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Demand forecasting and inventory control: A simulation study on automotive spare parts

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ABSTRACT

This paper presents results of a large-scale simulation study on spare parts demand forecasting and inventory control to select best policies within each SKU category. Simulations were conducted over 10,032 SKUs of an automaker that operates in Brazil, considering six years of demand data. Literature review drove the selection of different models simulated. The study included three alternatives to record demand data (individual orders data, weekly and monthly time buckets), three demand forecasting models (SMA – Simple Moving Average, SBA – Syntetos–Boylan Approximation and Bootstrapping) and six models for demand distribution during lead-time (Normal, Gamma, NBD-Negative Binomial Distribution, compound Poisson-Normal, compound Poisson-Gamma and Bootstrapping) resulting in 17 "combined" policies. These policies were applied under (s, nQ) inventory control (reorder point, multiples of fixed order quantity), considering two alternative frequencies for model parameters revision (monthly and semi-annually) and four Target-Fill-Rates (TFR=80%, 90%, 95% and 99%), totalizing 136 simulation runs over each SKU. Parameter values (s, Q) were calculated towards TFR using methods from literature. Performance of each combined policy was measured by total costs and RFR – Realized-Fill-Rate. Major contributions of the research are the policy recommendations within each SKU category, a new Bootstrapping procedure and the highlight of Single Demand Approach (SDA) as a promising area for future theoretical and empirical studies. Results shall be used as guideline for practitioners under similar operations.

1. Introduction

In many segments, including automotive, products have little differentiation among brands and other factors gained increased attention to maintain customer satisfaction and loyalty. After sales activities have received strong attention, as quick response and high quality services help companies to accomplish their objectives. Spare parts have significant impact over these services, so good management practices on inventory control are desired.

From practical and academic points of view, spare parts inventory control is a complex and defying activity, involving thousands of SKUs and demands spreading from thousand units per month to quite few units a year.

The literature shows several studies focusing on different aspects of spare parts demand forecasting and inventory control, including items classification (Eaves and Kingsman, 2004; Syntetos et al., 2005), time bucket selection (Krever et al., 2005; Bartezzaghi and Kalchsmidt, 2011), demand forecasting models (Croston, 1972; Syntetos and Boylan, 2005; Teunter and Duncan, 2009), Lead-Time Demand distribution (Porras and Dekker, 2008; Nenes et al., 2010; Bacchetti et al., 2013) and parameter revision frequencies (Babai et al., 2009; Syntetos et al., 2010).

Studies on spare parts inventory control considered different time buckets for demand recording, including individual orders and different time buckets (weekly, monthly, bi-monthly and quarterly). Examples of such approach are found on Eaves and Kingsman (2004), Krever et al. (2005), Porras and Dekker (2008), Boylan et al. (2008), Nenes et al. (2010), Syntetos et al. (2010) and Bacchetti et al. (2013). The study of Krever et al. (2005) proposed an interesting and different approach by using individual order data (called SDA – Single Demand Approach) and developed specific formulations to such cases. Krever et al. (2005) showed that SDA performed better than monthly data for highly sporadic items. On the other hand, Bartezzaghi and Kalchsmidt (2011) showed that larger time buckets (10 or 30 days) implied on lower inventory levels necessary to achieve a 94% TFR for most items. In the current study, simulations included procedures to deal with individual orders as developed by Krever et al. (2005) as well as weekly and monthly time buckets records.

Croston's (1972) seminal paper introduced the idea of separating demand sizes and time intervals to obtain forecasts better than

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traditional SES – Simple Exponential Smoothing. Later developments such as SBA – Syntetos–Boylan-Approximation, improved Croston's proposal, as confirmed on further comparison studies (Syntetos and Boylan, 2005, 2011). Many authors such as Johnston and Boylan (1996), Ghobbar and Friend (2003) and Teunter and Duncan (2009) report comparisons between different spare parts demand forecasting models. In this paper, SBA forecasts were included together with a base model (SMA – Simple Moving Average) and a Bootstrapping model (adaptation of Zhou and Viswanathan (2011)).

Efron (1979) developed the Bootstrapping technique, which was later used for demand distribution and inventory control as proposed by Bookbinder and Lordahl (1989). A milestone on such applications is the paper of Willemain et al. (2004), which showed the superior performance of their Bootstrapping forecasts when empirically compared with SES and Croston models. Porras and Dekker (2008) empirical study also compared Bootstrapping models with parametric (Normal and Poisson) alternatives and showed that Normal distribution provided slightly better results than Bootstrapping models.

Apart from the non-parametric alternative (Bootstrapping), several other parametric distributions are suggested on literature to model LTD. In current study, Normal, Gamma and Negative Binomial Distribution (NBD) were adopted together with SBA and SMA forecasting models. Under SDA, three compound distributions (demand intervals and demand sizes) were used: Poisson-Normal, Poisson-Gamma and Poisson-Logarithm (same as NBD).

Another important decision practitioners must take is how frequently to update the inventory control parameters (the reorder point and order size, for example). Although some previous case studies (Syntetos et al., 2009, 2010; Nenes et al., 2010) considered parameters revision on inventory control simulation, only Babai et al. (2009) actually compared dynamic updating (every period) against static parameters and showed superior performance for dynamic alternative, in spite of the additional computational effort required.

Among other alternatives (see Silver et al. (1998)), Target-Fill-Rates (TFR) are commonly used to set the parameters of the inventory control. Fill-Rate refers to the fraction of demand directly filled by the inventory, without backordering or stockout. An alternative performance measure can be the Cycle-Service-Level (CSL), the desired probability of not running out of stock in any one ordering cycle. In a (s, nQ) inventory control model, both parameters are linked to the TFR, while CSL is dependable only on the reorder point (s). In this paper, parameters are obtained considering four levels of TFR: 80%, 90%, 95% and 99% (analog to Nenes et al. (2010)). It is important to remark that SDA models developed by Krever et al. (2005) considered CSL objectives so a "conversion" to equivalent TFR was necessary for comparison purposes, as explained later.

This paper includes simultaneous evaluation of all above aspects by simulation over empirical data obtained from 10,032 SKUs from an automaker that operates in Brazil. Suitable choices of the above parameters and two revision frequencies alternatives lead to 34 combined policies. These 34 alternatives were simulated under the four different TFR, totalizing 136 simulation runs for each SKU. Empirical data included demand records from 6 years (2007–2013) of 10,032 SKUs. Performance under each policy was measured by its total costs, using TFR as a minimum requirement to RFR.

SKUs were classified within four categories according to the criteria defined by Syntetos et al. (2005) which considers demand size variability (measured by the CV^2 – square of coefficient of variation) and average demand inter-arrival interval (measured by the $\bar{I}$).

The remaining of this paper is organized as follows: Section 2 reviews the literature on spare parts inventory management, Section 3 presents the methodology and simulation model, Section 4 shows the results and Section 5 reports the conclusions.

2. Background

This section presents main references on each model simulated in current study. The sections refer to time bucket choice, demand forecasting, usage of Bootstrapping, reorder point and lot size calculation, Lead-Time-Demand (LTD) distribution, converting CSL objectives into TFR, parameter revision frequency and items classification.

2.1. Time bucket choice

Krever et al. (2005) developed SDA in a similar way as used by Croston (1972). While Croston (1972) developed a forecasting model splitting total demand into two variables (demand sizes and demand interval), Krever et al. (2005) applied same idea to develop order point (s) formulation linked to a CSL objective. They argue that usage of data grouped into time buckets can mislead demand sizes and tendencies, bringing poor performance on inventory control systems. Krever et al. (2005) developed analytical formulations for some special distributions (compound Poisson-Normal and compound Poisson-Gamma, as detailed on Section 2.5) and provided basic formulations to be adapted under other distributions. Their results showed superior performance of the proposed models when compared to traditional time bucket approach (which they called PDA – Period-Demand-Approach).

Traditionally, considering Lead-Time variability and its independence from demand distribution, LTD average and variance are expressed (Silver et al., 1998) as

$$\overline{D_L} = \overline{L} \cdot \overline{D} \tag{2.1}$$

$$\sigma^2_{D_L} = \overline{L} \cdot \sigma^2_D + (\overline{D})^2 \cdot \sigma^2_L \tag{2.2}$$

where $\overline{L}$ = lead-time average, $\overline{D}$ = demand average (per unit time), $\overline{D_L}$ = LTD average (units), σ^2_D = demand variance (per unit time), σ^2_L = lead-time variance, $\sigma^2_{D_L}$ = LTD variance.

By Krever et al.'s (2005) approach, the following formulations are obtained:

$$\overline{D_L} = \lambda_p \cdot \overline{L} \cdot \overline{q} \tag{2.3}$$

$$\sigma^2_{D_L} = \lambda_p \cdot \overline{L} \cdot \left[\sigma^2_q + (\overline{q})^2\right] + \lambda^2_p \cdot (\overline{q})^2 \cdot \sigma^2_L \tag{2.4}$$

where λ_p = average order arrivals per unit time (assumed Poisson), $\overline{q}$ = average order size (units), σ^2_q = order size variance, and other items as per traditional formulation.

In addition to SDA, the simulation herein includes time buckets approach with weekly and monthly records. The choice of such time buckets is supported by previous studies such as Eaves and Kingsman (2004), Porras and Dekker (2008), Boylan et al. (2008), Nenes et al. (2010), Syntetos et al. (2010) and Bacchetti et al. (2013). In a study comparing different time buckets (1, 2, 3, 10 and 30 days), Bartezzaghi and Kalchsmidt (2011) showed that larger time windows (10 or 30 days) implied on lower inventory levels to achieve a 94% TFR for most items. Exceptions occurred on items with high demand inter-arrival interval where increasing from 10 to 30 days implied on significantly higher inventory levels and over lumpy items where no significant conclusions were obtained.

2.2. Demand forecasting

Croston (1972) proposed to separate traditional Simple-Exponential-Smoothing (SES) forecasts into two basic components (demand sizes and demand interval), both modeled by exponential smoothing formulations. Croston (1972) showed that his model surpasses SES performance, although further studies revealed a bias on his model. To correct such bias, Syntetos and Boylan (2005) proposed the so called SBA (Syntetos–Boylan-Approximation), which works as follows:

$$Y'_t = \left(1-\frac{\alpha_s}{2}\right)\frac{Z'_t}{I'_t} \tag{2.5}$$

where Y'_t = demand forecast on period "t"; Z'_t = demand size forecast for period "t", updated with exponential weight "α_s" only when a demand occurs, as defined in Croston (1972), and I'_t = demand inter-arrival interval forecast on period "t", updated with exponential weight "α_s" only when a demand occurs, as defined in Croston (1972).

Joining Eqs. (2.1) and (2.5) brings

$$\overline{D_L} = \bar{L}\cdot\bar{D} = \bar{L}\cdot Y'_t = \left(1-\frac{\alpha_s}{2}\right)\cdot\frac{Z'_t}{I'_t}\cdot\bar{L} \tag{2.6}$$

Teunter and Duncan (2009) brought an interesting insight to this matter. They noticed that Realized-Fill-Rates (RFR) obtained in a comparison with different forecasting models were systematically below TFR. This result contradicts the expectations since RFR were supposed to be randomly distributed around TFR. Analyzing the procedure, Teunter and Duncan (2009) noticed that the models ignored the fact that all resupply cycles are triggered by an effective demand occurrence in the beginning time period. By correcting this, they observed Realized-Fill-Rates (RFR) closer to expected TFR. This simple insight has great impact on LTD computation. In the case of SBA, it means that instead of expecting demand as Z'_t/I'_t during the whole lead-time ($\bar{L}$), we should consider an average occurrence of size Z'_t on first time period and then Z'_t/I'_t on remaining. This insight changes Eq. (2.6) to the following:

$$\overline{D_L} = \left(1-\frac{\alpha_s}{2}\right)\cdot\left[Z'_t + (\bar{L}-1)\cdot\frac{Z'_t}{I'_t}\right] = \left(1-\frac{\alpha_s}{2}\right)\cdot\frac{Z'_t}{I'_t}\cdot(\bar{L}+I'_t-1) \tag{2.7}$$

It is important to notice that when $I'_t = 1$, Eq. (2.7) is reduced back to Eq. (2.6), which allows the conclusion that Teunter and Duncan's insight has impact only on intermittent items ($I'_t > 1$), increasing its effect as the demand occurrence interval (I'_t) increases.

Herein, Eq. (2.7) is applied to estimate LTD average with SBA forecasts. A baseline model (SMA – Simple Moving Average) was included on the simulation study. SMA forecasts were based on latest 6 months of data available (26 weeks, in case of weekly data). As SMA forecasts do not separate demand sizes and intervals, usage of Teunter and Duncan's insight is not feasible on this model. Next section details the usage of Bootstrapping, in which Teunter and Duncan's insight can also be applied to obtain LTD.

An estimate of LTD variance is also necessary to apply Eq. (2.2). The simulation in this study uses the Mean Square Error (MSE) to obtain such estimates, as done by Babai and Syntetos (2007).

2.3. Usage of Bootstrapping

According to Smith and Babai (2011), the use of parametric distributions (as Normal, Gamma, etc.) to model LTD is restricted when items are highly intermittent. As demand becomes more erratic, true demand size distribution may not be adherent to standard theoretical distributions. The Bootstrapping technique as proposed by Efron (1979) consists in using a sample of data to rebuild its original distribution. Bookbinder and Lordahl (1989) introduced the technique on inventory control to estimate LTD and determine reorder points. Bootstrapping procedures are therefore an alternative to the traditional process of obtaining demand forecasts and using such forecasts to fit standard theoretical distribution.

Willemain et al. (2004) proposed a new Bootstrapping model based on two state Markovian process and small perturbations on original data (called jittering process). They reported superior results from their model when compared with SES and Croston forecasts. Hua et al. (2007), Porras and Dekker (2008), Teunter and Duncan (2009) and Zhou and Viswanathan (2011) also compared Willemain's Bootstrapping to other Bootstrapping models and parametric alternatives. It is important to notice that these Bootstrapping models treat lead-time as deterministic, although small changes can adapt them to probabilistic lead-times, as proposed in this paper.

Zhou and Viswanathan (2011) proposed a simpler Bootstrapping procedure that does not include the jittering process. Their model (so called VZ model) showed superior performance when compared to Willemain's and other parametric alternatives, in a simulation study with only 50 SKU's. This paper adopts a Bootstrapping model similar to VZ but including probabilistic lead-times, adoption of Teunter and Duncan's insight and a jittering process slightly different from Willemain's proposal, as described on Methodology section below.

The studies from Porras and Dekker (2008) and Teunter and Duncan (2009) pointed out the need of a minimum of two demand occurrences to apply Bootstrapping models. Considering the way VZ model is constructed, the current study requires a minimum of three demand occurrences, allowing therefore a minimum of two demand intervals in-between. An unexplored area on all cited Bootstrapping models is the dynamic to rebuild the LTD in the long-term. This topic is discussed in Section 2.7 – Parameter revision frequency.

2.4. Reorder point and lot size calculation

This paper adopts a continuous review (s, nQ) model in which orders are made in multiples of lot size (Q) when inventory drops below reorder point (s). The number of lots is the minimum to raise the inventory again above reorder point (s). Literature presents many alternative models to spare parts inventory control, usually simple continuous or periodic review models (Sani and Kingsman, 1997; Silver et al., 1998; Porras and Dekker, 2008). Boylan et al. (2008) applied a continuous review (s, Q) model in their case study, but the authors state that no significant differences are expected if periodic review models were applied. In the same way, Sani and Kingsman (1997) pointed minimum differences when comparing different control models under intermittent demand.

There are different ways to model and derive expressions for the calculation of the inventory control parameters including costs minimization, customer service and other aggregate measures (Silver et al., 1998). On spare parts studies, some authors (Krever et al., 2005; Teunter and Duncan, 2009; and Syntetos et al., 2010) considered models based on CSL while others (Boylan et al., 2008; Syntetos et al., 2009; and Bacchetti et al., 2013) applied models based on TFR. According to Axsäter (2006), CSL should not be used to practical inventory control since it does not account for the effect of the order sizes (Q). Adopting TFR as performance measure, relationship between s, Q and TFR is expressed by (Silver et al., 1998) as follows:

$$\int_s^{\infty}(x-s)f(x)dx = Q\cdot(1-\mathrm{TFR}) \tag{2.8}$$

where $f(x)$ is the LTD probability density function

According to Tyworth and Ganesham (2000), simultaneous solution for s and Q on Eq. (2.8) is a complex task and adoption of Economic Order Quantity (EOQ) brings results close to optimal.

Platt et al. (1997) showed that there is a better solution than EOQ when lot size (Q) and reorder point (s) are determined sequentially under a TFR. Their solution is to calculate lot size (Q) as follows:

$$Q = \frac{1}{\text{TFR}}\sqrt{\frac{2 \cdot A \cdot \overline{D}}{C \cdot h} + \sigma^2_{D_L}} \tag{2.9}$$

where A=fixed order cost ($ per order), $\overline{D}$=average demand per unit time, C=SKU unit cost ($ per unit), h=inventory holding cost (% per unit time), and $\sigma^2_{D_L}$=LTD variance.

In this paper, rounding of lot size (Q) follows Axsäter (2006) rule (also used by Porras and Dekker (2008)) as follows:

1. calculate:	$m=Q$	integer part of Q
2. adjust:	$Q=1$	if: $m=0$
	$Q=m$	if: $m \neq 0$ and $\frac{Q}{m} \leq \frac{m+1}{Q}$
	$Q=m+1$	otherwise

After computing the lot size (Q), reorder point (s) is determined by the lowest value satisfying Eq. (2.8). Same equation has specific solutions for Normal and Gamma distributions, as well as alternative formulations obtained by Krever et al. (2005) for SDA (under Poisson-Normal and Poisson-Gamma compound distributions) as shown in Section 2.5 – LTD distribution models.

2.5. Lead-Time-Demand (LTD) distribution

Fig. 1 shows the different ways to model LTD from historical data, as adopted in this paper. Normal distribution is a natural choice in modeling LTD, especially on fast movers. Silver et al. (1998) argue that Normal distribution has good adherence to empirical data, is analytically convenient, easily tabulated and promptly available in commercial software. By splitting demand into distinct and independent components (demand sizes and demand intervals) as done by Croston (1972) and Krever et al. (2005), the usage of compound distributions is also a natural choice. In the compound distributions alternative, many authors (Ehrhardt, 1979; Agrawal and Smith, 1996; Boylan et al., 2008; Syntetos et al., 2011) suggest adoption of Negative Binomial Distribution (NBD). NBD results from a Poisson arrival process with logarithmic distributed demands.

In this paper we also adopt Gamma distribution due to many good properties: covers a wide range of distribution shapes, only exists for non-negative values, is easily analytically tractable and can be considered the continuous version of discrete NBD (Syntetos et al., 2011). Poisson-Normal and Poisson-Gamma compound distributions were also included in this paper since both have specific analytical formulations to determine reorder point (s) under SDA, as developed by Krever et al. (2005).

In addition to parametrical distributions, an empirical and non-parametrical Bootstrapping model (based on original VZ model) was included on the simulations.

LTD information is obtained from historical data as shown in Fig. 1 and then used to obtain reorder point (s) as summarized in Table 1. Most of these formulations are iterative, starting with $s=0$ and going up until the lowest satisfying value (Rego, 2014).

For compound Poisson-Normal and Poisson-Gamma, the reorder points are determined targeting a predefined CSL. In order to be compared to other models based on TFR, some "convertion" shall be made in these two situations, as detailed in Section 2.6.

2.6. Converting CSL objectives into TFR

The formulas for reorder point (s) adopted in this paper are based on TFR, except with compound Poisson-Normal and Poisson-Gamma, as derived by Krever et al. (2005). Considering the need to compare such different approaches to evaluate their practical usage, the following "conversion" procedure was adopted:

(1) obtain lot size (Q) for a specific TFR, by Eq. (2.9)
(2) start with 50% CSL
(3) reorder point (s) is calculated for current CSL, using Eqs. (2.13) or (2.14);
(4) from reorder point (s) and lot size (Q), calculate the Fill-Rate (FR) by Eq. (2.16) (derived from Eq. (2.10)):

$$FR = 1 - \frac{\sigma_{D_L}}{Q} \cdot \left\{ \varphi(z) - z \cdot [1 - \Phi(z)] - \varphi\left(z + \frac{Q}{\sigma_{D_L}}\right) + \left(z + \frac{Q}{\sigma_{D_L}}\right) \cdot \left[1 - \Phi\left(z + \frac{Q}{\sigma_{D_L}}\right)\right] \right\} \tag{2.16}$$

(5) in case FR ≥ TFR, adopt current reorder point (s), otherwise increase CSL by 1% and return to step (3).

Above procedure is a simplification since Eq. (2.16) considers LTD as Normal, which is not same as the assumption in Eqs. (2.13) and (2.14). This approximation, however, was made to allow comparison of promising results from Krever et al. (2005) to other models.

2.7. Parameter revision frequency

Inventory control parameters updating is surprisingly little discussed on academics papers over spare parts inventories. Bucher and Meissner (2011) suggest annual review of item categorization. In a similar way, Bacchetti et al. (2013) recognize that in real world items can move between demand categories,

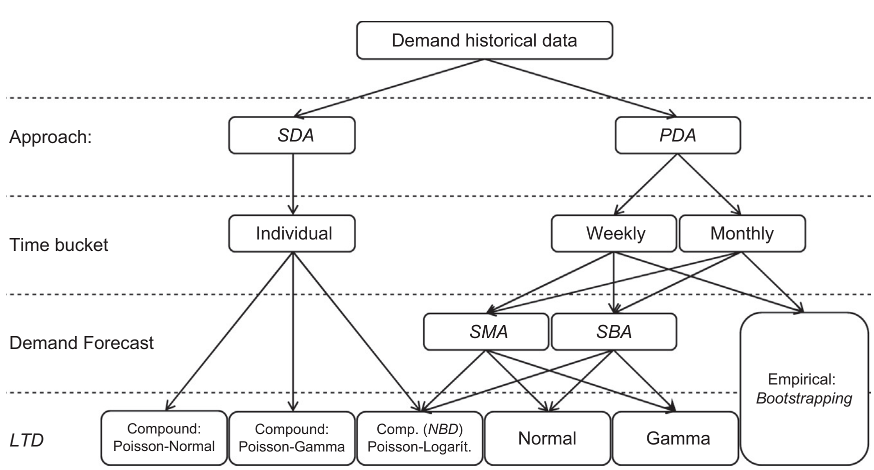

Fig. 1. Modeling LTD from historical data in current research.

Table 1
Reorder point (s) formulas under different LTD distributions.

LTD distribution (equation #)	Reorder point (s) formula	Parameters	References
Normal (2.10)	$\varphi(z)-z\cdot[1-\Phi(z)]-\varphi\left(z+\frac{Q}{\sigma_{D_L}}\right)+\left(z+\frac{Q}{\sigma_{D_L}}\right)\cdot\left[1-\Phi\left(z+\frac{Q}{\sigma_{D_L}}\right)\right]\le\frac{Q\cdot(1-\mathrm{TFR})}{\sigma_{D_L}}$	$z=\frac{s-\overline{D_L}}{\sigma_{D_L}}$	Chopra and Meindl (2003) and Silver et al. (1998)
Gamma (2.11)	$\alpha\beta[1-F_{GAMMA}(s;\ \alpha+1;\ \beta)]-s[1-F_{GAMMA}(s;\alpha;\beta)]\le Q\cdot(1-\mathrm{TFR})$	$\alpha=\frac{\overline{D_L}^2}{\sigma_{D_L}^2}$ $\beta=\frac{\sigma_{D_L}^2}{\overline{D_L}}$	Silver et al. (1998) and Tyworth and Ganesham (2000)
NBD (2.12)	$\sum_{x=s+1}^{\infty}(x-s)\frac{\Gamma(x+r)}{x!\Gamma(r)}(1-p)^r p^x\le Q\cdot(1-\mathrm{TFR})$	$r=\frac{\overline{D_L}^2}{\sigma_{D_L}^2-\overline{D_L}}$ $p=1-\frac{\overline{D_L}}{\sigma_{D_L}^2}$	Krever et al. (2005)
Poisson-Normal (2.13)	$e^{-\lambda_p\overline{L}}\left[1+\sum_{n=1}^{\infty}\frac{\left(\lambda_p\overline{L}\right)^n}{n!}\Phi\left(\frac{s-n\overline{q}}{\sigma_q\sqrt{n}}\right)\right]\ge\mathrm{CSL}$		Krever et al. (2005)
Poisson-Gamma (2.14)	$e^{-\lambda_p\overline{L}}\left[1+\sum_{n=1}^{\infty}\frac{\left(\lambda_p\overline{L}\right)^n}{n!}\int_0^s f_{GAMMA}(Q_n)dQ_n\right]\ge\mathrm{CSL}$	$\alpha=\frac{\overline{q}^2}{\sigma_q^2}$ $\beta=\frac{\sigma_q^2}{\overline{q}}$	Krever et al. (2005)
Bootstrapping (2.15)	$\sum_{x=s+1}^{\infty}(x-s)f(x)\le Q\cdot(1-\mathrm{TFR})$		Porras and Dekker (2008)

although they do not propose a specific procedure for such updates.

Demand forecasts can be re-evaluated after each new data is obtained. Inventory control parameters also can be revised since new forecasts arise. Not only the model parameters can be revised but even the model itself can be changed when new data are available.

Few studies address this issue. Syntetos et al. (2009) report quarterly parameter revision only for the most important (class A) items due to research limitations, though recommending revisions for all items as demand patterns evolves in the long term. Nenes et al. (2010) reported the development of a system where parameters are revised semi-annually based on latest two years of information available. Syntetos et al. (2010) reported inventory control parameters revision every time period (weeks, in this case), when new demand forecasts were generated.

In the same way, Babai et al. (2009) compare an every period updating model against static parameterization. They concluded that dynamic model brings inventory levels much lower than static, although both brought similar service levels.

Considering Bootstrapping models, none of previous papers (Willemain et al., 2004; Hua et al., 2007; Porras and Dekker, 2008; Teunter and Duncan, 2009; Zhou and Viswanathan, 2011) discussed the need of updating LTD distribution dynamically. The dynamic adopted in current study is detailed in Section 3.

2.8. Items classification methods

Huiskonen (2001) and Boylan et al. (2008) point that item classification for inventory management should allow determining adequate managerial attention, choosing demand forecast and inventory control models and establishing different service levels within categories. Bucher and Meissner (2011) emphasizes that an efficient selection of inventory management policies for different categories can bring benefits on both cost reduction and service levels. Syntetos et al. (2009) also mention that organizations with relevant spare parts inventories often classifies such items with different criteria, assuming different service levels within each category.

Literature has many contributions on spare parts classification. Williams (1984) was the first to propose demand variation partition in basic components towards item categorization. Eaves and Kingsman (2004) developed over Williams (1984) ideas and proposed new categorization scheme focused on demand forecast models selection. The classification of Syntetos et al. (2005) is as shown in Fig. 2. Four quadrants are determined based on two dimensions: demand size variability (measured by CV^2) and average demand inter-arrival interval (measured by the $\overline{I}$), considering cutoff values of 0.49 and 1.32 respectively. These cutoff limits were mathematically determined (instead of empirically as in previous papers) by comparing Mean Square Error (MSE) of different forecasting models. The cutoff limits derived by Syntetos et al. (2005) are expected to have general validity for a wide range of realistic control parameters (Bucher and Meissner, 2011).

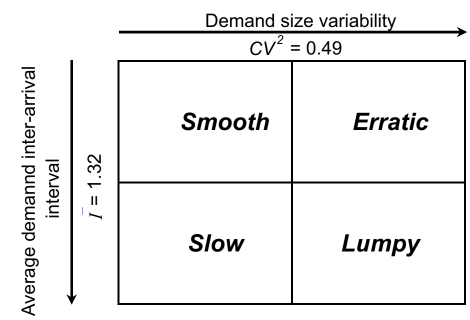

Fig. 2. SKU demand classification (Syntetos et al., 2005).

In addition to demand analysis, other authors suggest adoption of multiple criteria classification schemes. Life cycle phase is suggested as criteria by Yamashina (1989), Sharaf and Helmy (2001), Persson and Saccani (2009), Rego and Mesquita (2011) and Bucher and Meissner (2011). Other factors such as total revenues, total profits, unit cost, item criticality and logistic aspects are also mentioned by Sharaf and Helmy (2001), Huiskonen (2001), Eaves and Kingsman (2004), Chu et al. (2008), Porras and Dekker (2008) and Bacchetti and Saccani (2012).

3. Methodology

This section presents the simulation model proposed to set up the inventory control system. It includes database structure, outliers treatment, cost and lead-time structure, TFR and exponential smoothing factors choice, modified Bootstrapping model and simulation flow charts.

3.1. Database structure

Historical demand data was obtained for a 6-year period (from September 1st, 2007 until August 31st, 2013). First 3 years of data were considered an adjust period (for establishing initial parameters), next 6 months were used as warm-up period (to reduce impact of simulation start-up) and remaining 2.5 years were actually the test period (used to compare alternatives). From the automaker original database (approximately 90,000 SKUs) a cleaning process removed everything except spare parts (marketing promotional materials, accessories, special tools and technical manuals). A minimum of 6 months of life in the beginning of the simulation warm-up was also a restriction as well as a minimum of

Table 2
Database information compared to different upper limits.

Upper Limit (z-value)	2	3	4	6	8
SKUs with outlier demands	5084	3564	1951	515	123
% of weeks exceeding limit	0.62	0.22	0.08	0.02	0.004
% of demand exceeding limit	2.26	0.80	0.34	0.07	0.02
Standard normal: $1-\Phi(z)$ (%)	2.28	0.13	0.003	0.0000001	0.0000000

3 demand occurrences during the adjust period (necessary for Bootstrapping). Demands of superseded items were consolidated under the new items. After this cleaning process, database was reduced to 10,032 SKUs.

3.2. Outliers treatment

Krever et al. (2005), Porras and Dekker (2008) and Nenes et al. (2010) used filters to identify exceptionally high demands, usually associated to recall campaigns or special orders. Such exceptions are supposed to be pre-planned thus not affecting usual supply–consumption cycles.

Table 2 compares database information to expected values from a Normal distribution (demand was taken in weekly time buckets for this analysis). From Table 2, at the upper limit of two standard deviations, database percents remained equal or below what should be expected under a standard Normal distribution. For higher upper limits, however, database presented much more outliers than a Normal distribution should expect.

Although “cleaning” the outliers could bring a positive impact increasing RFR, the absence of explanations for the large majority of them was the reason for not excluding any record from the database, similarly to Nenes et al. (2010) study.

3.3. Cost and lead-time structures

The cost structure adopted in current study includes

- A fixed order cost ($ per order)
- b fixed backorder cost ($ per backorder occurrence)
- $\hat{b}$ unit backorder cost ($ per unit time)
- C SKU unit cost ($ per unit)
- h inventory holding cost (% per unit time)

Automaker managers estimated these costs for each SKU. Due to confidentiality agreement with the automaker, all costs were converted to a proper currency, so proportions were maintained while actual values kept undisclosed. Two fixed order costs (A) were established, depending on the SKU source: $1.00 for imported items and $7.92 for local suppliers. Difference on fixed order costs are due to different process handling purchase orders for such origins: imported items are mainly obtained from the automaker Headquarter abroad, so purchase process is very simplified while local purchases come from more than one hundred different suppliers.

Inventory holding costs (h) includes capital costs and several other components such as rental, handling, insurance, losses, etc. Total inventory holding costs were estimated to be 21.5% per year.

Fixed backorder cost (b) and unit backorder cost ($\hat{b}$) were established as being the double of fixed order cost (A) and unit inventory holding cost ($C \cdot h$), respectively. The automaker managers' believed backorder occurrences lead to several extra fixed costs related to urgent ordering and transporting, raising such costs.

For lead times, 170 different suppliers were identified in the 10,032 SKU database. Lead-times were treated as probabilistic during the simulation. For each supplier a minimum/optimistic, a maximum/pessimistic and a most probable/mode lead-time were estimated. Those values were used in a triangular distribution to obtain actual lead-times during the simulation. Triangular distribution was adopted due easy modeling of non-negative and asymmetric distribution of lead-times found within automaker's real data.

3.4. TFR and exponential smoothing factors choice

Simulations were conducted under four different values of TFR: 80%, 90%, 95% and 99%. These values are the same as adopted by Nenes et al. (2012). Automakers have strong responsibility in supplying spare parts to their dealer network (and to final customers, consequently) so usage of lower TFR is not usual or recommended. Companies usually establish different values of TFR depending on each SKU importance. Some previous studies adopted a single fixed TFR so their conclusions were limited to this specific value. In this paper we intend to understand if policy recommendations may vary when we simply vary the TFR, ceteris paribus.

Within SBA forecasts and also estimating demand variance (by MSE) researchers must define exponential smoothing factors. In this paper, exponential smoothing factors are fixed at a 20% default level.

3.5. Modified Bootstrapping model

Bootstrapping model adopted in this paper is an adaptation of the VZ model presented in Section 2.3. Modifications were made to adopt Teunter and Duncan's insight and to introduce probabilistic lead-times and jittering process. Table 3 shows the step-by-step procedure to build LTD from original sample.

According to Teunter and Duncan's insight, the procedure first selects a demand size as the one which triggered the replenishment cycle. After this, the corresponding interval to the next demand occurrence is accumulated in the time horizon and compared to the previously randomly generated lead-time. The jittering process also includes a modification (in steps 3 and 4, as shown in Table 4) over Willemain et al. (2004) original proposal.

For Bootstrapping, a minimum of three demand occurrences is required, providing three demand sizes and two intervals between such consecutive demands. Differently from previous studies, in this simulation we also established a maximum of 25 demand occurrences to be used in the Bootstrapping procedure. This maximum was necessary since Bootstrapping is also being applied for non-intermittent items, so taking more than 25 occurrences could mean taking more than 25 months of data (for monthly time buckets) or more than 25 weeks (equivalent to 6 months, if using weekly time buckets). Such large historical data put together could jeopardize LTD obtained from Bootstrapping procedure since “old” data would be mixed together with “new” information.

Every time Bootstrapping procedure is executed, most recent data are incorporated in the demand database. For any SKU, if database includes more than 25 periods with non-zero demand, only latest 25 records were used. Depending on the intermittency of demand, a SKU can reach the end of simulation considering the whole historical or at least the latest 25 time periods.

Table 3
Bootstrapping procedure.

Step	Description
1	Obtain historical demand data (including demand size and interarrival times) according to the time bucket in use.
2	Obtain Lead-time distribution. In current study this means establishing Lead-time parameters for triangular distribution modeling.
3	Randomly obtain a lead-time (for the next resupply cycle).
4	Randomly select a demand size from historical data. Perform the jittering process.
5	Take the interval from selected demand to the next one also from historical data. Increase time horizon with this interval (so it contains the time of next demand occurrence).
6	If time horizon is equal or lower than lead-time, return to step 4. Otherwise, sum the jittered demands during lead-time and obtain a LTD value.
7	Repeat steps 3–6, 2000 times.
8	Classify LTD data and generate LTD distribution.

Table 4
Jittering procedure.

Willemain et al. (2004)	Current paper
(1) randomly select an historical demand value X^*	(1) randomly select an historical demand value X^*
(2) generate random "z" from standard Normal (0;1) distribution	(2) generate random "z" from standard Normal (0;1) distribution
(3) calculate *jittered value* $=1+\text{integer}\left(X^*+z\sqrt{X^*}\right)$	(3) calculate *jittered value* $=\text{integer}\left(0.5+X^*+z\sqrt{X^*}\right)$
(4) if *jittered value* ≤ 0, then *jittered value* $=X^*$	(4) if *jittered value* ≤ 0, then *jittered value* $=1$

Approach	***Time Bucket***	***Forecasting model***	***LTD Distribution***	***#***
SDA	Individual	36 months moving base	Poisson-Normal	A
			Poisson-Gamma	B
			NBD	C
PDA	Weekly	SMA	Normal	D
			Gamma	E
			NBD	F
		SBA	Normal	G
			Gamma	H
			NBD	I
		Last 25 occurrences	Bootstrapping	J
	Monthly	SMA	Normal	K
			Gamma	L
			NBD	M
		SBA	Normal	N
			Gamma	O
			NBD	P
		Last 25 occurrences	Bootstrapping	Q

X

Revision dynamic
* Monthly
* Semi-annually

X

Target Fill Rates (TFR)
* 80%
* 90%
* 95%
*99%

Fig. 3. 136 Simulation runs (17 combined policies, 2 revision frequencies, 4 TFRs).

3.6. Simulation flow charts

Fig. 3 shows the breakdown of 136 simulation runs for each SKU. From original 6 year historical data, the first 3 years were used just for initialization of models. Simulation actually starts on September 1st, 2010 data, considering

- on-hand inventory $=s+Q$ (every SKU starts at maximum inventory level)
- backorder $=0$ (no pending quantity for customers)
- supplier backorder $=0$ (no pending quantity from suppliers)

Although these initial equal conditions to all SKUs may be unrealistic, they will not have influence on the simulation since the first six months were discarded as a warm-up. The remaining 2.5 years of simulation were divided into five semesters when results were analyzed.

Fig. 4 shows the main simulation flow chart. The process described in this flow chart basically organizes spare parts inventory management hierarchically: some decisions are taken less frequently (tactical level) while others (operational level) are taken on a daily basis.

Figs. 5 and 6 show the details of "Tactical" and "Operational" simulation, respectively.

The model was implemented in a spreadsheet using Microsoft Excel and Visual Basic for Applications.

4. Results and discussion

Comparisons between different policies were made by two measures: total costs incurred and RFR – Realized-Fill-Rate. We simplified the analysis by reducing the number of alternatives comparing paired results from parameter revision frequency (monthly $\times$ semi-annually, 17 *t*-tests for each TFR), time buckets (weekly $\times$ monthly, 7 t-tests for each TFR) and forecasting models (SBA $\times$ SMA, 3 *t*-tests for each TFR). From such comparisons, initial 34 combined policies for each TFR were reduced to 10. Further simplification (discarding two other alternatives) was possible by analyzing overlapping policies (in many cases different policies lead to similar results). The eight final combined policies were deeply analyzed under categories defined by Syntetos et al. (2005) criteria, generating recommendations for each category. The following sections include comparisons on general results, paired *t*-tests and overlapping policies discussion.

4.1. General results

Tables 5 and 6 summarize the results for 136 simulation runs. A simple analysis of these tables should drive to the following conflicting conclusions:

- cost results would drive to adoption of the simplest model: usage of Normal LTD combined with SMA forecasts;
- RFR results would drive to adoption of the most complex (considering computational effort) Bootstrapping model.

It is interesting to notice that none of RFR reached TFR when it was fixed on 95% and 99%. Only 9 within 34 surpassed the 80% TFR and only 5 within 34 surpassed 90% TFR. As demands are random, RFR also is expected to be randomly distributed around the TFR used to set up (s, Q) parameters. These negatively biased results are expected on models that do not account for Teunter and Duncan (2009) insight. In current research, SBA and Bootstrapping models included Teunter and Duncan's insight but still presented a negative bias (RFR often below TFR). The main reason for this may be the large amount of outlier demands (exceeding expected values) or even a non-stationary pattern of some demands. When an outlier demand shows up, inventory system presents poor performance and brings RFR down.

As expected, the "outlier" effect over RFR shows up stronger on all parametric distributions, while the non-parametric Bootstrapping brings RFR closer to TFR.

4.2. Parameter revision frequency effect

Firstly we evaluated the possibility to eliminate one of the parameter revision frequencies tested (monthly versus semi-annual cycles). Since every SKU was simulated under both frequencies for each combined policy, it is possible to compare cost results from each SKU through a paired *t*-test for difference between population means. Tables 7 and 8 show the results of 68 tests performed using R free software (R Core Team, 2013). Each combined policy is identified by a letter (A through Q) as described in Tables 5 and 6, while monthly revision is labeled as population 1 and semi-annual revision named population 2. Analyses were made using 95% confidence level. Differences on means are shown only if null hypothesis is rejected.

At 95% confidence level, 48 out of 68 tests did not reject the null hypothesis, which shows that there is no significant difference between both alternatives (monthly or semi-annual revisions), 13 tests pointed lower costs with monthly revisions and only 7 cases of better costs under semi-annual revisions. Four of these seven

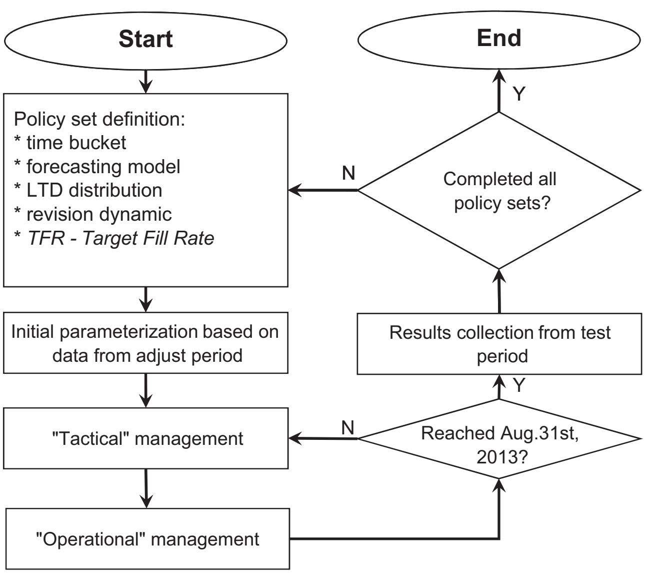

Fig. 4. Simulation main flow chart.

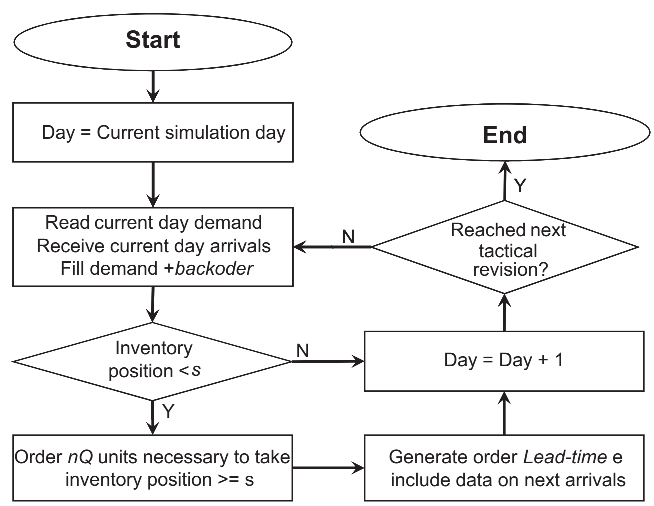

Fig. 6. "Operational" simulation flow chart.

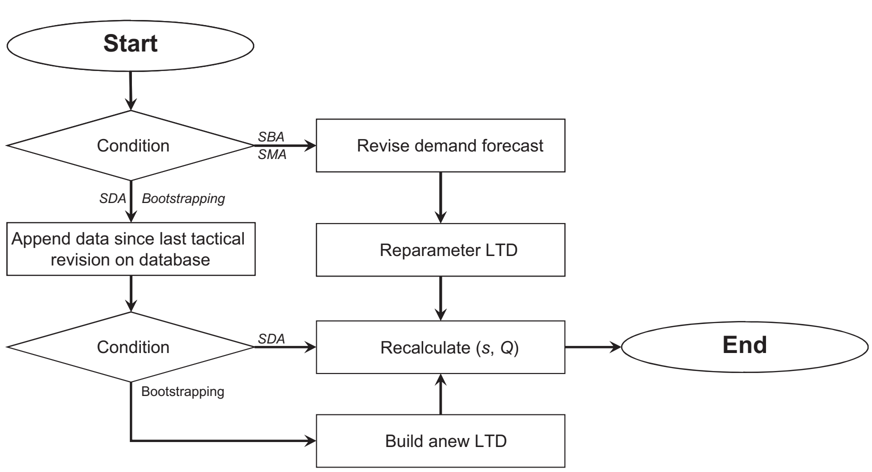

Fig. 5. "Tactical" simulation flow chart.

Table 5
Cost results for 10,032 SKUs × 136 simulation runs. Best results hachured.

Total cost average (per semester, in $1000):			LTD distribution	#	TFR=80%		TFR=90%		TFR=95%		TFR=99%	
					Revision dynamics							
Approach	Time bucket	Forecasting model			Monthly	6-Months	Monthly	6-Months	Monthly	6-Months	Monthly	6-Months
SDA	Individual	36 months moving base	Poisson-Normal	A	$3029	$3106	$3121	$3191	$3032	$3120	$3369	$3438
			Poisson-Gamma	B	$3154	$2834	$3171	$3197	$3214	$3263	$3398	$3425
			NBD	C	$3723	$3749	$3739	$3713	$3734	$3726	$3720	$3748
PDA	Weekly	SMA	Normal	D	$1467	$1463	$1398	$1459	$1376	$1427	$1354	$1414
			Gamma	E	$1618	$1625	$1908	$1875	$2216	$2149	$3056	$2790
			NBD	F	$3098	$3222	$3279	$3195	$3214	$3160	$3213	$3235
		SBA	Normal	G	$1577	$1604	$1474	$1527	$1432	$1528	$1451	$1520
			Gamma	H	$2532	$2511	$2899	$2817	$3264	$3106	$4122	$3813
			NBD	I	$3048	$3053	$3225	$3243	$3157	$3283	$3307	$3342
		Last 25 occurrences	Bootstrapping	J	$1903	$2280	$2351	$2297	$2383	$1745	$2407	$1839
	Monthly	SMA	Normal	K	$1429	$1485	$1410	$1477	$1354	$1435	$1352	$1402
			Gamma	L	$1667	$1680	$1928	$1964	$2196	$2226	$2956	$2950
			NBD	M	$3106	$3131	$3139	$3161	$3166	$3119	$3242	$3228
		SBA	Normal	N	$1612	$1653	$1500	$1633	$1454	$1580	$1438	$1553
			Gamma	O	$1661	$1789	$2010	$2028	$2303	$2395	$3091	$3223
			NBD	P	$3075	$3034	$3141	$3169	$3117	$3211	$3206	$3082
		Last 25 occurrences	Bootstrapping	Q	$2073	$1863	$2420	$1938	$2511	$1899	$2161	$2039

Table 6
RFR results for 10,032 SKUs × 136 simulation runs. Best results hachured.

Average Realized-Fill-Rate (RFR)			LTD distribution	#	TFR=80%		TFR=90%		TFR=95%		TFR=99%	
					Revision dynamics							
Approach	Time bucket	Forecasting model			Monthly (%)	6-Months (%)	Monthly (%)	6-Months (%)	Monthly (%)	6-Months (%)	Monthly (%)	6-Months (%)
SDA	Individual	36 Months moving base	Poisson-Normal	A	79.1	77.9	73.6	73.0	79.1	77.8	43.5	42.9
			Poisson-Gamma	B	79.2	82.6	73.9	69.0	67.5	58.4	43.8	43.2
			NBD	C	76.5	75.6	82.8	82.1	85.7	85.1	87.3	86.8
PDA	Weekly	SMA	Normal	D	63.2	60.9	67.3	64.7	69.2	66.4	71.2	68.4
			Gamma	E	86.0	76.6	86.8	83.5	90.8	87.4	94.3	91.7
			NBD	F	79.5	64.9	79.6	69.8	83.6	72.9	86.9	76.1
		SBA	Normal	G	72.6	71.8	75.5	74.4	77.1	75.9	78.3	77.2
			Gamma	H	72.9	84.4	90.7	88.6	93.1	91.1	95.6	94.2
			NBD	I	77.1	69.9	81.8	72.6	84.5	74.3	87.1	76.4
		Last 25 occurrences	Bootstrapping	J	89.2	87.0	91.1	90.4	92.2	91.9	94.1	94.7
	Monthly	SMA	Normal	K	61.8	60.1	66.3	64.7	67.9	66.4	69.7	67.9
			Gamma	L	79.5	77.6	86.5	84.5	90.4	88.3	94.0	92.6
			NBD	M	71.6	67.4	77.9	72.4	81.6	75.5	85.2	78.9
		SBA	Normal	N	68.9	68.9	71.7	71.4	73.3	72.9	74.5	74.2
			Gamma	O	83.6	82.7	88.8	87.8	91.9	90.9	95.1	94.4
			NBD	P	72.4	70.5	75.9	73.2	78.3	75.1	80.9	77.2
		Last 25 occurrences	Bootstrapping	Q	90.1	87.9	93.1	91.3	94.6	93.0	95.7	94.5

cases refers to Bootstrapping policies (named "J" and "Q") so monthly revision was discarded under such policies. The remaining 3 cases where semi-annual revisions surpassed monthly are as follows:

- under TFR=80%, policy B2 had lower costs than B1, but was discarded since policies O1 and O2 had similar RFR and lower costs;
- under TFR=99%, policy E2 and H2 had lower costs than E1 and H1 respectively, but were discarded since policies J2 and Q2 had RFR closer to TFR and lower costs.

So, from the original 34 alternatives for each TFR, we kept 17 for further analysis discarding semi-annual revision for all policies except for the Bootstrapping cases. Preference for monthly revisions was expected in accordance to Babai et al. (2009), and is probably due to non-stationary patterns on demand history.

4.3. PDA time bucket comparison

It is also possible to compare costs between weekly time buckets and monthly time buckets by pairing D1 × K1, E1 × L1, F1 × M1, G1 × N1, H1 × O1, I1 × P1 and J2 × Q2. Again, comparison is a paired t-test for the difference of means at 95% confidence level. Table 9 summarizes the results for different TFRs.

In every case where the null hypothesis was rejected it indicated better results with monthly time buckets. Looking at RFR, in most cases, weekly results were equal or worse than monthly, so further analysis discarded weekly time bucket alternatives and will concentrate now over 10 combined policies. Whether PDA has better results than SDA is still an open question at this point.

Preference for monthly data may have occurred due to presence of seasonal effects on weekly data. In fact, we noticed that average sales on last week of each month are twice the sales on

Table 7
Paired *t*-test for means (same combined policies, varying revision frequency) under 80% and 90% TFR. Null hypothesis is absence of difference.

	TFR=80%			TFR=90%		
	p-Value	$\mu 1-\mu 2$	Choice (alfa=5%)	*p-Value*	$\mu 1-\mu 2$	Choice (alfa=5%)
A	0.0106	−38.69	Monthly	0.0782		Either
B	0.0173	159.14	Semi-annual	0.8622		Either
C	0.1453		Either	0.4235		Either
D	0.9278		Either	0.0223	−30.52	Monthly
E	0.7831		Either	0.4744		Either
F	0.0630		Either	0.2121		Either
G	0.5291		Either	0.0592		Either
H	0.7585		Either	0.3204		Either
I	0.9265		Either	0.4931		Either
J	0.0570		Either	0.7156		Either
K	0.1234		Either	0.0001	−33.38	Monthly
L	0.6355		Either	0.2700		Either
M	0.6039		Either	0.8550		Either
N	0.2395		Either	0.0007	−66.03	Monthly
O	0.0236	−63.85	Monthly	0.7769		Either
P	0.5592		Either	0.5283		Either
Q	0.0505		Either	0.0029	24.13	Semi-annual

Table 8
Paired *t*-test for means (same combined policy, varying revision frequency) under 95% and 99% TFR. Null hypothesis is absence of difference.

	TFR=95%			TFR=99%		
	p-Value	$\mu 1-\mu 2$	Choice (alfa=5%)	*p-Value*	$\mu 1-\mu 2$	Choice (alfa=5%)
A	0.0029	−43.83	Monthly	0.0877		Either
B	0.7367		Either	0.1611		Either
C	0.5424		Either	0.1918		Either
D	0.0064	−25.40	Monthly	0.0009	−29.98	Monthly
E	0.0760		Either	0.0001	132.80	Semi-annual
F	0.5456		Either	0.6876		Either
G	0.0125	−47.64	Monthly	0.0791		Either
H	0.1011		Either	0.0123	15.40	Semi-annual
I	0.1964		Either	0.8104		Either
J	0.0034	31.76	Semi-annual	0.0083	28.30	Semi-annual
K	0.0119	−40.37	Monthly	0.0746		Either
L	0.5340		Either	0.8495		Either
M	0.5104		Either	0.7785		Either
N	0.0003	−62.93	Monthly	0.0136	−57.26	Monthly
O	0.0454	−45.91	Monthly	0.0679		Either
P	0.2537		Either	0.1445		Either
Q	0.0045	30.52	Semi-annual	0.2866		Either

first week of same month, and this pattern is repeated almost every month. Reason of such patterns is related to "end-of-month" efforts to accomplish sales targets, bringing high volume of sales to last days, and starting every month with low volumes. Gathering data on monthly time buckets "clean" this effect.

4.4. SMA versus SBA forecasting models comparison

Another attempt to reduce the alternatives can be done comparing same policies where the only difference is the forecasting model, as found on K1 × N1, L1 × O1 and M1 × P1. Table 10 summarizes the results of costs difference paired *t*-test for means.

Although K1 provided costs significantly lower than N1, its RFR was repeatedly worse than N1 for every TFR. Comparing L1 and O1 also shows the first with lower costs (although difference was not significant at 95% confidence level) but the second with better RFR. In case of M1 versus P1, no significant cost difference was shown and similar RFR were observed for 80%, 90% and 95% TFR (for TFR=99%,

Table 9
Monthly versus weekly time buckets cost comparison (paired *t*-test for means).

	TFR=80%			TFR=90%		
	p-Value	$\mu 1-\mu 2$	Choice (alfa=5%)	*p-Value*	$\mu 1-\mu 2$	Choice (alfa=5%)
D1 × K1	0.3668		Either	0.6678		Either
E1 × L1	0.0501		Either	0.4788		Either
F1 × M1	0.7549		Either	0.1701		Either
G1 × N1	0.4471		Either	0.4376		Either
H1 × O1	0.0017	434.16	Monthly	0.0001	442.73	Monthly
I1 × P1	0.6040		Either	0.0384	74.76	Monthly
J2 × Q2	0.0520		Either	0.0728		Either
	TFR=95%			TFR=99%		
D1 × K1	0.3458		Either	0.9219		Either
E1 × L1	0.6822		Either	0.0414	49.94	Montlhy
F1 × M1	0.5829		Either	0.7438		Either
G1 × N1	0.3396		Either	0.6332		Either
H1 × O1	0.0000	478.59	Monthly	0.0000	514.23	Montlhy
I1 × P1	0.5952		Either	0.0490	50.25	Montlhy
J2 × Q2	0.1416		Either	0.0568		Either

Table 10
SMA (K1, L1, M1) × SBA (N1, O1, P1) costs comparison (paired *t*-test for means).

	TFR=80%			TFR=90%		
	p-Value	$\mu 1-\mu 2$	Choice (alfa=5%)	*p-Value*	$\mu 1-\mu 2$	Choice (alfa=5%)
K1 × N1	0.0071	−91.44	SMA	0.0087	−44.92	SMA
L1 × O1	0.9190		Either	0.2294		Either
M1 × P1	0.5472		Either	0.9532		Either
	TFR=95%			TFR=99%		
K1 × N1	0.0275	−49.79	SMA	0.0103	−42.78	SMA
L1 × O1	0.2059		Either	0.2883		Either
M1 × P1	0.3064		Either	0.5537		Either

M1 showed better RFR than P1). Anyway, conflicting results from these comparisons does not allow discarding other alternatives.

4.5. Best policies overlapping

Looking at each SKU result, many policies presented similar performance (same RFR and cost in the semester). Analysis of policies overlapping was done to allow further simplification of the alternatives. Considering each SKU in each semester it is possible to identify "best" policy under each TFR. Best policies were set as follows:

- if RFR of a single policy surpassed TFR, this was the chosen one;
- if RFR of more than one policy surpassed TFR, the one with lower costs was chosen;
- if no RFR surpassed TFR, policy with higher RFR was chosen;
- in case of equal results, a policy was randomly chosen.

As a high amount of equal results was found, it was necessary to evaluate how many times a particular policy was the unique best, what we call degree of uniqueness. Discarding a policy with high overlapping results would cause minor losses since it would be replaced by another policy with same performance. Table 11 shows the quantity of SKUs pointing each policy as the unique best one under each TFR while Table 12 shows the degree of uniqueness of each policy. The degree of uniqueness was calculated as the quantity of SKUs from Table 11 divided by the total quantity of SKUs where each policy was chosen as the best one (either unique or not).

It is interesting to notice a high degree of uniqueness of Q2 policy and low results for A1, B1, L1 and M1. Detailed analysis reveals a remarkable overlapping between A1 and B1, as well between L1 and M1. Discard of B1 and L1 was decided due to smaller quantity of unique SKUs.

Table 13 shows the effect on the degree of uniqueness by discarding B1 and L1. There is only one value below 50% degree of uniqueness while previous table showed 10 cases. Improvement on policy discrimination (reduction of overlapping) shall further simplify the analysis.

4.6. Best policies for each SKU category

Weekly demand (average from 6 years of data) was used to classify 10.032 SKUs according to Syntetos et al. (2005) proposal, resulting in data shown in Fig. 7. Most of the SKUs (75.4%) have high inter-arrival interval but low size variability, being classified as Slow moving. The second class is Lumpy, which has both high inter-arrival interval and size variability. Slow and Lumpy Demand sum up more than 88% of all SKUs. On the other hand, only 1.3% are Smooth items with low inter-arrival interval and size variability.

Although Fig. 7 represents average classification during the whole 6 years of demand records, further analysis was done considering just the test period (last 2.5 years) and semester-wisely. As the purpose of this paper is the identification of best policies to be applied to each part, SKUs classification was done over previous semester demand records and policy performance measured over the following semester. Idea is that practitioners should revise classification semi-annually taking last 6 months of demand records to classify SKUs so recommendations can be done over the best policies observed in next semester. Detailed results (average RFR and costs over the 5 semesters of test period) for each policy within each category are shown in the Appendix A.

Figs. 8–11 present the simulation results by plotting RFR × costs for each policy within each category (Smooth, Erratic, Slow and Lumpy), under 80%, 90%, 95% and 99% TFR respectively.

A summary of recommendations is shown in Fig. 12. The recommendations include all different models simulated except adoption of three LTD distributions (Normal, compound Poisson-Normal and compound Poisson-Gamma). Although Normal distribution provided lowest costs among all others (Table 5) it was not recommended for any category since these low costs were paired with low RFR as well. In case of compound Poisson-Normal and Poisson-Gamma distributions associated with usage of individual demand data (SDA), the conversion process used to adapt such models to be TFR-driven may have jeopardized their performance.

From Fig. 12 we can extract following summary:

- PDA provided better results than Single-Demand-Approach (SDA) developed by Krever et al. (2005) for all categories except Slow items under 90% and 95% Target-Fill-Rate (TFR).
- regarding forecasting models, Bootstrapping was chosen for Lumpy items under any TFR, Smooth items under 80% and 90% TFR and Slow items under 99% TFR. Simple-Moving-Average (SMA) was only chosen for Slow items under 80% TFR while Syntetos–Boylan-Approximation (SBA) was the single choice for Erratic items (under any TFR) and also for Smooth items under 95% and 99% TFR.

Table 11
How many times (number of SKUs) a policy was the unique best solution.

Policy	TFR				Average
	80%	90%	95%	99%	
A1	384	782	1037	314	629
B1	387	481	360	229	364
C1	984	892	691	570	784
K1	1299	1796	2052	1962	1777
L1	666	504	481	354	501
M1	546	599	590	755	622
N1	1127	1444	1553	1572	1424
O1	530	509	475	424	485
P1	629	787	829	911	789
Q2	983	698	730	695	776

Table 12
Degree of uniqueness for each policy.

Policy	TFR (%)				Average (%)
	80%	90%	95%	99%	
A1	38.0	69.7	77.3	14.4	49.9
B1	38.6	98.3	57.9	11.0	51.5
C1	61.8	80.6	83.8	91.3	79.4
K1	48.3	64.6	72.6	77.6	65.8
L1	37.8	44.8	56.3	63.3	50.6
M1	30.8	42.8	53.2	72.8	49.9
N1	66.4	80.4	85.0	91.9	80.9
O1	62.9	82.2	88.3	95.7	82.3
P1	47.4	61.0	67.8	74.4	62.6
Q2	95.9	97.0	98.4	99.4	97.7

Table 13
Degree of uniqueness after discarding B1 and L1.

Policy	TFR (%)				Average (%)
	80%	90%	95%	99%	
A1	62.1	71.7	77.1	77.9	72.2
C1	63.6	82.0	84.4	91.7	80.4
K1	49.7	65.2	72.9	77.7	66.4
M1	41.2	53.6	60.0	75.7	57.6
N1	67.3	79.9	84.5	91.8	80.9
O1	69.2	84.8	90.4	97.1	85.4
P1	50.1	62.5	70.4	75.2	64.5
Q2	96.4	97.7	98.4	99.5	98.0

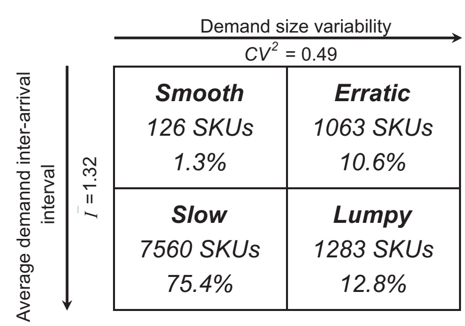

Fig. 7. SKU categorization according to Syntetos et al. (2005).

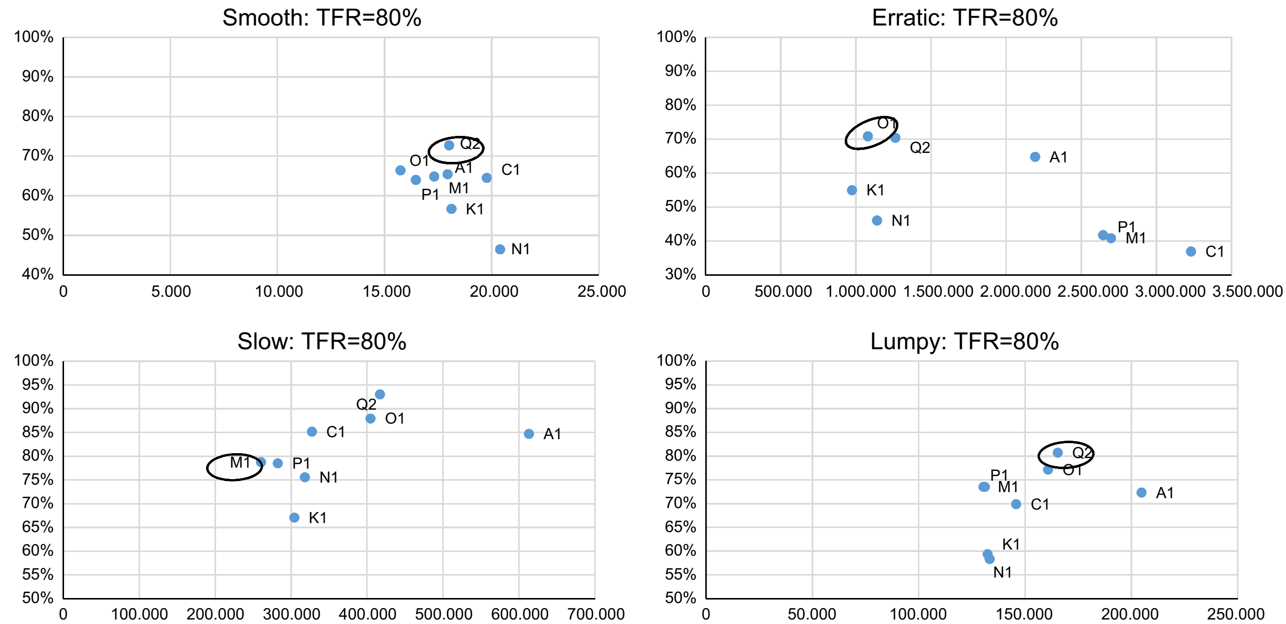

Fig. 8. RFR × cost for each policy within each category (TFR = 80%). Recommended policy is highlighted.

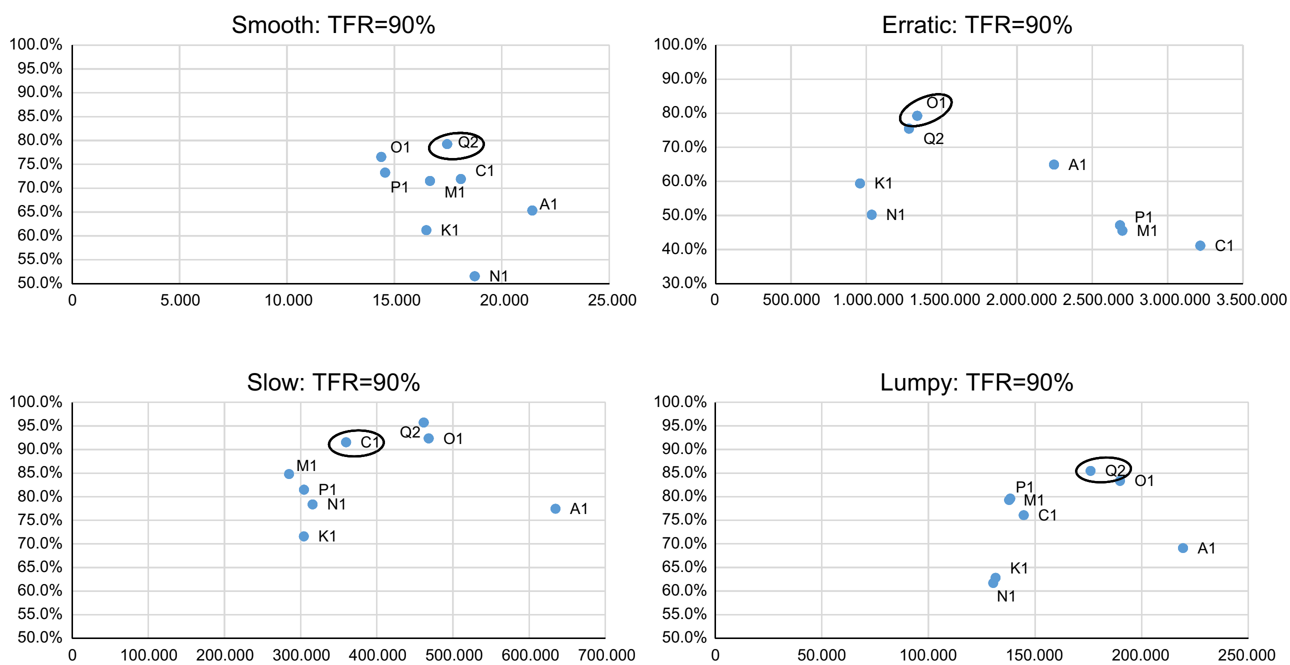

Fig. 9. RFR × cost for each policy within each category (TFR = 90%). Recommended policy is highlighted.

- regarding Lead-Time-Demand (LTD), Gamma distribution was chosen always associated with SBA forecasts. Negative-Binomial-Distribution (NBD) was chosen associated with SMA forecasts for Slow items under 80% TFR and associated to SDA for Slow items under 90% and 95% TFR. On remaining categories, the recommendation is to use Bootstrapping.

4.7. RFR × TFR discussion

As most RFR results did not reach TFR in current study, it is necessary to discuss its reasons and countermeasures. In a perfect world, we shall expect RFR to be randomly distributed around TFR. In real world, several aspects may influence RFR as follows:

(1) **the insight of Teunter and Duncan**: as these authors noticed, all replenishment cycles are triggered by an initial demand occurrence. Bootstrapping and SBA models can be adjusted to correct this effect, however it is not possible on SMA and SDA, so it explains part of the negative bias observed;

(2) **outliers × parametric LTD distributions**: all tested models (except Bootstrapping) consider parametric distributions for LTD. In such cases, distribution parameters are estimates from sales history. In fact, as we kept all outliers in demand history in the current study, all parametric distributions will underestimate upper tail of actual demand distributions. Not

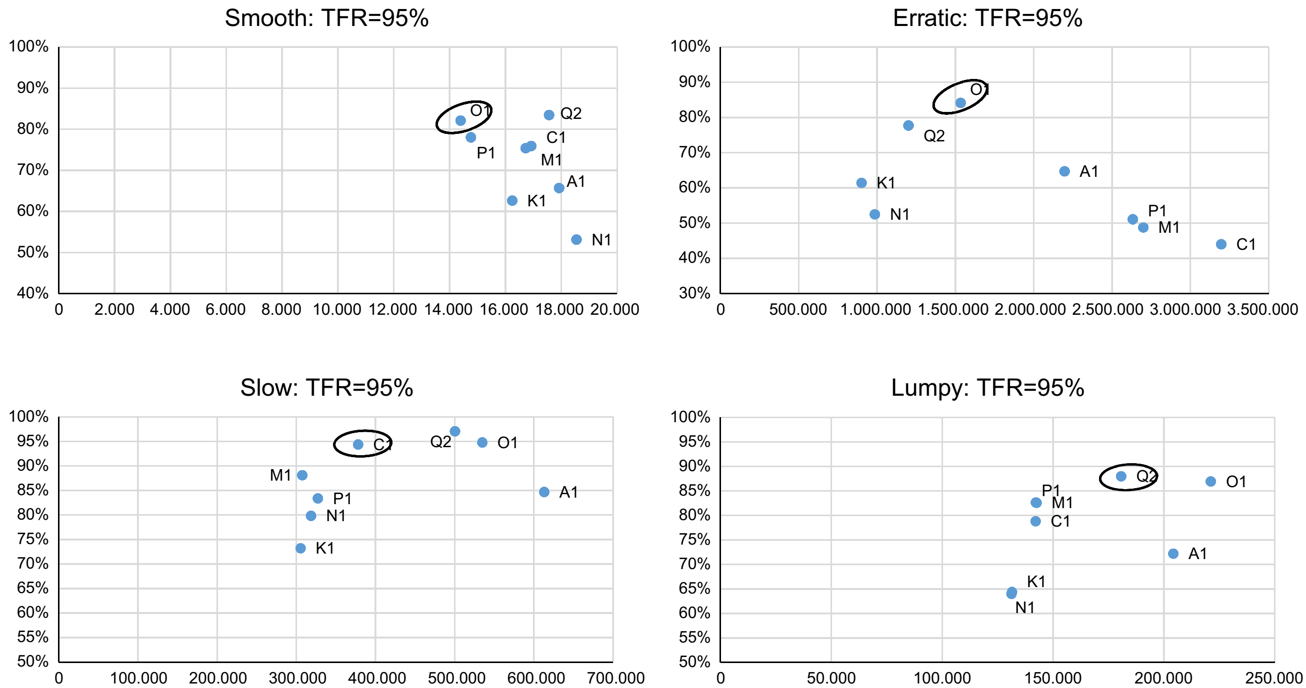

Fig. 10. RFR × cost for each policy within each category (TFR = 95%). Recommended policy is highlighted.

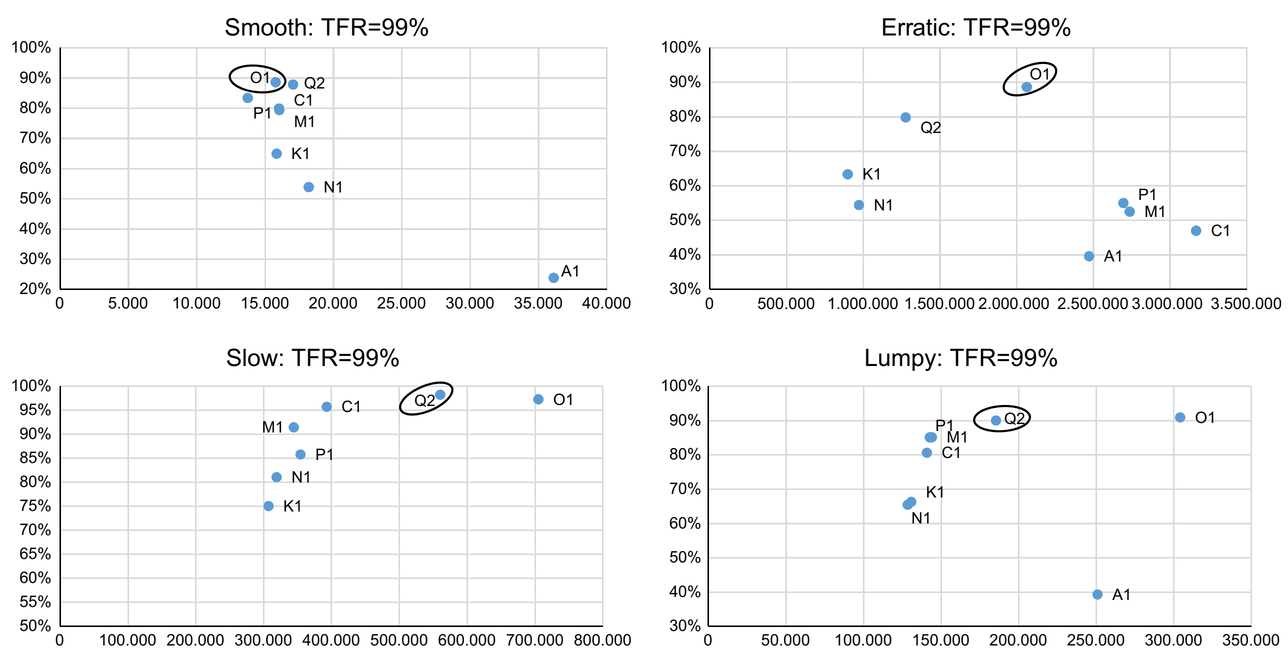

Fig. 11. RFR × cost for each policy within each category (TFR = 99%). Recommended policy is highlighted.

surprisingly, looking only at RFR results (Table 6), Bootstrapping brings the higher results, as it considers actual data distribution to rebuild original one. So, presence of outliers also explains part of negative bias observed;

(3) **data used to obtain parameters × data used to read RFR**: inventory control is a dynamic task so that parameters are obtained (by some past demand history) and then applied over new data coming from the field. Considering that running fleet of the studied automaker is increasing (and spare parts sales as well), most recent demands may present a positive tendency over past data, thus leading to negative bias on RFR.

We believe major reason of RFR poor performance in current study is due to large amount of outliers on demand data so we proposed some strategies to deal with such a problem:

- **previously identify exceptional demands (recall campaigns, promotions, etc.) and pre-order necessary quantities to support them (treat the cause).** Although studied automaker currently do such pre-orders whenever possible, they do not exclude such special demands from database. As historical data in current study did not record such exceptions, we cannot apply such a strategy on our simulation but we do recommend researchers and practitioners to do it;

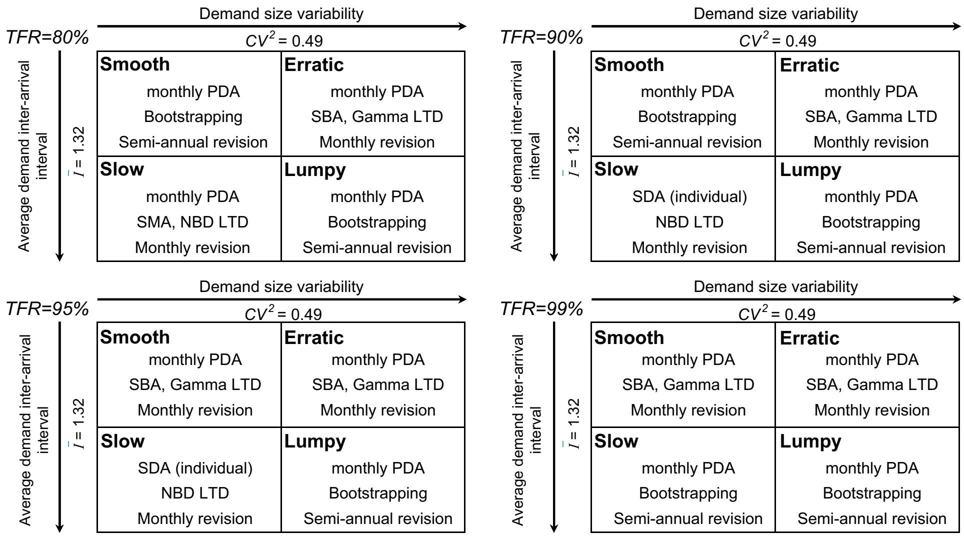

Fig. 12. Recommendations under each TFR.

- **align automaker customers (dealers) policies to avoid end-of-month peaks (treat the cause).** A phenomenon observed in such automaker (and common in many industries) is the strong difference on demand observed from the first week of any month and the last one on same month. Demand presents a peak on last week, driven by sales targets being pursued within automaker-dealer relationship. In order to accomplish their objectives, some dealer managers generate large orders and consequently introduce much more variation on demand read by automaker. For this study, average demand (units) on last week of each month is twice the demand of first week. This can explain why weekly data were discarded when compared to monthly records;
- **increase TFR used to obtain (s, Q) above the actual desired TFR (treat the effect).** This approach would be a target-correction: let us say we desire TFR of 90% but observed average RFR around 80%. In such an approach, we should increase TFR to something above 90% until we obtain average RFR greater than or equal to 90%. Total RFR is an average of RFR from each SKU, so a big quantity of them may be surpassing TFR while a larger quantity may be below. Choosing which SKU needs the target correction is not feasible since past results shall not be the same in the future. Simply applying increased TFR on all SKUs can bring undesired extra costs (so we do not suggest adoption of this strategy).

5. Conclusions

This paper presented a case study on spare parts inventory management, comparing several methods by simulation over field data (10.032 SKU's). Results of the simulation experiments allowed recommendation of best policies to be followed within each SKU category, and for four different values of TFR (80%, 90%, 95% and 99%).

Results pointed out that model parameters should be revised monthly, except for Bootstrapping, which should be revised semi-annually. For Erratic and Lumpy items recommended policies are independent from TFR, while for Smooth and Slow items such recommendations are TFR-sensitive.

Although generalization of results is limited in a case study, practitioners with similar operations shall follow two parallel routines. Every 6 months:

- classify SKU using last 6 months' data (grouped by weekly time buckets);
- adopt policies as recommended in Fig. 12;
- update LTD distribution and calculate (*s*, *Q*) for items driven by Bootstrapping;

and every month:

- update LTD (and forecasts) and calculate (*s*, *Q*) for all items except those driven by Bootstrapping.

It is necessary to emphasize that demand data should be recorded individually to allow SDA application. Same data shall be converted to weekly time buckets just for classification purposes and monthly time buckets to be used within all SKU categories except those driven by SDA. In addition, special care should be taken about strong influence of outliers in the results. Anyone interested in replicating the method proposed in this paper should give special attention to the database cleaning up.

Regarding SDA, promising results presented by Krever et al. (2005) were not confirmed in this paper. Although NBD associated with SDA was the best policy under 90% and 95% TFR, Krever et al. (2005) specific formulations using compound Poisson-Normal and Poisson-Gamma were not recommended in any case. SDA model's adherence to the present case was jeopardized by different objectives: while SDA is CSL-based all other models were TFR-driven and a "conversion" was necessary to allow in-between comparisons. Future research shall be done to develop TFR-driven models under SDA.

More research is also necessary to evaluate the performance of Bootstrapping model used in current study against other alternatives (Willemain et al., 2004; Zhou and Viswanathan, 2011). Although the approach proposed in this paper provided good results, it should be extensively tested and compared to other alternatives before concluding for its superiority.

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Appendix A

See Tables A1 and A2.

Table A1
RFR and costs for each policy within each category, under 80% and 90% TFR. Recommended policies highlighted.

80%	*Smooth*		80%	*Erratic*		90%	*Smooth*		90%	*Erratic*	
	RFR (%)	Cost		RFR (%)	Cost		RFR (%)	Cost		RFR (%)	Cost
A1	65.4	17,942	A1	64.8	2,192,940	A1	65.3	21,417	A1	64.9	2,245,696
C1	64.5	19,771	C1	36.9	3,230,131	C1	71.9	18,095	C1	41.1	3,216,596
K1	56.7	18,120	K1	55.0	974,017	K1	61.2	16,487	K1	59.4	958,019
M1	64.9	17,314	M1	40.8	2,698,075	M1	71.5	16,657	M1	45.5	2,699,276
N1	46.5	20,390	N1	46.0	1,140,426	N1	51.5	18,740	N1	50.2	1,035,529
O1	66.4	15,741	O1	70.9	1,079,672	O1	76.6	14,390	O1	79.3	1,337,906
P1	64.0	16,460	P1	41.7	2,644,999	P1	73.3	14,569	P1	47.1	2,683,816
Q2	72.7	18,012	Q2	70.4	1,262,446	Q2	79.3	17,456	Q2	75.5	1,283,152
	Slow			*Lumpy*			*Slow*			*Lumpy*	
A1	84.7	613,145	A1	72.3	204,766	A1	77.5	634,612	A1	69.1	219,369
C1	85.2	327,423	C1	69.9	145,817	C1	91.6	359,710	C1	76.1	144,676
K1	67.1	304,247	K1	59.4	132,409	K1	71.6	304,231	K1	62.8	131,498
M1	78.7	260,333	M1	73.5	130,445	M1	84.8	284,821	M1	79.3	137,836
N1	75.6	318,097	N1	58.3	133,349	N1	78.4	315,714	N1	61.7	130,388
O1	87.9	404,471	O1	77.2	160,826	O1	92.4	468,122	O1	83.4	189,813
P1	78.5	282,476	P1	73.5	131,081	P1	81.5	304,400	P1	79.6	138,374
Q2	93.0	416,934	Q2	80.7	165,378	Q2	95.7	461,628	Q2	85.5	176,010

Table A2
RFR and costs for each policy within each category, under 95% and 99% TFR. Recommended policies highlighted.

95%	*Smooth*		95%	*Erratic*		99%	*Smooth*		99%	*Erratic*	
	RFR (%)	Cost		RFR (%)	Cost		RFR (%)	Cost		RFR (%)	Cost
A1	65.7	17,921	A1	64.7	2,196,759	A1	23.8	36,110	A1	39.6	2,472,594
C1	75.9	16,924	C1	44.0	3,197,024	C1	79.9	16,013	C1	47.0	3,169,863
K1	62.6	16,242	K1	61.4	900,878	K1	65.0	15,840	K1	63.4	898,235
M1	75.4	16,725	M1	48.8	2,699,553	M1	79.4	16,036	M1	52.5	2,737,117
N1	53.1	18,544	N1	52.5	985,447	N1	53.9	18,189	N1	54.4	972,135
O1	82.1	14,393	O1	84.1	1,532,895	O1	88.7	15,763	O1	88.7	2,065,787
P1	78.0	14,763	P1	51.1	2,632,394	P1	83.4	13,722	P1	55.0	2,695,609
Q2	83.5	17,561	Q2	77.7	1,200,381	Q2	87.9	17,038	Q2	79.9	1,276,098
	Slow			*Lumpy*			*Slow*			*Lumpy*	
A1	84.7	613,074	A1	72.2	204,234	A1	44.6	609,902	A1	39.3	250,876
C1	94.4	378,051	C1	78.8	142,144	C1	95.7	393,018	C1	80.6	140,805
K1	73.2	305,380	K1	64.4	131,478	K1	75.1	307,206	K1	66.3	130,759
M1	88.1	307,549	M1	82.6	142,580	M1	91.5	344,351	M1	85.1	144,022
N1	79.8	318,641	N1	64.0	131,250	N1	81.1	319,071	N1	65.5	128,486
O1	94.8	534,843	O1	86.9	221,146	O1	97.3	704,911	O1	90.9	304,176
P1	83.4	327,268	P1	82.6	142,265	P1	85.8	354,284	P1	85.1	142,702
Q2	97.1	500,388	Q2	88.0	180,707	Q2	98.2	560,261	Q2	90.0	185,384

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