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# Classifying inventory using an artificial neural network approach

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**Abstract**

This paper presents artificial neural networks (ANNs) for ABC classification of stock keeping units (SKUs) in a pharmaceutical company. Two learning methods were utilized in the ANNs, namely back propagation (BP) and genetic algorithms (GA). The reliability of the models was tested by comparing their classification ability with two data sets (a hold-out sample and an external data set). Furthermore, the ANN models were compared with the multiple discriminate analysis (MDA) technique. The results showed that both ANN models had higher predictive accuracy than MDA. The results also indicate that there was no significant difference between the two learning methods used to develop the ANN. 

*Keywords*: ABC classification; Neural network; Genetic algorithms; Back propagation

## 1. Introduction

In organizations that have thousands of different types of materials and spare parts, it is easy to lose sight of effectively managing materials. To keep an organization from misallocating its material management resources, planning and control systems need to be implemented. An effective procurement planning and control system maintains a balance between two opposing forces. On one hand, it must protect a company against critical stock-outs of raw materials, work-in-progress inventories and finished goods. On the other hand, inventory costs must be held to the lowest possible level. ABC inventory classification is a widely used procurement planning and control method that is designed to achieve an appropriate balance between these valid but conflicting economic forces. ABC classification

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allows organizations to separate stock keeping units (SKUs) into three classes: A — very important; B — moderately important; and C — least important. The amount of time, effort, money and other resources spent on inventory planning and control should be in the relative importance of each item. Thus, the purpose of classifying items into groups is to establish appropriate levels of control over each item.

Traditionally, the classification of inventory into the A, B, or C categories has generally been based on dollar value per unit multiplied by annual usage rate, commonly known as dollar usage (Cohen & Ernst, 1988). In this classification, items are ordered in descending order of their annual dollar usage values. The relatively small number of items at the top of the list (approx. 10%) controlling the majority of the total annual dollar usage constitute class A, and the majority of the items at the bottom of the list (approx. 60%) controlling a relatively small portion of the total annual dollar usage constitute class C. Items between the two classes constitute class B (approx. 30%). These classifications may not always be exact, but they have been found to be close to the actual occurrence in firms with remarkable accuracy, Swamidass (2000). Class A items require tight inventory control because they represent such a large percentage of the total dollar value of inventory. This requires accurate demand forecasts and detailed record keeping. In addition, close attention should be given to purchasing policies and procedures if the inventory items are acquired from sources outside the firm. Class C items should receive a flexible control, such as a simple two bin system. Finally, class B items should have a control effort that lies between these two extremes. The reader interested in the details of inventory control policies for the above classes is referred to Silver, Pyke and Peterson (1998). For many items however, there may be other criteria that represent important considerations for management. The certainty of supply, the rate of obsolescence, the criticality of the item, and the impact of stock-out of the item are all examples of such considerations. Some of these may weigh more heavily than dollar usage in the procurement planning and control of the item.

Many companies tend to rely on ad hoc decisions of their inventory managers for considering other criteria for ABC classification. These decisions are based on the manager's experience, knowledge and judgement. Although sometimes very effective, this ad hoc decision making process may not always be consistent while considering all the criteria which may be relevant.

In recent years, several new multi-attribute approaches to inventory classification have been introduced. For example, Flores and Whybark (1986, 1987) have proposed the bi-criteria matrix approach. Essentially their approach is to use standard ABC classification on each of two criteria, and then combine the two single-criterion grouping through the use of a joint-criteria matrix. Though this approach is a step forward in multi-criteria ABC classification, it does have some limitations. First, there is no obvious way to extend the procedure to more than two criteria. Second, the weights of different criteria are assumed to be equal. Ernst and Cohen (1990), have proposed a methodology based on statistical clustering, which utilizes a full range of operationally significant attributes. The main advantage of this approach is that it can accommodate large numbers of combinations of attributes, which are significant for both strategic and operational purposes. However, this requires substantial data, the use of factor analysis, a clustering procedure, which may render it impractical in typical stockroom environments. Furthermore, the clusters themselves must be re-evaluated in order to classify new stock items, so there is a chance that previously classified stock may end up being classified differently every time new items are added: this may disturb the inventory control procedure. In short, their model may be too sophisticated for the average manager.

The analytic hierarchy process (AHP), which was introduced by Saaty (1980) has also been adopted by many authors for ABC classification (Gajpal, Ganesh & Rajendran, 1994; Partovi & Burton, 1993; Partovi & Hopton, 1994). The advantage of the AHP approach is that it can incorporate many relevant qualitative, and quantitative criteria when classifying inventory. Other advantages to this approach include ease of use and minimal reliance on a massive accounting and measurement system. Unfortunately, the use of AHP for ABC classification has some shortcomings: one of the important drawbacks of the method is that a significant amount of subjectivity is involved in pair wise comparisons of criteria, rating levels and assigning a rating level and associated weights. Today, with the advancement of computer technology, more data is available. Hence, advanced database methodologies have proliferated to help manage the decision-making process, thereby reducing the amount of human involvement in the process. This in turn enhances accuracy and consistency of the decision making process, while reducing the processing time. One of the methods for multi-criteria inventory classification is the use of artificial intelligence (Guvenir & Erel, 1998).

Artificial neural networks (ANNs) is an artificial intelligence based technique, which is applicable to the classification process. The ANN can simulate a manager's utilization of perceived relationships for both quantitative and qualitative cues that provide important intermediate steps towards reaching the decision maker's final judgment. These networks have at least two potential strengths over the more traditional model-fitting techniques such as regression (Bishop, 1995). First, ANNs are capable of detecting and extracting nonlinear relationships and interactions among predictor variables. Second, the inferred patterns and associated estimates of the precision of the ANN do not depend on the various assumptions about the distribution of variables. The purpose of this study is to examine the classification accuracy of ANN as an aid to facilitate the decision making process of classifying inventory items. More specifically two types of learning methods, namely back propagation (BP) and genetic algorithms (GA) are used to examine the ANNs classification ability. These models replicate the manager's decision, based on the data pertaining to the important criteria for classification of each SKU.

The rest of this paper is organized as follows. Section 2 reviews the concepts of ANN. This is followed by the research methodology, and the evaluation of the classifier models. The paper concludes with a summary of the findings and directions for future research.

## 2. Artificial neural networks

An ANN is a parallel, dynamic system of highly interconnected and interacting parts based on neurobiological models. The neurobiological nervous system consists of individual but highly interconnected nerve cells called neurons. These neurons typically receive information or stimuli from the external environment similar to the way the neuron in a biological eye registers the intensity of light in a room. These stimuli pass through the network by neurons releasing neurotransmitters to the neighboring neurons. The connections between the neurons are called synapses. The stimuli can either excite the neuron or inhibit it. If the receiving neuron is excited by the information it receives, it will fire when it gets the input and pass the information on to other neighboring neurons. If the neuron is inhibited, the input of the information is dampened and not passed on. In other words, the neurons process the information and pass it on only if it is considered important.

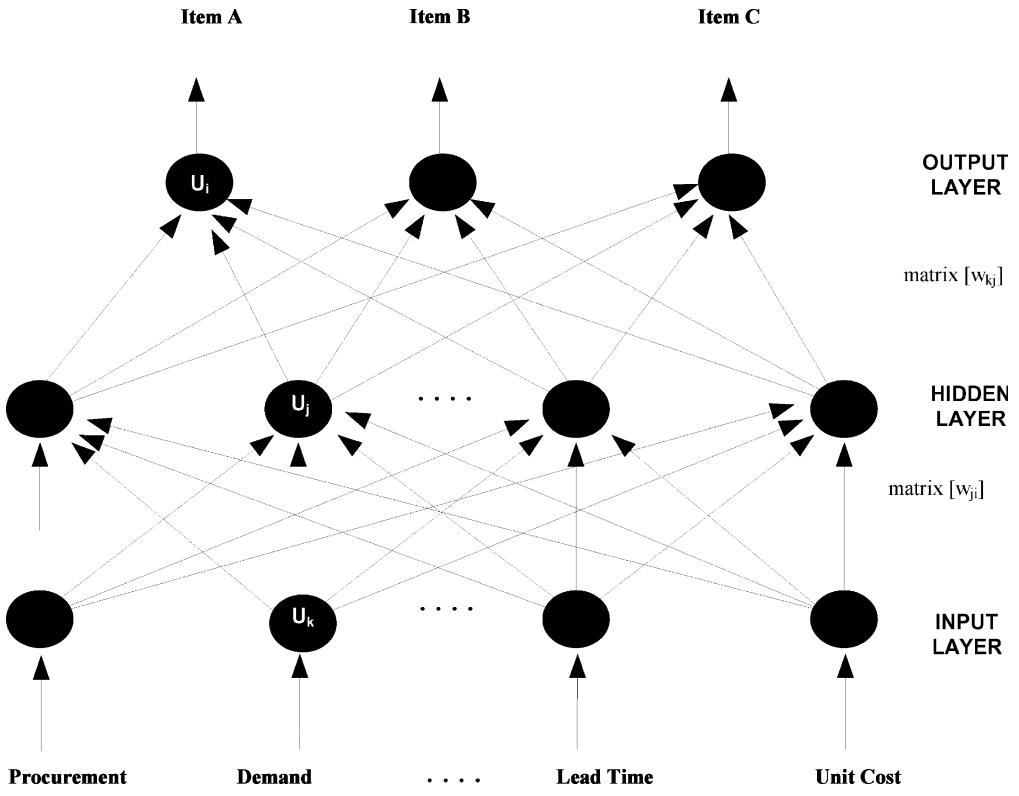


Fig. 1. Structure of the (MLP) ANN.

Like their biological counterpart, ANNs are designed to emulate the human pattern recognition function by the parallel processing of multiple inputs, and can capture the causal relationships between the independent and dependent variables in any given data set, i.e. ANNs have the ability to scan data for patterns which can be used to construct nonlinear models.

An ANN consists of a number of neurons, which are distributed in a number of hierarchical layers. One of the most widely implemented neural network architecture is the multilayer perceptrons (MLP) model. This network has a three-layer, feed forward, hierarchical structure. The total number of neurons, number of neurons on each layer, as well as the number of layers determine the accuracy of the network model. A typical MLP is shown in Fig. 1. The neurons in the input layer represent the attributes or stimuli in a data set. These inputs $(x_1, x_2, \ldots, x_n)$ initiate the activations into the network.

As illustrated in Fig. 1, these inputs are combined in the lower portion of the neuron. The upper portion of the neuron takes this sum and calculates the degree to which the sum is important using a transfer function $(f)$, producing an individual output,

$$f\left(\sum_{i=1}^{n} w_i x_i\right)$$

where, $w$ is weight vector $w \equiv [w_1, w_2, \ldots, w_n]$; and $x$ is the input vector $x \equiv [x_1, x_2, \ldots, x_n]$; for a specific neuron. The transfer function serves as a dimmer switch for turning on and off, depending on the input into the neurons. The selection of the transfer function typically depends on the nature of the output of the network (Fausett, 1994). In this regard, there are a number of alternatives, including the step function, sigmoid function, hyperbolic tangent function, and linear function among others. Because the output of this study is continuous in nature and ranges from 0 to 1, this study uses the *sigmoid transfer function* $f(w'x) = 1/(1 + e^{(-w'x)})$ as recommended by Zahedi (1994).

Since the inventory classification problems are inherently non-linear in nature, it is important to create an ANN, which can approximate complex non-linear functions. This is achieved by adding hidden layers (i.e. several layers of sigmoidal functions), which consist of neurons that receive inputs from the preceding cells and pass on outputs to subsequent cell layers. Although in theory a single hidden layer is sufficient to solve any function approximation problem, some problems may be easier to solve using more than a single hidden layer (Fausett, 1994).

In summary, each connection in the ANN has a weight, which is generated from the input values and then converted to an output value by a transfer function. The output value of a neuron is a function of the weighted sum of its inputs. The weights represent both the strength and nature of the connection between neurons. A large positive value will influence the next neuron to activate, while a large negative value will inhibit activation of the next neuron. The determination of these weights is a critical component of the learning process, and is generated by an iterative training process where case examples with known decision outputs are repeatedly presented to the network. A commonly used learning method in ANN is the BP algorithm.

### 2.1. Back propagation algorithm based learning

The essence of the BP learning algorithm is to load the input–output relations (which are represented by data sets) within the MLP topologies so that it is trained adequately about the past to generalize the future. During the feed forward, each input unit receives an input signal and sends the signal to each of the hidden units. The hidden unit then computes its activation and sends its signal to the output units. Each output unit computes its activation to form the response of the network for the given input pattern. These estimates are then compared to the desired output, and an error is computed for each given observation. This error is then transmitted backward from the output layer to each mode in the hidden layer. Each of the hidden nodes receives only a portion of the error, which is based upon the relative contribution of each of the hidden nodes to the given estimate. This process continues until each node has received its error contribution. The weights are then adjusted to converge towards a solution, and the network is considered trained. The steps used in the BP algorithm are summarized in Appendix C.

The BP algorithm however, may not provide the most efficient way to train neural networks and has in many instances resulted in inconsistent performance (Lenard, Alam & Madey, 1995). This can be attributed to the fact that gradient search techniques such as BP are designed for local search, i.e. they typically achieve the best solution in the region of their starting point. In other words, obtaining a global solution is often dependent on the choice of starting values (Schaffer, Whitley & Eshelman, 1992).

An alternate approach to learning is selectionism, i.e. a complete behavior system is generated,

Table 1
Univariate analysis of variables used in the study (SD: standard deviation)

| Variables | Classification of inventory items | | | | | | |
|---|---|---|---|---|---|---|---|
| | Item A (15 units) | | Item B (35 units) | | Item C (46 units) | | *F* |
| | Mean | SD | Mean | SD | Mean | SD | |
| Unit price | 1356.67 | 2097.37 | 147.02 | 275.53 | 16.42 | 33.04 | 15.19 |
| Ordering cost | 53.33 | 81.76 | 5.96 | 9.64 | 0.78 | 1.48 | 15.47 |
| Demand | 17.13 | 15.99 | 25.63 | 18.87 | 31.76 | 18.90 | 13.75 |
| Lead time | 31.20 | 21.04 | 10.88 | 11.96 | 3.05 | 2.46 | 36.91 |

by evolutionary process. Evolutionary development has been shown to be an extremely important source for generating more complexity in systems (Ray, 1992). Evolutionary development has been studied in great depth from a mathematical point of view, for instance (Baeck & Schwefel, 1993). A common variant for classifying systems is called GA. This learning technique is discussed below.

### *2.2. Genetic algorithm based learning*

GA is a stochastic heuristic optimization search technique designed following the natural selection process in biological evolution, i.e. it models the nature of sexual reproduction in which the genes of two parents combine to form those of their children. When this technique is applied to problem solving, the basic premise is that an initial population of individuals representing possible solutions to a problem is created. Each of these individuals has certain characteristics that make them more or less fit as members of the population. The most fit members will have a higher probability of mating than lesser fit members, to produce progeny that have a significant chance of retaining the desirable attributes of their parents.

This method is very effective at finding optimal or near optimal solutions to a wide variety of problems, because it does not impose many of the limitations required by traditional methods (Levitan & Gupta, 1996; Varetto, 1998). It is an elegant generate-and-test strategy that can identify and exploit regularities in the environment, and converges on solutions that are globally optimal or nearly so. The GA consists of four steps, namely: initialization, reproduction, selection and convergence. These stages are discussed in Appendix D.

## 3. Research methodology

The empirical investigation for this study was carried out using real-world data obtained from a large pharmaceutical company located in northeastern United States. A sample of 96 data sets (given in the Appendix A) was used to design the network. Each data set represented a spare part and contained four types of information: unit price ($/unit, ranging from $0.1 to $7132), ordering cost ($/order, ranging from $0.05 to $300), demand range (units/year, ranging from 1–75 units) and lead

time (days, ranging from 1–70 days). For the sake of brevity and confidentiality, product codes have been substituted for actual item names. A five-step procedure was used to design the ANN, as discussed below.

### 3.1. Representing inventory items in the ANN model

This study represented the classification of inventory items with regard to four attributes: *unit price, ordering cost, demand range and lead-time*. These criteria were selected based on their importance, as indicated by pharmaceutical company managers through personal interviews. Unit price was included as a criterion for classification because it directly affects the holding cost of the item, which includes the opportunity cost of capital invested in inventory stock. Ordering cost is in dollars per lot and represents the cost of replenishing the stock of inventory being held. Annual demand (usage) is important to monitor and control high-demand items so that shortages can be reduced. The final criterion for judging the SKUs is lead-time, which is the elapsed time between placing an order with a vendor and when that order arrives. Inventory managers use ad hoc techniques to integrate the above criteria for ABC classifications. The results of their classification for 96 randomly selected items are shown in Appendix A.

Univariate analyses were performed to determine whether the samples of three types of inventory were from different populations. As can be observed from the $F$-values in Table 1, all variables were significantly different for all the groups at $\alpha = 0.01$ level, indicating that the three groups represented different populations.

### 3.2. Developing the neural network classifier models

The ANNs for this study were developed on NeuroSolutions, a Windows based neural software application. Each of the networks consisted of 4 input neurons (one for each inventory characteristic), 16 hidden neurons, and 3 output neurons (namely, inventory items A, B or C). The momentum learning parameter was chosen for its simplicity and efficiency with respect to the standard gradient. The value of the learning parameters and the threshold function were kept constant, since the purpose of this study was to compare the performance of the ANN topologies.

### 3.3. Training the neural network classifier models

To assess the predictive accuracy of the ANN models the experimental sample was split into two distinct groups, namely, a training group (50 items) and a holdout data (46 items). Using the former, the network models were trained. Training is the learning process by which the input and output data are repeatedly presented to the network. This process is used to determine the best set of weights for the network, which allow the ANN to classify the input vectors with a satisfactory level of accuracy.

### 3.4. Validating the ANN classifier models

After the network was trained, the holdout data (consisting of the 46 data sets) was entered into the system, and the trained ANN was used to test the selection accuracy of the network for

Table 2
Prediction accuracy of ANN (BP and GA) vs MDA for inventory classification (*significant at 0.01, **significant at 0.001, n/s: non-significant)

| | ANN (%) | | MDA (%) | *t*-value | |
|---|---|---|---|---|---|
| | BP | GA | | Ho: $\mu_{BP} - \mu_{MDA}$ | Ho: $\mu_{GA} - \mu_{MDA}$ |
| Overall training sample | 80.00 | 82.30 | 66.70 | | |
| Holdout sample | | | | | |
| Overall classification | 83.70 | 84.30 | 64.60 | | |
| Item A | 75.00 | 60.00 | 60.00 | 5.71* | n/s |
| Item B | 88.80 | 91.30 | 31.40 | 5.32* | 6.01* |
| Item C | 88.90 | 82.60 | 91.30 | n/s | n/s |
| External sample | | | | | |
| Overall classification | 75.00 | 80.30 | 53.10 | | |
| Item A | 100.00 | 100.00 | 55.00 | 7.32** | 7.32** |
| Item B | 62.00 | 73.00 | 28.00 | 6.01* | 7.41** |
| Item C | 78.00 | 82.50 | 86.00 | n/s | n/s |

the 46 testing data sets. This is where the predictive accuracy of the machine learning techniques is measured.

This 'train-and-test' procedure was performed for five iterations, with a new, randomly selected sample of 50 training data sets and 46 testing data sets used each time. The data sets for the five iterations were created from the original sample using Jackknifing. The Jackknife technique is a useful statistical procedure that produces unbiased estimates for the probability of misclassification (Chen & Church, 1992). The predictive accuracy rate of the classification models was calculated as the sum of the outcomes from the five iterations of the testing sample.

### *3.5. Further validation of the classification models*

To validate the model's ability to classify data, another data set from another pharmaceutical company was obtained. This 'out of population' i.e. external data set consisting of 90 SKU data was inputted into the predictive model only for purposes of model validation (see Appendix B). The results were compared against the ABC classification as defined by the inventory managers.

## 4. Results

In order to study the effectiveness of the ANN based classifiers, their results were compared with the traditional multiple discriminate analysis (MDA) technique, which was suggested by Ernst and Cohen (1990) as an alternate technique for inventory classification. A common measure of predictive models is the percentage of observations correctly classified. Table 2 reports the predictive accuracy of the two data sets. Pair wise *t*-tests were conducted to assess the significance of the differences of the models. As can be observed, with the exception of one instance, the ANN

techniques outperformed the MDA technique at $\alpha = 0.01$ level. This can possibly be attributed to the MDAs stringent assumptions. Namely, the technique requires that the data set used to distinguish between the inventory items must be linearly separable, and second, MDA does not allow for each variables data signal to vacillate depending on its relationship to another variable(s). Thus, the complex nature of inventory classification makes MDA a poor classifier of inventory items as compared to the ANN classifiers.

Table 2 also indicates that ANN (GA) outperformed the ANN (BP) in the classification of the holdout datasets. In addition, the validity of the ANN models was further confirmed in that the ANN models outperformed MDA for the external sample.

These results support the recent findings by Sexton, Dorsey and Johnson (1998) who among others, have shown that for a variety of complex functions the ANN (GA) was able to achieve superior solutions for optimization when compared with ANN (BP). This could be attributed to the two techniques differing search techniques, i.e. while BP moves from one point to another, the GA searches the weight space from one set of weights to another set, thereby searching in many directions simultaneously which enhances the probability of finding a global optimum solution.

## 5. Summary and conclusions

In today's manufacturing environment, an organization needs to maintain the delicate balance between critical stock-outs and minimizing inventory costs. Researchers have developed various types of classification models to achieve this balance. This paper presents a new approach for ABC classification of various SKUs. We have used AI-based techniques in terms of ANN to classify SKUs in a pharmaceutical industry. Specifically, two learning methods were utilized in the ANNs, namely, BP and GA. The classification results of these methods were compared with the statistical techniques of MDA, which has been suggested as an alternate technique for inventory classification. The development of these models was based on the actual decisions of inventory managers. The reliability of the models was tested by comparing their classification ability with two data sets (a holdout data and an external data set). The results indicate that the ANN classifier models have a higher predictive accuracy than the MDA technique. In addition, the results indicate that the ANN (GA) network was a superior classifier than the ANN (BP) network.

The use of the ANN model can prove to be a persuasive analytical tool in deciding whether an SKU should be classified as a category A, B, or C item. However, although these classification models have several advantages, they also have their limitations. First, the number of variables which can be input into these models are limited. Second, the model cannot and should not entirely replace professional judgment. Many new important qualitative variables may be difficult to incorporate into the models. Furthermore, such models can be industry specific. Further research will be needed to examine this issue.

## Appendix A

Listing of spare part SKUs for a maintenance department of a pharmaceutical company (Table A1).

Table A1

| Item code | Unit price ($/lot) | Ordering cost ($/lot) | Demand (units/year) | Lead time (days) | Class |
|---|---|---|---|---|---|
| *aa* | 1.50 | 0.07 | 11–20 | 3–5 | B |
| *ab* | 27.00 | 1.00 | 41–75 | 15–35 | B |
| *ac* | 56.00 | 2.00 | 21–40 | 15–35 | B |
| *ad* | 54.00 | 2.50 | 41–75 | 1–2 | B |
| *ae* | 103.00 | 5.00 | 11–20 | 3–5 | C |
| *af* | 15.00 | 0.69 | 11–20 | 3–5 | C |
| *ag* | 0.10 | 0.05 | 41–75 | 1–2 | C |
| *ah* | 0.10 | 0.05 | 11–20 | 1–2 | C |
| *ai* | 0.10 | 0.50 | 41–75 | 1–2 | B |
| *aj* | 34.00 | 2.00 | 21–40 | 1–2 | C |
| *ak* | 48.00 | 2.00 | 11–20 | 1–2 | C |
| *al* | 0.10 | 0.05 | 41–75 | 1–2 | C |
| *am* | 0.10 | 0.05 | 41–75 | 1–2 | C |
| *an* | 600.00 | 25.00 | 3–10 | 6–14 | A |
| *ao* | 400.00 | 20.00 | 3–10 | 15–35 | A |
| *ap* | 0.10 | 0.05 | 41–75 | 1–2 | B |
| *aq* | 0.10 | 0.05 | 21–40 | 1–2 | C |
| *ar* | 3.75 | 0.20 | 41–75 | 3–5 | B |
| *as* | 0.10 | 0.05 | 11–20 | 3–5 | C |
| *at* | 100.00 | 5.00 | 21–40 | 15–35 | A |
| *au* | 13.00 | 0.40 | 41–75 | 1–2 | C |
| *av* | 0.10 | 0.05 | 21–40 | 1–2 | C |
| *aw* | 0.10 | 0.05 | 11–20 | 1–2 | C |
| *ax* | 150.00 | 7.00 | 11–20 | 3–5 | C |
| *ay* | 7.00 | 0.30 | 41–75 | 3–5 | B |
| *az* | 10.00 | 0.50 | 11–20 | 3–5 | B |
| *ba* | 30.00 | 1.50 | 21–40 | 15–35 | A |
| *bb* | 4570.00 | 150.00 | 3–10 | 36–70 | A |
| *bc* | 0.10 | 0.05 | 41–75 | 1–2 | C |
| *bd* | 4.50 | 0.25 | 11–20 | 1–2 | C |
| *be* | 5.00 | 0.25 | 11–20 | 3–5 | B |
| *bf* | 70.00 | 3.00 | 11–20 | 6–14 | C |
| *bg* | 18.00 | 0.75 | 3–10 | 1–2 | C |
| *bh* | 3.75 | 0.15 | 11–20 | 3–5 | B |
| *bi* | 5.00 | 0.01 | 11–20 | 3–5 | B |
| *bj* | 0.10 | 0.05 | 41–75 | 1–2 | B |
| *bk* | 75.00 | 3.00 | 41–75 | 3–5 | B |
| *bl* | 175.00 | 8.00 | 11–20 | 6–14 | B |
| *bm* | 300.00 | 15.00 | 11–20 | 6–14 | B |
| *bn* | 1.00 | 0.05 | 11–20 | 3–5 | B |
| *bo* | 87.00 | 4.00 | 3–10 | 3–5 | B |
| *bp* | 26.00 | 1.00 | 41–75 | 1–2 | C |
| *bq* | 0.10 | 0.05 | 21–40 | 1–2 | C |
| *br* | 120.00 | 5.00 | 3–10 | 6–14 | C |
| *bs* | 0.10 | 0.05 | 41–75 | 1–2 | C |
| *bt* | 0.10 | 0.05 | 3–10 | 3–5 | C |
| *bu* | 0.10 | 0.05 | 21–40 | 1–2 | C |
| *bv* | 0.10 | 0.05 | 41–75 | 1–2 | C |

Table A1 (*continued*)

| Item code | Unit price ($/lot) | Ordering cost ($/lot) | Demand (units/year) | Lead time (days) | Class |
|---|---|---|---|---|---|
| *bx* | 0.10 | 0.50 | 41–75 | 1–2 | C |
| *by* | 730.00 | 17.00 | 1–2 | 15–35 | B |
| *bz* | 0.10 | 0.50 | 41–75 | 1–2 | C |
| *ca* | 20.00 | 1.00 | 21–40 | 3–5 | C |
| *cb* | 520.00 | 10.00 | 11–20 | 15–35 | A |
| *cc* | 161.00 | 8.00 | 21–40 | 6–14 | B |
| *cd* | 7132.00 | 300.00 | 1–2 | 36–70 | A |
| *ce* | 400.00 | 20.00 | 3–10 | 15–35 | B |
| *cf* | 249.00 | 10.00 | 3–10 | 15–35 | B |
| *cg* | 2614.00 | 100.00 | 11–20 | 36–70 | A |
| *ch* | 0.10 | 0.05 | 41–75 | 1–2 | C |
| *ci* | 0.10 | 0.05 | 41–75 | 1–2 | C |
| *cj* | 1425.00 | 50.00 | 1–2 | 36–70 | B |
| *ck* | 72.00 | 3.00 | 3–10 | 6–14 | B |
| *cl* | 0.10 | 0.05 | 11–20 | 3–5 | C |
| *cm* | 0.10 | 0.05 | 21–40 | 3–5 | C |
| *cn* | 105.00 | 5.00 | 1–2 | 36–70 | A |
| *cp* | 12.00 | 0.50 | 21–40 | 1–2 | B |
| *cq* | 12.00 | 0.50 | 21–40 | 1–2 | B |
| *cr* | 108.00 | 5.00 | 21–40 | 15–35 | B |
| *cs* | 0.10 | 0.05 | 21–40 | 1–2 | C |
| *ct* | 3.40 | 0.15 | 11–20 | 6–14 | C |
| *cu* | 40.00 | 2.00 | 11–20 | 3–5 | C |
| *cv* | 40.00 | 2.00 | 11–20 | 3–5 | C |
| *cx* | 65.00 | 3.00 | 21–40 | 6–14 | B |
| *cy* | 116.00 | 30.00 | 21–40 | 3–5 | B |
| *cz* | 129.00 | 6.00 | 21–40 | 3–5 | B |
| *da* | 0.10 | 0.05 | 11–20 | 1–2 | C |
| *db* | 1.00 | 0.05 | 21–40 | 3–5 | C |
| *dc* | 12.00 | 0.60 | 11–20 | 3–5 | C |
| *dd* | 8.50 | 0.40 | 41–75 | 1–2 | C |
| *de* | 10.00 | 0.50 | 11–20 | 1–2 | C |
| *df* | 2995.00 | 100.00 | 1–2 | 36–70 | A |
| *dg* | 0.10 | 0.05 | 21–40 | 3–5 | C |
| *di* | 0.10 | 0.05 | 21–40 | 1–2 | C |
| *dj* | 35.00 | 2.00 | 3–10 | 15–35 | B |
| *dk* | 363.00 | 15.00 | 11–20 | 3–5 | C |
| *dl* | 150.00 | 7.00 | 11–20 | 3–5 | C |
| *dm* | 19.00 | 0.07 | 11–20 | 6–14 | C |
| *dn* | 0.10 | 0.05 | 41–75 | 1–2 | C |
| *dp* | 30.00 | 7.00 | 21–40 | 3–5 | B |
| *dq* | 34.00 | 1.50 | 21–40 | 1–2 | C |
| *dr* | 330.00 | 15.00 | 11–20 | 6–14 | B |
| *ds* | 140.00 | 7.00 | 11–20 | 15–35 | B |
| *dt* | 5.80 | 0.25 | 21–40 | 3–5 | C |
| *du* | 882.00 | 40.00 | 3–10 | 15–35 | A |
| *dv* | 17.00 | 0.50 | 41–75 | 6–14 | A |

## Appendix B

Listing of spare part SKUs for out-of-population validation from another pharmaceutical company (total units in class A — 35, class B — 15, class C — 40; the number of items in the sample may not be representative of population proportions) (Table B1).

Table B1

| Item code | Unit price ($/lot) | Ordering cost ($/lot) | Demand (units/year) | Lead time (days) | Class |
|---|---|---|---|---|---|
| 1527 | 86.40 | 6.43 | 16 | 6 | B |
| 1507 | 348.48 | 38.60 | 5 | 6 | C |
| 1495 | 11.81 | 2.97 | 8 | 45 | C |
| 1504 | 11.10 | 5.51 | 2 | 12 | C |
| 1506 | 25.44 | 3.51 | 14 | 30 | C |
| 1488 | 8.05 | 1.48 | 23 | 30 | A |
| 1535 | 9.00 | 3.86 | 4 | 14 | A |
| 1536 | 714.00 | 19.30 | 14 | 30 | C |
| 1519 | 1393.00 | 38.60 | 1 | 36 | A |
| 1576 | 250.00 | 38.60 | 2 | 90 | A |
| 1585 | 21.80 | 3.51 | 12 | 15 | A |
| 1586 | 16.60 | 7.72 | 2 | 15 | A |
| 1589 | 19.00 | 12.87 | 2 | 30 | C |
| 1590 | 12.45 | 9.96 | 1 | 30 | C |
| 1572 | 111.68 | 19.30 | 2 | 20 | B |
| 1575 | 294.00 | 38.60 | 2 | 90 | C |
| 1717 | 2099.00 | 38.60 | 2 | 30 | A |
| 1700 | 18.43 | 3.51 | 10 | 10 | B |
| 1708 | 92.00 | 5.51 | 22 | 60 | C |
| 1690 | 0.33 | 0.06 | 519 | 60 | C |
| 1674 | 15.79 | 2.57 | 17 | 5 | B |
| 1676 | 478.89 | 38.60 | 2 | 30 | C |
| 1710 | 685.00 | 12.87 | 29 | 60 | C |
| 1747 | 9.65 | 2.76 | 5 | 30 | A |
| 1735 | 34.00 | 2.97 | 23 | 10 | B |
| 1741 | 78.67 | 2.76 | 74 | 30 | A |
| 1744 | 46.40 | 1.93 | 14 | 30 | A |
| 1722 | 8.20 | 4.29 | 15 | 3 | A |
| 1816 | 38.70 | 19.30 | 1 | 14 | A |
| 1808 | 99.29 | 19.30 | 1 | 15 | A |
| 1795 | 53.55 | 6.43 | 7 | 3 | A |
| 1773 | 68.50 | 4.83 | 18 | 20 | A |
| 1776 | 32.92 | 7.72 | 4 | 16 | B |
| 1760 | 255.00 | 12.87 | 9 | 30 | C |
| 1814 | 259.00 | 19.30 | 6 | 30 | C |
| 1834 | 96.80 | 19.30 | 3 | 14 | A |
| 1837 | 30.00 | 12.87 | 1 | 7 | C |
| 1838 | 14.40 | 2.57 | 13 | 15 | B |
| 1821 | 323.20 | 19.30 | 4 | 45 | C |
| 1822 | 43.20 | 12.87 | 2 | 45 | A |
| 1824 | 252.00 | 38.60 | 1 | 30 | A |
| 1813 | 12.16 | 4.29 | 4 | 15 | B |

Table B1 (*continued*)

| Item code | Unit price ($/lot) | Ordering cost ($/lot) | Demand (units/year) | Lead time (days) | Class |
|---|---|---|---|---|---|
| 5835 | 293.00 | 6.43 | 48 | 30 | C |
| 5606 | 150.08 | 12.87 | 6 | 0 | C |
| 5612 | 474.85 | 38.60 | 4 | 0 | C |
| 5490 | 251.00 | 12.87 | 8 | 21 | C |
| 618 | 82.75 | 4.83 | 23 | 14 | A |
| 826 | 33.00 | 3.51 | 15 | 14 | C |
| 784 | 1.57 | 0.32 | 96 | 14 | C |
| 952 | 56.33 | 7.72 | 5 | 7 | A |
| 938 | 9.33 | 0.90 | 73 | 14 | C |
| 917 | 48.02 | 5.51 | 11 | 7 | B |
| 926 | 265.00 | 7.72 | 32 | 7 | C |
| 911 | 20.14 | 1.75 | 43 | 7 | B |
| 874 | 132.20 | 38.60 | 11 | 30 | A |
| 953 | 308.00 | 6.43 | 52 | 21 | A |
| 1088 | 14.27 | 0.84 | 125 | 25 | C |
| 1069 | 18.12 | 2.27 | 21 | 30 | C |
| 1078 | 203.29 | 19.30 | 2 | 30 | A |
| 1046 | 505.44 | 38.60 | 4 | 20 | C |
| 1211 | 845.00 | 38.60 | 1 | 30 | C |
| 1215 | 137.24 | 38.60 | 1 | 60 | A |
| 1218 | 70.32 | 7.72 | 7 | 15 | B |
| 1197 | 366.00 | 19.30 | 8 | 30 | C |
| 1203 | 127.10 | 19.30 | 1 | 5 | C |
| 1206 | 73.82 | 3.51 | 37 | 10 | A |
| 1186 | 146.98 | 38.60 | 1 | 30 | A |
| 1189 | 62.79 | 4.29 | 19 | 30 | C |
| 1220 | 24.82 | 12.87 | 1 | 21 | B |
| 1257 | 315.05 | 7.72 | 38 | 30 | C |
| 1231 | 341.00 | 38.60 | 2 | 14 | A |
| 1235 | 230.00 | 4.29 | 91 | 60 | C |
| 1238 | 5.50 | 0.42 | 198 | 30 | C |
| 1241 | 0.38 | 0.28 | 30 | 5 | A |
| 1408 | 6.25 | 0.68 | 84 | 12 | C |
| 1398 | 434.52 | 38.60 | 1 | 20 | C |
| 1383 | 7.98 | 3.51 | 4 | 30 | B |
| 1376 | 384.00 | 12.87 | 16 | 30 | C |
| 1367 | 107.80 | 12.87 | 11 | 30 | A |
| 1517 | 17.50 | 3.86 | 7 | 10 | A |
| 1746 | 10.87 | 1.68 | 5 | 30 | A |
| 1725 | 70.20 | 6.43 | 10 | 20 | A |
| 1734 | 84.20 | 4.83 | 20 | 12 | B |
| 1702 | 348.48 | 19.30 | 5 | 30 | C |
| 1675 | 19.76 | 2.57 | 18 | 15 | A |
| 1836 | 34.30 | 7.72 | 4 | 7 | C |
| 1823 | 432.25 | 19.30 | 11 | 15 | B |
| 1826 | 8.39 | 3.51 | 4 | 5 | C |
| 1793 | 47.30 | 12.87 | 2 | 15 | A |
| 3032 | 8.33 | 0.37 | 329 | 14 | A |

## Appendix C. Explanation of the back propagation learning algorithm

The mathematical basis for the BP algorithm is the optimization technique steepest gradient method. The steps of the BP algorithm is given below:

*Step* 1: initialize weights to connections $[w_{jk}]$ and $[v_{ij}]$ with random weights.

*Feed forward*

*Step* 2: input $X_i$ $(i = 1, \ldots, n)$ receives an input signal and passes the signal to the hidden units $Z_j$.

*Step* 3: each of the hidden units $Z_j$ $(j = 1, \ldots, n)$ sums the weighted input signals net input to $Z_j = \theta_j + \sum_{i=1}^{n} x_i v_{ij}$ and applies the activation function to compute the output signal, where $\theta_j =$ Bias on hidden unit $j$.

*Step* 4: each of the output units $Y_k$ $(k = 1, \ldots, n)$ sums the weighted input signals net input to $Y_k = \theta_k + \sum_{j=1}^{n} z_i w_{jk}$ and applies the activation function to compute the output signal, $\theta_k =$ Bias on output unit $k$.

*BP of error*

*Step* 5: each of the output units compares its computed activation with a target to determine association error $e_k = y_k - t_k$. Based on $e_k$, errors at output unit $Y_k$ is calculated as follows:

$$\delta_k = e_k \theta_k + \sum_{j=1}^{n} z_j w_{jk},$$

and sent to all neurons in the previous layers.

*Step* 6: when the training converges and the system error decreases below an acceptable threshold, the ANN is considered to be trained and then applied over the testing data set.

## Appendix D. Explanation of the genetic algorithm learning algorithm

*Step* 1: *initialize population and fitness evaluation*. In the initialization stage, the number of genes for each individual and the total number of chromosomes in the population size has to be determined. The starting point of the search is a randomly generated population of chromosomes. Each individual chromosome is evaluated using a predetermined objective function. The objective function in this study, the mean square error (MSE) was to be minimized.

*Step* 2: *reproduction stage*. Based on their relative fitness values, individuals in the current population are selected for reproduction. The selection strategy used in this study was the *Roulette wheel* selection, where a roulette wheel with slots ($F$) sized according to the total fitness function of the population is identified as,

$$F = \sum_{i=1}^{\text{popsize}} \text{fitness}\,(Z_i)$$

where $Z_i$ indicates the fitness value of the chromosome according to MSE. In other words, the candidate for the reproduction is chosen randomly, but the choice probability is proportional to the candidate's fitness. The higher the fitness function value of the individual, the higher its chances of being chosen for the reproduction process.

Each chromosome has a certain number of slots proportional to the fitness value. The roulette wheel selection process is based on spinning the wheel popsize times, and with each spin a single chromosome

is selected for a new population. In accordance to the theory of inheritance the most fit chromosomes will be selected more than once, while the least fit will die.

*Step* 3: *selection stage*. The selected parent individuals from the current generation are now recombined using the crossover operator to produce two new offspring. Generally, crossover draws only on the information present in the solutions of the current population in generating new solutions for evaluation. Typically, only one of the offspring passes over into the next generation. The mates for the crossover are chosen randomly from the initial population with the restriction that no individual can crossover with itself. The probability of crossover (Pc) is the probability that crossover will occur; and gives the expected number (Pc*popsize) of chromosomes that should undergo the crossover operation. The number of individuals undergoing the crossover operation increases as the population size and the number of chromosomes per individual increases.

The next recombination operator is the mutation operator, which arbitrarily alters one or more components of a selected structure, provides the means for introducing new information into the population. The probability of mutation (Pm) provides the expected number of mutated bits (Pm*m*popsize). The presence of mutation ensures that the probability of reaching any point in the search space is never zero.

*Step* 4: *convergence*. Following reproduction, crossover, and mutation, the new population is ready for its next generation. The remainder of the evolution are basically cyclic repetitions of the steps given above, until the system reaches a predetermined number of generations or convergence (i.e. no improvement in the overall fitness of the population).

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