and the rule is proved that

$$\frac{du^n}{dx} = nu^{n-1}\frac{du}{dx},$$

where n is a positive fraction whose numerator and denominator are integers. This rule has already been used in the solution of numerous exercises.

34. The Derivative of a Constant. Let $y = c$, where c is a constant. Corresponding to any Δx, $\Delta y = 0$, and consequently

$$\frac{\Delta y}{\Delta x} = 0,$$

and

$$\lim_{\Delta x \doteq 0} \frac{\Delta y}{\Delta x} = 0,$$

or

$$\frac{dy}{dx} = 0.$$

The derivative of a constant is zero.

Interpret this result geometrically.

35. The Derivative of the Sum of Two Functions. Let

$$y = u + v,$$

where u and v are functions of x. Let Δu, Δv, and Δy be the increments of u, v, and y, respectively, corresponding to the increment Δx.

$$\begin{aligned} y + \Delta y &= u + \Delta u + v + \Delta v \\ \Delta y &= \Delta u + \Delta v \\ \frac{\Delta y}{\Delta x} &= \frac{\Delta u}{\Delta x} + \frac{\Delta v}{\Delta x} \\ \frac{dy}{dx} &= \frac{du}{dx} + \frac{dv}{dx}, \end{aligned}$$

or

$$\frac{\mathbf{d(u+v)}}{\mathbf{dx}} = \frac{\mathbf{du}}{\mathbf{dx}} + \frac{\mathbf{dv}}{\mathbf{dx}}.$$

The derivative of the sum of two functions is equal to the sum of their derivatives.