Tarea Semanal
12
Síntesis de Cuadripolos

1)

a)

$$H(s) = \left.\frac{\frac{I_2}{V_1}}{\frac{I_1}{V_1}}\right|_{V_2=0} = \frac{I_2}{I_1} = \frac{s(s^2+1)}{3s\left(s^2+\frac{7}{3}\right)} = \frac{Y_{21}}{Y_{11}}$$

La transferencia de corriente se obtuvo a partir del cociente de los parámetros que están evaluados bajo la misma condición ($V_2=0$). De igual forma:

$$\frac{Z_{21}}{Z_{22}} = -\frac{Y_{12}}{Y_{11}}$$

Condiciones:

- Y_{12}: ESTABLE
- Y_{11}: FRP.

Como son parámetros Y (en corto circuito)

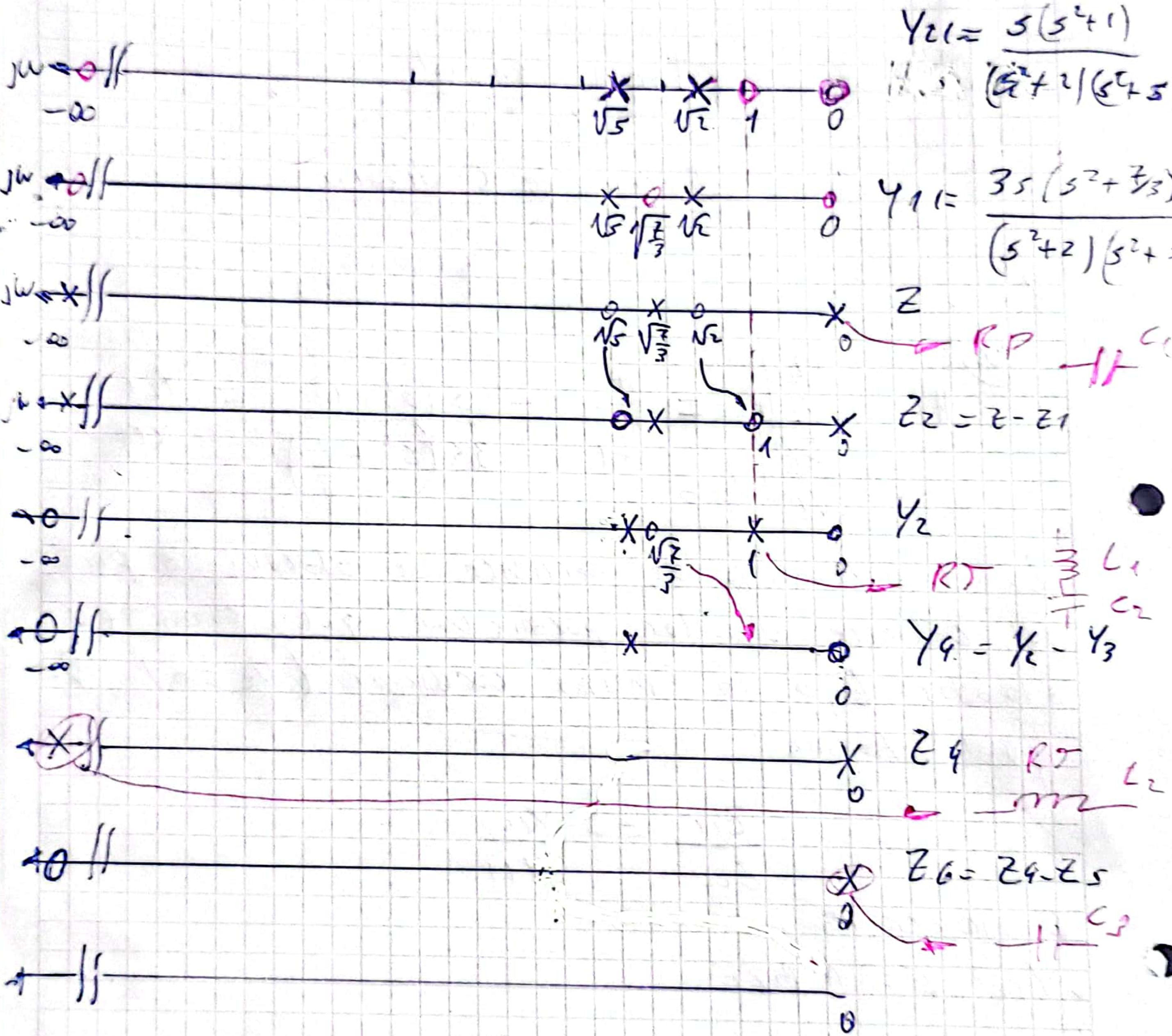
- Primer elemento puede estar en serie
- Último elemento debe estar en serie sino el cto. cto. lo anula.

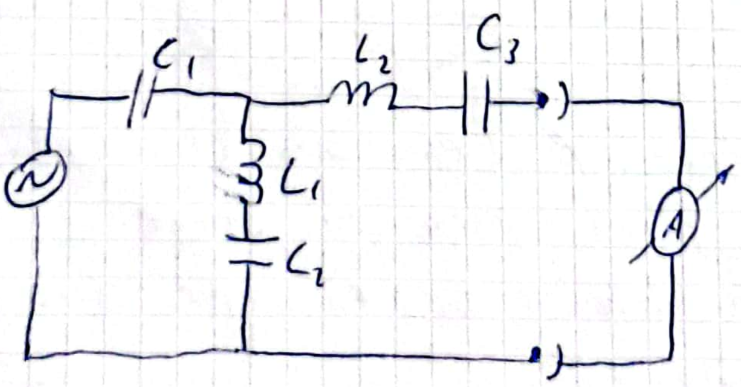
Remuevo en Y_{11} condicionando por los ceros de la transferencia que deberían cancelarse con los de Y_{21}

Metodo práctico

$$Y_{21} = \frac{s(s^2+1)}{(s^2+2)(s^2+5)}$$

$$Y_{11} = \frac{3s(s^2+7/3)}{(s^2+2)(s^2+5)}$$





(b) • Remoción parcial en 0

(K1)

$Z_2(j1) = 0 = Z - Z_1$

$Z_2 = Z - \frac{K_1}{s}$

$\lim_{s^2 \to -1} Z_2 = 0$

$K1 = \lim_{s^2 \to -1} Z \cdot s$

$K_1 = \lim_{s^2 \to -1} \frac{(s^2+2)(s^2+5)\cancel{s}}{3\cancel{s}(s^2+\frac{7}{3})} = \frac{4}{4} = 1$

$C_1 = 1F$

Remoción

$Z_2 = \frac{s^4+7s^2+10}{3s^3+7s} - \frac{1}{s} = \frac{s^4+4s^2+3}{(3s^3+7s)}$

$Z_2 = \frac{(s^2+3)(s^2+1)}{3s(s^2+\frac{7}{3})}$ $\quad Y_2 = \frac{3s(s^2+7/3)}{(s^2+3)(s^2+1)}$

• Remoción total en j1

(K2)

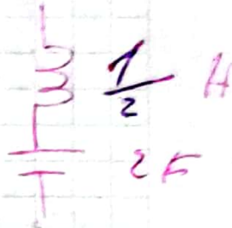
$Y_4 = Y_2 - \frac{2K_1 s}{s^2+\omega^2}$ $\to \infty$

$\lim_{s^2 \to -1} Y_4 = \lim_{s^2 \to -1} Y_2 - \lim_{s^2 \to -1} \frac{2K_1 s}{s^2+1^2}$

$$2K_1 = \lim_{s^2\to -1} Y_2 \frac{(s^2+1)}{s}$$

$$2K_1 = \lim_{s^2\to -1} \frac{3\cancel{s}(s^2+7/3)\cancel{(s^2+1)}}{(s^2+3)\cancel{(s^2+1)}\cancel{s}} = \frac{4}{2} = 2$$

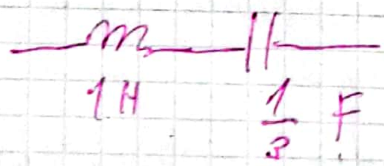
$$Y_4 = \frac{3s(s^2+7/3)}{(s^2+3)(s^2+1)} - \frac{2s}{(s^2+1)} \qquad \frac{1}{\frac{1}{2}s+\frac{1}{2s}}$$

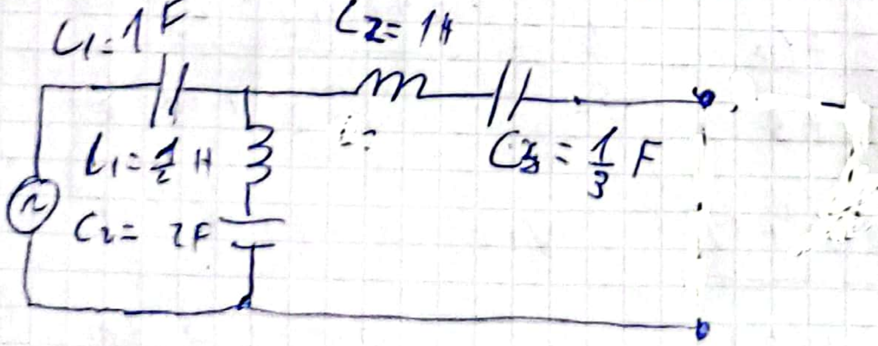


$$Y_4 = \frac{3s^3+7s-2s^3-6s}{(s^2+3)(s^2+1)} = \frac{s^3+s}{(s^2+3)(s^2+1)} = \frac{s\cancel{(s^2+1)}}{(s^2+3)\cancel{(s^2+1)}}$$

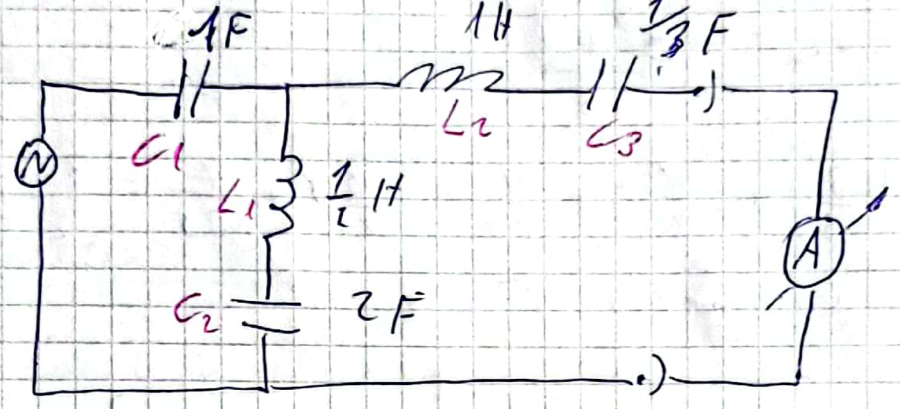
$$Y_4 = \frac{s}{s^2+3} \qquad Z_4 = \frac{s^2+3}{s} = s + \frac{3}{s}$$

Remoción total en ∞ y en 0





Verificación:



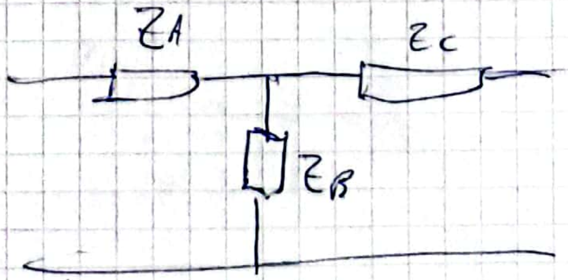
$s \to 0$ C_1 impone 0 de transmisión

$\omega = 1 \frac{rad}{s}$ (resuena) C_2 y L_1 imponen 0 de transmisión

$s \to \infty$ L_2 impone 0 de transmisión.

- Debo llegar a $H(s) = \dfrac{s^2+1}{3s^2+7}$

- Red "T" parámetros Z:



$Z_A = \frac{1}{s}$ $\qquad Z_B = \frac{1}{2}s + \frac{1}{2s}$ $\qquad Z_C = s + \frac{3}{s}$

$Z_B = \dfrac{s^2+1}{2s}$ $\qquad Z_C = \dfrac{s^2+3}{s}$

NOTA

$$Z_B = Z_{12} = \frac{s^2+1}{2s}$$

$$Z_C = Z_{22} - Z_{12} = \frac{s^2+3}{s}$$

$$Z_{22} = s + \frac{3}{s} + \frac{1}{2}s + \frac{1}{2s} = \frac{3}{2}s + \frac{7}{2s} = \frac{3s^2+7}{2s}$$

a. Conversión a parámetros T

$$D = \left.\frac{I_2}{I_1}\right|_{V_2=0} = \frac{Z_{21}}{Z_{22}} = \frac{\frac{s^2+1}{2s}}{\frac{3s^2+7}{2s}} = \boxed{\frac{s^2+1}{3s^2+7}}$$

VERIFICA

NOTA: $Z_{21} = Z_{12}$ por condición de reciprocidad que caracteriza los circuitos pasivos

② ⓐ

$$T(s) = \left.\frac{V_2}{V_1}\right|_{I_2=0} = \frac{K\,(s+1)}{(s+2)\,(s+4)}$$

Parametros Z

- Circuito abierto: Ultimo elemento en derivación, sino no influye en $T(s)$ ya que $I_2=0$
- Generador de tensión: Primer elemento en serie sino no influye en $T(s)$

- Z_{RC}: Para evitar los inductores

$$Z_0 > Z_\infty$$

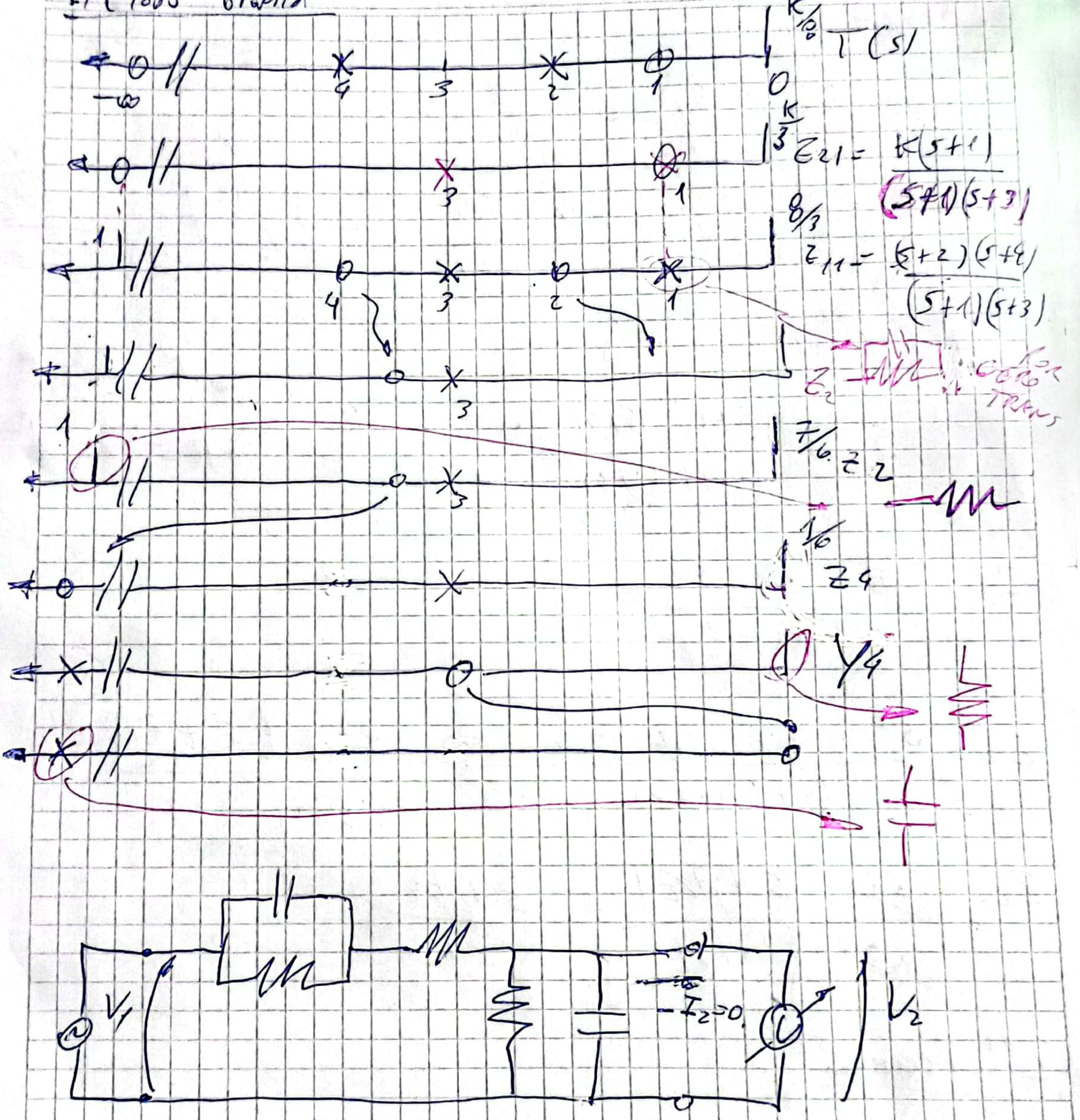
$$\begin{cases} V_1 = Z_{11} I_1 + Z_{12} I_2 \\ V_2 = Z_{21} I_1 + Z_{22} I_2 \end{cases}$$

$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} \qquad Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0}$$

$$T(s) = \frac{\left.\frac{V_2}{I_1}\right|_{I_2=0}}{\left.\frac{V_1}{I_1}\right|_{I_2=0}} = \left.\frac{V_2}{V_1}\right|_{I_2=0} = \frac{Z_{21}}{Z_{11}}$$

NOTA

Método Gráfico

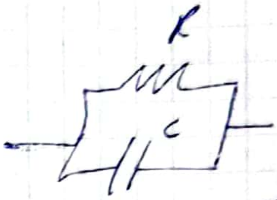


Cálculos

1era remoción

$$Z_2 = Z_{11} - \frac{K_1}{s+\sigma_1}$$

$$\lim_{s \to -1} Z_2 = 0 = Z_{11} - \frac{K_1}{s+1}$$



$$\frac{1/C}{\frac{1}{RC}+s}$$

$$\sigma \left\{ \frac{1}{RC} \right.$$

$$K_1 = \lim_{s \to -1} Z_{11}(s+1)$$

$$K_1 = \lim_{s \to -1} \frac{(s+2)(s+4)\cancel{(s+1)}}{\cancel{(s+1)}(s+3)} = \frac{3}{2}$$

$C = 2/3\ F$

$R = 3/2\ \Omega$

$$Z_2 = \frac{s^2+6s+8}{s^2+4s+3} - \frac{\frac{3}{2}}{s+1} =$$

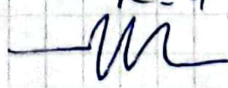
$$Z_2 = \frac{s^2+6s+8 - \frac{3}{2}s - \frac{9}{2}}{(s+1)(s+3)} = \frac{s^2 + \frac{9}{2}s + \frac{7}{2}}{(s+1)(s+3)}$$

$$Z_2 = \frac{\cancel{(s+1)}(s+7/2)}{\cancel{(s+1)}(s+3)} = \frac{s+7/2}{s+3}$$

2da Remoción

$$Z_4 = Z_2 - K_\infty$$

$$\lim_{s\to\infty} Z_4 = 0 \Rightarrow K_\infty = \lim_{s\to\infty} Z_2$$

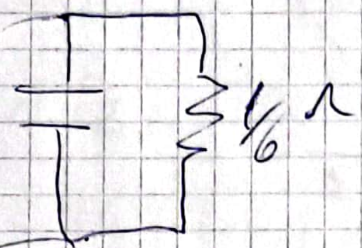
$K_\infty = 1$  $R = 1\ \Omega$

$$Z_4 = \frac{s+\frac{7}{2}}{s+3} - 1 = \frac{s+\frac{7}{2}-s-3}{s+3} = \frac{1/2}{s+7}$$

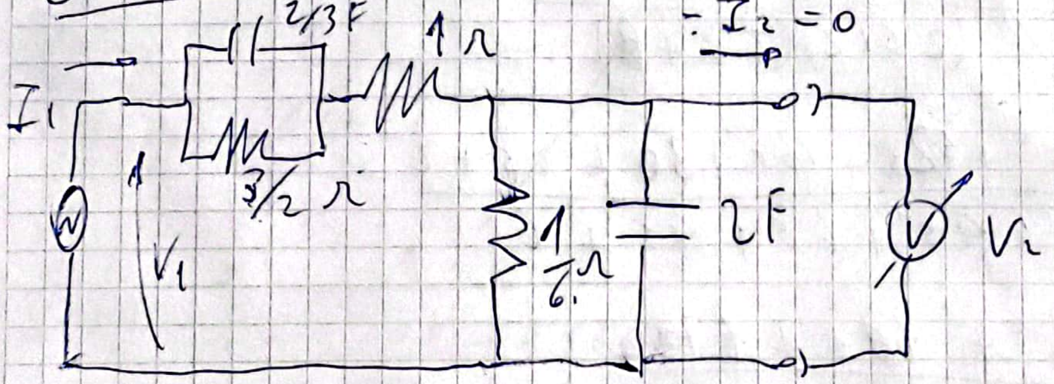
$$Z_4 = \frac{1/2}{s+3} \qquad Y_4 = \frac{s+3}{1/2} = 2s+6$$

3ra y 4ta remoción

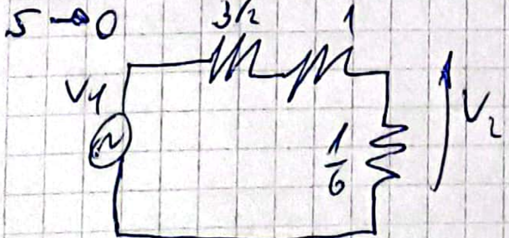
Y_4:



Circuito final



$$T(s) = K\frac{s+1}{(s+2)(s+4)}$$



$$\frac{V_2}{V_1} = \frac{1/6}{\frac{3}{2}+1+\frac{1}{6}} = \frac{1}{16}$$

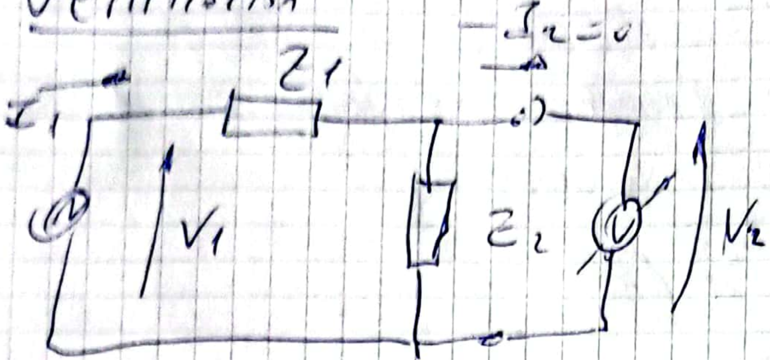
$$\frac{1}{16} = K\frac{1}{8}$$

Por

$K = \frac{1}{2} < 1$ PASIV

$$\boxed{T(s) = \frac{1}{2}\,\frac{s+1}{(s+2)(s+4)}}$$

Verificación



$$Z_1 = \frac{3}{2s+2} + 1$$

$$Z_2 = \frac{1}{2s+6}$$

Parámetros T (ABCD)

$$\begin{cases} V_1 = A\,V_2 + (-I_2)\,B \\ I_1 = C\,V_2 + (-I_2)\,D \end{cases}$$

$$A = \left.\frac{V_1}{V_2}\right|_{(-I_2)=0} = \frac{1}{T(s)}$$

$$\left.\frac{V_2}{V_1}\right|_{I_2=0} = \frac{Z_2}{Z_1+Z_2} = \frac{\frac{1}{2s+6}}{\frac{3}{2s+2} + 1 + \frac{1}{2s+6}}$$

$$\left.\frac{V_2}{V_1}\right|_{-I_2=0} = \frac{1}{\frac{3(2s+6)}{2s+2} + 2s+6+1}$$

$$\left.\frac{V_2}{V_1}\right|_{-I_2=0} = \frac{1}{\frac{6s+18+(2s+7)(2s+2)}{2s+2}}$$

$$\left.\frac{V_2}{V_1}\right|_{I_2=0} = \frac{2s+2}{4s^2+4s+14s+14+6s+18}$$

VERIFICA!

$$\left.\frac{V_2}{V_1}\right|_{I_2=0} = \frac{2}{4}\,\frac{s+1}{s^2+6s+8} = \frac{1}{2}\,\frac{s+1}{(s+2)(s+4)}$$

Parametros Y

- Condición de medición $I_2 = 0$: Último elemento en derivación
- Generador de tensión: Primer elemento serie
- Y_{RC} para evitar inductores

$$Y_\infty > Y_0$$

$$\begin{cases} I_1 = V_1 \cdot Y_{11} + V_2 Y_{12} \\ I_2 = V_1 Y_{21} + V_2 Y_{22} \end{cases}$$

$$I_2 = 0 \Rightarrow \frac{V_2}{V_1} = -\frac{Y_{21}}{Y_{22}} = T(s)$$

$$T(s) = K \frac{s+1}{(s+2)(s+4)} = \frac{\frac{s+1}{s+3}}{\frac{(s+2)(s+4)}{s+3}}$$

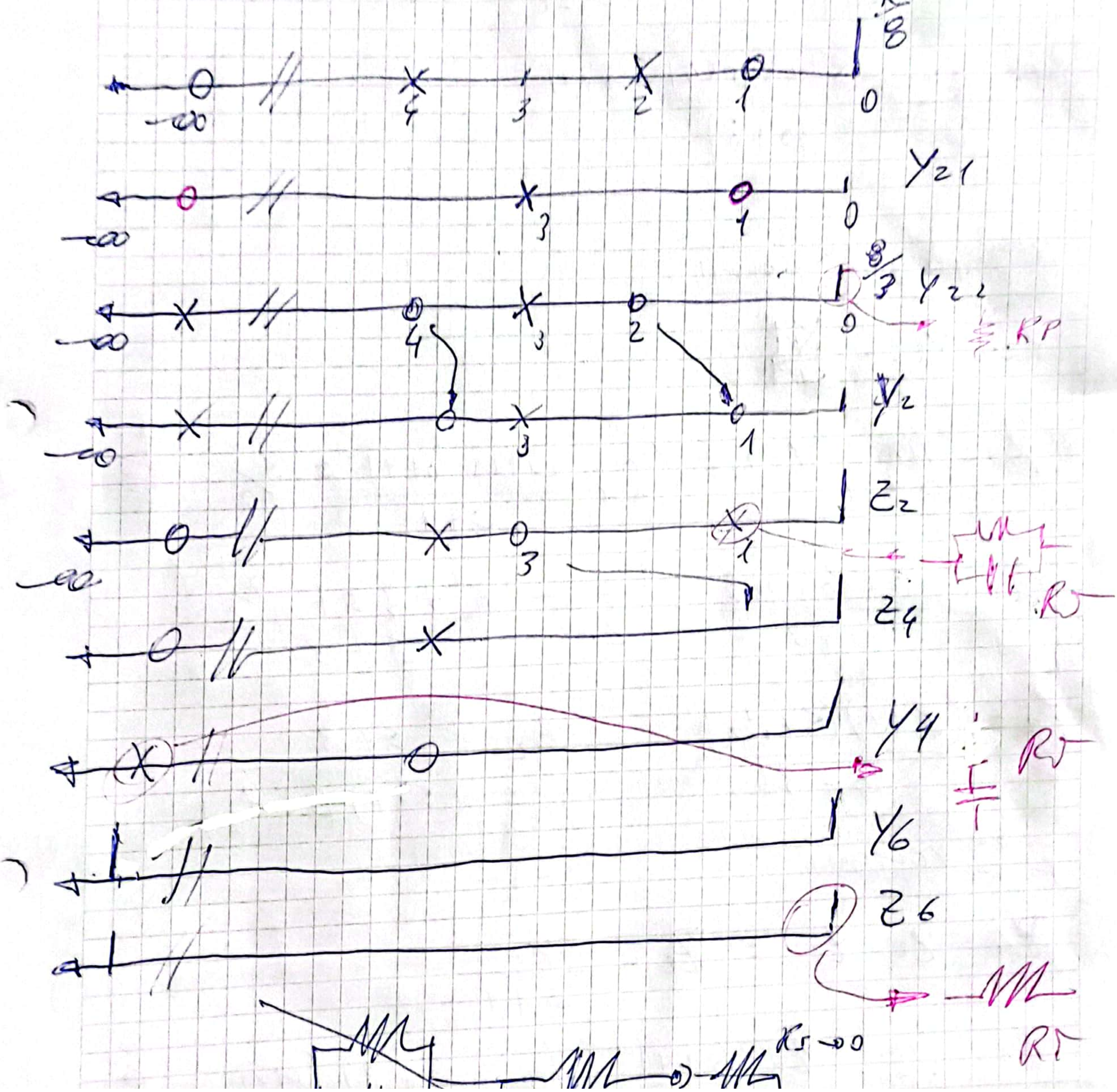
Polinomio auxiliar: $s+3$

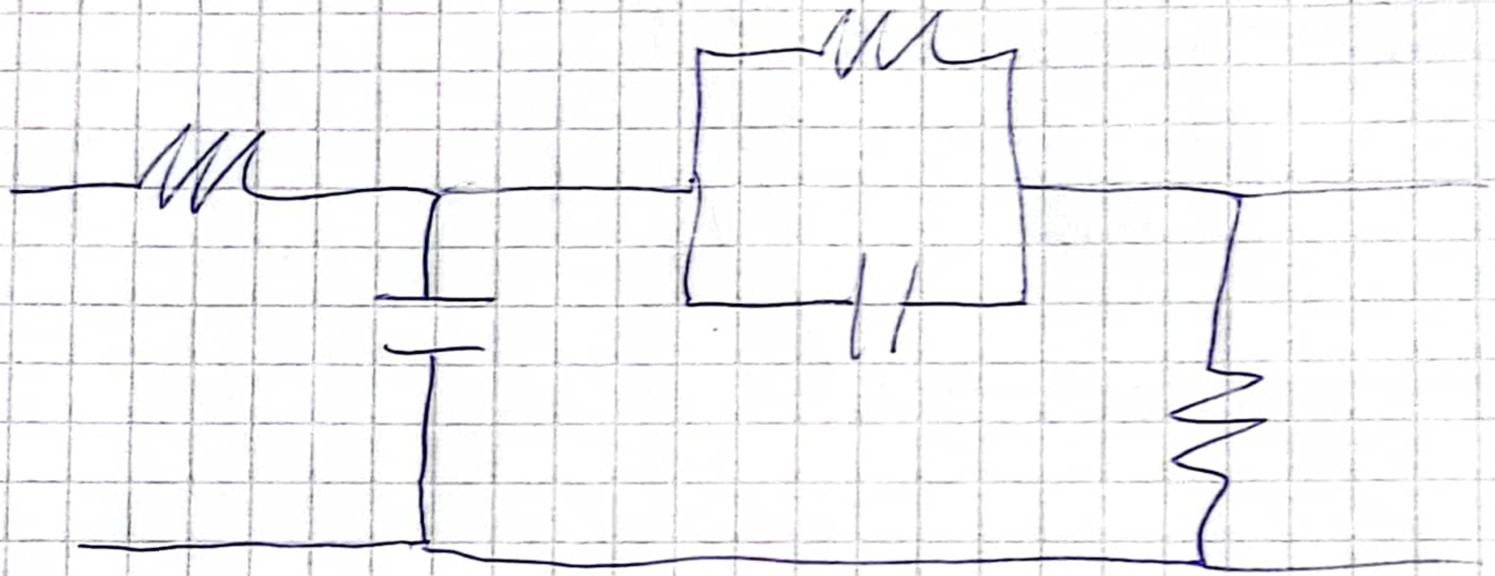
$$Y_{21} = \frac{s+1}{s+3}$$

$$Y_{22} = \frac{(s+2)(s+4)}{s+3}$$

NOTA

Metodo Grafico

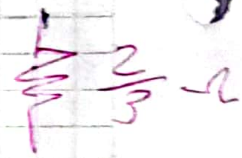




1era remoción parcial

$Y_2 = Y_{22} - K_1$

$K_1 = \lim_{s \to -1} Y_{22} = \lim_{s \to -1} \frac{(s+2)(s+4)}{s+3} = 3/2$



$Y_2 = \frac{s^2 + 6s + 8}{s+3} - \frac{3}{2} = \frac{s^2 + \frac{9}{2}s + \frac{7}{2}}{s+3}$

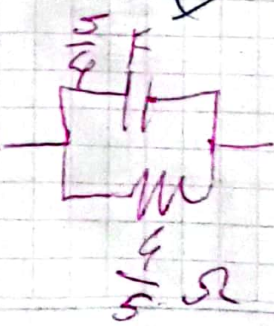
$Y_2 = \frac{(s+1)(s+7/2)}{s+3} \qquad Z_2 = \frac{s+3}{(s+1)(s+7/2)}$

2da remoción

$Z_4 = Z_2 - Z_3 = Z_2 - \frac{K_1}{s+1}$

$K_1 = \lim_{s \to -1} Z_2 (s+1) = \lim_{s \to -1} \frac{(s+3)\cancel{(s+1)}}{\cancel{(s+1)}(s+7/2)} = \frac{4}{5}$

$\frac{\frac{4}{5}}{s+1} = \frac{1}{\frac{5}{4}s + \frac{5}{4}}$



$\tau = RC = 1$

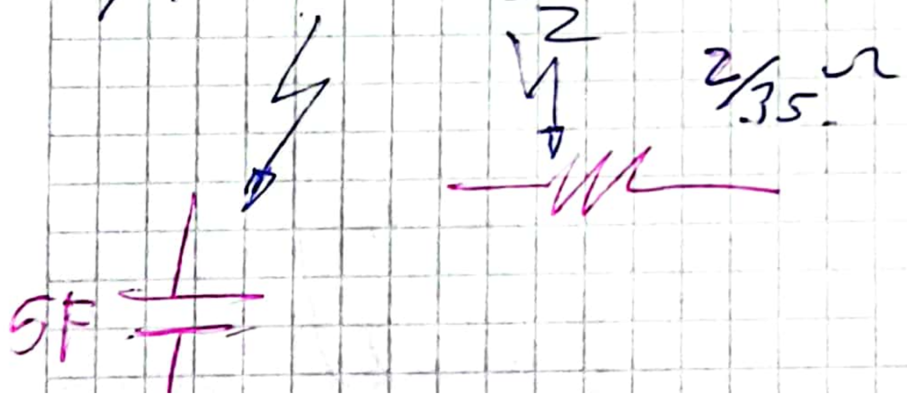
$$Z_4 = \frac{s+3}{s^2 + \frac{9}{2}s + \frac{7}{2}} - \frac{4/5}{s+1}$$

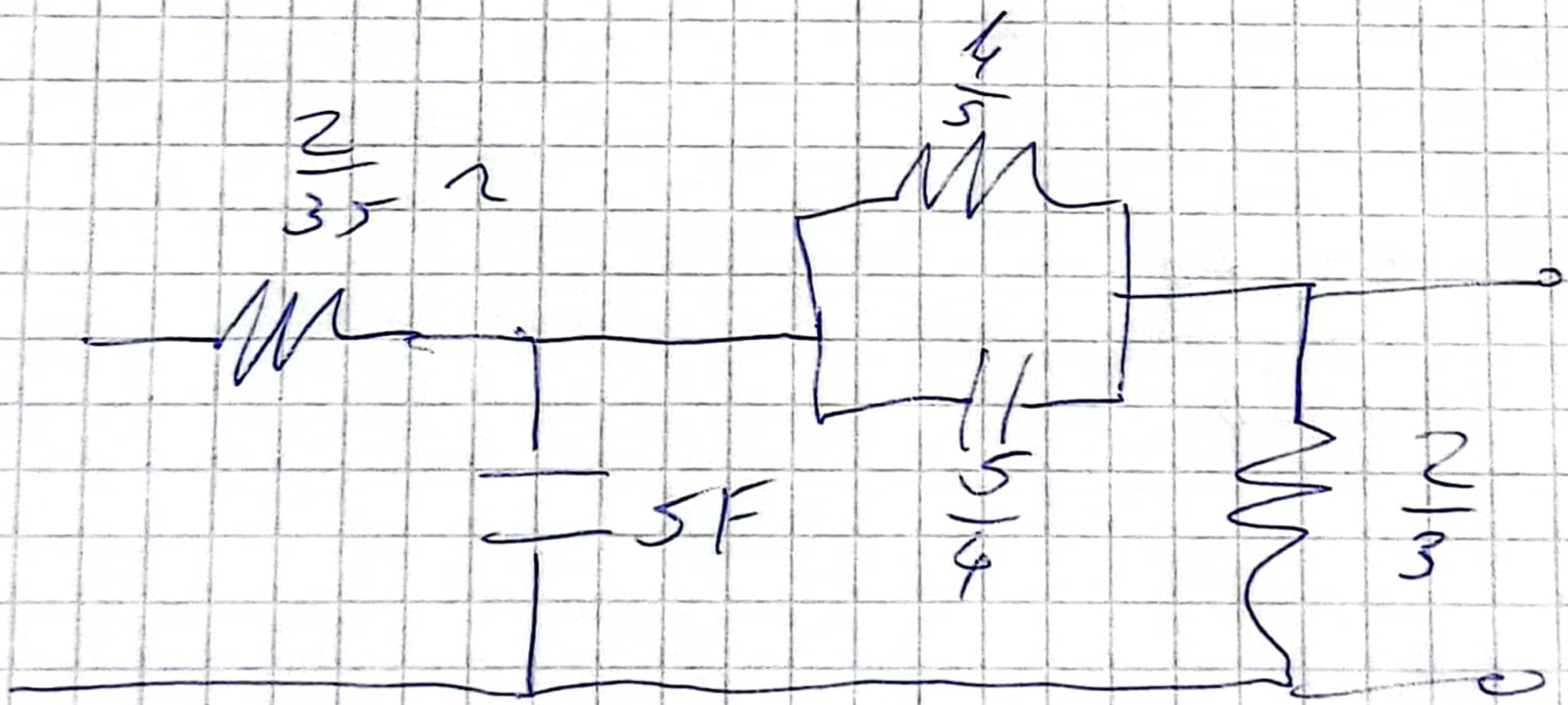
$$Z_4 = \frac{s + 3 - \frac{4}{5}s - \frac{14}{5}}{(s+1)\left(s+\frac{7}{2}\right)} = \frac{\frac{1}{5}s + \frac{1}{5}}{(s+1)\left(s+\frac{7}{2}\right)}$$

$$Z_4 = \frac{1}{5}\,\frac{\cancel{s+1}}{\cancel{(s+1)}\left(s+\frac{7}{2}\right)} = \frac{1/5}{s+\frac{7}{2}}$$

3ra remoción

$$Y_4 = 5s + \frac{3s}{2}$$




2/3
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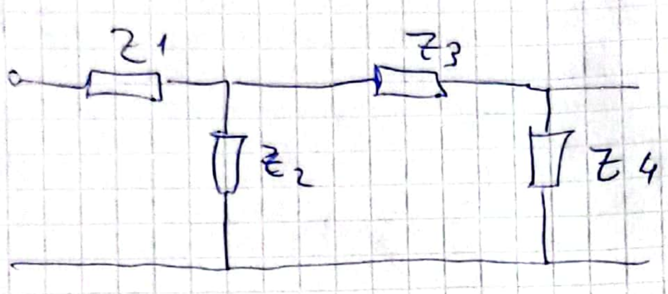
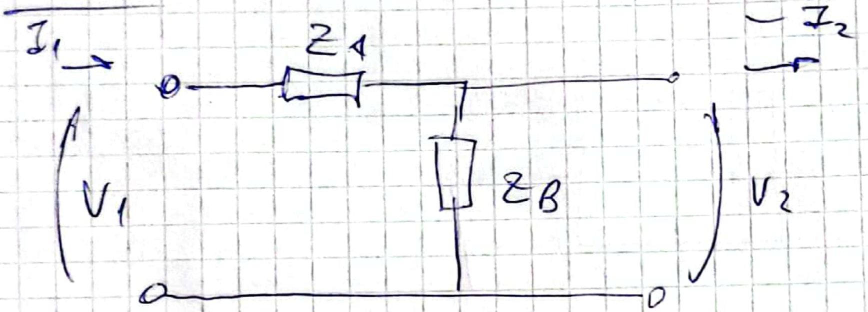
Verificación



TABCD



$$\begin{cases} V_1 = A\ V_2 + B\ (-I_2) \\ I_1 = C\ V_2 + D\ (-I_2) \end{cases}$$

$$A = \left.\frac{V_1}{V_2}\right|_{-I_2=0} = \frac{Z_A + Z_B}{Z_B}$$

$$B = \left.\frac{V_1}{-I_2}\right|_{V_2=0} = Z_A$$

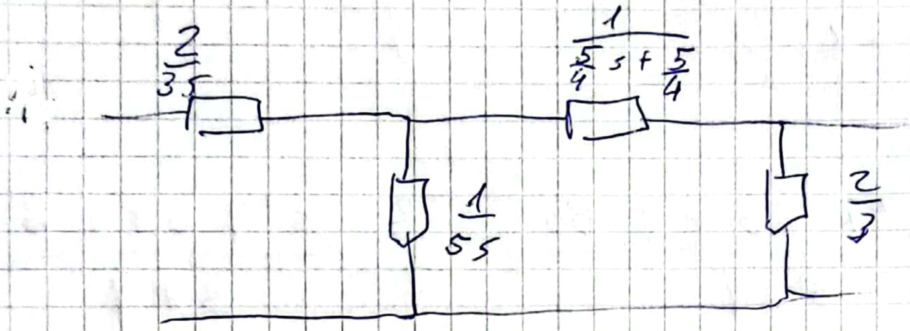
$$C = \left.\frac{I_1}{V_2}\right|_{-I_2=0} = \frac{1}{Z_B}$$

$$D = \left.\frac{I_1}{-I_2}\right|_{V_2=0} = 1$$

HOJA Nº

FECHA

$$T_R = \begin{pmatrix} A_1 & B_1 \\ C_1 & D_1 \end{pmatrix} \begin{pmatrix} A_2 & B_2 \\ C_2 & D_2 \end{pmatrix} = \begin{pmatrix} A_1 A_2 + B_1 C_2 & \dots \\ & \end{pmatrix}$$



$$A_1 = \frac{\frac{2}{3s} + \frac{1}{5s}}{\frac{1}{5s}} = \frac{10s}{3s} + 1 = \frac{2s+7}{7}$$

$$A_2 = \frac{\frac{\frac{4}{5}}{s+1} + \frac{2}{3}}{2/3} = \frac{6}{5} \frac{1}{s+1} + 1 = \frac{6 + 5s + 5}{5(s+1)}$$

$$A_2 = \frac{5s+11}{5s+5}$$

$$B_1 = \frac{2}{3s}$$

$$C_2 = \frac{3}{2}$$

$$A_R = \frac{(2s+7)}{7} \cdot \frac{(5s+11)}{(5s+5)} + \frac{3}{3s}$$

$$A_R = \frac{10s^2 + 22s + 35s + 77}{35s + 35} + \frac{3}{3s}$$

NOTA

$$A_R = \frac{1}{35}\left(\frac{10s^2+57s+77}{3 \cdot s+3\cdot 1} + \frac{3}{3s}\right)$$

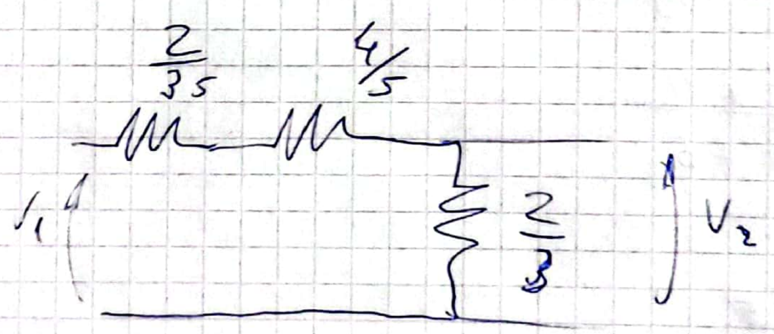
$$A_R = \frac{1}{35}\left(\frac{10s^2+57s+77+3s+3}{s+1}\right)$$

$$A_R = \frac{1}{35}\,\frac{10s^2+60s+80}{s+1} = \frac{2}{7}\,\frac{s^2+6s+8}{s+1}$$

$$\frac{V_2}{V_1} = \frac{1}{A_R} = \boxed{\frac{7}{2}\,\frac{s+1}{(s+2)(s+4)}}$$ Verifica!

Constante K

$s \to 0$:



$$T(s)=0 \Rightarrow k\,\frac{s+1}{(s+2)(s+4)} \Rightarrow \left.\frac{k}{8}\right|_{s=0}$$

$$\frac{V_2}{V_1} = \frac{2/3}{\frac{2}{3}+\frac{2}{35}+\frac{4}{5}} = \frac{7}{16} = \frac{k}{8} \Rightarrow \boxed{k = \frac{7}{2}}$$