

- [1] A real number, in mathematics, is a quantity **that** can be expressed as an infinite decimal expansion. Real numbers are used in measurements of continuously varying quantities such as size and time, in contrast to the natural numbers 1, 2, 3, ..., arising from counting. The word *real* distinguishes **them** from the complex numbers involving the symbol  $i$ , or  $\sqrt{-1}$ , used to simplify the mathematical interpretation of effects such as **those** occurring in electrical phenomena. The real numbers include the positive and negative integers and fractions (or rational numbers) and also the irrational numbers. The irrational numbers have decimal expansions **that** do not repeat **themselves**, in contrast to the rational numbers, the expansions of **which** always contain a digit or group of digits **that** repeats **itself**, as  $1/6 = 0.16666...$  or  $2/7 = 0.285714285714...$ . The decimal formed as 0.4244244424442... has no regularly repeating group and is thus irrational.
- [2] The most familiar irrational numbers are algebraic numbers, **which** are the roots of algebraic equations with integer coefficients. For example, the solution to the equation  $x^2 - 2 = 0$  is an algebraic irrational number, indicated by  $\sqrt{2}$ . Some numbers, such as  $\pi$  and  $e$ , are not the solutions of any such algebraic equation and are thus called transcendental irrational numbers. **These numbers** can often be represented as an infinite sum of fractions determined in some regular way, indeed the decimal expansion is one such sum.
- [3] The real numbers can be characterized by the important mathematical property of completeness, meaning that every nonempty set **that** has an upper bound has a smallest such bound, a property not possessed by the rational numbers. For example, the set of all rational numbers the squares of **which** are less than 2 has no smallest upper bound, because  $\sqrt{2}$  is not a rational number. The irrational and rational numbers are both infinitely numerous, but the infinity of irrationals is "greater" than the infinity of rationals, in the sense that the rationals can be paired off with a subset of the irrationals, while the reverse pairing is not possible.

1. **LECTOCOMPREENSIÓN.** (a) Consulte el texto e indique si las siguientes oraciones son verdaderas o falsas (V/F). Consigne los renglones de referencia.

V/F	ORACIÓN	Renglón
	1. Algunos números reciben la denominación de números irracionales trascendentales.	
	2. Los números algebraicos no se consideran como números irracionales más comunes.	
	3. Los enteros y fracciones positivos y negativos (racionales) y los números irracionales están incluidos dentro de los números reales.	
	4. No es posible caracterizar a los números reales mediante la propiedad de completitud.	
	5. Se emplea números reales en las mediciones de cantidades que no varían constantemente.	

**2. VOCABULARIO.(a). Encuentre sinónimos de las siguientes palabras:**

1. employed (r. 2) \_\_\_\_\_
2. as opposed (r. 3) \_\_\_\_\_
3. iterate (r. 9) \_\_\_\_\_
4. have (r. 10) \_\_\_\_\_
5. instance (r. 14) \_\_\_\_\_
6. therefore (r. 17) \_\_\_\_\_
7. established (r. 18) \_\_\_\_\_
8. combined (r. 26) \_\_\_\_\_

**3. VOCABULARIO.(b). Encuentre antónimos de las siguientes palabras:**

1. randomly (r. 2) \_\_\_\_\_
2. obscure (r. 5) \_\_\_\_\_
3. lack (r. 8) \_\_\_\_\_
4. other than (r. 16) \_\_\_\_\_
5. seldom (r. 17) \_\_\_\_\_
6. erratic (r. 18) \_\_\_\_\_
7. worthless (r. 20) \_\_\_\_\_
8. greater (r. 23) \_\_\_\_\_

**4. REFERENCIA EN EL CONTEXTO. Lea nuevamente el texto y consigne a qué hacen referencia las palabras dadas.**

1. that (r.1) \_\_\_\_\_
2. them (r.4) \_\_\_\_\_
3. those (r.6) \_\_\_\_\_
4. that (r.8) \_\_\_\_\_
5. themselves (r.9) \_\_\_\_\_
6. which (r.9) \_\_\_\_\_
7. that (r.10) \_\_\_\_\_
8. itself (r.10) \_\_\_\_\_
9. which (r.13) \_\_\_\_\_
10. these numbers (r.17) \_\_\_\_\_
11. that (r.21) \_\_\_\_\_
12. which (r.23) \_\_\_\_\_

5. **LECTOCOMPREENSIÓN. (b)** Consulte el texto y responda las siguientes preguntas en castellano. Indique las referencias de renglón.

1. ¿Para qué se usa a los números complejos?

Renglón ►



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2. ¿Cómo se representa a los números irracionales trascendentales?

Renglón ►



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3. ¿Qué es un número real?

Renglón ►



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6. **VERIFICACIÓN DEL CONTENIDO.** Lea minuciosamente el texto y recomponga los conceptos compatibilizando el contenido de ambas columnas.

	The decimal formed as...
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A	...is an algebraic irrational number.
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	The infinity of irrationals is...
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B	...are less than 2 has no smallest upper bound.
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	The irrational numbers have...
--	--------------------------------

C	...decimal expansions containing a digit or group of digits that repeats itself.
---	--

	The rational numbers have...
--	------------------------------

D	...decimal expansions that do not repeat themselves.
---	--

	The set of all rational numbers whose squares...
--	--

E	...greater than the infinity of rationals.
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	The solution to the equation $x^2 - 2 = 0$ ...
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F	... 0.42442444244442... is irrational.
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7. **CONCEPTOS BÁSICOS.** Hallar la correspondencia entre cada palabra y su definición.

	COMPLEX	<b>A</b>	Applied to an irrational number in which no finite sequence of algebraic operations on integers can be equal to its value.
	DIGIT	<b>B</b>	Collection of distinct objects or numbers.
	FRACTION	<b>C</b>	Pertaining to numbers expressed as an ordered pair comprising a real number and an imaginary number.
	INFINITY	<b>D</b>	Referring to numbers which may be thought of as all points on an infinitely long number line.
	NATURAL	<b>E</b>	Relating to numbers that can be expressed as a fraction (or ratio) of two integers.
	RATIONAL	<b>F</b>	Subsidiary collection of objects contained in, an original given set.
	REAL	<b>G</b>	Relating to the set of positive integers.
	SET	<b>H</b>	Quantity or set of numbers without bound, limit or end.
	SUBSET	<b>I</b>	Symbol used to write numbers.
	TRANSCENDENTAL	<b>J</b>	Way of writing rational numbers used to represent ratios or division.

8. **TÍTULO.** Seleccione el mejor título para el texto.

- ☐ a. Infinite Decimal Expansions      ☐ c. Irrational Numbers  
☐ b. Real Numbers      ☐ d. Complex Numbers

9. **IDEA PRINCIPAL.** Indique cuál oración expresa más acabadamente la idea principal del texto.

- ☐ a. Irrational numbers possess decimal expansions which do not replicate.  
☐ b. Rational numbers are likely to be paired off with a subset of the irrational numbers.  
☐ c. Real numbers are quantities capable of being expressed as infinite decimal expansions.  
☐ d. The solution of some equations are algebraic irrational numbers.

**10. FUNCIONES DEL LENGUAJE.** Identifique y transcriba el nexos según la referencia de renglón dada. Indique la relación lógica, el equivalente en español y las ideas relacionadas.

1. Renglón 9	<u>Nexo lógico</u>	<u>Rel. Lógica</u>	<u>Equivalente</u>

Ideas relacionadas

**Idea 1:**

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**Idea 2:**

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2. Renglón 14	<u>Nexo lógico</u>	<u>Rel. Lógica</u>	<u>Equivalente</u>

Ideas relacionadas

**Idea 1:**

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**Idea 2:**

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3. Renglón 24	<u>Nexo lógico</u>	<u>Rel. Lógica</u>	<u>Equivalente</u>

Ideas relacionadas

**Idea 1:**

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**Idea 2:**

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**11. FUNCIONES COMUNICATIVAS.**a. Indique la función comunicativa existente en los renglones 1/2.☐

DEFINICIÓN

☐

COMPARACIÓN

☐

NARRACIÓN

(Tache lo que  
no corresponda)

define?

¿Qué se compara? \_\_\_\_\_  
narra? \_\_\_\_\_b. Indique la función comunicativa existente en los renglones 20/22.☐

DESCRIPCIÓN

☐

CLASIFICACIÓN

☐

INSTRUCCIÓN

(Tache lo que  
no corresponda)

describe?

¿Qué se clasifica? \_\_\_\_\_  
instruye? \_\_\_\_\_**12. CLOZE. Complete el texto con las palabras dadas.**

accurate	approximations	because	calculator	consists
decimals	instrument	numbers	rational	sequences

The set  $\mathbb{R}$  of real numbers \_\_\_\_\_ of *all* numbers that can be written as (possibly non-terminating and possibly non-repeating) decimals.

This description is \_\_\_\_\_ and conceptually it is valuable, but it is not of much practical use \_\_\_\_\_. It is not possible to write out a non-terminating non-repeating decimal or do calculations with it. In reality, when we do calculations with \_\_\_\_\_ (either by hand or by machine), we truncate them at some point and work with \_\_\_\_\_ which are rational numbers. The set of numbers that a \_\_\_\_\_ works with is not the set of real numbers or even the set of rational numbers - it is some subset of  $\mathbb{Q}$  that depends on the precision of the \_\_\_\_\_.

This arithmetic description of the real numbers highlights the following point. *All* \_\_\_\_\_ that can be expressed as decimals means *all* numbers that can be written as \_\_\_\_\_ of the digits 0,1, . . . ,9 (with a decimal point somewhere) with no pattern of repetition necessary in the digits. In the universe of all such things, the ones that terminate (i.e. end in an infinite string of zeroes) or have a repeating pattern from some point onwards are special and rare. These are the \_\_\_\_\_ numbers. The ones that have all zeroes after the decimal point are even more special - these are the integers.

**13. UBICACIÓN DE LA INFORMACIÓN.** Proporcione los números de renglón en los cuales se expresa las siguientes ideas.

- \_\_\_\_\_ 1. The infinity of irrationals is greater than the infinity of rationals.
- \_\_\_\_\_ 2. The expansions of rational numbers always contain a digit or group of digits that repeats itself.
- \_\_\_\_\_ 3. The set of all rational numbers whose the squares are less than 2 has no smallest upper bound.
- \_\_\_\_\_ 4. The solution to the equation  $x^2 - 2 = 0$  is an algebraic irrational number.

**14. FORMACIÓN DE LAS PALABRAS**

Completar los espacios con la forma apropiada de las palabras dadas.

*express, expresses, expressed, expressing, expression, expressions*

1. A *factorization* of a matrix  $A$  is an equation that expresses  $A$  as a product of two or more matrices.
2. An equation is a statement that says that two mathematical \_\_\_\_\_ have the same value.
3. Exponential notation provides a way of \_\_\_\_\_ very big and very small numbers on computers.
4. Factoring is the process of splitting a complicated \_\_\_\_\_ into the product of two or more simpler \_\_\_\_\_, called factors.
5. In applications involving time, formulas for functions are often \_\_\_\_\_ in terms of a variable  $t$  whose starting value is taken to be  $t = 0$ .
6. Integrals that involve a quadratic \_\_\_\_\_  $ax^2 + bx + c$ , where  $a \neq 0$  and  $b \neq 0$ , can often be evaluated by first completing the square, then making an appropriate substitution.
7. Irrational numbers are also needed to \_\_\_\_\_ most of the values for trigonometric functions, and two special numbers,  $\pi = 3.14159 \dots$  and  $e = 2.71828 \dots$  are both irrational.
8. The fundamental theorem of arithmetic says that any natural number can be \_\_\_\_\_ as a unique product of prime numbers.
9. The Pythagorean theorem \_\_\_\_\_ a relationship between the three sides of a right triangle:  $c^2 = a^2 + b^2$  where  $a$  and  $b$  are the lengths of the two legs, and  $c$  is the length of the hypotenuse.
10. There is always more than one way to \_\_\_\_\_ a function as a composition. For example, here are two ways to \_\_\_\_\_  $(x^2 + 1)^{10}$  as a composition that differ from that shown in the Table.
11. These formulas can be used to find limits of the remaining trigonometric functions by \_\_\_\_\_ them in terms of  $\sin x$  and  $\cos x$ .