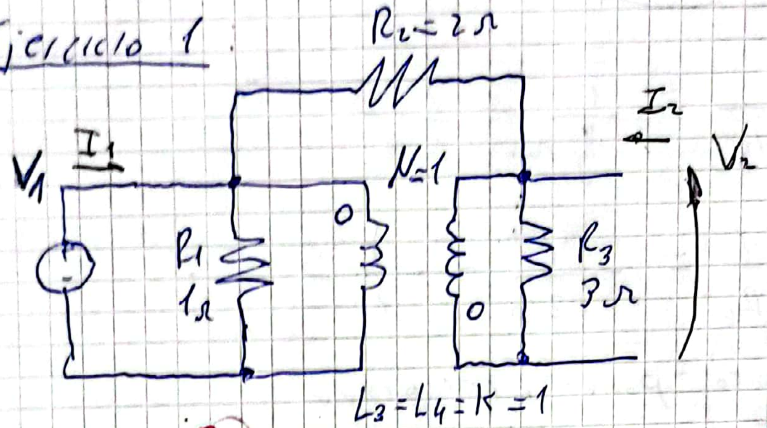
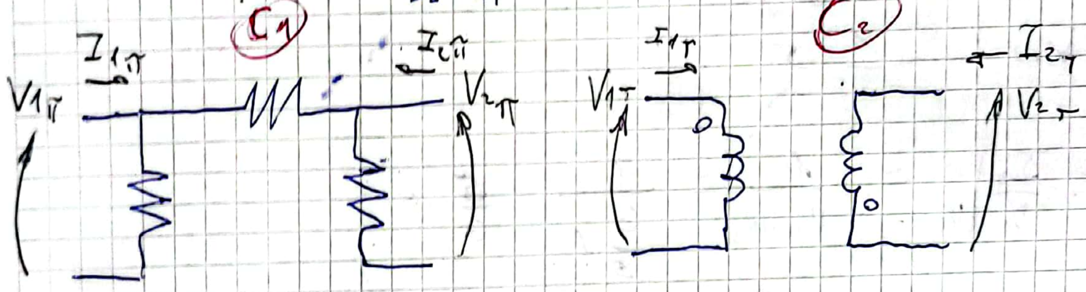
Tarea semanal
7: Cuadripolos

Ejercicio 1



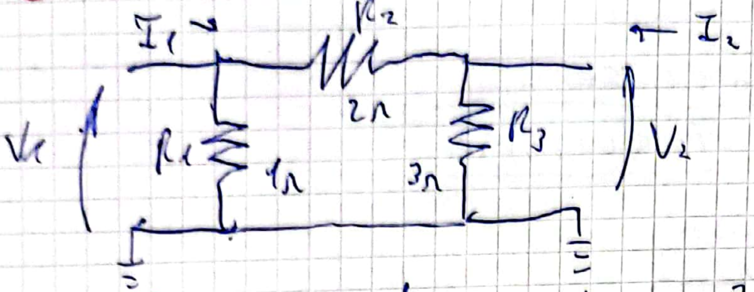


Como:

$$\begin{cases} V_{1\pi} = V_{1T} \\ V_{2\pi} = V_{2T} \end{cases} \quad y \quad \begin{cases} I_1 = I_{1\pi} + I_{1T} \\ I_2 = I_{2\pi} + I_{2T} \end{cases}$$

Están interconectados en paralelo, utilizo parámetros Y que se suman matricialmente y luego transformo a parámetros Z.

C1 Cuadripolo π



$$\begin{cases} I_1 = V_1 Y_{11} + V_2 Y_{12} \\ I_2 = V_1 Y_{21} + V_2 Y_{22} \end{cases}$$

$$Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} = \frac{1}{R_1 // R_2} = \frac{1}{\frac{2}{3}\Omega} = \frac{3}{2}\ S$$

$$Y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0} = -\frac{1}{R_2} = -\frac{1}{2\Omega} = -\frac{1}{2}\ S$$

Por ser un circuito pasivo / recíproco

$$Y_{21} = Y_{12} = -\frac{1}{2}\ S$$

$$Y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0} = \frac{1}{R_2 // R_3} = \frac{1}{\frac{6}{5}\Omega} = \frac{5}{6}\ S$$

Se verifica que cumple con los parámetros Y de un π

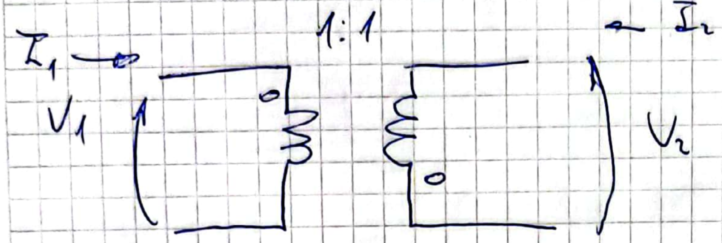
$$Y_1 = Y_{11} + Y_{12} = \frac{3}{2}\ S + \left(-\frac{1}{2}\ S\right) = \frac{1}{2}\ S$$

$$Y_2 = -Y_{12} = -Y_{21} = \frac{1}{2}\ S$$

$$Y_3 = Y_{22} + Y_{21} = \frac{5}{6}\ S + \left(-\frac{1}{2}\ S\right) = \frac{1}{3}\ S$$

$$Y_\pi = \begin{pmatrix} 3/2 & -\frac{1}{2} \\ -\frac{1}{2} & \frac{5}{6} \end{pmatrix}$$

C_2 Transformador ideal



Ecuaciones de un transformador ideal

$$\begin{cases} V_1 = -a\, V_2 \\ I_1 = -\frac{1}{a}(-I_2) \end{cases} \Rightarrow T_T = \begin{pmatrix} -a & 0 \\ 0 & -1/a \end{pmatrix}$$

$\Delta T = 1 \Rightarrow$ Condición de ~~polividad~~ reciprocidad

Bornes homólogos opuestos

Parámetros Y

$$Y_{11} = \frac{I_1}{V_1}\bigg|_{V_2=0} = \frac{-\frac{1}{a}(-I_2)}{-a\,\cancel{V_2}=0} \Rightarrow \infty$$

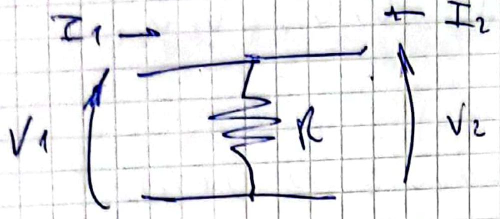
Como los parámetros Y no están definidos se procede a calcular por otro camino

Donde se observa que C_1, C_2 y C_4 están en cascada y luego en paralelo con C_3

Por lo tanto,

- Se calculan los parámetros T de C_1, C_2 y C_3
- El cuadripolo resultante se lo transforma a parámetros Y
- Se calcula los parámetros Y de C_3 y se suma la matriz.
- Se transforma a parámetros Z

Cuadripolos C_1 y C_4



$$T = \begin{cases} V_1 = A V_2 + B(-I_2) \\ I_1 = C V_2 + D(-I_2) \end{cases}$$

$$A = \left.\frac{V_1}{V_2}\right|_{-I_2=0} = 1$$ ya que $V_1 = V_2$

$$B = \left.\frac{V_1}{-I_2}\right|_{V_2=0} = 0$$ ya que $V_1 = V_2 = 0$

$$C = \left.\frac{I_1}{V_2}\right|_{-I_2=0} = \frac{1}{R}$$ ya que $V_1 = V_2$ y toda la I_1 pasa por R

$$D = \left.\frac{I_1}{-I_2}\right|_{V_2=0} = 1$$ ya que $I_1 = -I_2$

$$T_R = \begin{pmatrix} 1 & 0 \\ \frac{1}{R} & 1 \end{pmatrix}$$

$$\therefore T_{R_1} = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} \qquad T_{R_3} = \begin{pmatrix} 1 & 0 \\ \frac{1}{3} & 1 \end{pmatrix}$$

$\Delta T = 1$ $\qquad$ $\Delta T = 1$

Transformador ideal

Con bornes homologos opuestos

$$T_T = \begin{pmatrix} -a & 0 \\ 0 & -\frac{1}{a} \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$$

Cuadripolo C_3

$$\begin{cases} I_1 = V_1 Y_{11} + V_2 Y_{12} \\ I_2 = V_1 Y_{21} + V_2 Y_{22} \end{cases}$$

$$Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} = \frac{1}{R}$$ cortocircuito la salida

$$Y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0} = -\left.\frac{I_2}{V_2}\right|_{V_1=0} = -\frac{1}{R}$$ porque $I_1 = -I_2$

$$Y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0} = -\left.\frac{I_1}{V_1}\right|_{V_2=0} = -\frac{1}{R}$$ porque $I_2 = -I_1$

$$Y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0} = \frac{1}{R}$$ cortocircuitando la entrada

$$Y_R = \frac{1}{R}\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} \qquad Y_{R_2} = \begin{pmatrix} \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} \end{pmatrix}$$

$Y_{11} = Y_{22} \rightarrow$ simétrico

$Y_{12} = Y_{21} \rightarrow$ recíproco

Matriz resultante C_1, C_2, C_4

$$T_{124} = T_{R_1} \cdot T_{R_2} \cdot T_{R_3}$$

$$T_{124} = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 1/3 & 1 \end{pmatrix}$$

$$T_{124} = \begin{pmatrix} -1 & 0 \\ -1 & -1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 1/3 & 1 \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ -\frac{4}{3} & -1 \end{pmatrix}$$

Transformación de T a Y

$$T = \begin{cases} V_1 = V_2 \cdot A + (-I_2) B \\ I_1 = V_2 C + (-I_2) D \end{cases} \qquad Y = \begin{cases} I_1 = V_1 Y_{11} + V_2 Y_{12} \\ I_2 = V_1 Y_{21} + V_2 Y_{22} \end{cases}$$

$$A = \frac{V_1}{V_2}\Big|_{-I_2=0} \qquad B = \frac{V_1}{-I_2}\Big|_{V_2=0} \qquad Y_{11} = \frac{I_1}{V_1}\Big|_{V_2=0} \qquad Y_{12} = \frac{I_1}{V_2}\Big|_{V_1=0}$$

$$C = \frac{I_1}{V_2}\Big|_{-I_2=0} \qquad D = \frac{I_1}{-I_2}\Big|_{V_2=0} \qquad Y_{21} = \frac{I_2}{V_1}\Big|_{V_2=0} \qquad Y_{22} = \frac{I_2}{V_2}\Big|_{V_1=0}$$

- $\cancel{Y_{11} = \frac{I_1}{B(-I_2)}}$ $B = \frac{\cancel{V_1}}{-Y_{21}\cancel{V_1}} \Rightarrow \boxed{Y_{21} = -\frac{1}{B}}$

- $D = \frac{I_1}{-I_2} = \frac{Y_{11}\cancel{V_1}}{-Y_{21}\cancel{V_1}} \Rightarrow D = \frac{Y_{11}}{\frac{1}{B}} \Rightarrow \boxed{Y_{11} = \frac{D}{B}}$

- $I_2 = 0 \Rightarrow V_1 Y_{21} = -V_2 Y_{22}$

$$A = \frac{V_1}{V_2} \quad y \quad C = \frac{I_1}{V_2}$$

- $A = -\frac{\cancel{V_2} Y_{22}}{Y_{21}\cancel{V_2}} = -\frac{Y_{22}}{-\frac{1}{B}} \Rightarrow \boxed{Y_{22} = \frac{A}{B}}$

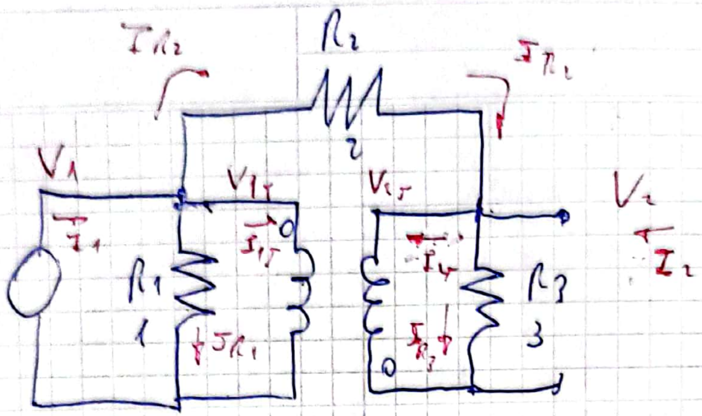
$$C = \frac{I_1}{V_2} = \frac{V_1 Y_{11} + V_2 Y_{12}}{-\frac{V_1 Y_{21}}{Y_{22}}} = -\left(\frac{\cancel{V_1} Y_{11} Y_{22}}{\cancel{V_1} Y_{21}} + \frac{V_2 Y_{12} Y_{22}}{V_1 Y_{21}}\right)$$

$$C = -\left(\frac{Y_{11} Y_{22}}{Y_{21}} + \frac{Y_{12} Y_{22}}{A\, Y_{21}}\right) = -\left(\frac{D A}{B \cancel{B}\left(-\frac{1}{\cancel{B}}\right)} + \frac{Y_{12} \cancel{A}}{\cancel{B}\cancel{A}\left(-\frac{1}{\cancel{B}}\right)}\right)$$

$$C = \frac{DA}{B} + Y_{12} \Rightarrow Y_{12} = C - \frac{DA}{B} = \boxed{\frac{BC - AD}{B} = -\frac{\Delta T}{B}}$$

Como $Y_{124} =$ P no está definido

Como $B = 0$ la matriz Y no está definida para este circuito.



$N = 1$

$$\begin{cases} V_{1T} = -V_{2T} & (1) \\ I_{1T} = I_{2T} & (2) \end{cases}$$

$$I_1 = I_{R_2} + I_{1T} + I_{R_1}$$

$$I_1 = \frac{V_1 - V_2}{2} + I_{1T} + V_1 \quad (3)$$

$$I_2 = -I_{R_2} + I_{2T} + I_{R_3}$$

$$I_2 = -\frac{V_1 - V_2}{2} + I_{2T} + \frac{V_2}{3} \quad (4)$$