Determine the gradient $\vec{g}_1$ in the starting point $\vec{x}_1$.

Step 1:

$$\vec{d}_1 = \vec{n}_1 = -\frac{\vec{g}_1}{|\vec{g}_1|}$$

$$\vec{x}_2 = \vec{x}_1 + \delta_1 \vec{d}_1$$

Determine the gradient $\vec{g}_2$ in the new point $\vec{x}_2$.

Step k>1:

$$\vec{n}_k^* = -\vec{g}_k + \gamma_{k,k-1}\vec{n}_{k-1}; \qquad \gamma_{k,k-1} = (\vec{g}_k, \vec{n}_{k-1});$$

Orthogonalize (option not needed):

$$\vec{n}_k^* = \vec{n}_k^* + \sum_{i=1}^{k-1} \gamma_{k,i}\vec{n}_i\,; \qquad \gamma_{k,i} = -(\vec{n}_k^*, \vec{n}_i)$$

We could perform the step into the point of minimum along direction $\vec{d}_{k-1}$:

$$\vec{x}_{k+1}^* = \vec{x}_k + \alpha_{k,k-1}\vec{d}_{k-1}; \qquad \alpha_{k,k-1} = -\frac{(\vec{g}_k, \vec{d}_{k-1})}{(\vec{g}_k, \vec{d}_{k-1}) - (\vec{g}_{k-1}, \vec{d}_{k-1})} \cdot \delta_{k-1}$$

The gradient norm will be

$$|\vec{g}_{k+1}^*| = |\vec{n}_k^*| \cdot \left|\frac{\delta_{k-1} + \alpha_{k,k-1}}{\delta_{k-1}}\right|$$

If it is satisfy the stopping criteria:

$$|\vec{g}_{k+1}^*| \le \varepsilon \cdot |\vec{g}_1|$$

The solution is found!

Otherwise we continue with contracting a new conjugate vector:

Normalize:

$$\vec{n}_k = \frac{\vec{n}_k^*}{|\vec{n}_k^*|}$$

A new conjugate vector

$$\vec{d}_k^* = \vec{n}_k + \beta_{k-1}\vec{d}_{k-1}; \qquad \beta_{k-1} = \frac{|\vec{n}_k^*|}{(\vec{g}_k, \vec{d}_{k-1}) - (\vec{g}_{k-1}, \vec{d}_{k-1})}; \qquad \vec{d}_k = \frac{\vec{d}_k^*}{|\vec{d}_k^*|}$$

Note

$$|\vec{d}_k^*| = \sqrt{1 + \beta_{k-1}^2}$$

The step will be:

$$\vec{x}_{k+1} = \vec{x}_{k+1}^* + \delta_k \vec{d}_k$$

Determine the gradient $\vec{g}_{k+1}$ in the new point $\vec{x}_{k+1}$.

Data structure:

<u>Vectors/scalars to keep from previous iteration:</u>

$\vec{d}_{k-1}$ – the conjugate vector

$\vec{n}_{k-1}$ – the normalized basis vector

δ_{k-1} - the step along last found vector

$(\vec{g}_{k-1}, \vec{d}_{k-1})$ – the scalar product

$\vec{x}_k$ – the current point

Vectors/scalars determined on new iteration:

$|\vec{n}_k^*|$ - the norm of the new basis vector

$\gamma_{k,k-1} = (\vec{g}_k, \vec{n}_{k-1})$ – the projection of the new gradient on the cv found on previous iteration

$\alpha_{k,k-1}$ – the step into the minimum along the cv found on previous iteration

$|\vec{g}_{k+1}^*|$ - norm of the gradient in the point of minimum along cv found on previous iteration

β_{k-1} – the projection of the new vector on the previous vector

$\vec{n}_k$ – the normalized basis vector

$\left|\vec{d}_k^*\right|$ - the norm of a new unnormalized conjugate vector

$\vec{d}_k$ – a new normalized conjugate vector

$\vec{x}_{k+1}$ – the new point

$\vec{g}_{k+1}$- new gradient