E281 Computational methods - HW 3

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Matlab Code: For the homework I make three new files: "shifts.m", "vfi_iteration_im.m", and "main_im.m". Below I explain what is contained in each file.

The function "shifts.m" has the same purpose of "vfi_upwind" used in class but with some key differences. First it saves the shifts $s_{i,B}$ and $s_{i,F}$ that later allow to generate the matrix A. Second, it takes "type" as an additional input that vary according to exercise 1 (type=endowment), 3 (type=production) and 4 (type=production-BG). This allow the income of the agent to be represented either by endowment y or by some production function with or without borrowing constraints. Finally, it also saves a vector "fchoice" that contains the choice of which production function the agent is using given initial wealth.

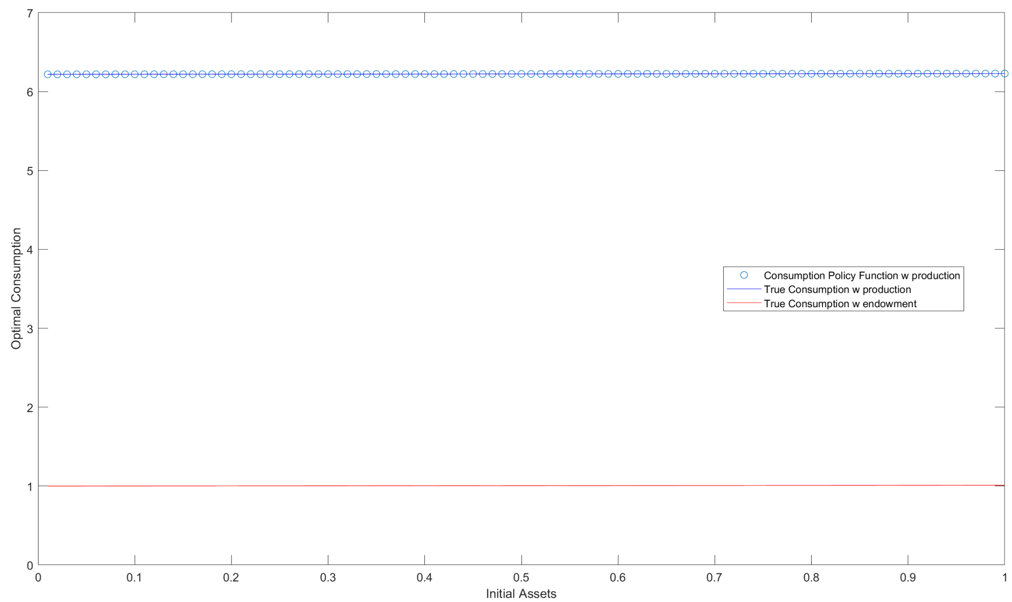
Given an initial guess of v, the function "vfi_iteration_im.m" uses results from "shift" to generate the matrix A and the updated values of v as $v^{n+1} = [(\rho+1/\Delta)I - A(v^n)]^{-1}[u(v^n) + v^n/\Delta]$ according to the implicit method.

Finally, "main_im.m" iterates value functions to find the solutions to each problem. It is also worth noting that some extra parameters are defined in "numerical_parameters.m"

Problem 1. The implicit method seems to be faster than the explicit method (0.26 seconds vs 3.5), but only because it takes less iterations to converge (2421 vs 81755).This seems to be strongly affected by our choice of Δ: using the same value as in the explicit method takes roughly the same amount of iterations (81764) but longer time (6.8 seconds).

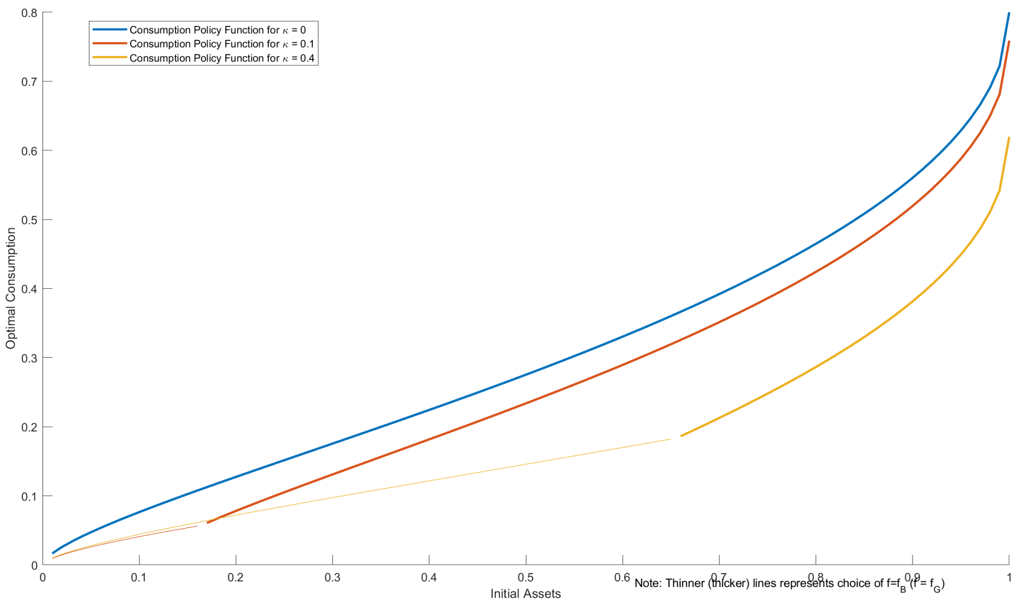
Problem 2. The problems are equivalent if we set correctly the discounting rates. We know that $\beta(\Delta) = e^{-\rho\Delta}$, and so if $\beta(1) = 0.99$ that implies $\rho = -ln(0.99)$. With this parametrization we get exactly the same solutions, and it is easy to see how it deviated when we chance slightly the choice of ρ.

Problem 3. In this problem choosing optimal capital and optimal savings are independent decisions. Each period, the agent will chose capital k such that $f'(k) = r$. Since interest rate is a parameter for now, k is a static choice and will always be the same, giving constant profits of $f(k*) - rk*$. Therefore this is just a variation of problem 1 where income is higher (around 6 instead of 1). The following figure shows optimal consupmtion with production (in blue) and with endowment (in red) to compare.



Problem 4. This problem is slightly different because, given the borrowing constraints, the technology available and profits will depend on the assets of each agent. Given a choice of production function, the agent will either put all her assets into production, or make the marginal productivity equal to the interest rate and save the rest in the safe asset. Of course, she will chose the technology that yields higher profit given this conditions. In this problem the agent has more incentives to save, as higher savings will allow for higher production in the future. Figure 2 plots optimal consumption for different values of κ .

The first thing we notice is that, given the borrowing constraints, the agent has stronger incentives to accumulate wealth (and consume less than the initial assets when assets are low). Consumption will clearly have a more pronounced slope in a world with borrowing constraints, but we can see the desire to save from the fact that the slope is less than 1 at most parts of the curve. As



κ increases, consumption is lower because the best technology available is less productive. Besides this, we can see that as κ increases there is also an additional return of savings coming from the possibility of reaching a more productive technology. We can see this without any extra plot since the consumption function is flatter when the agent is still using the bad technology.