# Homework2

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# Question 1

## Solution:

Consider $g'(x) = 6x - 2$

Let $|g'(x)| < 1$, then $|6x - 2| < 1$

we have the fixed point should located in $(\frac{1}{6}, \frac{1}{2})$, to use the fixed point iteration method.

Solve function $g(x) = x$:

Let

$$h(x) = g(x) - x$$

Let $h'(x) = 0$, we have $x = \frac{1}{2}$, so $h(x)$ has a local minimum at $x = \frac{1}{2}$.

Since

$$h(a) < 0, h(b) < 0, h(x_0) < 0$$

where $(a, b)$ is the interval of $(\frac{1}{6}, \frac{1}{2})$, and $x_0$ is the local minimum.

we can have $h(x) < 0$ in $(\frac{1}{6}, \frac{1}{2})$.

So the function $g(x) = x$ have no solution in $(\frac{1}{6}, \frac{1}{2})$, which means the fixed point iteration method will not converge.

# Question 2

Apply the Mean Value Theorem to $g$ on the interval between $p_0$ and $p_1$. There exists $\xi$ between $p_0$ and $p_1$ such that:

$g'(\xi) = \frac{g(p_1)-g(p_0)}{p_1-p_0}$.

Given $p_2 = g(p_1)$ and $p_1 = g(p_0)$, we have:

$g(p_1) - g(p_0) = p_2 - p_1$,

so:

$g'(\xi) = \frac{p_2-p_1}{p_1-p_0}$.

Since $|g'(\xi)| < K$:

$\left|\frac{p_2-p_1}{p_1-p_0}\right| < K$.

Then:

$|p_2 - p_1| < K|p_1 - p_0|$,

which is the desired inequality.

# Question 3

see question3.ipynb

the answer is
we have

$$c_0 = 1.2701241482595913$$
$$c_1 = 1.283232912796939$$
$$c_2 = 1.2834259276134252$$
$$c_3 = 1.283428701320108$$

# Question 4

see question4.ipynb

The answer is "*Cannot find answer*" for (a) and "*Can find answer*" for (b)

# Question 5

**Proof:**

Given $\frac{|b-a|}{2^{n+1}} < \delta$,

Take natural logarithms: $\ln\left(\frac{|b-a|}{\delta}\right) < \ln(2^{n+1})$,

Using logarithm properties: $\ln(|b-a|) - \ln(\delta) < (n+1)\ln(2)$,

Solve for $n+1$: $n+1 > \frac{\ln(|b-a|)-\ln(\delta)}{\ln(2)}$,

Thus, $n > \frac{\ln(|b-a|)-\ln(\delta)}{\ln(2)} - 1$,

Since $N$ is the smallest integer number of iterations, $N = \left\lceil \frac{\ln(b-a)-\ln(\delta)}{\ln(2)} \right\rceil$,

Given the problem's form, $N = \text{int}\left(\frac{\ln(b-a)-\ln(\delta)}{\ln(2)}\right)$, which approximates the ceiling for practical iteration count.

**Q.E.D.**

# Question 6

see question6.ipynb
the answer is 0.7390851974487305

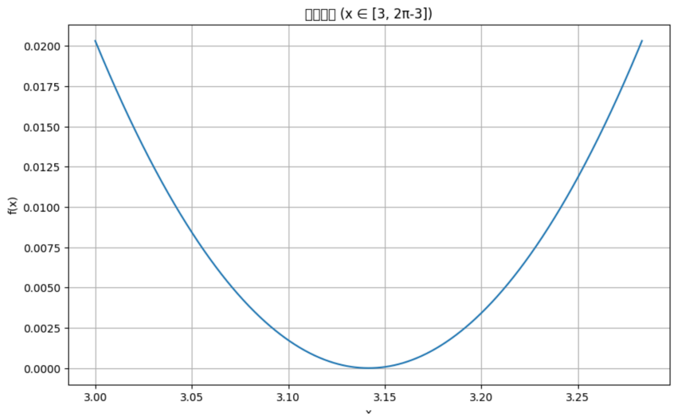
# Question 7

(a) formula $g(x) = x - sin(x)/cos(x) = x - tan(x)$

(b) No. $p_0 = 1$ will find root $x = 0$
Or, function $\frac{|f(x)\cdot f''(x)|}{(f'(x))^2}$
has value larger then 1 in the interval $(1, \pi + 1)$

(c) Yes. The function $\frac{|f(x)\cdot f''(x)|}{(f'(x))^2}$ is smaller than 1 in the interval $(3, 2 * \pi - 3)$

Here is the image:




# Question 8

see question8.py

the answer is