Homework 2 (Due on March 13th)

Question 1

Let $g(x) = 3x^2 - 2x - 5$. Can fixed-point iteration be used to find the solution(s) to the equation $x = g(x)$ and why?

Question 2

Suppose that $g(x)$ and $g'(x)$ are defined and continuous on (a, b); $p_0, p_1, p_2 \in (a, b)$; and $p_1 = g(p_0)$ and $p_2 = g(p_1)$. Also, assume that there exists a constant K such that $|g'(x)| < K$. Show that $|p_2 - p_1| < K|p_1 - p_0|$. **Hint** Use the mean value theorem.

Question 3

Start with $[a_0, b_0]$ and use the false position method to compute c_0, c_1, c_2 and c_3.

$$cos(x) + 1 - x = 0, \quad [a_0, b_0] = [0.8, 1.6] \tag{1}$$

Question 4

What will happen if the bisection method is used with the function $f(x) = 1/(x - 2)$ and

(a) the interval is $[3, 7]$ (b) the interval is $[1, 7]$

Question 5

Establish the following formula for determining the number of iterations required in the bisection method. **Hint** Use $|b - a|/2^{n+1} < \delta$ and take logarithms.

$$N = \mathbf{int}\left(\frac{ln(b-a) - ln(\delta)}{ln(2)}\right) \tag{2}$$

Question 6

Use a computer or graphics calculator to graphically determine the approximate location of the roots of $f(x) = x - cos(x)$ in interval $-2 \leq x \leq 2$. In each case, determine an interval $[a, b]$ over which Programs 2.2 and 2.3 (Textbook *Numerical methods Using MATLAB*, page 59-60) could be used to determine the roots (i.e., $f(a)f(b) < 0$).

Question 7

Consider the function $f(x) = sin(x)$.

(a) Find the Newton-Raphson formula $p_k = g(p_{k-1})$.

(b) We want to find the root $p = \pi$. Can we use $p_0 = 1$? Why?

(c) We want to find the root $p = \pi$. Can we use $p_0 = 3$? Why?

Question 8

Consider $f(x) = x^N + A$, where N is a positive integer.

(a) What real values are the solution to $f(x) = 0$ for the various choices of N and A that can arise?

(b) Derive the following recursive formula for finding the Nth rooth of A.

$$p_k = \frac{(N-1)p_{k-1} - A/p_{k-1}^{N-1}}{N}, \quad k = 1, 2, \ldots \tag{3}$$

Question 9 (Programming)

Please take a screenshot for your programming result and attach it in your HW file. Executable program files should also be submitted. Any programming language is acceptable.

Suppose that the equations of motion for a projectile are:

$$\begin{aligned} y = f(t) = 9500(1 - e^{-t/14}) - 470t, \\ x = r(t) = 2500(1 - e^{-t/14}). \end{aligned} \tag{4}$$

(a) Find the elapsed time until impact accurate to 10 decimal places.

(b) Find the range accurate to 10 decimal places.

<u>**Attention:**</u> Existing python/matlab... packages for solving nonlinear equations are **NOT ALLOWED** and will **LOSE ALL POINTS** for this question. (i.e., sympy.solve, scipy.fsolve, gekko.solve, matlab fsolve, solve, roots...).