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Inventory control system design by integrating inventory classification and policy selection

Davood Mohammaditabar[a], Seyed Hassan Ghodsypour[a,*], Chris O'Brien[b]

[a] Amirkabir University of Technology, No.424, Hafez Avenue, Tehran, Iran
[b] Nottingham University Business School, University Park, Nottingham NG72RD, UK

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Very large numbers of inventory items complicate the inventory control process. Companies categorize their inventory items into a few groups and take similar inventory control policies for the items in each group to overcome this problem. In this regard many grouping methods have been proposed. Some researchers have studied the appropriate inventory policy for each group. Since both the actions of categorization and policy selection are sub-optimal solutions for the original problem of efficient inventory control policy, this paper proposes an integrated model to categorize the items and find the best policy simultaneously. As it is difficult to find a global solution, simulated annealing is used to find appropriate solutions. The model results are compared with the findings of other methods both for dissimilarity and total inventory values.

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1. Introduction

Classification is widely proposed in the literature to tackle the problem of very large numbers of inventory items. Thousands of inventory items in companies even with moderate size increase the risk of losing sight of the most important items and spending unnecessary resources in controlling less important ones. Therefore companies try to classify the items and select appropriate control policies for each group. In this regard many authors have studied the classification process and proposed various exact and heuristic methods to classify inventories satisfying some criteria. ABC classification is the most widely employed technique, which in its basic form considers the only criteria of annual use value (Cohen and Ernst, 1988).

Some other authors have focused on the appropriate control policies for each group of the items. Reorder point, two-bin systems and material requirement planning (MRP) are some of the developed strategies (Hautaniemi and Pirttila, 1999).

Thus, in the literature, the original problem of making an effective inventory control system has been decomposed into two problems of classification of inventory items and finding appropriate strategies for each group. This has misled some authors to focus on decomposed sub-problems independently and forget the original goal. Although this may produce some sub-optimal solution for the original problem we should not forget that the aim of the classification is not solely to classify items but to excel in the performance of inventory control policy.

In this paper we propose a model that concurrently classifies inventory items and selects appropriate policies for each product group with the objective of having an effective inventory performance.

The paper is organized as follows. In Section 2 the related literature is reviewed. The developed model is described in Section 3. The coding of the problem in simulated annealing is described in Section 4. An illustrative example is solved in Section 5. Section 6 discusses the algorithm's time complexity and finally Section 7 presents the conclusions.

2. Literature review

The most important reason for applying ABC classification is that in most practical situations the number of inventory items is too large to implement a specific inventory control system for each item (Ernst and Cohen, 1990). However in practice, the traditional ABC classification may not be able to provide a good classification of inventory items (Partovi and Anandarajan, 2002; Huiskonen, 2001; Guvenir and Erel, 1998). Although ABC classification is simple to implement it classifies parts based only on the dollar value of the sales, which tends to allocate a large portion of the capital investment to expensive parts and also by itself does not eliminate the need for optimization of stocking parameters in each group (Zhang et al., 2001). Teunter et al. (2010)

* Corresponding author. Tel.: +98 216 4545390; fax: +98 21 22578713.
E-mail addresses: mohammaditabar@aut.ac.ir (D. Mohammaditabar), ghodsypo@aut.ac.ir (S. Hassan Ghodsypour), chris.obrien@nottingham.ac.uk (C. O'Brien).

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states that the criteria used in the ABC classification give results far from the cost optimal. They propose a single criterion to be used instead of the criteria used on the traditional ABC and claim it leads to a more cost efficient solution. Some researchers proposed multi-criteria ABC classification. Flores and Whybark (1986) provide a matrix based methodology for multi-criteria ABC classification. In the case of two criteria, a joint criteria matrix is developed though their methodology is relatively difficult to use when there are more than two criteria.

Gajpal et al. (1994), Partovi and Burton (1993) and Partovi and Hopton (1994) applied the analytical hierarchy process (AHP) to ABC analysis. The subjectivity involved in the analysis is the most important issue associated with AHP. Also the method is not easy to implement in the case of large numbers of inventory items.

Ernst and Cohen (1990) used cluster analysis to group similar items. The main advantage of this approach is that it can accommodate large combinations of attributes. The excessive statistical data necessary in their method is the most important weakness, which makes it impractical.

Heuristic approaches such as genetic algorithms and artificial neural networks have also been applied to address the problem (Partovi and Anandarajan, 2002; Guvenir and Erel, 1998). Clearly, these heuristic approaches may not provide optimal solutions in all environments.

Ramanathan (2006) developed a scheme using weighted linear optimization for ABC inventory classification with multiple criteria. An extended scheme was presented by Zhou and Fan (2007).

All the mentioned methods in inventory classification disregarded the original problem and focused on the classification process. The decision maker can only try to select appropriate criteria to help these methods produce good sub-optimal solutions.

Tsai and Yeh (2008) used a particle swarm optimization approach for inventory classification problems where inventory items are classified based on a specific objective or multiple objectives, such as minimizing costs, maximizing inventory turnover ratios and maximizing inventory correlation. They compare their model with classification by suppliers, ABC classification scheme, no grouping and placing all items in a single group. They conclude that their algorithm performs comparatively well with respect to those schemes that are commonly used in practice.

3. Model

An effective inventory system has many characteristics, including the simplicity of implementation, the service quality and cost effectiveness. The most mentioned criterion is cost effectiveness. We try to design an inventory control system by categorizing the items based on their similarity and the inventory cost. In order to achieve this goal, one of the objective functions of the model is the minimization of the costs. It is assumed that items classified in the same group have the same replenishment cycle. Therefore the distinction between inventory policies would be in the order interval of each group. The objective is to identify the optimal order interval for each group.

In selecting an inventory control policy it is important to know what kind of demand we are encountering. Demand could be dependent or independent. Reorder point systems are used for independent demand and MRP based systems for dependent demand. In addition the demand could be deterministic or stochastic. When the inventory parameters (demand, lead time, etc.) are deterministic both continuous and periodic reorder point policies produce similar results (quantity, reorder level, etc.) and the only difference would be in the managerial complications of the system. In general when there are stochastic parameters in the inventory model, the continuous review system needs to keep less safety stock than the periodic review. This usually results in lower inventory costs, unless the cost of having a continuous review of the system is much larger than for the periodic reviews. In most of the cases, for the very important items the continuous review is proposed and for less important items a periodic review system. Nowadays with new technologies it is more common for companies to have continuous review of their inventory items. Based on the issues discussed earlier, in our deterministic independent demand inventory environment a continuous review reorder point system is proposed for the items categorized as A and a periodic review reorder point system for items categorized as C. Also, considering the development of new technologies in inventory control, the class B items could be controlled by continuous review, even though periodic review would not be illogical for them.

Ordering and inventory holding cost are the most visible factors, which we could encounter in the model. However there are some other factors that affect the overall cost of the inventory system. For example items that are bought from the same supplier may have less ordering costs if they are classified in one group and have a joint order. However it is neither easy nor efficient to directly include all these factors in the model. To tackle this problem some researchers have used non-quantity factors in their proposed models.

In practice an efficient inventory control system is not based solely on the total inventory cost. There are other factors that might not be explicitly seen in the cost. For example when the items in each group with the same replenishment time are ordered from the same supplier it may create some flexibility in planning and also reductions in cost. When the items with similar ordering requirements (certificates, bank statements, etc.) or similar keeping characteristics are put in the same groups it creates some advantages for the companies. Therefore it is very sensible that in an appropriate solution, similar items should be classified into the same groups. This has been the basis of the classification proposed in the literature. As mentioned earlier ABC classification tries to put items with high annual value into the same group and items with low annual value into another group. Other classification techniques also try to classify similar items into the same groups. This similarity is based on the scores that items achieve in the criteria. The only difference could be in the way similarities are calculated. Our model defines a dissimilarity index for each pair of items and tries to minimize it in another objective function. In this regard many criteria can be defined and used in the algorithm. Based on the weight that we consider for this objective we might have a classification totally based on dissimilarity or in the other extreme totally based on the total inventory cost objective.

3.1. Notation

n	number of inventory items
m	number of criteria
U	number of classification groups
w_k	weight of criteria k
y_{ik}	performance score of the ith item in terms of the kth criterion
s_i	setup cost of item i
T_g	order interval of classification group g
D_i	demand per unit time of item i
h_i	holding cost per unit per unit of time of item i
$x_{ig}\begin{cases}1\\0\end{cases}$	If item i is classified in group g else
d_{ij}	dissimilarity index of item i and item j

At first performance scores of items are normalized to be between 0 and 1 as follows:

Normalized $y_{ik} = y_{ik} - \min_k / \max_k - \min_k$

where $\min_k$ and $\max_k$ are the minimal and maximal values of the performance of all items in criteria k. In our formulation we assume that all the performances have been normalized to avoid further variable definition.

It is assumed that items classified in the same group have the same replenishment cycle. Chakravarty (1985) showed that the optimal joint replenishment cycle of item group g can be expressed as

$$T_g = \sqrt{\frac{2\sum_{i \in \text{group}(g)} s_i}{\sum_{i \in \text{group}(g)} D_i h_i}} \tag{1}$$

Eq. (1) can be rewritten as

$$T_g = \sqrt{\frac{2\sum_i s_i x_{ig}}{\sum_i D_i h_i x_{ig}}} \tag{2}$$

Total relevant costs of item group g can be written as

$$TRC = \sum_g \left(\frac{\sum_{i \in \text{group}(g)} s_i}{T_g} + ½ T_g \sum_{i \in \text{group}(g)} D_i h_i \right) \tag{3}$$

The dissimilarity index is calculated by

$$d_{ij} = \left[\sum_{k=1}^{m} w_k (y_{ik} - y_{jk})^2 \right]^{1/2} \tag{4}$$

The model is as follows:

$$\min \sum_{i=1}^{n} \sum_{g=1}^{U} \left[\left(\frac{s_i}{T_g} + ½ T_g D_i h_i \right) x_{ig} \right] \tag{5}$$

$$\min \sum_{i=1}^{n} \sum_{j=1}^{n} d_{ij} x_{ig} x_{jg} \tag{6}$$

Subject to

$$\sum_{g=1}^{U} x_{ig} = 1 \forall i = 1 \ldots n, g = 1 \ldots U \tag{7}$$

$$x_{ig} \in \{0,1\} \forall i, g \tag{8}$$

Eq. (5) represents the objective function of minimizing total inventory system cost, which is composed of two main costs known as inventory holding cost and inventory replenishment cost.

Eq. (6) is the objective function to minimize the dissimilarity of items categorized in the same groups. In other words, Eq. (6) guarantees that most similar items are put together in the same groups.

This model is very difficult to solve optimally since the objective functions are not linear and the value of T_g is dependent on the value of x_{ig}.

4. Simulated annealing

The model is solved by simulated annealing. We assume that the items are to be categorized into three groups ($U=3$). The initial solution to be used in simulated annealing is constructed by the ABC method since it is very easy to produce and is a good start. Therefore our results will never be worse than the ABC method since it is our initial solution. Each solution is represented by a n*3 binary matrix in which each row contains a single 1 and two 0s. A neighbor for a solution is produced when only one of the items changes its group. Each time a neighbor produces the dissimilarity and total inventory cost is calculated. If the weighted sum of these two functions is less than that for the best known answer, the best answer is updated by the current solution and the search continues. Otherwise by a random chance the neighbor is accepted as the current solution.

The weights for each objective function is defined before the model is run and demonstrates the preference of the decision maker towards each objective and also is a factor to normalize the objective functions. Therefore the weight for each objective function is the preference of the decision maker (towards the other objective) divided by the best value for that objective (the value it reaches when it is the only objective).

When the algorithm ends, the group with the least order interval is named as group A, the group with the largest order interval is named as group C and the third group with the order interval in between is named as group B. The result of our algorithm is tested in a sample problem explained in Ng (2007), Zhou and Fan (2007), Ramanathan (2006) and Flores et al. (1992). Also in order to examine the time complexity of the algorithm, some test problems are produced and the run times are compared.

5. Illustrative example

We apply our new model to the same multi-criteria inventory classification problem discussed by Ng (2007), Zhou and Fan (2007), Ramanathan (2006) and Flores et al. (1992). The results of our model are compared with their proposed models.

In the beginning we compare our model with the classification resulting from annual dollar usage when it is the only criteria. As shown in Table 1 the classification is almost equivalent.

As shown in Table 1 our classification when having the sole criteria of cost gives very similar categorization to using annual dollar usage. It is obvious that since we are minimizing the total cost in this step, it would result in less total cost than the ABC method, which classifies items based on one criterion. The total inventory cost for the annual dollar method is 1187.7 compared with 1164.8 by our method. Although our model brought better results in this case, we compare it when there are other criteria and with other methods. Again it is obvious that since we are adding some other criteria in the classification (other than cost), the total inventory cost (Eq. (5)) might increase (in comparison to the previous step).

To compare our model with the other classifications we need to define the criteria. Flores et al. (1992) used three criteria of annual dollar usage, critical factor and lead time. Although these criteria may not necessarily be adequate to design an efficient inventory control system, in order to compare our results we also use them.

In order to calculate the total inventory cost we need to have setup costs and since setup cost was not included in Flores et al. (1992) we consider it equivalent to the lead time multiplied by a fixed coefficient for all items. Inventory holding cost is assumed to be 10% of the item average cost. Also the demand is calculated by dividing annual dollar usage with the average item cost.

The inventory control policy is integrated with the classification by calculating order intervals for each group. The group that has the smallest order interval is named as group A, group C has the largest order interval and other items not in A and C are in group B.

In Table 2 we compare our model with other classifications by considering three criteria of annual dollar usage, critical factor and lead time.

To compare the efficiency of each method the dissimilarity and total inventory cost, which resulted from each categorization, is

Table 1
Comparison of the classifications by annual dollar usage and our proposed model with one criteria and one objective function.

Item no.	Annual dollar usage	Average unit cost	Lead time	Traditional ABC	New model
1	5840.64	49.92	2	A	A
2	5670	210	5	A	A
3	5037.12	23.76	4	A	A
4	4769.56	27.73	1	A	A
5	3478.8	57.98	3	A	A
7	2936.67	31.24	3	A	A
8	2820	28.2	4	A	A
9	2640	55	6	A	B
10	2423.52	73.44	4	A	A
29	268.68	134.34	7	A	C
13	1038	86.5	7	B	B
14	1043.5	20.87	5	B	B
15	1038	86.5	3	B	B
17	883.2	110.4	4	B	B
18	854.4	71.2	6	B	B
20	810	45	4	B	B
21	703.68	14.66	4	B	B
22	594	49.5	4	B	B
28	313.6	78.4	6	B	C
37	150	30	5	B	C
40	103.36	51.68	6	B	C
43	59.78	29.89	5	B	C
45	34.4	34.4	4	B	C
47	25.38	8.46	5	B	C
6	370.5	37.05	3	C	A
11	1075.2	5.12	2	C	A
12	1043.5	20.87	5	C	B
16	313.6	78.4	3	C	B
19	268.68	134.34	5	C	B
23	224	56	4	C	B
24	216	72	3	C	B
25	370.5	37.05	1	C	B
26	338.4	33.84	3	C	B
27	336.12	84.03	1	C	B
30	224	56	1	C	B
31	216	72	5	C	C
32	212.08	53.02	2	C	B
33	197.92	49.48	5	C	C
34	190.89	7.07	7	C	C
35	181.8	60.6	3	C	C
36	163.28	40.82	3	C	C
38	134.8	67.4	3	C	C
39	119.2	59.6	5	C	C
41	79.2	19.8	2	C	C
42	75.4	37.7	2	C	C
44	48.3	48.3	3	C	C
46	28.8	28.8	3	C	C

Table 2
Comparison of the classifications by other methods and our model with three criteria.

Item no.	Traditional ABC	Zhang et al. (2001)	AHP weighted score	Optimal inventory score	New model
1	A	A	A	B	A
2	A	C	A	C	A
3	A	A	A	C	A
4	A	A	C	A	A
5	A	B	B	C	A
7	A	A	C	C	A
8	A	B	C	C	A
9	A	B	A	C	A
10	A	C	B	C	A
29	A	C	B	A	C
13	B	C	A	C	B
14	B	C	B	A	B
15	B	C	A	A	B
17	B	A	B	B	B
18	B	B	A	A	B
20	B	C	B	B	B
21	B	B	A	B	B
22	B	C	B	B	B
28	B	C	C	A	C
37	B	B	C	B	C
40	B	C	C	B	C
43	B	C	C	B	C
45	B	C	B	A	C
47	B	B	C	B	C
6	C	A	C	C	A
11	C	A	B	C	A
12	C	A	B	C	B
16	C	B	C	A	B
19	C	B	B	A	B
23	C	C	A	B	B
24	C	B	A	C	B
25	C	B	C	C	C
26	C	B	C	C	C
27	C	C	C	B	C
30	C	B	C	C	C
31	C	C	B	B	B
32	C	C	B	C	B
33	C	C	C	B	C
34	C	A	C	A	C
35	C	C	C	C	C
36	C	C	B	C	B
38	C	C	C	C	B
39	C	C	C	B	C
41	C	B	C	C	C
42	C	C	C	C	C
44	C	C	C	C	C
46	C	C	C	C	C

presented in Table 3. Our model has the least dissimilarity value and total inventory cost in comparison with other methods.

It is seen in Table 3 that our proposed model produces better results both in dissimilarity and total inventory cost.

Table 3
Comparison of dissimilarity and total inventory cost obtained by other methods and our proposed model under three criteria.

	Dissimilarity	Total inventory cost
Annual dollar usage	210.5902	1187.7
AHP weighted score	232.7131	1429.2
Optimal inventory score	301.0233	1424
Zhang et al. (2001)	282.66	1347.8
Our model	177.4595	1179.5

6. Time complexity of the algorithm

In this section the time complexity of the algorithm is discussed. In our model there should be n^2 calculations associated with dissimilarities when n is the number of inventory items. Also the simulated annealing runs for initial temperature are multiplied by iterations at each temperature. Therefore the algorithm has a polynomial time complexity. This is examined in some test problems, which are shown in Tables 4 and 5. The test problems' parameters are produced by normal distribution between the minimum and maximum values of the same parameters in the example defined in Section 5.

As it is clear from Table 4, the run time is not very sensitive to initial temperature. Also, from Table 5, it can be seen that iterations at each temperature and also sample size have a polynomial behavior of order two (see Fig. 1).

By a simple estimate it is clear that the algorithm will provide its answer for 10,000 inventory items in 2 h, which is a reasonable time for the decision of an inventory control system design with this size. The algorithm will provide the answer in less than a second for the size of problem explained in Section 5. In the case of large companies with more than 35,000 inventory items the

Table 4
Comparison of algorithm average time with respect to initial temperature.

Number of runs	Sample size	Iterations at each temperature	Initial temperature (50)	Initial temperature (500)	Initial temperature (1000)
			Avg. time (s)	Avg. time (s)	Avg. time (s)
10	250	50	4.087	6.117	6.6317
10	250	100	8.0409	12.1703	12.8897
10	250	200	16.2052	24.7924	26.7001

Table 5
Comparison of min, max and average algorithm time with respect to number of iterations in each temperature and sample size.

Number of runs	Sample size	Iterations at each temperature	Initial temperature (50)		
			Min time (s)	Max time (s)	Avg. Time (s)
10	250	50	4.0162	4.2185	4.087
10	250	100	7.9368	8.1599	8.0409
10	250	200	14.5514	17.9969	16.2052
10	500	50	20.3049	21.7245	20.9577
10	500	100	40.4384	42.2583	41.2364
10	500	200	81.4485	85.0551	82.8822
10	1000	50	94.6664	110.3187	98.2502
10	1000	100	188.6021	208.2236	195.3325
10	1000	200	328.5544	352.6335	338.0427
10	2000	50	344.6474	421.3307	360.9996

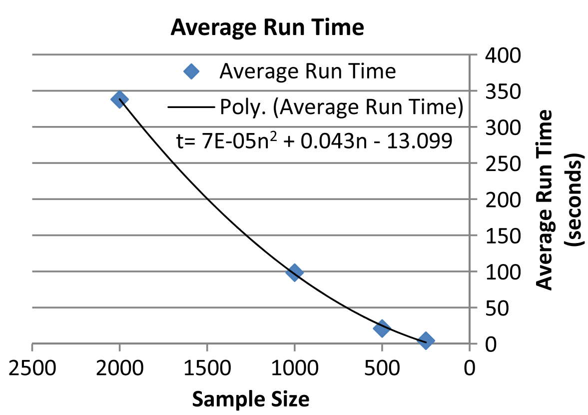

Fig. 1. Plot of the average run time versus sample size.

run time will take more than a day. Therefore large size companies need to compromise between run time and the quality of solution.

7. Conclusions

In the literature, the original goal of having an efficient inventory control policy has been of less concern in many papers than concentrating on sub-optimal problems of inventory items categorization and policy selection. In this paper an integrated method is proposed to simultaneously categorize inventory items and find an efficient control policy. The proposed model is compared with annual dollar usage, AHP weighted score, the method proposed by Zhang et al. (2001) and optimal inventory score, and exceeds all of them in minimizing both dissimilarity and total inventory value. Simulated annealing is used for the model to find a good solution. It is shown that the run time of the simulated annealing is a polynomial of order two with respect to sample size.

References

Cohen, M.A., Ernst, R., 1988. Multi-item classification and generic inventory stock control policies. Production and Inventory Management Journal 29 (3), 6–8.
Chakravarty, A.K., 1985. An optimal heuristic for coordinated multi-item inventory replenishments. Journal of the Operational Research Society 36 (11), 1027–1039.
Ernst, R., Cohen, M.A., 1990. Operations related groups (ORGs): a clustering procedure for production/inventory systems. Journal of Operations Management 9 (4), 574–598.
Flores, B.E., Whybark, D.C., 1986. Multiple criteria ABC analysis. International Journal of Operations and Production Management 6 (3), 38–46.
Flores, B.E., Olson, D.L., Dorai, V.K., 1992. Management of multicriteria inventory classification. Mathematical and Computer Modeling 16 (12), 71–82.
Guvenir, H.A., Erel, E., 1998. Multi criteria inventory classification using a genetic algorithm. European Journal of Operational Research 105, 29–37.
Gajpal, P.P., Ganesh, L.S., Rajendran, C., 1994. Criticality analysis of spare parts using the analytic hierarchy process. International Journal of Production Economics 35 (1-3), 293–297.
Hautaniemi, P., Pirttila, T., 1999. The choice of replenishment policies in an MRP environment. International Journal of Production Economics 59, 85–92.
Huiskonen, J., 2001. Maintanance spare parts logistics: special charactarestics and strategic choices. International Journal of Production Economics 71, 125–133.
Ng, W.L., 2007. A simple classifier for multiple criteria ABC analysis. European Journal of Operational Reseach 177, 344–353.
Partovi, F.Y., Anandarajan, M., 2002. Classifying inventory using an artificial neural network approach. Computers and Industrial Engineering 41, 389–404.
Partovi, F.Y., Burton, J., 1993. Using the analytic hierarchy process for ABC analysis. International Journal of Operations and Production Management 13 (9), 29–44.
Partovi, F.Y., Hopton, W.E., 1994. The analytic hierarchy process as applied to two types of inventory problems. Production and Inventory Management Journal 35 (1), 13–19.
Ramanathan, R., 2006. ABC inventory classification with multiple-criteria using weighted linear optimization. Computers & Operations Research 33, 695–700.
Tsai, C.Y., Yeh, S.W., 2008. A multiple objective particle swarm optimization approach for inventory classification. International Journal of Production Economics 114, 656–666.
Teunter, R.H., Babai, M.Z., Syntetos, A.A., 2010. ABC classification: service levels and inventory costs. Production and Operations Management 19 (3), 343–352.
Zhou, P., Fan, Liwei, 2007. A note on multi-criteria ABC inventory classification using weighted linear optimization. European Journal of Operational Research 182, 1488–1491.
Zhang, R.Q., Hopp, W.J., Supatgiat, C., 2001. Spreadsheet implementable inventory control for a distribution center. Journal of Heuristics 7, 185–203.