W \ K	1.5	2	3	4	5	10	20	50
0.5	1.042	1.125	1.333	1.562	1.800	3.025	5.512	13.005
0.2	1.027	1.080	1.213	1.360	1.512	2.296	3.888	8.683
0.1	1.015	1.040	1.120	1.202	1.288	1.729	2.624	5.322

TABLE 11.7.I Effect of Two Divergent Selection Rates.

The sampling rates and the weights are in the proportion $1:K$ in two strata, which have the sizes W and $(1 - W)$. The variance of oversampling is presented relative to proportionate sampling, when the latter is optimum (11.7.8). Note the large increases for large weight differences, especially when W is near 0.5.

to find the ratio of actual variance to proportionate variance, assuming that the latter is optimum.

1. For a rectangular distribution of stratum size, that is, for *uniform size strata*, where $W_h = 1/H$, *with the* k_h *increasing linearly from* 1 *to* H, the variance is

$$\sum W_h k_h \left(\sum \frac{W_h}{k_h}\right) = \frac{1}{H^2}(\sum h)\left(\sum \frac{1}{h}\right) = \frac{H(H+1)}{2H^2}\left(\sum \frac{1}{h}\right)$$
$$= \frac{H+1}{2H}\left(\sum \frac{1}{h}\right). \tag{11.7.9}$$

2. *If the strata are of uniform size,* $W_h = 1/H$, *and the* $k_h = 1$ *for all but one stratum in which* $k_h = H$, the variance is

$$\sum W_h k_h \left(\sum \frac{W_h}{k_h}\right) = \frac{1}{H^2}(\sum k_h)\left(\sum \frac{1}{k_h}\right)$$
$$= \frac{1}{H^2}(H - 1 + H)\left(H - 1 + \frac{1}{H}\right)$$
$$= 1 + \frac{(H-1)^3}{H^3}. \tag{11.7.10}$$

3. *For a triangular distribution of stratum size* we must have $W_h = h/\Sigma h = h/[H(H+1)/2]$, because $\Sigma W_h = 1$. Note that we get the same results, whether the weights are $k_h = h$, increasing linearly with stratum size from 1 to H, or $k_h = H/h$, decreasing linearly with stratum size; in either case we get

$$\sum W_h k_h \left(\sum \frac{W_h}{k_h}\right) = \frac{(\sum 1)(\sum h^2)}{[H(H+1)/2]^2} = \frac{H[H(H+1)(2H+1)/6]}{H^2(H+1)^2/4}$$
$$= 1 + \frac{H-1}{3(H+1)}. \tag{11.7.11}$$