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Newsvendor Problems with Demand Shocks and Unknown Demand Distributions

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ABSTRACT

In today's competitive market, demand volume and even the underlying demand distribution can change quickly for a newsvendor seller. We refer to sudden changes in demand distribution as demand shocks. When a newsvendor seller has limited demand distribution information and also experiences underlying demand shocks, the majority of existing methods for newsvendor problems may not work well since they either require demand distribution information or assume stationary demand distribution. We present a new, robust, and effective machine learning algorithm for newsvendor problems with demand shocks but without any demand distribution information. The algorithm needs only an approximate estimate of the lower and upper bounds of demand range; no other knowledge such as demand mean, variance, or distribution type is necessary. We establish the theoretical bounds that determine this machine learning algorithm's performance in handling demand shocks. Computational experiments show that this algorithm outperforms the traditional approaches in a variety of situations including large and frequent shocks of the demand mean. The method can also be used as a meta-algorithm by incorporating other traditional approaches as experts. Working together, the original algorithm and the extended meta-algorithm can help manufacturers and retailers better adapt their production and inventory control decisions in dynamic environments where demand information is limited and demand shocks are frequent [Submitted: October 31, 2013. Revised: July 16, 2014. Accepted: July 28, 2014.]

Subject Areas: Demand Forecasting, Demand Shocks, Inventory Management, Machine Learning Algorithm, Newsvendor Problem, and and Stochastic Demands.

INTRODUCTION AND MOTIVATION

In a newsvendor problem, a firm faces stochastic demand and has only one opportunity to supply the product. Overage of inventory must be salvaged or discarded, whereas underage costs a firm in lost sales and sometimes in lost customer loyalty.

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Therefore, it is critical for a firm facing the newsvendor problem to choose optimal order quantities. For example, in supermarkets, ever increasing product variety and assortment make it more difficult to match supply with demand. Furthermore, overstocking perishable products substantially increases the product spoilage rate.

The newsvendor problem is applicable to a wide variety of products in industry. It is commonly observed in managing goods with short life cycles and fast-changing demand, such as fashion apparel, seasonal toys, high-tech consumer electronics, and perishable fresh food (Silver, Pyke, & Peterson, 1998). For example, fashion goods are typically short lived (Raman & Fisher, 1996), consumer electronics have a short selling season due to their continuously evolving nature (e.g., cellular phones can have a life cycle as short as six months) (Barnes et al., 2000), and some vaccines such as those for influenza are only useful for a single season (Chick, Mamani, & Simchi-Levi, 2008). As indicated by Yue, Chen, and Wang (2006), the newsvendor problem has served as a building block for numerous problems and models in various streams of research.

The newsvendor problem is easier to solve when the firm can forecast the full demand distribution including type, mean, and variance, in which case it can apply the well-known "critical fractile" solution (Porteus, 2002). However, it is difficult to forecast the precise demand distribution in reality. In many highly competitive markets, practitioners confront fast-changing demand and can face frequent "demand shocks," indicating changes taking place in the mean or shape of the distribution. Demand shocks can have multiple causes, and thus they may or may not be noticed by the seller in advance. For example, a seller's sales promotion may alter his own demand distribution and cause a demand shock. The seller can predict and prepare for such a demand shock in advance. Competitors' promotion events may also alter the seller's demand distribution and thus cause demand shocks. However, competitors' promotional events are not usually known in advance, because sometimes such promotion plans are considered as business secrets. Then a seller may not know such demand shocks in advance. A forecasting method which uses few parameters to describe the demand distribution cannot possibly capture such changes (Murty, 2006). Today, even with the advent of information technology that collects point-of-sales data and analyzes them in real time, accurately forecasting a demand distribution remains a challenging task.

Moreover, the expected profit from the critical fractile solution decreases when demand variance grows, and in practice, demand variance is usually high. Working with retailers, Nahmias and Smith (1994) found that even for basic items in department stores, demand can vary dramatically even after adjusting for known causal factors such as promotions. They observed that in a major retail chain, the coefficient of variation (the variance to mean ratios) for weekly demand can range from 3 to 500 for items that can be replenished frequently. In addition, Graves (1999) has recognized that incorporating more realistic assumptions about product demand is a major theme in the continuing development of inventory theory. He has observed that in most industrial contexts, demands are uncertain and hard to predict, behaving like random walks that evolve over time with frequent changes in their directions and rates of growth or decline.

In this article, we study a repeated newsvendor problem when there exist demand shocks but there is no information about demand mean, variance, distribution type, or when demand shocks might occur. We assume that uncensored demand is revealed after each selling period. We present a new machine learning algorithm, called Weighted Majority Newsvendor Shifting (WMNS), and provide the theoretical foundation of the WMNS algorithm. The algorithm is unique as it only requires approximate estimates of the demand range and quickly adapts to demand shocks.

In addition, the development of our algorithm does not depend on the nature of demand shocks, which may alter the distribution shape or parameters such as mean or variance. In the numerical experiments, we focused on two types of demand shocks. The first is that demand mean suddenly jumps up to a second level, stays there for a while, and suddenly jumps down to a third level. The second one is that demand standard deviation jumps up (or down) and comes back to the base level. Our algorithm performs well in both types of demand shocks.

We develop two variants of the WMNS algorithm. First, we present WMNS–DSE, where DSE represents "diversity of static experts." WMNS–DSE is an expert advice algorithm. However, rather than human experts, it simulates static experts. A static expert predicts the same number every time. To test WMNS–DSE's performance, we conduct simulation experiments that compare WMNS–DSE to several well-known benchmarks, the Critical Fractile solution (Porteus, 2002), Scarf's solution (Scarf, 1958; Alfares & Elmorra, 2005), and approaches presented in Perakis and Roels (2008) and Yue, Wang, and Chen (2007), in scenarios with demand shocks. Key factors used in the experiments include the magnitude and frequency of demand shock, the standard deviation and distribution type of the demand, and the original cost and salvage value of the product. In most of the experimental situations, we find that WMNS–DSE performs well against the benchmarks in handling demand shocks, especially when shocks are strong or frequent. We demonstrate that the performance of WMNS–DSE is consistent across four tested types of distributions: normal, truncated normal, lognormal, and uniform.

Next, we develop another variant, WMNS–META, which utilizes as input the order suggestions provided by the various benchmark solutions themselves, from the classic Scarf's solution to contemporary approaches such as the hybrid distribution-free (q^{hyb}) solution of Yue et al. (2007). We find that WMNS–DSE performs better than WMNS–META when demand shocks are frequent, salvage value is low, or standard deviation is high. On the other hand, WMNS–META performs well in the face of extreme critical fractile values (e.g., when cost is low or salvage value is high), which are those situations where WMNS–DSE does not perform well. Thus, working in a complementary manner, these two machine learning variants can help manufacturers and retailers manage their production and inventory control decisions in dynamic environments where demand information is limited and shocks are frequent.

LITERATURE REVIEW

We are interested in the newsvendor problem with demand shocks in the absence of information on the mean, variance, or distribution of the demand. This problem is practically significant and has attracted extensive attention in the literature, which we review in first subsection. The algorithm we propose is based on machine learning. So we summarize the relevant known machine learning algorithms in second subsection.

Newsvendor Problem with Incomplete Demand Information

In the newsvendor and inventory management literature, there are two branches of research. The first branch assumes that although demand is unknown, demand distribution is known to a seller. The second branch considers more realistic situations where both demand and demand distribution are unknown to a seller. Our work belongs to the second branch and our literature review focuses on this branch.

Newsvendor problems without full demand distribution information have inspired multiple research papers. Most of these papers focus on stationary demand distribution, that is, without demand shocks. As far as we know, we are the first to develop an efficient and effective algorithm for newsvendor problems with unknown nonstationary demand distribution, that is, with demand shocks.

In the newsvendor literature without full demand distribution information, there are three research streams. The first adopts parametric approaches, the most popular of which is Bayesian update. These approaches require prior knowledge of a parametric family of distributions that the true distribution belongs to. However, Nahmias (1994) shows that it is hard to parsimoniously update the prior distributions. Liyanage and Shanthikumar (2005) propose a new parametic approach called operational statistics, which integrates parameter estimation and profit optimization. Such an approach does not need to know either distribution parameters or prior distribution, but it still requires knowing the distribution family that demand belongs to.

Another stream of papers adopts nonparametric data-driven approaches. A nonparametric approach does not require knowing the demand distribution or the distribution family to which the demand belongs. It may use an adaptive algorithm or make use of the gradient information in each iteration by recognizing the convexity of the cost function or concavity of the profit (value) function. For example, Levi, Roundy, and Shmoys (2007), Burnetas and Smith (2000), Godfrey and Powell (2001), Huh, Levi, Rusmevichientong, and Orlin (2011), and Huh and Rusmevichientong (2009) belong to this stream. As far as we know, however, none of these approaches consider nonstationary demand, that is, demand shocks.

The third stream of papers adopts the robust or the min-max optimization approach. Our article belongs to this stream and we are the first to consider demand shocks. This min-max optimization approach requires even less general assumptions than previously mentioned approaches. For example, existing nonparametric data-driven approaches at least require demands in different time periods to be independent. Below we survey this research stream in detail.

In his seminal work, Scarf (1958) adopts the maximin approach to derive the order quantity that maximizes the worst case profits given only the knowledge about the mean and variance of the demand. Gallego and Moon (1993) and Gallego (1998) use the maximin approach to solve the multiple period newsvendor problem with limited demand information. Burnetas and Smith (2000) apply a stochastic approximation method to the newsvendor problem where demand distribution is unknown and demand observations are censored due to lost sales. They employ an adaptive ordering scheme that only depends on the history of orders and sales and the cost/profit data in the past, but not on the unknown demand distribution. Bertsimas and Thiele (2005) develop a data-driven approach that only needs the ranked previous demands and incorporates risk preference by a scale parameter.

Maximin approaches are risk averse, in some situations leading to order quantities much lower than optimal. A less conservative approach is to optimize according to the minimax regret, which aims to minimize the maximum loss from not being able to make optimal decisions because of limited demand information. Chamberlain (2000) applies a minimax regret approach to the newsvendor problem when only the distribution type is known, and not the mean or variance. Yue et al. (2006) recognize the importance of measuring the robustness of the order quantity decision in the newsvendor problem facing limited information. They develop a procedure to compute the maximum expected value over a class of probability distribution functions with the same mean and variance. The procedure obtains an order quantity solution that corresponds to the minimax regret decision. The same authors also develop several procedures to find the optimal order quantity when only the mean and range of the random demand are available. Perakis and Roels (2008) and Levi, Perakis, and Uichanco (2013) analyze the newsvendor problem with a minimax regret approach when only partial demand information (e.g., mean, variance, symmetry, and unimodality) is known.

Vairaktarakis (2000) describes solutions for several performance criteria including minimax regret in the setting of multiple item types per period and a budget constraint, where the only restriction is a lower and upper bound on demand. Like others mentioned, this solution is risk averse because of the strict minimax criterion and the loose restrictions placed on the demand. In this article, we also adopt a variation on the minimax regret strategy and also only require an estimate of the demand range. However, unlike the solutions presented in Vairaktarakis (2000) (when applied repeatedly to the single-period problem), we do not attempt to minimize the maximum regret in each period. We, instead, attempt to minimize the maximum regret from the profit-maximizing single repeated order quantity over a subsequence of periods. Thus, even under such loose restrictions on the demand, our approach does not suffer from being particularly risk averse.

Machine Learning and Expert Advice Problems

Learning from expert advice (Blum, 1996; Kalai & Vempala, 2005) is an important topic in machine learning. At the beginning of each period j, $j \in 1, \ldots, t$, an expert advice algorithm seeks to predict the value of an unknown variable $label_j$ that is revealed at the end of period j. To do so, the algorithm may make use of the predictions of a number of "experts," where expert i predicts $pred_{i,j}$ in period j. The value of $pred_{i,j}$ is considered as expert i's prediction advice in period j. An expert's advice can be a simple static value repeated in every period, a random value, or an output value from some prediction algorithm. An expert is considered "weak," if the expert advice is either a static constant or random variable. However, weak experts often can be used effectively, if they represent a broad ensemble of specialized predictors (Tuv, 2006). An expert's advice in our algorithm

WMNS–DSE is a simple static value repeated over t periods, while an expert advice in our variant algorithm WMNS–META represents a suggested order quantity from an existing newsvendor algorithm in the literature.

At the beginning of each period j, the algorithm gets inputs of expert advice $pred_{i,j}$ from all experts i. Based on these experts' advice, the algorithm determines a coalesced prediction $prediction_j$. At the end of each period j, the value of $label_j$ is revealed and the algorithm suffers some cost as a function of $prediction_j$ and $label_j$. The error incurred by expert i in period j can also be computed as the same function applied to $pred_{i,j}$ and $label_j$. An expert is called the *regret-minimizing expert in hindsight*, if the expert has the smallest sum error over t periods. The objective of the algorithm is to minimize the algorithm's sum error, usually to an amount comparable to the sum error incurred by the regret-minimizing expert in hindsight.

Our article is closely related to two algorithms presented in Littlestone and Warmuth (1994), algorithm WMC (for continuous expert advice) and algorithm WML (for the shifting target scenario on binary predictions). Algorithm WMC provides $prediction_j \in [0, 1]$ in every period j as a weighted average of the N experts' predictions $pred_{i,j}$, where $pred_{i,j} \in [0, 1]$. The true value, $label_j \in [0, 1]$, is revealed after each prediction and the error of each expert in period j is computed. Finally, the weight of each expert is updated by multiplying it by a factor related to his error. Let $\sum_j |prediction_j - label_j|$ denote the sum of absolute errors over all the periods generated by Algorithm WMC. According to corollary 5.2 in Littlestone and Warmuth (1994), Algorithm WMC achieves the following bound:

$$\sum_j |prediction_j - label_j| \le \frac{ln(n) + ln\frac{1}{\beta} * \sum_j |pred_{i,j} - label_j|}{1-\beta}, \forall i,$$

where $\beta \in (0, 1)$ is a parameter used in the weight update in algorithm WMC, n is the number of experts, and $\sum_j |pred_{i,j} - label_j|$ is the sum of errors made by expert i over all the periods. Since the above holds for any expert i, it also holds for the regret-minimizing expert. That is, Algorithm WMC's performance is comparable to the performance of the expert who minimizes the sum of error over all periods.

Weighted Majority and variants thereof have been applied to a wide variety of problems including online portfolio selection (Cover & Ordentlich, 1996a, b), robust option pricing (Demarzo, Kremer, & Mansour, 2006), predicting user actions on the world wide web (Armstrong, Freitag, Joachims, & Mitchell, 1995), and tracking concept drift in online learning (Kolter & Maloof, 2003).

An important variant of predicting based on expert advice considers a "shifting target." Littlestone and Warmuth (1994) also describe an algorithm called WML for this variant. Under shifting target, for different subsequences of labels different experts perform well. For example, one expert is highly accurate in periods 1 through s and another expert is highly accurate during periods $s + 1$ to j. In this case, the algorithm needs to be able to adjust its predictions quickly based on rapid changes in expert performance that are unknown *a priori*. By modifying the weight update method for experts, WML shifts its own predictions rapidly, either when experts change behaviors, or when the sequence of labels changes so that a different expert should be preferred. Littlestone and Warmuth (1994) show that algorithm WML has a total cost bounded in a similar fashion to algorithm WMC, namely, by the total cost that would be suffered by the most accurate expert for any subsequence regardless of labels or experts' prior accuracies.

However, Algorithm WML assumes that labels and predictions are discrete in the set $\{0,1\}$. That is, all predictions and labels in algorithm WML are binary, that is, are either 0 or 1 and the cost is simply a count of incorrect predictions. Littlestone and Warmuth (1994) mentioned the possibility of continuous solutions in a shifting target scenario, but they did not describe any.

In this article, we extend the WML algorithm for the continuous case for any convex regret function. In fact, our algorithm combines the features from WMC and WML to consider both continuous predictions and demand shocks. This algorithm achieves a bound (as demonstrated in Theorem 1) that has a similar format as the aforementioned bound for algorithm WMC.

Our algorithm also requires that the operative regret function produces continuous values in [0,1]. To adapt the newsvendor problem to this scenario, we divide newsvendor regrets by a value $\mathbf{C}$, which represents the maximum possible newsvendor regret. This requires an estimate of the range of possible demand values $[m, M]$. However, the algorithm continues to operate if demands are seen outside this range (resulting in regrets larger than $\mathbf{C}$). Our experiments show that the method continues to perform well.

On the other hand, if all demands are in the range $[m, M]$, then all of our theoretical results hold even if the demands are not "random" in nature but are drawn to maximize the algorithm's total regret. This is in contrast to other adaptive approaches, such as the stochastic approximation procedure of Burnetas and Smith (2000), which converges to optimal order quantities assuming demands are randomly draw from a continuous distribution. In practice, demands may represent random samples, but the distribution may change, representing a demand shock. Such scenarios are also not considered by existing literature.

ALGORITHM DESCRIPTION AND THEORETICAL BOUNDS

Table 1 summarizes the major notation used in this article. In studying the newsvendor problem, we suppose there are t time periods. At the beginning of each period, the firm needs to decide an order quantity to stock. At the end of each period, the demand realizes and unsold products have to be salvaged. The purchase cost, selling price, and salvage price per item are c, r, and s, respectively. Following the newsvendor literature, we also assume an additional cost c_u per unit of demand not met in case of stockout. Our theoretical results assume that all demands are drawn from the *a priori* known range $[m, M]$.

We define several useful functions. $\pi(q, d)$ is the profit obtained by ordering q with realized demand d. $R(q, q^*, d)$ represents the regret suffered by ordering q as opposed to q^* with a realized demand d, and $R_a(q, d)$ is the "absolute" regret suffered by ordering q as opposed to the actual realized demand d itself. We also define a constant $\mathbf{C}$, which is the maximum possible absolute regret in a single period under the assumption that all demands and orders are in the range $[m, M]$.

Table 1: Major notation.

c	Per-item cost
r	Per-item revenue
s	Per-item salvage value
c_u	Per-item understock cost
m	Assumed demand range minimum
M	Assumed demand range maximum
$\pi(q, d)$	Profit obtained by ordering q with realized demand d
$R(q, q^*, d)$	Regret suffered by ordering q as opposed to q^* with realized demand d
$R_a(q, d)$	Regret of ordering q as opposed to an amount equal to the realized demand d
$\mathbf{C}$	Maximum possible regret sufferable in a single demand period under the assumption that order q and realized demand d are both in $[m, M]$
K	Number of experts giving order recommendations to algorithm WMNS
$p_i^{(j)}$	Recommended order quantity of expert i in demand period j
$d^{(j)}$	Realized demand in demand period j
$\gamma^{(j)}$	Order quantity used by WMNS after aggregating expert predictions $p_i^{(j)}, i \in \{1, \ldots, K\}$
$w_i^{(j)}$	Weight assigned to expert i by WMNS in demand period j, in the range (0, 1)
β	Weight update parameter used by WMNS, in the range (0, 1)
δ	Weight limiting parameter used by WMNS, in the range (0, 1)
$p_i^{(stat)}$	Order recommendation of static expert i; invariant over demand periods, used by WMNS variant WMNS–DSE
$q^{(j)}$	Order quantity used by hypothetical benchmark SSTOPT in demand period j

Note that $q \in [m, M]$, $(r - c + c_u)$ is the underage cost, and $(c - s)$ is the overage cost. Thus, if $q = M$ and $d = m$, then the maximum possible overage total cost is $(M - m)(c - s)$. If $q = m$ and $d = M$, then the maximum possible underage total cost is $(M - m)(r - c + c_u)$. The cost **C**equals the maximum value of the above two costs. That is,

$$\pi(q, d) = r \cdot \min\{d, q\} - c \cdot q - c_u \cdot \max\{0, d - q\} + s \cdot \max\{0, q - d\},$$

$$R\left(q, q^*, d\right) = \pi\left(q^*, d\right) - \pi(q, d),$$

$$R_a(q, d) = R(q, d, d),$$

$$\mathbf{C} = \max_{d,q \in [m,M]} R_a(q, d) = \max\{(M - m)(r - c + c_u), (M - m)(c - s)\}.$$

Suppose we had the benefit of expert advice in solving the newsvendor problem: at the beginning of each period j each expert i of K experts gives a recommendation of order quantity $p_i^{(j)} \in [m, M]$ and we use those to choose an order quantity $\gamma^{(j)} \in [m, M]$. When demand $d^{(j)}$ is realized, the regret suffered with respect to each expert i's prediction is $R(\gamma^{(j)}, p_i^{(j)}, d^{(j)})$ and the absolute regret suffered is $R_a(\gamma^{(j)}, d^{(j)})$.

The main algorithm, called WMNS, is given below. In the first subsection, we show how WMNS's regret can be bounded in terms of the regret of the regret-minimizing expert in hindsight for a subsequence. Second, we show in the next subsection how to adapt WMNS when expert advice is not available (as is frequently the case), by simulating a diverse pool of weak experts. For this adaptation, in the last subsection we bound the regret in terms of the regret of an optimal hypothetical algorithm which knows the entire demand sequence but is not allowed to change its order quantity more than k times, called SSTOPT (note that k determines the power of SSTOPT) (see SSTOPT in the last subsection).

WMNS, essentially, maintains weights on the experts based on past performance. It works as follows: Every period, only experts with sufficiently large weights (those whose normalized weights exceed a threshold δ) are considered. These experts' predictions, averaged according their normalized weights, yield the order quantity γ_j for period j. If an expert's prediction has been used in a given period, then the expert's weight is adjusted in the next period to account for the expert's accuracy. When doing such updating, the parameter β reflects how sensitive the updated weight is to the expert's accuracy. Experts whose predictions have not been utilized in a given period retain the same weight in the next period. To initialize, all experts' weights start at 1.

Algorithm 1. WMNS

Step 1. **Initialize experts, weights, constants, and parameters.** Initialize the maximum single period regret $\mathbf{C} = \max\{(M - m)(r - c + c_u), (M - m)(c - s)\}$. For each expert $i \in \{1, \ldots, K\}$, set i's initial weight w_i to 1.0. Choose a weight reduction parameter β and weight limiting parameter δ from the range (0,1).

Repeat steps 2 and 3 for each newsvendor period $j \in \{1, \ldots, t\}$:

Step 2. **Determine order quantity.** Let expert i's prescribed order quantity in period j be $p_i^{(j)}$ and let $w_i^{(j)}$ represent expert i's weight at the start of period j. Let $\mathcal{U}^{(j)}$ be the subset of experts whose weight is larger than δ times the average weight: $\mathcal{U}^{(j)} = \{i \mid w_i^{(j)} > \delta(\sum_{i=1}^{K} w_i^{(j)}/K)\}$. Order the quantity $\gamma^{(j)} = (\sum_{i\in\mathcal{U}^{(j)}} p_i^{(j)} w_i^{(j)})/(\sum_{i\in\mathcal{U}^{(j)}} w_i^{(j)})$, which is the weighted average prediction of those experts in $\mathcal{U}^{(j)}$.

Step 3. **Discover realized demand and update weights.** For each expert $i \in \mathcal{U}^{(j)}$, let $w_i^{(j+1)} = w_i^{(j)}[1 - (1 - \beta)R_a(p_i^{(j)}, d^{(j)})/\mathbf{C}]$. For each expert $i \notin \mathcal{U}^{(j)}$, $w_i^{(j+1)} = w_i^{(j)}$.

Regarding step 3 in algorithm WMNS, we have followed the framework of weight update in algorithm WMC introduced in Littlestone and Warmuth (1994). One benefit of such a weight update is to connect the weight change in each period with individual expert's regret in that period, which is represented by the term $R_a(p_i^{(j)}, d^{(j)})/C$. Thus, the regret of the algorithm can be bounded in terms of the total changes in weight over the subsequence. By further bounding the total change in weight, that is, the final total weight minus the initial total weight, in terms of any given expert's regret, we can bound the overall regret of the algorithm in terms of any given expert's regret.

In Step 3, if all $p_i^{(j)}, d^{(j)} \in [m, M]$, then $R_a(p_i^{(j)}, d^{(j)})/\mathbf{C} \in [0, 1]$. In actual practice, if $p_i^{(j)}$ or $d^{(j)} \notin [m, M]$, then $R_a(p_i^{(j)}, d^{(j)})/\mathbf{C}$ may be greater than 1. In this case, the practitioner would update w_i according to $w_i^{(j+1)} = w_i^{(j)}[1 - (1 - \beta)]$, and the theoretical results of subsections in this section do not hold.

Shifting Target/Demand Shocks

WMNS employs a weight limiting parameter δ similar to that used in the "shifting target" version of the expert advice algorithm developed by Littlestone and Warmuth (1994). Based on this parameter, WMNS ensures that no expert's weight becomes arbitrarily small compared to others', and that experts who give poor predictions are, temporarily, ignored in the computation of the order quantity $\gamma^{(j)}$. The advantage of this mechanism over nonshifting target versions of the weighted majority approach (or more generalized online gradient descent approaches such as those developed by Zinkevich, 2003) is that it allows for bounds not just with respect to the performance of the regret-minimizing expert over all t periods, but rather for *any* consecutive subsequence of periods. Theorem 1 formalizes this notion for WMNS:

Theorem 1: Let $\gamma^{(j)}$ be the quantity ordered by WMNS in period j given K expert order predictions $p_i^{(j)} \in [m, M]$, $i \in \{1, \ldots, K\}$, with subsequent realized demand $d^{(j)} \in [m, M]$. For any subsequence of newsvendor periods indexed from init to fin,

$$\sum_{j=init}^{fin} R_a\left(\gamma^{(j)}, d^{(j)}\right) \leq \frac{\mathbf{C} \ln\left(\frac{K}{\beta\delta}\right)}{(1-\beta)(1-\delta)} + \frac{\ln\left(\frac{1}{\beta}\right) \min_i \sum_{j=init}^{fin} R_a\left(p_i^{(j)}, d^{(j)}\right)}{(1-\beta)(1-\delta)}. \tag{1}$$

The bound provided in Theorem 1 is consistent with the mistake bound of a weighted majority algorithm as a predicting algorithm (e.g., Littlestone & Warmuth, 1994). Theorem 1 shows that the total absolute regret of WMNS for any subsequence of periods can be bounded in terms of the absolute regret of the regret-minimizing expert during that subsequence. The proof can be found in Supporting Information Appendix A.

The bound in Theorem 1 is useful for the following reasons. In the context of newsvendor problems with demand shocks, different experts may perform well in different time periods. For example, expert 1 may be the best expert with minimum regret before a demand shock and expert 2 can become the best expert after a demand shock in the next time period. Without prior information about demand distributions or shocks, it is difficult for a company to forecast which of the K experts would perform best in the next time period. According to Theorem 1, WMNS helps a company achieve the performance comparable to the regret-minimizing expert in each period, even if the regret-minimizing expert may change rapidly and be different during each period.

The intuition of this result lies in the use of the weight-limiting parameter δ. Suppose that before a demand shock, expert i makes the regret-minimizing predictions, whereas after the shock expert i' makes the regret-minimizing predictions. Just prior to the demand shock, w_i will be large relative to $w_{i'}$, and after the shock w_i will be adjusted downward until $w_{i'}$ is relatively larger. Unless $\delta = 0$, the mechanism of WMNS ensures that w_i is never "arbitrarily" large in comparison to $w_{i'}$, so this adjustment can be performed quickly.

Here, we also note that the proof of Theorem 1 makes no assumption about realized demands or expert predictions other than that they are all in the positive range $[m, M]$. Indeed, even if demands and expert predictions were chosen by some malicious "adversary" specifically to maximize WMNS's absolute regret, that regret would still conform to Theorem 1.

It is possible, in theory, to numerically optimize the parameters β and δ in Theorem 1. However, this would require an estimate of the regret suffered by the regret-minimizing expert, which would in turn require an estimate of details about the upcoming demand sequence or distribution. Because this is the very problem the WMNS mechanism seeks to avoid, we do not consider numerical optimization of β and δ in this article. Practitioners, however, may choose to optimize these values based on past distribution information, if they believe that such information will be valid for future applications.

DSE

As an expert advice algorithm, WMNS is able to utilize predictions from a panel of actual human experts (or traditional, possibly dynamic order policies, see the second last subsection in next section), and we have shown that the regret suffered can be bounded in terms of the regret of the regret-minimizing expert on any subsequence. When such a panel of experts may not be available, we give an alternate approach wherein we simulate "static" experts. A static expert predicts the same thing every period. For example, a simulated expert A might always predict a demand of 10 units, expert B might always predict a demand of 20 units, and so on. Clearly, these simulated experts would be less powerful in their predictive abilities than human experts. However, as has been noted in the machine learning literature, intelligently aggregating a pool of "weak" predictors can produce good results if the pools represent sufficient diversity such that at least one will perform well in any given situation (Tuv, 2006). We call this modified approach WMNS–DSE, for WMNS–Diversity of Static Experts.

We ensure diversity in the pool by evenly dividing the possible demand space $[m, M]$ into K disjoint ranges $[e_0, e_1]$, $(e_1, e_2]$, $(e_{K-1}, e_K]$. A single expert predicts a static value from each range, which we denote by p_i^{stat} for $i \in 1, \ldots, K$.

Recall that r, c, s, and c_u are the per-item value, cost, salvage value, and understock cost such as loss of goodwill. The per-item shortage cost is thus $(c + r - s)$ and the per-item overstock cost is $(c - s)$. According to Vairaktarakis (2000), if the demand in period j, $d_{(j)}$ occurs in the range $(e_{i-1}, e_i]$, then then order that minimizes the maximum potential loss from ordering the demand itself is

$$p_i^{stat} = e_i \frac{r - c + c_u}{r - s + c_u} + e_{i-1} \frac{c - s}{r - s + c_u}.$$

After $d^{(j)}$ is realized, expert i realizes either shortage or overstock cost, which are maximized when $d^{(j)}$ is either at e_i or e_{i-1}. If the demand occurs at the high endpoint e_i, the expert faces maximum shortage cost as follows:

$$\begin{aligned} R_a(q_i^{stat}, e_i) &= (r - c + c_u)\left(e_i - \left(e_i \frac{r - c + c_u}{r - s + c_u} + e_{i-1}\frac{c - s}{r - s + c_u}\right)\right) \\ &= (e_i - e_{i-1})\frac{(r - c + c_u)(c - s)}{r - s + c_u} \ . \end{aligned}$$

If the demand occurs at the other low end e_{i-1}, the expert faces maximum overstock cost as follows:

$$\begin{aligned} R_a(q_i^{stat}, e_{i-1}) &= (c - s)\left(e_i \frac{r - c + c_u}{r - s + c_u} + e_{i-1}\frac{c - s}{r - s + c_u} - e_{i-1}\right) \\ &= (e_i - e_{i-1})\frac{(r - c + c_u)(c - s)}{r - s + c_u} \ . \end{aligned}$$

In both cases, the cost simplifies to the maximum regret (as a minimax regret order should):

$$\max_{d^{(j)} \in [e_i, e_{i-1}]} R_a(q_i^{stat}, d^{(j)}) = (e_i - e_{i-1})\frac{(r - c + c_u)(c - s)}{r - s + c_u} \ .$$

To determine endpoints e_i and e_{i-1} for each range i such that the minimax regret value is equal for all, we set equal the minimax regret for two neighboring ranges and solve for bordering endpoint. It can easily be shown that this results in choosing endpoints

$$e_i = \frac{i(M - m)}{K} + m, \ i \in \{0, \ldots, K\}.$$

Correspondingly, the minimax regret order decision for range i, p_i^{stat}, can be solved for as follows:

$$\begin{aligned} p_i^{stat} &= e_i \frac{r - c + c_u}{r - s + c_u} + e_{i-1}\frac{c - s}{r - s + c_u} \\ &= \frac{i(M - m)}{K} - \frac{(M - m)(c - s)}{K(r - s + c_u)} + m. \end{aligned}$$

Similarly, we can solve for the minimax regret given these range endpoints assuming demand falls in this range:

$$\begin{aligned} \max_{d^{(j)} \in [e_i, e_{i-1}]} R_a(q_i^{stat}, d^{(j)}) &= (e_i - e_{i-1})\frac{(r - c + c_u)(c - s)}{r - s + c_u} \\ &= \frac{(M - m)(r - c + c_u)(c - s)}{K(r - s + c_u)}. \end{aligned}$$

Note that given any demand in $d^{(j)} \in [m, M]$, there will be a static expert with minimum regret, and that will correspond to expert i with range $d^{(j)} \in (e_i, e_{i-1}]$. Thus, the minimax regret suffered by the regret-minimizing expert in a single period (or any sequence of periods) can be reduced by increasing K.

WMNS–DSE **Regret Bounds**

While Theorem 1 characterizes an upper bound for the regret of WMNS in terms of the best expert for any given subsequence of demands, it is not specific for WMNS–DSE where demand predictions are fully specified. To fully characterize the regret of WMNS–DSE, we need a theoretical method that we can compare against and that performs optimally in demand-shock scenarios. We call this theoretical method SSTOPT, for "Semi-Static Optimal." This restricted clairvoyant approach is given full and perfect information about the demand sequence (to the level of individual demands) before the first demand is seen, and chooses order quantities to minimize the sum of absolute regret under the restriction that it can only change order quantities $k-1$ times.

Definition 1: For demand sequence $d^{(1)}, \ldots, d^{(t)}$, algorithm SSTOPT chooses orders $q^{(1)}, \ldots, q^{(t)}$ and k periods $l_1 = 1, l_2, \ldots l_k \in 2, \ldots, t$. It may choose a new order quantity if and only if it is in period $l_2, \ldots l_k$, that is, $q^{(j)} = q^{(j-1)}$ unless $j \in l_2, \ldots, l_k$. (The first-order quantity, $q^{(1)}$, may be any value.) Subject to these constraints, order quantities and order change periods are chosen to minimize total absolute regret: $\sum_{j=1}^{t} R_a(q^{(j)}, d^{(j)})$.

Note that the same choices also maximize the total profit, $\sum_{j=1}^{t} \pi(q^{(j)}, d^{(j)})$. For the purposes of proofs in this article, it is not necessary to know how SSTOPT might reach its decisions, though such a study would be interesting in a theoretical sense. An extension to the work of Bertsimas and Thiele (2005) shows that when $k = 1$, SSTOPT orders the $\lceil (r-c+c_u)/(r-s+c_u) \cdot t \rceil$th order statistic of the demand sequence for all periods. In a sense, the parameter k represents the power of SSTOPT: if $k = 1$, SSTOPT must use a single order for all demands, $k = 2$ allows SSTOPT to change its order one time, and so on.

Although SSTOPT is not a distributional approach, if demands are drawn from k fixed distributions in sequence (due to demand shocks), it will slightly outperform a "distributionally perfect" approach that knows merely the demand "distribution" at any given time and orders the critical fractile, which we call PERFECT.

Definition 2: For demand sequence of length t drawn from distinct distributions: $d^{(1)} \sim \Phi^{(1)}, \ldots, d^{(t)} \sim \Phi^{(t)}$, algorithm PERFECT chooses orders $q^{(1)}, \ldots, q^{(t)}$ according to:

$$q^{(j)} = (\Phi^{(j)})^{-1} \left(\frac{r-c+c_u}{r-s+c_u} \right),$$

which is the critical fractile solution for period j (Porteus, 2002).

PERFECT uses the correct critical fractile solution for each subsequence where a single distribution is used, thus approximating SSTOPT. However, SSTOPT, by definition, outperforms such an *a posteriori* critical fractile solution. These "best in hindsight" comparisons, which are common among machine learning approaches (e.g., Littlestone & Warmuth, 1994; Zinkevich, 2003), are natural fits for inventory management problems: by bounding regret with respect to SSTOPT, we simultaneously bound regret with respect to PERFECT in demand-shock scenarios.

Lemma 1 summarizes the results of the previous subsection and it can be applied to any subsequence of demands. Further details can be found in Supporting Information Appendix B.

Lemma 1: For a subsequence of newsvendor periods indexed from init to fin and any static order quantity $q^{(stat)}$, there exists a simulated expert i whose regret from $q^{(stat)}$ is no more than $(fin - init + 1)$ times the single range maximum regret:

$$\forall q^{(stat)} \in [m, M], \exists i \sum_{j=init}^{fin} R\left(p_i^{(stat)}, q^{(stat)}, d^{(j)}\right)$$
$$\leq \frac{M-m}{K} \frac{(r-c+c_u)(c-s)}{r-s+c_u}(fin - init + 1)\,. \tag{2}$$

Finally, by combining Theorem 1 and Lemma 1, we can bound the regret suffered by WMNS–DSE in terms of the parameter choices for WMNS–DSE and the regret of SSTOPT. Details can be found in Supporting Information Appendix C.

Theorem 2: For a t-period newsvendor problem with per item cost c, revenue r, understock cost c_u, and salvage value s, if $d^{(j)} \in [m, M]$, $\forall j$, then the total absolute regret suffered by WMNS–DSE satisfies

$$\sum_{j=1}^{t} R_a\left(\gamma^{(j)}, d^{(j)}\right) \leq \frac{k\mathbf{C}\ln\left(\frac{K}{\beta\delta}\right)}{(1-\beta)(1-\delta)} + \frac{\ln\left(\frac{1}{\beta}\right)(M-m)(c-s)(r-c+c_u)t}{K(r-s+c_u)(1-\beta)(1-\delta)}$$
$$+\frac{\ln\left(\frac{1}{\beta}\right)\sum_{j=1}^{t} R_a\left(q^{(j)}, d^{(j)}\right)}{(1-\beta)(1-\delta)},$$

where $\mathbf{C} = \max\{(M-m)(r-c+c_u), (M-m)(c-s)\}$, and $K \in \mathbb{N}$, $\beta \in (0, 1)$, and $\delta \in (0, 1)$ are parameters of WMNS. Values k and $q^{(j)}$ are those used by SSTOPT (Definition 1).

Because Theorem 2 bounds the regret of WMNS–DSE in terms of the regret of SSTOPT (and thus also PERFECT, if demands are drawn from k fixed distributions in order), this result indicates that WMNS–DSE should perform well under demand shocks. In the bound shown in Theorem 2, the second term captures the worst-case potential loss of the best expert in each period. The third term is a function of the SSTOPT's regret, $\sum_{j=1}^{t} R_a(q^{(j)}, d^{(j)})$.

Although the bound in Theorem 2 may be important for practitioners requiring strict guarantees for algorithmic performance independent of statistical assumptions, experimental evaluation reveals that WMNS–DSE performs better than the bound would indicate. Furthermore, when the demands are in fact drawn from distributions such as the normal, lognormal, or uniform, worst-case per-period effects such as the second term in Theorem 2 appear to be dominated by the first and third terms.

Table 2: Relative regrets (in percent) in the default scenario.

Approach	Relative Regret (%)	C.I. Margin (%)
FRACT–W12	1.707	0.137
FRACT–W30	2.210	0.160
FRACT–EX2	1.900	0.129
FRACT–EX0	2.535	0.161
SCARF–W12	1.774	0.140
SCARF–W30	2.278	0.161
SCARF–EX2	1.964	0.129
SCARF–EX0	2.506	0.162
MUS–W12	2.273	0.156
MUS–W30	2.814	0.176
MUS–EX2	2.514	0.143
MUS–EX0	2.785	0.167
QHYB–W12	4.976	0.247
QHYB–W30	5.244	0.267
QHYB–EX2	5.508	0.262
QHYB–EX0	6.578	0.270
WMNS–DSE	1.478	0.048

EXPERIMENT DESIGN AND RESULTS

Experiment Design

In order to evaluate the effectiveness of WMNS–DSE, we simulate demand sequences describing a variety of conditions, comparing the results to benchmarks that estimate demand distribution parameters based on recent order observations.

We give the performance of an algorithm ALG in terms of simulated relative regret from the critical fractile solution PERFECT that uses perfect information on demand distribution for each period j:

$$Relative\ Regret(\mathrm{ALG}) = \frac{\sum_j \pi(\mathrm{PERFECT}^{(j)}, d^{(j)}) - \sum_j \pi(\mathrm{ALG}^{(j)}, d^{(j)})}{\sum_j \pi(\mathrm{PERFECT}^{(j)}, d^{(j)})} * 100\%, \tag{3}$$

where $\mathrm{PERFECT}^{(j)}$ is the critical fractile solution derived using the distribution from which demand $d^{(j)}$ is drawn from in period j, and $\mathrm{ALG}^{(j)}$ is the order quantity used by ALGin period j. Relative regret is similar to absolute regret, which WMNS–DSE is designed to minimize; however, it is easier to compare performance results across experiments because it is scaled with respect to PERFECT's profit.

For all experiments, we evaluate the performance of algorithms via the average relative regret over 200 trials. This number of trials allows for robust comparison between tests (see Table 2).

The demand sequences used for testing consist of 240 demands, each drawn from one of two distributions. We use a choice of 240 demands as this number is evenly divisible for a variety of demand-shock frequencies. If there are to be $k - 1$

demand shocks, the first $240/k$ demands are drawn from the first distribution, the second $240/k$ from the second, the third $240/k$ again from the first, and so on until 240 demands have been generated. Thus, every $240/k$ demands simulates a demand shock, and k can be varied to adjust the frequency of shocks.

As a basic scenario for all experiments, the first distribution is normal with a mean of 600 and a standard deviation of 200. The second distribution is also normal with an identical standard deviation of 200, but has a mean of 900. Any demand generated below 0 is resampled so demands are always nonnegative. The default scenario includes two demand shocks. That is, there are 80 periods from a distribution with mean of 600, 80 periods from a distribution with mean of 900, and 80 periods from a distribution with mean of 600. The per item cost, selling price, and salvage values are \$20, \$40, and \$8.50, respectively. The understock cost c_u is \$0 for all tests. In third subsection, we study changes in performance while varying shock magnitude, shock number, demand variance, per-item cost, per-item salvage value, and distribution type.

Approaches Tested

Because the WMNS–DSE algorithm is the first one designed for handling unknown demand information as well as demand shocks, it is hard to compare its performance against the performances of the traditional algorithms available. Reviewing the literature, we found that Scarf's rule has been frequently used as a benchmark when evaluating a new algorithm's performance. In addition, we include the critical fractile solution, representing the optimal solution for known demand distributions. For more comprehensive comparison, we also include two contemporary approaches from Perakis and Roels (2008) and Yue et al. (2007) as benchmarks.

Both the critical fractile and Scarf's solution require estimating the demand mean μ and standard deviation σ, while the two contemporary approaches require estimating the demand mean μ. We use two popular methods to estimate the demand mean and standard deviation for the benchmark approaches. One is moving window and another one is exponential smoothing. Below we describe in details the approaches tested, as well as the two methods for parameter estimation where necessary.

Critical fractile: FRACT

Given the shape of the demand distribution and estimated mean $\hat{\mu}$ and standard deviation $\hat{\sigma}$, the critical fractile solution prescribes an order that balances the expected costs of overstocking and understocking (Porteus, 2002):

$$order = \Phi^{-1}_{\hat{\mu},\hat{\sigma}}\left(\frac{r - c + c_u}{r - s + c_u}\right),$$

where $\Phi_{\hat{\mu},\hat{\sigma}}$ is the estimated cumulative distribution function of the demand.

In these tests, FRACT uses the distribution shape information (e.g., normal, lognormal, or uniform) when computing the critical fractile. However, realistically, practitioners may not know the correct shape of the demand distribution in advance.

Scarf's rule: SCARF

Given a similar estimate of mean $\hat{\mu}$ and standard deviation $\hat{\sigma}$, Scarf's rule prescribes ordering the quantity that maximizes the worst case profit over all distributions with those parameters (Scarf, 1958). Alfares and Elmorra (2005) extend Scarf's solution to the case of understock costs when $c_u < c^2$:

$$\text{If } \left(\frac{(r-c)\hat{\mu}}{c\hat{\sigma}}\right)^2 > \frac{(c-s)(r-c+c_u)}{c^2}, \text{ order } = \hat{\mu} + \frac{\hat{\sigma}}{2}\left(\sqrt{\frac{r-c+c_u}{c-s}} - \sqrt{\frac{c-s}{r-c+c_u}}\right).$$

An order of 0 is used if the condition is not met.

Known mean, unimodality, and symmetry: MUS

Perakis and Roels (2008) describe minimax expected regret solutions given limited demand distribution information such as mean, median, mode, symmetry, variance, and unimodality. Here, we apply a solution assuming it is known that demand distributions are unimodal and symmetric, using only an estimated demand mean $\hat{\mu}$.

$$order = \begin{cases} 2\hat{\mu}\sqrt{\beta(1-\beta)} & \text{if } \beta \geq 1/2, \\ 2\hat{\mu}\left(1-\sqrt{\beta(1-\beta)}\right) & \text{if } \beta \leq 1/2, \end{cases}$$

where $\beta = (c-s)/(r-s+c_u)$.

Known mean and range: QHYB

Yue et al. (2007) also derive solutions assuming limited demand information—in this case, estimated distribution mean $\hat{\mu}$ and range $[m_q, M_q]$. Here, we apply the suggested approach for linear cost functions, q^{hyb}:

$$order = \begin{cases} \frac{\gamma}{2}\left[M_q + \hat{\mu} - \frac{p}{t}\left(M_q - m_q\right)\right] + (1-\gamma)\left[(1-\gamma)M_q + \gamma\hat{\mu}\right] & \text{if } \gamma < 1 \\ \frac{1}{2\gamma}\left[m_q + \hat{\mu} + \frac{t}{p}(\hat{\mu} - m_q)\right] + \left(1-\frac{1}{\gamma}\right)\left[\left(1-\frac{1}{\gamma}\right)m_q + \frac{1}{\gamma}\hat{\mu}\right] & \text{if } \gamma > 1 \\ \frac{1}{2}\left(M_q + m_q\right) & \text{if } \gamma = 1, \end{cases}$$

where $p = c - s$, $t = r - c + c_u$, and $\gamma = p(M_q - \hat{\mu})/t(\hat{\mu} - m_q)$.

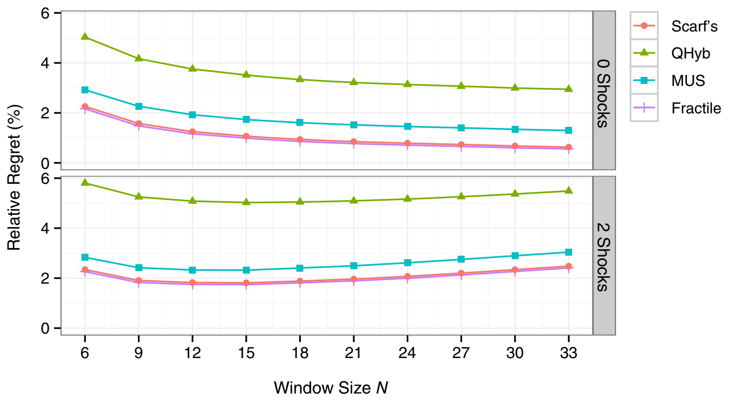
If the estimate of $\hat{\mu}$ falls outside the assumed range $[m_q, M_q]$, then the order of QHYB is undefined. To ensure that proper order quantities are generated by QHYB (even for distributions such as the normal, there is always a nonzero probability of the mean estimate falling outside any finite range), for each generated 240 period demand sequence, the actual minimum and maximum demand are assigned to $[m_q, M_q]$.

WMNS–DSE

WMNS–DSE utilizes a number of decision variables: a weight update factor $\beta \in (0, 1]$, weight limiting factor $\delta \in (0, 1]$, the number of simulated experts to create K, and the "estimated demand range" $[m, M]$.

To facilitate relatively large expert weight adjustments, we use $\beta = .1$ for all tests. (Large weight adjustments are appropriate when demands are stochastic. If the demands are adversarial, a larger value of β would minimize the bound of Theorem 2). Because the weight limiting factor δ is most crucial for determining

Figure 1: Relative regret of FRACT, SCARF, MUS, and QHYB in the default two-shock scenario (bottom) and a zero-shock scenario (top), varying the number of recent demand points N used in the estimation of $\hat{\mu}$ and $\hat{\sigma}$.



how quickly the algorithm will adjust to demand shock, we choose $\delta = .5$ as a naïve balanced choice. We also (arbitrarily) let the number of simulated experts K be 64.

As a conservative estimate of the demand range $[m, M]$, we use [300, 1, 200], which represents 1.5 standard deviations from the means in the default scenario. Note, however, that in one experiment where the standard deviation is increased to 300 units, this assumption is frequently violated with approximately 16% of demands falling outside the range. Nevertheless, we find that $\mathrm{WMNS–DSE}$ still performs well, and we discuss the intuition behind this important result in the "Effect of Demand Standard Deviation" section below.

Moving window parameter estimation method

For approaches requiring estimates of mean $\hat{\mu}$ or standard deviation $\hat{\sigma}$, one parameter estimation method is moving window, which estimates distribution parameters from the most recent N demands. Using a larger window size provides for more robust estimates, but smaller window sizes provide for more responsive adjustments to changes in the underlying demand distribution. When fewer than N demand observations are available, the entire available history is used. For all tests, prior to any demand observations the initial $\hat{\mu}$ is set to 750.0 and the initial $\hat{\sigma}$ is set to 200.0.

Figure 1 shows average relative regrets for FRACT, SCARF, MUS, and QHYB with various window sizes N, in both the default scenario with two demand shocks and a scenario with zero demand shocks. According to Figure 1, the window size $N = 12$ is optimal for the two-shock demand scenario since the four benchmark approaches seem to have the lowest relative regrets when $N = 12$. In the zero-shock scenario, the window size $N = 30$ is large enough to perform well since the relative regrets decreases very slowly in the window size for $N \geq 30$. Therefore, based on these results, we implement versions of FRACT, SCARF, MUS, and QHYB with N equal to 12 and N equal to 30, naming these specific approaches $\mathrm{FRACT{-}W12}$, $\mathrm{SCARF{-}W12}$, $\mathrm{MUS{-}W12}$, $\mathrm{QHYB{-}W12}$, $\mathrm{FRACT{-}W30}$, $\mathrm{SCARF{-}W30}$, $\mathrm{MUS{-}W30}$, and $\mathrm{QHYB{-}W30}$, respectively.

Exponential smoothing parameter estimation method

A common alternative method for estimating demand distribution mean $\hat{\mu}$ is exponential smoothing. In the simplest version, a weight $\alpha \in (0, 1)$ is chosen, and $\hat{\mu}$ is updated in each period j according to the demand seen in the most recent period, $d^{(j-1)}$: $\hat{\mu}^{(j)} = \alpha d^{(j-1)} + (1-\alpha)\hat{\mu}^{(j-1)}$.

A variant of exponential smoothing that can detect bias in the forecast and respond quickly to shifts in demand mean is described by Trigg and Leach (1967). Here, a different smoothing value $\alpha^{(j)}$ is used in each update, and these values are defined according to $d^{(j-1)}$, a parameter $\gamma \in (0, 1)$, and smoothed error and absolute error values $e^{(j)}$ and $a^{(j)}$ (initially set to 1):

$$\begin{aligned}
e^{(j)} &= \gamma\left(d^{(j-1)} - \hat{\mu}^{(j-1)}\right) + (1-\gamma)e^{(j-1)} \ , \\
a^{(j)} &= \gamma\left|d^{(j-1)} - \hat{\mu}^{(j-1)}\right| + (1-\gamma)a^{(j-1)} \ , \\
\alpha^{(j)} &= \left|e^{(j)}/a^{(j)}\right| \ , \\
\hat{\mu}^{(j)} &= \alpha^{(j)}d^{(j-1)} + (1-\alpha^{(j)})\hat{\mu}^{(j-1)} \ .
\end{aligned}$$

This method provides a robust exponentially smoothed estimate for $\hat{\mu}^{(j)}$ in any given period j, but does not provide an estimate for $\hat{\sigma}^{(j)}$. To compute a similarly smoothed $\hat{\sigma}^{(j)}$, we note that exponential smoothing, in general, can be considered a weighted average of all previous demands (generally higher weights are given to more recent demand observations). Given the equations above, we can rewrite the mean estimate as

$$\hat{\mu}^{(j)} = \sum_{i=1}^{j-1} d^{(i)}\alpha^{(i)} \prod_{l=i+1}^{j-1}\left(1-\alpha^{(l)}\right) \ .$$

Thus, in period j, the weight given to previous seen demand $d^{(i)}$, which we denote by $w^{(j,i)}$, is given by

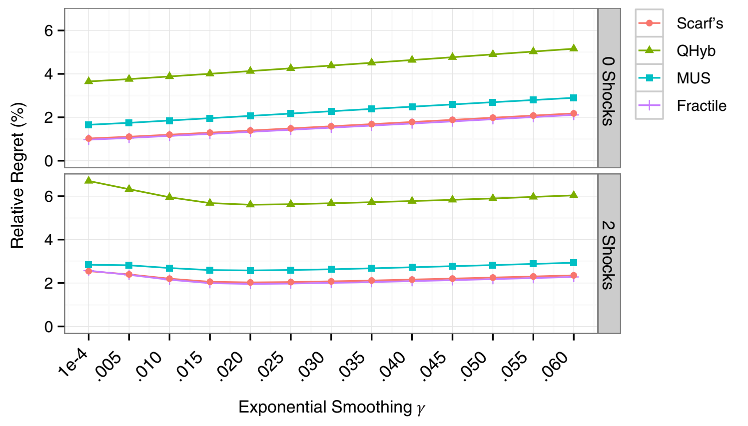
$$w^{(j,i)} = \alpha^{(i)} \prod_{l=i+1}^{j-1}\left(1-\alpha^{(l)}\right) \ ,$$

and we can define the exponentially smoothed estimate for standard deviation, $\hat{\sigma}^{(j)}$, as the weighted standard deviation using the weights ascribed above:

$$\hat{\sigma}^{(j)} = \sqrt{\frac{\sum_{i=1}^{j-1} w^{(j,i)}\left(d^{(i)} - \hat{\mu}^{(j)}\right)^2}{\sum_{i=1}^{j-1} w^{(j,i)}}} \ .$$

As with the moving window method, the initial estimates for $\hat{\mu}$ and $\hat{\sigma}$ (before any demands are seen) are 750.0 and 200.0, respectively, but these values are not

Figure 2: Relative regret of FRACT, SCARF, MUS, and QHYB in the default two-shock scenario (bottom) and a zero-shock scenario (top), varying γ used in the estimation of $\hat{\mu}$ and $\hat{\sigma}$.



given any weight once the first demand is seen. Figure 2 shows average relative regrets for FRACT, SCARF, MUS, and QHYB with various values for the parameter γ, in both the default two-shock demand scenario, and a scenario with zero demand shocks. Based on these results, we implement optimized versions of FRACT, SCARF, MUS, and QHYB with γ equal to .0001 and .02, naming these specific approaches FRACT–EX0, SCARF–EX0, MUS–EX0, QHYB–EX0, FRACT–EX2, SCARF–EX2, MUS–EX2, and QHYB–EX2, respectively.

Experimental Results

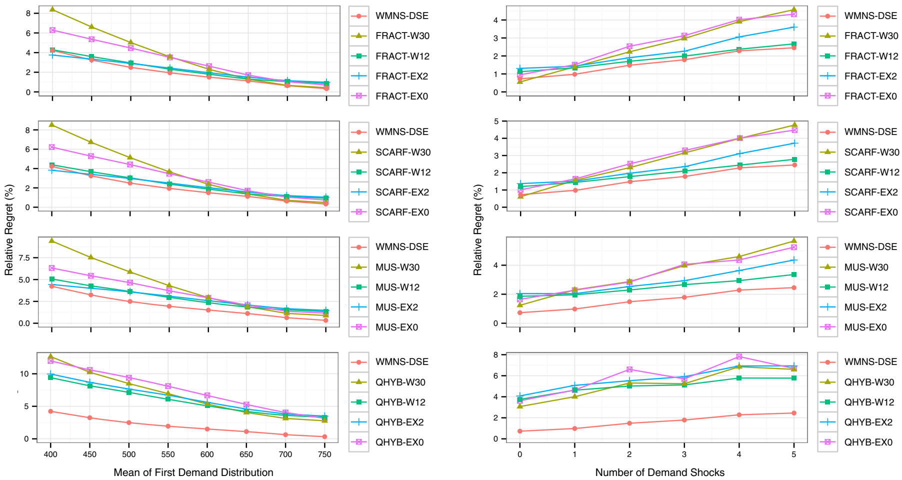
Table 2 gives the mean relative regret over 200 trials for the approaches we test in the default scenario: a normal distribution with a standard deviation 200 experiencing two demand shocks such that the mean increases then decreases from 600 to 900 to 600. Also shown are 95% confidence interval margins for this mean, calculated from a t-test with 199 degrees of freedom.

FRACT–W12 and SCARF–W12 perform well, as do FRACT–EX2 and SCARF–EX2; recall that a window size of 12 and γ value of .02 were chosen as the experimentally determined optimals for these approaches in this default test. Even so, WMNS–DSE outperforms these approaches.

SCARF and FRACT seem to perform better than MUS and QHYB. This may be caused by the fact that MUS and QHYB only use information on the mean, whereas SCARF and FRACT use both the mean and standard deviation. Although none of them are designed for the scenarios with demand shocks, estimating and using both the mean and the standard deviation helps to improve performance during demand shocks than only estimating and using the mean.

Note that the demand mean jumps up to a second level, stays there for a while, and then jumps down to a third level. We note that the first base mean level

Figure 3: Effects of shock magnitude and frequency.



(a) The mean of the first distribution μ_1 is varied from 400 to 750. The mean of the second distribution μ_2 is computed as $\mu_2 = 1500 - \mu_1$, thus, $\mu_1 = 400$ represents the largest demand shocks, while $\mu_1 = 750$ represents a no-shock scenario.

(b) The number of demand shocks in the 240 period demand sequence varies from 0 to 5.

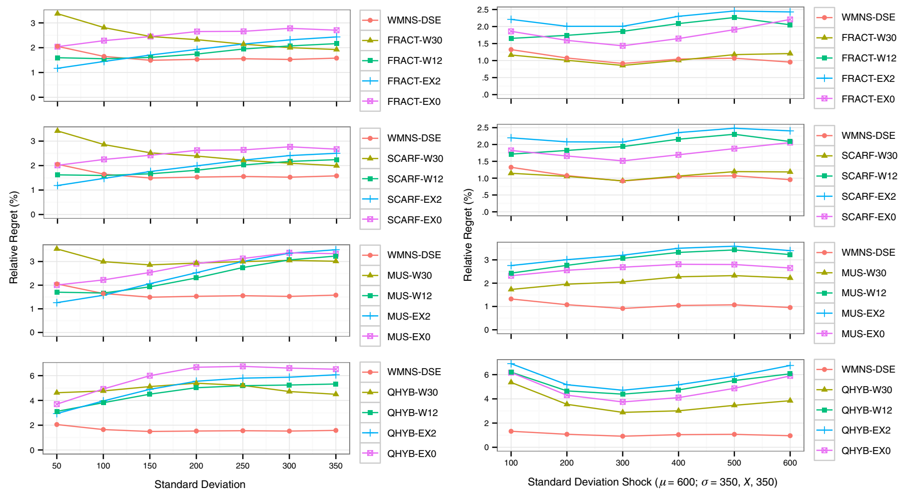
and the third level are the same in our experiments. This symmetric setting (e.g., 600–900–600) was intentionally selected to eliminate any confounding effects. Specifically, the 600–900–600 setting implies that there are total 600 demand changes (i.e., 300 increase and then 300 decrease). While there are many possible settings for 600 demand changes (500–900–700, 400–900–800, etc.), the selected setting enables us to examine the effect of demand shock magnitude clearly. However, we emphasize that our algorithm does not require that the demand mean eventually comes back to the base level. Depending on the shock magnitude that may be random in practice, the third mean level can be higher or lower than the base mean level. Our results still hold in such scenarios.

Effect of shock magnitude

Here we study the effect on relative regret as we vary the magnitude of the demand shock in the default scenario. Letting the mean of the first distribution be μ_1 and the mean of the second be $\mu_2 = 1,500 - \mu_1$, we vary μ_1 resulting in demand shocks of $400/1,100/400$, $450/1,050/450$, $500/1,000/500$, $550/950/550$, $600/900/600$, $650/850/650$, $700/800/700$, and $750/750/750$, the last of which represents a lack of demand shocks altogether.

As shown in Figure 3(a), for these windowed approaches, larger windows produce better results when the shock is small, and smaller windows produce better results when the shock is large. WMNS–DSE performs very well over all tests, and the relative regret is also minimized when the demand shock is least severe.

Figure 4: Effects of demand standard deviation and different shock type.



(a) Effects of standard deviation when facing demand shocks from changes in demand mean

(b) Effects of demand shocks from changes in demand standard deviation

Effect of shock frequency

We vary the number of demand shocks in the 240 demand sequence from 0 to 5, which corresponds to no shock and a shock in mean demand every 120, 80, 60, and 48 periods.

The results in Figure 3(b) reinforce the pattern that larger windows generally perform better in no-shock situations, whereas as shocks become more frequent the smaller window size becomes increasingly beneficial. WMNS–DSE is slightly outperformed by both FRACT–W30 and SCARF–W30 in the zero-shock scenario. For all approaches, relative regret increases as demand shocks become more frequent.

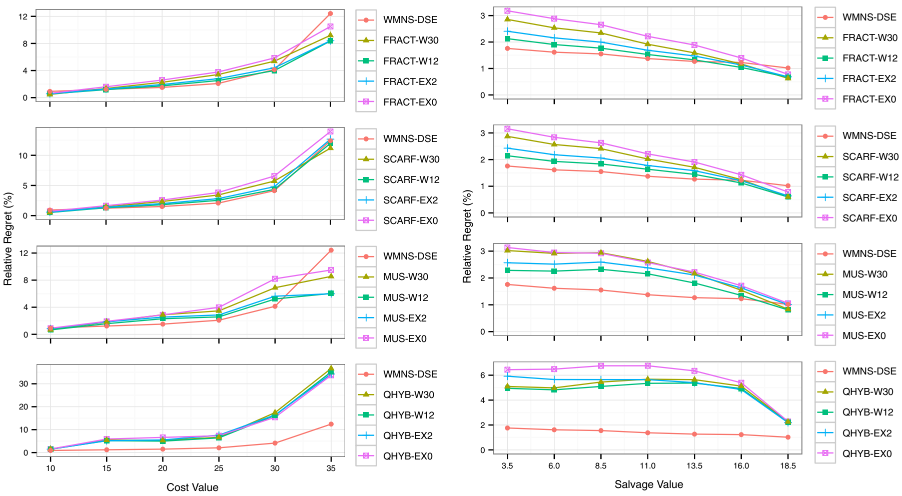
Effect of demand standard deviation

Varying the standard deviation of the default scenario from 100 to 300 shows how each approach copes with different levels of uncertainty in the demand distribution. When the mean is 600, this represents variance/mean ratios of 16.6 to 150; when the mean is 900, it represents variance/mean ratios of 11.1 to 100. These ratios agree with those found in the departmental store data studied by Nahmias and Smith (1994).

Figure 4(a) shows that in a demand-shock scenario, small window sizes become decreasingly beneficial as demand variance increases. This is because large window sizes serve to smooth out the variation when estimating distribution parameters.

WMNS–DSE is outperformed by FRACT–W12, FRACT–EX0, SCARF–W12, SCARF–EX0, MUS–W12, and MUS–EX0 when the standard

Figure 5: Effects of cost and salvage value.



(a) The per-item cost is varied from \$10 to \$35.

(b) The per-item salvage value is varied from \$3.5 to \$18.5.

deviation is below 100. Otherwise, WMNS–DSE performs best, and its relative regret is stable. Because the distributions are normal, when the standard deviation is 300 approximately 16% of the demands fall outside of WMNS–DSE's assumed demand range of [300, 1, 200]. Even in this case WMNS–DSE outperforms the other approaches.

Effect of different demand shock type

We study a different demand shock caused by changes in demand standard deviation. We fix the demand mean at $\mu = 600$. Letting the standard deviation of the first distribution be $\sigma_1 = 350$, we vary the standard deviation of the second distribution from 100 to 600 in the increment of 100. Therefore, the seller face demand shocks (from changes in standard deviation) of 350/100/350, 350/200/350, 350/300/350, 350/400/350, 350/500/350, and 350/600/350. Figure 4(b) shows that WMNS–DSE is outperformed by FRACT–W12 and SCARF–W12 when the standard deviation of the second distribution drops to 100. Otherwise, WMNS–DSE performs best, and its relative regret is stable.

Effect of cost value

In Figure 5(a), we vary the per-item cost value from the default scenario of \$20 from \$10 to \$35. Here, we see that all approaches suffer significantly when the per-item cost is very close to the per-item revenue. Although WMNS–DSE performs well for most cost values, the other approaches perform better when per-item cost is very small or very large, that is, when the critical fractile value is close to the extremes (0 or 1).

Effect of salvage value

We also investigate the effect of the per-item salvage value. While increasing the per-item cost results in critical fractile values that are very small (close to 0), increasing the per-item salvage produces large critical fractile values (close to 1).

As depicted in Figure 5(b), all approaches are able to reduce their relative regret as the salvage value increases. As we saw for the cost dimension, WMNS–DSE performs best except in the extreme of the critical fractile range: most approaches outperform WMNS–DSE when the salvage value is \$18.5.

Effect of distribution type

Finally, we also test the approaches on a number of distribution types. These other distributions, including truncated normal, lognormal, and uniform, are commonly found in the literature (Godfrey & Powell, 2001; Perakis & Roels, 2008).

For the lognormal and uniform distributions, just as in the default scenario there are demand shocks such that the mean changes from 600 to 900 and back to 600, with a constant standard deviation of 200 (recall that the uniform distribution is characterized by having a range of $\mu \pm \sigma\sqrt{3}$). The truncated normal distributions are the same as in the default scenario, but truncated to a [400, 1, 100] range via resampling, corresponding to actual means of 653.75 and 846.25. Figure 6 shows that WMNS–DSE outperforms the other approaches in these tests.

WMNS **as a Meta-Approach**

Although we have primarily focused on WMNS–DSE, the WMNS algorithm can aggregate order predictions from any number of approaches, including the traditional approaches such as SCARF and FRACT themselves. To this end, we implement WMNS–META, which is the WMNS algorithm utilizing all 16 traditional and contemporary approaches, including each version of FRACT, SCARF, MUS, and QHYB as experts.

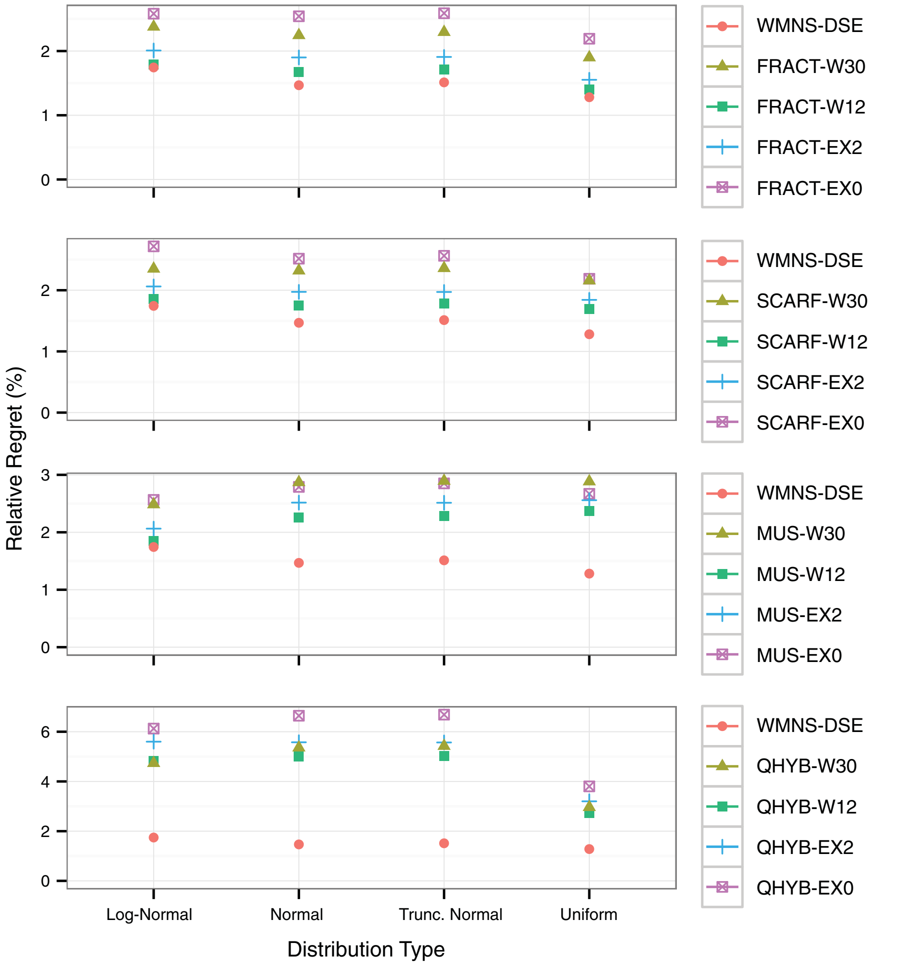
Figure 7 shows the relative regret of WMNS–META for all tests as compared to WMNS–DSE. Overall, WMNS–META is able to aggregate the traditional approaches well and produce results that are on par with WMNS–DSE. However, for the most part WMNS–DSE still slightly outperforms WMNS–META.

Counterexamples to this trend are precisely those situations where WMNS–DSE performed poorly compared to traditional approaches: when per-item costs or salvage values were very large, that is, the critical fractile value is near 0 or 1. Therefore, WMNS–META can serve as a complimentary algorithm for WMNS–DSE.

The Choice of β, δ, and $[m, M]$ for WMNS-DSE

An advantage to our algorithm WMNS–DSE in general is as follows. Traditionally, practitioners separate forecasting and policy optimization. For example, traditional newsvendor techniques in practice estimate the distribution, then use this knowledge to calculate the optimal order amount. Our algorithm WMNS–DSE does not estimate the distribution but directly estimates the optimal order amount. For the real world that has messy distributions and demand shocks, this

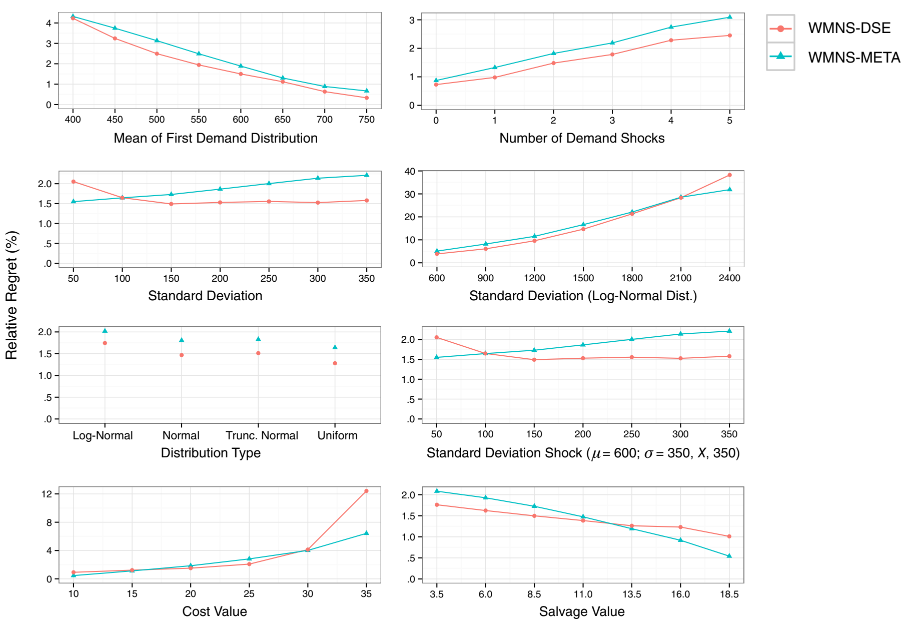
Figure 6: Relative regret with four different demand distribution types.



is an advantage. Therefore, when the underlying demand distribution is messy or when a practitioner has reasons to suspect that demand shocks may occur, he/she should consider using our algorithm WMNS–DSE.

Because WMNS-DSE utilizes a number of parameters, guidance for setting these parameters will be of particular interest to practitioners. We provide guidance on good choices for β and δ through numerical studies. We have explored the effect of both β and δ in .1 increments from .1 to .9, as well as the effect of the range $[m, M]$ spanning 900 units (as in other tests) but also 1,200 and 600 unit ranges. For all of these algorithmic choices, we explore the average relative regret with the default scenario of two shocks, but also no shocks and four shocks to see if shock number influences the results on these algorithm parameter choices. We find the choice of β affects the choice of δ if there are demand shocks, however the

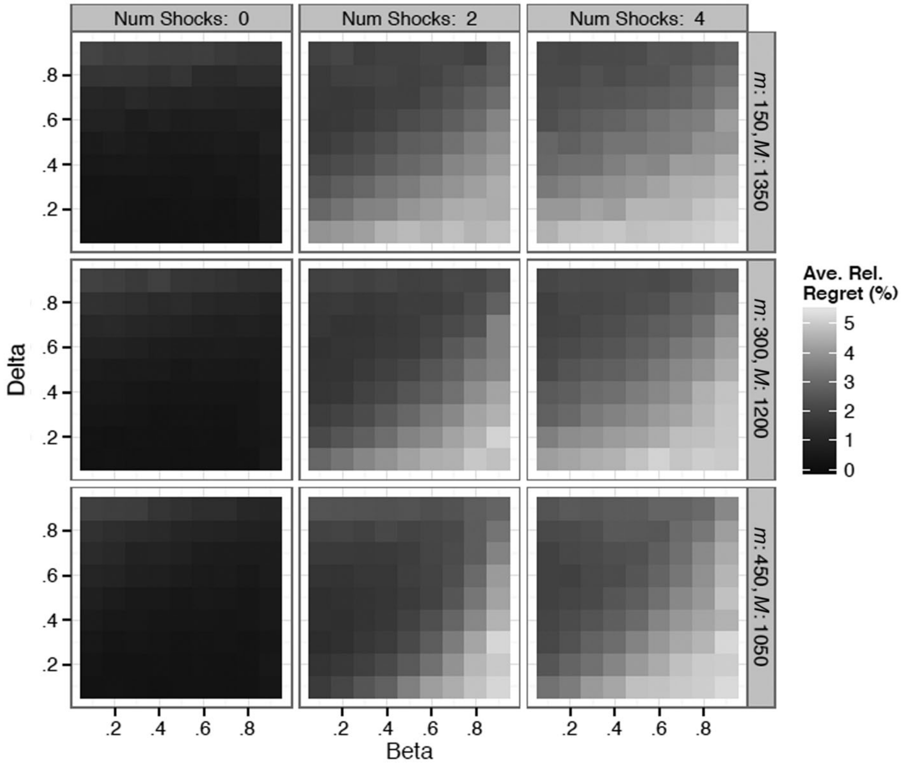
Figure 7: Relative regret of WMNS–META as compared to WMNS–DSE for all tests. While WMNS–DSE slightly outperforms WMNS–META in general, where the critical fractile value is extreme (near 0 or 1, when per-item costs or salvage values are high, respectively) WMNS–META obtains lower relative regret than WMNS–DSE.



transitions are smooth across the changes in these variables. The algorithm performs very well for any parameter settings when there is no demand shock. When there exist demand shocks, the algorithm performs well in general with small values of β (around .1) and medium or high values of δ (.4 to .8). In addition, although our algorithm requires an estimation of demand range $[m, M]$, the estimation of the range $[m, M]$ does not strongly impact the algorithm's performance. The results are summarized in Figure 8.

Based on the algorithm described in the previous section, we provide some insightful explanation for the optimal choices of β and δ demonstrated in Figure 7. In algorithm WMNS–DSE, the parameter β controls the weight reduction level based on an expert's performance in the most recent period. The lower the β value, the higher the weight reduction for an expert. Therefore, when a low β value is used, experts' weights are updated more dramatically according to their performance in the most recent period. On the other hand, the parameter δ determines the threshold of the minimum weight that an expert needs to have in order to be considered in determining the order quantity. The higher the δ value, the higher the weight threshold. Therefore, when there exist demand shocks, a practitioner should choose a small β value (such that those experts who perform well under

Figure 8: The impact of interval range, β value, and δ value on algorithm performance.



the new demand distribution can be quickly identified) and a high δ value (such that the identified experts' opinions will be focused and noises from other experts will be reduced).

CONCLUDING REMARKS

In this article, we propose a new machine learning method, called WMNS, which utilizes expert order predictions to solve a newsvendor problem with incomplete demand information. An extension of the method with predefined static experts, called WMNS–DSE, requires very little information about the demand distribution but performs very well in our numerical experiments. We compare WMNS–DSE's performance against those of data-estimated Scarf's solutions and data-estimated critical fractile solutions in a variety of situations, including varying shock magnitude, number of shocks, demand variation, per-item cost, per-item salvage value, and distribution type. Through numerical experiments, we find that WMNS–DSE performs better than or similar to the two benchmark approaches in most settings, except when the critical fractile value is near 0 or 1.

We also demonstrate the use of the WMNS mechanism to act as a meta-algorithm WMNS–META, aggregating a number of traditional approaches to produce an algorithm that also perform well in nearly all scenarios. While WMNS–META is only slightly outperformed by WMNS–DSE overall, it does very well when the critical fractile value is near 0 or 1, thus complementing the weakness of WMNS–DSE.

As retailing channels become more competitive and complex, an efficient and effective method to handle uncertain demand in an automated fashion becomes more important. This is evident given the increasing research attention paid to newsvendor problems with incomplete demand information. Furthermore, fierce competition in retailing, as well as unpredictable demand changes, cause demand shocks frequently. When full demand distribution information is not available, practitioners have few choices, such as Scarf's rule and some forecasting techniques, many of which still require estimating the demand mean and variance. As demonstrated earlier, performances of two variants of WMNS were outstanding, as compared to the traditional approaches, even in the presence of demand shocks. Therefore, we believe WMNS can be successfully applied to handle complicated problems faced by many practitioners today.

When demand shocks are unpredictable, our algorithm helps a seller to automatically catch up with demand shocks and adjust order quantities correspondingly without human intervention and prediction. Retailers offering thousands of different products subject to any number of distributions or demand shocks should also find the consistent performance of these approaches with minimal adjustments quite useful.

We can also extend the algorithm to consider predictable demand shocks. If the timing of events is known to a seller, our algorithm can take advantage of such information and reinitialize experts' weights. That is, while the traditional methods may prepare for the demand shock through incorporating additive or multiplicative adjustment to their average demand, our algorithm can update the experts' weights to achieve the same effect. Updating or reinitializing the experts' weights right before the events is similar to incorporating additive or multiplicative adjustments to the average demand. Meanwhile, after the demand shock and before the next demand shock, that is, when there are no demand shocks, our numerical experiments in Figure 2(a) show that our algorithm outperforms other methods. That is, when $\mu = 750$ (so there is no demand shock) in Figure 2(a), WMNS–DSE outperforms others.

Furthermore, our algorithm can be easily generalized. In the generalized version, any convex loss function can be scaled to the [0,1] interval by normalizing the loss by **C**, the maximum possible loss in any single period. The static experts should choose a prediction quantity in each interval to minimize the loss defined by the generic loss function over all periods. In this case, the total regret suffered by the algorithm will be bounded in terms of the regret of the best expert on any subsequence, but as with Theorem 1, this bound will depend on β, δ, the number of experts, and the size of **C**.

There are several important future directions of our study to overcome some limitations. First, we limited our discussion to a repeated newsvendor problem, but applying machine learning algorithms to a general dynamic inventory problem would be an important extension of our study. Second, we tested WMNS's performance with four popular distributions: normal, truncated normal, lognormal, and uniform. Testing its performance with a wider variety of distributions and detailed sensitivity analysis may reveal more insights into whether the method's performance is sensitive to certain types of demand functions. Finally, investigating the performance of our algorithms when there is only censored demand information would increase our understanding of the applicability of machine learning algorithms to many inventory problems.

SUPPORTING INFORMATION

Additional supporting information may be found in the online version of this article at the publisher's website:

Appendix

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