# Biological robustness of the hippocampal spatial map

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*Yu. DABAGHIAN[1 *], F. MEMOLI [2], L. M. FRANK[1], G. CARLSSON [2]

[(1)]Department of Physiology and Keck Centre for Integrative Neuroscience, University of California, San Francisco, CA

[(2)]Department of Mathematics, Stanford University, Palo Alto, CA

(*) Current affiliation: Jan and Dan Duncan Neurological Research Institute, BCM, and the Department of the Computational and Applied Math., Rice University, Houston, TX

## Abstract

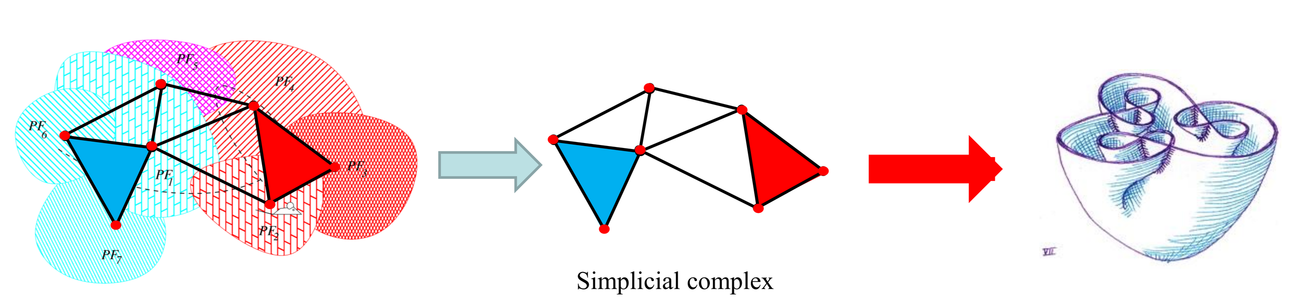
It is well known that the hippocampus plays crucial role in creating a spatial representation of the environment and in forming spatial memories. A number of experiments [1] suggests that the hippocampal map remains largely invariant under a wide range of spatial transformations of the environment and hence the temporal structure of neuronal firing in the hippocampus, which is the key determinant of the spatial information communicated to downstream neurons, provides predominantly topological information. This implies that the hippocampus encodes topological maps, based in the patterns of temporal overlap between the spike trains produced by the hippocampal neurons, which can be used to build a computational model of hippocampal activity. Clearly, the mechanism of global topological information encoding must be robust with respect to biological variability of the population activity in the hippocampus, i.e. it must persist within the observed range of parameters that characterize place cell firing.

To verify this, we build a computational model of the hippocampal topological map and investigate its robustness with respect to independent variations of the biological parameters, e.g. the firing rates, the of sizes of the firing fields, the number of active cell, etc., using the persistent homology method [2]. Using the simulated data is essential in our approach because it allows scanning the full range for each parameter independently and hence establishing the theoretical boundaries of topological stability regime for the hippocampal map.

After establishing the range of topological stability of the hippocampal map, we compare it with the experimentally observed values [3] which show consistency with our theoretical model.

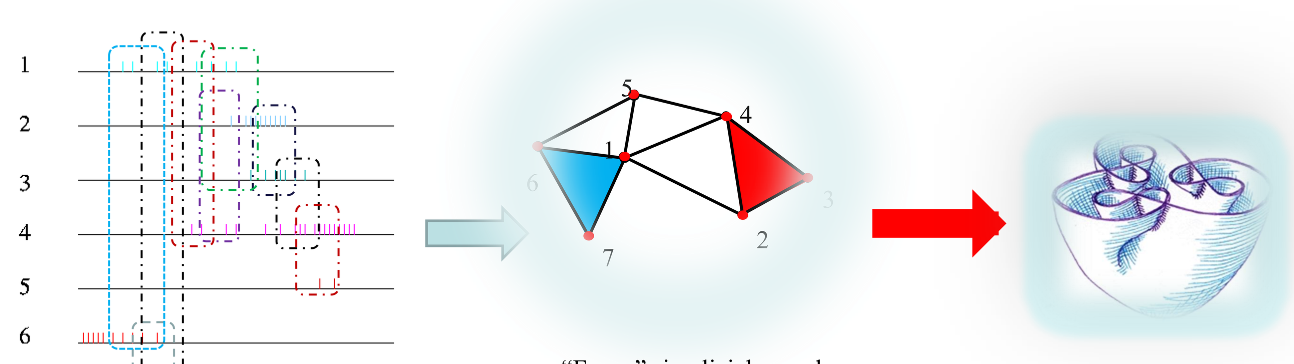
## Computational model

**A. In standard theory, the topological structure of space can be deduced from the pattern of spatial overlaps between regions that cover it**



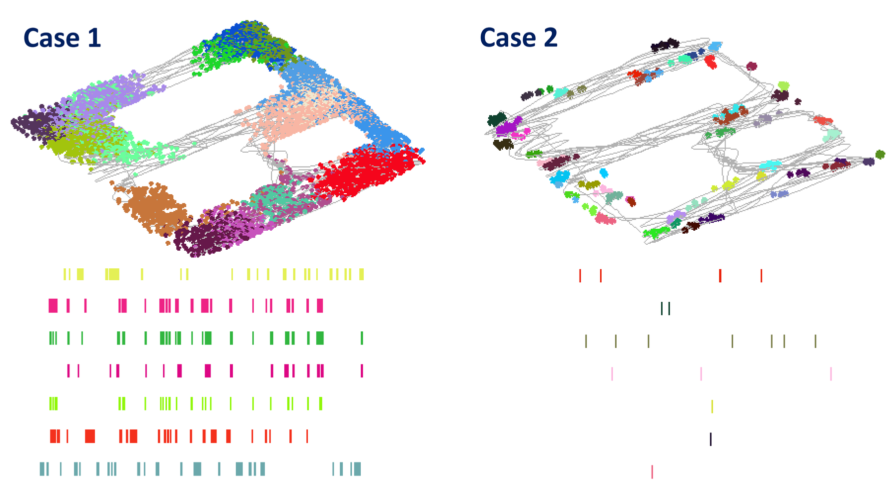
Simplicial complex

**B. In time code approach, the topology *may* be extractable from the spike trains temporal overlap pattern**



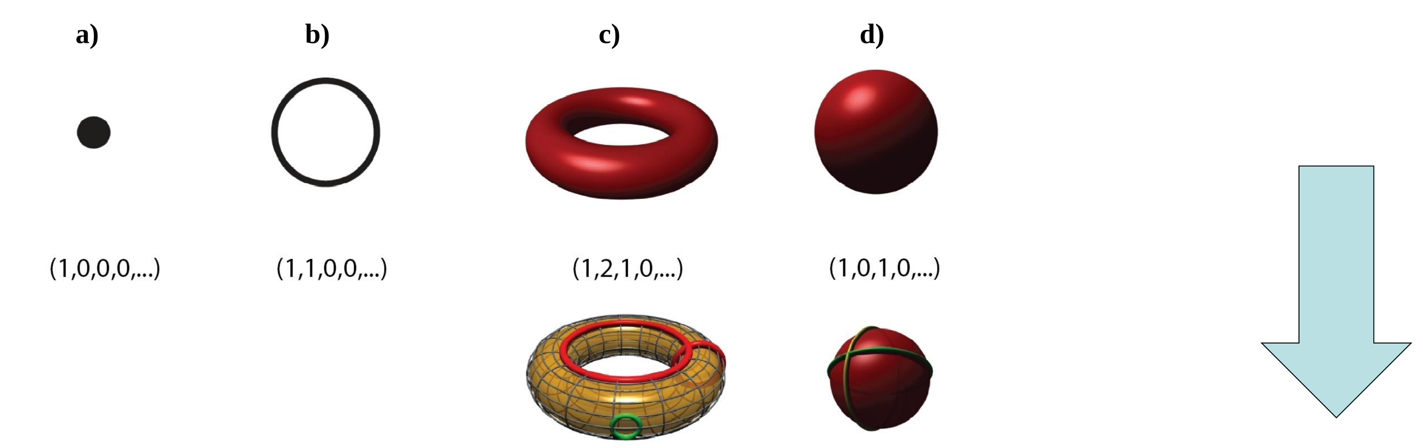
"Fuzzy" simplicial complex

**Whether the topological information can be extracted from neuronal spikes depends on the "quality" of the map, the characteristics of neuronal activity**




*What matters*: Firing rates, $f_i$, sizes of the place fields, $s_i$, place cell ensemble size, $N_{pc}$, *etc., etc…*

## To characterize spaces and shapes: count loops




a) **Point** – a 0-dimensional (*0d*) loop. **Topological barcode: (1,0,0,…)**

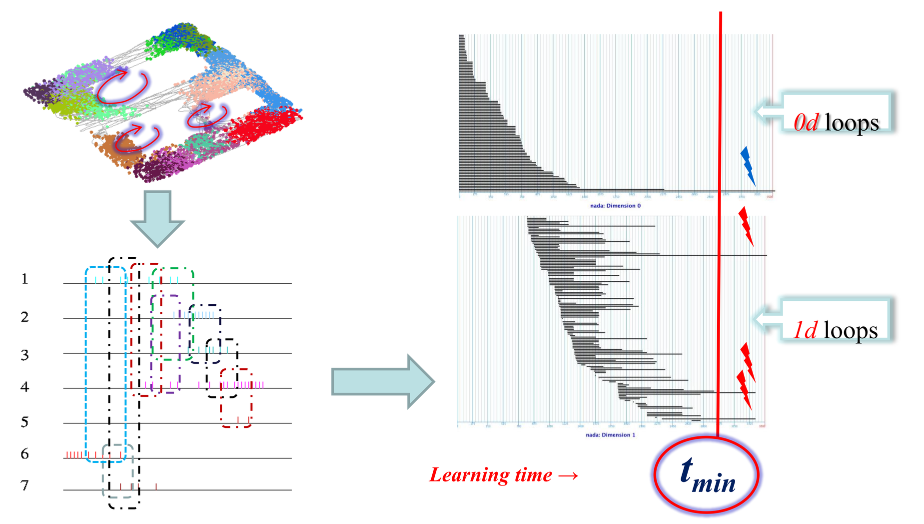
b) **Circle** – a *0d* loop plus a *1d* loop. **Topological barcode: (1,1,0,…)**

c) **Sphere** a *0d* loop plus a *2d* sheet "looped" onto itself. All *1D* loops contract into points. **Topological barcode: (1,0,2,…)**

d) **Torus** – a *0d* loop plus two *1d* loops plus a *2d* loop. The green loop shrink to a point, red loops do not. **Topological barcode: (1,2,1,…)**

**Persistent Homology method (Edelsbrunner et al.)**

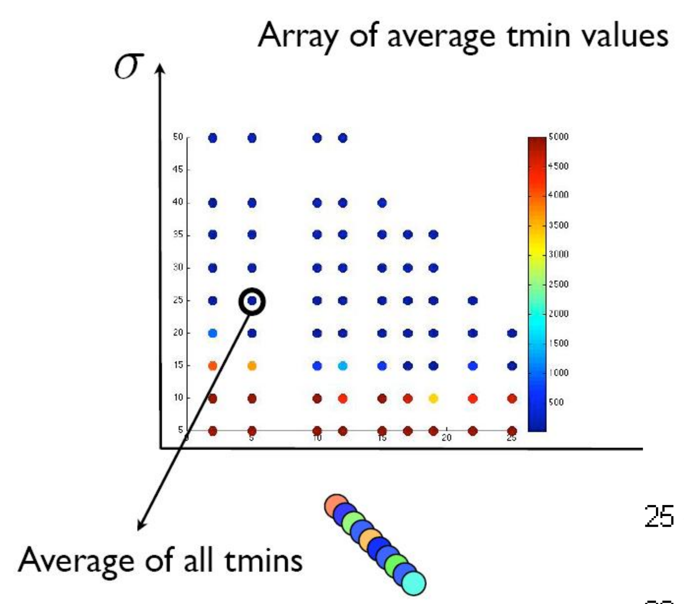
## How follow map learning: follow loops




**The algorithm:**

1. Follow how connectivity loops develop in time.
2. Record the first time $t_{min}$ when the topological barcode is correct

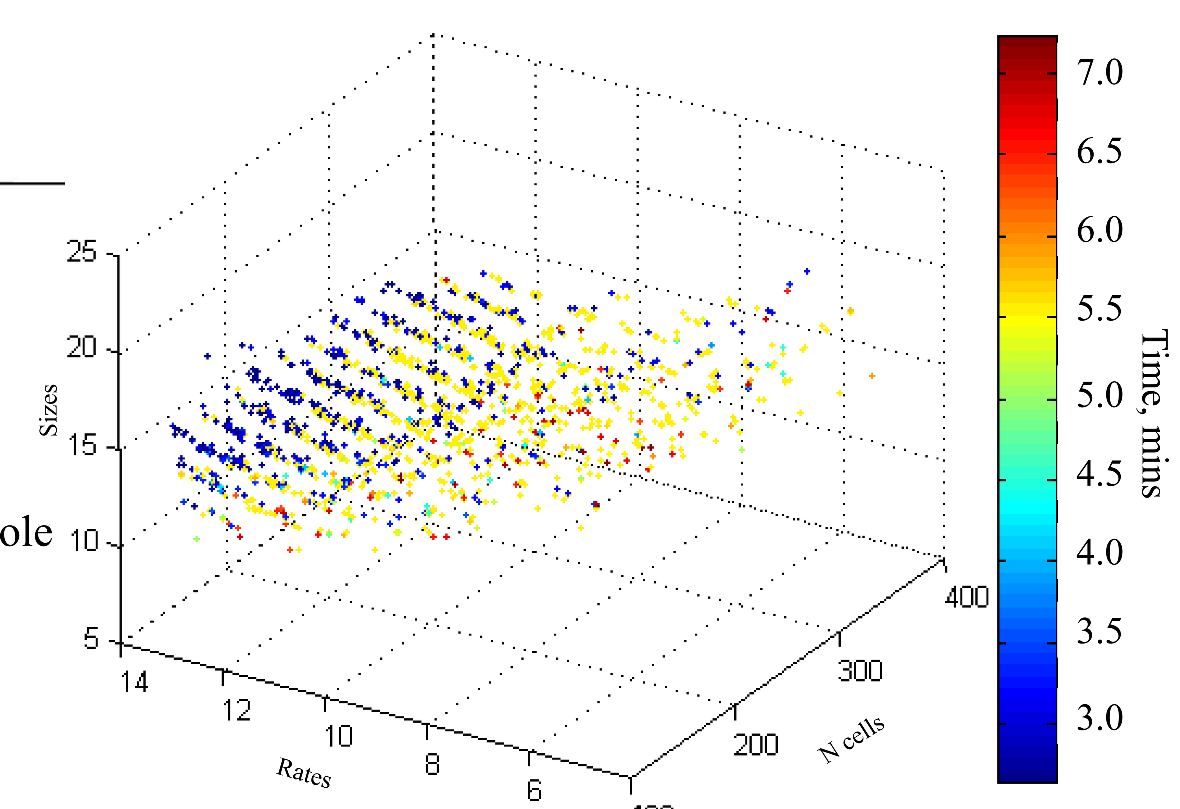
## Stability of the topological map: *loops must persist over a large range of scales*



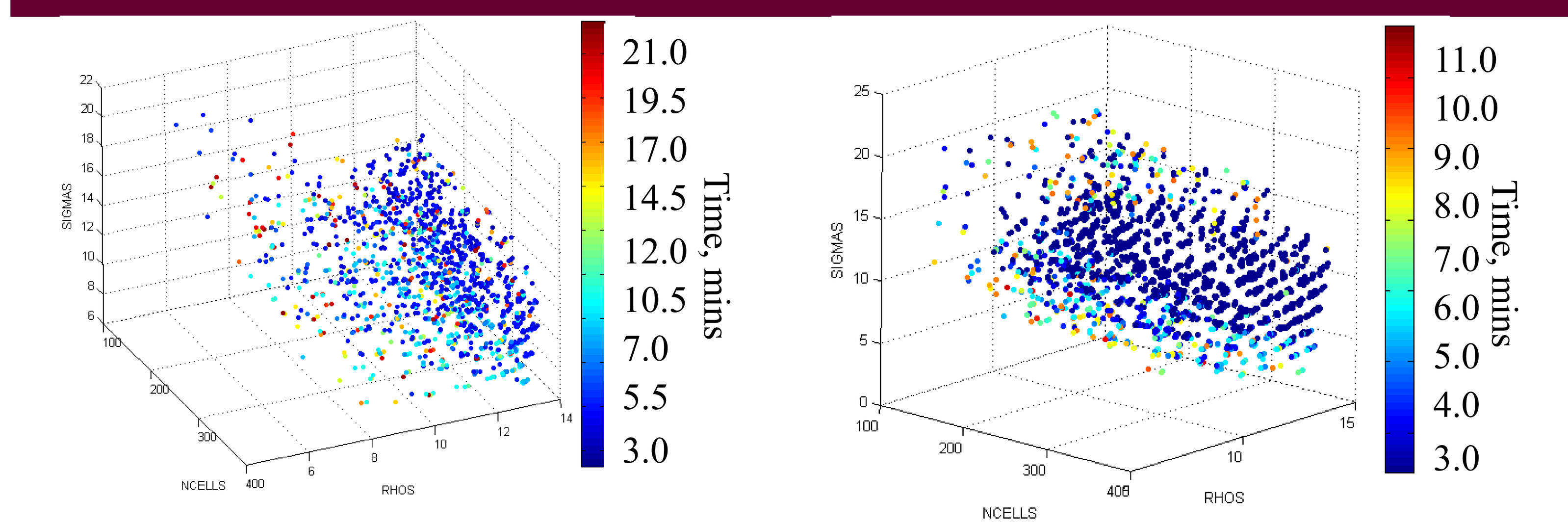

1. There is a clearly defined region of robust map representation.
2. For the values of place cell and place field parameters outside of the stability region the correct map can not be learned reliably

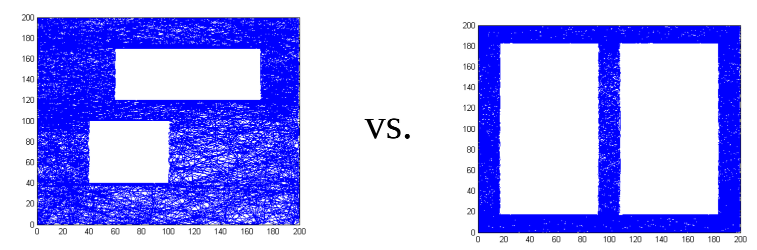
Learning time maps

An environment with one hole



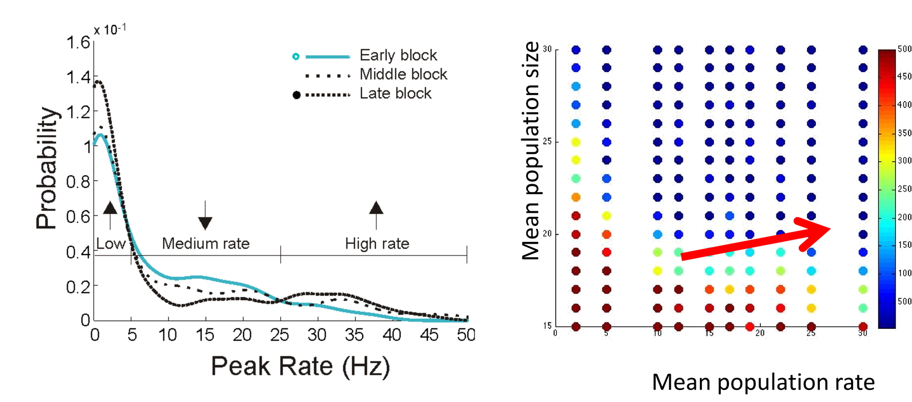

## Stability diagrams in different spaces




Geometry also matters.
Learning connectivity in a larger space takes time!

## Following the CA1 population dynamics




- Active CA1 place cells form a low rate, a medium rate and a high rate subpopulations
- Robustly firing, high rate cells recorded fall into the stability region
- The proportion of the high rate cells increases with experience

**M. Karlsson JN, 2008**

- **Dynamics: the maps moves from a quasi-stable regime into a stable regime**
- **Learning increases the stability of the map**

## Results

1. The topological stability of the place field map is achieved only for a specific range of parameters of the place cell firing activity.
2. This range depends on:
   a. The biological parameters
   b. The geometry and topology of space
3. The experimentally observed firing activity parameters of a subpopulation of the CA1 place cells *and their variability range* fall within the anticipated stability region

## References

1. K. Gothard et al, J. Neurosci., (1996); K. Diba & Buszaki, (2008)
2. H. Edelsbrunner, et al, Topological Persistence and Simplification, FOCS 2000: 454-63
3. M. Karlsson and L. Frank, The Journal of Neuroscience, 28(52):14271–81 (2008)

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