M(M+1)/2 \tag{2}$$

Question 4

Questions: Solve, using four-digit rounding arithmetic, the following linear systems using:

(i) Gaussian elimination with partial pivoting.

(ii) Gaussian elimination with scaled partial pivoting.

Both a and b should be solved using the above two methods

$$
(a)\begin{cases} 2x_1 - 4x_2 + 150x_3 = 1 \\ x_1 + 12x_2 - 0.01x_3 = 0 \\ 3x_1 - 150x_2 + 0.3x_3 = 0 \end{cases} \qquad (b)\begin{cases} x_1 + 10x_2 - x_3 + 0.001x_4 = 0 \\ 2x_1 - 5x_2 + 35x_3 - 0.1x_4 = 1 \\ 5x_1 + x_2 - 120x_3 - 10x_4 = 0 \\ 2x_1 - 100x_2 - 3x_3 + x_4 = 0 \end{cases} \tag{3}
$$

Question 5

Questions: Find the triangular factorization $A = LU$ for the matrix

Detailed calculating process is needed for full scores

$$\begin{bmatrix} 1 & 1 & 0 & 3 \\ 2 & -1 & 5 & -2 \\ 3 & -3 & 6 & -17 \\ -3 & 0 & -19 & -24 \end{bmatrix} \tag{4}$$

Question 6

Questions: (a) Start with $P_0 = 0$ and use Jacobi iteration to find P_k for $k = 1, 2, 3$. Will Jacobi iteration converge to the solution?

(b) Start with $P_0 = 0$ and use Gauss-Seidel iteration to find P_k for $k = 1, 2, 3$. Will Gauss Seidel iteration converge to the solution?

$$\begin{cases} x - 5y - z = -8 \\ 4x + y - z = 13 \\ 2x - y - 6z = -2 \end{cases} \tag{5}$$

Question 7 (Programming)

Please take a screenshot for your programming result and attach it in your HW file. Executable program files should also be submitted. Any programming language is acceptable

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Questions: In calculus the following integral would be found by the technique of partial fractions:

$$\int \frac{x^2 + x + 1}{(x-1)(x-3)(x-2)^2(x^2+1)} dx \tag{6}$$

This would require finding the coefficients A_i, for $i = 1, 2, \ldots, 6$ in the expression

$$\frac{x^2 + x + 1}{(x-1)(x-3)(x-2)^2(x^2+1)} = \frac{A_1}{(x-1)} + \frac{A_2}{(x-3)} + \frac{A_3}{(x-2)^2} + \frac{A_4}{(x-2)} + \frac{A_5 x + A_6}{(x^2+1)} \tag{7}$$

Use your program to find the partial fraction coefficients.

Question 8 (Programming)

Please take a screenshot for your programming result and attach it in your HW file. Executable program files should also be submitted. Any programming language is acceptable

Questions: Use Gauss-Seidel iteration to solve the following band system $AX = B$ and use the norm of the residual $AX_k - B$ as the stopping criterion. X_k is the k th iterate from the Gauss-Seidel iteration procedure.

$$
\begin{aligned}
12x_1 - 2x_2 + x_3 &= 5 \\
-2x_1 + 12x_2 - 2x_3 + x_4 &= 5 \\
x_1 - 2x_2 + 12x_3 - 2x_4 + x_5 &= 5 \\
x_2 - 2x_3 + 12x_4 - 2x_5 + x_6 &= 5 \\
\vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad &\vdots \\
x_{46} - 2x_{47} + 12x_{48} - 2x_{49} + x_{50} &= 5 \\
x_{47} - 2x_{48} + 12x_{49} - 2x_{50} &= 5 \\
x_{48} - 2x_{49} + 12x_{50} &= 5
\end{aligned}
$$