be represented as $\Sigma W_{h'}\bar{y}_h = K\Sigma W_h\bar{y}_h(k_h/K)$. The weighting can be accomplished by eliminating the proportions $e_h = (1 - k_h/K)$, so that the retained portions, $(1 - e_h) = k_h/K$, will accomplish the desired weights. The increase of the unit variance in each stratum is

$$\frac{e_h}{1 - e_h} = \frac{1 - k_h/K}{k_h/K} = \frac{K - k_h}{k_h}. \tag{11.7.4'}$$

11.7C Losses from Oversampling Strata

If the sampling fractions of a stratified sample diverge from optimum allocation, the variance is increased. We express the increase in variance as a function of the divergence of sampling rates. We emphasize the common situation when optimum allocation would be proportionate: How great is the increase in the variance due to unequal sampling rates? It can be large if the rates vary considerably; for example, if in two halves the rates are in the proportion 1:4, the increase in variance is 1.56. For an unrestricted random sample the increase is similar.

The weights k_h in a stratified sample are inversely proportional to the sampling fractions $f_h = f/k_h$; hence $n_h = f_h N_h = fN_h/k_h$. The stratum sizes are assumed proportional to the numbers of elements, hence $W_h = N_h/N$ and $n_h = mW_h/k_h$, where $m = fN = \Sigma fN_h = \Sigma n_h k_h$ denotes the properly weighted total sample size. The number of elements in the sample is $n = \Sigma n_h = m\Sigma W_h/k_h$.

The element variance for the mean of a stratified sample may be denoted

$$\begin{aligned} nV^2 = n \operatorname{Var}\left(\sum W_h\bar{y}_h\right) &= n\sum W_h^2\frac{(1 - f_h)}{n_h} S_h^2 \\ &= \frac{n}{m}\sum W_h k_h S_h^2\left[1 - \frac{f}{k_h}\right] \\ &= \left[\sum W_h k_h S_h^2\left(1 - \frac{f}{k_h}\right)\right]\left[\sum W_h/k_h\right]. \end{aligned} \tag{11.7.5}$$

Instead of fixing n, the variances may be compared for a fixed cost $C = \Sigma J_h n_h = m\Sigma J_h W_h/k_n$. Therefore, the variance obtained for a unit cost C/m is

$$V^2C = \left[\sum W_h k_h S_h^2\left(1 - \frac{f}{k_h}\right)\right]\left(\frac{\sum J_h W_h}{k_h}\right). \tag{11.7.5'}$$

This form is useful for investigating the efficiency of various allocations, including the proportionate: $(1 - f)(\Sigma W_h S_h^2)(\Sigma J_h n_h)$. But now we want to investigate the results of different allocations when the S_h^2 and J_h are equal. This case is approximated in many situations where the cost J_h