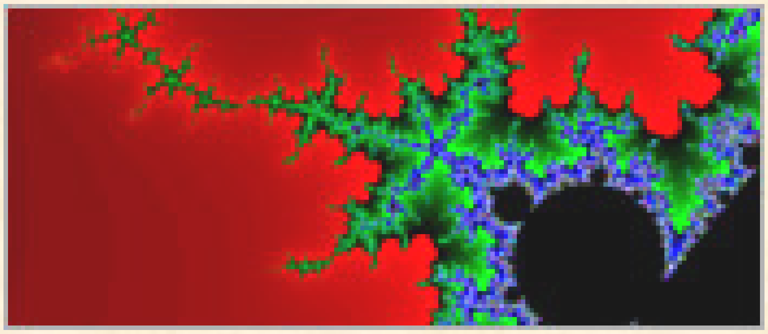
Benoit Mandelbrot's Fractal Geometry



A Radical Mind

"Clouds are not spheres, mountains are not cones, coastlines are not circles. And bark is not smooth, nor does lightning travel in a straight line." So writes acclaimed mathematician **Benoit Mandelbrot** in his path-breaking book The Fractal Geometry of Nature.

Instead, such natural forms -- and many man-made creations as well -- are "rough," he says. To study and learn from such roughness for which he invented the term "**fractal**", Mandelbrot devised a new kind of visual mathematics based on such irregular shapes.

Fractal geometry -- as he called this new math -- is worlds apart from the Euclidean variety we all learn in school. It has sparked discoveries in myriad fields from finance to metallurgy, cosmology to medicine.

In this interview, hear from the father of fractals about why he disdains rules; why he considers himself a philosopher; and why he abandons work on any given advance in fractals as soon as it becomes popular.

Savoring the Unsmooth

Q: You've said "My whole career is an ardent pursuit of the concept of roughness." What exactly do you mean by that?

[Mandelbrot]: Actually, this word "roughness" has different meanings according to context. Until I turned 20 and World War II ended, my life was becoming increasingly "rough" because of historical events over which no one I knew had any control. But after I turned 20, things changed.

Without any clear plan or conscious decision, I became fascinated by -- and then almost exclusively devoted to -- all kinds of phenomena in which irregularity and variability dominate but are so great that they don't average out. This led me first to disbelieve and then to contradict in a radical fashion what everybody else was saying about those phenomena.

Q: In what way?

[Mandelbrot]: The predominant view of irregularity continues to follow Galileo's famous saying that the Great Book of Nature is written in the language of mathematics with the characters being circles, triangles, and other such shapes. A circle is perfectly regular. A triangle has 3 corners and is otherwise very smooth. And the great bulk of Science studies smooth behavior -- in particular using equations that assume that everything evolves in a very regular fashion.

Q: And you were studying the unsmooth?

[Mandelbrot]: That's right. I soon came to devote my life to phenomena that may belong to very different organized sciences but have the common characteristic of being irregular and fragmented at many scales. Like the weather, for instance. I could not possibly anticipate the level of complication this youthful choice would bring to my life trajectory. Each year of my life brought change in my interests, my tools, and my self-confidence. And I greatly learned from both the kind and the unkind comments I continually received.

But I was so alone that the direction I was following was not described by any existing word. In 1975, my work forced me to coin one: **fractal**. The Latin adjective "fractus" can denote anything that is like a broken-up stone -- irregular and fragmented. The sudden realization that "fractal" deserved to be put in a book's title changed nothing in the substance but brought considerable change in the perception of my work. The word is now found in many dictionaries.

"With two hands, you can count all the simple shapes of Nature. Everything else is rough."

Q: And did that one word change your own perception of your work?

[Mandelbrot]: Yes. And it provided a degree of order to a myriad of other objects that I went on to study. In a way, I became attracted to a host of phenomena that for me smelled the same. My friends praised my vision, but I rather felt that I was good at sniffing *{laughs}*. The word "vision" only became appropriate when I had reached the end and could look back.

Myriad Applications

Q: Fractals are beautiful. But they're much more than that. The uses for fractal geometry just keep growing, don't they?

[Mandelbrot]: And how! I heard fractals described endless times as "pretty pictures but pretty useless." Ridiculous! To give only one example, my study of fractals began with the stock market which certainly deserves close attention.

Q: And today there are numerous other applications.

[Mandelbrot]: So numerous and diverse you wouldn't believe it. Many came together within just the last few years. Some of them may seem obvious, but they're extremely important.

For example, take wireless antennas. They used to be sticks. Then sticks with bends and crosses. Such more complicated antennas were not good ideas because one could not make calculations to see how they perform.

Then at one point, enough young people active in that field read my books. Why not make an antenna fractal? Fractal antennas are now almost routine.

Q: And concrete. Concrete has gone fractal, right?

[Mandelbrot]: Yes. Concrete was known to the Romans, then was forgotten and had to be reinvented. The result was not impermeable. Water that came through dissolved it and chunks of it were falling off.

Now concrete has come into its own. Many former physicists moved into studying it because it's a very well-supported field. At a conference on fractal applications that took place around my birthday in 2004, a speaker introduced an altogether new kind of concrete that is enormously stronger and more durable. A form entirely based upon his deep understanding of fractals.

Take something else that wasn't anticipated but is very simple. People living along highways scream about noise. But the flat walls put in place to placate them were very ineffective, because the noise that hit them simply bounced off. Responding to some political pressure, a friend of mine had the brilliant idea that a wall having a fractal surface would be far better because it would absorb the noise. That insight works and underlies the current technology.

Q: These walls are rough -- kind of like Nature would have made them.

[Mandelbrot]: Precisely. In raw Nature, very few shapes are simple. The pupil, the iris, the Moon … With two hands, you can count all the simple shapes of Nature. Everything else is rough. But if you look around us, almost everything industrial is very smooth, round, flat, corrugated, and so on. Now that is changing. Engineers everywhere know how to use fractals.

Q: Even medicine is embracing fractals, such as in interpreting illness, right?

[Mandelbrot]: Yes. Anybody who has had a mild operation may realize how uneven a healthy heartbeat is. I knew this only theoretically until a special dental surgeon put a little thing on my finger to see how my heart was reacting. So I could listen to my heartbeat. I'm a healthy old man. But my heartbeat is very irregular in a way very well described by fractals. Many people hope that this will help enormously in following the progression of illness.

Q: Do you keep up on advances that use fractals?

[Mandelbrot]: Read everything written about fractals? I don't even try. People very often report to me "Here's something you may have missed that is very interesting." I do go to meetings and hear new things. But there is no way to follow everything. I abandon problems when a constituency gets created around them.

Q: Why?

[Mandelbrot]: Well, for example, the "Mandelbrot set" took off like a rocket. Within a year, millions of people had become involved. I felt overgunned by their number and tenacity.

Besides, I can stand loneliness. In fact, I'm rarely comfortable in a big crowd,because big crowds automatically are very specifically organized by dates, by tradition, by training.

And I don't sound like a mathematician. I don't sound like a physicist either. Nor do I sound like an art critic. There's very great strength in being a stranger if one brings something new.

<u>Early Years</u>

Q: You call yourself a "maverick". Why did you choose a lifestyle that's "dangerous"?

[Mandelbrot]: Earlier European politics handed me an extremely complicated youth. My parents were from Lithuania but I was born in Warsaw, Poland. I was at least twice a foreigner, which made an enormous difference. I was forced very early to take very big risks. Often big gambles.

Q: Is that why your family moved to France in 1936 when you were 11?

[Mandelbrot]: My mother was a highly-educated woman. One of the first few Russian doctors in medicine. She had enormous ambitions for her sons. But we lived in a world where everything was unstable. We didn't read newspapers by curiosity but because -- within a week or a month -- what we read could affect our whole life.

We moved to France when my mother at age 50 made a decision I still find admirable and incomprehensible. She abandoned her profession, her roots, her friends, and her place in a well-defined society to become a lonely housewife in a slum in Paris. My parents figured this gave us a better chance to survive.

"I don't only study books. I study Nature."

Q: And what was the War like for you <u>personally</u>?

[Mandelbrot]: The critical year was 1944 when I was 19. I turned 20 just too late to be in the military during the War. But certainly not too late to watch what was happening. A constant feeling of anxiety made me develop what many people call a survival instinct. In a dangerous situation, I was very careful to abstain from intervening. Or even to skip away.

Q: What kind of education did you get early on?

[Mandelbrot]: I don't remember about learning to read and write. But I do remember very well learning to play chess very early. This is a very <u>geometric</u> game. And geometric memory is essential. In Poland, chess is very popular. I was a local champion and soon would have been pushed to compete seriously.

Also, my father was a passionate collector of maps. So I could read them as far back as I can remember. In fact, I learned to crawl on a Caucasian rug that my parents had received as a wedding present. Caucasian rugs are very <u>geometric</u>.

Q: You've said you played "intuitive chess". What do you mean by that?

[Mandelbrot]: Well, I had an uncle who -- like most people in Poland at that time -- seemed to be chronically unemployed. He was kept alive by the family. In particular, he was my tutor. Today this would be called "home-schooling". A very cultured person and a truly nice man, yet very ill-prepared.

But he taught me the rules of chess and played with me. Not according to a textbook but in a very, very informal fashion. He didn't explain to me the famous games with all the old champions' names but instead kept asking "What do you think is the best move?"

It was my good luck to leave Poland and abandon chess about the time when I would have started reading the books about the famous games, the championship of 1870 when so-and-so beat so-and-so, and so on. I never did that.

Free of Constraints

Q: You've said there is a danger of making rules absolute.

[Mandelbrot]: In France, one must belong to one well-defined and very stable sub-community. You do your work and don't look "over the fence" or at what your neighbor does. This is an extraordinary burden largely established in Napoleonic times. By the late 19th Century, it was deeply rooted. The French school of mathematics was very strong and gifted young people were virtually pushed into joining. To the contrary, France had no counterpart to James Maxwell in Britain or Max Planck in Germany who were creating theoretical physics.

Q: But you didn't let such constraints hold you back.

[Mandelbrot]: I didn't. Perhaps my early rootlessness gave me an awareness that one can live without being so completely specified.

Q: You've been interested in the revolution in thinking that took place during the Renaissance. I love the term "natural philosophy" from that period.

[Mandelbrot]: It is lovely indeed. Too bad it hasn't been used since the 18th Century.

Q: What does that term mean to you?

[Mandelbrot]: Before Galileo, a philosopher was somebody who studied the great books. Many of those people were extraordinarily brilliant. But their absolute obedience to books was destructive. What Galileo did was to say that natural philosophy is written in the Great Book of Nature and that one must move from reading the books in the library to reading the books around us. That is, use the experimental method and believe in the power of the eye. That was the big thing. Newton was called a natural philosopher. And in the 18th Century, the professions of Mathematics and Physics were not deeply distinguished. But now they are.

I'm certainly a philosopher entranced with unifying ideas. However, I don't only study books; I study Nature. Also art of the past for the purpose of finding artifacts that I could embrace.

Q: Doesn't such a stance have dangers of its own? Like being too much of a generalist, perhaps?

[Mandelbrot]: This is a fundamental question that deserves a very careful answer. Close and competent authority has continually reminded me that every generalist is a gambler and faces very serious professional threats. Perhaps so, but I was lucky. I am glad to be able to say that many distinct fields forgave me the time I spent on other projects and gave me a very nice share of very nice awards. Had I been nothing but a gambler, I would have vanished absolutely. But I have always been extremely disciplined and conservative.

"What I did was totally despised by my peers who felt that I'd completely destroyed my promise."

Let me elaborate. I did start as a gambler but soon enough realized that all my successful "plays" had a strong common feel, a common flavor. As a result, I gradually made myself into a red-blooded true innovator/specialist. The organizer of a fractal theory of roughness as a new field following its own ways.

Back to your question, which I understand perfectly. Many people I have known were blessed with comparable intelligence, memory, and independence of mind but just abandoned themselves to the pleasure of commenting about everything that comes out. On new discoveries made in, say, Biology or Cosmology. They are the ones who live a very "dangerous" life. One very close friend -- whom I admired endlessly and rated as superior to me in many of the basic tools of our trade -- never knew how to discipline himself. By the end of his life, he had already been forgotten.

A Freewheeling Atmosphere

Q: Is this point of view what led you in 1958 to consider working as a scientist at IBM's Thomas J. Watson Research Center?

[Mandelbrot]: No. Going to IBM was initially pure accident. And it took me many years to realize how lucky I'd been. In fact, my story provides an interesting reflection on what it is to be lucky. I had the chance of enjoying a very long and varied apprenticeship which included working with people nobody would expect me to work with. For example, I spent 2 years in Geneva as an associate of Jean Piaget, the child psychologist.

However, in the mood that prevailed in France, I belonged nowhere. What I did was totally despised by my peers who felt that I'd completely destroyed my promise. That I was just playing around with things of no particular interest. A good older friend commented that my Ph.D. was one-half in a field that didn't yet exist and one-half in a field that no longer existed. But that didn't disturb me.

Q: Why not?

[Mandelbrot]: I was trying to tell my colleagues and everybody that assuming there is a choice, one should not force people to move as fast as possible and decide as early as possible where to go. One should not force people to classify themselves before they are ready. But it was talking to the deaf. In France, if you tried to change from field-to-field, you had to start from scratch.

Q: You've said you were forced out of France.

[Mandelbrot]: I was referring to an intense feeling of not belonging. Also, I was a junior professor of mathematics. But my senior professor -- whom I liked very much -- was about to retire. And the idea of working for his replacement was totally unbearable. I could have stuck around, I'm sure, but I would not have liked it.

Q: How did you get to IBM, then?

[Mandelbrot]: Pure luck. Some people I knew were joining IBM Research. They invited me for a summer in 1958, just after I had started teaching in France and was very dubious about my prospects there. IBM was changing all the time. So I thought I might -- perhaps for a couple of years -- do enough to please the Establishment and yet save enough time to do some gambles of mine.

Q: Wasn't IBM the image of a highly structured environment, though?

[Mandelbrot]: This image was strong and pervasive. But the reality was different. The goal was to create a freewheeling academic atmosphere. While they lasted, IBM's laboratories were highly renowned.

That summer was very nice. So after a substantial discussion, my wife and I decided to ask France for a leave and stay at IBM for a year-or-two. During the first 2 years at IBM, I hit a very conspicuous jackpot and was invited by Harvard as Visiting Professor of Economics *{laughs}*. So we postponed our return to France.

"It dawned on him that I was unpredictable and might do things about which he didn't give a hoot."

Q: Which you eventually postponed indefinitely.

[Mandelbrot]: Yes. After a year as Visiting Professor of Economics, I hit a second jackpot. Harvard asked me to return as a Visiting Professor of Applied Physics. So while I had left France temporarily, I eventually faced the reality of the situation.

I was helped by Jerry Weisner -- a very remarkable fellow and good friend who later became President of M.I.T. I asked him for advice about what to do next. It's a long story. But his conclusion was that IBM was no longer a chancy situation. There was no other place in any country where I could do my kind of work.

Q: Because of that freewheeling academic atmosphere?

[Mandelbrot]: Yes. IBM was very keen to achieve intellectual renown and technological success. So they wanted me back while everywhere else I had this problem of being unclassifiable. People would tell me: "If we knew that you were going to stop running around and -- from today on -- become an honest economist or an honest electrical engineer or an honest this-or-that, we would offer you a job instantly."

At one point, a university offered me a very high position. But the next day, the Dean called to withdraw the offer because it dawned on him that I was unpredictable and might do things about which he didn't give a hoot.

Q: *{laughs}* You've had a lot of opposition over the years, haven't you?

[Mandelbrot]: I have. For example, a proposal I made to the National Science Foundation was turned down. I was very surprised when a man telephoned and told me, "In order to fund this thing, we need six Outstanding reviews. You have five Outstanding and one Excellent. So you can't be funded."

I was so shocked that I raised my voice. In Europe, to raise one's voice is okay. But not in America. I told this man, "Please consider the fact that what I propose is completely at variance with everything else you're supporting. In a certain sense, this proposal is criticizing my peers because I think they do things in a narrow fashion and I'm offering a new way. So five Outstanding and one Excellent should be viewed not as a failure but as something of a success."

To his credit, he agreed … sort of. "Yes, okay," he said. "I'll fund you. But I'm going to cut your grant in half."

<u>Asking Questions</u>

Q: Now in Mathematics, your main contribution has not been proofs but <u>new</u> <u>questions</u>, correct?

[Mandelbrot]: That's been the case in pure mathematics in which the overwhelming bulk of mathematical work consists of proving or extending existing statements. But my work in other fields has had a very, very different aspect. In Economics, there are no proofs. Science is "proven" by its applicability.

Now my uncle -- who was a mathematician given to strong opinions -- was very scornful of some of his peers. He said that they were very, very good but they were just theorem-provers. They have an extraordinary arsenal of techniques, remember many previous results, and put them together in new ways. But they don't have the creativity to ask new questions.

So in Mathematics, there has been historically this more-or-less sharp distinction between those who are best known for asking questions and those who are best known for proving theorems that others have conjectured.

Q: Who among mathematicians do you most admire for asking questions?

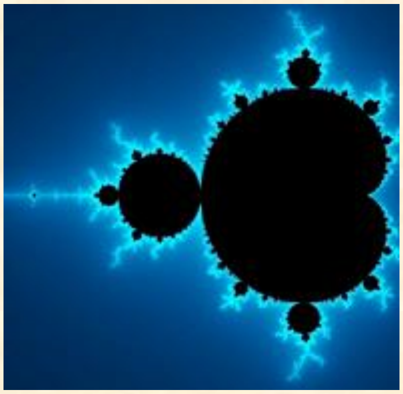
[Mandelbrot]: The greatest mathematician in my private pantheon has been Henri Poincaré. Altogether a very great man, he started many branches of Mathematics from scratch. But he acknowledged himself that he didn't prove any difficult theorem and cared about proofs less than about concepts.

I'm nowhere near Poincaré {*laughs}*. Don't misunderstand me. My point is that a large number of truths that I discovered did not result from purely mathematical deduction but from skilled examination of mathematical pictures.

Q: And do you see the World differently now because of those mathematical pictures? Because of fractals?

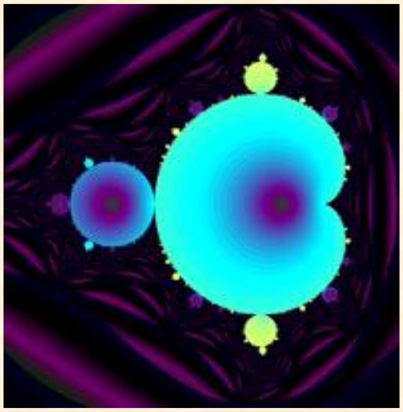
[Mandelbrot]: I certainly see the World today differently from the way I saw it early on. And friends of mine who are mountain climbers tell me they see mountains differently now than before.

People who just like to look out the window when they fly … They tell me that they see mountains <u>differently</u> now than before. They see an <u>orderliness</u> to mountains, piles-upon-piles of pyramids that before they did not see.

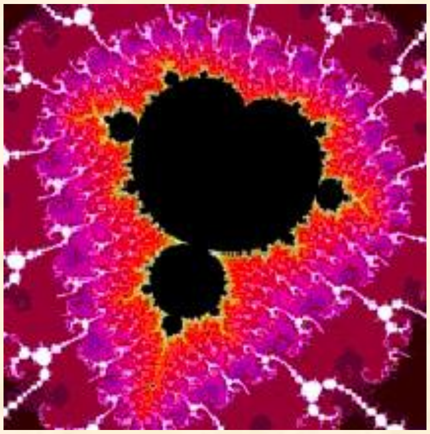


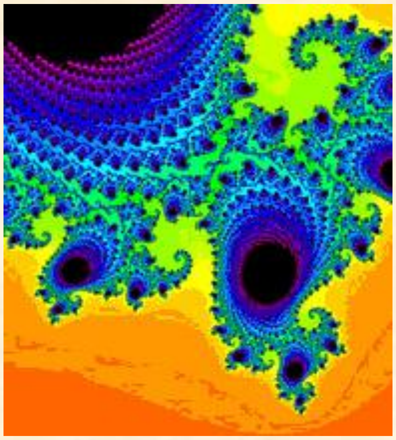
The Mandelbrot set seen here in an image generated by NOVA epitomizes the fractal.

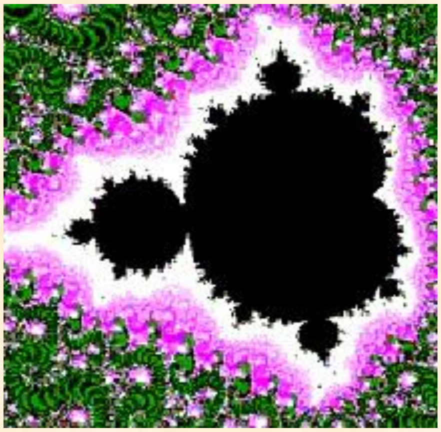
For more on the set, see **The Most Famous Fractal** and **A Sense of Scale** which takes the left image and zooms into it over-and-over, finally reaching a size 250 million times smaller than the original.

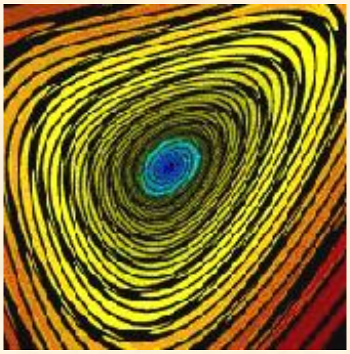


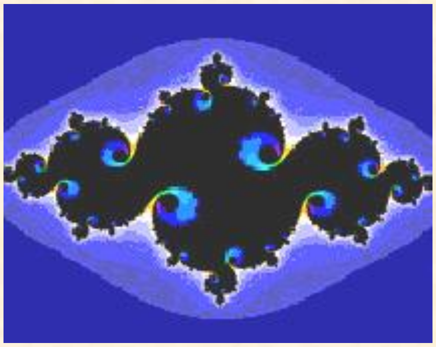
This fractal image and all the ones to follow were created at Art Matrix which is part of the Lightlink Supercomputer Facility in Ithaca, New York. Thanks to Art Matrix's Homer Smith and Jane Staller for supplying the images.

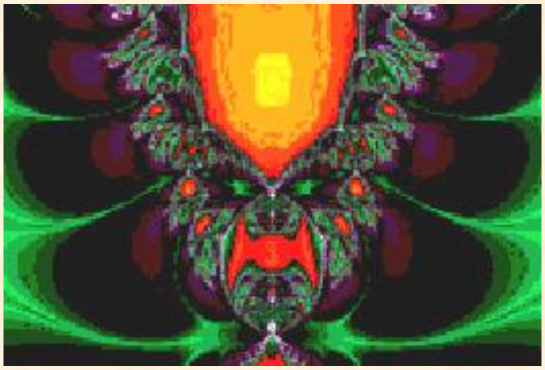
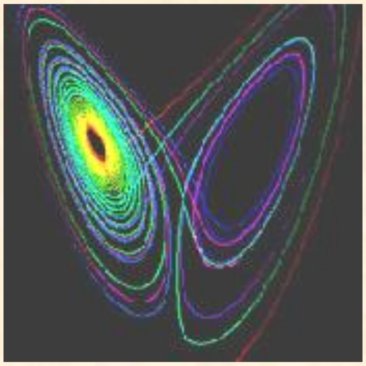


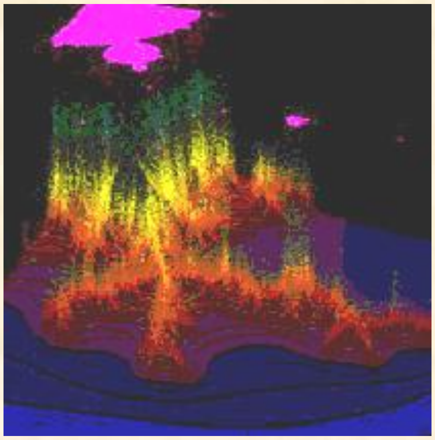
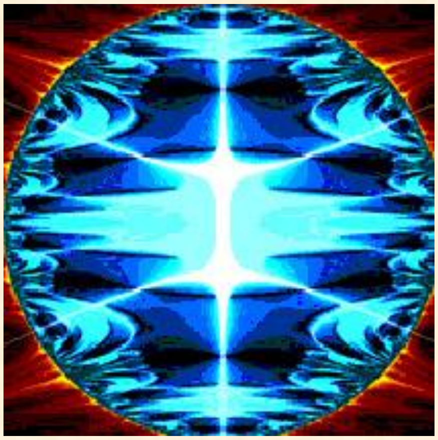
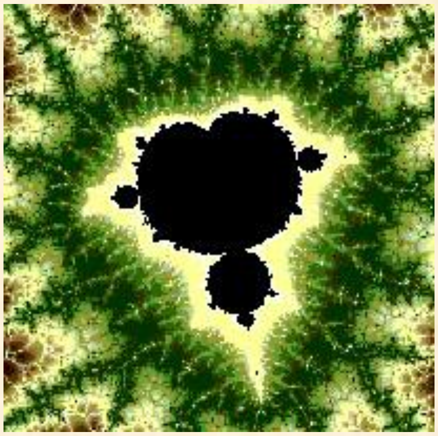
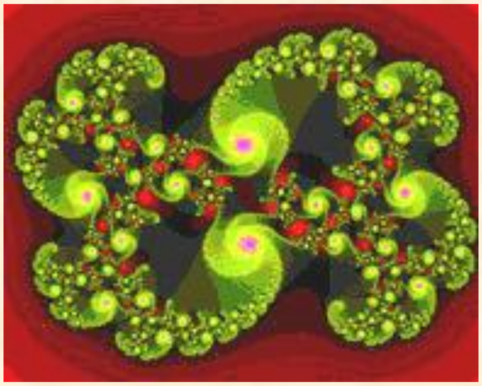








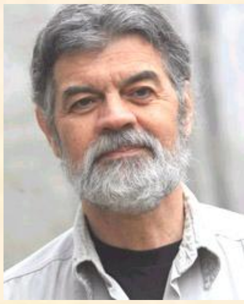



http://www.pbs.org/wgbh/nova/fractals/set.html

The Most Famous Fractal

by John Briggs



John Briggs is author of *Fractals: The Patterns of Chaos* *(Simon & Schuster, 1992), from which this article was excerpted with kind permission of the author and publisher.*

Largely because of its haunting beauty, the **Mandelbrot set** has become the most famous object in modern mathematics. It is also the breeding ground for the World's most famous fractals. Since 1980, the set has provided an inspiration for artists, a source of wonder for schoolchildren, and a fertile testing ground for the science of linear dynamics.

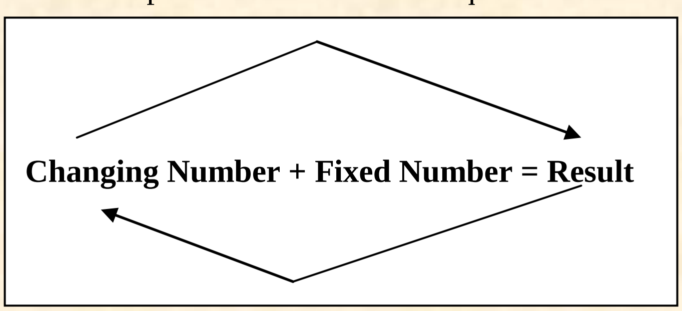
The set itself is a mathematical artifact. An odd-shaped infinite swarm of points clustered on what is known as the "complex number plane." Let's try to visualize it.

Picturing the plane

To make them tangible, we imagine real numbers like 1, 2, 3... as spaced out along a number line. Because complex numbers have 2 parts to them (called their "real" and "imaginary" parts),making complex numbers tangible requires 2 lines (or axes). Which means a plane. Picture the plane dotted by complex numbers as a computer screen (which is just where the visual form of the Mandelbrot set was discovered).

Like the screen of your television set, a computer screen is covered with a host of very tiny, evenly-spaced points called "pixels". The moving image on the screen is made when patterns of pixels are excited (made to glow) by a fast-moving scanning beam of electrons. Think of each pixel as a complex number. The pixels in any neighborhood are numerically close to each other, just as 3 and 4 are numerically close to each other on the real number line. Pixels (numbers) are made to glow by applying an iterative equation to them.

In the late 1970s and early 1980s, Benoit Mandelbrot (the inventor of fractal geometry) and several others were using simple iterative equations to explore the behavior of numbers on the complex plane. A very simple way to view the operation of an iterative equation is as follows:



Start with one of the numbers on the complex plane and put its value in the "Fixed Number" slot of the equation. In the "Changing Number" slot, put zero. Now calculate the equation; take the "Result"; and slip it into the "Changing Number" slot. Repeat the whole operation again (in other words, recalculate and "iterate" the equation) and watch what happens to the "Result". Does it hover around a fixed value? Does it spiral toward infinity quickly? Or does it stagger upward by a slower expansion?

A Mathematical Marvel

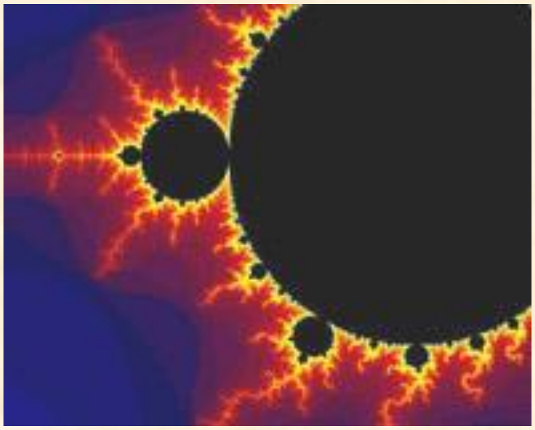
When iterative equations are applied to points in a certain region of the complex plane, the results are spectacular. By treating the pixels on computer screens as points on the plane, even nonmathematicians can now admire this marvel. In fact, without computers, only the most intuitive mathematicians could have glimpsed what was there. With the computer, it works like this:

Starting with the value of a point (or pixel) and applying the equation to it, iterate the equation perhaps 1,000 times. If the "Result" remains stable, color the pixel 'black'. If the number heads at one speed or another toward infinity, paint it a different color, assigning colors for each rate of movement.

The points (pixels) representing the fastest-expanding numbers might be colored 'red'; slightly slower ones 'magenta'; very slow ones 'blue' … whatever color scheme that the fractal explorer decides. Now move on to the next pixel and do the same thing with the color palette until all the pixels on the screen have been colored.

Artists and the public have been attracted by the set's haunting beauty.

When all the pixels (or points representing complex numbers) have been iterated by the equation, a pattern emerges. The pattern that Mandelbrot and others discovered in one region of the complex plane was a long-proboscidean insect shape of stable points -- the Mandelbrot set itself -- usually shown in black surrounded by a flaming boundary of filigreed detail that includes miniature, slightly distorted replicas of the insect shape and layer-upon-layer of self-similar forms.



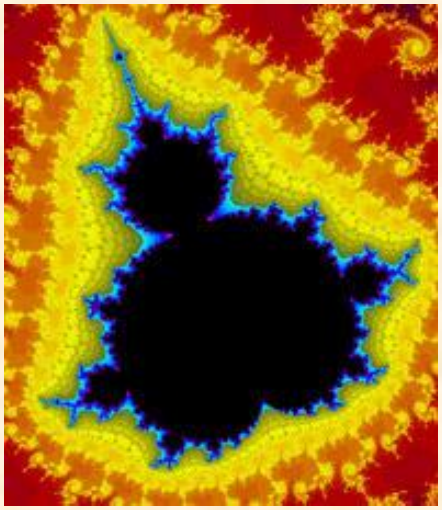
In this detail of the Mandelbrot set, the set itself appears in black with the fractal boundary alive with color.

Infinitely fine

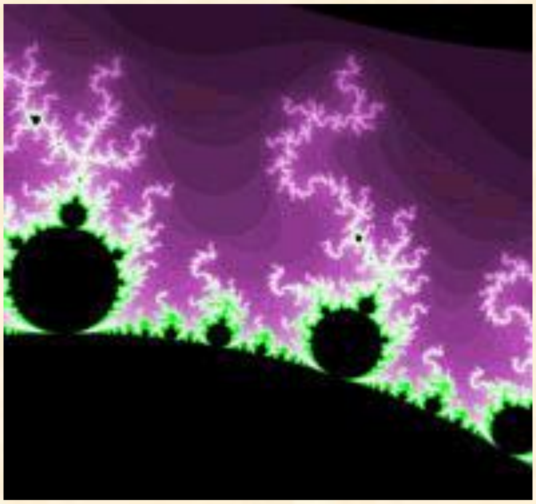
The boundary area of the set is infinitely complex (therefore fractal) because it is possible to bring out finer-and-finer detail. Computer graphics artists call the process of unfolding the detail "zooming in" on the set's boundary or "magnifying" it. It's fairly easy to grasp what this means.

On the real-number line, we routinely imagine that between the numbers '1' and '2' are other numbers. 1.5, for example, or 1.6. (We encounter this every time we pick up a ruler.) Of course, between those numbers are still more numbers (1.53 and 1.54, for example) and so on, indefinitely.

Because an infinite number of points exist between any two points on the number plane, the Mandelbrot set's detail is infinite. This image is a tiny part of the previous image magnified many thousands of times over.



The same is true for the numbers on the complex plane. Between any two of them are many more. And between those many more are many more still ad infinitum. These numbers between numbers allow us to use the computer like a microscope to dive into increasingly deeper detail.



"A flaming boundary of filigreed detail" is how Briggs aptly describes the border of the Mandelbrot set.

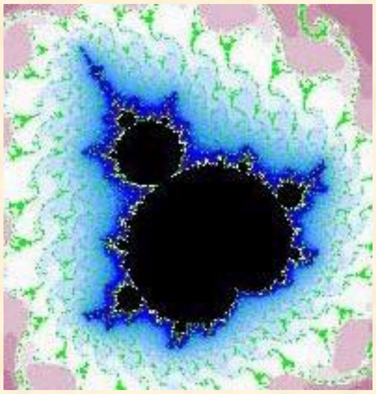
To extend our analogy, if the numbers we were examining on the complex plane were all like the numbers at the level of say, 1, 2, 3, etc. on a ruler, then we would be examining the largest scale of numbers. But we could also go to a smaller scale and examine the numbers at the level of 1.5, 1.6. Between those will be yet a smaller scale (including the numbers 1.53 and 1.54, for example). And so in any region of the complex plane, we could move downward (or inward) to smaller-and-smaller scales.

Similarly, explorers of the Mandelbrot set can zoom in to study finer and finer detail as they examine the ever smaller scales of numbers between numbers on the complex plane. Indeed, a home computer can examine numbers out to 15 decimal points.

To complete the microscope analogy, if the numbers '1' and '2' were the equivalent of objects the size of human beings and trees, a number '15' decimal points smaller would be an object tinier than an atom. More powerful computers can go into even finer (or deeper) detail. In addition, different styles of iterative equations can act as prisms to display varying facets of the behavior of the complex numbers around the set.

Applying zoom-ins and different iterative prisms to the numbers in the boundary area of the Mandelbrot set has revealed that this region is a **mathematical strange attractor**. The "strange attractor" name here applies to the set because it is self-similar at many scales; is infinitely detailed; and attracts points (numbers) to certain recurrent behavior.

Scientists study the set for insights into the nonlinear (chaotic) dynamics of real systems. For example, the wildly different behavior exhibited when 2 numbers with almost the same starting value and lying next to each other in the set's boundary are iterated is similar to the behavior of systems like the weather undergoing dynamic flux because of its "sensitive dependence on initial conditions."



The "self-similar" nature of fractals means that particular elements (such as the Mandelbrot set) reappear over-and-over again no matter how "deep" one goes into the image through magnification.

Strange Attraction

But a major importance of the set may be that it has become a strange attractor for scientists, artists, and the public, though each may be drawn to it for quite different reasons. Scientists have found themselves attracted -- often with childlike delight -- to a new aesthetic that involves the artistic choices of color and detail they must make when exploring the set. Artists and the public have been attracted by the set's haunting beauty and the idea of abstract mathematics turned into tangible pleasures.

Homer Smith (cofounder of an independent research group based at Cornell University) produces fractal images with the aim of attracting young children to mathematics. "We hope that fractals show up in early classrooms to get kids interested in Mathematics very early," Smith says, "because it really opens the eyes of children who haven't been turned off by education... We hope by the time they get up to the 10^{th} grade, they'll have seen these things and say, 'There's something here in math, science, and computers that I want to learn.'"

http://www.pbs.org/wgbh/nova/fractals/design.html

Design A Fractal

-- Rachel VanCott

The image of the Mandelbrot set is one of the most recognizable representations of a fractal. But what's behind the entrancing picture?

In this interactive, learn a bit about how we generated our version of the iconic image and make and save your own versions of the set. If you want to explore the set a bit more deeply, check out "The Most Famous Fractal" and "A Sense of Scale".

<Launch interactive>

http://www.pbs.org/wgbh/nova/fractals/scale.html

A Sense of Scale

-- Peter Tyson

So, naturalists observe, a flea
Has smaller fleas that on him prey;
And these have smaller still to bite 'em,
And so proceed ad infinitum.
-- Jonathan Swift from "On Poetry: A Rhapsody"

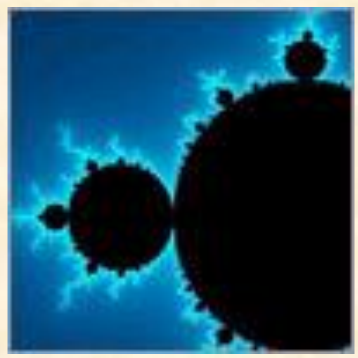
The satirist and author of Gulliver's Travels might have been talking about fractals when he made this oft-quoted observation if he hadn't lived 2 centuries before fractals were discovered. (In fact, Swift was complaining about lesser poets criticizing better ones like pestering fleas.)

As it happens, these 4 lines can serve as a perfect metaphor for the infinitely detailed, "self-similar" nature of fractals.

In this interactive, zoom deep into a Mandelbrot set (the most famous of fractals) to the mind-bending magnification of 250,000,000x. Along the way, you'll see what is meant by self-similarity; how the iconic Mandelbrot-set shape keeps turning up at smaller scales like one of Swift's ever-tinier fleas; and why the 18th-century wit's metaphor suits fractals to a tee.—

[Note: NOVA Online's Rachel VanCott used the program "Ultra Fractal 5" to generate these images of the Mandelbrot set.]

<Launch Interactive>



1x

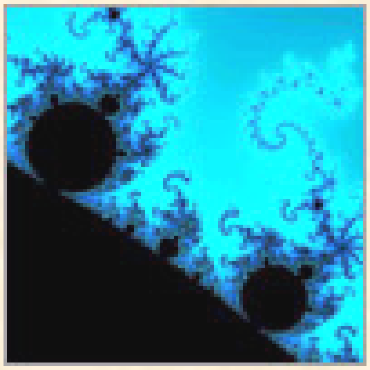
The black, flea-like shape you see here is the classic Mandelbrot-set icon. Okay, it looks more like "Frosty the Snowman" tipped over into an inkwell. But in honor of Jonathan Swift (see intro), suspend disbelief for a moment and imagine it as a kind of fractal flea seen from above with its bristly head facing left. Now see the next image to zoom in on our fractal flea's head.

10x

Having magnified the image 10 times, the first thing you might notice beyond the great "V" of blue is that the icon -- our fractal flea -- reappears in smaller form all along the edge of the flea.

Our flea is positively infested with Mandelbrot-set icons which get tinier and tinier as your eye drifts down into the crease. Swift would like where this is going, don't you think?

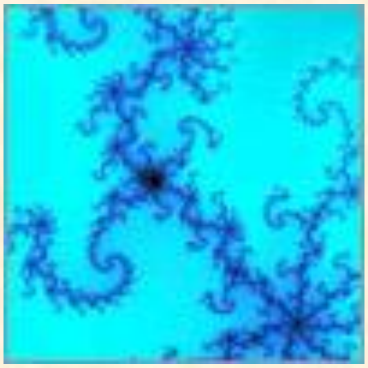
50x



Now you can see those tiny fractal fleas more closely. They're not exact replicas of the Big Flea at the start. But they're pretty close.

In fact, this fractal -- like all fractals -- is "self-similar". No matter how much you magnify certain portions of the colorful boundary area of a Mandelbrot set, you will come upon particular shapes (like our fractal flea) that closely resemble shapes you've already seen at lower magnification.

500x

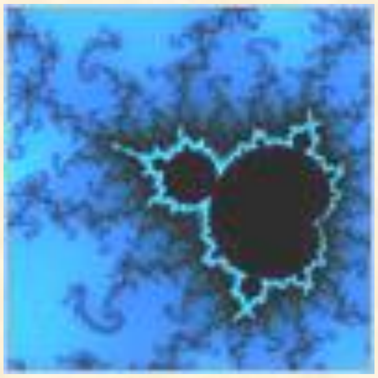


Wow! Rather frondy, if that's a word. Where did all the fractal fleas go?

Well, a Mandelbrot set includes more than just the classic icon we started with. In fact, any portion of the images you see in this feature that is 'black' belongs to the Mandelbrot set that was generated by the iterative equation we used. (For more on iterative equations and how they generate fractals, see "The Most Famous Fractal".

But wait... What is that hidden amidst this filamentous frond? You got it! Another Mandelbrot-set icon.

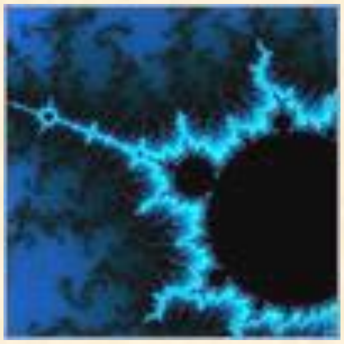
5,000x



See how similar it is to our Big Flea? It just looks a little frothier around the edges, perhaps.

Amazingly, the simple equation that furnished our original Mandelbrot-set icon created this one as well as all the swirly filaments and colors at this and all other powers of magnification. Who knew math could be so artistic?

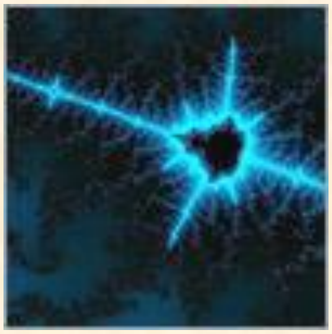
20,000x



We've now magnified our view 20,000 times. The big round shape to the right is the head of our previous fractal flea.

Like the 60-foot-tall farmer who finds Gulliver when he's marooned in the Land of Brobdingnag (we can't forget Swift, can we?), the larger flea dwarfs yet another, even tinier flea clinging to its long "mouthparts." (Run with us here a bit.)

100,000x

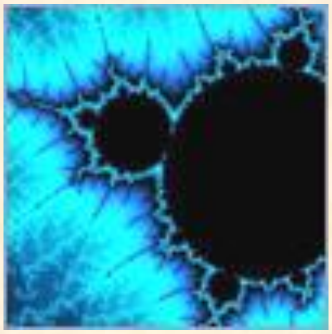


The edges of this fractal flea are a little less well-defined than those of earlier ones. But it is still easily recognizable as a Mandelbrot-set icon. So how can we keep zooming in like this and still find such vivid detail?

Well, each of these images represents a graph of the complex number plane. And between any two numbers on the plane exists an infinite number of numbers.

So a Mandelbrot set, by definition, is infinitely detailed.

500,000x

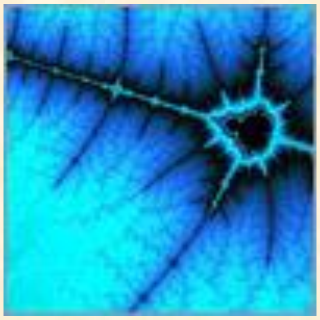


We are now half-a-million times smaller than when we started.

And Swift's ever-smaller fleas? Still here, pressed up against this particular flea like …

Well, like Lilliputians tossing ropes over the unconscious Gulliver.

5,000,000x



Is your head hurting yet? Five million times smaller. Our image is beginning to look like a leaf with its delicate tracery of veins. That's fitting because a tree is a good example of the fractal nature of Nature.

Branches branch into smaller branches which themselves branch into yet smaller branches, all roughly similar to any other branch on the tree. Same goes for the veins in a leaf.

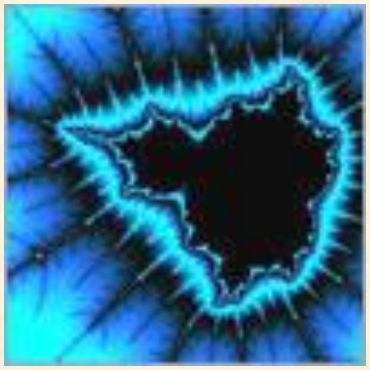
50,000,000x

Even more like we're inside a leaf, veins shooting out everywhere. And, of course, the ubiquitous icon is here. (It may look more like a mini space shuttle at this point. But it's still a Mandelbrot-set icon.)

Though you can't see 'em, the icon has its own "smaller fleas that on him prey", of course.

"Thus every poet, in his kind/Is bit by him that comes behind," writes Swift in the lines that follow his flea metaphor, "Who, though too little to be seen/Can teaze, and gall, and give the spleen..."

250,000,000x



We could proceed ad infinitum -- magnifying ever further -- but we don't want to give you the spleen. So just how tiny have we gotten at a magnification of 250,000,000x?

Well, if our Big Flea was an actual flea, you are now looking at something smaller than an atom of Carbon within that flea. Or, if what appears here is our actual flea, then our Big Flea is about the size of Switzerland.

That's one Brobdingnagian flea, eh Mr. Swift?!

if on the Internet, Press <BACK> on your browser to return to the previous page (or go to www.stealthskater.com)

else if accessing these files from the CD in a MS-Word session, simply <CLOSE> this file's window-session; the previous window-session should still remain 'active'