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A fuzzy echelon approach for inventory management in supply chains

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Abstract

This paper presents a methodology to define a supply chain (SC) inventory management policy, which is based on the concept of echelon stock and fuzzy set theory. In particular, the echelon stock concept is adopted to manage the SC inventory in an integrated manner, whereas fuzzy set theory is used to properly model the uncertainty associated with both market demand and inventory costs (e.g. holding and backorder costs). The methodology is applied on a three stage SC so as to show the ease of implementation. Finally, by adopting simulation, the performance of the three stage SC is assessed and shown to be superior to that, which the adoption of a local inventory management policy would guarantee.

Keywords: Supply chain management; Inventory policies; Echelon stock; Fuzzy set theory

1. Introduction

Inventory usually represents from 20% to 60% of the total assets of manufacturing firms. Therefore, inventory management policies prove critical in determining the profit of such firms [1]. Inventory management is to a greater extent relevant when a whole supply chain (SC), namely a network of procurement, transformation, and delivering firms, is considered. Inventory management is indeed a major issue in supply chain management (SCM), i.e. an approach that addresses SC issues under an integrated perspective [2,3].

When inventory decisions in SCs are made independently at each stage (as it is often the case), they are usually based on the local inventory status and local performance objectives (*local policies*). These policies are simple to be defined and implemented, but ignore the implications that decisions at one stage can have on the others, let alone the fact that local objectives are often conflicting among each other, which often leads to suboptimize the SC performance. In addition, the lack of a coordinated inventory management throughout the SC often causes the *bullwhip effect* [4], namely an amplification of demand variability moving toward the upstream stages. This causes excessive

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inventory investments, lost revenues, misguided capacity plans, ineffective transportation, missed production schedules, and poor customer service [3]. In addition, the time necessary to have the upstream SC stages perceive changes in market demand increases, so making the whole SC less responsive to customer requirements [2]. These inefficiencies reveal dangerous in both mature markets, where customers value the service as much as (or even more than) the good itself, and innovative markets, such as fashion or computers, where the lack of responsiveness may lead to either lost sales or obsolete stock [5].

Tracing back to Forrester [6], many scholars have studied these problems as well as emphasized the need of integration among SC stages to make the chain effectively and efficiently satisfy customer requests (e.g. [7]). Clark and Scarf [8] introduced the concept of *echelon stock* as opposed to *installation stock*. In echelon stock policies, ordering decisions at a given stage are based on the echelon inventory position, which is the sum of the inventory position at the considered stage and at all the downstream stages [9]. This requires a tight coordination and information sharing within the chain.

Beside the integration issue, uncertainty has to be dealt with in order to define an effective SC inventory policy. In addition to the uncertainty on supply (e.g. lead times) and demand, information delays associated with the manufacturing and distribution processes characterize SCs [10]. Moreover, uncertainty is associated with the estimates of inventory costs as well as backorder costs [11,12]. Typically, stochastic techniques have been used to cope with these problems (e.g. [13]). In such cases the sources of uncertainty are modeled by probabilistic distributions that derive from the analysis of past cases. However, past data are not always available or reliable (e.g. due to market turbulence). Moreover, also when they can be utilized, the integrated inventory policies so determined are usually difficult to be implemented, which makes them unattractive even if efficiently computed [14]. As a result, research has been focused on heuristic policies that even though suboptimal are easier to be implemented [15].

This paper addresses both the integration and the uncertainty issues in SCs. A methodology to determine a global inventory policy, based on the concept of echelon stock, that is near to optimal and easy to be implemented is proposed. The global inventory policy definition requires that a periodic control time T and an order-up-to level (or target level) S that are optimal with respect to the whole SC be determined at each stage. Uncertainty is dealt with by modeling market demand as well as holding and backorder costs through fuzzy set theory as in Petrovic et al. [16], rather than by probability theory. The latter has been mainly used to address inventory problems [13] whereas, to our knowledge, few studies addressed the uncertainty issue in SCs through fuzzy set theory [12,16–18]. Besides, most of them resort to fuzzy set theory to model inventory costs only.

Shorter and shorter product life cycles as well as growing innovation rates make demand extremely variable and the collection of statistics (which are required by stochastic models) less and less reliable [19]. Therefore, we argue that probability theory is not appropriate to assess market demand and inventory related costs. Moreover, when uncertainty is also due to the ambiguity of the available information, the existence of conflicting evidence or measurement problems (such as in the case of market demand or some inventory costs), possibility rather than probability theory should be used to model uncertainty [20]. Possibility theory in fact deals with the analysis of similarities between an object and some given properties and does not convey information about relative frequencies, as probabilities does [21]. Among methods that address possibility, techniques that allow to model management judgements, based on intuition and experience, may reveal more appropriate. In this paper fuzzy set theory has been adopted because of its potential to deal with the concept of possibility (rather than probability) and linguistic expressions (i.e. management judgements) so as to properly address the uncertainty issue.

The paper is organized as follows. A brief introduction to supply chain inventory management (SCIM) is given in Section 2. The proposed methodology for managing inventory in SCs is presented in Section 3. In Section 4 the methodology is tested on a three stages SC, and the SC performance under the obtained global policy is compared via simulation to that provided by a local inventory policy. Finally, in Section 5, some conclusions about future research are drawn.

2. Supply chain inventory management

SCIM is an integrated approach to the planning and control of inventory, throughout the entire network of cooperating organizations from the source of supply to the end user. SCIM is focused on the end-customer demand and aims at improving customer service, increasing product variety, and lowering costs [10]. Below, the main attributes of SCIM policies are briefly discussed.

An inventory policy can possess local or global objectives. In the former case, the SC inventory policy results from a collection of local policies in which every SC actor tends to make decisions on its own inventory solely based on local performance criteria. On the contrary, under a global policy inventory decisions tend to optimize global performance criteria. However, by using effective incentive systems (such as accounting methods, transfer pricing schemes, quantity discount, etc.) every actor's objective can be aligned to that of the SC as a whole. Hence, also a collection of local policies can be considered as part of SCIM approaches.

Two different strategies can be pursued for managing SC inventory, namely centralized and decentralized inventory control [16]. In the former case, a central decision-maker determines the policy that minimizes the whole SC costs. A centralized control requires a high degree of coordination and communication between the SC actors, i.e. the firms involved in the SC. In most cases SCs adopt a decentralized inventory control. That especially occurs when actors represent several different organizations, each characterized by a relevant dimension and a large amount of information is necessary to inventory management. In a decentralized control scheme, each stage monitors the status of its own local inventory and places orders to its predecessors based on its own performance objectives. Recent studies demonstrate that, thanks to incentive mechanisms, decentralized inventory policies can achieve performance levels almost as high as centralized optimal policies do [22]. Inventory management policies can also differ in the inventory position review frequency (what hereby is called inventory control type). Under periodic-review control policies, the inventory position is reviewed at every stage on a constant time interval T. At each review, should the inventory position be decreased with respect to a target level S, a replenishment order can be issued, which amounts to the quantity Q necessary to raise the stock up to S. Under continuous-review control policies, a replenishment order is issued when the inventory position at the considered stage falls below a predetermined level (reorder point r). In that case a fixed quantity Q is ordered. A hybrid control policy may also be used. The most common are the (s, S) and the base-stock policies. Under the (s, S) policy, at each time interval T, if the inventory position falls below the reorder point s, an order is issued to raise the stock up to a target level S. Under the base-stock policy, a replenishment order is issued any time a withdrawal is made. The ordered amount equals the withdrawn quantity. Such a policy is also referred to as $(S-1, S)$ policy.

Strategic choices in SCIM also regard the adopted approach to manage material flows along the SC. A proactive or a reactive planning approach can be used. Proactive planning approaches, such as material requirement planning, require future demand information per time phase and estimate projected inventory levels. On the other hand, reactive approaches are based on current inventory level and instantaneous demand [10].

Finally, inventory policies can be characterized based on the stock level measurement approach, i.e. installation or echelon stock. At a certain stock point the installation stock is the local on hand inventory, whereas the integral inventory level (which is named echelon stock) is the sum of the local inventory at the considered and all the downstream stock points. Echelon stock inventory policies are more coherent with a centralized decision-making and require intense information sharing among the SC actors. As shown by Axsater and Rosling [9], in serial SCs echelon stock policies achieve better performance than installation ones do. In their seminal work on centralized inventory management policies, Clark and Scarf [8] showed that echelon base-stock policies are optimal in a periodic-review, finite-horizon setting. This result was generalized to infinite horizon by Federgruen and Zipkin [23] and for continuous-review models by Chen and Zheng [24]. Thanks to the development of information and communication technologies, echelon stock inventory policies have been lately increasingly adopted in SCs, as demonstrated by the diffusion of quick response and efficient customer response. This allows to increasingly substitute information for inventory [2].

To sum up, inventory management policies in SCs can be classified according to the following criteria (Table 1):

- *Optimization goal*, by which it is described whether inventory at each SC stage is managed independently, i.e. adopting local inventory policies (*local*), or in an integrated manner through a global inventory policy (*global*);
- *Control type*, which can be *centralized*, if a central decision-maker exists in the SC, or *decentralized*, if decision-makers for the various stages differ from each other;
- *Inventory control frequency*, which can be *periodic*, *continuous*, or *hybrid*, according to inventory position control frequency;
- *Temporal information requirements*, by which it is described whether inventory planning adopts a proactive approach that requires an estimate of future demand based on forecast or customer orders (*time-phased*), or a reactive approach based on the current consumption of inventory stocks (*instantaneous*);
- *Spatial information requirements*, which characterizes the way in which the inventory status in the SC is measured, namely installation stock (*installation*) or echelon stock (*echelon*).

Table 1
Criteria to characterize inventory management policies

Inventory management criteria	Options
Optimization goal	Local; global
Control type	Decentralized; centralized
Inventory control	Periodic; hybrid; continuous
Temporal information requirement	Time-phased; instantaneous
Spatial information requirement	Installation; echelon

3. A fuzzy echelon methodology for SCIM

In this paper a fuzzy echelon methodology for SCIM is proposed. Two basic ideas characterize this methodology, namely the use of fuzzy set theory to model the uncertainty associated with market demand, inventory holding costs, and backorder costs and the adoption of a global perspective to manage SC inventory.

3.1. Uncertainty and fuzzy set theory

Three main demand forecasting models can be found in the literature, namely qualitative, intrinsic, and extrinsic models [1]. The last two usually forecast demand based on historical data, statistical techniques, and possible further economic indicators. However, due to a competitive environment characterized by shorter and shorter product life cycles, often these data either are unreliable or do not exist. Similar difficulties arise in estimating costs associated with inventory holding and backorder. Moreover a growing quota of holding costs depends on product obsolescence that can be hardly expressed in quantitative terms. Likewise, inadequate market responsiveness involves costs (e.g. costs due to an image loss) that go well beyond the mere delay in revenues and a more complex materials management. As a result, qualitative demand forecasts and cost estimates based on managers' judgements, intuitions, and experience may reveal more appropriate.

To properly model uncertainty several aspects need to be addressed: the causes of uncertainty, the characteristics of both the available (input) and required information (output), and the scale level of the numerical information [20]. In the case of market demand and inventory costs, uncertainty causes are often ambiguity, unreliability of statistics, and difficulties in measurement whereas available information is set- or interval-valued or linguistic-type. In these cases probability is not an adequate tool. Values these variables can assume depend on possibilities rather than probability (because statistics are unreliable).

To model these variables fuzzy set theory has been adopted, given that it is based on the concept of the possibility and allows to deal with qualitative data (e.g. linguistic expressions such as *the demand rate will be about x units per time unit* and *the unit holding cost will be more or less y monetary units per time unit and product unit* [25–27]) in a rigorous mathematical way. For a more detailed analysis of the differences between probability and possibility, refer to [21,27].

A fuzzy set is characterized by fuzzy boundaries: unlike crisp sets in which a given element does or does not belong to a given set, in fuzzy sets each element belongs to a set with a certain membership degree. The function that returns the membership degree of each fuzzy set element is called *membership function*. In this paper, triangular membership functions have been adopted because they are considered the most suitable form to model market demand, inventory holding costs, and backorder costs [18]. Fig. 1 depicts a possible fuzzy market demand.

According to Zadeh [25,26] a membership function should be designed by the expert, i.e.

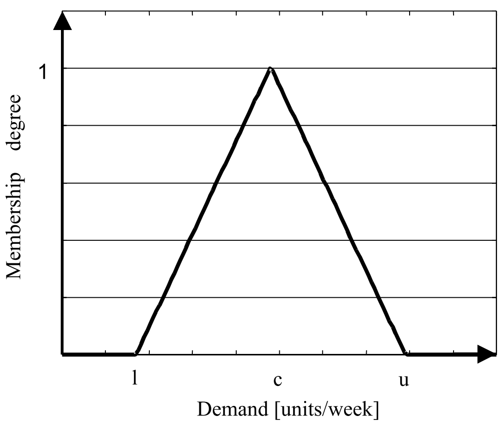

Fig. 1. Fuzzy membership function: *about c units per week*.

managers, that gives the estimate. This is quite simpler than asking managers to design a probability function. To model any variable, say market demand, by a triangular membership function managers only need to estimates the values that do or do not belong to its domain (fuzzy set). In particular, they should estimate which values are not possible at all for demand (so resulting in the lower and upper limits of the set, l and u respectively) as well as the value that better represents the set (c). An equivalent linguistic expression could be: "demand will amount about to c, ranging from l to u".

As discussed, a membership function does not model probability, which is why it is easier to be defined. It indeed models the uncertainty associated with linguistic variables through the concept of possibility [26]. In some cases only *possibility* can be estimated, even though *probability* would be desirable. In turbulent markets the possible demand rather than *probable* demand should be assessed. Moreover possible demand is more reliable over shorter periods. Accordingly, in the paper a given time interval, e.g. a month, is divided into smaller time units, e.g. weeks, and managers are asked to make forecasts over these units. Then, the fuzzy sum operator is used to evaluate the demand rate per month.

Fuzzy set theory is also used to better model the uncertainty associated to holding and backorder costs by simply using linguistic expressions. In fact, as in the case of market demand, managers can find it difficult to estimate costs through crisp numbers because they mostly depend on factors that can be hardly quantified. More reliable cost values can be obtained by modeling them through fuzzy sets and using fuzzy operators where necessary.

However, crisp quantities are at the end required to manage inventory: every SC stage is required to order exactly x units (rather than about x units). Fuzzy values are then to be transformed into crisp numbers. This process is called defuzzification. More details on defuzzification are given in Appendix A. To sum up, the purpose of fuzzy set theory is to make the final crisp values more reliable (by adopting possibility rather than probability in order to better model their uncertainty) and easy to be determined based on linguistic expressions.

So far, the proposed methodology resembles that by Petrovic et al. [16], which is to our knowledge one of the few applications of the fuzzy set theory to inventory management in SCs. However, as explained in the next section our methodology differs in the adopted inventory management policy.

3.2. Integration and the echelon stock concept

A key point of the proposed methodology is the definition of a SC inventory policy that minimizes the total SC costs (global policy). This is pursued by using a periodic-review control policy based on the echelon stock concept. A periodic rather than a continuous-review control policy has been chosen as it is easier to be implemented in real cases whereas the echelon stock concept allows the best SC performance to be achieved, as discussed in Section 2.

According to the echelon periodic-review control policy, at each SC stage the echelon stock is reviewed at constant time intervals and an order is issued to the upstream stage to raise the echelon stock up to a target level. Therefore, to completely define the SC inventory policy the optimal review time intervals and the attendant target levels at every stage need to be defined.

The proposed inventory policy can then be classified according to the criteria discussed in Section 2 as reported in Table 2.

Below the proposed methodology is presented with regard to the case of a serial SC.

Consider a serial production system of N stages (Fig. 2). Each stage has been modeled as a stock point that feeds the downstream stage and is fed by the upstream stage. The Nth stage only is assumed to be fed by an external supplier with unlimited stock. Market demand at stage 1 is expressed as the fuzzy set *about x unit per time unit*. When market demand exceeds the available stock the difference is backordered.

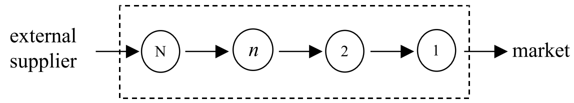

Fig. 2. The SC model.

The objective of the inventory policy is to minimize the average total SC cost over an infinite time horizon.

The considered costs are: (i) a fixed order cost K_n at every stage n, which includes the transportation cost and occurs any time stage n issues an order; (ii) the installation holding cost H_n at every stage n, which is the cost of keeping a stock unit per time unit at that stage; (iii) the backorder cost p at stage 1, which is a per unit and per time penalty incurred when market demand can not be immediately satisfied.

In particular, holding costs have been calculated with regard to the echelon rather than the installation stock. The echelon holding cost h_n at stage n is given by:

$$h_n = H_n - H_{n+1} \tag{1}$$

h_n is the incremental cost of keeping a stock unit at a given stage rather than at the previous one. It mainly takes into account the value added to the product when moving it from a stage to another closer to the customer.

When the echelon stock is used as a basis for charging holding costs, the average total cost in an N stage serial system is given by [28]:

$$C(\overline{T}) = \sum_{n=2}^{N}\left[\frac{K_n}{T_n} + \frac{1}{2}\lambda h_n T_n\right] + \left[\frac{K_1}{T_1} + \frac{1}{2}\lambda(H_0 - H_2)T_1\right] \tag{2}$$

being $\overline{T} = (T_1, \ldots, T_N)$, λ is the demand rate [unit/time], h_n is the echelon holding cost at stage n [cost/(time $\times$ unit)], H_n is the installation holding cost at stage n [cost/(time $\times$ unit)], p is the backorder cost

Table 2
The adopted global policy

Inventory management criteria	Policy
Optimization goal	Global
Control type	Centralized
Inventory control	Periodic
Temporal information requirement	Instantaneous
Spatial information requirement	Echelon

at stage 1 [cost/(time × unit)], $H_0 = pH_1/(p + H_1)$, K_n is the per order cost at stage n [cost].

Eq. (2) corresponds to the well-known economic order quantity model for a single stage, extended to a serial SC with backorder. Further details on multi-stage inventory models subjected to deterministic demand are given by [29].

The stochastic nature of the considered problem would suggest stochastic inventory models to be used. This in turn would require the probability distribution of the problem variables to be known, which is not the case. The fuzzy approach allows on the one hand uncertainty to be properly modeled, on the other hand the deterministic equation (2) to be utilized as long as defuzzified values are used in the place of deterministic ones.

In particular, all the coefficients in Eq. (2) are crisp values obtained by defuzzification through the moment rule, after that any necessary arithmetic operation (sum, subtraction, product, and division) is effected over the fuzzy sets according to fuzzy mathematics [30]. Further details are given in Appendix A.

The optimal stationary inventory policy is given by the vector $(T_1, T_2, \ldots, T_N)$ that solves the non linear programming problem that minimizes (2) subject to the integer ratio constraint:

$$T_n = m_n T_{n-1} \quad \text{for } n = 2, \ldots, N \tag{3}$$

where m_n is a positive integer [28].

Once vector $(T_1, T_2, \ldots, T_N)$ is determined, the target levels S_n $(n = 1, \ldots, N)$ are computed as follows:

$$S_n = \lambda(T_n + \mathrm{LT}) \quad \text{for } n = 1, \ldots, N \tag{4}$$

being LT the constant transportation lead time. In fact, S_n must cover demand during the review time interval plus the transportation lead time.

Moreover, the implementation of the echelon periodic-review control policy requires the echelon inventory position IP^e_n be monitored at each stage, which is given by (5) [9]

$$\mathrm{IP}^e_n = \sum_{j=1}^{n} \mathrm{IP}^i_j \quad \text{for } n = 1, \ldots, N \tag{5}$$

being IP^i_j the installation inventory position at stage j, which depends on: on hand inventory OH_j, outstanding backlogged orders at the previous stage SH_j, and backorders BO_j. In particular:

$$\mathrm{IP}^i_j = \mathrm{OH}_j + \mathrm{SH}_j - \mathrm{BO}_j \tag{6}$$

The installation inventory position IP^i_j measures the capacity of a given stage j to cope with future demand based on the inventory available at that stage. However, when a multi-echelon system is considered, the key issue is the capacity of any stage to cope with the final customer demand, which depends on inventory existing at the given stage and at all the downstream stages. Hence, the echelon inventory position IP^e_n proves a more appropriate measurement of inventory.

The order quantity at stage n amounts to the difference between the target level S_n and the echelon inventory position IP^e_n.

4. An application

In this section, it is shown the way in which the proposed methodology is applied. To this end a simple three stage SC is considered. After that the global policy is determined, a simulation analysis is carried out to assess the advantage of a global inventory policy relative to a local one. A sensitivity analysis is determined, by considering two demand patterns, i.e. low and high demand, and three patterns of the holding costs. In particular, the increase of the per unit and per time holding cost H_n when moving downstream in the SC is assumed to be low (L), medium (M), and high (H). Notice that the average holding cost averaged over the SC stages increases when the holding cost pattern passes from L to H.

With regard to the local policy, which is used as a benchmark, it pursues a local optimization under a decentralized control type, adopts a periodic-review inventory control, and is based on the installation stock concept.

Under the local policy, the SC stages manage their own inventory independently from each other: at the review time interval T_n the installation stock is reviewed and, if necessary, an order is issued to raise the installation inventory position up to the target level S_n. The vector (T_1, T_2, T_3) is computed by independently minimizing the total

Table 3
Cost and time variables

Cost variables
Order cost ($ per unit)
$K_1 = 45$, $K_2 = 40$, $K_3 = 35$
Installation holding cost ($ per month per unit)
Pattern L
$H_1 =$ about $30 =$ triangle $[x, 15, 30, 40]$; $H_2 =$ about $15 =$ triangle $[x, 10, 15, 25]$; $H_3 =$ about $7.5 =$ triangle $[x, 4, 7.5, 10]$
Pattern M
$H_1 =$ about $75 =$ triangle $[x, 65, 75, 90]$; $H_2 =$ about $15 =$ triangle $[x, 10, 15, 25]$; $H_3 =$ about $3 =$ triangle $[x, 0, 3, 7]$
Pattern H
$H_1 =$ about $150 =$ triangle $[x, 140, 150, 165]$; $H_2 =$ about $15 =$ triangle $[x, 10, 15, 25]$; $H_3 =$ about $1.5 =$ triangle $[x, 0, 1.5, 10]$
Backorder cost ($ per month per unit)
$p =$ about $50 =$ triangle $[x, 40, 50, 60]$
Time variables
Transportation time (month)
$\mathrm{LT}_1 = 0.015$ month, $\mathrm{LT}_2 = 0.015$, $\mathrm{LT}_3 = 0.015$

Table 4
Market demand

Market demand (units per week)
Pattern L
Week no. 1: about $75 =$ triangle $[x, 45, 75, 90]$
Week no. 2: about $80 =$ triangle $[x, 60, 80, 120]$
Week no. 3: about $85 =$ triangle $[x, 55, 85, 105]$
Week no. 4: about $90 =$ triangle $[x, 50, 90, 123]$
Pattern H
Week no. 1:
about 75 units per week $=$ triangle $[x, 50, 80, 95]$
Week no. 2:
about 80 units per week $=$ triangle $[x, 65, 85, 125]$
Week no. 3:
about 85 units per week $=$ triangle $[x, 60, 90, 115]$
Week no. 4:
about 90 units per week $=$ triangle $[x, 55, 95, 128]$

Table 5
Reorder time intervals (months)

Holding cost pattern	SC stage	Demand pattern			
		L		H	
		Global policy	Local policy	Global policy	Local policy
L	1	0.197	0.120	0.193	0.117
	2	0.197	0.127	0.193	0.123
	3	0.197	0.169	0.193	0.164
M	1	0.157	0.095	0.151	0.093
	2	0.157	0.127	0.151	0.123
	3	0.157	0.265	0.151	0.258
H	1	0.139	0.090	0.135	0.090
	2	0.139	0.127	0.135	0.123
	3	0.139	0.368	0.135	0.358

costs at each SC stage. Then, the target level S_n is determined by (4). The installation inventory position is given by (6).

Cost and time variables and market demand are reported in Tables 3 and 4, wherein the notation *about* $c =$ *triangle* $[x, l, c, u]$ stands for:

$$\text{triangle } [x, l, c, u] = \begin{cases} 0 & x \leqslant l \\ \dfrac{x-l}{c-l} & l \leqslant x \leqslant c \\ \dfrac{u-x}{u-c} & c \leqslant x \leqslant u \\ 0 & x \geqslant u \end{cases} \tag{7}$$

As described in Section 3.1, the monthly demand rate has been obtained by adding the weekly fuzzy demands as reported in Table 4.

Based on the above data, the global and local policies for the six cases are defined by the reorder time intervals T_n as reported in Table 5.

The performance of the two policies have been measured via simulation over a time horizon of 1000 time units, by the commercial package ARENA [31]. Simulation requires that probability distributions be used to model uncertain variables.

However the problem uncertain variables (i.e. market demand and inventory related costs) have been described by membership rather than probability functions. Therefore, the given *possibility* distributions need to be transformed into *probability* distributions. This task, which is not trivial due to the substantial difference between membership and probability functions, has been accomplished following Mizumoto and Tanaka [32]: given that, for every uncertain problem variable, information on probability is not available, the most reasonable assumption is that all the *possible* values (all the elements that belong to a set) are equally *probable*. As a result, uniform distributions defined over the possibility function domains have been utilized to model all the uncertain problem variables.

Furthermore, it must be emphasized that simulation is aimed at comparing the *global* vs. the *local policies*, whereas it does not pretend (and there is no need) to analyze the advantages due to the use of fuzzy set theory. Thus, the assumptions on the probability distributions are not relevant in this sense.

The SC costs observed during simulation are reported in Table 6.

The global inventory policy is as easy to be computed and implemented as the local inventory policy and is always characterized by a total cost lower than the local policies.

Table 6
The global vs. the local policy

Holding cost pattern	Costs ($/month)	Demand pattern			
		L		H	
		Global policy	Local policy	Global policy	Local policy
L	Total	1881	2328	1862	2332
	Order	608	897	622	923
	Holding	1064	1263	984	1168
	Backorder	208	167	256	241
M	Total	2982	3103	2755	3051
	Order	764	918	794	947
	Holding	2050	1962	1765	1827
	Backorder	167	222	195	277
H	Total	4253	4395	4111	4324
	Order	866	910	883	923
	Holding	3245	3248	3045	3122
	Backorder	142	237	183	279

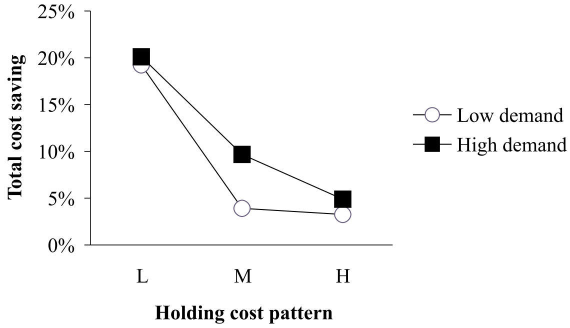

Fig. 3. Global vs. local policy.

In Fig. 3 the performance improvement (percentage of total cost saving) of the global policy with respect to the local one is depicted.

The benefits from using the global policy diminish when the holding cost pattern passes from L to H. In fact, this makes the inventory related costs change from being quite uniformly to unevenly distributed among the SC stages. In particular, in the latter case (pattern H) most of the costs are localized within stage 1. In such a situation, independently minimizing costs (particularly the costs at stage 1) tends to achieve results as globally minimizing does.

With regard to the influence of market demand, the relative cost saving increases with demand. This is probably due to the fact that the performance differences are magnified by the high volume of material flows.

5. Conclusions

This paper has addressed two key issues of inventory management in SCs, namely the uncertainty associated with market demand and inventory related costs and the need of a tight integration among the SC stages. In particular, a methodology has been proposed to define global inventory management policies that are both easy to be implemented and near optimal for the whole SC.

The methodology addresses uncertainty through fuzzy set theory, which is more appropriate than stochastic techniques to deal with market demand and inventory related costs, especially when the environment (e.g. the market) is complex and turbulent. The need of integration is taken into account by the adoption of the echelon concept to measure inventory and the holding costs.

An example of the proposed methodology is given to clarify its application. Then, a simulation analysis is carried out for evaluating the advantage associated with a tight integration among the SC stages. To this end, the *global* inventory policy has been compared against a *local* inventory policy, which is defined by independently minimizing the total cost associated with every SC stage. The global policy performs always better that the local policy does. Furthermore, the influence of different patterns of holding costs and market demand on performance has been analyzed. The benefits from integration increase when inventory costs do not differ much among SC stages as well as when demand is high.

The contribution of fuzzy set theory for dealing with uncertainty has been discussed under a theoretical perspective, pointing out its appropriateness to model market demand and inventory costs. Further research will address other uncertainty sources, such as suppliers' lead times or the quantities they actually deliver. Also, real cases (i.e. real SCs) will be analyzed to assess the performance enhancement achievable through the proposed methodology.

Appendix A

In this section some basic concepts of the fuzzy set theory are reviewed. These concepts are utilized within the paper to model demand and inventory related costs. In particular, the following is aimed at clarifying the way in which fuzzy arithmetic operations and defuzzification are accomplished in the paper. Further details can be found in [27,33,34].

A.1. Fuzzy theory basics concepts

First, let us note that in the fuzzy theory literature the term *crisp* usually refers to deterministic objects (i.e. deterministic sets, quantities, numbers) as opposed to fuzzy ones. Given a set U consisting of some elements, a fuzzy subset A of U ($A \subset U$), is characterized by the membership function μ_A: $U \to [0,1]$ such that $\forall x \in U$:

$$\mu_A(x) = 0 \quad \text{if } x \notin A$$

$$\mu_A(x) \in [0,1] \quad \text{otherwise}$$

Let A and B be two fuzzy subsets of U ($A \subset U$ and $B \subset U$), R the set of all real numbers, and R_0 the set of all non-zero real numbers. The following relations hold:

The *complement* of A is the fuzzy set $\overline{A}$ such that

$$\mu\overline{A}(x) = 1 - \mu_A(x) \quad \forall x \in U$$

The *union* of A and B is the fuzzy set $C = A \cup B$ such that

$$\mu_C(x) = \max(\mu_A(x), \mu_B(x)) \quad \forall x \in U$$

The *intersection* of A and B is the fuzzy set $D = A \cap B$ such that

$$\mu_D(x) = \min(\mu_A(x), \mu_B(x)) \quad \forall x \in U$$

The *opposite fuzzy set* of A is the fuzzy set $-A$ such that

$$\mu_{-A}(x) = \mu_A(-x) \quad \forall x \in U$$

The *reciprocal fuzzy set* of A the fuzzy set $1/A$ such that

$$\begin{array}{ll} \mu_{1/A}(x) = \mu_A(1/x) & \text{for } x \neq 0 \quad \forall x \in U \\ \mu_{1/A}(x) = 0 & \text{for } x = 0 \end{array}$$

The *defuzzification* of A consists in the conversion of the set into a crisp number. There exist different defuzzification techniques. In this paper the *moment rule* is adopted. Given a fuzzy set, such a rule returns the mean of the set elements that assume the maximum value as the crisp number associated with the set.

A.2. Fuzzy arithmetic

If A and B are fuzzy quantities, namely any vague numerical quantity whose possible values are real numbers, the following expressions hold:

The *sum* of A and B is also the fuzzy set $S = A \oplus B$ such that

$$\mu_S(x) = \sup_{y \in R} (\min(\mu_A(y), \mu_B(x - y)))$$

The *product* of A and B is the fuzzy set $P = A \odot B$ such that

$$\mu_P(x) = \sup_{y \in R} (\min(\mu_A(y), \mu_B(x/y))) \quad \text{for } x \neq 0 \ \forall x \in R$$

and

$$\mu_P(0) = \max(\mu_A(0), \mu_B(0)) \quad \text{for } x = 0$$

The sum and the product of fuzzy sets are two applications of a general principle called *representation principle*:

$$\mu_C(z) = \sup[\min(\mu_A(x), \mu_B(y)) : x, y \in R, z = f(x, y)] \quad \forall z \in R$$

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